Area Spectral Efficiency of Soft-Decision Space-Time-Frequency Shift Keying Aided Slow Frequency Hopping Multiple Access

Hoang Anh Ngo† and Lajos Hanzo‡
† School of Electronics & Computer Science, University of Southampton
Email: †{han08r, lh}@ecs.soton.ac.uk; http://www-mobile.ecs.soton.ac.uk

Abstract—Slow Frequency Hopping Multiple Access (SFHMA) is capable of providing inherent frequency diversity and of beneficially randomising the effects of cochannel interference. It may also be advantageously combined with our novel Space-Time-Shift Keying (STFSK) scheme. The proposed system’s area spectral efficiency is investigated in various cellular frequency reuse structures. Furthermore, it is compared to both classic Gaussian Minimum Shift Keying (GMSK) aided SFHMA as well as to GMSK assisted Time Division/ Frequency Division Multiple Access (TD/FDMA). The more sophisticated third-generation Wideband Code Division Multiple Access (WCDMA) and the fourth-generation Long Term Evolution (LTE) systems were also included in our comparisons. We demonstrate that the area spectral efficiency of the STFSK aided SFHMA system is higher than that of the GMSK aided SFHMA and TD/FDMA systems as well as that of the WCDMA, but it is only 60% that of the LTE system.

I. INTRODUCTION

Slow Frequency Hopping Multiple Access (SFHMA) [1] not only provides inherent frequency diversity but also advantageously randomises the effects of cochannel interference. Furthermore, it is capable of avoiding the problem of prolonged fades typically experienced by stationary or slow moving Mobile Stations (MS) since hopping to another independently faded frequency might curtail fading. The classic SFH900 mobile system [2], [3] which was based on “mixed” Slow Frequency Hopping (SFH) combined with time division demonstrated the benefits of SFHMA.

Owing to its potential to increase the attainable system capacity without requiring additional bandwidth, Multiple-Input-Multiple-Output (MIMO) techniques have been extensively investigated. The family of MIMO systems typically employs Spatial Division Multiplexing (SDM), such as the BLAST schemes of [4], [5] for increasing the transmission rate or Space-Time Codes (STC) [6], [7] for maximizing the attainable diversity order. More recently, Spatial Modulation [8] and Space Shift Keying [9] were proposed for avoiding any inter-antenna interference and inter-antenna synchronization. As a further advance, Space-Time-Frequency Shift Keying (STFSK) [10] was designed in order to achieve a beneficial diversity gain, which may be gleaned from three different domains, namely the space-, time- and frequency-domain.

All the above-mentioned techniques play an important role in the overall system design, but they have typically been investigated independently. Therefore, in this paper we develop the philosophy of STFSK into a multi-user, multi-cell SFHMA system in order to investigate the inter-play of these techniques on the performance of the holistically optimized system. The novelty and rational of our proposed scheme is summarized as follows:

1) First, we intrinsically amalgamate the STFSK with SFHMA and then investigate the achievable Area Spectral Efficiency (ASE) of a realistic multi-user, multi-cell wireless environment under different cellular frequency reuse structures.

2) The study in Chapter 7 of [11] showed that the SFHMA system is superior in comparison to the Time Division/ Frequency Division Multiple Access (TD/FDMA) benchmarker for a low-complexity 16 kbit/s subband speech codec combined with Reed-Solomon coding. By contrast, in this paper we demonstrate that the SFHMA system becomes inferior to the TD/FDMA system when state-of-the-art components, such as the Advanced Multi-rate (AMR) speech codec and the convolutional coding are employed.

3) Finally, we demonstrate that by intrinsically amalgamating low-complexity STFSK and SFHMA, we attained an improved ASE, compared to those of TD/FDMA and WCDMA. However, this ASE remains lower than that of the more complex LTE system, when the same convolutional channel code and the same system bandwidth are employed.

The outline of this paper is as follows. The SFHMA’s frequency reuse structures are presented in Section II. In Section III, we investigate the spectral efficiency of the STFSK aided SFHMA. Finally, our concluding remarks are offered in Section IV.

II. SLOW FREQUENCY HOPPING MULTIPLE ACCESS (SFHMA)

The SFHMA technique employs SFH relying on unique, user-specific pseudo-random hopping sequences for supporting multiple users. The SFH family may be divided into three categories, namely the orthogonal, random and mixed protocols [11]. As a benefit of providing an increased freedom in system design and of its improved spectral efficiency [2], [3], the mixed protocol will be considered in the following sections.

The basic form of the reuse structure has three basic frequency-sets, also often referred to as ‘colours’ [2], [3]. Each colour is represented by one of the letters A, B, C and each contains a set of $N_c$ frequencies. For a given reference cell associated with colour A, all other cells marked A are full-reuse cells, relying on the same set of frequencies. For the full-reuse structure, an active user in the reference cell will experience interference from an active user in the reuse cell, during one of each sequence of $N_c$ hops, when they happen to use the same frequency. Each additional active user from the same or other reuse cell will impose the same frequency collision probability, but at a different frequency in the reference user’s sequence. Therefore, the so-called ‘frequency collision rate’ is defined as the proportion of the hopping sequence, when a ‘hit’ by an identical-frequency active user is encountered. For a full reuse cluster size $C$, the frequency collision rate equals to $C/3N_c$.

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By contrast, in a fractional reuse structure, each colour is divided into $M_c$ overlapping sub-sets of $L_c$ groups of frequencies, which are referred to as ‘shades’ of that colour, or ‘pseudo-colour’ [2]. Hence, the frequency-reuse cluster contains $3M_c$ cells divided into $M_c$ sub-clusters of size 3, each using a different shade of 3 colours. This is known as a $3M_c/L_c$ fractional structure [2], where $L_c/M_c$ is the fraction of the $N_c$ frequencies making up a colour, which is constituted by the set of shades. Fig 1 shows an example of a $21/3$ reuse structure, where we have $M_c = 7$ and $L_c = 3$. Subsequently, the frequency collision rate is expressed as $y = (M_c k) / (L_c N_c)$, where $k$ is the fractional overlap factor. Note that we have $k = 0$ for cells of different colours while $k = 1$ for cells of the same colour and shade. Other values of $k$ may be found in [11].

![Frequency reuse cells of a $21/3$ fractional reuse structure](image)

Fig. 1. Frequency reuse cells of a $21/3$ fractional reuse structure

### III. AREA SPECTRAL EFFICIENCY OF STFSK AIDED SFHMA

#### A. Area Spectral Efficiency of the SFHMA system

In this section, the method proposed in [3], [11] will be employed for evaluating the ASE of the proposed STFSK aided SFHMA system. Furthermore, the spectral efficiency of the STFSK aided SFHMA, the GMSK aided SFHMA and the conventional GMSK aided TD/FDMA systems will be compared. In this study we focus our attention on the base-to-mobile DownLink (DL).

In order to calculate the ASE of the entire system, we first consider the probability of correct reception of a hop, $q_c$, ensured by the FEC coding and by the audio/video codec of a multimedia session for the sake of guaranteeing an acceptable service quality. According to [3], a $q_c$ value of 0.7 is capable of guaranteeing an acceptable speech quality for a system utilizing the Reed-Solomon (RS) (8,4) channel code and a 16 kbit/s sub-band codec, resulting in an RS-decoded BER of approximately $3 \times 10^{-3}$. It is worth noting that the state-of-the-art Advanced Multi-Rate (AMR) speech codec [12] provides a good speech quality even at $BER = 10^{-2}$. However, in order to facilitate reliable system acquisition and synchronization, the target BER of $10^{-3}$ is chosen in this paper. The probability $q_c$ of correct reception during a hop may be formulated as:

$$q_c = \text{Prob}(\lambda \geq \gamma), \quad (1)$$

where $\gamma$ represents the Energy per Bit to Noise power spectral density ratio ($E_b/N_0$) corresponding to the Bit Error Ratio (BER) threshold and $\lambda$ is the Carrier-to-Interference Ratio (CIR) at the receiver considered, where the interference is imposed by all the adjacent cells. Note that the CIR is defined as the received power, on the shadowing and fast-fading parameters, as well as on the interference imposed by other cells in the network. According to [11], Eq. (1) may be expressed as

$$q_c = \prod_{i=1}^{M_i} \left(1 - \frac{p_i \gamma}{\Lambda_i + \gamma}\right), \quad (2)$$

where $M_i$ denotes the number of cells imposing interference on the cell considered, while $\Lambda_i$ is the CIR corresponding to the interference from the $i^{th}$ cell. Finally, $p_i$ presents the frequency-collision probability.

In line with [11], we assume that the locations of MSs are random in the reference cell. Therefore, $q_c$ may be calculated for various values of the tele-traffic load, $X$, which is also known as the average number of users actively engaged in calls per MHz per cell. In the analysis of [11], the ‘90% worst case value’, $q_{90}$, defined as the specific value of $q_c$, which is exceeded with a probability of 90%, is utilized. The study of [3] suggested that the specific operating point where we have $q_{90} = 0.7$ in the graph of $q_{90}$ versus $X$ should be selected in order to ensure an acceptable reception quality. However, the analysis of [11] indicated that $q_{90} = 0.8$ might be a better choice for the minimum $q$ to determine the ASE. Hence, the value of $q_{90} = 0.8$ is considered in our following analysis.

The ASE calculation may then be simplified to:

$$\eta = \frac{a_{cell}}{W} = X, \quad (3)$$

where $a_{cell}$ is the traffic load and $W$ is the total allocated bandwidth. Furthermore, the frequency-collision probability may be expressed as

$$p_i = \frac{y_i r_s W X}{n_t n_{call}}, \quad (4)$$

where $n_t$ is the number of slots per TDM frame and $n_{call}$ is the number of calls per timeslot, while $r_s$ is the Voice Activity Ratio (VAR), typically assumed to be 0.5 for the downlink [3]. For a full-reuse cell cluster of size $C_c$, the frequency-collision rate $y_i$ becomes $C_c / 3N_c$. By contrast, we have $y_i = k_i M_c / L_c N_c$ for the $3M_c/L_c$ fractional reuse structure. As regards to the channel spacing $f_s$, the number of hopping frequencies assigned to each of the three basic colours is given by $N = W / (3f_s)$.

Hence, Eq. (4) may be simplified to

$$p_i = \frac{C_c X r_s f_s}{n_t n_{call}}, \quad (5)$$

for the full-reuse cells and

$$p_i = \frac{3k_i M_c X r_s f_s}{L_c n_{call}}, \quad (6)$$

for fractional reuse.

In reality, most cellular systems employ fixed beam-forming-based angular sectorization, at least near the cell-edge. Thus, in the scenario of $120^\circ$ sectorization only 3 of 6 neighbour cells impose interference on the reference cell. However, there are only two cells, which contribute the full interference. Moreover, the authors of [3] demonstrated that the probability of the event, when the worst-case value $q_{90}$ occurs is comparable to the probability of $\Lambda_{90}$, which is defined as the ‘90% worst-case value’ CIR. In other words,

$$\text{Prob}(q_c > q_{90}) = \text{Prob}(\Lambda > \Lambda_{90}) = 90\%.$$ 

Hence, $q_{90}$ may be expressed as

$$q_{90} = \prod_{i=1}^{K_1} \left(1 - \frac{p_i \gamma}{2\Lambda_i + \gamma}\right)^2, \quad (7)$$
where $K_I$ is the number of rings using a different frequency set around the reference cell.

Fig. 2 shows the BER performance of both GMSK and STFSK(4/1/4-2), where we employ $M/N/T/Q = 4/1/4/2$, 4-PSK and 2-FSK modulation for transmission over the COST-207 rural area channel model [13], when assuming a maximum multipath delay of $20 \mu s$, corresponding to an Inter-Symbol Interference (ISI) of 6 symbols at a bit rate of about 300 kbits/s. The half-rate channel codes RS(8,4) [11] and the Recursive Systematic Convolutional (RSC) code RSC(23,33) using the octally represented generator polynomials of 23 and 33 [14] are considered. For RSC-coded system, soft-decision were used at the receiver. According to Fig. 2, the target BER of 23 and 33 [14] are considered. For RSC-coded system, soft-decision were used at the receiver. According to Fig. 2, the target BER of $10^{-3}$ was achieved at the $E_b/N_0$ value of $\gamma = 26.0$ dB, 19.0 dB and 8.5 dB for the uncoded, R5 coded and RSC coded GMSK schemes, respectively. For the same coding schemes invoked for the STFSK transceiver, the corresponding values are $\gamma = 9.5$ dB, 7.5 dB and 1.5 dB, respectively. All of these values will be used for computing the system’s $q_{90}$ values as well as its ASE.

For a basic 3-cell structure using the above values of $\gamma$ as well as $\Lambda_{90}$ from Table 7.4 of [11] and assuming a frequency spacing of $f_s = 1/7$ MHz, the values of $q_{90}$ versus the average number of users, $X$, are presented in Fig. 3. The curves were plotted for the following conditions:

- Nearest reuse ring (cluster-size 3) only is marked by the circles.
- Nearest reuse ring plus two nearest rings of cells of the other colours which impose adjacent-channel interference, for which the values of $\Lambda_{90}$ taken from [3] are $R_{AC} = 2$ dB and $R_{AC} + 9$ dB, where $R_{AC} = 17$ dB is the adjacent-channel interference rejection for $f_s = 1/7$ MHz. This scenario is indicated by the down-facing triangles.
- The second reuse ring (size 9) only, as indicated by the upwards triangles.
- The first five reuse rings (size 3,9,12,21,27), as marked by the squares.

As seen in the figure, the tiny gaps between the circle-marked curves and the down-facing-marked curves indicate that the adjacent channel interference insignificantly affect the cell’s performance. By contrast, due to the co-channel interference, the cell’s tele-traffic load $X$ is reduced by approximately 1 users/cell/MHz at $q_{90} = 0.8$ when the number of reuse rings increases from one to five. Based on the $q_{90}$ curves, it is possible to find the system’s ASE, $\eta_{sys}$, by spotting the $X$ value, where we have $q_{90} = 0.8$. Note that the ASE is quantified in terms of $Erl/cell/MHz$, or as $Erl/cell/MHz$ for short.

Theoretically, the probability of frequency collision, $p_i$, depends on three factors, namely the fractional overlap between the sets of frequencies in the reuse and reference cell, the VAR and the mean channel utilization, $U$. Thus, $p_i$ can be expressed as

$$p_i = k_i U r_a,$$

Note that $U = 1.0$, when all channels are occupied. Using the condition $U = 1.0$ and Eqs. (3-4), the maximum ASE may be obtained as

$$\eta_{max} = \frac{\eta_i n_{call}}{C_v f_s}$$

for the full-reuse structure and as

$$\eta_{max} = \frac{L_s n_{call}}{3 M_e f_s}$$

for fractional reuse structure.

In practice, the achievable spectral efficiency, $\eta_{ach}$, of a system, which may be achieved with the aid of the Erlang B formula [15], depends additionally on the number of channels per cell, $n_{ch}$, and on the tolerable blocking probability, $P_B$. The achievable ASE may be obtained as

$$\eta_{ach} = U \times \eta_{max},$$

where $U$ may be determined by Erlang B equation. Table I shows the maximum as well as the achievable spectral efficiency of various reuse structures, where we have $W = 24$ MHz, $f_s = 1/7$ MHz and $P_B = 2\%$.

Consequently, the attainable ASE of a specific system is given by

$$\eta_{att} = \min\{\eta_{sys}, \eta_{ach}\}.$$

The attainable ASE of various reuse structures is provided in Table I. As seen in the table, the STFSK aided SFHMA may triple the ASE compared to the one employing classic GMSK, when no channel coding is employed. When the hard-decision RS(8,4) channel code was employed, the ASE of the STFSK system is still as twice as high as that of GMSK. Furthermore, with the aid of the soft-decision RSC(23,33) the STFSK may approach the maximum achievable ASE for all the reuse structures considered. By contrast, this was only possible for the full-reuse cluster size of 9 and for the fractional-reuse cluster size 21/3 in case of the RSC(23,33) coded GMSK.
In summary, the attainable ASE can be estimated by the following steps:

- **Step 1**: Determine the $E_b/N_0$ threshold, $\gamma$, from Fig. 2 which offers the target BER.
- **Step 2**: Compute the frequency collision probability, $p_i$, from Eq. (5) or Eq. (6).
- **Step 3**: Calculate $q_{th}$ from Eq. (7).
- **Step 4**: Determine the system's ASE, $\eta_{sys}$, by looking for the operating point in Fig. 3, where we have the target $q_{th}$, i.e. $q_{th} = 0.8$ in the graph of $q_{th}$ versus $X$.
- **Step 5**: Calculate the achievable spectral efficiency, $\eta_{ach}$, from Eq. (11).
- **Step 6**: Determine the attainable ASE, $\eta_{att}$, which is the lower of the pair ($\eta_{sys}$, $\eta_{ach}$).

B. Comparison between the SFHMA and TD/FDMA systems

Finally, we compare the attainable performance of STFSK aided SFHMA to that of conventional GMSK aided TD/FDMA, to that of the third-generation WCDMA system, as well as to the performance of the fourth-generation LTE system. The system parameters considered are provided in Table II. In order to support a voice channel employing the AMR speech codec operated for example at 12.2 kbps, in the 5 MHz-bandwidth WCDMA system a maximum of 98 users can be separated when using a spreading factor of 128 [16]. For the same bandwidth of 5 MHz, the LTE system is capable of supporting up to 200 users [17]. Naturally, both the WCDMA and the LTE as well as the STFSK system are capable of supporting a significantly higher data rate than the 12.2 kbps speech rate. However, in this investigation we focussed our attention on the telephony service. Therefore, the voice data rate of 12.2 kbps is selected for all in this investigation.

In Table I, we compare the performance of the three systems recorded at an $E_b/N_0$ of 30 dB is shown in Fig. 4 in terms of the BER versus the mean CIR.

Applying the 6-step process outlined above, the various systems’ ASEs are shown in the last line of Table II. According to the table, the GMSK aided TD/FDMA system does not perform as well as the GMSK aided SFHMA arrangement in terms of the ASE, which is in contrast to the results of Chapter 7 in [11]. The reason for this fact is that when strong channel codes, such as the RSC or turbo codes, are employed, the reference BER value can be reduced, resulting in smaller reuse cluster sizes. The advantage of the small reuse cluster sizes in the SFHMA system becomes less dominant. Moreover, owing to the employment of SFH, the SFHMA system requires a longer time slot for each user. Hence, the GMSK aided SFHMA system performs less efficiently than GMSK aided TD/FDMA. However, the cell’s performance may be significantly improved by employing STFSK. Quantitatively, STFSK aided SFHMA attained a spectral efficiency of 24.7 Erl/cell/MHz, outperforming the ASE of 15.0 Erl/cell/MHz recorded for the GMSK aided TD/FDMA system. This ASE is also significantly higher than the WCDMA’s ASE of 16.9 Erl/cell/MHz, which is only slightly better than that of the GMSK aided TD/FDMA system. Finally, the LTE system achieves an ASE of 36.5 Erl/cell/MHz, hence exhibiting a substantially better ASE than all the remaining systems considered.
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### IV. Conclusions

In this paper we investigated the ASE of the proposed STFSK aided SFHMA system in various frequency reuse structures. The results showed that the proposed system may double the attainable ASE compared to GMSK aided SFHMA, when the RS(8,4) channel codes are employed for transmission over the 6-tap COST-207 rural area channel model associated with Rayleigh fading and AWGN. Additionally, the soft-decision RSC(23,33) coded STFSK aided SFHMA may approach the maximum achievable ASE in various frequency reuse cluster sizes. By contrast, this is only possible for the full-reuse cluster size 9 and for the fractional-reuse cluster size 2/3 in case of the soft-decision RSC(23,33) coded STFSK aided SFHMA. Furthermore, the system’s ASE was compared to that of the soft-decision RSC coded GMSK aided TD/FDMA. We demonstrated that the soft-decision RSC(23,33) coded STFSK aided SFHMA system is capable of outperforming the ASE of the RSC(23,33) coded GMSK aided TD/FDMA as well as that of WCDMA. Despite this significant improvement, the ASE of the STFSK aided SFHMA remains only 60% of that of the more complex LTE system, when the same RSC(23,33) channel code and a system bandwidth of 5 MHz are employed. Hence in our future research we will find appropriate upper-layer techniques for STFSK in the interest of increasing its ASE.

### References


