Reduced-Complexity Soft STBC Detection

Chao Xu[1], Dandan Liang[1], Shinya Sugiura[2], Soon Xin Ng[1] and Lajos Hanzo[1]
[1]School of ECS, University of Southampton, SO17 1BJ, United Kingdom. Email: {cx1g08,dl4e08,sxn,lh}@ecs.soton.ac.uk.
[2]Toyota Central R&D Labs, Inc., Aichi, 480-1192, Japan. Email: sugiura@ieee.org

***Abstract*—In this paper, we propose to reduce the complexity of both the Approx-Log-MAP algorithm as well as of the Max-Log-MAP algorithm, which were designed for soft-decision-aided Space-Time Block Code (STBC) detectors. First of all, we review the STBC design, which enables regular L-PSK/QAM detectors to be invoked in order to detect STBCs on a symbol-by-symbol basis. Secondly, we propose to operate the L-PSK/QAM aided STBC detection on a bit-by-bit basis, so that the complexity may be reduced from the order of $O(L)$ to $O(\text{BPS} = \log_2 L)$. Our simulation results demonstrate that a near-capacity performance may be achieved by the proposed detectors at a substantially reduced detection complexity. For example, a factor six complexity reduction was achieved by the proposed algorithms, when they were invoked for detecting Alamouti's Square 16QAM aided G2 scheme.**

I. Introduction

Inspired by the scientific breakthrough of Turbo Codes (TCs) proposed in [1], a near-capacity performance has been pursued by numerous researchers in diverse system contexts [2]–[4]. However, as the researchers have inched closer and closer to the channel capacity, the complexity of the resultant communication systems was increased. The soft-decision aided MIMO detection typically contributes a large portion of the total complexity, owing to the fact that the conventional MIMO detection is typically operated on a matrix-by-matrix basis [4]. Among all the generalized MIMO schemes presented in [5], both the Spatial Modulation (SM) [6] as well as the Space-Time Shift Keying (STSK) [7] may be detected on a symbol-by-symbol basis, because only a single L-PSK/QAM symbol is transmitted in a SM/STSK transmission matrix. It was proposed in [8] that the complexity of the soft SM/STSK detection may be further reduced by operating the detection on a bit-by-bit basis.

However, most MIMO schemes, including V-BLAST [9], Linear Dispersion Codes (LDCs) [10] and non-orthogonal Space-Time Block Codes (STBCs) [11] impose certain correlation between the transmitted symbols, which hence have to be detected jointly and/or iteratively in order to approach the maximum attainable performance. By contrast, the orthogonal STBC [12] does not impose any correlation between the transmitted symbols, which enables the regular PSK/QAM demodulators to be invoked in order to carry out STBC detection on a symbol-by-symbol basis. As a result, the complexity of STBC detection algorithms may be reduced from the order of $O(L^Q)$ to the order of $O(L)$, where Q denotes the number of transmitted symbols per STBC block. Against this background, in this paper, we further propose to operate the PSK/QAM aided STBC detection on a bit-by-bit basis, so that the complexity order may be further reduced to $O(\text{BPS} = \log_2 L)$. *More explicitly, the novel contributions of this paper are as follows:*

1) *The conventional Max-Log-MAP algorithm has to compare all the combinations of the a posteriori symbol probabilities in order to make a soft bit decision. However, we observe that the Max-Log-MAP algorithm aims for finding the maximum probabilities, which is similar to the action of hard PSK/QAM aided STBC detection, when searching for their optimum metrics. Therefore, the Max-Log-MAP algorithm may be operated on a bit-by-bit basis, in a similar manner to the classic hard-decision detection.*
2) *Furthermore, the corresponding reduced-complexity Approx-Log-MAP algorithm is also conceived by compensating for the Max-Log-MAP algorithm using the widely-used Jacobian algorithm relying on a lookup table.*

The remainder of this paper is organized as follows. The conventional soft-decision-aided STBC detection is summarized in Section II. In Section III, the reduced-complexity PSK/QAM aided STBC detection algorithms are proposed. Our performance results are provided in Section IV, while our conclusions are offered in Section V.

In this paper, we focus our attention on Alamouti's G2 STBC, where ($M = 2$) transmit antennas and a variable number of N receive antennas are employed, while ($Q = 2$) modulated L-PSK/QAM symbols are transmitted over ($T = 2$) symbol periods. The extension of our work to the other orthogonal STBCs of [13] is straightforward.

II. Conventional STBC Detection

In this section, we briefly review the universal matrix-by-matrix based MIMO detection. Since STBC does not impose any correlation on the transmitted symbols, the detection may be operated on a symbol-by-symbol basis at a lower complexity.

A. Matrix-by-Matrix Based MIMO Detection

The transmitter firstly encodes the $(Q \log_2 L)$ bits to an L-PSK/QAM symbols vector of $\mathbf{s}_n = \left[s_n^1, s_n^2\right]^T$. During the following T symbol periods, the symbol-matrix transmitted from the M transmit antennas may be formulated for G2 STBC as [12]:

$$\mathbf{S}_n = \frac{1}{\sqrt{M}} G_M(\mathbf{s}_n) = \frac{1}{\sqrt{M}} \begin{bmatrix} s_n^1 & s_n^2 \\ -(s_n^2)^* & (s_n^1)^* \end{bmatrix}. \tag{1}$$

For a MIMO system, the signal received by the N receive antennas may be expressed as:

$$\mathbf{Y}_n = \mathbf{S}_n \mathbf{H}_n + \mathbf{V}_n, \tag{2}$$

where the $(T \times N)$-element matrices $\mathbf{Y}_n$ and $\mathbf{V}_n$ refer to the received signal matrix and the Additive White Gaussian Noise (AWGN) matrix, which has a zero mean and a variance of N_0, respectively, while the $(M \times N)$-element matrix $\mathbf{H}_n$ models the Rayleigh fading channel.

The Log-MAP algorithm based on the received signal model of Eq. (2) may be expressed as [14]:

$$L_p(b_k|\mathbf{Y}) = \ln \left[\frac{\sum_{\mathbf{S}_i \in \mathbf{S}_{b_k=1}} \exp(d_i)}{\sum_{\mathbf{S}_i \in \mathbf{S}_{b_k=0}} \exp(d_i)} \right], \tag{3}$$

where $\mathbf{S}_{b_k=1}$ and $\mathbf{S}_{b_k=0}$ denote the STBC codewords sets, when the specific bit b_k is fixed to 1 and 0, respectively. The *a posteriori* probability d_i in Eq. (3) is given by:

$$d_i = -\frac{\|\mathbf{Y}_n - \mathbf{S}_i \mathbf{H}_n\|^2}{N_0} + \sum_{j=1}^{\log_2 I} b_j L_a(b_j), \tag{4}$$

The financial support of the RC-UK under the auspices of the India-UK Advanced Technology Centre (IU-ATC) and that of the EPSRC under the China-UK science bridge as well as that of the EU's Concerto project is gratefully acknowledged.

where we have $I = L^Q$, which is the total number of STBC codewords.

The Log-MAP algorithm of Eq. (3) may be simplified by the Max-Log-MAP algorithm as [14]:

$$L_p(b_k|\mathbf{Y}) = \max_{\mathbf{S}_i \in \mathbf{S}_{b_k=1}} (d_i) - \max_{\mathbf{S}_i \in \mathbf{S}_{b_k=0}} (d_i). \quad (5)$$

Since only two maximum *a posteriori* symbol probabilities are taken into account in Eq. (5), the Max-Log-MAP algorithm imposes a slight performance degradation.

In order to compensate for this performance loss, the Approx-Log-MAP algorithm is introduced as [15]:

$$L_p(b_k|\mathbf{Y}) = \text{jac}_{\mathbf{S}_i \in \mathbf{S}_{b_k=1}} (d_i) - \text{jac}_{\mathbf{S}_i \in \mathbf{S}_{b_k=0}} (d_i), \quad (6)$$

where *jac* denotes the Jacobian logarithm, which compensates for the differences between the candidate *a posteriori* probabilities according to a look-up table [15].

The complexity of the detection algorithms invoking the probability calculation of Eq. (4) is a function of $\left(I = L^Q\right)$, where all the *a posteriori* probabilities of $\{d_i\}_{i=1}^I$, which correspond to the STBC codewords $\{\mathbf{S}_i\}_{i=1}^I$, have to be evaluated and compared by the detection algorithms for producing a single soft bit output.

B. Symbol-by-Symbol Based STBC Detection

According to the STBC structure of Eq. (1), the equivalent received symbol may be defined as:

$$z_n^q = s_n^q \cdot \widetilde{h}_n + \widetilde{v}_n^q. \quad (7)$$

More explicitly, the decorrelating variables $\{z_n^q\}_{q=1}^Q$ are obtained by [12]:

$$\begin{aligned} z_n^1 &= \mathbf{Y}_n^1 \left(\mathbf{H}_n^1\right)^H + \mathbf{H}_n^2 \left(\mathbf{Y}_n^2\right)^H = s_n^1 \cdot \widetilde{h}_n + \widetilde{v}_n^1, \\ z_n^2 &= \mathbf{Y}_n^1 \left(\mathbf{H}_n^2\right)^H - \mathbf{H}_n^1 \left(\mathbf{Y}_n^2\right)^H = s_n^2 \cdot \widetilde{h}_n + \widetilde{v}_n^2, \end{aligned} \quad (8)$$

where the equivalent fading factor is given by $\left(\widetilde{h}_n = \frac{\|\mathbf{H}_n\|^2}{\sqrt{2}}\right)$, while the equivalent noise factors $\{\widetilde{v}_n^q\}_{q=1}^Q$ have a new variance of $\left(\|\mathbf{H}_n\|^2 \cdot N_0\right)$. The $(1 \times N)$-element vectors $\{\mathbf{Y}_n^i\}_{i=1}^T$ and $\{\mathbf{H}_n^i\}_{i=1}^M$ denotes the i-th row in the matrices $\mathbf{Y}_n$ and $\mathbf{H}_n$, respectively.

Based on the new equivalent received signal model of Eq. (7), the *a posteriori* probability of Eq. (4) may be further modified to be operated on a symbol-by-symbol basis as:

$$d_l^q = -\frac{|\widetilde{z}_n^q - s_l|^2}{\widetilde{N}_0} + \sum_{j=(q-1)\times\log_2 L+1}^{q\times\log_2 L} b_j L_a(b_j), \quad (9)$$

where the normalized decorrelating variable is $\left(\widetilde{z}_n^q = z_n^q / \widetilde{h}_n\right)$, while the normalized noise power is $\left(\widetilde{N}_0 = 2N_0 / \|\mathbf{H}_n\|^2\right)$. When calculating $\{d_l^q\}_{l=1}^L$ for a specific symbol index q, the rest of $\{d_l^{q'}\}_{q' \neq q}$ remains unchanged, which is ignored by the detection algorithms. Therefore, the STBC detection may be completed by invoking the regular L-PSK/QAM detector using the following symbol probability:

$$d_l = -\frac{|\widetilde{z}_n - s_l|^2}{\widetilde{N}_0} + \sum_{j=1}^{\log_2 L} b_j L_a(b_j), \quad (10)$$

where the index q in Eq. (9) is discarded. More explicitly, the symbol-by-symbol based Max-Log-MAP algorithm may be expressed as:

$$L_p(b_k|\mathbf{Y}) = \max_{s_l \in \mathbf{s}_{b_k=1}} (d_l) - \max_{s_l \in \mathbf{s}_{b_k=0}} (d_l), \quad (11)$$

where $\mathbf{s}_{b_k=1}$ and $\mathbf{s}_{b_k=0}$ denote the L-PSK/QAM symbols set, when the specific bit b_k is fixed to 1 and 0, respectively.

Similarly, the symbol-by-symbol based Approx-Log-MAP algorithm may be obtained by replacing the maximization operation of $\max$ in Eq. (11) by the Jacobian algorithm of jac.

The complexity of the detection algorithms using Eq. (10) is now reduced from the order of $O(I = L^Q)$ to the order of $O(L)$, where the L-PSK/QAM detector only evaluates and compares the *a posteriori* symbol probabilities of $\{d_l\}_{l=1}^L$, which correspond to the L-PSK/QAM symbols $\{s_l\}_{l=1}^L$, for producing a single soft decision.

III. Reduced-Complexity STBC Detection

In this section, we now proceed further by proposing to operate the L-PSK/QAM aided STBC detection on a bit-by-bit basis, so that the detection complexity may be further reduced from the order of $O(L)$ to $O(\text{BPS} = \log_2 L)$.

A. Reduced-Complexity PSK Aided STBC Detection

The general concept of both the Max-Log-MAP algorithm as well as of the Approx-Log-MAP algorithm is that these algorithms have to estimate all the *a posteriori* symbol probabilities of $\{d_l\}_{l=1}^L$ in order to produce a single soft bit decision. Observe for the Max-Log-MAP algorithm of Eq. (11) that it aims for finding the maximum symbol probabilities, which is similar to the optimum metric search conducted by the symbol-by-symbol based hard-decision detector as:

$$\hat{s}_n = \arg \min_{s_l \in \mathbf{s}} |\widetilde{z}_n - s_l|^2. \quad (12)$$

However, it is well-known that the hard-decision L-PSK/QAM detector of Eq. (12) may be operated on a bit-by-bit basis. Considering QPSK as an example, the classic bit-by-bit based hard-decision-aided detection may be formulated as:

$$\hat{b}_1 = \begin{cases} 1, & \text{if } \text{Im}(\widetilde{z}_n) < 0 \\ 0, & \text{otherwise} \end{cases}, \quad \hat{b}_2 = \begin{cases} 1, & \text{if } \text{Re}(\widetilde{z}_n) < 0 \\ 0, & \text{otherwise} \end{cases}. \quad (13)$$

We note that we deliberately rotated all the constellations of L-PSK ($L \geq 4$) in [3] anti-clockwise by a phase of $\frac{\pi}{L}$, so that there are exactly $L/4$ constellation points in each quadrant. This feature will be beneficial for reducing the soft PSK detectors' complexity.

Motivated by this, we aim for proposing a bit-by-bit based Max-Log-MAP algorithm, which operates in a similar manner to Eq. (13). Therefore, we further extend the *a posteriori* probability of Eq. (10) as:

$$\begin{aligned} d_l &= -\frac{|\widetilde{z}_n|^2 + |s_l|^2 - 2\text{Re}\left[(s_l)^* \widetilde{z}_n\right]}{\widetilde{N}_0} + \sum_{j=1}^{\log_2 L} b_j L_a(b_j) \\ &= -\frac{|\widetilde{z}_n|^2}{\widetilde{N}_0} - \frac{|s_l|^2}{\widetilde{N}_0} + \frac{\text{Re}(z_n)\text{Re}(s_l) + \text{Im}(z_n)\text{Im}(s_l)}{\overline{N}_0} \\ &\quad + \sum_{j=1}^{\log_2 L} b_j L_a(b_j), \end{aligned} \quad (14)$$

where the new equivalent noise power is $(\overline{N}_0 = \frac{N_0}{\sqrt{2}})$. Furthermore, both $\left(-\frac{|\widetilde{z}_n|^2}{\widetilde{N}_0}\right)$ and $\left(-\frac{|s_l|^2}{\widetilde{N}_0}\right)$ in Eq. (14) are invariant over the variable $\{s_l\}_{l=1}^L$, and hence they are ignored by the detection algorithms. To sum up, Eq. (14) may be simplified as:

$$d_l = \frac{\text{Re}(s_l)}{\overline{N}_0}\text{Re}(z_n) + \frac{\text{Im}(s_l)}{\overline{N}_0}\text{Im}(z_n) + \sum_{j=1}^{\log_2 L} b_j L_a(b_j). \quad (15)$$

Let us now consider the QPSK constellation of Fig. 1 as an example, where the *a posteriori* symbol probabilities of Eq. (15)

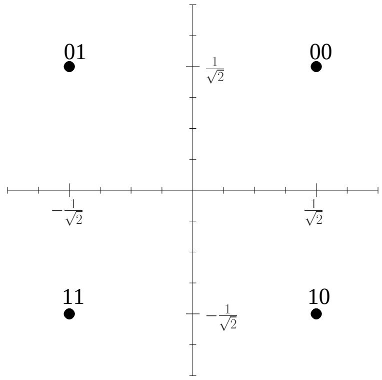

Fig. 1. Constellation diagram of QPSK.

may be expressed as:

$$\begin{aligned} d_1 &= \frac{\text{Re}(z_n)}{\sqrt{2}\cdot\overline{N}_0} + \frac{\text{Im}(z_n)}{\sqrt{2}\cdot\overline{N}_0} = t_{\text{Re}} + t_{\text{Im}} + C_{\text{QPSK}}, \\ d_2 &= -\frac{\text{Re}(z_n)}{\sqrt{2}\cdot\overline{N}_0} + \frac{\text{Im}(z_n)}{\sqrt{2}\cdot\overline{N}_0} + L_a(b_2) = -t_{\text{Re}} + t_{\text{Im}} + C_{\text{QPSK}}, \\ d_3 &= \frac{\text{Re}(z_n)}{\sqrt{2}\cdot\overline{N}_0} - \frac{\text{Im}(z_n)}{\sqrt{2}\cdot\overline{N}_0} + L_a(b_1) = t_{\text{Re}} - t_{\text{Im}} + C_{\text{QPSK}}, \\ d_4 &= -\frac{\text{Re}(z_n)}{\sqrt{2}\cdot\overline{N}_0} - \frac{\text{Im}(z_n)}{\sqrt{2}\cdot\overline{N}_0} + L_a(b_1) + L_a(b_2) \\ &= -t_{\text{Re}} - t_{\text{Im}} + C_{\text{QPSK}}, \end{aligned} \tag{16}$$

where we define the following variables for testing the real and the imaginary parts of z_n as:

$$\begin{aligned} t_{\text{Re}} &= \frac{\text{Re}(z_n)}{\sqrt{2}\cdot\overline{N}_0} - \frac{L_a(b_2)}{2}, \\ t_{\text{Im}} &= \frac{\text{Im}(z_n)}{\sqrt{2}\cdot\overline{N}_0} - \frac{L_a(b_1)}{2}, \end{aligned} \tag{17}$$

while the common constant in Eq. (16) is given by $C_{\text{QPSK}} = \frac{L_a(b_1)+L_a(b_2)}{2}$. As a result, the maximum probability d_{max} over these four candidates $\{d_l\}_{l=1}^4$, which is pursued by the Max-Log-MAP algorithm of Eq. (11), may be obtained in a single step:

$$\begin{aligned} d_{\text{max}} &= \max_{l\in\{1-4\}} d_l \\ &= |t_{\text{Re}}| + |t_{\text{Im}}| + C_{\text{QPSK}}. \end{aligned} \tag{18}$$

Therefore, instead of evaluating Eq. (15) $(L = 4)$ times in Eq. (16) and comparing these $(L = 4)$ candidates in order to find the maximum probability d_{max}, Eq. (18) only has to be evaluated once, without any comparison. According to this maximum distance search approach demonstrated in Eq. (18), the first soft bit produced by the reduced-complexity Max-Log-MAP algorithm may be obtained as[1]:

$$\begin{aligned} L_p(b_1) &= \max_{l\in\{3,4\}} d_l - \max_{l\in\{1,2\}} d_l \\ &= (|t_{\text{Re}}| - t_{\text{Im}}) - (|t_{\text{Re}}| + t_{\text{Im}}) \\ &= -2t_{\text{Im}}. \end{aligned} \tag{19}$$

Similarly, the second soft bit output may be obtained as:

$$\begin{aligned} L_p(b_2) &= \max_{l\in\{2,4\}} d_l - \max_{l\in\{1,3\}} d_l \\ &= (-t_{\text{Re}} + |t_{\text{Im}}|) - (t_{\text{Re}} + |t_{\text{Im}}|) \\ &= -2t_{\text{Re}}. \end{aligned} \tag{20}$$

The soft decisions of Eqs. (19) and (20) are obtained on a bit-by-bit basis, which is equivalent to the classic hard-decision detection of

[1]The common constant C_{QPSK} is ignored by the Max-Log-MAP algorithm.

TABLE I
THE REDUCED-COMPLEXITY MAX-LOG-MAP ALGORITHM FOR BPSK/8PSK AIDED STBC DETECTION.

BPSK	$t_{\text{Re}} = \frac{\text{Re}(z_n)}{\overline{N}_0} - \frac{L_a(b_1)}{2}, \quad L_p(b_1) = -2t_{\text{Re}}$.
8PSK	$t_{\text{Re1}} = \frac{\cos(\frac{\pi}{8})}{\overline{N}_0}\text{Re}(z_n) - \frac{L_a(b_2)}{2}, \quad t_{\text{Re2}} = \frac{\sin(\frac{\pi}{8})}{\overline{N}_0}\text{Re}(z_n) - \frac{L_a(b_2)}{2},$
	$t_{\text{Im1}} = \frac{\sin(\frac{\pi}{8})}{\overline{N}_0}\text{Im}(z_n) - \frac{L_a(b_1)}{2}, \quad t_{\text{Im2}} = \frac{\cos(\frac{\pi}{8})}{\overline{N}_0}\text{Im}(z_n) - \frac{L_a(b_1)}{2},$
	$L_p(b_1) = \max\left\{\begin{array}{l} \lvert t_{\text{Re1}}\rvert - t_{\text{Im1}} \\ \lvert t_{\text{Re2}}\rvert - t_{\text{Im2}} + L_a(b_3) \end{array}\right\} - \max\left\{\begin{array}{l} \lvert t_{\text{Re1}}\rvert + t_{\text{Im1}} \\ \lvert t_{\text{Re2}}\rvert + t_{\text{Im2}} + L_a(b_3) \end{array}\right\},$
	$L_p(b_2) = \max\left\{\begin{array}{l} -t_{\text{Re1}} + \lvert t_{\text{Im1}}\rvert \\ -t_{\text{Re2}} + \lvert t_{\text{Im2}}\rvert + L_a(b_3) \end{array}\right\} - \max\left\{\begin{array}{l} t_{\text{Re1}} + \lvert t_{\text{Im1}}\rvert \\ t_{\text{Re2}} + \lvert t_{\text{Im2}}\rvert + L_a(b_3) \end{array}\right\},$
	$L_p(b_3) = \lvert t_{\text{Re2}}\rvert + \lvert t_{\text{Im2}}\rvert + L_a(b_3) - \lvert t_{\text{Re1}}\rvert - \lvert t_{\text{Im1}}\rvert.$

Eq. (13), with the absence of the *a priori* probabilities, where only either the real part or the imaginary part of z_n is tested for producing a single-bit decision. As a result, the complexity of producing a single soft bit is reduced from the order of $O(L = 4)$ to $O(\log_2 L = 2)$, which suggests that the complexity should no longer increase exponentially with the BPS throughput.

We summarized our reduced-complexity Max-Log-MAP algorithm conceived for BPSK aided as well as for 8PSK aided STBC detection in Table I. For the corresponding Approx-Log-MAP algorithm, considering QPSK aided STBC detection as an example, the first soft bit output is given by:

$$\begin{aligned} L_p(b_1) &= \text{jac}_{l\in\{3,4\}} d_l - \text{jac}_{l\in\{1,2\}} d_l \\ &= [\text{jac}(t_{\text{Re}}, -t_{\text{Re}}) - t_{\text{Im}}] - [\text{jac}(t_{\text{Re}}, -t_{\text{Re}}) + t_{\text{Im}}] \\ &= -2t_{\text{Im}}, \end{aligned} \tag{21}$$

while the second soft bit output may be obtained in a similar way. It can be seen in Eq. (21) that the specific part of $\text{jac}(t_{\text{Re}}, -t_{\text{Re}})$ that was deleted by the Approx-Log-MAP algorithm corresponds to the particular part of $|t_{\text{Re}}|$, which is deleted by the Max-Log-MAP algorithm in Eq. (19). This implies that whatever is ignored by the Max-Log-MAP algorithm, its corresponding counterpart may also be ignored by the Approx-Log-MAP algorithm. Therefore, the reduced-complexity Approx-Log-MAP algorithm conceived for L-PSK aided STBC detection may be completed by correcting the Max-Log-MAP algorithm shown in Table I, where all the maximization operations denoted as $\max$ should be replaced by the Jacobian algorithm of jac. Furthermore, all the absolute value calculations in Table I are obtained according to $(|t| = \max\{t, -t\})$, and hence they should be replaced by the corresponding Jacobian algorithm as $(\text{jac}\{t, -t\})$.

Since the L-PSK $(L > 4)$ schemes encode their information bits onto the phase, which means that both the real part and the imaginary part of the L-PSK symbols are joinly encoded, they have to be jointly detected. Therefore, in the following sections we continue by applying our design to Square L-QAM schemes, which encode the real part and the imaginary part of the transmitted symbols separately.

B. Reduced-Complexity QAM Aided STBC Detection

When QAM-aided STBC is employed, the *a posteriori* probability of Eq. (14) may be simplified to:

$$d_l = \frac{\text{Re}(s_l)}{\overline{N}_0}\text{Re}(z_n) + \frac{\text{Im}(s_l)}{\overline{N}_0}\text{Im}(z_n) - \frac{|s_l|^2}{\widetilde{N}_0} + \sum_{j=1}^{\log_2 L} b_j L_a(b_j). \tag{22}$$

Since $\{|s_l|^2\}_{l=1}^L$ is no longer a constant, it cannot be omitted from Eq. (22).

Taking Square 16QAM as an example, we arrange the four constellation points, which share the same coordinate magnitude but are associated with different signs to a group. Similar to the case of QPSK encapsulated in Eq. (18), the maximum *a posteriori* probabilities found in each group are given by:

$$\begin{aligned}
d_{\max}^{\{6,8,14,16\}} &= |t_{\text{Re1}}| + |t_{\text{Im1}}| + \left[L_a(b_2) + L_a(b_4) + C_{\text{16QAM}}\right], \\
d_{\max}^{\{5,7,13,15\}} &= |t_{\text{Re2}}| + |t_{\text{Im1}}| + \left[L_a(b_2) - \frac{4}{5\widetilde{N}_0} + C_{\text{16QAM}}\right], \\
d_{\max}^{\{2,4,10,12\}} &= |t_{\text{Re1}}| + |t_{\text{Im2}}| + \left[-\frac{4}{5\widetilde{N}_0} + L_a(b_4) + C_{\text{16QAM}}\right], \\
d_{\max}^{\{1,3,9,11\}} &= |t_{\text{Re2}}| + |t_{\text{Im2}}| + \left[-\frac{8}{5\widetilde{N}_0} + C_{\text{16QAM}}\right],
\end{aligned} \tag{23}$$

where the decision variables test the real part and the imaginary part of z_n separately as:

$$\begin{aligned}
t_{\text{Re1}} &= \frac{\text{Re}(z_n)}{\sqrt{10}\cdot\widetilde{N}_0} - \frac{L_a(b_3)}{2}, \quad t_{\text{Re2}} = \frac{3\text{Re}(z_n)}{\sqrt{10}\cdot\widetilde{N}_0} - \frac{L_a(b_3)}{2}, \\
t_{\text{Im1}} &= \frac{\text{Im}(z_n)}{\sqrt{10}\cdot\widetilde{N}_0} - \frac{L_a(b_1)}{2}, \quad t_{\text{Im2}} = \frac{3\text{Im}(z_n)}{\sqrt{10}\cdot\widetilde{N}_0} - \frac{L_a(b_1)}{2},
\end{aligned} \tag{24}$$

while the common constant in Eq. (23) is given by $C_{\text{16QAM}} = \left[-\frac{1}{5\widetilde{N}_0} + \frac{L_a(b_1)+L_a(b_3)}{2}\right]$. Considering the magnitude differences, we may organize the four maximum *a posteriori* probabilities in Eq. (23) as:

$$\begin{aligned}
d_{\max}^{\{6,8,14,16\}} &= \left[|t_{\text{Re1}}| + L_a(b_4)\right] + \left[|t_{\text{Im1}}| + L_a(b_2)\right], \\
d_{\max}^{\{5,7,13,15\}} &= \left[|t_{\text{Re2}}| - \frac{4}{5\widetilde{N}_0}\right] + \left[|t_{\text{Im1}}| + L_a(b_2)\right], \\
d_{\max}^{\{2,4,10,12\}} &= \left[|t_{\text{Re1}}| + L_a(b_4)\right] + \left[|t_{\text{Im2}}| - \frac{4}{5\widetilde{N}_0}\right], \\
d_{\max}^{\{1,3,9,11\}} &= \left[|t_{\text{Re2}}| - \frac{4}{5\widetilde{N}_0}\right] + \left[|t_{\text{Im2}}| - \frac{4}{5\widetilde{N}_0}\right],
\end{aligned} \tag{25}$$

where the common constant C_{16QAM} in Eq. (23) is omitted. It can be seen that all the four *a posteriori* probabilities are constituted by two parts. The maximum value of the first parts is the higher one between $\left[|t_{\text{Re1}}| + L_a(b_4)\right]$ and $\left[|t_{\text{Re2}}| - \frac{4}{5\widetilde{N}_0}\right]$, while the maximum value of the last parts is given by the higher one of $\left[|t_{\text{Im1}}| + L_a(b_2)\right]$ and $\left[|t_{\text{Im2}}| - \frac{4}{5\widetilde{N}_0}\right]$. In summary, the maximum *a posteriori* probability $d_{\max}$ over all the $(L = 16)$ candidates $\{d_l\}_{l=1}^{16}$ is given by:

$$\begin{aligned}
d_{\max} &= \max_{l\in\{1-16\}} d_l \\
&= \max\left\{\begin{array}{l} |t_{\text{Re1}}| + L_a(b_4) \\ |t_{\text{Re2}}| - \frac{4}{5\widetilde{N}_0} \end{array}\right\} + \max\left\{\begin{array}{l} |t_{\text{Im1}}| + L_a(b_2) \\ |t_{\text{Im2}}| - \frac{4}{5\widetilde{N}_0} \end{array}\right\},
\end{aligned} \tag{26}$$

where the maximum *a posteriori* probability $d_{\max}$ in Eq. (26) is obtained by comparing the magnitudes of the real part and the imaginary part separately, instead of evaluating Eq. (22) $(L = 16)$ times for $\{d_l\}_{l=1}^{16}$ and comparing all the candidates as a whole. Therefore, its complexity is reduced from the order of $O(L = 16)$ to $O(\log_2 L = 4)$. In more detail, the reduced-complexity Max-Log-MAP algorithm may make the decisions on a bit-by-bit basis as:

$$\begin{aligned}
L_p(b_1) &= \max\left\{\begin{array}{l} -t_{\text{Im1}} + L_a(b_2) \\ -t_{\text{Im2}} - \frac{4}{5\widetilde{N}_0} \end{array}\right\} - \max\left\{\begin{array}{l} t_{\text{Im1}} + L_a(b_2) \\ t_{\text{Im2}} - \frac{4}{5\widetilde{N}_0} \end{array}\right\}, \\
L_p(b_2) &= |t_{\text{Im1}}| + L_a(b_2) - |t_{\text{Im2}}| + \frac{4}{5\widetilde{N}_0}, \\
L_p(b_3) &= \max\left\{\begin{array}{l} -t_{\text{Re1}} + L_a(b_4) \\ -t_{\text{Re2}} - \frac{4}{5\widetilde{N}_0} \end{array}\right\} - \max\left\{\begin{array}{l} t_{\text{Re1}} + L_a(b_4) \\ t_{\text{Re2}} - \frac{4}{5\widetilde{N}_0} \end{array}\right\}, \\
L_p(b_4) &= |t_{\text{Re1}}| + L_a(b_4) - |t_{\text{Re2}}| + \frac{4}{5\widetilde{N}_0},
\end{aligned} \tag{27}$$

which is also equivalent to the classic hard-decision-aided Square 16QAM detection in [3] with the absence of the *a priori* LLRs.

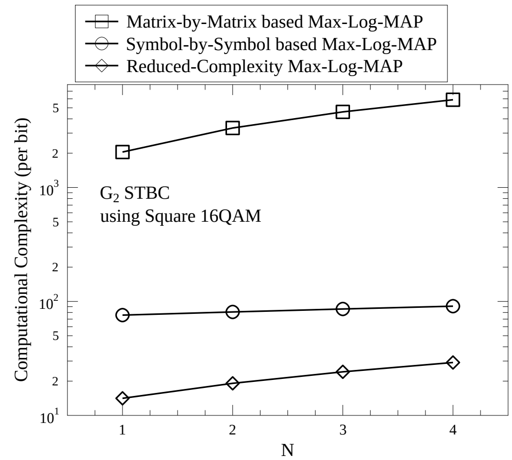

Fig. 2. Complexity comparison between different STBC detection algorithms when different number of receive antennas are equipped.

Similarly, the corresponding reduced-complexity Approx-Log-MAP algorithm may be obtained by appropriately modifying the Max-Log-MAP algorithm of Eq. (27), where the $\max$ operation should be replaced by jac operation, while the operation $(|t|)$ should be replaced by the $(\text{jac}\{t, -t\})$.

IV. Performance Results

We provide our simulation results in this section. We note that our proposed Approx-Log-MAP algorithm as well as Max-Log-MAP algorithm in Section III have the same detection capability as the conventional STBC detection introduced in Section II. We have arranged both detectors to detect the same channel output associated with the same *a priori* LLRs, and found that they always produce exactly the same *a posteriori* LLR values.

We quantify the complexity imposed by the mathematical measure of the total number of real-valued calculations required for producing a single soft bit output. Our complexity comparison between the conventional STBC detectors and the reduced-complexity STBC detector is portrayed in Fig. 2, for different number of receive antennas N. It can be seen in Fig. 2 that the symbol-by-symbol based STBC detector introduced in Section II-B has a lower complexity, which grows less significantly as N increases, compared to the matrix-by-matrix based STBC detector introduced in Section II-A. Furthermore, our bit-by-bit based STBC detector proposed in Section III imposes a further reduced complexity, as evidenced by Fig. 2.

The complexity of the G2 STBC detector is portrayed in Fig. 3, for different L-PSK/QAM schemes. It can be seen that the Approx-Log-MAP algorithm generally has a higher complexity than the Max-Log-MAP algorithm, except for BPSK and QPSK signalling. This is because neither the Jacobian algorithm nor the maximization operation is invoked by the algorithms conceived for the QPSK aided STBC detector, which was demonstrated in Section III-A. Fig. 3 also shows that the complexity imposed by generating a soft bit output by the QPSK aided G2 STBC detector is lower than that of the BPSK aided G2 STBC scheme. This is because both the BPSK and QPSK aided G2 STBC detectors have a similar complexity, but the QPSK scheme has twice the number of bits. Moreover, as expected, the complexity of our proposed detection algorithms is lower in Fig. 3 for Square 16QAM than for 8PSK. This is because both the magnitudes of the real and of the imaginary parts of a Square 16QAM symbol are detected separately, as demonstrated in Section III-B. Furthermore, Fig. 3 demonstrates that our bit-by-bit based detectors proposed in

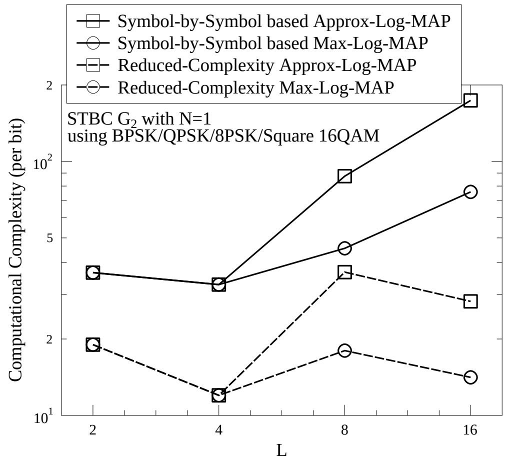

Fig. 3. Complexity comparison between different STBC detection algorithms for different L-PSK/QAM schemes.

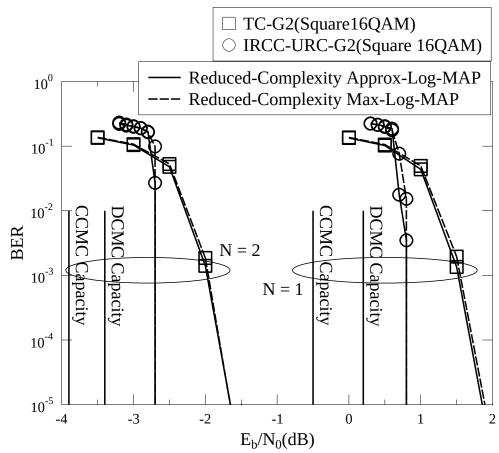

Fig. 4. BER performance of the TC coded, and IRCC-URC coded Square 16QAM aided G2 STBC detection.

Section III exhibit a lower complexity than the conventional symbol-by-symbol based STBC detector of Section II. It can be seen in Fig. 3 that a factor three complexity reduction is achieved for QPSK aided G2 STBC detection, while a factor six complexity reduction is achieved for Square 16QAM aided G2 STBC detection.

It is noteworthy that even when these detectors are invoked more than once in the context of turbo detection, the sophisticated coded systems explored in [2]–[4] may become more realistic, as a benefit of their substantially reduced complexity.

Fig. 4 portrays the BER performance of our proposed soft detectors, when they are applied to various coded systems. The TC schematic and the three-component serially concatenated IRCC-URC-STBC schematic may be found in [1] and [16], respectively. The Continuous-input Continuous-output Memoryless Channel's (CCMC) capacity as well as the Discrete-input Continuous-output Memoryless Channel's (DCMC) capacity are evaluated according to [2]. It is evidenced by Fig. 4 that a near-capacity performance may be achieved by the proposed reduced-complexity Square 16QAM aided G2 STBC detectors with the aid of channel coding and turbo detection. Moreover, the slight performance loss imposed by the Max-Log-MAP algorithm is negligible, as seen in Fig. 4.

V. Conclusions

A new method of reducing the complexity of the soft-decision aided STBC detection was proposed. We first revisited the conventional STBC detector and emphasized that the orthogonal STBC structure facilitates symbol-by-symbol based detection, which reduced the detection complexity from the order of $O(L^Q)$ to $O(L)$. We then conceived a new Max-Log-MAP algorithm as well as a new Approx-Log-MAP algorithm for PSK/QAM aided STBC detection, which operated on a bit-by-bit basis, so that the complexity order may be further reduced to the order of $O(\text{BPS} = \log_2 L)$. Most importantly, our proposed algorithms retain their original unimpaired detection capabilities.

Our simulation results demonstrated that the proposed reduced-complexity detection algorithms exhibit a substantially reduced complexity. Furthermore, we applied the proposed soft detection algorithms to a variety of coded systems. Our simulation results comfirmed that both the proposed Approx-Log-MAP algorithm and the Max-Log-MAP algorithm are capable of achieving a near-capacity performance at a reduced complexity in channel coding aided turbo detection assisted communication systems.

References

[1] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: Turbo-codes," *IEEE Transactions on Communications*, vol. 44, pp. 1261 –1271, Oct. 1996.

[2] S. X. Ng and L. Hanzo, "On the MIMO channel capacity of multidimensional signal sets," *IEEE Transactions on Vehicular Technology*, vol. 55, pp. 528 – 536, Mar. 2006.

[3] L. Hanzo, S. X. Ng, W. T. Webb, and T. Keller, *Quadrature Amplitude Modulation: From Basics to Adaptive Trellis-Coded, Turbo-Equalised and Space-Time Coded OFDM, CDMA and MC-CDMA Systems, 3rd Edition*. John Wiley & Sons, Sept. 2004.

[4] L. Hanzo, T. Liew, B. Yeap, R. Tee, and S. Ng, *Turbo Coding, Turbo Equalisation and Space-Time Coding(EXIT-Chart-Aided Near-Capacity Designs for Wireless Channels)*. 2011.

[5] S. Sugiura, S. Chen, and L. Hanzo, "Generalized space-time shift keying designed for flexible diversity-, multiplexing- and complexity-tradeoffs," *IEEE Transactions on Wireless Communications*, vol. 10, pp. 1144 – 1153, Apr. 2011.

[6] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Transactions on Vehicular Technology*, vol. 57, pp. 2228 –2241, July 2008.

[7] S. Sugiura, S. Chen, and L. Hanzo, "Coherent and differential space-time shift keying: A dispersion matrix approach," *IEEE Transactions on Communications*, vol. 58, pp. 3219 –3230, Nov. 2010.

[8] C. Xu, S. Sugiura, S. X. Ng, and L. Hanzo, "Reduced-complexity soft-decision aided space-time shift keying," *IEEE Signal Processing Letters*, vol. 18, pp. 547 –550, Oct. 2011.

[9] C. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multiple antennas," *Bell Labs. Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.

[10] J. Heath, R.W. and A. Paulraj, "Linear dispersion codes for MIMO systems based on frame theory," *IEEE Transactions on Signal Processing*, vol. 50, pp. 2429 – 2441, Oct. 2002.

[11] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Transactions onCommunications*, vol. 49, pp. 1 –4, Jan. 2001.

[12] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1451 –1458, Oct. 1998.

[13] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information Theory*, vol. 45, pp. 1456 –1467, July 1999.

[14] W. Koch and A. Baier, "Optimum and sub-optimum detection of coded data disturbed by time-varying intersymbol interference," in *IEEE Global Telecommunications Conference (GLOBECOM'90)*, pp. 1679 –1684 vol.3, Dec. 1990.

[15] P. Robertson, E. Villebrun, and P. Hoeher, "A comparison of optimal and sub-optimal MAP decoding algorithms operating in the log domain," in *IEEE International Conference on Communications (ICC'95)*, vol. 2, pp. 1009 –1013, June 1995.

[16] L. Kong, S. Ng, R. Tee, R. Maunder, and L. Hanzo, "Reduced-complexity near-capacity downlink iteratively decoded generalized multi-layer space-time coding using irregular convolutional codes," *IEEE Transactions on Wireless Communications*, vol. 9, pp. 684 –695, Feb. 2010.