Polarisation Effects in Optical Microcoil Resonators

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Optical microcoil resonators (OMRs) fabricated by wrapping a microfibre around a rod to allow evanescent coupling between adjacent turns as in Fig 1. (a) have recently attracted much interest due to their high Q-factor and large extinction ratios resonances, low input and output coupling losses, large evanescent field and compactness [1,2], with applications such as sensing [3] and signal processing [4]. However, theoretical models published so far have neglected polarisation effects, and hence in order to develop a more detailed understanding we have modelled the OMR with polarisation-dependent coupled mode equations in the linear [5] and nonlinear regimes.

In the linear regime there are two cases to consider. Firstly when the fibre is birefringent and the second when the fibre retains its circular symmetry but the geometry allows for an addition polarisation rotation (a Berry's phase). We first consider the case of a birefringent microcoil and discuss it's linear and nonlinear properties. The geometry for this case is shown in Fig. 1(a) where we have allowed in addition a small twist to fibre's axes as it is coiled up. This has the effect of allowing coupling between the two polarisation modes and even though the coupling might be small the effect of it on resonance can be large due to repeated passes through the resonator. As the fibre is assumed to be birefringent and slightly twisted, we can define a local reference frame (x, y) aligned with the fibre's slow/fast axes which rotates with the twisting of the fibre. In the j^{th} turn of the coil, the corresponding field amplitudes A_j^x and A_j^y are governed by the differential equations:

$$\begin{aligned}\frac{dA_j^x}{ds} =& i\kappa\Big[A_{j-1}^x\cos(\theta^-) + A_{j-1}^y\sin(\theta^-) + A_{j+1}^x\cos(\theta^+) - A_{j+1}^y\sin(\theta^+)\Big] \\ &- i\alpha A_j^x + i\gamma(|A_j^x|^2 + \frac{2}{3}|A_j^y|^2)A_j^x + i\kappa_{xy}A_j^y\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{dA_j^y}{ds} =& i\kappa\Big[A_{j-1}^y\cos(\theta^-) + A_{j-1}^x\sin(\theta^-) + A_{j+1}^y\cos(\theta^+) - A_{j+1}^x\sin(\theta^+)\Big] \\ &- i\alpha A_j^y + i\gamma(|A_j^y|^2 + \frac{2}{3}|A_j^x|^2)A_j^y + i\kappa_{yx}A_j^x + i\Delta\beta A_j^y\end{aligned} \tag{2}$$

Here, κ is the coupling coefficient between adjacent turns, α is the loss coefficient, and θ^-, θ^+ are the twist angles between the lower and upper turns respectively. The sine and cosine terms describe the twist dependent coupling between the neighbouring x and y polarisations and the nonlinear γ term models the self and cross phase modulation. Polarisation rotation from twist induced elasto-optic effects is accounted for by the κ_{yx} and κ_{xy} terms. The pitch between turns is assumed to be much smaller than the rod diameter, so geometric phase effects are not considered here.

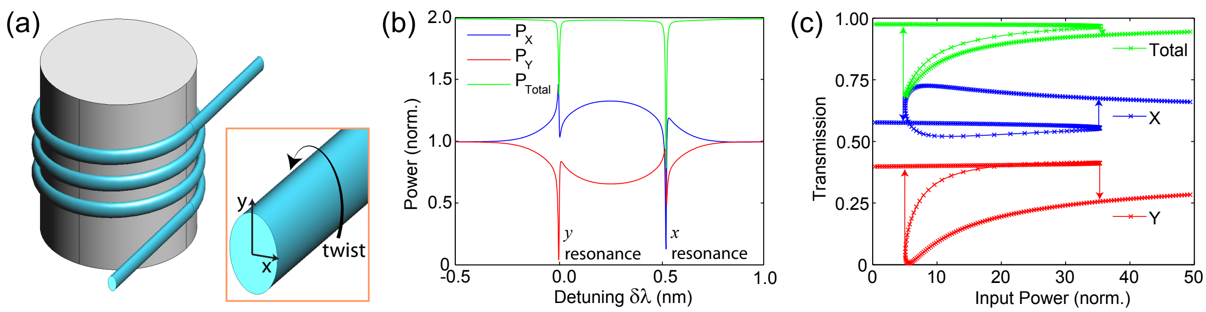

Fig. 1. (a) Microcoil structure showing the local (x, y) axes and fibre twist. (b) Typical linear output spectrum over one FSR for a 3 turn OMR with an input polarised at $\pi/4$ rad and fibre twist of $\tau = 30$ rad/m. (c) Bistability when detuned 25pm from the y resonance, showing hysteresis in the transmission and x and y components. Power is normalised for $\gamma = 1$/W/m.

Note that as expected twisting the fibre couples the two orthogonal polarisations which can be seen in Fig. 1(b) which shows linear response of a twisted OMR. Here the x polarisations resonances are detectable in the y polarised output and vice versa. There is also now an asymmetry in the polarisation dependent response which depends on the angle of the twist. If the twist is anti-clockwise then for near the resonance for the y-polarised light, light is lost from the y-polarisation while additional light is coupled in from the x-polarisation. In contrast near a resonance for the x-polarised light, light that couples to the y-polarisation avoids the resonance and so is transmitted. Thus the total loss of the system is higher for resonances corresponding to y polarised light. This behaviour is of course reversed if the twist is clockwise.

Once the linear behaviour is understood we can increase the input power to examine the nonlinear response of twisted OMRs. Typical results for this is shown in Fig. 1(c). As for the case of a non-brifringent OMR [1] when red-detuned from resonance, the transmission shows bistability with nonlinear switching powers typically in the range 10-100W. However, the hysteresis characteristics strongly depend on whether the wavelength is detuned from an x or y resonance. With the former, the contrast between the high and low transmission states is larger and the switching threshold power is lower, since coupling from the y to x polarisation state is favoured due to the choice of left-handed twist in this example. For similar reasons, the input polarisation angle also affects the switching power and contrast.

Berry Phase Magnification in OMRs

Finally we have looked at polarisation rotation in OMRs made from circularly symmetric fibres. This is due to geometric path effects as the light now propagates along a closed curve in the OMR and is better known as the Berry phase[6]. In this case the relevant coupled equations are[6]:

$$\frac{dA_j^x}{ds} = i\kappa_x\left(A_{j-1}^x + A_{j+1}^x\right) + \tau A_j^y + \left(i\Delta\beta_b - i\frac{C^2}{2\beta} - \alpha\right)A_j^x \tag{3a}$$

$$\frac{dA_j^y}{ds} = i\kappa_y\left(A_{j-1}^y + A_{j+1}^y\right) - \tau A_j^x - \alpha A_j^y \tag{3b}$$

Where τ describes the Berry's phase effect. We have also included a smallpropagation mismatch term due to the fibre curvature given by C [7]. Physically τ depends on the pitch of the coil and on the diameter of the OMR. In order for τ to become significant the OMR diameter must be below $1\,\mathrm{mm}$ since in this regime coils with a large relative pitch can be made. We modelled an OMR with a diameter of $0.2\,\mu\mathrm{m}$ and the pitch (i.e. the distance between one coil and the next) is in the range between 2 and $3\,\mu\mathrm{m}$. In this regime there is a small difference between the coupling coefficients for the x and y polarisations and we have calculated their values using FEM modelling of the fibres. Note that for the values we have used the coefficient τ is typically 1000 times smaller than the coupling coefficient which is why it was neglected in earlier work, however as we will show it's effects can still be observed. The additional birefringent term C is calculated from the elasto-optic coefficient for silica and the details are in reference [7].

From Eq. (3) It can be seen that in the absence of the Berry phase ($\tau = 0$) no light would couple from one polarisation to another and so the Berry phase is immediately observable as a rotation of the angle of the polarisation. In the absence of any coupling between the turns of the microcoil ($\kappa_{x,y} = 0$) the Berry's phase leads to a polarisation rotation of about 10^{-5} degrees which is far to small to be measurable. However it is expected that near resonance when light is trapped in the resonator and undergoes a large number of round trips the total acculumated Berry's phase can be extremely large and easily measurable. Indeed this can be seen in Fig. 2(a,b) which shows the power in each mode for frequencies near resonance. The amount of polarisation rotation as the pitch of the coil is varied is given in Fig. 2(c) and it can be seen that complete polarisation rotation can be achieved near resonance.

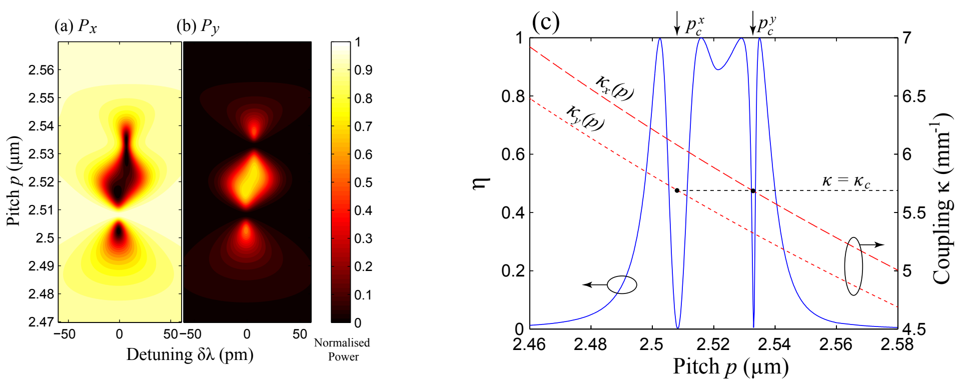

Fig. 2. (a,b) Coupled power due to Berry phase effects near the x and y mode resonances. (c) Fraction of coupled power as a function of the pitch of the microcoil. The parameters used were that it was a 3 turn OMR made from a 1 micron diameter fibre with a coil diameter of $0.1\,\mathrm{mm}$. The torsion of the coil varies between 39 and 41 inverse metres and so $\tau \approx 40\,\mathrm{m}^{-1}$ while the curvature $C = 10^4\,\mathrm{m}^{-1}$.

An interesting feature of the polarisation rotation is that it goes identically to zero exactly at the critical coupling point. The critical coupling point corresponds to the combination of κ and λ that allows light to be trapped indefinitely inside a micro-coil resonator. Mathematically at this point the equations become il-defined since the electric field inside the coil is not uniquely specified by the input to the coil but instead is a combination of the input light plus an arbitrary amount of stored light. However near to the resonance the fields are well defined and there is a wide range of parameters that would allow significant polarisation rotation.

In conclusion we have looked at both the linear and nonlinear polarisation properties of optical microcoil resonators. In the linear regime we have showed that twisting the fibre leads to interesting asymmetric responses while for a circularly symmetric fibre contrary to previous statements the Berry phase can be magnified on resonance leading to a measurable response. In the nonlinear regime birefringent OMRs are bistable and this bistability can be exploited for different applications. In particular we are looking at using one polarisation to switch a bistable state in the opposite polarisation thus allowing the polarisation sensitive Boolean logic operations.

References

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