STRATEGIES FOR BALANCING EXPLORATION AND EXPLOITATION IN ELECTROMAGNETIC OPTIMISATION

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***Abstract* – The paper focuses on the advantages and drawbacks of different strategies which may be used to assist kriging surrogate modelling with the purpose of selecting multiple design vectors for evaluation when stepping forward in optimisation routines. The combined criteria include the efficiency of finding the global optimum but also the quality of the approximation of the shape of the objective function; the latter may be used to make judgements about the robustness of the optimised design.**

I. INTRODUCTION

Design problems in electromagnetic devices are commonly solved using time-consuming numerical techniques, such as the finite element method. In order to relieve the heavy burden of computation in such designs, kriging has been suggested as one of the reliable surrogate models with low computational cost and good accuracy of predicting the shape of the objective function. In the optimisation task, the main target is using as few 'expensive' objective function calls as possible to find the global optimum. The balance between exploration (searching the region with high uncertainty) and exploitation (searching the highly confident space) has been discussed before [1-4]. This paper puts main emphasis on improving the existing strategies to predict the shape of the objective function as accurately as possible – in addition to locating the global optimum – in order to assess the robustness of the solution.

II. KRIGING WITH DIFFERENT STRATEGIES

A .Kriging and different strategies

Kriging [1] can exploit the spatial correlation of data in order to predict the shape of the objective function based only on limited information and estimates the accuracy of this prediction, which is helpful in assisting the main decision of the optimisation process how to choose the next design vector for evaluation. In general, an estimate of the accuracy (called the potential error) by the kriging model is commonly used to build a range of different 'utility functions' such as the Expected Improvement Function (EI) [2], or Weighted Expected Improvement (WEI) [3]. The EI function is defined as

$$\mathit{If}\quad s(x)>0$$

$$EI=(f_{\min}-\hat{y}(x))\Phi\left(\frac{f_{\min}-\hat{y}(x)}{s(x)}\right)+s(x)\phi\left(\frac{f_{\min}-\hat{y}(x)}{s(x)}\right)\qquad(1)$$

$$\mathit{If}\quad s(x)>0$$

$$EI=0\qquad(2)$$

where $\hat{y}(x)$ is the predicted value of objective function by the kriging model, and *s(x)* is the root mean squared error in this prediction. The first term called the Gaussian density favours searching promising regions, whereas the second term (Gaussian distribution function) is related to exploration, which favours searching regions with high uncertainty. Finding the global optimum of objective function is one of the significant aims for an optimization problem. In practical experiments, the exploration term performs dramatically better in terms of finding the global optimum of the objective function, while the exploitation often can only find the local minimum. Since EI applies equal weights on the two terms, it may be seen as a fixed compromise between exploration and exploitation. The WEI is derived from EI by adding a tuneable parameter which can adjust the weights on exploration and exploitation.

As suggested by previous tests [4], the optimal choice of the weights is known as critical in terms of the ability of the algorithm to achieve global optimum and doing it efficiently; unfortunately the optimal weights are normally hard to find and require numerous tests. Therefore two novel algorithms using reinforcement learning [5] called Adaptive Weighted Expected Improvement (AWEI) and Surrogate Model based Weighted Expected Improvement approach with rewards [6] (SMWEI) [4] have been proposed to make the process of tuning weights more intelligent and self-guiding.

The Mean Square Error (MSE) from the kriging model is used to calculate the rewards. The AWEI can tune the weights automatically based on the comparison between the potential rewards from two different weight distributions emphasising exploitation and exploration, respectively. After comparison, the weights are redistributed on the two terms of (1) to encourage exploration or exploitation depending of the results of the initial pre-test. However, the AWEI only takes account of the short term rewards at a given iteration step, whereas the SMWEI can predict the cumulative rewards likely to occur in long term as a consequence of a particular choice of actions. Furthermore, the SMWEI creates a surrogate model based on potential error and kriging prediction to use in a pre-test rather than using the information from the time-consuming finite element modelling software. In the pre-test, two distinct weights are used – one favouring exploration and the other one exploitation – and iterations continue using the surrogate model independently in parallel until overall rewards have been found. The optional weight with better reward of the two is then used to feed back – via the FEM module – into the main iterative loop of the design process.

B. The SMWEI with Multi-weights in pre-test

In practical electromagnetic problems, the robustness of the design is a significant requirement that needs to be considered. Through testing it has been found that the SMWEI algorithm with certain pairs of weights in the pre-test performs better in terms of estimating the shape of the objective function, a feature which might be helpful when assessing the robustness of the design. As SMWEI is limited by the pre-set pair of weights in the pre-test, a number of experiments may be necessary to find the pair resulting in more faithful representation of the shape. As the pre-test is 'cheap', more weights can be selected to broaden the base for comparisons. The new version of SMWEI with multi-weights is described in Fig. 1. In the pre-tests, if one of the rewards is not assessed properly or fails, the remaining rewards still participate in the comparison until an action with the biggest reward is chosen.

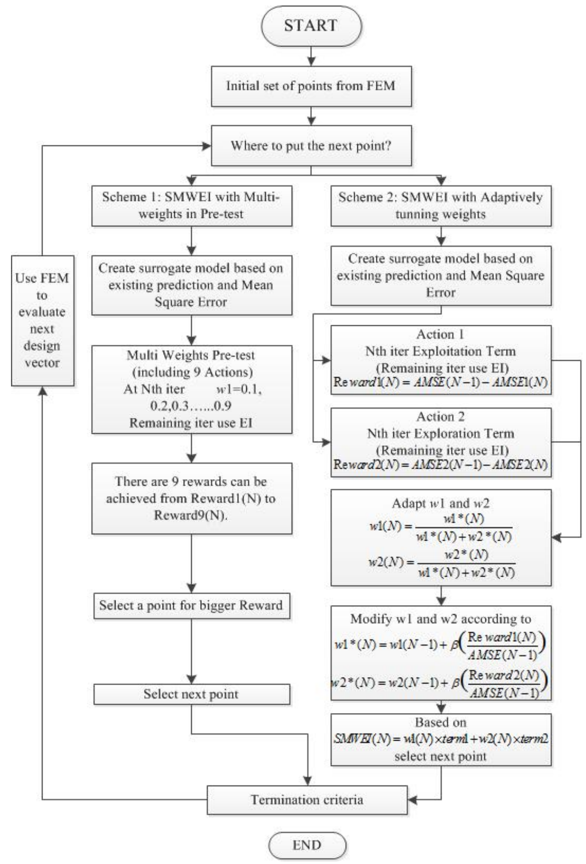

Fig. 1. The flowchart for SMWEI with multi weights in pre-test

C. The SMWEI with the strategy of adaptively tuning weights

In the pre-test of SMWEI, a pair of fixed weights (one emphasising exploration and the other one exploitation) needs to be set initially. The guidelines how to select such weights are subject to further experiments. However, the strategy of tuning the weights automatically and adaptively in the pre-test of AWEI can also be used in SMWEI in order to avoid the need for setting initial optional weights. The decision-making chart of the actual implementation is shown in Fig. 1.

Because all pre-tests in SMWEI apply 'cheap' simplified surrogate model based on the specific prediction and potential error produced by kriging, the Mean Square Error might be directly used in each pre-test's remaining iterations instead of the Expected Improvement. The simplified surrogate model in the pre-test, quite rough initially, is increasingly accurate as a result of adding objective function calls; therefore the MSE might guide the kriging model directly to search the region of the simplified surrogate model with high uncertainty.

III. RESULTS

Fig. 2(a) shows one of the test results to approximate the two variables Schwefel function [7], which is a two-objective task. The global minimum has been found after 9 iterations, but the quality of the shape representation of the objective function is poor in Fig. 2(b). Complete results will be reported in the full version.

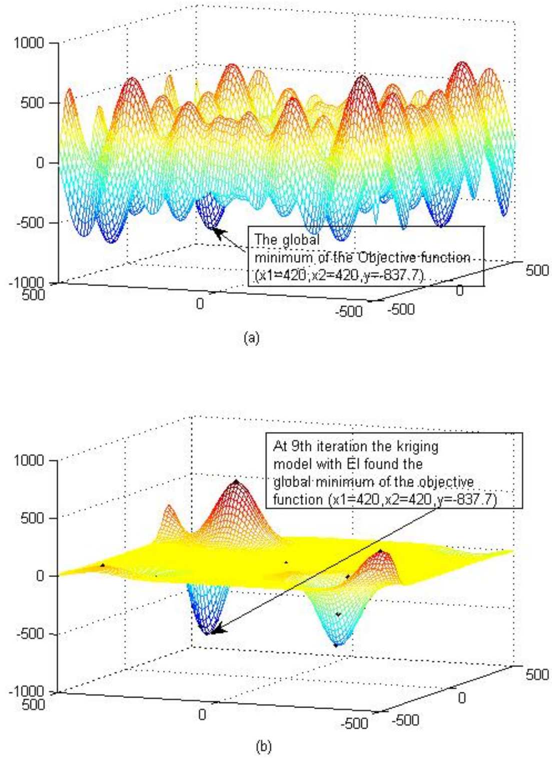

Fig. 2: a) Two variables Schwefel function in the $x\epsilon$[-500 500] and $y\epsilon$[-500 500] domain. b) The approximation found by kriging with EI.

IV. CONCLUSIONS

It is argued that a multi reward scheme based on a surrogate model may provide the best prediction of long term benefits for achieving best balance between exploration and exploitation.

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