UNIVERSITY OF SOUTHAMPTON

FACULTY OF ART BUSINESS & LAW
School of Management

FORECASTING OF DAILY DYNAMIC HEDGE RATIO IN AGRICULTURAL AND COMMODITIES' FUTURES MARKETS: EVIDENCE FROM GARCH MODELS

By

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Thesis for the degree of Doctor of Philosophy

April 2012

To my dear parents
and sister

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

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This thesis investigates the predictive power of six bivariate GARCH-CCC (constant conditional correlation) models; the GARCH (1, 1), BEKK GARCH (1, 1), GARCH-X (1, 1), BEKK-X (1, 1), GARCH-GJR (1, 1) and QGARCH (1, 1) based on both normal and student's t distributions. Empirical investigations are conducted by forecasting the daily hedge ratios from agricultural futures markets using one-step-ahead over 1 year and 2 year out-of-sample period. The forecasting of OHR in agricultural and commodities' futures markets has not been studied thoroughly and few publications are available in literature. My work enriches the literature and will hopefully provide guidance for hedging in these markets.

To forecast the OHR, we apply data from three storable commodities, coffee, wheat and soybean and two non-storable commodities, live cattle and live hog. Four tests are conducted to evaluate the forecasting errors of out-of-sample forecasted return of the portfolio based on the forecasted OHR.

Our study shows that the asymmetric GARCH model outperforms other models, and the standard GARCH is the weakest for 1-year forecast. However, the standard GARCH model performs well for 2-year forecast of live cattle with student's *t* distributed residuals. More generally, the BEKK and asymmetric GJR and QGARCH models are recommended to forecast OHR on both 1-year and 2-year horizons with normal and student's *t* distributions for storable products and the asymmetric models for non-storable commodities. Furthermore, our study demonstrates that the predictive power of GARCH models depends on the distribution of residuals, the commodity and also the length of the forecast horizons. This result is consistent with the those from Poon and Granger (2003) and Chen et.al (2003). Given accurately forecasted OHR, investors can determine appropriate hedging strategies for portfolio management to reduce or transfer risks, and prepare for the capital needed for hedging.

Key Words: Forecasting, Hedge Ratio, GARCH, Futures Market, Volatility.

Content

Content of Figures

Content of Tables

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Acknowledgements

It is my pleasure to send my warm thanks to these people.

First and foremost I would like to thank my supervisor Professor Taufiq Choudhry. I am sincere thankful to him for his encouragement and guidance throughout the past few years. He has been fully supportive to me. I greatly appreciate all his contributions of time, patience, advice to make my Ph.D. experience enjoyable and productive. This study would not have been possible without him.

I am also grateful to my friends Grant Wu and Frank Wang at the Management School for their support and friendship. I send my warm appreciation to the School of Management, University of Southampton for the full PhD scholarship for three years. I must express my gratitude to all the staff and colleagues at the School of Management who have provided great resource for graduate students and made the research painless and enjoyable.

A special thanks goes to my parents, sister and Wang Qi for their valuable love and moral support during the past few years.

Acronyms and Abbreviations

OHR	Optimal hedge ratio
OLS	Ordinary least square
ML	Maximum likelihood
ECM	Error correction model
CBOT	Chicago Board of Trade
LME	London Metals Exchange
CME	Chicago Mercantile Exchange
NYBOT	New York Board of trade
NYME	New York Mercantile Exchange
NFA	National Futures Association
CFTC	Commodity Futures Trading Commission
IFSL	International Financial Services London
JB	Jarque-Bera normality test
LM	Lagrange Multiplier ARCH effect test
LB	Liung-Box Autocorrelation test
LLF	Log-likelihood
AIC	Akaike's (1974) information criteria
BIC	Schwarz's (1978) information criteria
MAE	Mean absolute error
MSE	Mean square error
MDM	Modified Diebold Mariano test

1 Introduction

1.1 Motivation

The past few decades witnessed the dramatic increasing of futures trading in numbers and types. Hedging is a strategy to reduce or transfer unacceptable price risk of an asset (Johnson, 1960). A trader can hedge against possible uncertainties and price risk by taking out an offsetting position in futures, options contracts or other related derivative securities. Optimal hedge ratio (OHR hereafter) is the number of futures contracts used to hedge a particular exposure with a unit in the spot market (Chance, 1989). It is too risky to hedge if one cannot estimate the value of hedge ratio, since hedgers cannot be certain the number of futures contract. Accurate forecasting of hedge ratio helps investor to apply appropriate hedging strategies and technique to minimize price risk and protect their investment from unacceptable loss. One of the most important implications of forecasting is in planning or decision-making of investment. The forecasting of OHR helps hedger choose appropriate portfolio and allows for portfolio adjustment in dynamic hedging.

Over the past few years, the estimation of hedge ratio has attracted the attentions of many great researchers. In the field, various statistical techniques have been established and developed for this purpose including Ordinary Least Square (OLS), Error Correction Model (ECM), Cointegration (CI), Exponentially weighted Moving Average (EWMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. (Almost) Undoubtedly the GARCH models (Engle (1982), Bollerslev (1986)) shines in this family like a super star. In this thesis, using six different constant conditional correlation (CCC)-GARCH models, we estimate and forecast the OHR in agricultural and commodities' futures markets, which, to our knowledge, has not been studied in previous research. Thus, this thesis contributes to the OHR literature by being one of the first few studies to forecast the OHR in agricultural commodities market.

According to the Efficient Market Hypothesis (EMH) of Fama (1970), price series should be unpredictable in an efficient market, which means, one cannot consistently achieve returns in excess of average market returns on a risk-adjusted basis, given the information available at the time the investment is made. The EMH holds in a perfect market in which transaction costs are negligible and there is no arbitrage opportunity. However, the market is not perfect due to trading cost, information asymmetry, window dressing and other uncertain factors in financial statement. On the other hand, in the markets of weak or semi-strong forms of EMH[1], the return is predictable in some sense (Watts and Zimmerman, 1986, Timmermann and Granger, 2004). In our study, the estimated OHR series from all GARCH models for all five commodities are stationary, which makes the OHR predictable and the predictability of OHR does not violate the EMH.

The OLS was proposed by Ederington (1979) to estimate a constant hedge ratio. However, the constant hedge ratio from the simple regression ignores the information effect in previous time period on hedge ratio. Cecchetti et al. (1988) introduced dynamic hedge ratio to overcome this shortcoming. In the time since, the univariate GARCH model, multivariate GARCH model (Engle and Kroner (1991) and Bera and Higgins (1993)) and its various extensions are employed for OHR estimation and prediction. For example, the GARCH-X model proposed by Lee (1994) incorporates the effect of short-term derivations on variance in time series (deviation from the cointegration relationship). The debate about the presence of cointegration in agricultural markets took a new turn when Ghosh (1993, 1995) and Yang et al. (2001) demonstrated cointegration using commodity futures data. They also showed that the cointegration between cash and futures prices in commodity markets is necessary to ensure an optimal hedging decision. Kroner and Sultan (1993) claimed that the long-run cointegration relationship between financial assets and dynamic distribution of the assets was consequential to estimate an accurate OHR. Ghosh (1993), Lien (1996) and Lien (2004) showed that ignoring cointegration tends to produce a smaller hedge ratio; additionally the cointegration improves estimation and forecasting of OHR. This thesis takes one step further by showing whether cointegration relationship between cash and futures prices has any effect on the forecast of OHR.

The GARCH-GJR and QGARCH models take into account the leverage effect of information on the financial data detected by Black's (1976). Brooks and Henry (2002)

[1] The EMH is defined that if all information available only comprises past and current asset prices (trading volume and others), the market is the weak form of EMH; if the market contains all published information, it is the semi-strong form of EMH; if the market includes all available information which is exposed to anyone in this market, it is the strong form of EMH and it is also called perfect market.

suggested that the asymmetric BEKK GARCH model was appropriate for hedging and forecasting hedge ratio for short-term investment, but not for the investment with investment time horizon beyond 1 month when they investigated the asymmetric impact of information on OHR in the UK stock futures market.

The study of OHR attracted the attention of many scholars over the past few years. One of the main results is the majority of them agree that the GARCH family model is superior in estimating OHR in financial markets (see Baillie (1991), Kroner and Sultan (1993), Park and Switzer (1995), Lien (1996), Floros and Vougas (2004, 2006)). However, it's quite different a story for the research concerning the agricultural and commodities' futures markets, which has not been studied thoroughly and only limited publications are available, such as Baillie and Myers (1991) , Roh et al. (1995), Tse et al. (2002) and Choudhry (2003). As stated earlier, this is especially true with the forecasting of OHR in agricultural and commodity's future markets. This study applies six different versions of the GARCH models to forecast the daily OHR in the agricultural commodities market. An interesting question arises from the forecasting exercise, which of the six models employed provides the best forecast for the different commodities under study, storable or non-storable. An asset is not perfectly storable or non-storable (Covey and Bessler 1995), where soybean, wheat and coffee are typical storable and live hog and live cattle are among non-storable commodities. It also attracts our interests to study the effects of storability, volatility of price basis and demand-supply relationship on the forecasting powers of the models.

We employ the standard GARCH, BEKK, GARCH-X, BEKK-GARCH-X, GARCH-GJR and QGARCH models to estimate and forecast the OHR for storable coffee, wheat, soybean, and non-storable live cattle and live hog. The storable and non-storable products should provide different results based on difference in terms of storability, volatility of price basis and demand-supply relationship. The commodities we study are five of the most popularly traded agricultural products in futures market worldwide. The six GARCH family models are chosen because of their abilities of dealing with heteroskedasticity dynamically in time series, while the hedge ratio is developed mathematically as a typical time-varying series.

It is widely accepted that the student's *t*, generalised Gaussian and other non-normal distributions are more appropriate to describe the behaviour of residuals in regression between returns. However, referring to Brooks (2008) who stated that a certain amount of extreme value or heteroskedasticity will lead to non-normality of time series for large sample, thus we employ both normal and student's *t* distributed for forecasting and comparison of forecasting ability.

In this empirical study, we apply in-sample estimation of OHR, 1-year and 2-year out-of-sample OHR forecast based on the daily cash and futures price from 01/01/1980 to 23/03/2006 and 01/01/1980 to 14/01/2008 for the three storable and the two non-storable agricultural products respectively. We compare the forecasting performance of these six GARCH models with both normal and student's *t* distributed residuals on two forecast length horizons. In order to make this study more robust, we evaluate the forecast error and accuracy by different measures (MAE, MSE, Theil's U and Modified Diebold Mariano tests).

1.2 Aims, Objective and Research Questions

Although optimal hedge ratio (OHR) has been investigated extensively over the past few years, the forecasting of OHR, in particular that of agricultural and commodities' futures markets, lacks such intensive investigation in the past. To our best knowledge, this is the first study to forecast OHR while comparing the forecasting ability of these six GARCH models at the same time. Thus the forecasting of agricultural futures OHR makes this thesis' contribution unique to the literature. In this study, we are aiming at forecasting the daily OHR in agricultural futures market by six different GARCH models and comparing the predictive power of these models. Comparison of the forecasting ability of the six GARCH models based on OHR forecast is also unique and makes a substantial contribution to the OHR and forecasting literature. This study was inspired by Choudhry (2009)'s research and results. The data in this thesis are the same as those employed by Choudhry (2009), while our work differs from Choudhry (2009)'s research for two aspects. First of all, we estimate and forecast the daily OHR whereas Choudhry (2009) investigated the hedging effectiveness of the OHR. Second of all, the models that we apply are different from those in Choudhry (2009) in which the standard GARCH, BEKK-GARCH, GARCH-X and BEKK-GARCH-X models were applied to estimate and forecast the OHR. Specifically, the six models we utilize are standard GARCH, BEKK-GARCH, GARCH-X, BEKK-

GARCH-X, GJR-GARCH and QGARCH.

The thesis attempts to answer the following questions:

1. Can the GARCH family models estimate and forecast OHR in agricultural and commodities' futures markets?
2. Which GARCH model among the six models outperforms in OHR forecasting for coffee, wheat, soybean, live cattle and live hog based on
   a) 1-year and 2-year out-of-sample forecasting
   b) 4 evaluation methods on forecasted return
3. Is the forecasting performance of each GARCH model the same with normal and student's t distributions on 1-year and 2-year forecast horizons?
4. What are the differences of predictive power of the six GARCH models for storable and non-storable agricultural commodities?

If the answer to the first question is yes, the implications of GARCH type model in OHR estimation and forecast expand to agricultural and commodities' futures markets. The next 3 questions are based on the validity of the six chosen GARCH model on estimation and prediction of OHR in this futures market. The forecasting performance of the six GARCH models on the non-overlapping 1-year and 2-year forecast with normal and student's *t* distributions will hopefully provide a guideline for the investor. Furthermore, we are interested in the effect of the storability of agricultural products on the predictive power of these GARCH models. Once such an effect is confirmed and established, it will also provide investors guidance on the choice of forecasting model. Generally speaking, we aim at finding the best forecasting model among the six GARCH models on OHR prediction based on 2 distributions, 2 forecast horizons and 2 types of evaluation methods, and offer the investors some guidance in long term investment in agricultural futures markets with suitable hedging strategies to reduce and/or transfer of risk.

1.3 Structure of Thesis

This thesis is organized as follows. In the chapter 2, we introduce the development and mechanism of futures markets, and especially on those of agricultural and commodity's futures markets. The hedging with futures is also discussed briefly, together with the proposal of the concept of hedge ratios. Chapter 3 reviews some important literatures about the deviation, estimation and forecasting of hedge ratio. A number of articles about prediction of volatility are cited due to the lack of publications on hedge ratio forecasting. Chapter 4 is devoted to describe the methodology and data that we use for in-sample estimation and out-of-sample forecasting of OHR in this study, where the characters of the six GARCH models, involved statistical tests, evaluation method, source of data, and the way to deal with data are also presented in details. In the following chapter 5, we describe the empirical findings on the forecasting power of six GARCH models for OHR prediction in agricultural commodities' futures market based on normal and student's *t* distributions respectively. The chapter of result interpretation provides outputs of OHR estimation and 1-year and 2-year out-of-sample prediction of OHR and return with normal and student's *t* distributions in part A and part B, respectively. Furthermore, we explore the practical implication of dynamic OHR forecasting with these six GARCH models for investors in part C. A cross comparison between normal and student's *t* distribution is reported and interpreted in section 5.12. Finally, we discuss the major findings and problems solved, also the main contributions of this study, and also the potential prospective research in chapter 6.

2 Futures Markets and Hedging

2.1 Introduction

Futures markets[2] have expanded dramatically over the past decade and the trading scope of the underlying asset in futures markets is remarkably thrived from original agricultural goods to interest rates, treasury bonds, equity index, precious metal products, energy and other commodities and financial products.

Futures trading volume has increased strikingly since its appearance in 1884. The annual average volume of grain futures trading on U.S. Exchange was 23,600 million bushels between 1884 and 1888, while that number of futures volume of grain in CBOT (excludes Barley and Rye) boosted up to 125,000 in 2002[3]. In ICE Group (Intercontinental Exchange), the ICE futures US witnessed a 50.5% increase which enhanced 80.954 million in 2008 from its counterpart of 53.782 million in 2007[4].

With the development of financial futures contracts, such as interest rate, treasury bonds, Euro, dollars, they emerge to take much larger share than agricultural commodities and dominate contemporary futures industry. For example, Eurodollar futures counted 23 percent of trading volume in Chicago Mercantile Exchange in 1999 and this figure is increasing each year. In 2008, Equity Index, Individual equity combined with interest rates expend more than 80% of the global futures trading (Carter, 1999).

In finance, two parties (buyer and seller) sign a standardized contract, called future contract, to exchange specified trading items under specific terms of quantity, quality, price, and also the precise delivery date, through a broker who is a member of future exchange. This specified price locks the exchange price of the asset and reduces the uncertainty of asset price in future since the buyer and seller have the obligation to execute the contract on maturity. If one party (buyer or seller) is not aiming to purchase or deliver actual products, the other can cancel out the existed contract before the

[2] Futures market is an auction market in which participants buy and sell commodity and future contracts for delivery on a specified future date.
http://www.investopedia.com/terms/f/futuresmarket.asp

[3] Source: Hieronymus (1977), p.23; CFTC (2003), p, A2; NASS (2004) and Santos, Joseph. "A History of Futures Trading in the United States", EH.Net Encyclopedia, edited by Robert Whaples. March 16, 2008. http://eh.net/encyclopedia/article/Santos.futures

[4] Data from FIA: 2008 volume data and rankings.

delivery month. In addition, one can trade in futures markets even if the trader does not have or own the products, as long as the trader offsets all contracts and the net position is zero before the maturity. Thus hedger and speculator may play important roles in the process of such offsetting position.

This chapter is organized as follows. In the first part we introduce the foundations of futures, the mechanism of a futures market and also the agricultural futures markets. Second, hedging and hedge ratio are introduced, while we proceed in part three to discuss the details of a hedging, such as how to derive, estimate and forecast hedge ratio, particularly in commodities' and agricultural futures markets.

2.2 Futures Markets and Their Functions

The original futures trading, in the modern sense of word, can be traced at least to Babylonian times, when people made trading of livestock, such as pigs, goats and sheep, and some other items as 'commodity money'. During the 19th century, future contracts evolved to the form that farmers made handshake agreements to sell their crops or livestock to buyers and deliver the goods in some future time, which was the earliest futures contract, the so-called forward contract.

The prototype of modern futures market initiated in Midwestern United States with contracts on grain and pork products in the early 1800s. Since then, more and more commodities emerged in futures markets and the trading value in 1929 reached to an estimated $42 billion dollars (Hoffman, 1932).

Modern futures' trading includes that of the financial futures, energy products, metal and agricultural products throughout the world. In 1985, the futures' trading volume of Treasury bond in the US exceeded the total trading volume in all agricultural commodities (Leuthold, Junkus, 1989), while the financial futures counted over half of the total volume in all exchanges (Weller, 1992).

The futures' trading has increased dramatically in volume and becomes increasingly complex over the last decade, with the establishment and development of modern future exchanges, such as the Chicago Board of Trade (CBOT), the Chicago Mercantile Exchange (CME), the ICE Futures, the London Metal Exchange, and the Tokyo Commodity Exchange, where instantaneous electronic trading and management become possible. These exchanges provide centralized markets for the futures trading.

2.2.1 Futures Contracts

A futures contract is an agreement between the buyer and the seller, in which the trading date, price, quality, quantity and delivery method of the underlying asset are determined and specified. This contract contains standardized terms and requires margin and daily settlement under regulations of exchange, clearinghouse, NFA and CFTC. Futures contracts are traded on centralized and organized futures exchanges (Chance, 1989).

The term futures contracts stems from forwards contracts, and they share certain similarities. A forward contract contains nearly the same terms as a future contract, except that the former is customized and traded over-the-counter without margin requirement, daily settlement policies and formal regulations. On the other hand, a futures contract is more strictly regulated in the sense that the terms and conditions, including size, quotation unit, minimum price fluctuation, grades and delivery terms must be approved by the Commodity Futures Trading Commission (CFTC).

There are different types of futures contracts and they are divided into four main categories with respect to their underlying assets: Physical Commodity, Foreign Currency, Interest-Earning Asset and Index (Stock Index). There were around 85 different contracts in the US trading market in 1986 (Leuthold et al., 1989).

2.2.2 Functions of Futures Markets

A futures market or futures exchange is a central financial market where participants buy and sell underlying asset or futures contracts on a specified date in future according to the specifications of the contracts[5]. Futures markets facilitate the marketing and transference of ownership of goods and services, and increase the information content of spot market prices. The applications of futures are the information discovery and facilitation of risk management.

Price discovery, one of main usages of futures markets, is the process of determining the price of an asset through the futures markets (Kolb and Overdahl, 2007). Futures market

[5] Refer to http://www.investopedia.com/terms/f/futuresmarket.asp

keeps updating the latest trading information instantly twenty-four hours a day, which includes the trading price, the amount, location, quality of underlying asset, etc.

It is widely accepted that futures prices converge to cash prices of underlying assets on maturity. Alternatively speaking, the future cash prices might be obtained approximately through futures prices with an unbiased estimation. Actually the estimated values would be more accurate when they are closer to maturity. Mathematically the estimation reads

$$E(P_t^c) = P_t^f , \tag{2-1}$$

where P_t^c is the cash price of a underlying asset at time t, and P_t^f is its corresponding futures price at time t.

Additionally, given further information such as trading amount, traders can make optimal decisions, while price discovery provides a guidance of investment. For example, farmers can set an appropriate selling price of goods for next season by doing price discovery even before they enter into the futures markets. In particular, the latest 6 months prices information of grain would be useful for estimating and forecasting the future price of grain in the following year. The increase in price may secure more investment in the farms. On the other hand, speculators can collect costly information much cheaper and easier from the futures markets, while they may put themselves in risk otherwise (Leuthold et al., 1989). Real-time updated information also helps investors decide if to take short or long position. In general, prices discovery of underlying assets plays as a guideline for manufacturers, farmers, speculators and other traders in futures market.

In addition to the price discovery, most investors move into futures markets with motivations for reducing trading risks. Hedging is such a strategy to transfer or reduce unacceptable price risk, and its executer is called a hedger who trades futures in futures market to substitute for a cash market transaction (Kolb and Overdahl, 2007).

For instance, a farmer plants grain and expects to harvest in a half year time. If the farmer is concerned that the grain price will be lower than the current price at the time of harvest, he will probably suffer from the loss. Thus in order to minimize the potential for any loss, one can hold a short futures contract now and sells the futures contract at the harvest time to lock-in the price. In this transaction, the buying of futures contract is offset by the selling of futures, and no real grain is traded in futures market. The net income of trading in futures market compensates the price change of grain in cash market between the current time and the trading time at harvest. Alternatively speaking, the price fluctuation and uncertainty are decreased by hedging with futures and the farmer faces less price risk in cash market.

In addition, if no corresponding futures contact of such underlying asset exists in market, the farmer can use the so-called cross-hedging strategy. Generally speaking, one hedges the price risk by trading futures of other goods with the same or opposite price movement tendency in the future markets. A good example is cross hedging crude oil futures with a short position in natural gas. Feasibility of cross hedging cottonseed meal with futures of soybean meal was also demonstrated in Rahman and Turner's (2001) paper. Many firms get into futures markets to manage price risk when they buy/sell products or offer a service. With proper hedging strategies, the price risk can be reduced or transferred to others (Leuthold, Junkus et al., 1989).

2.3 Commodities and Agricultural Futures Markets

2.3.1 Introduction to Commodities and Agricultural Futures Markets

A commodity futures market acts as a market place where raw, primary[6] and other commodities, such as corn, wheat, oats, soybeans, and sugar, as well as crude oil, natural gas, live cattle, and pork bellies are sold or bought through a standardized futures contract. Wheat and corn, cattle and pigs were widely traded in the 19th century in the US through standard instruments.

At present, the Chicago Board of Trade (CBOT), the London Metals Exchange (LME), the Chicago Mercantile Exchange (CME), the New York Board of trade (NYBOT) and New York Mercantile Exchange (NYME) are main commodity futures exchanges which trade various kinds of raw commodities (Phlips, 1991). The CBOT is the most popular trading exchange for commodities where about 90% grain futures in the U.S. are traded, while the major commodities traded in CME are currency exchange and livestock. The NYBOT is main futures and cash markets for coffee, sugar, cotton and the NYME and CME trade copper, gold, oil and gas and the LME is for trading of non-ferrous base metals.

[6] Raw and primary commodities are in its natural state, without being processed, such as grain, iron, sliver, natural gas and other agricultural products, metals and energy products.

In 2007, the global volume of futures contracts of physical commodities and derivatives on commodities on exchanges traded reached 1,684 million, in which the agricultural contracts trading grew up by 32% from 2006, energy by 29% and industrial metals by 30%. However, the growth rate for precious metals trading during the same period was only 3%, because high trading volume in New York was partially offset by declining volume in Tokyo[7].

Same as that of the normal futures contracts, the trading price, quantity, and delivery terms at maturity have to be specified in commodities' futures contract. Additionally, the delivery grade of the commodity and the delivery terms must be stated in details, since the delivery date of commodity or agricultural goods highly depends on their natural characters that some commodities have seasonality and multi deliverable grades for exchange or delivery. For example, the best harvest for wheat is in July, September, December, the optimal delivery season in March and May, while the harvest time of Oats, barley and cotton is from July to September in the United States. After the harvest of Oat, barley and cotton, it is the time for harvesting soybeans during October through November. The U.S. corn crop has to be harvested between October 1st and November 15th (Labys and Granger 1970). The grade and its corresponding price must be stated in the futures contracts (Kroll and Shishko, 1972).

2.3.2 Storable and No-storable Commodities

Commodities are homogenous with similar nature that they have uncertain demand-supply in cash market and have at least limited storage capability (Chance, 1989). Based on these characters, commodities are divided into two categories, storable and non-storable. For example, the grains, metals, petroleum products are categorized as storable which can be stored for years, and sheep, goat, cattle and other livestock, eggs and treasury bills are non-storable whose storage is physically or economically infeasible (Richard J., 1969). However, cattle will not be kept for a long time because it is not worth feeding cattle while they are mature for sale. In general, commodities and services which are produced and consumed at the same time are technically referred as non-storable (Leuthold et al., 1989). However, most perishable fresh fruits and vegetables, such as

[7] Source from the IFSL (International Financial Services London) Research: Commodities Trading 2008. A report from Mount Lucas Management Corp.: The Mechanics of the Commodity Futures Markets.

apple, orange, are not allowed into to futures trading, while certain semi-perishable commodities (say eggs) meet this exceptions. (Kroll and Shishko, 1972).

The 'Livestock' is an important part of non-storable commodities in which the live cattle, lean hog, feeder cattle and frozen pork bellies via futures and options are widely traded in the CME Group.

Besides the effect of the demands for futures, their prices are greatly affected by the raising costs, government policies and international trading. Furthermore, price of forage and policies of livestock trade and worldwide business are other main factors that influence the futures prices of livestock (Chance, 1989).

Siegel and Siegel (1990) stated that the livestock in futures markets should always be at full carry since the livestock contracts differ from those of the grain whose underlying assets are transformable. However, since the livestock grows and changes over time, one cannot sell it which is deliverable today and hold it to deliver in the long future (Siegel and Siegel, 1990). Notwithstanding, the full-carry is only realized for the trader who needs physical products, and it not compatible in case of hedging, speculating or arbitraging.

Generally speaking, as discussed above, there are obvious differences between storable and non-storable goods. First of all, price basis of storable product tends to be stable as time goes by. The storable commodity futures markets is intend to stabilize the period-to-period fluctuations of commodity prices in both short run and long run, conditioning that storage speculators remain their risk attitudes and futures speculators are risk averse in the cash market (Sarris, 1984). Phlips (1991) described the price changes of storable and non-storable goods at peak time and off-peak time that the price basis depends on the cost of carrying or storing physical instrument over time and stably moves to zero at maturity. Since the storage cost for non-storable goods is infinitesimal, this basis cannot be expressed functionally for non-storable commodities. Though the basis has to be zero at delivery day in the delivery month, the cash and futures prices for non-storable products move relatively independently without maximum or minimum limit (Leuthold, Junkus et al., 1989).

Second of all, the supply can be estimated by approximating the demand for non-storable commodities; however, for storable goods, the supply-demand relationship is relatively not stable. Although Leuthold and Junkus (1989) stated that the production of livestock is hard to estimate due to the daily change of their weight and composition, and Schwager (1984) argued that because the inventory of non-storable is almost zero and thus their prices would not change dramatically.

Despite of the randomness of harvesting seasons for livestock, there is a yearly period of high production. For seasonal non-storable commodities, the entire amount of production is the seasonal amount.

Thirdly, one can buy the commodity grade that is deliverable right away and hold it to deliver in the future for storable goods; while for non-storable goods, they will be delivered according to the contract that expires in a certain short period of time. Also need to notice that even Treasury bills would literally perish as Leuthold et al. (1989) stated.

Because of the relative high volatility of price basis for livestock, the growers may wish to hedge the values of their livestock with a short inventory hedge. Growers who purchase feeder cattle or hog and sell mature cattle or hog may wish to hedge their margins. Livestock purchases (who have fixed contracts to sell the processed meat) may wish to hedge their costs with a long anticipatory hedge (Siegel and Siegel, 1990).

2.3.3 Transaction Costs

Though the low trading costs in futures markets is one of its advantages, one cannot simply ignore its effect on the markets. Transaction costs are the extra costs in an underlying exchange and the higher the transaction costs are, there would be less opportunities for traders to make profits. Beyond the cost of underlying asset in an exchange are the transaction costs, which consist of bid-ask spread, margins, short-selling costs, differential borrowing and lending rate, commission fees, delivery cost and information search costs[8].

Bid-ask Spread

The bid-ask spread is the difference between the bid price and the ask price. (Chance,

[8] The concepts of factors of transaction costs are mainly from books: Chance (1989, p280) and Siegel (1990, p63), see reference for detail.

1989). The bid-ask spread is not captured, observed, and reported, particularly in futures markets and it is also called a tick, the value of a minimum price fluctuation.

Margins and Short-selling Costs

Short-selling trader has to borrow assets or securities from a broker, and sell the assets/securities, and then buy them back to return to broker. In this process, the trader faces margins and short-selling costs.

Differential Borrowing and Lending Rate

Market participants pay rates for borrowing that are higher than those they receive for lending. Borrowing and lending transaction fee can be calculated through borrowing and lending rate.

Commission Fees

The total brokerage fee paid for buying and selling a futures contract is called the round-trip fee. No certain commission rate exists in futures market and it is negotiable between trader and clearinghouse. All traders incur a minimum charge including exchange fee paid to clearinghouse. For floor traders, the commission fee is $1 or $2 per contract.

Delivery Costs

In futures market, the party who holds a position to delivery is obligated to deliver the underlying asset, though the delivery possibility is slim. For storable commodity, such as wheat, the party who pays delivery fee also has responsibility to pay all warehousing costs[9] (Hull, 2008). For non-storable commodity, the delivery possibility is even smaller and delivery costs are more than that of storable commodity (Chance and Brooks, 2009).

Information Search Costs

Information search costs are fees to search and collect information for trading, such as the cost to do price discovery, and who has the lowest price, etc. Since this kind of cost is hard to statistically computed, it is always ignored in futures related research.

Empirically, inferential problems occur if we directly measure transaction costs. Locke and Venkatesh (1997) compare estimators of transaction costs of studies of Roll (1984) and Smith and Whaley (1994), and conclude that common time series estimators of

[9] When a party takes delivery of a commodity, he is seen to accept a warehouse receipt in return for immediate payment as default (Hull, 2008).

transaction costs are not suitable in futures market because assumptions of the estimators are not reasonable in real market.

All types of transaction costs tend to be stable except the bid-ask spread. Estimators of bid-ask spread have been investigated and employed as transaction costs proxy in financial and commodity markets. Alternatively, fixed transaction cost or a fixed rate of transaction cost plus a proportion of asset's value is considered as transaction costs. Fixed transaction costs, proportional to the value transacted and bid-ask spread proxy of transaction costs methods are widely used. In cocoa and coffee markets, it appears a positive relationship between spread and volatility (Myers and Thompson, 1989), even after controlling spread determinants (Bryant and Haigh, 2004). George and Longstaff (1993) find a negative relationship between transaction rates and bid–ask spread for the S&P 100 index options. There is an inverse relationship between trading volume and bid-ask spreads, after controlling other factors (Wang et al., 1997). If two market structures are similar in all respects, the market with lower transaction costs charge is more efficient (Locke and Venkatesh, 1997).

2.4 Hedging and Hedge Ratio

Risk management is one of the most important applications of futures contract, and hedging is a risk management technique to offset the price fluctuations. The hedging strategy is employed to augment the probability of success by lessening instability of prices and protecting against losses from high price fluctuation in futures markets (Koziol, 1990). In this section, we will introduce the function of hedging and hedge ratio in futures markets, particularly in commodities and agricultural futures markets.

2.4.1 Hedging

Hedging is a strategy using financial instruments, futures, options and other derivatives to neutralize the systematic risk[10] of price changes (Cusatis and Thomas, 2005) and has been an approach of risk management in futures markets. A trader takes a position on futures that is opposite to either an existed position or a future position to incur in the cash market, in order to close out futures position before maturity (Edwards and Ma, 1992). Thus the net position in futures market is zero and net gain or loss from futures

[10] Systematic risk is a risk of security that cannot be reduced through diversification.

market can partly or fully offset the net loss or gain from cash markets. In other words, gain or loss due to price fluctuation in cash market is offset by the gain or loss from futures markets. The price risk is minimized when the quantity of contract in futures position for hedging is calculated from an OHR.

Hedging protects investment from high price risk. For instance, when an investment on a commodity is made, the investor can take a futures position on this commodity, and close out the futures position with an opposite futures position; hence the gain from futures market at least partially offsets the loss of the investment in cash market. This hedging strategy reduces the price risk resulting from the uncertainty in futures. It locks the prices, and the price risk in cash market is reduced or transferred to speculators who are willing to bear the risks. Hedging is considered as 'insurance' sometimes which protects people against negative events, but it cannot stop negative events.

The primary objective of hedging is to minimize price risk and to protect trader's profit. Due to various unpredictable and uncontrolled factors, managers are always looking for a tool to reduce, control or transfer risks. The futures markets provide them the tool 'hedging' to reduce or transfer risk. However, edging deals with the price risk, not quantity risk. For financial goods, hedge-able risks are interest rate risk, equity risk, securities lending risk, credit risk, currency risk; for commodities' and agricultural products, the only hedge-able risk is price risk. By temporarily offsetting a position on one product with an economically opposite position on the same or related product, price risk of the former is reduced or transferred (DeCovny and Tacchi, 1991). In the risk transformation process, both speculators and Exchanges play important roles. In an efficient exchange with viable hedging mechanism, speculators invest capitals by taking the risk which is transferred from hedgers, and they enhance the liquidity of the exchange (Koziol 1990, p14).

The hedging process can be divided into two main steps[11]. First of all, a hedger enters into futures market, and there is a broker or a brokerage firm who has exchange membership to help the hedger to trade in futures market. Broker will open an account for the hedger and ask for a deposit of margins to put into this new opened account, and

[11] Source from: Short Hedge Example with Futures, Joe Parcell and Vern Pierce, Department of Agricultural Economics, University of Missouri-Columbia.

then place a futures position in an exchange according to the hedger's own situation and aim. If the hedger has or will have a long position in cash market in future, correspondingly short futures should be taken. In this process, at least one of other traders takes a long position or a speculator takes unfavorable risk. If the trader is willing to hedge a short position in spot market, he/she holds long futures.

The second step is selling/buying some underlying asset in cash market and closing out the existed position in futures market. The broker will keep the hedger informed with the latest news and will help him to choose the appropriate time to make hedge and offset futures position. By buying/selling the initial futures contracts with the same transaction terms, the futures positions are cleared out. The hedging process is done.

For agricultural commodities, production uncertainty, increase/decrease in demand, international production and some other unpredictable factors make the prices of commodities fluctuate widely and capriciously. Agricultural producers can use the commodity futures markets to hedge the potential loss from price volatility of commodity. For example, a farmer who plants corn expects a decreasing price of corn at harvest in six months. To reduce the potential loss, the farmer shorts futures against the future position in cash market. During the half year, the farmer will buy the futures contract back, and the gain from futures trading would offset the potential loss of selling corn in cash market. Hedging can lock in the price at which they will be able to sell in the future and hence to reduce the price volatility. However, at the same time as reducing price risk, the farmer gives up the chance of reaping more revenue from a sudden price increase of corn. This is a payment of risk reduction and also the main flaw of hedging (Edwards and Ma 1992).

For a firm, the greater the growth opportunities of the profit, the more it will depend on external finance in the absence of hedging. Hedging is commonly desirable because firms can lower their cost of capital by hedging risk. As Duffie (1989) says, hedging has low transaction costs, low default risk, and it is more convenient to hedge and make adjustment in futures market. Any decrease in a firm's cost of capital from hedging will be exactly offset by a decrease in its expected cash flows (Siegel and Siegel, 1990).

2.4.2 Hedging Strategy

Hedgers trade using 'hedge strategies' to protect against losses and unexpected risk of unfavourable price fluctuations. Long hedge, short hedge and perfect hedge are main hedge strategies.

A long futures hedge is appropriate when you buy or will buy an underlying asset in spot market and want to lock in the price (Hull, 2008). The long hedger puts a long position in futures market, and shorts the futures contract to clear out futures position during the life of contract in order to offset the expected or potential risk. The net gain or loss offsets cash price fluctuation and locks it in regardless of movement of the cash price.

The short hedge process is similar. Taking short hedge is reasonable when you are willing to sell an underlying asset in spot market in future and want to lock in the price. The short hedger holds a short futures contract, and buys the futures contract back during the life of contract to clear futures position. The selling price in cash market is somehow set in advance, and the effect of cash price fluctuation on the net gain or loss for trader is reduced by hedging.

The so called 'perfect hedge' is a special hedge situation in which all risks are completely eliminated. The ultimate objective of hedge is to achieve this perfect situation. However, it is impossible in real life since loads of uncontrollable factors lead to price risk which cannot be eliminated completely with a hedging strategy. Generally speaking, the perfect risk-elimination is not realistic in any active derivatives' market.

For most studies, the hedging strategies would be used with several assumptions. Firstly, all observations have to be independent. Secondly, changes of futures and cash prices are (log) normally or student's t distributed. Thirdly, prices are not sensitive to up, down, or neutral market environments, and would not change due to market environments. Fourthly, outlier occurrences are extremely remote and independent of prices fluctuations. Fifthly, market changes are predominantly small and continuous. Relatively stable market condition is important. Sixthly, low variability of basis and price volatility are required, because high basis and price volatility result in high price risks which make the market unstable and more risky for trader. The final assumption is that the sample size has to be large enough to analyze statistically (Koziol 1990). Observation independence, normality property of prices changes and large size assumptions make the statistical analysis easier. Stability of markets guarantees that the trading is smoothly running (Koziol 1990, p37).

Basis

Instead of gaining benefits, hedgers are aiming to reduce or transfer unfavorable risks by converting price-level risk into basis-level risk which is smaller and more stable. Empirically, basis is defined as difference between cash price and futures price and the basis-level risk is formulated as

$$B = (S_1 - S_0) + (F_0 - F_1) = (S_1 - F_1) - (S_0 - F_0)$$
$$= B_1 - B_0 \qquad (2\text{-}2)$$

where B is basis risk, S_0, S_1 are cash prices and F_0, F_1 are corresponding futures prices at time 0 and 1. Obviously, the basis change is much smaller than price-level risk because the effective price of the spot commodity being hedged is much more predictable than if no hedge had been created. By transforming risk, the hedgers now have tighter boundaries of basis-level risk. Though the basis risk cannot be eliminated, it is lower than price-level risk. Hence the hedged position benefits trader more than the unhedged position (Leuthold, Junkus et al., 1989, Chance, 1989).

Basis becomes zero at maturity but the basis can be either positive or negative prior to maturity, thereby creating some opportunities for hedger to earn a small profit during the life of the hedge as the basis fluctuates. If the basis constantly moves after putting a hedge, the price movements of cash and futures are in the same direction, and this hedge is considered as effective hedge (Stoll and Whaley, 1993).

The price of deferred futures contract is price of futures contract farther away from maturity than the nearby futures contract, and it can be used to compute its basis. When you are willing to compute basis for corn in October, you can choose the futures price in July instead of the nearby contract in December. Change of basis is far less volatile than change of spot prices since the futures price will tend to converge to the spot price of the commodity, the hedged position is less risky than the unhedged position (Chance, 1989).

For livestock, basis is the difference between local supply and demand in a local location[12]. When calculating basis of livestock, the futures price is obtained from the nearby futures

[12] Source from: An Introduction to Basis, Joe Parcell and Vern Pierce, Electronic Bulletin Board (AGEBB) of University of Missouri.

contract since livestock cannot be stored to the maturity of futures contract[13].

Basis risk is the variance of the basis in mathematics. The higher the correlation between cash and futures prices, the less the basis risk is. The basis movement is a sign to show who can get profit in hedge because its widening or narrowing results in gains or losses. The updating basis includes much information, such as size of cash and futures position, the gain or loss from cash and futures position before and after hedge, amount of margins, consistency of basis movement, and new information leading to hedge adjustment. In other words, by monitoring the change in the basis, hedgers can know the effective financial value of hedged position any minute. Hence, the basis plays an important role in hedging process (Edwards and Ma 1992).

Transforming price-level risk into the more manageable basis-level risk, the transformation establishes a criterion for hedge or not-to-hedge decision: when the dollar risk for the basis is less than the dollar risk for a no-hedge, the hedge should be established (Koziol 1990).

Dynamic Hedging

The price of underlying asset in cash market is not static and it moves nonlinearly with its futures price. As the price of underlying asset changes, the size of futures position for hedging should be readjusted to fit the original optimal hedging strategy, otherwise the hedge position is not optimal anymore. In this case, dynamic hedging is more appropriate to be employed. A dynamic hedge is a more complicated hedge strategy that the hedger continuously adjusts the size of futures position before the expiry date. By readjusting the hedged futures position, the hedger can compensate the loss which is because of price changes in cash market. The adjusting process is continuous until the futures expires (Duffie, 1989).

Hedging is meant to reduce price risks, and it would be terminated once the risk is perceived. However, as long as price of underlying asset moves during the period before the spot commitment date, the risk is still existed, and the hedge needs to be constantly readjusted in order to minimize this price risk. Put it in another way, the broker readjusts

[13] Source from: 'livestock basis' of Cooperative Extension Work in Agriculture and Home Economics, Acts of Congress of May 8, 1914, as amended, and June 30, 1914, in cooperation with the United States Department of Agriculture. Texas Agricultural Extension Service, the Texas A&M University System. 1M, New

the hedge position according to the price change of asset. The dynamic hedge instantly locks in the price in spot market. When the cash price suddenly jumps, a rehedge is necessary to avoid substantial loss; otherwise it would be a disaster.

In a static hedge, the hedge sets a certain decision horizon, such as five months, and will liquidate both cash and futures positions at the end of the horizon. This model gets criticism that hedgers have no chance to adjust the hedge position when they receive new information or cash price changes. However, in the case of dynamic hedge, when the updating new information shows that the originally hedge position is no longer optimal, the hedger can readjust the hedge position, and even revise cash position before the maturity to keep the hedger holding a profitable position.

For example, a farmer harvests corn in December and is willing to sell it with a satisfying profit in six months horizon. The farmer receives some new information, such as change of demand and supply, change of cash price change after placing a hedge position in December. Thus the farmer tends to adjust the prior hedge position and to rebalance the selling amount and quantity for storage of corn. If a static hedge is taken, the hedge position cannot be changed till maturity, even though there are some better chances to gain before the end of June. However, if the farmer takes a dynamic hedge, he would not miss an opportunity to get more returns by readjusting his hedge and cash positions before June. Dynamic hedge is more appropriate the dynamic nature of trading in futures markets and provides more chances to reduce price risk and gain more during the time of placing the hedge and the time when it is liquidated(Lence et al., 1993).

Factors effecting Hedging

Many factors affect hedging programs. Some external factors are out of control of hedger, such as market condition; policy changes and transaction costs (Koziol 1990). However, some other internal factors are under control of a hedger, for instance, the hedger determinates what when and how much to hedge and which hedge method to use.

The market has to be relatively stable and efficient. In a stable market, trading is smooth running and market changes are predominantly small and continuous. With a good market environment, basis moves consistently and converges to zero at maturity.

Transaction costs are considered as one of the major impediments of dynamic hedging

(Albanese and Tompaidis, 2008). As transaction costs increase, the dynamic hedging is degraded and the hedging error becomes larger than that of hedging with lower transaction costs. As a part of transaction costs, margin requirement of hedge is smaller than that of speculation, and therefore margin is not a major effecting factor in hedging. The effect of marking-to-market and the margin calls are two important factors. Closing out futures position is a way to not meet a margin call (Kolb and Overdahl, 2007).

Commission fee in futures market is the extra fee paid to brokerage for buying and selling a future contract. Different exchange charges different rate of commission fee. The low commission fee is relatively stable and has slim effect on hedging.

Delivery costs are fees of storage and transportation of commodity. The costs might be substantially high for some commodities, especially for livestock with extra costs on feeding and taking care of them. However, it is not considered as major part of transaction costs since the delivery possibility is extremely low.

The bid-ask spread takes a big portion in transaction costs and its estimator is seen as proxy of transaction costs in some empirical studies. From another perspective, the bid-ask spread is also the cost of guaranteeing future market's liquidity (Chance, 1989).

Moschini and Myers (2002) mention that in the premise of necessary convergence of futures price and cash price in the delivery month, hedging can be used to offset the change of cash prices by the price change of futures position. The convergence will not happen, if arbitrage transaction costs are high (Siegel and Siegel, 1990). In other words, high arbitrage transaction costs make hedging less useful.

Though those external factors have effect on hedging, policies would not be changed quite differently in an efficient and stable market. If the hedgers are able to control the internal factors well, the hedging can be a functional tool for risk management.

2.4.3 Hedge Ratios

Hedge ratio is the number of futures contracts that used to hedge a particular exposure with a unit in the spot market (Chance, 1989). When the quantity of underlying asset is known in cash market, the size of futures position can be calculated out with this ratio.

2.4.3.1 Optimal Hedge Ratio (OHR)

During a hedge process, the hedger determines the underlying asset, the futures hedge position and decides the number of futures contracts which would be used for hedging; and then the hedger can choose the right time to hedge according to basis movement (Duffie, 1989, p204). How to determine the amount of futures position in hedge? The 'Hedge ratio' concept is employed to represent the proportion of the quantity of futures contracts in the size of the entire cash position of commodity. As long as the hedge ratio is known, the size of futures position can be settled. Owing to its importance on hedging, this rising financial production attracts discussions and debates from deriving hedge ratio to its forecasts. In this section, we introduce hedge ratio and its derivation, estimation, forecast, application and usage in commodities futures markets.

2.4.3.2 Hedge ratio and its derivation

To minimize price risk, the hedger must take futures position to maximize the possibility of reducing the price fluctuation of the commodity in cash market (Edwards and Ma 1992). There is a slim possibility to hedge if we can not estimate the value of hedge ratio, since hedgers cannot certain the number of futures contract (Chance, 1989). Exact estimation and forecast of hedge ratio helps investor to minimize basis risk and apply appropriate hedging strategies and technique, such as transaction exposure, economic exposure, and translation exposure. Hence, the estimation and prediction of Optimal Hedge Ratio (OHR) becomes particular important.

Originally, hedge ratio is represented as: Hedge ratio=Quantity of futures position/Quantity of cash position

$$h = {}^{Q_f}\!/_{Q_c} \tag{2-3}$$

where Q_f is quantity of futures for hedging, Q_c is quantity of underlying asset to be hedged in spot market and assumed to be fixed. $Q_f = Q_c * h$, once we get the hedge ratio, we can work out the number of futures contracts needed.

There are many ways to estimate hedge ratio. With different objective functions, optimal hedge ratio is derived diversely. The variance-minimizing (MV), mean-variance, maximizing expected utility, mean extended-GINI coefficient (MEG) and generalized semi-variance (GSV) approaches are employed based on their portfolios. The MV method proposed by Johnson (1960) aims to minimize the portfolio risk which is considered as the variance of the hedged portfolio. However, the MV method gets criticized because it ignores expected return as a factor in portfolio. The Mean-variance strategy was proposed to take expected return and risk (variance) into the hedged portfolio. Even though it avoids disadvantages of MV, it is consistent with the expected utility maximization method which requires a quadratic utility function and jointly normal distributed return. MEG and GSV have been proposed to obtain hedge ratios that are consistent with the concept of stochastic dominance. MEG does not require some specific assumptions about utility functions and distribution of return.

The MV method is most widely used because its hedge ratio is easy to understand and estimate with econometric methods, though its reliability of MV method is suspected. All these methods based hedge ratios would be the same as MV hedge ratio, if futures and spot returns are jointly normally distributed and/or if the futures price follows a pure martingale process, except for MEG and minimum-GSV (Chen et al., 2003).

For each approach, it is necessary to build an appropriate portfolio which combines investments in cash and futures markets to reduce price fluctuation and minimize the risk (Carter, 1999). The hedged portfolio consists at least a long or short position in spot market and an opposite position in futures market to obtain a less-risk-return.

Based on the fact that movements of spot and future prices and are not parallel, the easiest way to measure their relationship and construct a portfolio properly is to run a 'linear regression'.

$$P_c = a + bP_f + e \tag{2-4}$$

where P_c is the price of the asset in spot market, P_f is the futures price of underlying asset. The 'price-level regression' was launched to minimise the potential risk for hedgers according to 'portfolio theory of hedging' with a hedge ratio of

$$b = \frac{\Delta P_c}{\Delta P_f} \tag{2-5}$$

The spot price moves b times of the movement of futures price. When $b = 1$, the spot price and futures price move exactly the same, the same direction and proportions. The net value from futures positions completely offset the price change in spot market. This one-for-one correspondence situation is called ‘perfect hedge’. And it is also said to be ‘naïve’ hedge because the futures and spot price may not fluctuate in the same proportions. The ‘perfect hedge’ situation is not possibly happening as an ‘eliminate a business risk’ strategy in practice, especially for cross hedging (Chance, 1989, Siegel and Siegel, 1990).

The price change ($\Delta P_c, \Delta P_f$), percentage change ($\Delta P_c / P_c$, $\Delta P_f / P_f$) and log difference ($\log P_c^2 - \log P_c^1$, $\log P_f^2 - \log P_f^1$) [14] level regressions are suggested to estimate OHR in various fields. There is a debate about the appropriateness of price level regression. Siegel and Siegel (1990) find that the price level only accounts for the hedging motives, not includes speculative motives. From perspective of statistics, the price level regression shows high correlation between futures and cash prices, but not accounts for the correlation between spot and futures price changes, which violates OLS assumption. Furthermore, most time series data has significant degrees of autocorrelation in the residuals, and has heteroscedasticity problem (Brown, 1985). The price level does not fit real time series. Myer (1989) generalizes an approach to estimate the OHR with simple regression of price change level, and point out that price-level regression is inappropriate, but price-change-level provides good estimates for corn, soybean and wheat in Michigan from July 1977 to July 1985. Witt, Schroeder, and Hayenga (1987) defend for price level model. They claim that the price level regression is sound and appropriate for anticipatory hedges, and there is no clear evidence to show that price change and price percentage change regressions are better than price-level change. Later on, Siegel and Siegel (1991) generalize that price percentage change regression is proper for financial futures, and price-change regression is appropriate for commodities futures. Recently, most researchers (Baillie and Myers, 1991 , Kroner and Sultan, 1993, Choudhry, 2003) prefer ‘log difference’ regression than others, since the non-linear regression fits the relationship between cash and futures prices better.

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[14] $\Delta P_c = P_c^2 - P_c^1$, $\Delta P_f = P_f^2 - P_f^1$ where P_c^1, P_c^2, are cash prices at the time the hedge is placed and lifted respectively; P_f^1, P_f^2, are futures prices at the time the hedge is placed and lifted respectively

For MV method, we choose 'log-difference' regression to derive hedge ratio. Since the return of the hedged portfolio is indicated as risk[15] that a hedger faces (Cecchetti et al., 1988). By minimising the variance of the return portfolio, the price risk is minimised. The return of the hedged portfolio is given by:

$$r_t = r_c^t - \beta_{t-1} r_f^t \qquad (2\text{-}6)$$

Where β_{t-1} is hedge ratio, r_c^t and r_f^t are $\log P_c^2 - \log P_c^1$ and $\log P_f^2 - \log P_f^1$ respectively, and they represent the corresponding returns from cash market and futures market between time $t-1$ to t.

The expectation of the return portfolio

$$E(r)_t = E(r_c^t) - \beta_{t-1} E(r_f^t) \qquad (2\text{-}7)$$

It shows that basis is not affected by the hedge ratio.

The variance of the return on the hedged portfolio is stated as:

$$\operatorname{var}(r_t) = \operatorname{var}(r_c^t) + \beta_{t-1}^2 \operatorname{var}(r_f^t) - 2\beta_{t-1} \operatorname{cov}(r_c^t, r_f^t), \qquad (2\text{-}8)$$

where $\operatorname{cov}(r_c^t, r_f^t)$ is correlation between futures and cash price. We derive 'return equation' with respect to β_{t-1} and set it as zero. By minimising variance, the hedge ratio is produced as:

$$\beta_{t-1} = \operatorname{cov}(r_c^t, r_f^t) \big/ \operatorname{var}(r_f^t) \qquad (2\text{-}9)$$

The optimal hedge ratio depends on log return of futures price and the correlation between log returns in cash and futures markets. The OHR is always less than 1 because of the higher volatility of futures relative to cash price (Choudhry, 2003).

2.4.3.3 Forecasting of Optimal Hedge Ratio

The theory that hedge ratio is time dependent is widely accepted in empirical research (Bollerslev, 1986, Cecchetti, 1988, Baillie and Myers, 1991, Kroner and Sultan, 1993 and etc.). As one of the main theoretical issues in hedging, forecasting of time-varying optimal hedge ratio (OHR) attracts great attention of academicians and practitioners. As mentioned before, the main hedgers in futures market are producers, buyers and users of the goods, instrument or whose prices are linked with prices on the goods or instrument traded in this market. How can they benefits from forecasting of hedge ratio?

[15] Here the 'risk' means price risk, since hedge only can reduce or transfer price risks.

In this section, we will discuss the potential benefit of forecasting hedge ratio for different hedgers in futures market, especially in commodities' and agricultural futures markets, and some conceivable negative influences on them.

One of the most important implications of forecasting is in planning or decision-making of investment. The forecasting of OHR helps hedger choose appropriate portfolio and allows for portfolio adjustment in dynamic hedging.

Forecasts of hedge ratio provide a reference for decision-maker who has desire to make most profitable decisions with low systematic risk. Once hedge ratio is predicted, the size of futures contracts which is used to hedge is easy to get from $Q_f = Q_c * h$. The investor can determine how much capital should be put into this investment in an optimal way. Higher the hedge ratio, the greater the risk of the price to be hedged, because more futures contracts are needed to hedge the greater risk. As we mentioned before, hedge ratio is normally less than unity. In an extreme case, if the forecasted hedge ratio is more than 1, is it worth paying one time, double, triple or even more times amount of fund of the investment to reduce systematic risk? The answer would be 'no' for most investor, alternatively, they might consider choosing other asset's futures to cross hedge and reconstruct the investment portfolio with much less fund. More generally speaking, given that hedge ratios of various portfolios are predictable, an investor always prefers a portfolio with lower financial capital to reach the maximum of risk reduction. The forecasts of optimal hedge ratio helps investor choose optimal portfolio with suitable futures and reasonable number of futures contract.

Given a specific horizon, the best time to hedge can be worked out. For the investor, the timing of investment is one of crucial elements of success. If the optimal hedge time is at the beginning of the time horizon, the investor has to prepare capital at or even before taking the first position in spot market. If the optimal time is at the very end of the horizon, the investor has no rush to allocate fund before the hedge. In other words, the investor can arrange and distribute capital to make other investments before the hedge instead of putting all money needed at the right beginning of the investment. Apparently, the prediction of hedge ratio makes capital more flexible.

During the period that the hedge is lifted to the maturity, the situation would be different when new information is announced, and hence there is a high possibility that the original hedge is no longer optimal since then. The time-varying hedge ratio enables the investor to adjust and readjust investment decisions again and again to keep hedging strategy optimal. According to the updating prediction of hedge ratio, the readjustment is continuous to reflect the changing situation. If the forecast is fixed and not adjusted, this guidance is a misleading for investor, and the effectiveness of hedging would be low (Makridakis and Wheelwright, 1989).

In commodities' and agricultural futures markets, most hedgers are producer, manufacture, merchants and dealers. They wish to reduce risk by stabilizing price change, adjusting commodities inventory to demand (Chance, 1989). For an oil wholesaler who holds a substantial inventory of gasoline, the wholesaler needs the inventory as a stock from which to service retail customers. If the wholesaler simply holds the stock of gasoline, she can sell crude oil futures as a substitute for selling the gasoline itself. By holding gasoline in her business inventory and selling crude oil futures to offset the risk associated with the gasoline. In this cross hedge, accurately predict optimal hedge ratio, the wholesaler can reduce her business risk and have the gasoline inventory that is essential to her entire business. Selling futures substitutes for the risk-reducing transaction of selling her entire inventory[16] .

Forecasts of hedge ratio have great applications in futures market as described above; nevertheless, there are also some flaws on its prediction which is easily effected by many events. The first important effect involves valid input data that is necessary for forecasts. Incorrect, imprecise and insufficient raw data and processed data would lead an inaccurate forecasting. When analyst collects data, he/she has to make sure that it is a valid data source and all raw data are correct and accurate; when analyst deals with raw data, he/she must guarantee that the method he/she uses is reasonable and reliable (Clements and Hendry, 1998).

Choosing appropriate forecasting methods is an important issue since models have different forecasting ability on different asset in different area and no superiority is suggested for any particular forecasting model in predicting hedge ratio so far.

[16] This example is quoted from: Robert W. Kolb, James A. Overdahl (2007), Futures, options, and swaps, the 5th edition.

On the other hand, the reasonable assumptions are essential for forecasting. For example, the relationship between futures and cash prices for most raw materials is not assured, and if this presupposition is not tenable, forecasts of hedge ratio is not reliable.

Time horizon influences hedge ratio forecasts. Different hedging horizon might affect forecasting accuracy for various forecasting methods(Chen et al., 2004). The longer the forecasting horizon, the more data are included and the more accurate the forecasts is. Nevertheless, because of forecasting ability of statistical method, the accuracy might decreases as more data being involved. Furthermore, the market environment might change or unexpected events happen which occurs assumptions less reasonable in a long time horizon (Clements and Hendry, 1998). Some activities of competitors will affect forecasting accuracy as well. The more competition in the market, the more difficult it is to forecast hedge ratio. In the market that has great competition, competitors can change the course of future events after they make forecast. In order to make themselves more competitive, which makes forecasts invalid (Makridakis and Wheelwright, 1989).

Forecasting has limitations on its accuracy, but we cannot deny the significance of forecast and neglect its merits with this shadow. In the active derivatives' market, decision-making depends on the quality of the forecasts, and hence forecast of hedge ratio is important and meaningful for hedgers (Chisholm and Whitaker, 1971).

As Waddell and Sohal.(1994) say:

'Explicit systematic forecasting approaches can provide substantial benefits when used properly, but it is illusory to believe that omniscient powers are plumbed by such approaches.'

This thesis will test and evaluate the forecasting accuracy of six different GARCH models on predicting hedge ratio with both normal and student's *t* distributions in agricultural and commodities' futures markets. The forecasting performance of symmetric and asymmetric GARCH models on hedge ratio for both storable and non-storable agricultural commodities will be analysed in detail. A comprehensive result will be shown in order to provide an overall cognition of forecasting ability of GARCH models in agriculture area.

3 Literature Review

3.1 Introduction

In this chapter, we review some contributive articles about hedge ratio that covers debate on derivation of hedge ratio and hedge ratio estimation, and important studies on hedge ratio forecasting. The main body of this chapter is divided into three parts. The derivation of hedge ratio is the first part; estimation and forecasting of hedge ratio respectively take the second and third parts. In the part of hedge ratio forecasting, we cover several pertinent articles of forecasting models on volatility due to limited source of prediction of hedge ratio.

Hedge ratio is derived using diversified methods in accordance with different objectives of hedging. The MV (minimum variance) method is the popular one among all approaches which include the MV, Mean-variance, Sharpe, MEG (mean extended Gini), M-MEG (mean-MEG), GSV (generalized semi-variance) and M-GSV (mean-generalized semi-variance). Every approach has its advantage and disadvantage. Numerous scholars approve the superiority of the mean-variance framework over MV since the mean-variance calculates expected return which is ignored by MV method. Nevertheless, in the computation of Mean-variance hedge ratio, the individual's risk aversion parameter has to be known, and this factor would be different for different person. In the case of infinitely risk averse or 0 of the expected return of futures, the mean-variance method is line with the MV approach. Other methods are somehow consistent with either MV or mean-variance approach. In the first part, the 6 methods will be described in detail.

Referring to the estimation of hedge ratio based on MV method, the OLS, ECM (error correction model), CI (cointegration) and GARCH family models are far-ranging dominating estimation methods. The estimation ability of GARCH family models gains coherent affirmation in financial market since the ARCH has been proposed. However, for different assets in different areas, an overall best-of-all model has not been found so far. In this part, some empirical studies about the debate of superiority of various models on estimating hedge ratio are reviewed.

The third part is comparison of hedge ratio forecasting. As the extension of the second part, the forecasting ability of GARCH models takes a large body of this part. The forecasting of volatility is involved in for reference since volatility prediction is much more discussed by scholars than hedge ratio forecasting. Volatility and hedge ratio has similarity in financial and commodity's markets, and forecasting models of these two time series are highly overlapped. For instance, the OLS, Kalman filter, GARCHs models are widely used on both time series. The comparison of predictive ability of various forecasting models is always a flourishing topic academically and practically.

3.2 Derivation of Hedge Ratio

According to the different definition of concept of hedging, the objective of hedging is either risk minimising or profit maximising (mean-variance). Based on these two different targets, the hedge ratio is derived with various methods. The MV, GSV and methods are categorized into risk minimising hedge ratio, in which the risk of hedged position is measured and minimised; the optimum mean variance, M-MEG, M-GSV belong to mean-variance category that takes both risk and return into account and maximising profit is ultimate goal (Chen et al., 2001).

The MV method is widely used for its financial meaning and numerical simplicity. The MV gets criticised because of ignoring expected return. Nevertheless, the mean-variance framework compensates the drawback of MV. In mean-variance method, computation of risk aversion takes more complicity since risk aversion depends on individual. The Sharpe hedge ratio is a highly non-linear function and it is improper if the second derivative of the Sharpe ratio in terms of the hedge ratio is positive (Chen et al., 2001). The maximum expected utility method is consistent with mean-variance framework. It requires a quadratic utility function or normal distributed joint return to fit the principle of expected utility maximization. The MEG and M-MEG are the same when the future price follows a martingale process. And the MEG converges to MV if the joint distribution of returns follows normal distribution. Allowing for the 'martingale' condition, the M-GSV converges to GSV technique. Comparably, Mean-variance, Sharpe, MEG, M-MEG, GSV and M-GSV methods are more advanced than MV from different aspects to some extent. But from another perspective, the above five methods are more complicated than MV either statistically or theoretically. The versatility of estimation method of MV approach also enhances the application of this method.

Holding the aim of minimising risk, Johnson (1960) proposed the MV method. The MV hedge ratio is obtained by minimising the risk that is the variance of price change of the hedged portfolio. Myers and Thompson (1989) pointed out that the price change regression is appropriate when returns follow a random walk, while price level and return level are inappropriate because of the possibility of arbitrage opportunity and inconsistency. In practice, all three level regressions lead to large errors. More recently, some scholars use 'log difference' of cash and futures price to better model their non-stationality (see Bailled and Myers (1991), Choudhry (2003, 2004, 2009)) .

The variance of the return on the hedged portfolio is as follows:

$$\mathrm{var}(r_t) = \mathrm{var}(r_c^t) + \beta_{t-1}^2 \mathrm{var}(r_f^t) - 2\beta_{t-1} \mathrm{cov}(r_c^t, r_f^t) , \tag{3-1}$$

where $\mathrm{cov}(r_c^t, r_f^t)$ is correlation between log-returns in futures and cash markets. By minimising variance of the portfolio in equation (3-1), the risk is minimised and the optimal hedge ratio is obtained:

$$\beta_{t-1} = \mathrm{cov}(r_c^t, r_f^t) \Big/ \mathrm{var}(r_f^t) \tag{3-2}$$

The measurements of risk vary depending upon the way they construct the regression, for example, the percentage change regression was employed in Johnson (1960), which is different from the return regression. A debate about appropriateness of returns, price change, percentage change, and logarithm of price change regression turns out that the suitability varies relying on the types of product.

The reliability of MV method is boosted since Engle (1982) initially proposed ARCH model. As one of estimation methods of MV, the expanding use of the GARCH family models definitely enhances the application of MV method in wider area. The more properties of time series are captured by GARCH-type models, the better the behaviour of time series observations are explained.

Cheung, Kwan and Yip (1990) proposed mean GINI approach as an alternative risk measure. This approach is consistent with stochastic dominance and eliminates assumptions of utility function and normal distribution of return that are restrictions of mean-variance method. Kolb and Okunev (1992) developed the extended mean-GINI model (MEG). By minimizing the MEG coefficient, the mean-variance based risk is minimized. In this article, they use the daily prices of gold, corn, copper, German mark and S&P 500 during1989 as sample. The MEG hedge ratios increase as risk aversion increases for gold, corn and S&P 500, yet adverse results for remaining goods, provided the risk aversion parameter from 2 to 200. When the risk aversion parameter is between 2 and 5, the MEG hedge ratio is close to the minimum variance hedge ratio. The higher the risk level, the more stable the hedge ratio.

Shalit (1995) tried to relate the MEG approach to the MV method by estimating MEG using the instrumental variable method and comparing them empirically and practically. As shown in Shalit (1995), the MEG approach incorporates the dispersion of risk-aversion which might attract great attentions from practitioners. Shalit (1995) stated that, when the prices of futures and cash are jointly normally distributed, the MEG and MV methods are the same; but when the normal distribution is not hold, they are significantly different. Sampled on all monthly futures contracts for metals (gold, silver, copper and aluminum) traded on the New York's Commodity Exchange (COMEX) between Jan 1977 and Dec 1990, normality hypothesis of the prices of metal contracts is rejected, and the MEG hedge ratio is said to be more benefitable than MV hedge ratio because the OLS estimation leads to inconsistent estimator for MV approach given non-normality of prices.

As Chen (2001) said, the GSV can be considered as an extension of risk minimising method. The risk is defined as real return below the target return. Some empirical articles (Fishburn, 1977, etc.) proved that the GSV is consistent with stochastic dominance. If the joint distribution of cash and futures prices is normal or the futures prices follow a pure martingale process, the minimum GSV converges to MV approach.

Jong and Roon (1997) investigated the hedging effectiveness of three hedging strategies and models with out-of-sample tests on three currencies (U.S. dollar futures on the British pound, German mark, and Japanese yen) in futures markets. Based on MV method, the GSV that was proposed by Fishburn (1977) and Sharpe-ratio models (Howard and D'Antonio, 1984), the out-of-sample hedging effectiveness was perceived using naïve hedge, model-based dynamic hedging and long-term average concerned hedge, respectively. This paper used a non-overlapping 90-day sample in which hedge ratios were estimated using the first 60-days sample and the daily post-sample hedging effectiveness of the remaining 30 days are computed. In terms of hedging effectiveness, the minimum-variance model and the GSV models are effective, but Sharpe-ratio model is not because it decreases the utility, along with worsening trade-off. For the two effective models, Naïve hedges provide best hedging in model-based hedges, constant hedge ratio (long-term average of the model based hedge ratio) and unhedged position; for Sharpe-ratio model, utility of hedger is not increased on currency futures, and any hedges are not more useful than unhedged position.

Cecchetti, Cumby *et al*. (1988) claimed that the minimizing risk is not all objective of hedging, and the expected return should be another main purpose of hedging. As an improvement of MV method, the OHR was derived by bivariate ARCH model from the expected utility function which depends on both risk and expected return. By implementing to Treasury Bonds during the period of October 1977 to May 1986, the performance of utility maximizing hedge is preferable to MV hedge for both in-sample and out-of-sample tests.

Lence (1995, 1996) discussed the relaxing of restrictions on minimizing variance hedge (MVH). The findings showed that no better MVH is available and the MVH is not optimal without its usual restrictions in real life. In the two literatures, Lence proposed expected utility maximizing hedge ratio in which the terminal wealth depends on the return of a diversified portfolio. Particularly, the production of a commodity, investment in a risk-free asset, investment in a risky asset, borrowing and the transaction costs are all considered and tested in this portfolio. Overall, the production and amount of initial wealth of investor are deterministic factors effecting hedging behaviour. The results well explained the little use of hedges for farmer and high hedging possibility for some firms. The more practical utility maximising approach better explains the behaviour of investor.

The mean-variance becomes another popular theory of hedging. The optimum mean variance, M-MEG and M-GSV are consistent with mean-variance framework. Hsln and Kuo et al. (1994) incorporated the return into objective function and construct a mean-variance framework to compare hedging effectiveness in currency exchange futures and options markets. Using daily data of exchange rate among British pound, German mark British pound, German mark, Yen, Swiss franc during time period from1986 to 1989, the hedging effectiveness under five horizons of 14, 30, 60, 90 and 120 calendar days were tested in both markets. The better performance of hedging in futures market suggested that hedging with futures is more efficient than that with options.

Kolb and Okunev (1993) studied the utility maximizing hedge ratios with MEG framework on cocoa in Ghana, Nigeria, Ivory Coast, and Brazil. The mean-MEG model was estimated, and compared with MV- and optimum mean-variance based hedge ratio with wide range of risk aversion. With 24 years data of cocoa (1952 to 1976) in these countries, the hedge ratios lead to long futures hedging strategy when risk aversion is less than 1.24, and hedge ratios converges to constant as risk becomes higher.

Risk minimizing is a reasonable objective of hedging, especially for weakly risk-averse investors (see Cecchetti (1988) and etc.). A number of articles consider utility functions (quadratic, logarithmic and exponential functions). The quadratic utility function was questioned by Hanoch and Levy (1969) that it implies disutility to incremental wealth at some wealth levels for any set of parameter values.

Followed by Kolb and Okunev (1992), Lieo and Luo (1993), they studied the stability of the optimal hedge ratio on S&P 500 using extended mean-GINI (EMG) method[17]. Firstly, using the empirical estimation method of Ederington (1979), Lien and Luo (1993) found that the EMG hedge ratio is quite closed to the MV hedge ratio when the risk aversion parameter is v = 9, and that the higher the risk aversion parameter, the lower the OHR (optimal hedge ratio) in S&P 500 market during the period from January 1984 to December 1988. The outcome denies the result of Kolb and Okunev (1992). As risk aversion parameter approach infinity, the OHR tends to be constant. With the same moving window tests as Kolb and Okunev, the instability of EMG OHR in terms of large risk aversion parameter was demonstrated. In addition, the MV and EMG hedge ratios are more stable with low risk aversion parameter rather than large values of it. Nonparametric kernel estimation was also employed to retest the stability, but it yielded a similar result at a certain confidence level.

Chen and Lee (2001) incorporated expected return into the GSV model (see Jong et al.(1997), Lien and Tse (1998, 2000)) and derived the M-GSV approach. The derivative of GSV model produces mean-risk hedge ratio. To tell the difference between considering expected return into portfolio and without it, the M-GSV and GSV models were compared. Furthermore, the MV, optimum mean-variance, Sharpe, MEG, GSV, M-MEG hedge ratios were also estimated. They tested the normality and martingale properties of

[17] The extended mean-GINI (EMG) and the mean extended-GINI (MEG) method are the same.

weekly price changes of Standard & Poor's (S&P) 500 futures from April 21, 1982 to December 27, 1991. It turns out that S&P 500 futures do not held normality and martingale assumptions. Alternatively, the data does not fulfil the convergence conditions to MV method. They proved that the M-GSV is consistent with stochastic dominance, but the GSV does not follow this track. The optimum mean-variance approaches the MV for larger value of risk aversion than 3. The M-GSV model yields different HR from other six models for high risk aversion. Compared with GSV hedge ratio, hedge ratio estimated from M-GSV is smaller and more stable than that from GSV model for lower risk aversion, since the higher the risk aversion, the less important the expected return. The M-GSV hedge ratio is higher than M-MEG ratio, and they both converge to standard MV hedge ratio, yet with different values, provided that the joint distribution of prices is symmetric.

3.3 Estimation of Hedge Ratio

The conventional ordinary least squared (OLS) technique of estimating OHR was proposed by Ederington (1979). This simple method involves regression of spot prices on futures prices (Ederington, 1979, Witt, 1987); furthermore, the regression approaches of price change (Myers and Thompson, 1989) and of percentage price change (Brown, 1985) were developed. The superiority of one of the three regressions brings on strong debate. The OLS method is not proper because of ignoring heteroscedasticity of variance of residual term, and not using current available information to construct conditional (co) variance. To fill these gaps, Cecchetti et al.(1988) suggested estimating dynamic OHR for Treasury bonds using Autoregressive conditional heteroscedasticity (ARCH) framework of Engle (1982) which depicted the time variation of OHR given current available information. Followed by Robert J. Myers (1991), traditional constant hedge ratio and time-varying generalized autoregressive conditional heteroscedasticity (GARCH) hedge ratio are compared on commodity futures market. With conclusion that GARCH-based OHR is slightly better than constant hedge ratio based on OLS method on wheat, and the GARCH model is a more flexible structure on the relationship between past and current volatilities.

In Cecchetti et al.(1988), the time-dependence of OHR was demonstrated for Treasury

Bonds with T-bond futures from October 1977 and to May 1986[18]. Time varying distribution of cash and futures price changes was allowed. In terms of profit-optimizing, a new utility-maximizing approach is invented which accounts for expected return as well as the risk. To employ time-varying conditional variance and covariance, a third-order linear bivariate ARCH model was used to estimate OHRs based on utility function and MV approaches in which the conditional correlation between cash and futures prices was assumed to be constant. The certainty-equivalent return of the utility maximizing method leads to 18 basis points better than MV hedge. On the other hand, the utility maximizing method yields 20 basis points better than MV method in average by a certainty-equivalent return over post-sample testing on OHR for the period between January 1984 and May 1986. The results implied that the time-varying HR of utility maximizing hedge is optimal to that of MV hedge for both in-sample estimation and out-of-sample forecasts, when a hedger set maximizing profit as a target.

Baillie and Myers (1991) further investigated the distribution of commodity cash and futures prices with bivariate GARCH models. Based on this study, the OHRs were estimated for six commodities, coffee, cotton, beef, gold, soybeans and corn. The cash and futures prices on each trading day of six commodities were collected from Columbia Centre for the Study of Futures Markets data tape. The GARCH model describes the non-normality of unconditional distribution of commodity price changes. This model also implies time variation in the conditional covariance matrix for all six commodities and result shows that the model fits the data. The OHRs from GARCH model perform nearly the same as the constant OHR for beef, gold and soybean, but they significantly outperform others. The GARCH-based OHRs demonstrate their superiority in reducing the conditional variance of the portfolio returns for all six commodities. For out-of-sample tests in 1986, the GARCH-based OHRs perform much better than the constant OHR for all commodities, except for gold. Generally speaking, the GARCH model provides more accurate descriptions of the non-normality of changes of cash and futures prices, the autocorrelation and conditional heteroscedasticity of commodity prices. Because of

[18] Treasury Bonds which would be short held for one month were observations. The cash returns of Treasury Bonds from October 1977 and to May 1986 were obtained from the Salomon Brothers Analytical Record of Yields and Yield Spreads, June 1986. And the corresponding futures prices were from the Interactive Data Corporation.

failure in capturing all features of data, the traditional OLS regression is inappropriate for estimating OHR.

Engle and Kroner (1991) and Bera and Higgins (1993) worked on multivariate GARCH models in detail. Baillie and Myers (1991) demonstrated the advantages of multivariate GARCH over univariate model since the multi version GARCH model provides a better description of autocorrelation between commodity prices and it is substantially more efficient and more accurate to model the joint distribution of cash and futures prices. Grounded upon the strongpoint, the multi-GARCH OHR outperforms those from the traditional OLS regression approaches and the multivariate GARCH model is recommended to estimate OHR on commodity market.

In 1987, Engle and Granger suggested incorporating cointegration that is the effect of short-term derivations of long run relationship between time series. They successfully developed an autoregressive representation and an error-correction representation for cointegration in time series, especially in the condition that series are non-stationary and conditional heteroscedasticity exists. The long-run relationship between cash and futures prices was tested using two-step cointegration tests by Engle and Granger (1987) and extended to multivariate version by Engle and Yoo (1987a). Lee (1994) proposed GARCH-X model to incorporate the effect of short-term derivations on variance in time series. Since then, the cointegration relationship in the long-run is widely tested in financial and commodity futures market. Baillie and Myers (1991) and Sephton (2002) failed to find the cointegration in commodity market. Kroner and Sultan (1993), Brenner and Kroner (1995) and Lien (1996) found the existence of cointegration in currency markets. Error correction model with GARCH structure was employed and Kroner and Sultan indicated that long-run cointegration relationship between financial assets and dynamic distribution of the assets is an important factor and is not negligible on estimating an accurate OHR. However, the cointegration theory obtains ample empirical support. Ghosh (1993), Kroner and Sultan (1993) (1993a) (1993a) (1993a) (1993a) (1993a) (1993a) and Lien (1996) showed that the cointegration relationship leads to smaller hedge ratio, and improves estimation and forecasting of OHR. Ghosh (1993, 1995) and Yang et al.(2001) further claimed that cointegration between cash and futures prices on commodity markets is necessary to ensure an optimal hedging decision.

Kroner and Sultan (1993) proposed a bivariate error correction model (ECM with conditional hedge) with a GARCH error structure to compute risk-minimizing hedge ratio for the British pound (BP), the Canadian dollar (CD), the German mark (DM), the Japanese yen (JY), and the Swiss franc (SF). Both long-run cointegration relationship between financial assets and dynamic distribution of the assets were taken into account in this model. The futures rates used are IMM[19] closing prices of five currencies from February 8, 1985 to February 23, 1990 and the spot exchange rates applied are Thursday's closing prices in New York. They compared hedge ratios estimated from conventional method, error correction model (CI) and bivariate conditional models (ECM-GARCH). Drawing a conclusion that hedge ratio from the conventional model has smaller variance than other models within sample for all five currencies. For out-of-sample, the one-year forecast was executed for one-period-ahead hedge ratio from July 10 to July 17, 1987. The conditional hedge outperforms all other models in terms of variance reduction for all five currencies, except for BP. They found that the hedge ratio varies when new information arrives in currency markets. In other words, hedge ratio is sensitive to new information and is time varying. Even if the investor readjusts and rebalances hedging portfolio, and more transaction costs (example of 0.01, 0.0125 and 0.015 return reduction due to transaction costs) are involved in, the conditional hedge increases the utility slightly with the given mean-variance utility function. In this study, they approve the advantage of ECM-GARCH model in estimating and forecasting hedge ratio, and its improvement in dynamic hedging.

Floros and Vougas (2004, 2006) investigated hedge ratio and hedging effectiveness in Greek stock index futures market. The multivariate GARCH model provides more accurate time-varying hedge ratio and greater risk reduction than OLS, ECM, VECM models on FTSE/ASE-20 index and FTSE/ASE Mid 40 index in this study. This result is consistent to Choudhry (2004).

Park and Switzer (1995) employed a model combining the OLS with cointegration (CI) and compared it with bivariate GARCH model, one-to-one Naïve, and OLS models on estimating the OHR for U.S and Canada stock index futures. The evident out-performance of GARCH model over other three models was found. As the most endemic stock index,

[19] IMM: International Monetary Market

the S&P 500 and Toronto 35 represent U.S and Canada stock markets. 185 weekly spot and futures prices for the S&P 500 and 184 prices for Toronto 35 from June 8, 1988 to December 18, 1991 are used. Given that futures prices are martingale, coefficients for each model are estimated using the first 92 and 91 observations. They used the one-period-ahead rolling forecasting with size of window 93 for the remaining period. Based on the objective of maximising utility, all models significantly reduce variance of portfolio relative to unhedged position for both index futures. The GARCH-based hedge provides best hedging effectiveness with variance reduction of GARCH over naïve hedge by 3.762%, over OLS by 8.870%, over OLS-CI by 5.840%, and over unhedged position as much as 97.916% for S&P 500. The GARCH model outperforms for mean-variance expected utility maximising, even when transaction costs are taken into account. Empirically, the transaction cost for a round trip is around $15-23 for one stock index futures contract. The transaction cost of rebalancing the hedge position was assumed as 0.0125% and the trader would adjust hedge position when the potential gains from readjusting are sufficient to cover the rebalancing costs. In this case, the GARCH model produces higher utility than OLS and OLS-CI models, and therefore it improves hedging strategy for both index futures. In dynamic hedging strategy, the bivariate GARCH model yields a time-varying OHR and gives a remarkable improvement on maximising profit with or without transaction costs for U.S and Canada stock index futures markets.

More recently, Kavussanos and Nomikos (2000) and Choudhry (2003, 2009) investigated impact of cointegration on hedging effectiveness in futures market. Kavussanos and Nomikos (2000) associated ECM model and GARCH-X model as a ECM-GARCH-X model. This new model takes into account the error correction term in mean and the long-run relationship between spot and futures price to estimate time-varying hedge ratio on 11 component routes in BIFFEX[20]. The ECM-GARCH-X model yields larger variance reduction than unhedged position, naïve hedge, conventional hedge and ECM-GARCH-based hedge. In addition, the model is also capable of forecasting hedge ratio since it provides the most accurate forecasts on hedge ratio. The ECM-GARCH-X model is strongly

[20] BIFFEX (Baltic International Freight Futures Exchange) was a London-based exchange for trading ocean freight futures contracts with settlement based on the Baltic Freight Index. It started trading dry cargo freight futures contracts in 1985. Trading volume in the dry cargo contracts dwindled over the years, and the contracts ceased trading due to lack of liquidity in 2001. Source: The Baltic Exchange.

recommended to estimate time-varying hedge ratio in the BIFFEX market.

Choudhry (2003) looked into the effect of short-run cointegration on hedging effectiveness in stock and agricultural futures markets. Choudhry (2003) studied European futures markets, including Australia, Germany, Hong Kong, Japan, South Africa and United Kingdom. The agricultural commodities, such as corn, coffee, wheat, sugar and soybeans are sampled for hedging effectiveness test in Choudhry (2009). Working on two different markets, surely two conclusions are not the same. In the case of European futures market, the GARCH-X outperforms standard bivariate GARCH model in two out of five cases in out-of-sample forecasting, which indicates the potential improvement on hedging effectiveness when cointegration is considered. However, In the case of agriculture market, the inverse result is obtained. The GARCH-X and BEKK-GARCH-X models never perform better than other GARCH models. Overall the GARCH family models provide better hedging effectiveness than constant hedge ratios. Taking the cointegration into account does not improve the hedging effectiveness in agricultural futures market, but it works well in European stock futures markets. In this study, we extend to forecast OHR and investigate forecasting power of six GARCH models.

The conclusions of Choudhry (2003, 2009) are in accordance with the study of Kroner and Sultan (1993) which demonstrated the great use of M-GARCH models in capturing time-variation and estimating variable hedge ratio. Additionally, Choudhry (2009) emphasized that whether the cointegration produces better hedging effectiveness or not somehow depends on the market and frequency of data.

Yang and Allen (2004) estimated OHR and hedging effectiveness using OLS, Vector autoregression (VAR), vector error-correction (VECM) and D-VEC multivariate GARCH(MGARCH) models in Australian futures markets. The better performance of VECM over VAR supports Lien (1996) and Ghosh (1993) that cointegration relationship between spot and futures price is an important factor in OHR estimation. Meanwhile, the out-of-sample tests yield that the MGARCH model outperforms in four models and the time-varying conditional hedge ratio is supposed to be more reasonable and appropriate.

Lien (2004) generalized the effect of omitting cointegration relationship between spot and future prices on optimal hedge ratio and hedging effectiveness. Though ignoring cointegration produces a smaller hedge ratio; the negative effect on hedging effectiveness could be minimal with less than 20% loss of hedging effectiveness. In previous study, Moosa (2003) found that considering cointegration has no significant impact on hedge ratio and the cointegration is a negligible factor. Lien (2004) reached a similar result. Theoretically, optimal hedge ratio increases when the cointegration is not taken into account. On the other hand, ignoring cointegration leads to a slightly smaller the hedging effectiveness which is less than 20 percent in empirical study. These two literatures did not support 'important cointegration' theory theoretically and practically which is contrast to voluminous empirically scholars (see Ghosh (1993); Kroner and Sultan (1993) and Choudhry (2003, 2009)).

Inspired by the leverage effect theory of Black (1976) and Christie (1982), Kim and Kon (1994) compared the performance of inter-temporal GARCH, asymmetry-GJR-GARCH and EGARCH-in-mean models on describing stock returns. Results show the descriptive power of asymmetry-GARCH models on time series. Furthermore, three time-independent models and the inter-temporal GARCH models based on conditional student's t distribution are also applied. The sample contains 30 stocks traded in the Dow Jones Industrial Average and 3 stock indexes[21]. Daily returns for the period of July 2, 1962 to December 31, 1990 are provided by the CRSP at the University of Chicago (CRSP tapes). The GARCH models which allow time-dependent mean and variance are called inter-temporal dependence models. Among the three inter-temporal dependence models, the GJR (1, 3)-M model which describes both volatility clustering and the leverage effect generates best description for returns of stocks, followed by GARCH (1, 3)-M and EGARCH (1, 3)-M models. The GARCH (1, 3)-M fits returns of stock indexes better than other two. For time-independent models, the generalized discrete mixture-of-normal distributions model captures kurtosis and skewness, and better explains observations than student's t and Poisson jump models. The conditional student's *t* based GJR (1, 3)-M model is suggested as the best alternative of data description of other three time-dependent GARCH-M models. Overall, the inter-temporal GJR well explains the behavior of stock

[21] All data: 30 stocks are Allied Signal, Alcoa, American Express, AT&T, Bethlehem Steel, Boeing, Caterpillar, Chevron, Coca Cola, Disney, Walt, Du Pont, Eastman Kodak, Exxon, General Electric, Gneral Motors, Goodyear, IBM, International paper, McDonalds, Morgan, J.P., Merck, 3M, Phillip Morris, P&G, Sears, Texaco, Union Carbide, United Technology, Westinghouse, Woolworth; and 3 stock index include Standard and Poor's (S&P) 500, Center for Research in Security Prices (CRSP) Equally Weighted (EW), and CRSP Value Weighted (VW).

return over others, and the EGARCH most comprehensively captures properties of stock index returns. The inter-temporal asymmetry-GARCH models have supreme descriptive ability on stock and stock index returns. Nevertheless, the student's t inter-temporal model performs poorly and time-independent models have the poorest performance.

3.4 Forecasting of Hedge Ratio

Referring to forecasting models, the models used in volatility prediction are generalized to almost all time series, and therefore forecasting of hedge ratio with these models is permeated. Comparing with vast majority of studies on volatility forecasting, the volume of scholars who research hedge ratio prediction is small. Therefore, I am quoting several literatures about volatility forecasting for analogy.

3.4.1 Volatility Forecasting

Poon and Granger (2003) reviewed 93 papers on forecasting volatility in financial markets. The financial market volatility was proved to be clearly forecastable. Since scholars have their own objectives, and they study different asset, using different data sets, various reference literatures, and diverse models and evaluation techniques, it is hard to generalize an overall conclusion about rank of the forecasting power of models.

West and Cho (1994) investigated predictive ability of several models on volatility of exchange rate. Six models capturing homoscedasticity and heteroscedasticity (GARCH), AR and nonparametric models (Gaussian Kernel) were compared on forecasting volatilities (conditional variance) on dollar against five currencies of Canada, France, Germany, Japan and United Kingdom. With total weekly data during the period from 14, March, 1973 to 20, September, 1989, the weekly conditional variances are estimated and forecasted with horizons of one-week, 12-week and 24-week. The univariate homoscedasticity model is discarded, given the poor performance in estimation. For out-of-sample forecasting, the predictive ability of models varies relying on the evaluation techniques. Under MSPE (mean squared prediction error) statistic, GARCH models provide slightly smaller error than other model for one-week-ahead forecasts, though the error is not statistically different from others. For other two horizons, no single winner in conventional test of forecast efficiency. Under the RMSPE (root mean squared prediction error) test, all models are statistically the same. In this study, the forecasting performance of GARCH does not definitely exceed other models applied here; however, we have to admit that incorporating heteroskedasticity should be seriously considered when dealing with time series.

Yu (2002) forecasted volatility in the new Zealand stock market. Yu compareed nine high frequently used models for volatility estimation and prediction, including random walk (RW), historical average, moving average, simple regression, exponential smoothing, EWMA, ARCH, GARCH and stochastic (SV) models. Yu used most popular and weighted NZSE40[22] on the New Zealand Stock market Exchange (NZSE) to represent New Zealand stock market. Daily data is converted in to monthly log return from 1 January 1980 to 31 December 1998. In order to make a relatively comprehensive comparison of forecasting ability of these nine models, four different error evaluation techniques are employed, including RMSE, MAE, U-statistic and LINEX loss function. The symmetry property of the RMSE and MAE measurements somehow limit their implication in evaluating asymmetric time series. The SV model outperforms under RMSE statistics. Surprisingly, the SV ranks first in U-statistic and three LINEX which capture asymmetry in volatility. In MAE statistics, the SV only underperforms GARCH (3, 2). In other words, the traditional SV model has a stably high rank in above nine models on forecasting volatility. The rankings of GARCH-family models vary depending on error statistics. The GARCH (3, 2) performs better than other simpler GARCH members, such as GARCH and ARCH models. Yu explains the outperformance of SV models that the SV model involves one more noise process than GARCHs do and it is granted as a better forecasting model in volatility prediction. The SV model ranks second after exponential smoothing model in MAE test. The EMA models perform poorly in all cases. Generally, the SV model provides the most accurate forecasts of volatility and GARCH models are sensitive to evaluation techniques. Moreover, regression and EWM models do not perform well in New Zealand stock market.

Hansen and Lunde (2005) compared predictive power of 330 different volatility GARCH family models on exchange rate and IBM stock prices. In order for more precise volatility comparison, the intra-day (one-minute and five-minute returns) estimation of realized volatility is computed on daily exchange rate data (DM/$) from October 1, 1987 to

[22] NZSE40 cover 40 largest and most liquid stocks listed and quoted on the New Zealand Stock Market Exchange (NZSE), weighted by the market capitalisation without dividends reinvested(Yu, 2002).

September 30, 1992 and IBM stock prices for the period from January 2, 1990 to May 28, 1999. Because of variety of 330 models, Hansen and Lunde did a lot of work on specify reasonable pre-assumptions about benchmarks, distribution of return, and mean function. Obviously, it is more appropriate to employ more than one error statistics to evaluate forecast accuracy. In this paper, Hansen and Lunde used seven popular evaluation techniques, corresponding six loss functions, including MSE1, MSE2, QLIKE, R^2 LOG, MAD1, and MAD2[23]. In addition, the reality check (RC) data snooping[24] is used for evaluating performances of models, and it turns out that it is not a capable measure in this study. The ARCH (1, 1) and GARCH (1, 1) are two benchmarks employed. The Gaussian and *t*-distributed specifications are assumed for the density function. The lags of models are limited to (2, 2) since short lags models might produce more accurate forecasts than complicated models in some cases. In the superior predictive ability (SPA) test, not a single model outperforms for both assets with different pre-assumptions. In the case of exchange rate, the GARCH (2, 2), the LOG-GARCH (2, 2), and the GQ-ARCH (2, 1)[25] models rank first amongst all 330 models with GARCH (1, 1) as benchmark. In the case of IBM stock, the A-PARCH (2, 2) model with *t*-distributed residuals and mean zero performs best. The EGARCH model does a good work with *t*-distributed standardized residuals, but not for Gaussian errors. Based on these two data sets, they supported the appropriateness of PSE error statistics. However, an overall conclusion can be reached respectively. Take benchmarks into account, the ARCH is generally the worst model, while GARCH (1, 1) cannot be beaten by others candidates statistically, although the original GARCH model is simpler than other GARCH models.

Andersen and Bollerslev (1998) demonstrated forecasting ability of GARCH family models in volatility prediction. They defended that both ARCH and stochastic volatility models have forecasting power, and produce accurate inter-daily forecasts theoretically and practically. They forecasted volatility of exchange rates of Deutschemark-U.S. Dollar (DM-$) and Japanese Yen -U.S. Dollar (¥-$) from October, 1987 to September, 1992. The one-step-ahead daily, monthly, hourly, minutely (5 minutes) forecasts on both exchange rates

[23] MSE2 and R2LOG are in accordance with MZ (see note 9); QLIKE is the loss function in terms of Gaussian likelihood. For more detail of MSE2, QLIKE and R^2LOG criteria, please read Bollerslev (1994).
[24] See Hansen (2001) for detail of reality check of data snooping.
[25] GQ-GARCH model is generalized Quadratic GARCH model.

are tested. The MSE measure was employed for evaluating error of forecasts. The 20-day-ahead estimation from GARCH model is the least biased using rolling data, and the forecasting is superior in the case of high volatility. The one-day-ahead GARCH forecast provides a notable tracking measure of ex-post volatility in both markets. The R^2 increases from 10% of daily squared return to 50% for 5 min square return. Theoretically, the 1-day-ahead out-of-sample using 5 min return for Yen vs. Dollar is supposed to yield higher R^2 than it does in reality, and they explained the partial difference as a consequence of the abnormality of apparent volume expansion in this period. As the data frequency increases, the R^2 monotonically increases and the forecasts error dramatically reduces. In addition, the forecast based on the continuous sample provides much more efficiency than daily forecast.

Brailsford and Faff (1996) investigated relative forecasting ability of various forecasting models on predicting monthly volatility on Australia stock market, including random walk model, historical mean model, moving average model, exponential smoothing model, exponentially weighted moving average model, simple regression model, standard GARCH models and asymmetric GARCH-GJR models. They studied Statex-Actuaries Accumulation Index on Australian stock market and collected 4900 observations from 1 January 1974 to 30 June 1993, and the data of 1987’s energy crisis is included. From simple regression models to more complicated GARCH family models, 90 monthly forecast errors of all models were generated using different empirical error statistics, which are mean error (ME), the mean absolute error (MAE), the root mean squared error (RMSE), the mean absolute percentage error (MAPE) and Mean mixed error (MME) for asymmetric models. The results show inconsistence depending upon error statistics and corresponding benchmark. The under-predict volatility for all models except exponential smoothing model and no single superior model is found in the ME method. The GARCH-GJR is preferred for MAE, MME (O), MAPE error statistics; RMSE is in favour of historical mean and simple regression models; and MME (U) identifies the accuracy of standard GARCH (1, 1) model. The above mixed results reveal the sensitivity on error statistic technique of evaluating the accuracy of forecasting. Alternatively, none of a single model is superior to others and it is not doable to rank these models according to their forecasting ability. Hence, they concluded that GARCH family models and simple regression model overall outperform in all models on forecasting monthly volatility on Australia stock market. They also pointed out that forecasting ability of models is sensitive to error statistics to some extent, and no any single model is overall superior than others. However, the conclusion is questioned because the speciality of the Australian Stock Exchange or the 1987's energy crisis is suspected to make inconsistent results, yet not error statistic technique leads to.

The impact of negative and positive news on volatility becomes a popular topic since Black (1976) discovers the asymmetric effect of information on volatility of financial time series. Motivated by Ross (1989) who found that variance of price changes is related directly to the information, Conrad and Gultekin (1991) studied the differential asymmetric predictability of volatility and suggest to incorporating asymmetry in predictability of conditional variances. The model considering asymmetric effect of information is not always desired. Day and Lewis (1993) indicated that traditional implied volatility provides a better predictions of volatility on crude oil futures than Naïve historical volatility, GARCH and EGARCH models, although GARCH-type models capture some information that neither implied volatility nor historical volatility models contain. Franses and Van Dijk (1996) found the sensitivity of nonlinear GARCH family models on extreme observations with-in sample tests. Filtering extreme observations of stock market crash in 1987, the QGARCH model is superior to Random walk and GARCH, GJR-GARCH models to forecast return volatility for all sub samples for five European countries, and the GJR model is suggested not appropriate in forecasting. Nevertheless, Engle and Ng (1993) defended that the GJR model is the best parametric model since the GJR provides low conditional variance and most accurate volatility forecasts in parametric models.

Lee (1991) declared that linear GARCH model is not always a better forecasting tool than nonlinear GARCH models under RMSE and MAE criterion with rolling and recursive samples. Lee (1991) tested the out-of-sample forecasting performance of GARCH models on predicting volatility. The forecasting ability of linear and nonlinear GARCH models are examined on volatility prediction for exchange rate volatility of US dollar against Japanese Yen, German Mark, British Pound, French Franc and Canadian Dollar from 1973 to 1989. Since their performance depends upon the measure criteria, both RMSE (root mean square error) and MAE (mean absolute error) criterions are used in this study. For comparison, both rolling and recursive forecast methods are employed to measure the forecasting ability of linear GARCH models (standard GARCH, IGARCH with trend) and nonlinear GARCH (EGARCH with trend, nonparametric kernel models, ARMA (GARCH) index) models. The mixed results are received under two criterions. Rolling forecasting is preferred in RMSE, but kernel models perform better under MAE with recursive forecast. The GARCH (1, 1) model may explain high volatility better than the nonparametric kernel model under RMSE criterion. However, the nonparametric kernel model outperforms the GARCH and IGARCH models in the multi-steps-ahead out-of-sample performance. The results indicate that no preference of linear or non-linear GARCH models on forecasting exchange rate volatility.

Gokcan (2000) argued that linear GARCH model overall has better forecasting performance than non-linear EGARCH model on volatility in emerging countries. The comparison of forecasting ability on volatility of linear and non-linear GARCH models is extended from European futures market to emerging countries' futures market.

Monthly return of stock market is applied from February 1988 to December 1996 from seven emerging countries, including Argentina, Brazil, Colombia, Malaysia, Mexico, Philippines and Taiwan. The returns are found to be leptokurtic for all countries except for Argentina. Results show that the GARCH (1, 1) yields lower AIC in in-sample test, and outperforms EGARCH for all countries. In one-month-ahead forecasting, the GARCH (1, 1) provides smaller forecasting error for all except Brazil. Generally speaking, the GARCH captures dynamic volatility of returns and has better forecasting ability for emerging countries than non-linear EGARCH model.

Conrad and Gultekin (1991) further studied the predictability of conditional variance of return for different capital sized companies. In this paper, Conrad and Gultekin (1991) combined the daily returns of all securities in the American and the New York Stock Exchanges according to the size of companies into three value weighted portfolios from 1962 to 1988. They took account the turn-of-the-year effect and the remaining asymmetric lagged cross-correlations into both univariate and multivariate ARMA(1,1)-GARCH(1,1)-M models to examine the effect of asymmetry on returns. The evidence shows the superiority of multivariate GARCH model that captures entire information for variance-covariance matrix of the errors and estimates all parameters in the meantime with higher accuracy than univariate model. More importantly, the results indicate that there presents a significant asymmetry in the predictability of the volatilities of large companies. Hence, it is more reasonable to incorporate an asymmetry effect on mean and variance for modeling time-varying returns and volatilities.

Engle and Ng (1993) expanded the asymmetric information effect on volatility from U.S and U.K. financial markets (see French, 1987, Sentana, 1993) to Japanese stock market. The daily returns series of the Japanese TOPIX index from January 1, 1980 to December 31, 1988 were tested. Engle and Ng (1993) found that negative news increases volatility more than positive news using diagnostic tests and a partially nonparametric model. In addition, the accuracy of testing impact of news on volatility through seven GARCH type models, the EGARCH, the AGARCH (Asymmetric-GARCH), the VGARCH(1, 1), standard GARCH, the NGARCH (Nonlinear Asymmetric GARCH), the partially nonparametric (PNP) and the GJR models were compared. The GJR is the best parametric model since the GJR provides low conditional variance and the most accurate volatility forecasts in parametric models. At the same time, the PNP model is also preferred because it adequately simulates the news impact curve.

Ulu (2005) explored the forecasting performance of QGARCH model on volatility prediction. Ulu (2005) use R^2 from MZ (Mincer-Zarnowitz volatility forecast) regression as a measure of forecasts performance. The higher the value of R^2, the better forecasting performance of the model. The asymmetry in conditional variance enhances the predictive power of forecasts volatility. Ulu derived the formula of R^2 for QGARCH (1, 1) model which allows for leverage effect of negative and positive information on conditional variance. Under assumptions that the innovation factor is symmetric distributed; the fourth moment of return in MZ regression is infinite and population value of R^2 for GARCH (1, 1) model has boundary of the reciprocal of the innovation kurtosis. Ulu found that the R^2 is greater than that of standard GARCH (1, 1). The asymmetric model yields a greater R^2, which means accounting for asymmetry in the conditional variance process can increase the predictive power of volatility forecasts. Notwithstanding, the leverage effect impact on predictive power becomes smaller and smaller and finally can be negligible as the unconditional variance of the series increases.

As the constant term in conditional variance formula approaches infinite, the R^2 of GARCH and QGARCH models are equal good, and their predictive power is the same. Measure with the value of R^2 from the MZ volatility forecast regression, a reasonable specified QGARCH model yields great value of R^2 which results in a high predictive power[26].

Traditional volatility forecasting models, such as implied volatility, SV and others are compared with GARCH family models. The predictive power of GARCH models are great in most cases (see Hansen & Lunde (2005), Engle and Ng (1993), Ulu (2005)), though some scholars cast doubt that GARCH models somewhat are sensitive to extreme observations (see Yu (2002)) and evaluation techniques (see west & Cho (1994)). The better forecasting performance between linear and non-linear GARCH models on volatility is an unsolved problem.

3.4.2 Hedge Ratio Forecasting

The GARCH model is popular in modelling time series, and it is especially useful on simulating and forecasting time series which has dependence on first-order mean and second-order variance. Since GARCH models fits commodity price quite satisfactorily and explains well the relationship between cash price and its futures price, the GARCH family models are taken for granted as proper measure of hedge ratio (Baillie, 1991).

The bivariate GARCH models were employed for OHR estimation for U.S and Canada stock index futures in Park and Switzer (1995). The results of one-period ahead rolling forecasting showed that the bivariate GARCH model has forecasting ability and best hedging effectiveness among to one-to-one Naïve, OLS, and OLS-CI (cointegration) models.

Brooks and Henry (2002) studied the asymmetric impact of information on optimal hedge ratios in UK stock futures market. They took into account the cointegration relationship between spot and futures prices with an error correction term in mean equation. Specifically, they allowed different effect of good and bad news on return volatility and

[26] The R^2 is derived from the Mincer-Zarnowitz (MZ) volatility forecast regression for a QGARCH(1,1).The Mincer-Zarnowitz (MZ) regression for the return volatility can be written as: $x_t^2 = a + b\sigma_t^2 + v_t$, where x_t is return series, v_t is a zero mean error term. If the model is correctly specified, the population value of a is zero and the population value of b is one. The R^2 from this regression measures how well the model forecasts is.

variance-covariance matrix in asymmetric BEKK-GARCH model. The OHRs based on naïve hedge model, BEKK-GARCH and asymmetric BEKK-GARCH models are tested and compared in terms of value. They studied FTSE 100 stock index with corresponding FTSE 100 stock index futures from 1st, January, 1985 to 9th, April, 1999. It turns out that any hedge produces significantly lower return than unhedged position, and asymmetric model yields the largest variance reduction for both in-sample and out-of-sample tests. The hedging effectiveness is evaluated by the minimum capital risk requirements (MCRRs) which is similar with Hsieh (1993)'s measurement[27]. The evaluation of hedging effectiveness is tested for 1-day, 10-day, 30-day, 3-month and 6-month investments. The results show that time-varying asymmetric BEKK GARCH model fits data better, substantially reduces portfolio risk and improves forecast accuracy of hedge ratio than other models in sample when the investment is less than 1 month. On the contrary, the benefit of long-term hedging with asymmetric model dramatically reduces. Additionally, out-of-sample forecasting of long hedging increases risk rather than diminishes it. Asymmetric models are prized for considering negative and positive news impact on hedge ratio, but its forecasting ability is limited. Hence, Brooks and Henry (2002) suggested that the asymmetric BEKK GARCH model is appropriate for hedging and forecasting hedge ratio of short term investment, but not for the investment beyond 1 month.

The bivariate GARCH model is famous for its ability of describing time-varying OHR, autocorrelation and conditional heteroscedasticity of commodity prices. Baillie and Myers (1991) estimated optimal hedge ratio within-sample and out-of-sample using bivariate GARCH model, providing the estimated distribution of cash and futures prices on six commodities. The post-sample forecasts of GARCH OHR fits the data better than constant OHR and it explains the non-normality of commodity price changes in futures and cash markets better.

Garcia, Roh et al. (1995) explored the appropriation of the time-varying hedge ratios on capturing dynamic characteristics of prices in the soybean complex. Hedge ratios which are estimated by multivariate constant covariance model (MCCM), time-varying BGARCH,

[27] MCRRs measurement is the minimum amount of capital required to absorb all with a specified percentage of potential losses (Brooks and Henry, 2002).

naïve hedging and constant correlation MGARCH models are compared with unhedged position for soybean complex for the period January 1983 to December 1990[28]. Under the maximum likelihood estimation, the MGARCH produces more variable hedge ratio than BGARCH model does. The multivariate models (MCCM and MGARCH) provide the largest risk reduction among five models, and they dramatically reduce variance by around 45% for in-sample test. The time-varying BGARCH does not significantly improve hedging performance. However, multivariate models delicately increase variance of portfolio for out-of-sample test. The results reveal the importance of simultaneous estimation of hedge ratio and the effectiveness of time-varying GARCH models on estimating hedge ratio for soybean complex. The BGARCH model under-performs due to its limited ability of variance reduction for soybean complex. In further study of BGARCH model, Garcia claimed its strength on estimating time-varying hedge ratios for corn and soybean.

As an alternative of testing hedging performance, Yeh and Gannon (2000) studied the trading performance and profit effects on portfolio of constant and dynamic hedge model in the presence of transaction costs for Sydney futures market. The OHRs are estimated and forecasted based on naïve hedge model, two versions of two-stage constant hedging models proposed by Ghosh (1993) and constant correlation bivariate GARCH model. The daily spot and futures prices of Australian All Ordinaries Index (AOI) are collected from EQUINET time series Database and Sydney Futures Exchange (SFE) for the period of January, 1988 and June, 1996), respectively. They employed the real transaction costs of each trading from McIntosh Futures limited instead of getting the bid-ask spread as a proxy[29] or taking a fixed proportion of trading value as transaction costs. In the estimation of OHR, all three constant models generate significantly greater profit than never rebalanced position and the BGARCH model produces the largest profit than other three constant hedging models when readjustment of position is allowed. The extra transaction cost charged for rebalancing portfolio is substantially less than the profit made from readjustment. On the other hand, in the one-step-ahead forecasting of

[28] Cash prices of soybean and soybean oil are from Illinois quotes and futures prices are from Chicago Borad of Trade yearbook).

[29] In this paper, the transaction costs consist of brokerage, trading fees, and clearing house fees. And the trader would be charged whenever he/she readjust the existed position.

OHR, the BGARCH model far outperforms constant models in terms of profit and loss, given that new positions are opened when rebalance the position each week. The profit made by BGARCH model in the condition of opening new position is more than 3 times of that only rebalance strategy is utilized. Considering transaction costs, the hedging performance of constant correlation BGARCH model is not weakened in terms of profit maximization.

Kroner and Sultan (1993) suggested the ECM-GARCH-X model which is used to present both long-run cointegration relationship between financial assets and dynamic distribution of the British pound (BP), the Canadian dollar (CD), the German mark (DM), the Japanese yen (JY), and the Swiss franc (SF). The ECM-GARCH model outperforms over conventional and an error correction model (CI) on estimating and forecasting OHR. Likewise, Kavussanos and Nomikos (2000) compared the estimating and forecasting performance of unhedged position, naïve hedge, conventional hedge, ECM-GARCH, and combined ECM-GARCH-X models on time-varying hedge ratio in Baltic International Financial Futures Exchange (BIFFEX). In this market, the ECM-GARCH-X produces largest risk reduction and most accurate forecasting in all models, followed by ECM-GARCH for both in sample and out-of-sample tests. The superb performance of ECM-GARCH-X states that taking account cointegration both in mean equation and variance may improves hedging effectiveness. More recently, the predictive power of GARCH-type models, Kalman filter and rolling OLS models are tested (see Lien et al. (2002), Choudhry (2003, 2009), Moon (2009), Liu and Hung (2010)).

Lien, Tse et al. (2002) evaluated the hedging performance of the Constant-correlation vector GARCH (1,1) (CC-VGARCH) for ten futures markets, and the simple OLS method were also employed for comparison. One-day-ahead forecasting of hedge ratio with recursive daily spot and futures prices for currency market, commodity and stock index markets. British Pound (BP), Deutschmark (DM) and Japanese Yen (JY), soy bean oil (BO), wheat (KW), crude oil (CL), corn (C) and cotton (CT), NYSE composite (YX) and S&P 500 (SP)) markets for the period January 1988 through June 1998 are sampled. Referring to the LMC statistic[30] (Tse, 2000), the constant correlation hypothesis is accepted for eight

[30] The LMC is asymptotically distributed as χ^2 with 1 degree of freedom under the null of constant correlation.

out of ten markets. In the case of currency markets, the OLS method produces larger hedge ratios than CC-VGARCH does. In the case of commodity and stock index futures market, a uniform conclusion could not be reached. Overall, the CC-VGARCH model attempts to lead to smaller and asymptotic stable hedge ratios and the OLS makes less volatile hedge ratio. Nevertheless, the CC-VGARCH model results in slightly larger portfolio variance than that of OLS approach; as a special case, the GARCH makes 20% higher risk than OLS method for crude oil. Evidently, the OLS method outperforms CC-VGARCH model for the ten futures markets. Based on the ten futures market, the GARCH does not make improvement on hedge ratio forecasting and hedging performance over OLS method in currency, commodity and stock index futures markets in some extent.

However, GARCH family models show strong predictive power on hedge ratio forecasting in many other cases. Choudhry (2003, 2009) demonstrated the powerful forecasting ability of GARCH family models, especially GARCHs with cointegration.

Choudhry (2003) examined the effect of cointegration relationship between stock returns from cash price and futures prices on hedging effectiveness in Australia, Germany, Hong Kong, Japan, South Africa and the United Kingdom stock market. Two time-varying hedge ratios based on GARCH and GARCH-X are compared with three constant hedge ratios which are estimated using traditional hedge strategy, minimum variance and unhedged position. Daily stock returns between January 1990 and December 1999 of All Ordinary price and futures index from Australia, Dax 30 index and the Eurex-Dax futures index from Germany, Hang Seng price index and Hang Seng index futures from Hong kong, Nikkei 225 price index and Nikkei stock average futures prices from Japan, JSE industrial index and the Industrial 25 index from South Africa and FTSE-100 index and the FTSE-100 futures index from the United Kingdom stock markets are samples for in-sample estimation (January1990-December 1999) and out-of-sample (Jan 1998-Dec 1999 and Jan 1999-Dec 1999) forecasts. For in-sample estimation, the constant minimum variance hedge ratio has higher hedging effectiveness than others in most cases. For out-of-sample forecasts, the hedging effectiveness of time-varying hedge ratios outperforms that of constant hedge ratio for all five stock markets, and the standard GARCH model performs better than GARCH-X in three out of five cases. The cointegration relationship between cash and futures prices exists for all cases and the short-run deviation of cointegration do have impact on hedging effectiveness for some extent, providing some evidence that GARCH-X model is superior to others in some cases.

Commodity price is one of the most volatile time series and it is even more non-stable than exchange rates and interest rates in some periods due to disturbance in demand and supply (Kroner et al., 2006). Lately, Choudhry (2009) extended the research of short-run deviation of long-run relationship between cash and futures prices on hedging effectiveness to agricultural futures market. The GARCH-X and BEKK-GARCH-X incorporating the cointegration were employed to compare with standard GARCH and BEKK-GARCH models. The in-sample estimation and two out-of-sample forecasts of hedge ratio for corn, coffee, wheat, sugar and soybean are tested, respectively. With weekly cash and futures prices of these five commodities from August 1980 to July 2004, comparison of hedging effectiveness is performed for both in-sample and out-of-sample tests. For all sub samples, the GARCH-X and BEKK-GARCH-X never perform better than other two GARCH models. The results are inconsistent with that in stock futures markets (Choudhry, 2003), and indicate that taking account short-run deviation will not help improving hedging effectiveness for the five agricultural futures. As pointed out in Chouldhry (2003), the frequency of data and the market difference might be the reasons of leading contrary results.

Most literatures cited above work on hedge ratio forecasting in financial market, yet Garcia (1995) and Choudhry (2009) studied hedge ratio estimation and hedging effectiveness in agricultural market. However, to my knowledge, rarely articles study the prediction of OHR of agriculture to the more volatile prices. This thesis investigates the forecasting ability of six popular GARCH models on OHR prediction in agricultural and commodities' futures markets. So far, it is the first study to compare predictive power of bivariate GARCH, GARCH with cointegration and GARCH with asymmetric effect on OHR in these two futures markets. This research will contribute to compensate the lack of investigation of OHR forecasting in agricultural and commodities' markets.

3.5 Conclusions

This chapter reviews most frequently used estimating and forecasting models of optimal hedge ratio (OHR). In the part about forecasting, the GARCH family models, SV, EWMA and other models are mainly employed on volatility and hedge ratio prediction. Due to the lack of research on hedge ratio forecast, sufficient articles involved in the prediction of volatility are also reviewed.

The second section introduces several derivation approaches of hedge ratio with detail. The cons and pros of these methods are analyzed and competed with each other. Eventually the MV method is relatively more appropriate for hedge ratio derivation than others because of its simplicity in financial theory and statistical computation.

In the third section, empirical researches on estimating hedge ratio in various areas are illustrated. Ederington (1979) initially applied the OLS technique in OHR estimation. Sequentially, Cecchetti et al. (1988) suggested dynamic OHR. Since the proposal of ARCH model by Engle (1982) and Bollerslev (1986), there is a dramatic increasing study on time series with GARCH family model. The time-varying property of hedge ratio is strongly supported by voluminous scholars, such as Robert J. Myers (1991), Baille and Myers (1991, 1993), Kroner (1991) and Bera, Higgins (1993) and Choudhry (2003, 2009). The following studies are extended to other properties of time series, covering the asymmetric effect of information, cointegration relationship between two time series.

The fourth section is the main part. The literatures on forecasting of volatility and hedge ratio in stock and commodity futures markets are widely discussed. The predictive power tests of GARCH-type models and other well-perform estimation models take sufficient share. The forecasting performance and forecast error of models are examined, and result differs depending on the research object, the frequency of data, forecasting horizon and the evaluation techniques of forecast error. The debate of 'best forecasting model' continues. Though GARCH family models are not obviously outperforms over other models on predicting both hedge ratio and volatility, their predictive ability is definitely worth gaining more attention and further research in various area.

4 Methodology and Data

4.1 Introduction

As Chen (2001) states the main methods of estimating the MV (minimum variance) hedge ratio are OLS, ARCH and GARCH models, Random coefficient and cointegration/error correction approaches. In OHR estimation, the OLS, and random coefficient methods are not preferred since they do not provide dynamic hedge ratio and constant hedge ratio is not true in practice. Furthermore, the ARCH and GARCH models provide estimates of the OHR more efficiently when heteroskedasticity shows up (Bollerslev et al., 1992). In terms of forecasting of OHR, the OLS, and random coefficient methods are not capable of offering time-varying forecasts. While many empirical researches demonstrate the forecasting ability of ARCH and GARCH-type models on time series prediction, such as volatility, hedge ratio (see Baillie and Myers, 1991; Brooks and Henry, 2002).

In my study, I investigate and compare the predictive power of six GARCH models on OHR forecasting. A comprehensive description of six GARCH-CCC (constant conditional covariance) family models (standard GARCH, BEKK-GARCH, GARCH-X, BEKK-GARCH-X, GARCH-GJR and QGARCH) is provided in this chapter. The second section describes the properties of ARCH/GARCH models and its estimation; the third and fourth sections explore the way the univariate and multivariate GARCH-type models forecast. The fifth section involves the evaluation of forecast accuracy and forecast error of GARCH forecasts. The last part of this chapter is about the data employed in this study. Basic statistics of samples are presented.

4.2 Time Series

A time series is a sequence of observations over time in identical time interval, such as daily stock price, interest rate, seasonal crop production (Makridakis et al., 1998). Many time series show (non)-stationarity, or/and periodic or/and cyclical feature or/and trend. The last three properties are visible from graph of time series, but the stationarity is not as obvious as others.

4.2.1 Basic Properties

Stationarity and Unit Root Tests

A stationary time series moves around a constant value without obvious variation apart from it. The stationarity is divided into strict and weak stationarity. The stationarity significantly affects the behavior of time series. Stationary time series has time-independent mean, variance and covariance which make the forecast more doable. However, for a non-stationary time series, the forecast is valid only for a particular time period without generality (Gujarati, 2003).

A weak stationary process has constant mean, constant variance over time and constant autocorrelation which depends only on the time difference. Statistically, the three assumptions of weak stationarity are expressed as (Brooks, 2008):

1. $E(y_t) = \mu$

2. $E(y_t - \mu)(y_t - \mu) = \sigma^2 < \infty$ (4-1)

3. $E(y_{t_1} - \mu)(y_{t_2} - \mu) = \gamma_{t_2 - t_1}$ for $\forall t_1, t_2$

For a time series $\{y_t\}$, the first and second moments of time series remain constant as time goes by. Time independence property of weak stationary process, as Tsay (2005, p30) says,

'*Weak stationarity `enables one to make inferences concerning future observations (prediction)*'.

In other words, it is definitely possible to forecast or predict how a time series goes in the future given a weakly stationary time series. The so called 'stationary' generally refers to 'weak stationary'.

Strict stationarity means the joint distribution of $(y_{t_1}, y_{t_2}, \cdots y_{t_n})$ is the same as that of $(y_{t_{1+h}}, y_{t_{2+h}}, \cdots y_{t_{n+h}})$, and the joint distribution only relies on the value of h without varying over time. A strictly stationary process does not necessarily have finite variance. In the case of normal distributed time series, the weak stationarity is equivalent to strict stationarity (Makridakis et al., 1998).

There are some empirical methods of testing the stationarity of time series, say DF and ADF ((Dickey and Fuller, 1979, 1981)), KPSS method (Kwiatkowski et al., 1992)

DF and ADF

The basic DF (Dickey and Fuller (1979)) statistic tests the null hypothesis of the non-stationary (presence of unit root) against stationary time series. For the simple regression $y_t = \mu + \rho y_{t-1} + \varepsilon_t$, the DF test has hypothesis that $\rho = 1$ alternative $\rho < 1$. For the first-difference (return) regression,

$$\Delta y_t = \mu + \theta y_{t-1} + \varepsilon_t, \qquad (4\text{-}2)$$

The equivalent null hypothesis is that $\theta = 0$ against $\theta < 0$. The DF test is:

$$\textbf{DF}\ statistic = \frac{\hat{\theta}}{s\hat{e}(\hat{\theta})} \qquad (4\text{-}3)$$

Where $\hat{\theta}$ is the estimated θ and $s\hat{e}(\hat{\theta})$ represents the standard error of $\hat{\theta}$. The critical value of DF test is a simulation of a non-stationary regression, and it is not from a standard distribution. Hence, the DF test is not capable of examining stationarity and unit root of student's t distributed time series. Response to this weakness, Dickey and Fuller (1981) propose the augmented DF (ADF) test with the same null hypothesis. The ADF adds a lagged difference term to examine possible autocorrelation for more complicated time series, such as a series with deterministic trend, correlated residual and etc. Schwert (1989) shows evidence of advantage of ADF test that it adequately captures the moving average of residuals of random walk and is more useful than non-parametrically adjusted DF statistics (Banerjee, 1993). However, it provides a root close to the non-stationary boundary even when the time series is stationary. Alternatively, it fails to reject the null hypothesis more often than it should. The disadvantage of ADF should be noticed. On the other hand, the null hypothesis is easily rejected when the null is right or sample size is too small without sufficient information (Brooks, 2008, Wang, 2008).

KPSS Test

The Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test (Kwiatkowski et al., 1992) is an alternative of ADF test for unit root test. The null hypothesis of this test is the time series is trend stationary with alternative of unit root.

$H_0 : \sigma_t^2 = 0$ against $H_A : \sigma_t^2 > 0$

The KPSS statistic is $$KPSS = \frac{1}{T^2} \cdot \frac{\sum_{t=1}^{T} S_t^2}{\hat{\sigma}_\infty^2} \qquad (4\text{-}4)$$

where T is the sample size, $S_t = \sum_{s=1}^{t} \hat{e}_s$ is a sum of $\hat{e}_t$, $\hat{\sigma}_\infty^2$ is the estimator of the variance of residuals $\hat{e}_t$. This statistic is an LM test and the asymptotic distribution. A large KPSS will reject the null of stationarity.

Kwiatkowski, Phillips, Schmidt and Shin (1992) states that the KPSS method not only test the stationarity of time series, but it also can distinguish if a series appears to be stationary, or series appears to have a unit root, or series hold insufficient information to determine its stationarity. Furthermore, the KPSS has an advantage that it is more powerful to reject null hypothesis in the presence of breaks (Chen, 2002).

The DF and ADF in the case of heteroskedasticity, these statistics are similar and tests are suffered from high probability of potential acceptance of null hypothesis. Nevertheless, there is not a superior test for unit root hypothesis (Wang, 2008, Gujarati 2003). Thus, the DF, ADF and KPSS statistics are applied in this study.

Random walk with drift and trend-stationary process are two main categories of non-stationary time series. Non-stationary time series show positive autocorrelation and result in spurious regressions which increase difficulty of estimating or forecasting. Taking the first difference of the time series can remove non-stationarity in most cases, while rare series requires two or more difference to reach stationarity. The process reaches a difference stationarity. Stationary process has more desirable properties statistically. It somehow explains the theory of Campbell, Lo and Mackinlay (1997) that it is more appropriate to use return instead of price in most financial studies (Tsay, 2005).

Stationary time series is noted as $I(0)$. If the series makes stationarity at the kth difference, the series is defined as kth integrated $I(k)$. If two non-stationary time series have a linear relationship between them, and if their difference achieves stationarity at order of m and n respectively, they are cointegrated, probably with lower order of integration than m and n.

Apart from the above two tests, the Bayesian (Sims, 1988) method is also commonly used, and it is especially useful when the series is not standard-Dickey-Fuller distribution.

Notwithstanding, it is not as practical as PP or DF tests since there is not a simple way to generalize intercept or trend in programming[31].

Skewness

Skewness is the standardized third moment testing symmetry of a time series around its mean. Normal distribution has zero value of skewness, but that the value of skewness is zero does not necessarily result in normal or symmetric distribution. A likelihood-symmetric distribution has a value of 0 in skewness. A series that shows skewness is not normally distributed and has asymmetric distribution. When skewness presents, a negative skewness indicates longer tail on the left-hand side of its mean where mass observations lies in this side; otherwise, it has longer tail on right-hand side. Most time series do not follow normal distribution. Hence it is important to consider skewness and to do normality test (Brooks, 2008).

Kurtosis

Kurtosis is the ratio of the conditional fourth moment to the square of the conditional variance of a normally distributed time series and tests the peakedness of it.

$$\gamma = E(u_t^{\ 4} \mid \Omega_{t-1}) / 3[E(u_t^{\ 2} \mid \Omega_{t-1})]^2 \qquad (4\text{-}5)$$

Where the γ is kurtosis, the $E(u_t^{\ 4})$ and $E(u_t^{\ 2})$ are the fourth and second moments of the time series. To make the measure simpler, the statistic is reset as $(\gamma - 3)$. The normal distribution has value of kurtosis of 0 that is called as mesokurtic. If the value of kurtosis is larger than 0, the time series is called leptokurtic and it has fatter tail and higher peak value around the mean than normal distributed series does. Otherwise, less than 0 of kurtosis refers to a platykurtic time series that exhibits lower and wider peak with thinner tail than normal distribution.

The higher the kurtosis, the more infrequent extreme deviations on variance and the more extreme values in this time series, and vice versa. As increasing of kurtosis, the difference between unconditional and conditional distribution becomes more obvious (Gouriéroux 1997). The student's *t* distribution and Laplace distribution are leptokurtic distributions with fatter tails. While the uniform distribution and Bernoulli distributions show platykurtosis. In most cases, financial and economic time series have leptokurtic

[31] From Rats 6.2 user guide (1)

fatter tail distribution rather than normal distribution, especially for high-frequency data. In other words, leptokurtic distribution is more appropriate to fit economic time series and residuals of them than normal distribution. The student's *t* and generalized error distribution (GED) strengthen the description of conditional leptokurtosis (Lütkepohl and Krätzig, 2004). Statistically, the student's *t* distribution has a greater likelihood value compared with normal distribution (Brooks, 2008, Enders, 2009).

4.2.2 Time Series Forecasting

Many of time series models are proposed for forecasting and prediction in latest two decades. Time series forecasting is used to discover the future movement of time series based on historical data. As long as historical data provides available numerical information and it is assumed that the future pattern of the time series is the extension of past pattern, the forecasting of price, return, volatility and others which involve some uncontrollable events is meaningful for successful decision-making (Gouriéroux 1997).

It is infeasible to investigate how and why the forecasted time series behavior this way, since the volatility of many time series (say financial time series) significantly depends on the economy, the policy, wars and other events happening in the world. If the forecast is aiming to predict the future value without analyzing the reason for it, for example, the corresponding events that take place in reality, the quantitative forecast is appropriate for this purpose (Makridakis et al., 1998).

Two decades ago, a class of models attracts some attention of scholars, such as the autoregressive (AR) and moving average (MA). The AR model composes a linear regression of the current value Y_t and its past value. The error terms in AR are weak white noise and not autocorrelated. The MA model takes the moving average of past error terms to explicate Y_t. Combining the AR and MA obtains the autoregressive moving average processes (ARMA) model. The ARMA (p, q) model can be written as

$$Y_t = c + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \cdots - \theta_q e_{t-q} \tag{4-6}$$

where c is constant term; e_t is white noise error term; the ϕ_1 and θ_1 are assumed to be between -1 and +1 to guarantee the stationarity of the time series. If the ARMA process is not stationary, we can take the k th difference to a ARIMA (p, k, q) process. In the ARMA model, the Y_t depends upon its previous value and white noise in previous time period.

This linear process is parsimonious and tractable in which the estimation and forecast are easily obtained. The ARMA model makes success over other models because of its various applications in 1970s. However, this model could not fit financial data well because it simulates dynamics using linear autoregressive and moving average terms without some specific constraints. Additionally, the linear ARMA model is not capable of capturing some nonlinear properties of financial data, say heteroskedasticity of residual term[32] (Gouriéroux, 1997).

In 1982, Engle proposes the autoregressive conditional heteroskedasticity (ARCH) model to interpret some nonlinear features of financial and other time series. I will bring in some more detail in next section.

Normal and student's *t* distributions

In this study, the sample size of the five commodities is moderate large. Based on the Central limit theorem (CLT) which states that the pattern of large sample approximately follows normal distribution statistically (Parks, 1992), the u_t in the mean equation is assumed as conditionally normally distributed with mean 0 and conditional variance H_t.

However, financial and economic time series generally have leptokurtic fatter tail distribution than normal distribution for high-frequency data (see DeGennaro, 1990, Lütkepohl and Krätzig, 2004). Based on the evidence of non-normality, skewness and kurtosis of returns and log returns for all five commodities, the conditional normal u_t does not well fit the observations. Additionally, kurtosis and skewness show up in both log returns and log-difference returns for all five commodities. A better choice to model u_t is employing conditional student's *t* distribution which is managed to capture fatter/thinner tail and higher/lower peak than normal distribution by changing degree of freedom. The student's *t* distribution is equivalent to normal distribution when the degree of freedom approaches a sufficient large number.

The initial value of the degree of freedom of student's distribution is set, and the most appropriate degree would be worked out for the purpose of getting the most accurate simulation of observations. From this perspective, the student's *t* distribution is more

[32] Heteroscedasticity is the contrast of homoscedasticity in which the variance of disturbance term is consistent over time. Heteroscedasticity means a time-varying variance.

proper than normal distribution as the presence of skewness and kurtosis.

The Normal distribution cannot adequately take into account all leptokurtosis. Baillie and DeGennaro (1990) approves the success of GARCH in mean with student's *t* residual innovation on fitting daily and monthly stock return. Ignoring or non-adequately capturing the conditional leptokurtosis may result in spurious outcome. The exponential GARCH (EGARCH) model, the jump-diffusion process with ARCH errors and semi-nonparametric method are alternative approaches to model the leptokurtosis (Bollerslev et al., 1992).

Brooks (2008) argues that the non-normality of some time series may due to the effect of extreme value or high heteroskedasticity when the sample size is large. Put it in another way, the rejection of normality in the Jarque-Bera test does not necessarily imply non-normality for sufficient large sample. Yet based on the basic statistics of log returns and difference-log returns, the appropriation of conditional student's *t* distribution is rather theoretical so far. Hence forecasting with both normally and student's *t* distributed residuals are applied in this study.

4.3 Univariate ARCH/GARCH Models

Most economic time series show unusual jumps with high volatility and then move back to normal condition. The presence of sudden 'jump' generally indicates that the constant variance assumption is violated in most cases. The ARCH model is the first framework to capture the inconsistent variance in time series.

Heteroskedasticity

For an AR (1) process $Y_t = \mu + \varphi Y_{t-1} + u_t$ (for $\forall t, |\varphi| < 1$), the conditional variance of residual term u_t which is also the conditional variance of Y_t varies over time, and the time-dependent conditional variance is called heteroskedastic variance. The following scatter plots of Y_t show homoskedasticity and heteroskedasticity of disturbance.

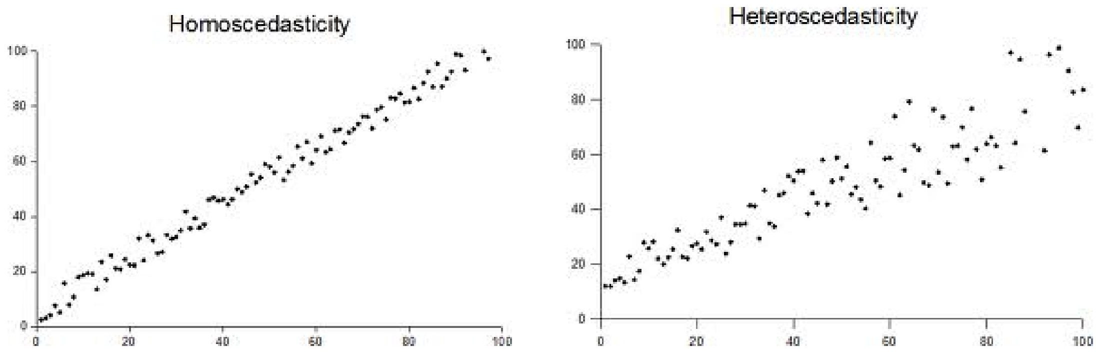


Figure 4-1 plots of homoscedasticity and heteroskedasticity of disturbance

The observations of time series that have homoscedasticity disturbance gather together around the mean; but some observations of time series with heteroscedastic residuals spread away from the mean. The main reasons of existence of heteroskedasticity would be a decreasing variance from better data collecting techniques, or/and presence of outlier, or/and skewness[33] of regressor(s), or/and misspecification or/and incorrect data transformation or functional form, especially in cross-sectional data (Gouriéroux, 1997).

4.3.1 ARCH/GARCH Models

ARCH Model

The traditional OLS is said to be the best unbiased estimation method, whatsoever, it is not able to properly estimate coefficient in case of heteroskedasticity. However, the ARCH model that describes the heteroskedasticity of conditional variance by taking autoregressive square errors of previous periods is a much more efficient approach for estimation (Engle, 1983).

The ARCH (autoregressive conditional heteroskedasticity model) framework considers the influence of new arrived information on current mean and variance. For an AR (1) model, the conditional mean and conditional variance of residual term u_t are $E(u_t \mid \Omega_{t-1}) = 0, \forall t$ and $V(u_t \mid \Omega_{t-1}) = \sigma_t^2 = H_t$, where $u_t \mid \Omega_t \sim N(0, H_t)$ and Ω_t is cumulate information until time t. The conditional variance of u_t is time-dependent of past history of u_{t-i}. Base on this theory, the ARCH (q) is defined as (Engle, 1982):

[33] Skewed distribution is asymmetric about mean and it has a longer tail than normal distribution.

$$H_t = c + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + \cdots + a_q u = c + \sum_{i=1}^{q} a_i u_{t-i}^2 \quad (4\text{-}7)$$

where q is the number of lags of previous residuals that ARCH model accounts for; the u_t is written as $u_t = \sigma_t \varepsilon_t$ sometimes where the ε_t is a white noise and independent and identically normal distributed; to ensure positive variance and avoid meaningless H_t, parameters c and a_i are assumed to be positive for $0 < i \le q$. The value of a_i reflects how much the ith past squared residual contribute to the current conditional variance. The a_1 is restricted to be $0 < a_1 < 1$ in order to ensure the stability of AR process in ARCH model. The residuals $\{u_t\}$ are uncorrelated, but the realized values of squared residuals u_{t-i}^2 are not independent with each other. Furthermore, the larger the u_{t-i}^2, the higher the value of conditional variance H_t. The effect of past squared residuals on conditional variance exhibits volatility clustering that large/small shock is followed by another large/small shock (Brooks, 2008).

The conditional variance from ARCH framework offers great potential to forecast, but the traditional unconditional variance does not. For an AR (1) process, the one-step ahead forecasted variance of y_{t+1} is $v(y_{t+1}) = E(y_{t+1} - E(y_{t+1}))^2$, by arranging, it is written as

$$v(y_{t+1}) = E(y_{t+1} - a_0 - a_1 y_t)^2 = \sigma^2 \quad (16)$$

In the case of unconditional variance, the forecasted variance is

$$v(y_{t+1}) = E(y_{t+1} - a_0/(1-a_1))^2 = \sigma^2/1 - a_1^2 \quad (17)$$

Obviously, the unconditional variance is larger than conditional variance. The ARCH model has advantages of providing better forecasts and smaller the conditional variance (Enders, 2009).

The ARCH model is initialed to measure the dynamic price movement of underlying assets and it is a simple and functional model in practice. The ARCH model describes conditional mean and variance at the same time which boosts the application in estimating and forecasting volatility, return and others relevant to investment and other financial behaviors in various economic markets (Gouriéroux, 1997).

Empirically, most researches test the number of lags q using partial autocorrelation function (PACF) of u_t^2, since there is no best proper technique to calculate the q. The number of parameters increases as the raise of order the ARCH, and hence the simple ARCH model with high order becomes no longer parsimonious with complicated computation process. In ARCH (1) model, the fourth moment exists if and only if the a_1^2 is $0 \le a_1^2 \le 1/3$; for the higher order ARCH model, the constraint on a_1^2 might be too complicated to reach. Tsay (2005) points out that the ARCH model probably yields over-prediction on volatility when a single sudden jump takes place to which the ARCH could not instantly respond. The ARCH model could not reflect the different effects of good and bad news on underlying asset. These drawbacks indicate its limitation on fitting return and other similar time series (Enders, 2009, Brooks, 2008).

GARCH Model

Bollerslev (1986) suggests the generalized ARCH (GARCH) model to modify the ARCH and the GARCH model successfully broads its implications. Based on ARCH framework, the GARCH model incorporates the past conditional variance of residuals which is called GARCH effect. The GARCH (p, q) model is:

$$H_t = \sigma_t^2 = c + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{j=1}^{p} b_j \sigma_{t-j}^2 \tag{4-8}$$

where the $\sum_{i=1}^{q} a_i u_{t-i}^2$ is ARCH term; p is the number of lags of past conditional variance and the $\sum_{j=1}^{p} b_j \sigma_{t-j}^2$ term represent the GARCH effect. For GARCH (1, 1) model, the weak stationary requires $0 < a_1 + b_1 < 1$. Nesting test of the conditional variance makes the process more traceable. In other words, the GARCH model suffices the conditions of prediction. If $p = 0$, the GARCH(p, q) model converges to ARCH(q) model.

The same with ARCH model, all coefficients of GARCH are required to be positive for non-negative variance, yet the GARCH itself is less likely to break this assumption (Brooks, 2008). According to Degiannakis and Xekalaki (2004), the special case is that if slightly negative parameters show up for high order lags, it does not necessarily produce negative conditional variance. All characteristic roots have to be less than 1 in order to guarantee a finite variance. Adding the moving-average variance of previous period makes the GARCH model more parsimonious because the GARCH model can be deduced to an infinite order of ARCH model provided that $b_1 < 1$.

$$H_t = \frac{a_0}{1-b_1} + a_1 \sum_{i=1}^{\infty} b_1^{i-1} u_{t-i}^2 \qquad (4\text{-}9)$$

The current conditional variance can be explained as infinite cumulative impacts of previous shocks (squared residuals). When the ARCH term has higher order, the GARCH framework has a more concise formula. Apart from autocorrelated squared residuals, the variance of residuals is assumed to be correlated due to volatility clustering (Enders, 2009, Gujarati 2003). The conditional variance at a time point can be obtained from both ARCH and GARCH models. The GARCH fills in the gap between discrete and continuous time series models. When the time interval is sufficiently small, the discrete variance of GARCH converges to a continuous conditional variance time diffusion series (Bollerslev et al., 1992).

Comparing with ARCH model, the GARCH model also produces unbiased and more accurate estimation and forecasting with more flexible lags, since it allows for longer memory, fits heavier tail distribution than normal distribution and has simpler parametric representation than ARCH model (Tsay, 2005). However, GARCH takes equal weight of past conditional variance, which means that the different-type of news has equivalent effect on current variance. Consequently, the GARCH model has the same drawback as ARCH model on asymmetric information issue.

As early as 1976, Black finds the existence of unequal influence of good and bad news of stock price. In 1990s, Nelson (1991), Gloston, Jagannathan and Runkle (1993) and other scholars suggest EGARCH, GARCH-GJR models and other extensions based on GARCH model to capture the asymmetric information effect. From another perspective, Kroner and Sultan (1993) propose GARCH-X model to take into account short-term deviation in the long run relationship between time series. These developments of GARCH model somehow improve the performance of GARCH theoretically, but the empirical implications could not indicate a consistent result based on controversial support of standard GARCH and other extensions. Due to the computation complication, the GARCH family models usually apply low orders, and the p and q are less than 3 (Tsay, 2005). More detail about others GARCH models based on standard GARCH model will be introduced in next section.

4.3.2 Diagnostic Tests

Some preliminary diagnostic tests are essential for model selection and enhancing the reliability of employed models. For example, if the normality test does not support the normal distribution of residual innovation, it is reasonable to consider student's *t*, or GED distribution or others.

Normality Test

Jarque-Bera (1981) initializes the JB normality tests. This method is widely used in practice, and the Bayesian test is a frequently applied alternative in most cases. The null hypothesis of JB test is that the time series is symmetrically distributed and mesokurtic. The statistic is

$$W = T[\frac{b_1^2}{6} + \frac{(b_2 - 3)^2}{24}] \tag{4-10}$$

Where T is the sample size; $b_1 = \frac{E[u^3]}{(\sigma^2)^{3/2}}$ and $b_2 = \frac{E[u^4]}{(\sigma^2)^2}$. The statistic also can be written as $W = T[\frac{skewness^2}{6} + \frac{(kurtosis - 3)^2}{24}]$ which is relevant to skewness and kurtosis. The test statistic asymptotically follows a χ^2 distribution with order of 2. If the p-value of the JB test is larger than 5%, it fails to reject the normality of series at significant level 5%. Large sample has low potential of following non-normal distribution, yet that non-normality may results from a certain amount of extreme value, or heteroskedasticity (Brooks, 2008).

Lagrange Multiplier ARCH Effect Test

Engle (1982) suggest the Lagrange Multiplier (LM) test which is a test for presence of ARCH effect under conditional normality. It is usually applied for testing raw return data. The null hypothesis assumes that there is no ARCH effect. The statistics is

$$\xi_{LM} = TR^2 \tag{4-11}$$

Where T is the sample size; the R^2 is estimated from the regression model to show how much of the sum of squares has been explained by a regression model and the total sum of squares around the mean; the statistic follows a χ^2 distribution with order of restriction number. For ARCH and GARCH models, the null hypothesis is equivalent to that all coefficients of q lags of squared residuals are not significantly from zero (Brooks, 2002).

Low TR^2 values indicates a failure of reject the null hypothesis, while sufficiently large value of TR^2 deduces the existence of ARCH effect. It is also employed in the case of non-Gaussian distributed residuals (Gouriéroux 1997).

From another perspective, the LM test also checks the adequacy of MGARCH (multivariate GARCH) specification (Bauwens et al., 2006). The Wald test, Portmanteau tests of the Box-Pierce-Ljung type and residual-based diagnostics are alternatives commonly applied in conditional heteroskedasticity models. When the residuals are not normally distributed, the Wald test is more robust than the LM method (Ding and Engle, 2001).

Liung-Box Autocorrelation test

Ljung and Box (1978) modify the overall criterion (Box & Pierce,1970) of testing adequacy of fit and propose the Ljung-Box Q test for examining the serial correlation with null hypothesis of no autocorrelation.

$$Q(\hat{r}) = n^2 \sum_{k=0}^{m} \frac{(\hat{r}_k^*)^2}{(n-k)} \tag{4-12}$$

Where $\hat{r}_k^* = \sum_{t=k+1}^{n} \alpha_{t-k}\hat{a}_t \Big/ (\sum_{t=1}^{n} \alpha_t^2 \sum_{t=1}^{n} \hat{a}_t^2)$ which is an approximate χ^2_{m-p-q} distribution for the model who has $p+q$ parameters; k means testing the variance of k th sample serial correlation between $\{a_t\}$ and $\{\alpha_t\}$; m is the number of lags involved. Comparing with the traditional criterion, the modified Q test makes improvement by providing smaller variance. And the modified approach better fits the data in Box and Jenkins (1970). However, if the a_t is not normally distributed, the Q statistic is less sensitive to the observations that deviate from normal distribution.

Model Selection Criteria

The autocorrelation function (ACF) and partial autocorrelation function (PACF) are two measures on correlation between lags and the number of lags. The ACF and PACF coefficient will be non-zero till the appropriate order of lags.

$$\text{ACF } r_k = \text{cov}(Y_t, Y_{t-k}) / \text{var}(Y_t) \tag{4-13}$$

The ACF is equivalent to PACF at the first lag, yet for the second and larger lags, the PACF formulas are getting more complicated (Gujarati, 2003). The ACF and PACF cannot capture all features of data if the data series cannot be shown with simple graphs. The alternative AIC (Akaike's (1974) information criteria) and BIC (Schwarz's (1978) information criteria) methods are managed to interpret odd series since they get rid of some subjective unexplainable factor. The statistics of AIC and BIC are:

$$AIC = \ln(\hat{\sigma}^2) + \frac{2k}{T}, \quad BIC = \ln(\hat{\sigma}^2) + \frac{k}{T}\ln T \tag{4-14}$$

where $\hat{\sigma}^2$ is estimated variance, k is the number of parameters, and T is the number of observations. The value of AIC and BIC can be negative. A regression with proper number of parameters yields minimum information criteria. In the forecasting, the AIC and BIC are used to test adequacy of order p and q. High p and q will lead to small conditional variance, but the parameter error from estimation is also high which contributes larger error in terms of mean squared error (MSE) (Brockwell and Davis, 2002). In other words, adding additional regressor will result in larger value of AIC and BIC, and the increased value of AIC and BIC is called penalty. As the model get improved, the AIC and BIC may approach negative infinity. For the above formulas, it is easy to see that information criteria decrease as the reduction of sample size. But it is not reasonable by reducing the number of usable observations in order to get smaller AIC and BIC value (Enders, 2009).

The value of information criteria is lower for short regression than that of long regression. Both criterions can be used for in-sample and out-of-sample regressions. Additionally, the AIC can be employed for testing nested and non-nested regression models. The BIC offers proper order of lags asymptotically with harsher and more consistent penalty than that of AIC, but the AIC is more efficient by returning order for moderate size, large size and even infinite large data (Gujarati 2003, Brooks 2008).

4.3.3 Parameter Estimation

Maximum Likelihood

The most widely used method for parameter estimation of ARCH and GARCH models is maximum likelihood (ML). The ML method estimates parameters by maximizing the (log) likelihood function based on the given distribution and models.

The ML approach is more complicated, but more popular than ordinary least square (OLS)[34] estimation because the inconsistent OLS estimator does not fit time series and it

[34] Ordinary least square (OLS) is a popular estimate method which estimates parameters by minimising the sum of squared residuals.

might lead to ‘spurious regression’ because of the ‘significance’ of some explanatory variables which is should not be. On the other hand, the ML approach works well on time series for both linear and non-linear models (Makridakis, Wheelwright et al. 1998, Brooks, 2008).

The log-likelihood function (LLF) is another method to write joint density function. the joint density function of error term is needed because the log-likelihood is more appropriate than likelihood which has stationarity problem of some time series. Hence we use the log-likelihood function (LLF) in this study. For an AR (1) model, the error terms u_t are identical independent distributed (i.i.d.). Under the GARCH (1, 1) model, the joint density function of errors u_t for $t > 1$ is written as $f(u_1, u_2, u_3 \cdots u_n \mid \theta) = \prod_{i=1}^{n} f(u_i \mid \theta)$

The log-likelihood function in this case will be:

$$\ln L(\theta \mid u_1, u_2, u_3 \cdots u_n) = \sum_{i=1}^{n} \ln f(u_i \mid \theta) \qquad (4\text{-}15)$$

Substitute the σ_i^2 with h_t at time t, the LLF of normal distributed residuals is given as:

$$\ln L = -\frac{TN}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log h_t - \frac{1}{2}\sum_{t=1}^{T}\frac{u_t^2}{h_t} \qquad (4\text{-}16)$$

Because most time series do not follow normal distribution and have fatter tails, the student’s *t* distribution that describes fatter/thinner tail than normal distribution is employed (Tsay, 2005). Under student’s *t* distribution, the LLF is derived as follows:

$$\ln L' = N(\log(\Gamma(\frac{v+1}{2}) - \log(\Gamma(\frac{v}{2})) - \frac{1}{2}\sum_{t=1}^{N}\log(v-2)h_t \qquad (4\text{-}17)$$

where $\Gamma(\cdot)$ is a gamma function that $\Gamma(h) = \int_0^\infty x^{h-1}\exp(-x)dx$ and v is degree of freedom ($v > 2$). When the v goes to infinity, the LLF of student’s t distribution converges to that of generalized error distribution (GED)[35]. Since $\varepsilon_t = u_t\sqrt{h_t}$ and the ε_t is the residual at time t from mean equation, we can substitute the h_t and y_t with form of ε_t and take differential of LLF with respect to unknown parameters. Different from the case of independent random variables, we set the partial differential equations equal to zero and iteratively compute to reach the optimization. Hence the estimated parameters can be

[35] Some empirical articles find the insufficient of student’s t distributed in capturing fat tails, and hence the GED is introduced. More detail about the analogy of students t and GED can be found in Lütkepohl, H. and M. Krätzig (2004).

obtained (Enders, 2009).

4.4 Multivariate GARCH Model

Volatilities of economic time series move fairly independently, but the patterns show that they will band together eventually in financial markets. A single univariate GARCH model cannot capture this feature. Simultaneous equations which construct separate univariate models at the same time can capture the relationship among variables (such as volatilities), but the OLS estimation will lead to biased estimation of coefficient due to the inconsistent estimator. In other words, the simultaneous system is not the best choice in economics and finance (Brooks, 2008).

It is necessary to employ multivariate models to incorporate related variables of time series because multivariate model provides a more straightforward way to describe the relationship among them. The implication of multivariate models broadens the research perspective in real investment and risk management selection (Bauwens et al., 2006). For example, the multivariate GARCH models result in more reasonable and accurate computation on the hedge ratio, option pricing. Additionally, the precise estimation of time-dependent covariance between returns or volatilities is helpful for portfolio (Degiannakis and Xekalaki, 2004).

4.4.1 Representations of GARCH Model

All multivariate GARCH models we will introduce in this study have constant conditional correlation (CCC).

4.4.1.1 Full Parameterisation

VECH Model

Bollerslev (1988) suggests a general representation of GARCH model, which is called VECH model. The VECH writes the covariance matrix as vectors. This formulation reduces the dimension of GARCH model from $N \times N$ to $N(N+1)/2 \times 1$ due to the property of covariance matrix that $\text{cov}(x_i, x_j) = \text{cov}(x_j, x_i)$.

The VECH (p, q) model is

$$vech(H_t) = vech(CC') + \sum_{i=1}^{q} Avech(u_t u_t') + \sum_{j=1}^{p} Bvech(H_{t-j}) \tag{4-18}$$

Where $vech(.)$ is a $N(N+1)/2 \times 1$ vector, and the dimension of $vech(.)$ is only 3 for bivariate GARCH model; p and q are non-negative integers, C is the parameter vector for constant term, A and B are symmetric parameter vectors with order of $N(N+1)/2$ for cross product of residuals and lagged conditional covariance respectively. The total number of parameter is $N(N+1)(N(N+1)+1)/2$ (Wang, 2008).

The VECH representation of multivariate GARCH model makes the model more visible and straightforward. The bivariate GARCH (1, 1) is written as

$$H_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{01} \\ c_{02} \\ c_{03} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \quad \text{(4-19)}$$

where $h_{11,t}$ and $h_{22,t}$ are conditional variance of time series 1 and 2; the $h_{12,t}$ is the covariance between the two series; a_{ij} represents the effect of squared residuals and cross impact of past residuals on the current variance or covariance, b_{ij} explains the influence of previous conditional variance and covariance on the covariance matrix in the same way. The $h_{21,t}$ is not shown in the vector matrix because it has the same parameter with $h_{12,t}$ and the $h_{21,t}$ is reasonably omitted to avoid redundant. In this case, the number of parameters is 21 for bivariate GARCH model; for higher order, it increases dramatically with complicated computation problem (Bauwens et al., 2006). The VECH-GARCH model cannot ensure the positivity of variance-covarance matrix which is supposed to be positive semi-definite (Brooks, 2008).

DVECH Model

Respect to the problem of large number of parameters, Bollerslev and Engle (1988) propose the diagonal VECH (DVECH) model. In this representation, the number of parameters decreases, while some constraints are imposed.

The DVECH GARCH (1, 1) is

$$H_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{12} & 0 \\ 0 & 0 & a_{22} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{12} & 0 \\ 0 & 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \quad \text{(4-20)}$$

Where the parameter matrices A and B are assumed to be diagonal and each covariance only depends on cross-product of residual terms and its own lag. In this simpler representation, interpretation of parameters is easier, i.e. a_{11} and a_{33} represent the effect of squared residuals of two series on the corresponding conditional variance of series; and a_{22} is cross effect of the first lag of residuals on conditional variance between two series. If a_{22} has different sign from a_{33}, it implies a high potential increase of future uncertainty (Wang 2008).

The bivariate DVECH GARCH model reduces the number of parameters from 21 to 9. Generally speaking, it reduces the parameter number from $N(N+1)(N(N+1)+1)/2$ to $N(N+5)/2$. The 'diagonal' parameterization is useful for higher number of series via repeated computation, for example, more commodities or securities are added to the portfolio (Giannopoulos, 1995). This model releases the computation problem of loads of parameters of VECH model in some extent, but the parameter size still increases notably as the increase of order. Nevertheless, the DVECH model is widely used in low order of multivariate GARCH, say 2 or 3 (Tsay, 2005).

4.4.1.2 Positive Definite Parameterisation

The multivariate DVECH GARCH model sets constraint on VECH to successfully decrease the number of parameters in some extent. Another drawback of VECH framework is still not resolved that the conditional covariance matrix might be negative if other restriction is not added on (Degiannakis and Xekalaki, 2004). It is hard to ensure a positive definite conditional covariance matrix in VECH or DVECH models. Baba, Engle, Kraft and Kroner (1991) suggest the BEKK and DBEKK models to guarantee the positivity of conditional covariance matrix.

BEKK Model

The BEKK model is proposed with purpose of avoiding non-negative conditional covariance matrix. The new parameterization is written as

$$H_t = CC' + \sum_{i=1}^{q} A_i (u_{t-i} u'_{t-i}) A'_i + \sum_{j=1}^{p} B_j H_{t-j} B'_j \qquad (4\text{-}21)$$

The BEKK model is not a vector framework. Where C is a lower triangular matrix and A_i, B_j are $N \times N$ symmetric parameter matrices. As long as the constant term CC' is positive, the H_t is guaranteed to be non-negative. If the summation of eigenvalues of $\sum_{i=1}^{q} A_i A_i{}' + \sum_{j=1}^{p} B_j B_j{}'$ is less than 1, the covariance is stationary.

Comparing with DCC-GARCH model, the BEKK model is capable of obtaining consistent estimates of dynamic conditional correlation.

The bivariate BEKK GARCH (1, 1) is

$$H_t = CC' + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} u_{1,t-1}^2 & u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}u_{1,t-1} & u_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' H_{t-1} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \quad \text{(4-22)}$$

Suppressing the time subscripts and the GARCH terms, the BEKK GARCH (1, 1) will be

$$\begin{aligned} h_{11} &= c_{11} + a_{11}^2 u_1^2 + 2a_{11}a_{21}u_1u_2 + a_{21}^2 u_2^2 \\ h_{12} &= c_{12} + a_{11}a_{21}u_1^2 + (a_{21}a_{12} + a_{11}a_{22})u_1u_2 + a_{21}a_{22}u_2^2 \\ h_{22} &= c_{13} + a_{12}^2 u_1^2 + 2a_{12}a_{22}u_1u_2 + a_{22}^2 u_2^2 \end{aligned} \quad \text{(4-23)}$$

In this representation, the diagonal elements in C, the parameters a_{11} and b_{11} are restricted to be positive. Notice that the BEKK model has more parameters than DVECH model because the BEKK allows for time-varying dependence among series. In the case of bivariate GARCH, the BEKK has 11 parameters rather than 9 of DVECH approach. The number of parameters of BEKK is generalized as $N^3 + N(N+1)/2$, and it increases strikingly with the increasing order of N (Tsay, 2005). In the BEKK model, the parameters do not have actual meaning; in other words, they do not explain how much impact and shocks of squared residuals, cross product of residuals and its lags on the conditional covariance (Engle et al., 1991).

DBEKK Model

In order to reduce the size of parameter, the diagonal BEKK (DBEKK) model is given by Engle and Kroner (1995). The diagonal BEKK GARCH (1, 1) is written as

$$\begin{aligned} h_{11,t} &= c_{11}^2 + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1} \\ h_{12,t} &= c_{11}c_{12} + a_{11}a_{22}u_{1,t-1}u_{2,t-1} + b_{11}b_{22}h_{12,t-1} \\ h_{22,t} &= c_{12}^2 + c_{22}^2 + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} \end{aligned} \quad \text{(4-24)}$$

The difference between BEKK and DBEKK is similar with VECH and DVECH that the a_{ji} and b_{ji} are omitted for $i \neq j$. The dependence among series is captured by taking products of corresponding parameters of two variances which lead to reduction of 2 parameters (Bauwens et al., 2006).

Wang (2008) states that the VECH and BEKK models are equivalent provided some non-linear restrictions. Parameters of BEKK model represent the similar meaning of those of

VECH with restrictions. The BEKK has apparent advantages. It allows for dynamic dependence among series, and it makes the parameters problem less severe. Last but not the least, the BEKK are able to ensure positivity of conditional covariance under weak conditions. On the other hand, some researchers point out that the estimated parameters from BEKK and DBEKK models are insignificant in some cases. However, the small amount of negative evidences could not halt the expanding application of (D) BEKK models (Wang, 2008).

4.4.2 Variations of GARCH Models

Besides the various representations of multivariate GARCH models, the GARCH models are extended to many other frameworks through considering other factors, such as asymmetric effect of news, cointegration and so on.

4.4.2.1 Asymmetric GARCH Models

Black (1965) finds that bad news has larger effect on return volatility than good news does in stock market. The asymmetric impact of negative and positive news with the same absolute value is commonly explained as leverage effect[36](Bauwens et al., 2006). Negative news is a sign of higher volatility that may result in more risk. For investors, it becomes more necessary to simulate this asymmetric effect. The standard GARCH models could not capture this asymmetry of news impacts. In 1990's, some scholars suggest asymmetric GARCH models to incorporate the news impact, such as EGARCH by Nelson (1991), GJR-GARCH by Glosten, Jagannathan and Runkle (1993), QGARCH by Sentana (1995) and so on (Lütkepohl and Krätzig, 2004).

The GJR-GARCH Model

The GJR-GARCH (Glosten, Jagannathan and Runkle, 1993) model adds on a squared residual term with dummy variable to capture the leverage effects of good and bad news on conditional variance. The GJR (p, q) is expressed as

$$\sigma_t^2 = c + \sum_{i=1}^{q} (a_i + \gamma_i I_{t-i}) u_{t-i}^2 + \sum_{j=1}^{p} b_j \sigma_{t-j}^2 \qquad (4\text{-}25)$$

where I_{t-i} is a dummy variable which takes value 1 when the last residual is negative,

[36] The leverage effect describes the phenomenon that returns rise will lead to decreasing volatility in future and the lower return will result in higher future volatility for many stocks. The negative relationship between return and volatility is called leverage effect (Enders, 2010).

otherwise, it takes 0. In the case of bad news, the potential ARCH effect of the negative information is $(a_i+\gamma_i)u_{t-i}^2$; in the case of good news, the ARCH effect is $a_iu_{t-i}^2$. For the purpose of getting positive conditional variance, the a_i , b_j , γ_i and $a_i+\gamma_i$ are all required to be positive (Glosten et al., 1993). In the recently study, the asymmetry is found not to be leverage effect (Bollerslev, 2008).

The GJR is quite similar with threshold GARCH (TGARCH) model which is proposed by Zakoian (1994). In both GJR and TGARCH model, they add the squared residual term for asymmetric news impact. It is easy to find that they are equivalent by rearranging their formula. In most cases, they are considered as the same method, called GJR-GARCH model (Zakoian, 1994).

As I know, the complete standard formula of multivariate GJR model is not presented in any previous publications. In my study, the bivariate GJR-GARCH (1, 1) model is derived as diagonal framework (DVECH) as follows:

$$H_t=\begin{bmatrix}h_{11,t}\\h_{12,t}\\h_{22,t}\end{bmatrix}=\begin{bmatrix}c_{11}\\c_{12}\\c_{13}\end{bmatrix}+\begin{bmatrix}a_{11}+\gamma_1 I_{t-1}&0&0\\0&a_{22}&0\\0&0&a_{33}+\gamma_3 I_{t-1}\end{bmatrix}\begin{bmatrix}u_{1,t-1}^2\\u_{1,t-1}u_{2,t-1}\\u_{2,t-1}^2\end{bmatrix}+\begin{bmatrix}b_{11}&0&0\\0&b_{22}&0\\0&0&b_{33}\end{bmatrix}\begin{bmatrix}h_{11,t-1}\\h_{12,t-1}\\h_{22,t-1}\end{bmatrix} \tag{4-26}$$

The asymmetry is on squared residuals of two series, but not on cross-product of residuals. This form of asymmetric effect adapts the principle of GJR model that applying dummy variable on past squared residuals simulates the asymmetric impact of good and bad news on current conditional covariance.

Quadratic GARCH models

The QGARCH model incorporates an additional past residual term in GARCH model as a proxy of asymmetric effect of negative and positive shocks on conditional (co) variance. Sentana (1995) believed that the past residual is a proper factor to capture the leverage effect.

The univariate QGARCH (1, 1) is

$$\sigma_t^2=c+a_1u_{t-1}^2+b_1\sigma_{t-1}^2+d_1u_{t-1} \tag{4-27}$$

Where the d_1u_{t-1} measures the asymmetric effect, and the generalized QGARCH (p, q) is written as

$$\sigma_t^2 = c + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{j=1}^{p} b_j \sigma_{t-j}^2 + \sum_{i=1}^{q} d_i u_{t-i} + 2\sum_{i=1}^{q} \sum_{j=i+1}^{p} e_{il} u_{t-i} u_{t-j} \tag{4-28}$$

In the higher order univariate QGARCH model, the term $\sum_{i=1}^{q} \sum_{j=i+1}^{p} e_{il} u_{t-i} u_{t-j}$ shows that the cross-product of previous residuals and the $\sum_{i=1}^{q} d_i u_{t-i}$ error term have impacts on current conditional variance. The QGARCH uses a simple way to test the asymmetric impact of news on conditional variance, even in the high order model. Similar with TGARCH (GJR) model, the QGARCH also ensures the positivity of conditional covariance. Yet it is widely accepted that all parameters are assumed to be non-negative to avoid complicated statistic problem. The conditional variance from QGARCH is consistent with other economic and financial models theoretically, such as Black-Scholes and Monte Carol methods. The QGARCH improves ARCH model on asymmetric new impact and it is easily used in financial market. Comparing with standard GARCH model, the QGARCH captures the asymmetry information impact and higher kurtosis which better fits features of the economic and financial data (Sentana, 1995).

The DVECH parameterization of bivariate QGARCH (1, 1) is easily derived.

$$\begin{aligned} h_{11,t} &= c_{11} + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1} + d_{11} u_{1,t-1} \\ h_{12,t} &= c_{12} + a_{12} u_{1,t-1} u_{2,t-1} + b_{12} h_{12,t-1} \\ h_{22,t} &= c_{22} + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} + d_{22} u_{2,t-1} \end{aligned} \tag{4-29}$$

The multivariate QGARCH model benefits from relatively small computational burden as increase of order compared with standard GARCH model. In the empirical study of Franses and Van Dijk (1996), the QGARCH is sensitive to extreme values, however, it performs best in volatility forecasting in random walk, standard GARCH and GJR-GARCH models after filtering extreme observations in stock market.

Differences among Three Asymmetric Models

Bollerslev (2007) proposes a uni-form APGARCH (asymmetric power GARCH) model for asymmetric GARCH models:

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{q} \alpha_i (|u_{t-i}| - \gamma_i u_{t-i})^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \tag{4-30}$$

where the γ_i contributes to the asymmetric impacts, such as $\gamma_i = 0$ for the standard GARCH model. For the asymmetric GJR(TGARCH) model, $\delta = 2$, $0 \le \gamma_i \le 1$; and for the

QGARCH model, $\delta = 2$, $\gamma_i \neq 0$ (Bollerslev, 2008).

The asymmetric GJR and QGARCH models consider asymmetric responses of negative and positive shocks on conditional covariance in two different ways. In theory, the two frameworks have the same favorable characters on capturing asymmetric information effect, but their performances in various cases are different in empirical studies. The exponential GARCH (EGARCH) model proposed by Nelson (1991) is included for comparing asymmetric GARCH models.

They all relax the non-negative restriction on parameters, but all parameters are required to be non-negative in terms of statistic interpretation. The GJR (TGARCH) has advantages over standard GARCH and EGARCH models in different aspects. Firstly, the GJR (TGARCH) is able to capture inverted asymmetries that is small positive values might response strongly than small negative values with the same absolute value, and sometimes it is also true for large values (Rabemananjara and Zakoian, 1993). Secondly, the EGARCH considers natural logarithm of conditional deviation which makes it easier to have convergence problem than GJR and QGARCH models due to the complex exponential formula when the order is increasing (Degiannakis and Xekalaki, 2004). The severe convergence issue of EGARCH is not resolved so far, and many researchers are in favor of GJR, QGARCH and other asymmetric GARCH models.

The GJR model is less sensitive to extreme values than QGARCH. Taking evidence from Franses and Van Dijk (1996), asymmetric GARCH models (say GJR and QGARCH) are sensitive to extreme stock returns in with-in sample tests, and the GJR is relatively less responsive to extreme observations than QGARCH. However, in the condition of extreme value free, the QGARCH provides the most precise out-of-sample volatility prediction in random walk, standard GARCH, GJR-GARCH and QGARCH models; and the forecasting power of GJR is demonstrated on volatility prediction in this study. Notwithstanding, Engle and Ng (1993) showed that GJR parametric model produces the most accurate prediction on volatility and the lowest conditional variance of portfolio for Japanese stock return from 1980 to 1988. Ulu (2005) tested the forecasting power of QGARCH on volatility and demonstrates its high prediction ability in terms of MZ[37] volatility forecast

[37] MZ, see footnote 44 in Literature Review.

regression.

It is hard to define a single model as 'the best' forecasting asymmetric model in general regarding to the above evidence. The more reasonable way to investigate the prediction power of them is measuring forecasting models on specific assets and in a specific market and revealing the most powerful forecasting model in the specific case. Apart from asymmetric models, some other linear/nonlinear symmetric GARCH models are powerful in time series forecasting. It is worth to putting more effort to study and compare their prediction ability.

4.4.2.2 Cointegration and Error Correction Models

Cointegration

Cointegration states that two related series may move apart in a short term, but they will move back together in the long-run. This cointegration binds them together at some particular time point in future. This phenomenon frequently appears in economic and financial markets, such as cash price of underlying asset and futures prices.

The integration of a series shows the number of times of differencing that is needed for the series to achieve stationarity, for example, stationary series has no integration with order zero, written as $I(0)$; a series reaches stationarity by differencing n times, which means it is an integrated process $I(n)$ and called an integrated series with order of n. If two series are non-stationary integrated processes with $I(m)$ and $I(n)$, and if the residuals of their linear regression is a stationary series, the two series are called 'cointegrated'. Statistically, the cointegration is described as follows:

$$y_{1t} = \beta y_{2t} + \varepsilon_t \tag{4-31}$$

If the series y_{1t} and y_{2t} are $I(1)$ processes, and the residual term ε_t is $I(0)$, the two series are said to be cointegrated (Maddala and Kim, 1998). Because the order of cointegration is always equal to or less than the integrated order of any of series, differencing individual series to obtain stationary for cointegrated system may lead to over-differencing problem which increases estimation burden (Tsay, 2005).

Error-Correction Model

The over-differencing issue is solved by Engle and Granger (1987) who propose the error-correction model. Assume that $y_{1,t}$ and $y_{2,t}$ are two $I(1)$ non-stationary processes, and they are cointegrated with coefficient γ. The linear regression between the differences of the two series $y_{1,t}$ and $y_{2,t}$ is constructed as 'error correction model ':

$$\Delta y_{1,t} = \beta_1 \Delta y_{2,t} + \beta_2 (y_{1,t-1} - \gamma y_{2,t-1}) + \varepsilon_t \qquad (4\text{-}32)$$

where β_1 represents the short-run relationship between differences of two series; β_2 reflects the adjustment speed from disequilibrium of previous period back to equilibrium[38]; the $y_{1,t-1} - \gamma y_{2,t-1}$ term is named 'error correction term' which is stationary. We should notice that the error correction term involves the first lag of two series to reach equilibrium (Brooks, 2008).

It is supportive that the error-correction mechanism (ECM) and the cointegration cause each other and the lead-lag issue can be ignored. The ECM is broadly used for multi variables. As Tsay (2005) says, the error correction term can be considered as a compensation for the over-differencing of $\Delta y_{1,t}$. Basically, this is also the principle of the ECM model to overcome over-differencing problem.

There is sufficient theoretical and empirical evidence to emphasize the importance of cointegration in economic and financial markets. In theory, the feature that the order of cointegration is not higher than any of integration series diminishes computation problem of high order. Taking the cointegration as a proxy of equilibrium makes the simple linear regression of cointegration is more meaningful. The ECM combines the short run and long-run relationship among variables. It contributes to model changes and to adjust the disequilibrium to equilibrium of past period in a long run. Furthermore, the ECM is mathematically simple since it decreases the potential of presence of multicollinearity[39], and the error correction term is easy to obtain from estimated equation (Özkaya and Korürek, 2010). In financial market, since the cash and futures prices of underlying asset are closely relevant, there must be a constraint to tie them together at maturity; otherwise, the cash and futures prices may deviate apart and never move back at some time in future which is not real in practice (Brooks, 2008).

The tests of cointegration are various. All Unit root and stationarity tests are suitable for

[38] The cointegration is also called equilibrium phenomenon, because the cointegration forces them to move together in a long run, no matter how far they move apart in a short term that will not persistent.

[39] Multicollinearity is the high correlation among n ($n \geq 2$) time series.

cointegration test because stationary residual term implies the existence of cointegration. The ADF, DW and PP approaches have null hypothesis of non-stationarity, and the acceptance of null hypothesis suggest the presence of cointegration (Brooks, 2008). The Engle and Granger (1987) state that the ADF can be the rough test in cointegration test. Some other representations of cointegration are employed; say VAR (Banerjee, 1993), Engle-Granger 2-step test and Engle & Yoo method.

Engle-Granger 2-step Cointegration Test

Engle and Granger (1987) introduce the 'Engle-Granger 2-step' method (EG) to test the cointegration. The first step is testing the existence of 'cointegrated relationship' between two time series $y_{1,t}$ and $y_{2,t}$ [40]. If unit root shows up in both time series, the two series are non-stationary. Then test the stationarity of difference between $y_{1,t}$ and $y_{1,t-1}$, and between $y_{2,t}$ and $y_{2,t-1}$. If the difference series are stationary and the residuals of a linear regression $y_{1t} = \alpha + \beta y_{2t} + \varepsilon_t$ is also stationary, the series $y_{2,t}$ and $y_{2,t-1}$ are said to be cointegrated with order 1. Brooks (2008) suggests to further test the relationship by adding more lags to take into account autocorrelation of residual term.

In the second step, an error-correction term is plugged in as a variable to the model.

$$\Delta y_{1t} = \alpha + \beta_1 \Delta y_{2t} + \beta_2 z_{t-1} + u_t \tag{4-33}$$

where the z_{t-1} is the first lag of residuals and called the error correction term. A significant positive $\hat{\beta}_2$ implies a decrease of $y_{2,t}$ in next period if two price differences are positive in this period. This test is criticized because the hypothesis-free test may cause the insufficient power in unit root and stationary tests, and biasness for small sample size due to non-normality of estimator of cointegrated vector.

Engle and Yoo (1987b) add one more step to modify the Engle-Granger 2-step test. The modification addresses the two problems, but it is still a hypothesis-free test. Engle and Yoo add a third step in order to update the estimates of the cointegrated vector and its standard residuals. An additional regression is involved

$$u_t = \phi(-\beta_2 y_{2t}) + v_t \tag{4-34}$$

And the correction on linear regression is $\beta = \hat{\beta} + \phi$. The adjusted linear regression is

[40] $y_{1,t}$ and $y_{2,t}$ are log-price series.

more efficient, even though the Engle and Yoo method (EY) increases the computational complication and has the problem of hypothesis-free.

In my study, the Engle-Granger 2-step test is applied for cointegration between cash and futures log returns because the two problems about lack of power in unit root test and potential biasness are addressed in large size samples. Additionally, the Engle-Granger 2-step test is easy to understand in both mathematical and financial theory (Brooks, 2008).

GARCH-X Model

Accounting for the error-correction term, the GARCH model is developed by Lee (1994) as GARCH-X model. Lee considers adding an error-correction term to variance for accounting the potential influence of short-run deviations on both conditional mean and variance. This model takes into account the effect of short-run deviation from the long-run cointegration relationship between log cash and futures prices on conditional covariance. Recall the equation 3-2, the hedge ratio is covariance between cash and futures log-return divided by variance of futures log-return, and it will be affected by the short-run deviation from the log-run cointegration relationship between log cash and futures prices. Lee finds that the GARCH-X model provides better fit for daily, short- and long-term monthly exchange, based on the evidence of no serial correlation and skewness and kurtosis reduction and it may boost the prediction power of GARCH model(Lee, 1994).

The GARCH-X model incorporates the long-run cointegration relationship among time series on conditional covariance (covariance for multivariate GARCH-X). The VECH form of AR (1)-GARCH (p, q)-X model is

$$Y_t = \mu + \varphi Y_{t-1} + \beta z_{t-1} + u_t \tag{4-35}$$

$$u_t \mid \Omega_t \sim N(0, H_t) \tag{4-36}$$

$$vech(H_t) = C + \sum_{i=1}^{q} A_i vech(u_{t-i})^2 + \sum_{i=1}^{p} B_i vech(H_{t-i}) + \sum_{i=1}^{k} D_i vech(z_{t-i})^2 \tag{4-37}$$

The error-correction term z_{t-1} in equation (4-35) is the first lag of residuals from linear regression $z_t = y_{1,t} - \gamma y_{2,t}$ and it is stationary in levels. The z_{t-1} measures the long-run equilibrium relationship between log cash and futures prices and the deviation between short-run disequilibrium and its expected value. Hence Lee (1994) states that the cointegration between log cash and futures prices may have important effect on conditional variance and covariance of log-returns in cash and futures markets.

The cointegration term Z_{t-1}^2 is employed as the squared first lag of error correction in the second moment.

Parameters C and D are upper triangular matrices. The extra conditional heteroscedasticity which is un-captured by GARCH model can be interpreted by the square of z_{t-1} in equation (4-37). The GARCH-X model is capable of explaining some special features of cointegrated series (Lee, 1994). In this research, based on the fact that the error-correction term has influence on mean equation for all samples, the potential relationship between the disequilibrium measured by error-correction term and uncertainly examined by conditional variance can be captured by the lagged error-correction terms.

The bivariate GARCH (1, 1)-X model is

$$\begin{aligned} H_{11,t} &= c_{11} + a_{11}u_{1,t-1}^2 + b_{11}H_{11,t-1} + d_{11}z_{t-1}^2 \\ H_{12,t} &= c_{12} + a_{12}u_{1,t-1}u_{2,t-1} + b_{12}H_{12,t-1} + d_{12}z_{t-1}^2 \\ H_{22,t} &= c_{22} + a_{22}u_{2,t-1}^2 + b_{22}H_{22,t-1} + d_{22}z_{t-1}^2 \end{aligned} \tag{4-38}$$

Here the c_{ij}, a_{ij} and b_{ij} are parameters of constant term, ARCH effect and GARCH effect, respectively, and all of them are restricted to be non-negative. The d_{ij} represents impact of short-run deviation of long-run cointegration between two series on conditional variance and covariance. If d_{ij} are not significant different from zero, the GARCH-X model is the same as GARCH model and the effect of short-run deviation can be ignored. If short-run deviation is too large, the prediction of conditional variance becomes less doable.

Brenner and Harjes (1996) suggest an alternative of GARCH-X model to capture the relationship between short-term interest rate and volatility. It is assumed that $E(u_t \mid \Omega_{t-1}) = 0$, $V(u_t \mid \Omega_{t-1}) = \sigma_t^2 r_{t-1}^{2\gamma}$, and the conditional variance is

$$\sigma_t^2 = c + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{j=1}^{p} b_j \sigma_{t-j}^2 \tag{4-39}$$

Similarly, all parameters are required to be non-negative and $\gamma > 0$. Larger shocks have more important influence on conditional variance of volatility. If the $\gamma = 0$, this version of GARCH-X is equivalent with standard GARCH model (Brenner et al., 1996). Basically, the principles of two GARCH-X models are the same that they incorporate effect of short-run deviations of long-run cointegration between series on conditional variance, and they both emphasize the importance of cointegration on variance prediction.

BEKK-GARCH-X

The BEKK representation of GARCH-X is called BEKK-GARCH-X models. The BEKK-GARCH-X (p, q) model is expressed as:

$$H_t = CC' + \sum_{i=1}^{q} A_i (u_{t-i} u'_{t-i}) A'_i + \sum_{j=1}^{p} B_j H_{t-j} B'_j + \sum_{j=1}^{p} D_j z^2_{t-j} D'_j \tag{4-40}$$

Where C and D are upper triangular parameter matrices. The BEKK parameterization of GARCH-X ensures the positivity of H_t (Lee, 1994).

The simplest BEKK-X (1, 1) is

$$H_t = CC' + A_1 (u_{t-1} u'_{t-1}) A'_1 + B_1 H_{t-1} B'_1 + D_1 Z^2_{t-1} D'_1 \tag{4-41}$$

Here A_1 and B_1 are diagonal matrices, and C_1 and D_1 are upper triangular matrices. The bivariate BEKK-GARCH-X model can be rewritten as an equation group as follows:

$$\begin{aligned} H_{11,t} &= c^2_{11} + a^2_{11} u^2_{11,t-1} + b^2_{11} H_{11,t-1} + d^2_{11} z^2_{t-1} \\ H_{12,t} &= c_{11} c_{12} + a_{11} a_{22} u_{1,t-1} u_{2,t-1} + b_{11} b_{22} H_{12,t-1} + d_{11} d_{22} z^2_{t-1} \\ H_{22,t} &= c^2_{12} + c^2_{22} + a^2_{22} u^2_{2,t-1} + b^2_{22} H_{22,t-1} + (d^2_{12} + d^2_{22}) z^2_{t-1} \end{aligned} \tag{4-42}$$

For the first series, the short-run deviation of long-run cointegration of past period brings in d^2_{11} and $d_{11}d_{22}$ to variance of series 1 and covariance between two series, respectively. For the second series, it has impact $(d^2_{12} + d^2_{22})$ on corresponding current conditional variance.

Since the GARCH-X model considers one more factor, the number of parameters of the BEKK representation of GARCH-X increases, and the computation burden becomes heavier due to the more complex covariance matrix. In addition, the probability of getting convergence problem rises up. Lee (1994) tests the presence of cointegration in conditional covariance using Lagrange Multiplier, Wald test and AIC methods and recommends that the AIC is a more robust approach.

Models Comparison

In my study, I try to find out the most powerful forecasting model on OHR among 6 models, containing standard GARCH, BEKK-GARCH, asymmetric GARCHs (GJR, QGARCH), Cointegration-models (GARCH and GARCH-X) in agricultural market. As described, the BEKK decreases the number of parameter, and hence reduces some computational burden. The asymmetric models suggest larger impact of negative shocks than positive shocks on conditional variance in distinct ways, and this feature better fits the time series in economic and financial markets. The GARCH-X and BEKK-GARCH-X models are special since they take into account cointegration with an error-correction term. As lee (1994) says, the cointegration-models are aiming to adequately explain the relationship among cointegrated series.

4.4.3 Forecasting OHR with GARCH Models

4.4.3.1 Parameter Estimation with Multivariate GARCH Models

Some preliminary tests are employed before employing the model, such as the unit-root, cointegration and LM (Lagrange Multiplier) tests. Kroner and Sultan (1993) suggested not incorporating error correction term for commodities as cointegration is not found between cash and futures prices in some previous literatures (see Brenner and Kroner (1993), Baillie and Myers (1991)). However, the cointegration is present for all five commodities in my study and more detail is presented in the next section. The error correction term is included in mean equation for every commodity. The positive and significant test-statistic from LM test implies the validity of incorporating ARCH effect.

In the optimal hedge ratio forecasting, the mean equation with an error correction term and the second moment in a standard bivariate GARCH model are assumed similarly as Kroner and Sultan (1993) (1993a)(1993a)(1993a)(1993a)(1993a)(1993a) that

$$\begin{gathered} Y_t = \mu + \varphi z_{t-1} + u_t \\ u_t \mid \Omega_t \sim N(0, H_t) \\ vech(H_t) = C + \sum_{i=1}^{q} A_i vech(u_{t-i})^2 + \sum_{i=1}^{p} B_i vech(H_{t-i}) \end{gathered} \qquad (4\text{-}43)$$

where $Y_t = \begin{bmatrix} R_t^c \\ R_t^f \end{bmatrix}$ is a 2×1 matrix for bivariate case, the R_t^c and R_t^f are logarithmic returns in cash and futures markets that $R_t^c = \log P_t^c - \log P_{t-1}^c$ and $R_t^f = \log P_t^f - \log P_{t-1}^f$; μ and φ are 2×1 parameter metrics for the mean; the error correction term z_{t-1} is from the linear regression between log price in cash and futures markets. The conditional variance of residuals u_t is constructed in the GARCH framework.

In the estimation process, the maximum likelihood method on multivariate GARCH models produces a similar LLF with univariate GARCH model, yet the computation is much more demanding.

$$\ln L = -\frac{TN}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log h_t - \frac{1}{2}\sum_{t=1}^{T}\frac{u_t^2}{h_t} \qquad (4\text{-}44)$$

The LLF of student's t distribution is even more complicated and takes more effort to derive the LLF in multivariate case. More information about multivariate student's t distribution can be found in Tsay (2005, p482).

The (Quasi) maximum likelihood (QML) estimators get first and second order derivations of LLF which are easy or hard to obtain depending on different types of GARCH models. Response to the computation issue of QML, optimization algorithms BHHH (Berndt, B. Hall, R. Hall, and Jerry Hausman) and BFGS (Broyden–Fletcher–Goldfarb–Shanno) are employed in WinRats software for the OHR prediction. These two algorithms provide most accurate estimation in case of nonlinear model by optimizing the LLF (Lütkepohl and Krätzig, 2004). The two methods are asymptotically equivalent due to the analogy of structure. The BHHH method is widely used because it ensures convergence under certain weak conditions. Although the BFGS approach does not produce convergence unless the quadratic Taylor expansion is available, it offers decent result for large number of variables (Avriel, 2003).

Unavoidably, the estimation has some error, minor or large because the original data are transformed for convenience in models and statistical computation. For large-size sample, the error might not influence the prediction significantly; however, for small-size sample, the MSE value of forecasts could be far smaller than the actual error. Brockwell and Davis (2002) suggested testing the goodness of fit of models in terms of estimation. Fitting graphs is a visible way to show the appropriation of models. The AIC, BIC methods and residuals statistics are applied here to provide numerical evidence. If the estimation method does not fit the series, it is no need to forecast using this model; otherwise, it is worth predicting future value of the series (Brockwell and Davis, 2002).

4.4.3.2 Forecasting using GARCH

Once the appropriateness of estimation models is demonstrated, forecasting based on these models can be carried out (Brockwell and Davis, 2002). Numerous scholars employ GARCH family models to forecast return (Premaratne and Bera, 2000), variance and covariance for volatility (see Engle and Ng, 1993; West and Cho, 1994; Yu, 2002), option pricing (Duan, 1995; Ritchken and Trevor, 1999), Beta prediction (Choudhry, 2008), but a small amount of literatures exists for hedge ratio forecasting (see Kroner & Sultan, 1993; Yang, 2001).

For the standard GARCH model, the one-step ahead forecast of conditional variance based on GARCH (1, 1) model is

$$E(\sigma_h^2(1) \mid \Omega) = c + a_1 u_h^2 + b_1 \sigma_h^2 \tag{4-45}$$

where $\sigma_h^2(1)$ is the one-step-ahead predictor based on previous available information; $a_1 + b_1 < 1$ is restricted for positivity of conditional variance. In the same way, the two-step ahead forecast is

$$E(\sigma_h^2(2) \mid \Omega_{h+1}) = c + a_1 E(u_{h+1}^2 \mid \Omega_h) + b_1 \sigma_h^2(1) \tag{4-46}$$

where $E(u_{h+1}^2 \mid \Omega_h) = \sigma_h^1(1)$, $\sigma_h^2(1)$ replaces the $\sigma_h^2(2)$, the two-step ahead forecast can easily be derived to the formula of $\sigma_h^2(1)$.

$$E(\sigma_h^2(2) \mid \Omega_{h+1}) = c + (a_1 + b_1)\sigma_h^2(1) \tag{4-47}$$

In general, in l-step ahead prediction, we substitute $\sigma_h^2(l)$ with formula of $\sigma_h^2(l-1)$, and then replace $\sigma_h^2(l-1)$ with a form of $\sigma_h^2(l-2)$, by repeating the process, the predictor $\sigma_h^2(l)$ can be written as an equation of $\sigma_h^2(1)$.

$$E(\sigma_h^2(l) \mid \Omega_{h+l-1}) = \frac{c[1-(a_1+b_1)^{l-1}]}{1-a_1-b_1} + (a_1+b_1)^{l-1}\sigma_h^2(1) \tag{4-48}$$

When the l is approaching infinity, the $\sigma_h^2(l)$ converges to a constant $c/1-a_1-b_1$ which is the unconditional variance of residual term u_t (Tsay, 2005).

For asymmetric and X-GARCH models, the forecasts are more complicated because of more involved factors. Nevertheless, the one-step-ahead forecasts are quite straightforward. For example, the one-step-ahead forecast of conditional variance based on GARCH-GJR (1, 1) model is

$$E(\sigma_h^2(1)\,|\,\Omega_h) = c + (a_1 + \gamma_1 I_h)u_h^2 + b_1\sigma_h^2 \tag{4-49}$$

where I_h is a dummy variable which is 1 for negative shocks, and 0 for positive shocks.

Recursive and Rolling

Recursive and rolling forecasts are two different ways to employ data for forecasting. For one-step-ahead forecasting, recursive method forecasts the y_{n+1} using the previous n observations, and predicts y_{n+2} by utilizing the past $n+1$ data. This approach recursively adds all past observations to forecast the next one. Apparently, the sample size gradually increases by 1 each time for one-step prediction. On the other hand, the rolling forecast has fixed sample size n and with a rolling window. The observations from the first one to n are samples for forecasting the y_{n+1}, but to forecast the y_{n+2}, samples are observations from the second one to $n+1$th data (Brockwell and Davis, 2002). For h-step ahead forecasting, the y_{n+h} is predicted based on the first n observations for both methods; for y_{n+h+1}, the first $n+1$ observations are used in with recursive forecasting, and the second to $n+1$ data are applied with rolling forecasting (Brooks, 2008).

For the in-sample estimation with recursive method, the WinRats sends back the optimal estimated parameters for the following out-of-sample forecasting. For rolling forecasting, the program produces one optimal estimated parameter at each time point for future prediction. In this method, not a single optimized estimated parameters present, but it involves a group of corresponding parameters.

4.5 Forecasting Accuracy

Forecasting accuracy is an equivalence of 'goodness of fit' of the forecasts to their actual value. More accurate prediction a model produces, the superior the model is in future forecast. Choosing appropriate measurement of forecasting accuracy is one of the most important factors to find the best forecasting model (Makridakis et al., 1998).

In this thesis, I apply four measures to test the forecasting accuracy of six GARCH models, including MAE (mean absolute error), MSE (mean square error), Theil's U and MDM (Modified Diebold Mariano) tests. These models are appropriate for testing forecast accuracy because they measure the error of out-of-sample forecasts, and do not involved in forecast process. It escapes some serious issue in forecasting based on minimization of a single evaluation (Makridakis et al., 1998). According to the different research perspective of measures, the measures are divided into two categories, forecast error tests and forecast accuracy tests.

4.5.1 Forecast Error Tests

The evaluations of forecast error which test how much the forecasts deviate from the actual value based on difference GARCH models are MAE, MSE, and Theil U tests.

These tests examine the forecast error of each single model, and the one who produces smaller error is superior. In order to eliminate any biasness of various evaluations, I put equal weight for each method. In other words, if a model offers smallest error for all 3 evaluations, overall, it is definitely the most powerful forecasting model. In analogy, the model whose forecasts provide smallest error under more evaluations, the model has the best forecasting power.

We set the one-step ahead forecast error as $e_t = Y_t - F_t$, where Y_t is the actual value at time t and F_t is the corresponding forecast.

MAE and MSE

At first, I will introduce the mean error (ME) evaluation and its mathematically expressions is written as

$$ME = \frac{1}{n}\sum_{t=1}^{n} e_t \tag{4-50}$$

From its definition, the ME gets the average error for the whole forecast period. Apparently, the positive and negative errors offset each other in this method. To overcome the problem of the ME that it probably turns out small because of neutralizing positive and negative errors, the MAE (mean absolute error) and MSE (mean square error) ensure the non-negativity of effect of every single error.

$$MAE = \frac{1}{n}\sum_{t=1}^{n} | e_t | \tag{4-51}$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n} {e_t}^2 \tag{4-52}$$

The MAE obtains the average absolute error in which negative and positive errors have equal portion as long as their absolute values are equivalent. Alternatively, the sign of forecasts do not make any difference on the weight of impact that only depends on the absolute value. The MSE refers to average square error of the error series in which large error takes larger impact than small one. Both of them are simple and non-negative, and this property makes the evaluation more meaningful than ME. The MAE entitles weight proportionately to the error size, but clearly the MSE utilizes a quadratic loss function to put more weight for the large errors than small ones. In theory, the MSE may not work well in the case that large error would not produce essentially stronger influence than that of small error does (Brooks, 2008). By comparing MSE with MAE method, the MSE is simpler in terms of mathematical computation and interpretation (Makridakis et al., 1998).

Theil's U

The Theil's U statistic is proposed in 1996. Simply speaking, the Theil's U statistic gives the square root of relative percentage change of forecasted error to that of actual error. It compares the relative percentage change of forecast error with that of benchmark model. The Large error takes more weight than small error does in this test.

$$U = \sqrt{\frac{\sum_{t=1}^{n-1}(FPE_{t+1} - APE_{t+1})^2}{\sum_{t=1}^{n-1}(APE_{t+1})^2}} \tag{4-53}$$

Where $FPE_{t+1} = \frac{F_{t+1} - Y_t}{Y_t}$ represents the forecast relative change and $APE_{t+1} = \frac{Y_{t+1} - Y_t}{Y_t}$ is actual relative change from benchmark model. The choice of benchmark model relies on the forecasting model and the market (Brooks, 2008).

The U-statistic is a symmetric evaluation that the sign direction does not affect the relative forecasting comparison (Yu, 2002). It is in a similar vein to MSE that large errors weight much more than small errors. The value of U statistics is non-negative. When the forecast is perfect, the relative change of forecast equals to actual relative change $FPE_{t+1} = APE_{t+1}$ which results to 0 in U since the numerator of U is 0. When $U < 1$, the forecasting model is said to be superior to benchmark model. Generally speaking, the smaller the U value, the better the forecasting model. If $U = 1$, the model for predicting is equal good/bad as the same as the benchmark model. Put it in another way, the performance of forecasting model somehow depends on the prediction-performance of benchmark model when $U = 1$. Otherwise, if $U > 1$, the benchmark model has more preferable prediction than the forecasting model and the forecasting model is not recommended for prediction in this case (Makridakis et al., 1998).

4.5.2 Forecast Accuracy Tests

Diebold Mariano (DM) Method

Diebold and Mariano (1995) propose a forecast accuracy evaluation that compares prediction accuracy of two forecasting series. This method tests the deviation of forecasts away from the real value according to a specific loss function.

For two forecasts $y_{1,t}$ and $y_{2,t}$, the $e_{1,t}$ and $e_{2,t}$ are defined as corresponding forecast residuals which are serially correlated in h-step ahead forecasts because of overlapping data. However, the forecast errors are assumed to be uncorrelated and not autocorrelated. Another assumption is that the forecasts are unbiased to ensure the validity of this method. The loss function is either squared error loss or absolute error loss at that time. For example, under the squared error loss function, loss function of the Diebold Mariano test for h-step ahead forecast is

$$d_{t+h} \equiv L(e_{1,t+h}) - L(e_{2,t+h}) \quad \text{(4-54)}$$

where $L(e_{1,t+h}) = e_{1,t+h}^2$ and $L(e_{2,t+h}) = e_{2,t+h}^2$; if the $d_{t+h} = 0$, the prediction power of two forecasts equals to each other. Hence the null hypothesis (H_0) of the Diebold Mariano test is $E(d_{t+h}) = 0$ that two forecasts are equally accurate against the alternative H_1 that $E(L(e_{1,t+h})) \neq E(L(e_{2,t+h}))$ and their forecasting abilities are different. It is easy to obtain that $\bar{d} = \frac{1}{N}\sum_{h=1}^{N}(L(e_{1,t+h}) - L(e_{2,t+h}))$ and its variance is $V(\bar{d}) = \frac{1}{N}(\gamma_0 + 2\sum_{d=1}^{h-1}\gamma_d)$; additionally the asymptotic distribution of $\bar{d}$ is

$$\sqrt{N}(\bar{d} - \mu) \rightarrow N(0, 2\pi f_d(0)) \quad \text{(4-55)}$$

where $f_d(0) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty}\gamma_d(\tau)$, and γ_d is the auto-covariance function.

The null hypothesis is written mathematically as

$$S = \frac{\bar{d}}{\sqrt{N^{-1}2\pi\hat{f}_d(0)}} \sim N(0,1) \quad \text{(4-56)}$$

where $\hat{f}_d(0)$ is an estimator of $f_d(0)$. If the S follows normal distribution, the null hypothesis holds that two forecasts are equal good/bad. Otherwise, they have different forecasting ability. It is worth mentioning that the loss function can also be gained from other measures of testing forecast error, for example RMSE (Clements and Hendry, 1998).

Modified Diebold Mariano

In the DM method, the statistic value for forecast error is compared with critical value of normal distribution. This principle results in an acute over-sized problem since the distribution of errors are more likely to be a fatter tailed distribution, rather than normal distribution. Harvey and Leybourne (1997) analyze the pros and cons of the DM method in detail and propose a modified Diebold Mariano (MDM) measure (Clements and Hendry, 1998).

The DM method is a successful measure to compare the prediction errors of two forecasts; however, it has several assumptions that they are free of autocorrelation and are unbiased forecasts which are not practical in economic and financial markets. In addition, the DM is not appropriate for small sample, and even if it performs well in large sample, it is still oversized if the residual series is not normally distributed. The over-size problem becomes more severe for h ($h>1$)-step ahead forecasting because the DM is more over-size as the h increases. These disadvantages of DM inspire Harvey and Leybourne (1997) to make modification of this method.

The distribution of forecasts in assumption of the DM method is modified as student's *t* with ($n-1$) degree of freedom to form the MDM approach. The variance estimator of $\bar{d}$ is proved to be unbiased ($E(\hat{V}(\bar{d}))\approx\hat{V}(\bar{d})$) under student's *t* distributed residuals. And thus the statistic of MDM method is written as below:

$$S'=[\frac{n+1-2h+n^{-1}h(h-1)}{n}]^{1/2}S \qquad (4\text{-}57)$$

where h is the forecast horizon and $h\geq 1$; S is the statistic of DM method and S' is its modification. For the 1-step ahead forecast, $S'=\sqrt{\frac{n-1}{n}}S$, and they are roughly equal if the sample size is large enough.

The MDM measure releases restrictions of DM, and it allows for correlated residual series and autocorrelation of forecast errors. Even if the forecasts are biased, the variance estimator of $\bar{d}$ is unbiased. The MDM broadens the implication to test heavy tailed series, not only for normal distributed residuals. The fatter tailed distribution (i.e.

student's *t*) is evidently more appropriate than normal distribution in reality. Harvey and Leybourne (1997) test the performances of DM and MDM approaches for the sample size ranging from 8 to 512, the results imply the severe over-size problem of DM method. On the other hand, the MDM method is capable of comparing forecast accuracy of two forecasts for either small or large samples.

The MDM approach is also over-sized sometimes, but this problem is much less severe than that of DM test. The slight over-sized method is acceptable for practitioners. Overall, the benefit of using MDM far over covers its drawback in terms of reliability. Harvey and Leybourne (1997) recommend the MDM for comparing prediction-accuracy between two forecasts.

4.6 Data

In this section, I will introduce the data I use for the study. Five agricultural commodities coffee, wheat, soybean, live cattle and live hog traded in the US markets are used in the samples. Description of the data and the basic statistics of all data are provided in the following subsections.

4.6.1 Data Description

This study applies the daily closing futures and cash prices of coffee, wheat, soybean, live cattle and live hog from U.S. cash and futures. The five commodities from the U.S market are chosen since they are the most popular agricultural products traded worldwide, and the U.S. futures market conducts the majority of futures trading on agricultural goods.

The futures price of coffee is from Coffee, Sugar and Cocoa Exchange (CSCE); both the wheat and soybean futures prices are from the Chicago Board of Trade (CBOT). The live cattle and live hog futures prices are from Chicago Mercantile Exchange (CME). Correspondingly, the cash price of Santos coffee is in the New York board of trade; the wheat cash prices are from the CBOT; the soybeans cash price is the price of soybeans in Southeast Iowa, the live cattle cash price is taken from the Commodity research bureau and ICX index, and the hog cash price are is taken from IHX hog index. All data are obtained from *Global Financial Data*[41]. The cash and futures prices are recorded at the

[41] The Global Financial Data is a database which provides continuous financial data series in various financial areas, commodities, stock, equity and other financial related historical data.

same time of the day, at the end of the trading day. Futures prices are continuous series. The price of the nearest futures contract is obtained for the first value of futures price series. The new contract is not brought in until the expiry date of the previous contract or the first business day of the last contract month. The 'rollover' date of futures contract is typically based on the timing when the trading volume tends to switch from the nearest expiry into the later expiry contract. In this study, the 'rollover' is done on the first day of the expiry month or the expiry date. The 'rollover' date is more likely to be the day before expiry for stock index futures.The data of storable commodities (coffee, wheat and soybean) starts from 1st January, 1980 to 23rd June 2006; but the data for livestock (live cattle and live hog) is from the 1st January, 1980 to 14th January 2008.

Futures prices are continuous series. Apart from the public holiday, the commodity trading in cash market is not exactly daily trading due to the higher flexibility of cash market, and therefore there are some data missing of cash prices. To remove the mismatch of cash and futures prices and keep the continuity of price series, the cash price is sorted by inserting proxy. The insertion method is useful for keeping the integrity of the time series. The x_1th, x_2th spot prices $f(x_1)$ and $f(x_2)$ are known, and the unknown x_ith price is calculated with the following formula

$$f(x_i) = \frac{x_i - x_1}{x_2 - x_1} \times (f(x_2) - f(x_1)) + f(x_1) \quad \text{(4-58)}$$

where $x_1 < x_i < x_2$. If the x_i is closer to x_1; the value of $f(x_i)$ depends on the $f(x_1)$ more heavily and vice versa. In other words, the distance between the known and unknown point determines the weight of known point affecting the value of the unknown point. To avoid too many inserted data and feint on real data, we remove the missing cash price and corresponding futures price when $i > 2$. And then the following data set moves up to fill the gap which is caused by deleting mismatched data.

The insertion method somewhat improves the continuity and completion of the data series, though it may result in misleading of forecasting because of information mismatch between trading prices and trading date. Notwithstanding, it is an inevitable problem no matter what kind of method dealing with data due to the particularity of price data of spot and futures markets. Furthermore, the flaw is accepted for practitioners and the

http://www.globalfinancialdata.com/index.html guidance of accurate forecasting of OHR is still essential for investors.

We show the prices patterns of five commodities in the whole period as following in figures 4-2 to 4-6. The blue line represents futures price over time, and the red line is cash price. The storable commodities have similar trend in terms of seasonality. The price pattern of Brazail Santos Arabicas coffee is presented in Figure 4-2. The increase and decease of spot and futures prices of coffee highly depend on the supply of coffee. The harvest season of Brazail Santos Arabicas coffee is summer, and the price will fall down in summer due to increasing supply. In spring and winter, coffee from other places, such as Mexico, Costa Rica and Panama are harvested, and results in the price drop of Brazail coffee.

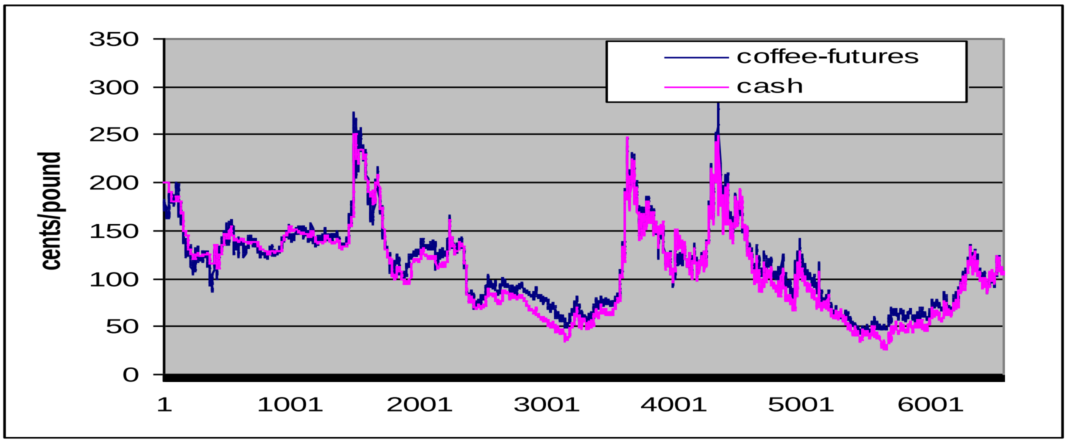


Figure 4-2 Price pattern of coffee

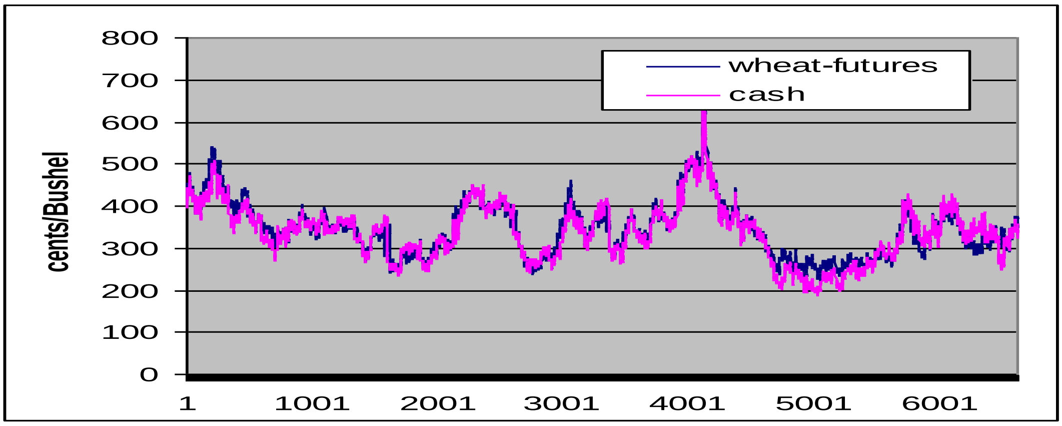


Figure 4-3 Price pattern of wheat

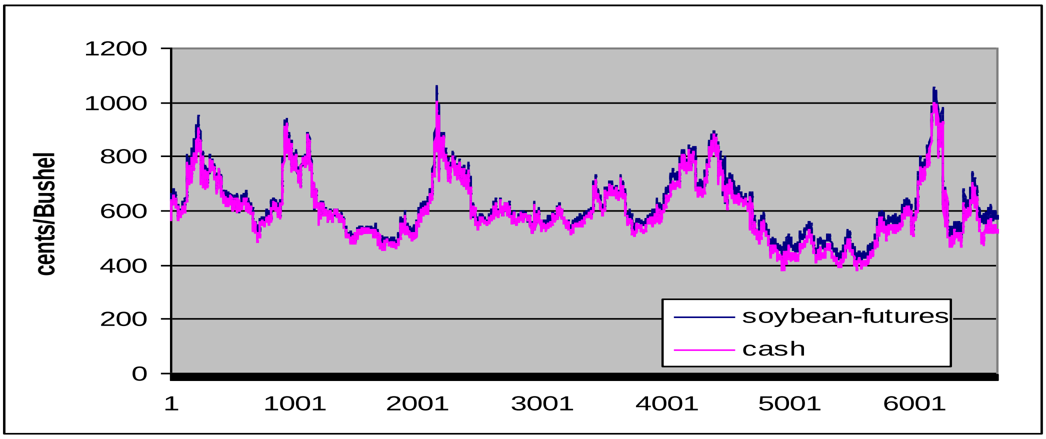


Figure 4-4 Price pattern of soybean

The figures 4-3 and 4-4 show the price pattern of wheat and soybean, respectively. In the harvest season, the price decreases due to high trading volume. Inversely, the price rises up to the peak at the middle of two harvest seasons.

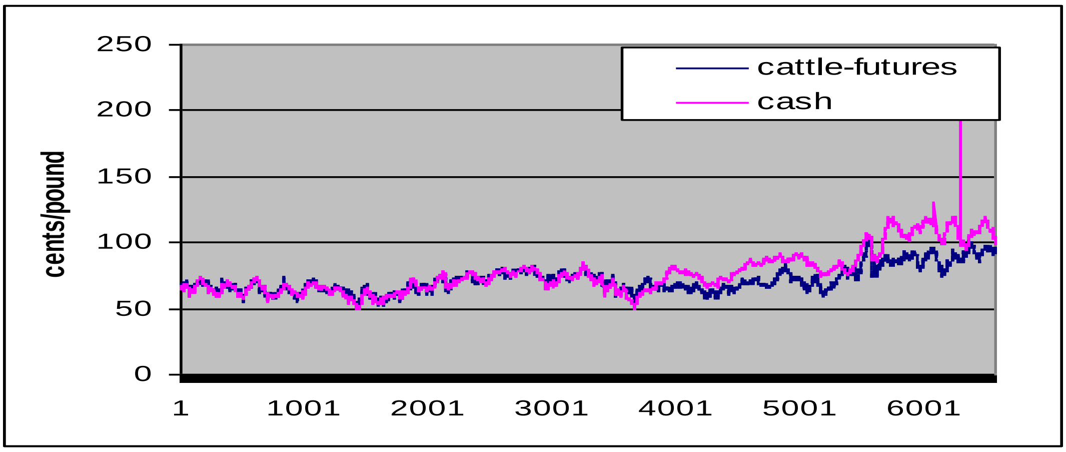


Figure 4-5 Price pattern of live cattle

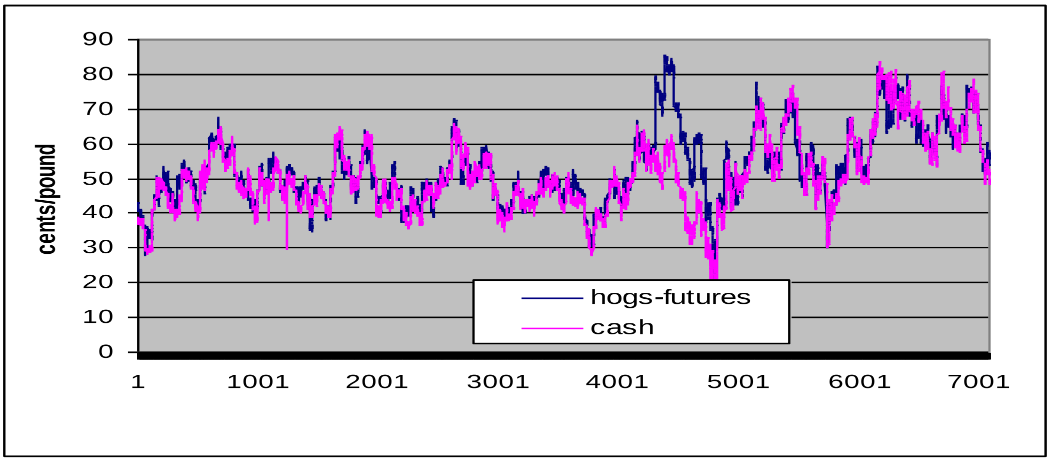


Figure 4-6 Price pattern of live hog

The price pattern of live hog in figures 4-5 and 4-6 implies high price volatility. The cash and future prices of live hog move apart in the long run between 1997 and 2000. Live cattle prices normally get higher from January through May. Prices for live cattle reach the seasonal peak in May and then usually begin a downtrend that extends through the end of the year (price of feeding). In the last ten years, the cash and futures prices hardly move together at maturity.

Judging from the visible price pattern, the storable commodities seem likely to have more volatile prices, but they do move together at maturity; price pattern of non-storable commodities exhibits much high probability of price deviation at maturity which is consistent with the statement in section 2.3.3.. In addition, we find the presence of volatility clustering in price pattern that large changes tend to follow large change and then followed by relatively small changes in certain periods. This is a type of heteroscedasticity which will be dealt with by GARCH-family model in this study. The basic statistics in next section will show more features of price series based on mathematical evidence.

4.6.2 Basic Statistics

In this study, the estimation and the forecasting of the OHR are conducted by using the returns data from the cash and the futures markets. The cointegration tests are conducted using the cash and futures prices in log level. The cash and futures prices are closing prices of each day. The returns are estimated by taking the first difference of the price in log level. Two in-sample estimations and two corresponding out-of-sample forecast periods are chosen. One in sample estimation uses 25 years log-return from 1st January, 1980 to 23rd March, 2005 for storable commodities and 27-year log-return from 1st January, 1980 to 14th January, 2007 for non-storable goods. Correspondingly, the 1-year out-of-sample forecasting is for the period of 24th Mar. 2005 to 23rd Mar. 2006 for three storable commodities and from 15th Jan. 2007 to 14th Jan. 2008 for non-storable commodities. Another in sample estimation employs the first 24-year log-return from 1st Jan. 1980 to 23rd Mar. 2003 and 26 years log-return from 1st Jan. 1980 to 14th Jan. 2005 for storable and non-storable goods respectively. Corresponding to the estimation periods, 2-year out-of-sample forecasting from 24th Mar. 2003 to 23rd Mar. 2005 and 15th Jan. 2005 to 14th Jan. 2007 for storable and non-storable commodities respectively are applied using six GARCH-type models.

A short-term and a long-term prediction are employed to test the forecasting power of different models when forecast horizon changes, based on the finding that both the in-sample estimation and out-of-sample forecasting of OHR vary with increase and decrease of length of horizons in terms of hedging effectiveness (see Geppert (1995), Malliaris and Urrutia (1991)). These are non-overlapping 1-year and 2-year forecasting horizons in this study to avoid the sample effect and overlapping issue. If the two forecasting periods are over-lapping, the data for in-sample estimation is contaminated due to a certain amount of mutual data which may result in non-robust forecasting in our case. Consequently, it becomes hard to tell the real influential factors that cause the result. For instance, if it turns out that the same GARCH model is the most powerful forecasting model on two horizons, it is possibly because of too many mutual data or the non-important horizon effect. The over-lapping data makes the result more confusing. Thus non-overlapping sub-samples can enhance the reliability and robustness of outcome, and simplify the result interpretation, although may not have significant influence on forecast accuracy in some cases (Harri and Brorsen, 2002).

The cash and futures prices in the raw data (price in log level) and the returns from the cash and futures markets are tested for stationarity and autocorrelation. We apply the KPSS unit root test and the Ljung-Box Q autocorrelation test (up to 24 lags) for the full samples and present the result in Table 4-1 and Table 4-5. All significance level are tested at the 5% level minimum. With presence of unit root in log-prices series for all commodities, we conclude that log-price series are non-stationary. However for log-return series, all of them are stationary. On the other hand, the raw data shows autocorrelation for all five products, but log-return series generally fail to show autocorrelation.

In addition, we also test the randomness of raw data for all five commodities. We find that all series are random processes. Among them, all series are non-stationary processes with a deterministic trend, except for cash and futures log-prices series for live cattle and futures price of live hog which have no significant deterministic trend at 5% significant level. In other words, most price series have mean that grow around a fixed trend without depending on time, while price series for live cattle and live hog show insignificant trend.

Hence we find that log cash and/or futures price series for all five commodities are non-stationary with existence of unit root and the result L-B Q test on log-returns fails to reject the null of no autocorrelation in both cash and futures markets. As Baillie and Myers (1991) state, the martingale data is consistent with efficient market hypothesis in the weak form. As described earlier, prediction is possible under conditions of weak form of EMH.

We present the results of basic statistics and cointegration tests for the full sample and two in-sample periods in this section. Basic statistics of log-price and log-return in cash and futures markets in full sample are in the table 4-2. The kurtosis test of log price in cash and futures markets are positive and significant for all except for coffee at 1% significant level. The leptokurtosis implies the presence of fatter tail and higher peak value around the mean than normal distribution in four commodities except for coffee. For the case of coffee, the negative significant kurtosis shows that the price pattern has thinner tail and lower peak value than normal distribution does. In terms of skewness, all shows longer tail on the left-hand side except for live cattle which has longer tail on another side. Log futures price pattern of live hog is relatively symmetric and close to the normal distribution. Additionally, both cash and futures log returns produce similar results. The significant Jarque-Bera test indicates non-normality of cash and futures prices for all five agricultural commodities. As described, most time series do not follow normal distribution in reality.

The sub-sample covers first 25 and 27 –year data from 1st Jan. 1980 to 23rd Mar. 2005 for storable commodities and 27-year data from 1st Jan. 1980 to 14th Jan for non-storable products, and leaves the last 1-year for out-of-sample forecasting. The result of the basic statistic for sub-sample in table 4-3 is slightly different from that of full period data. Log cash and futures prices of storable goods have thinner tail and lower peak value than normal distribution, but non-storable products show fatter tail and higher peak value. Positive and significant skewness for log cash price of soybean presents long tail on right-hand side. Distributions of log cash prices for other commodities are as the same as those of full sample. Log futures price of wheat and soybean show long tail on right-hand side. The Jarque-Bera test indicates non-normal distributed log cash and futures prices for all five commodities.

Table 4-4 reports the basic statistics for the 2-year sub-sample on first 24 and 26 –year data from 24th Mar. 2003 to 23rd Mar. 2005 and 15th Jan. 2005 to 14th Jan. 2007 for storable and non-storable agricultural commodities respectively. The result is quite similar with the first sub-sample that kurtosis and skewness present in log-price and log returns for all commodities, yet with subtle distinction in significance level. Similarly, the results of Jarque-Bera test are positive and significant which show non-normal distribution of log-price series for all products.

4.6.3 Cointegration Test

The existence of cointegration relationship between log cash and futures prices are tested for all commodities in this subsection. Cointegration relationship widely exists in financial and commodity markets. Kroner and Sultan (1993), Brenner and Kroner (1995) and Lien (1996) found the cointegration in currency markets. Ghosh (1993), Kroner and Sultan (1993) and Lien (1996) showed that the cointegration relationship leads to smaller hedge ratio, and improves estimation and forecasting of OHR. Ghosh (1993, 1995) and Yang et al.(2001) further claimed that cointegration between cash and futures prices on commodity markets is necessary to ensure an optimal hedging decision.

The Engle-Granger two-step method is applied to measure the validity of taking into account the cointegration relationship into the model.

In the first step, the KPSS stationary method is applied for $\log(P_c)$, $\log(P_f)$, $R(P_c)$, $R(P_f)$. The result in Table 4-5 denotes that one unit root shows up in all log price levels for all five commodities at 1% significant level, even when the lags are up to 9. Permanent effect from shocks on log price level would not decay in the non-stationary series (Choudhry and Wu, 2008). However, the unit-root tests of the log-return imply a rejection of presence of unit root at 95% confidence level. The first difference of log price for all products are stationary which implies that the log price are all $I(1)$ processes and there is no need to further test the higher order difference.

In the second step, the results of cointegration test in Table 4-6 show that error correction term from the Engle-Granger regressions are stationary based on the DF and ADF tests for all five cases at all lags length. In other words, results show the long-run cointegration relationship between log-price of cash and futures markets for all five commodities under study. It indicates that the error correction term should be incorporated in the mean equation and the conditional variance equation of the GARCH models to capture the effect of the short-term deviations of the cash and futures price relationship on the conditional variance and covariance.

4.7 Conclusion

In this chapter, the main methodology is introduced. More specifically, this chapter covers basic theory about standard ARCH and GARCH models, six GARCH-type models, relevant diagnostic tests, estimation and forecasting using GARCH models, accuracy evaluation and data description.

A lot of effort is put on description of the models employed in section 4.2, 4.3 and 4.4. The ARCH/GARCH model is a breakthrough in forecasting economic and financial time series. They capture the heteroskedasticity in time series. The development from univariate to multivariate GARCH model is managed to describe the relationship among variables using matrix in which variables are contained instead of simultaneous equations. Based on the GARCH structure, derivations of it are proposed to better fit different types of time series. Asymmetric effect of negative and positive shocks is measured by putting heavier weight on the impact of negative shocks. The GJR incorporates a dummy variable to account for the excess effect of bad news over good news. The QGARCH adds an extra term as impact of information asymmetry. The cointegration relationship is widely found and the GARCH-X, BEKK-GARCH-X and other ECM-type models are applied in various financial and commodities' markets (see Kroner and sultan (1993), Tse et al. (2002), Choudhry (2003, 2009), Moon (2009), Liu and Hung (2010)). The way they estimate and forecast is also introduced, specifically for OHR. The evaluation of forecasting accuracy becomes an influential element in measuring the forecasting power of different models. To reduce the side effects (such as biasness) due to evaluation techniques, four approaches are addressed in section 4.5. Thus, the reliability of prediction comparison is solid in this study.

The description of data employed is presented in section 4.6. Basic features of log spot and futures prices, and log returns have been explained. It is not surprising when the log spot and futures prices are found to be non-normal distributed with fatter tail and leptokurtosis. Additionally, the presence of cointegration is confirmed with evidence from Engle-Granger two-step test. And hence taking the error-correction term into the mean equation is rational. The LM test also indicates the validity of using GARCH-family models for OHR forecasting

Table 4-1 The autocorrelation tests for all samples

Commodity	variable	Q(24) Full sample		Q(24) Full sample
Coffee	$\log(P_c)$	149841 (0.0000)	$R(P_c)$	10.7353 (0.0011)
	$\log(P_f)$	147091 (0.0000)	$R(P_f)$	35.5379 (0.0459)
Wheat	$\log(P_c)$	145712 (0.0000)	$R(P_c)$	28.3637 (0.0567)
	$\log(P_f)$	145204 (0.0000)	$R(P_f)$	24.9030 (0.0355)
Soybean	$\log(P_c)$	146349 (0.0000)	$R(P_c)$	71.1479 (0.0001)
	$\log(P_f)$	144986 (0.0000)	$R(P_f)$	32.1694 (0.0967)
Live cattle	$\log(P_c)$	152588 (0.0000)	$R(P_c)$	96.0650 (0.0000)
	$\log(P_f)$	144433 (0.0000)	$R(P_f)$	9.20730 (0.9952)
Live hog	$\log(P_c)$	145930 (0.0000)	$R(P_c)$	238.983 (0.0000)
	$\log(P_f)$	148274 (0.0000)	$R(P_f)$	26.2760 (0.2880)

Note: The Liung-Box Q statistic is for testing serial correlation for up to 24 lags, and its null hypothesis is no existence of autocorrelation. The test is at 5% significant level.

Basic statistics

Table 4-2 Basic statistic of log price and log return for storable goods from 1980 to 2006 (1980-2008 for non-storable goods)

$\log(P_c)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	4.574919	0.209889	-0.717691*	-0.344480*	281.4589500*
wheat	5.754019	0.115839	11.224731*	-2.886865*	45309.62490*
Soybean	6.325504	0.146695	20.195478*	-4.018150*	134349.9660*
Live cattle	4.346798	0.088669	21.412241*	3.750245*	146379.5900*
Live hog	3.909154	0.045493	0.7672480*	-0.244703*	244.1430380*
$\log(P_f)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	4.647264	0.152220	-0.723982*	-0.158073*	177.4777870*
wheat	5.775451	0.112436	13.113634*	-3.107983*	59890.97120*
Soybean	6.359038	0.148869	21.872095*	-4.250099*	156588.7883*
Live cattle	4.289970	0.074832	41.472024*	5.825859*	527712.2127*
Live hog	3.953263	0.038622	0.117773**	0.036459	5.65630000**
$d\log(P_c)$	mean	variance	kurtosis	skewness	Jarque-Bera
Coffee	-0.000181	0.000619	55.179836*	-1.440836*	868103.3001*
Wheat	-0.000295	0.000746	2442.6498*	-38.19316*	1698144701*
Soybean	-0.000346	0.000853	3574.4214*	-50.93086*	3635731763*
Live cattle	-0.000019	0.001249	2497.1286*	-12.46837*	1773180373*
Live hog	0.000036	0.000472	102.36090*	-0.17070*	3088351.993*
$d\log(P_f)$	mean	variance	Kurtosis	skewness	Jarque-Bera
coffee	-0.000154	0.000621	28.636131*	-0.985944*	234266.8784*
wheat	-0.000289	0.000727	2761.7353*	-42.23345*	2170690731*
Soybean	-0.000343	0.000882	3940.3350*	-54.82438*	4418045991*
Live cattle	-0.000014	0.001312	2784.5699*	-10.63524*	2204800765*
Live hog	0.000032	0.000431	29.439328*	-0.283689*	255547.0538*

Note: '*' represents that it shows significance at 1% significant level
'**' represents that it shows significance at 5% significant level
'***' represents that it shows significance at 10% significant level

Table 4-3 Basic statistic of log price and log return for storable goods from 1980 to 2005 (1980-2007 for non-storable goods)

$\log(P_c)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	4.588617	0.218831	-0.7214170*	-0.405427*	309.745388*
wheat	5.804106	0.041851	-0.1898580*	-0.309719*	111.215862*
Soybean	6.380358	0.036144	-0.3346030*	0.201822*	73.2575940*
Live cattle	4.338507	0.092149	22.131762*	3.956697*	147224.378*
Live hog	3.900193	0.044247	0.8986120*	-0.240600*	295.481866*
$\log(P_f)$	mean	variance	Kurtosis	skewness	Jarque-Bera
coffee	4.666362	0.153776	-0.673344*	-0.229130*	174.445165*
wheat	5.826430	0.035204	-0.246388*	0.259049*	87.2062530*
Soybean	6.413474	0.032301	-0.307146*	0.383682*	182.068850*
Live cattle	4.283903	0.078850	39.68492*	5.840897*	456076.650*
Live hog	3.945174	0.037691	0.233969*	0.068125**	20.8462360*
$d\log(P_c)$	mean	variance	Kurtosis	skewness	Jarque-Bera
coffee	-0.000089	0.000589	14.218815*	0.491234*	53408.8789*
wheat	-0.000035	0.000299	14.422739*	0.05269***	55109.6852*
Soybean	-0.000010	0.000238	9.3331780*	-0.487181*	23463.6789*
Live cattle	0.000091	0.000220	1504.4725*	-0.294123*	603111836*
Live hog	0.000065	0.000488	99.578010*	-0.172157*	2819420.52*
$d\log(P_f)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	-0.000061	0.000599	8.4323380*	0.0859***	18702.2765*
wheat	-0.000044	0.000263	32.128610*	-1.872780*	277176.107*
Soybean	-0.000004	0.000209	8.9804850*	-0.845554*	22251.6588*
Live cattle	0.000065	0.000138	9.3203450*	-1.110773*	24461.9676*
Live hog	0.000053	0.000436	29.362900*	-0.356393*	245290.917*

Note: all symbols are the same with table 4-2.

Table 4-4 Basic statistic of log price and log return for storable goods from 1980 to 2003 (1980-2005 for non-storable goods)

$\log(P_c)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	4.626046	0.214759	-0.4635040*	-0.565001*	360.5038940*
wheat	5.798726	0.044560	-0.3468790*	-0.246248*	88.37561600*
Soybean	6.372413	0.034219	-0.4418340*	0.100178**	57.66315000*
Live cattle	4.292672	0.054813	38.257680*	4.839439*	381546.0363*
Live hog	3.877268	0.039922	1.4120830*	-0.268448*	601.3692320*
$\log(P_f)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	4.699039	0.150438	-0.4437710*	-0.379141*	186.5485910*
wheat	5.827063	0.037282	-0.3477270*	0.250088*	90.37599400*
Soybean	6.404688	0.030348	-0.4332570*	0.299162*	133.6972510*
Live cattle	4.251160	0.044893	70.194722*	7.380274*	1260567.341*
Live hog	3.926033	0.035163	0.6395080*	0.161898*	135.3901970*
$d\log(P_c)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	-0.000247	0.000582	15.514878*	0.493371*	58397.15799*
wheat	-0.000058	0.000285	16.622170*	0.020228	67278.60538*
Soybean	-0.000019	0.000223	10.037332*	-0.385884*	24824.97613*
Live cattle	0.000089	0.000062	12.663936*	0.203927*	39326.00815*
Live hog	0.000104	0.000520	94.464699*	0.174478*	2351023.415*
$d\log(P_f)$	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	-0.000183	0.000609	8.6995460*	0.0603***	18290.2039
wheat	-0.000076	0.000256	36.652868*	-2.184059*	331771.954*
Soybean	-0.000018	0.000191	8.4228100*	-0.680722*	17832.2968*
Live cattle	0.000059	0.000139	9.7132750*	-1.173234*	24459.9789*
Live hog	0.000089	0.000443	29.361159*	-0.360610*	227258.581*

Note: all symbols are the same with table 4-2.

Table 4-5 Unit root tests for log cash and futures prices and log cash and futures returns

Commodity	variable	KPSS test: H0: Stationary		
		Lags=3	Lags=6	Lags=9
Coffee	$\log(P_c)$	49.760	28.483	19.970
	$\log(P_f)$	46.184	26.447	18.550
	$R(P_c)$	0.075	0.077	0.078
	$R(P_f)$	0.046	0.048	0.049
Wheat	$\log(P_c)$	16.276	9.335	6.558
	$\log(P_f)$	17.120	9.822	6.902
	$R(P_c)$	0.030	0.032	0.032
	$R(P_f)$	0.031	0.032	0.033
soybean	$\log(P_c)$	17.174	9.846	6.913
	$\log(P_f)$	11.279	6.469	4.544
	$R(P_c)$	0.023	0.024	0.024
	$R(P_f)$	0.021	0.021	0.021
Live cattle	$\log(P_c)$	75.408	43.358	30.535
	$\log(P_f)$	37.193	21.44	15.139
	$R(P_c)$	0.273	0.283	0.290
	$R(P_f)$	0.299	0.305	0.311
Live hog	$\log(P_c)$	35.437	20.346	14.315
	$\log(P_f)$	43.256	24.873	17.521
	$R(P_c)$	0.020	0.020	0.020
	$R(P_f)$	0.011	0.011	0.011
Critical level	0.10	0.05	0.025	0.01
Critical value	0.347	0.463	0.574	0.739

Table 4-6 Cointegration tests

Commodity	DF	ADF (lags=3)	ADF (lags=6)	ADF (lags=9)
Coffee	-0.0241077* (0.0026)	-0.0196284* (0.0027)	-0.0191612* (0.00270)	-0.0185748* (0.0027)
Wheat	-0.0243444* (0.0027)	-0.0222303* (0.0027)	-0.0224715* (0.0027)	-0.0227807* (0.0027)
Soybean	-0.0704680* (0.0045)	-0.0509501* (0.00450)	-0.0481752* (0.0046)	-0.0408313* (0.0047)
Live cattle	-0.0146056* (0.0020)	-0.0099687* (0.0020)	-0.0086947* (0.0020)	-0.0076294* (0.0020)
Live hog	-0.0322501* (0.0030)	-0.0287813* (0.0030)	-0.0262965* (0.0031)	-0.0238825* (0.0031)

Note: This method tests coefficients of error correction term in regressions; '*' represents that the test statistic is significant at 95% significant level with standard errors in parentheses.

5 Result Analysis

5.1 Introduction

The GARCH family model is popular in financial and economic market due to its nature of dealing with dynamic heteroskedasticity in time series. The hedge ratio is developed mathematically as a typical time-varying series and estimated using GARCH type model. However, few studies pay attention to hedge ratio estimation and forecast with GARCH in agricultural market. We contribute to the literature by studying estimation and forecasting of hedge ratio in the agricultural market.

As described, the residual term in regression between financial time series follows fatter-tailed distribution more frequently than normal distribution theoretically, such as student's *t*, generalised Gaussian distributions. Nevertheless, it is rational to measure the residuals with conditional normal distribution provided that a certain amount of extreme value or heteroskedasticity will lead to non-normality of time series for large sample (Brooks, 2008). To take into account the possibility of conditional normal distribution, both conditional normal and student's *t* distributed residuals are tested for comparison. This chapter consists of part A, B and C. The result of the normal distribution is presented in this chapter as the part A of result analysis and the part B is the outcome using student's *t* distribution. In the part C, we assess the economic benefits of dynamic hedging strategy based on forecasted OHR from these six models for investors, when transaction costs are taken into account.

Part A

In this part, we report and interpret the result of estimation and forecast of OHR and return from six GARCH models (GARCH, BEKK-GARCH, GARCH-X, BEKK-GARCH-X, GARCH-GJR and QGARCH) based on conditional normal distributed residuals.

We present the result of estimation of OHR from six GARCH models with normally distributed residuals in section 5.2. With the full sample of five agricultural commodities, the main features of model-based estimated OHR are analysed with basic statistic and stationary tests. Furthermore, the autocorrelation and white tests on residuals are employed for measuring the remaining autocorrelation and heteroscedasticity in residuals after estimation. Based on these tests, the validity and sufficiency of order of six models are demonstrated, and thus the forecast based on the estimated parameters is reliable. Sections 5.3 and 5.4 are parallel which introduce the one-year and two-year forecast of OHR and return, respectively. The return prediction is necessary for this study due to the lack of benchmark of real OHR, and the alternatively forecasting the return based on forecasted OHR is a more rational approach given that the return of portfolio can be easily worked out. The forecasted return is obtained based on the portfolio with known forecasted OHR. To avoid biasness of a single measure and assure the robustness of result, the MAE, MSE, Theil's U and modified Diebold Mariano (MDM) tests are employed for forecast accuracy of forecasted return. Furthermore, the percentage dominance of each model for storable and non-storable commodities is summarized in simple statistic. The main finding and conclusion is stated in section 5.5.

5.2 Estimated OHR

The OHR is estimated by employing six diagonal bivariate GARCH (1, 1) model using either BFGS or BHHH methods for full sample ((1980-2006 for coffee, wheat and soybean; 1980-2008 for live cattle and live hog)). We interpret the coefficients estimated in WinRats programme with recursive data for each variable with detail. The model-based OHRs show some similarity and difference which shall be presented in following part.

With evidence of presence of error term in mean equation, we bring in an error-correction term in the mean equation which is defined as

$$Y_t = \mu + \varphi z_{t-1} + u_t, \quad u_t \mid \Omega_t \sim N(0, H_t) \qquad (5\text{-}1)$$

for all models and all commodities; where z_{t+1} is error term from the regression of log returns in cash and futures markets.

1. Diagonal Bivariate GARCH (1, 1)

The diagonal bi-GARCH model is a simpler form than bi-VECH-GARCH. This representation sets the value of off-diagonal coefficients as 0 so that it leaves only 3 and 3 coefficients for ARCH and GARCH terms, respectively. The numerical burden is somewhat relieved.

Estimated Parameters

The table 5-1 contains results of coefficient from bivariate diagonal GARCH. There are 9 parameters in GARCH model, and 4 parameters of mean equations. Significant φ_1 and φ_2 imply significant effect of short-run deviation between log cash and futures prices on log returns in cash and futures markets for all five commodities. Positive and significant φ_1 and φ_2 present a positive relationship between the short-run deviation and log difference of returns and vice versa. However, the insignificant φ_2 for coffee and live hog indicate that the error correction term is not helpful in interpreting the log return of futures price.

The ARCH and GARCH effects on conditional (co)variance are generally found with evidence of significant coefficients of ARCH (a_{11}, a_{12}, a_{22}) and GARCH (b_{11}, b_{12}, b_{22}) terms for all products except for live cattle. These coefficients indicate that the future conditional variance depends on previous square residual and past variance terms and the pattern is traceable providing the past information. For live cattle, the ARCH effect turns out to be insignificant with insignificant coefficients of ARCH term. Fortunately the GARCH term (b_{22} is significant at 10% significant level) has impact on conditional variance of log futures return. Namely, the previous lagged conditional variance of futures price has significant influence on the futures conditional variance which is not relevant to its square residuals of log returns and cross-produce of residuals in the case of live cattle. The sum of coefficients of ARCH and corresponding GARCH coefficients is close to unity, such as $a_{11} + b_{11} = 1.0143$, $a_{22} + b_{22} = 0.9766$. This suggests that the impact of ARCH and GARCH on current conditional variance is persistent and the current and past information remain important for variance forecasting. The general significant a_{12} and b_{12} show the evident interaction between cash and futures markets.

Analysis of estimated OHR and residuals from standard GARCH models (normal distribution)

In order to analyze properties of estimated OHR and residuals, basic statistics, stationarity and correlation tests are employed with results in tables 5-2, 5-3 and 5-4.

The basic statistics of estimated OHR from standard bivariate GARCH (1, 1) model is carried out for analyzing the basic characters of estimated OHR in full sample and the result is presented in table 5-2. In the second column, the mean shows the average hedge ratio for commodities. The average estimated OHR for storable products (coffee, wheat and soybean) ranges from 0.54 to 0.82. The soybean has as high as 0.81 hedge ratio, but for non-storable products (live cattle and live hog), the hedge ratio is as low as 0.08. Put it in another way, the result implies that the risk is minimized, when the investor takes futures position with amount of 8% of cash position for live cattle and live hog. Referring to the variance, the volatility for OHR for wheat is the smallest in five cases. The estimated OHR series for all five commodities are non-normally distributed according to significant statistic value of Jarque-Bera (JB) test at 1% significant level. The OHR for coffee is slightly skewed with a minor longer and fatter tail on the right-hand side and has delicately higher peak than normal distribution in terms of small positive kurtosis and skewness. In the case of wheat, soybean, live cattle and live hog, the OHR series have significant fatter tails and higher peak with longer tail either in the right- or left- hand side than normal distribution. In other words, a certain amount of extreme value of hedge ratio presents for all cases.

The DF/ADF unit root test is applied for stationary test of estimated OHR from bivariate GARCH (1, 1) model with result in table 5-3. Both the one- and two-unit root tests are using 3, 6 and 9 lags for five commodities. In all cases, the larger value of T test-statistic than critical value result in rejection of null hypothesis of existence of unit root in estimated OHR time series. The absence of unit root implies that the estimated OHR time series with bivariate GARCH (1, 1) model are stationary for all agricultural products.

The Ljung-Box test is employed to test the serial correlation of residuals and examine the adequacy of order of ARCH for the full sample. In other words, if autocorrelation presents in residual series, a higher order of ARCH term is needed. Since the disturbance is not observable, the autocorrelation test is passed to the residuals u_{t1} and u_{t2} from regressions (Brooks, 2005). The squared standardized residuals u_{t1}^2/h_t and u_{t2}^2/h_t are tested up to 24th order of the L-B test. Significant test statistic indicates that autocorrelation presents in residual series for coffee and wheat. However, the autocorrelation shows up in 3 out of 10 tests and we can draw a general conclusion that the order of ARCH term is sufficient in capturing the properties of estimated OHR in general. Otherwise, a higher order of ARCH term is required to adequately explain the estimated OHR.

We present the results of Ljung-Box autocorrelation and white tests on residuals after estimating by the bivariate GARCH (1, 1) model in table 5-4. In general, the absence of autocorrelation for all five commodities implies that the order of ARCH is sufficient in the bivariate GARCH (1, 1). As Giannopoulos (1995) described, because of the low ratio of number of presence of autocorrelation to the total number of tests, there is no need to encompass higher order of ARCH and the bivariate GARCH model is satisfactory. In the white test, the remaining heteroscedasticity for wheat and soybean is statistically significant at 5% significant level. In this case, we test the stationarity of residual series with ADF test for wheat and soybean. As Qiu (2008) states that weakly stationary series allows low persistent heteroskedasticity which has no significant effect on asymptotically distribution for large sample (Qiu, 2008). The result shows that the residual series are stationary. Hence, we conclude that the conditional heteroskedasticity in residuals is not persistent enough to affect the validity of GARCH. Overall, the autocorrelation and heteroscedasticity results indicate that the assumption and order of the bivariate GARCH (1, 1) model are valid for our sample.

2. Diagonal Bivariate BEKK-GARCH (1, 1)

The DBEKK-GARCH model is an alternative representation of bivariate GARCH. In the bivariate DBEKK (1, 1) framework, the number of coefficients reduces to 11 (4 for mean equation and 7 for GARCH model) from 13 of bivariate GARCH (1, 1) and the positivity of variance-covariance matrix is guaranteed.

Estimated Parameters

Table 5-5 presents the estimated coefficients in OHR estimation based on DBEKK-GARCH (1, 1) model. Roughly all value of φ_1 and φ_2 for five commodities is significant in DBEKK-GARCH. Alternatively, the short-run deviation is helpful in explaining the log-return in cash and futures markets and there is a negative relationship between short-run deviation and log returns in general.

Coefficients a_{11}, a_{12}, a_{22} and b_{11}, b_{12}, b_{22} are significant at 1% significant level for all goods except the insignificant a_{11} for live cattle. It is worth noticing that coefficients of the ARCH and GARCH effects are ($a_{11}^2, a_{11}a_{22}, a_{22}^2$) and ($b_{11}^2$, $b_{11}b_{22}$, b_{22}^2) in the BEKK-GARCH model which uses a different way to define the coefficients of parameters. The coefficients of

ARCH and GARCH terms are squared or cross-product coefficients which entitle positive value of conditional variance-covariance matrix. However, it is hard to define the significance of coefficients of ARCH and GARCH terms, even though a_{11}, a_{12}, a_{22} and b_{11}, b_{12}, b_{22} are significance. On the other hand, fairly large value of a_{11}, a_{12}, a_{22} and b_{11}, b_{12}, b_{22} result in a large coefficient of ARCH and GARCH terms which generally demonstrate the influential roles of past values of ARCH and GARCH effects on impacting the current conditional variance. Besides the live cattle, the summations of coefficients of ARCH and GARCH effects $a_{11}^2 + b_{11}^2$ and $a_{22}^2 + b_{22}^2$ are approaching unity which implies that the persistent volatility will die out slowly for other four commodities. The coefficients in conditional covariance are significant for all five cases which reveal that there is a certain interaction between cash and futures markets. The BEKK-GARCH model improves the significance of coefficients of ARCH and GARCH effects for live cattle comparing with the standard GARCH model, although the live cattle is still the particular case.

Analysis of estimated OHR and residuals on diagonal BEKK GARCH models (normal distribution)

We show the basic statistic of estimated OHR from DBEKK GARCH (1, 1) model in table 5-6. Estimated by BEKK-GARCH model, the OHR series for the five commodities present some different features comparing to the OHR series estimated from GARCH model. The upper bound of average OHR for storable commodities slightly increases to 0.86. The mean of estimated OHR series is low for non-storable products and the variance of OHR series for non-storable products is ten times smaller than that of storable commodities. Although the OHR series are all non-normally distributed with significant kurtosis and skewness at 5% significant level, skewness for all is not typical sign of non-normality providing the small values from skewness test. Additionally, the apparent fatter tail and higher peak of OHR series than normal distribution do not show up except for the case of live cattle. Comparing to the OHR estimated by bivariate standard GARCH (1, 1), the BEKK downgrades the level of skewness and kurtosis for the estimated OHR and the non-normality in the OHR becomes less evident for the BEKK model.

As presented in table 5-7, the null hypothesis of presence of one or two unit roots is rejected at 5% significant level in both one- and two- unit root DF/ADF tests, even when the 3, 6 and 9 lags are accounted for. The OHR series from the BEKK-GARCH (1, 1) model for all five commodities are stationary with evidence of lack of unit roots. The result is identical to that of OHR estimated by standard GARCH model.

Judging from the output of test statistic in table 5-8, it is roughly support the null of the no serial correlation in residual series for all products. Serial correlation presents in 1 out of 10 series, the low ratio of autocorrelation supports the general situation that residual series are free of autocorrelation. On the other hand, the White and ADF tests demonstrate the existence of weak heteroscedasticity in residuals for coffee and other products. As describe earlier, weak heteroscedasticity has no significant effect on the estimation. In all, and the order of ARCH in the BEKK-GARCH model is validity because of no presence of auto-correlation and heteroscedasticity in the estimated OHR for five commodities in this study.

3. Diagonal Bivariate GARCH-X (1, 1)

The GARCH-X model adjoins an error-correction term in conditional variance and covariance. The error-correction term is measuring the influence of short-run deviation in the long-run cointegration relationship between log returns in cash and futures markets on the one-step-ahead forecast of conditional (co)variance.

Estimated Parameters

The number of parameters increases by 3 in diagonal bivariate GARCH-X (1, 1) model along with the error-correction term in which the numerical difficulty enlarges. The d_{11}, d_{22} and d_{12} represent coefficients of error-correction.

In this diagonal bivariate GARCH-X (1, 1) model, the positive and significant value of φ_1 imply that the error-correction term has non-negligible influence on log-return in cash market. As the φ_1 increases, the cash log-return will increase. The φ_2 is positive and significant in 2 out of 5 cases which indicates that the short-term deviation generally does not affect the log-return in futures market.

The ARCH and GARCH effects in table 5-9 present in all cases based on their significant coefficients, yet the insignificant coefficient of ARCH term for live cattle is an exception. In the case of significant ARCH and GARCH terms, the value of $a_{11} + b_{11}$ and $a_{22} + b_{22}$ are around 1 which reveals that the volatility clustering dies out tardily. The effect of error-correction term on conditional variance is significant because 4 out of 5 cases have statistically significant coefficient (d_{11}, d_{22}, d_{12}) of error-correction term. However, the short-run deviation from the long-run cointegration relationship has no meaning effect on conditional variance for soybean. The live cattle is another special case in terms of error-correction term in which conditional variance of live cattle does not depend on its past residuals, but rely on its past conditional variance and the long-run cointegration relationship between log returns in cash and futures markets. It is rational to reckon that for some special case, considering the factor of error-correction term is conducive when the ARCH effect is not significant. A close relationship between cash and futures markets is found based on significant a_{12} and b_{12}. In this study, the GARCH-X (1, 1) model is competent for more comprehensive features capturing of log returns.

Analysis of estimated OHR and residuals on GARCH-X models (normal distribution)

In basic statistics in table 5-10, the mean for storable commodities ranges from 0.59 to 0.86. The average estimated OHR for live hog exceeds 0.10, but it remains the similar low value for the case of live cattle. Test on skewness shows longer tail on left-hand side for all products except for the case of live hog. The estimated OHR series for live hog is not significantly different from normal distribution in terms of skewness, but it might follow a likelihood-symmetric distribution instead of normal distribution (Brooks, 2008). Positive value of kurtosis presents in OHR series of wheat, soybean and live cattle which have fatter tail and higher peak than normal distribution in OHR series. High kurtosis of OHR for live cattle probably results from a certain large amount of extreme observations in this OHR series. However, negative kurtosis shows up in the rest of two cases which is an evidence of thinner tail and lower and wider peak compared with normal distribution. The J-B test produces significant value for all five agricultural products which is a sign of non-normal distribution.

In table 5-11, we report the results of one or two unit root(s) test for estimated OHR from bivariate GARCH-X (1, 1). The hypothesis of existence of one unit root in OHR series is rejected for all products except for the case of live hog. For the first three GARCH models, it is the first time to find the presence of unit root in estimated OHR series. Nevertheless, we fail to accept the hypothesis of two unit roots for live hog, namely, the estimated OHR for live hog is first-difference stationary. For other four commodities, unit root (s) does not present based on significant test statistics and hence the estimated OHR series are stationary.

The table 5-12 contains result of L-B Q correlation and White tests of residuals from bivariate GARCH-X (1, 1). Significant test statistics in L-B test fails to reject the null hypothesis of no autocorrelation for all cases. Tests on residual series confirm the statement that the residual series are free of autocorrelation for each commodity. From another perspective, the significant but weak heteroscedasticity in residuals for coffee is economically negligible. In other words, the validity of bivariate GARCH-X (1, 1) model is confirmed in this study.

4. Diagonal Bivariate BEKK-GARCH-X (1, 1)

The BEKK-GARCH-X model is the BEKK parameterization of GARCH-X and it is an alternate of VECH framework. In the case of bivariate, the diagonal BEKK framework has advantages of reducing the number of parameters from and it is managed to degrade 16 parameters of diagonal VECH GARCH-X (1, 1) to 14 in DBEKK-GARCH-X (1, 1).

Estimated Parameters

We present the estimated coefficients of parameters with test statistic in parenthesis in table 5-13. The error–correction term in mean equation is not helpful for the case of live hog since the coefficients φ_1, φ_2 for this term are insignificant. However, the error-correction term in mean equations claims a negative relationship between short-run deviation of long-run cointegration relationship and log return in cash market for the rest cases, and positive relationship between cointegration and futures log-return.

Similar with BEKK-GARCH model, the coefficients of ARCH and GARCH effects on conditional variance and covariance in the BEKK-GARCH-X (1, 1) model are a_{11}^2, $a_{11}a_{12}$, a_{22}^2 and $b_{11}^2, b_{11}b_{12}, b_{22}^2$. The coefficients of error-correction terms in conditional covariance matrix are denoted as d_{11}^2, $d_{11}d_{22}$ and $d_{22}^2 + d_{12}^2$. Roughly all coefficients relevant to ARCH and GARCH $a_{11}, a_{22}, b_{11}, b_{12}$ are positive and significant, but it is still hard to tell whether the ARCH and GARCH effects are significant or not due to the uncertainty of significance of ARCH/GARCH terms. The value of joint ARCH and GARCH effects $a_{11}^2 + b_{11}^2$, $a_{22}^2 + b_{22}^2$ for almost all cases is fairly 1 that is the hint of sluggishly extinct volatility clustering. The summation of $a_{22}^2 + b_{22}^2$ for live hog is 0.0822 which is an evidence of a quick dying out volatility. With respect to the error-correction term, even though d_{11}, d_{12} and d_{22} are significant for coffee and wheat, they are insignificant in other 3 cases. The small value of d_{11}, d_{12} and implies there is a small influence of error-correction term on conditional covariance matrix in the DBEKK-GARCH-X (1, 1) model. Additionally, interaction exists between cash and futures markets. In both BEKK and VECH parameterizations of GARCH-X (1, 1), the effect of short-run deviation of long-run cointegration relationship between log spot and futures returns on future conditional variance exists. Moreover, the BEKK-GARCH-X model helps us to better assay the impact of long-run cointegration relationship on variance prediction in two different ways for agricultural products.

Analysis of estimated OHR and residuals on BEKK GARCH-X models (normal distribution)
The basic analysis of estimated OHR on BEKK-GARCH-X (1, 1) is reported in table 5-14, 5-15 and 5-16. The average OHR for storable commodities is located in the same range as previous models, while the average estimated OHR for non-storable products reduces to around 0.065. Namely, less futures contracts are needed for non-storable goods based on estimation from this model. The estimated OHR series for all commodities are found to be non-normally distributed with evidence of significant test statistic in J-B normality test. All non-normally distributed OHR series are asymmetric with fatter or thinner and longer or shorter tails than normal distribution does. In terms of skewness, the OHRs of storable commodities have slightly longer tail than normal distribution on the left-hand side, yet there shows an opposite way of estimated OHR for the non-storable products with positive and significant skewness. All estimated OHR series have fatter tail and lower and wider peak than normal distribution except for the case of coffee. The OHR of coffee shows thinner tail and lower peak relative to normal distribution. Additionally, the OHR series of non-storable commodities contains much more extreme value of OHR refereeing to apparently large kurtosis.

The DF/ADF unit root test is carried out for the estimated OHR series for all five commodities. With output in table 5-15, the null hypothesis of DF/ADF is rejected because all test statistic are large than those of critical statistic in 1- and 2- unit root tests with 3, 6 and 9. The absence of unit root in the estimated OHR series infers that all OHR series for five agricultural goods are stationary.

We show the results of L-B autocorrelation and White tests relevant to the residual series in table 5-16. The serial correlation presents in one of two residual series for the case of coffee at 5% significant level. Nevertheless, there is no presence of autocorrelation for other residual series. In other words, the result overall supports the fact of the lack of autocorrelation in the residual series for all five agricultural goods. Again, the remaining heteroscedasticity in residual series for coffee is not completely moved after estimation. However, the ADF stationary test demonstrates the weakness of heteroscedasticity. Consequently the order of ARCH term in the BEKK-GARCH-X (1, 1) is said to be adequacy for totally carry ARCH effect of estimated OHR.

5. Diagonal Bivariate GARCH-GJR (1, 1)

The GARCH-GJR model is one of two asymmetric GARCH models on OHR forecasting in this study. The GARCH-GJR model reaps asymmetric impact of negative and positive news on the future conditional variance. When the asymmetric effect shows up, the bad news always takes more weight and have larger effect than good news does (see Brooks and Henry, 2002). A dummy variable I_t is applied in ARCH term for capturing the leverage effect. The I_t takes value of 0 for good news while it takes 1 for bad news.

Estimated Parameters

There are two coefficients γ_1 and γ_2 for the dummy variable I_t in the diagonal bivariate GARCH-GJR (1, 1) model. In table 5-17, the coefficients of error-correction term are generally significant in the mean equation. The φ_1 are negative for all cases which infer that an increase of short-run deviation from the long-run cointegration relationship between log-return in cash and futures markets leads to a decrease of cash log-return. Positive and significant φ_2 for non-storable goods and insignificant φ_2 for storable products indicate that cointegration has strong effect on futures log-run for non-storable commodities, but not for storable agricultural products in this study.

In the GARCH-GJR model, the coefficient for ARCH term is either $a_{11}+\gamma_1$ or a_{11} and $a_{22}+\gamma_2$ or a_{22} depending on the sign of lagged square residuals and the value of I_t . There are two significant coefficients for the term u_{t-1}^2 which describes leverage effect for the case of wheat and soybean. The presence of leverage effects is demonstrated in the case of wheat and soybean provided positive coefficient γ_1 of the additional squared residual term. The significant negative γ_2 exhibits an abnormal phenomenon that the negative shock has less impact than positive shock for the case of soybean. The γ_2 is negative for the case of soybean, and the coefficient for the ARCH term $a_{22}+\gamma_2$ is positive since a_{22} is larger than the absolute value of γ_2. Additionally, larger effect of good news than bad news occurs for the cases of wheat and live hog. Although other coefficients of the additional u_{t-1}^2 term are insignificant statistically, there has presence of asymmetric effect. And hence it is worth investigating the leverage effect in this study using GARCH-GJR model. The sum of ARCH and GARCH coefficient is approaching 1 for the cases of coffee, wheat and soybean, but it is evidently lower than 1 for the rest cases in which the volatility of returns is temporary and returns are mean reverting.

Analysis of estimated OHR and residuals on GARCH-GJR models (normal distribution)

The table 5-18 presents the basic statistics for estimated OHR based on the bivariate GARCH-GJR (1, 1) model. In line with the basic statistics on OHR series from the previous four GARCH models, the average estimated OHR (mean) for storable and non-storable commodities are in the same range. All five estimated OHR series follow evident non-normal distribution with evidence from J-B normality test. The OHR of coffee has negative but insignificant skewness that is a sign of symmetric distribution with the equal length of tails on right and left hand side. However, the OHR series for other commodities are found to be asymmetric distributed and the lengths of tails on both sides are not equal. Most estimated OHR for each day locates on the left-hand side for wheat and soybean, but more OHR distributes on the right-hand side for other products. Test statistics on kurtosis are all significant. The negative kurtosis of OHR for coffee shows that the symmetric distribution of OHR series has lower and wider peak than normal distribution does. For the rest cases, the estimated OHR series are asymmetric distributed with higher and thinner peak than that of normal distribution.

The result of one- and two- unit root tests using DF and ADF methods is in table 5-19. We apply stationarity test of OHR series and its lagged value on this series. All test statistics are larger than critical value, and thus the null hypothesis of presence of one unit root is rejected. In the two unit roots test, again, substantially large value from test for the OHR series rejects the null hypothesis. Thus no sign of unit root implies that the estimated OHR series for all five commodities are stationary.

The Ljung-Box correlation and White heteroscedasticity tests on residual series are applied with results in table 5-20. Except for 2 correlated series for coffee and wheat, all other residual series are free of autocorrelation. The approximately free of autocorrelation for residual series is a hint that the ARCH term in the bivariate GARCH-GJR model is sufficient to totally explain the ARCH effect in series. There are two significant residual series for coffee and one residual in cash market for soybean. Furthermore, the ADF test reveals that residual series are stationary. In other words, the stationary weak heteroscedastic residual series also indicate that the order of ARCH in this GARCH model is adequate for all five commodities.

6. Diagonal Bivariate QGARCH (1, 1)

The QGARCH model uses a different way to describe asymmetric impact of news. In QGARCH model, the past residual term is added on and this additional term represents asymmetric effect of good and bad news on current/future conditional covariance matrix. The diagonal bivariate QGARCH (1, 1) model has two more coefficients than standard diagonal bivariate GARCH (1, 1) model, and the d_{11} and d_{22} are coefficients for the past residual term.

Estimated Parameters

We present result of estimated coefficients for variables in diagonal bivariate QGARCH (1, 1) model for the full sample in table 5-21. In the mean equation, the sign of significant φ_1 for all five commodities represents a negative relationship between short-run deviation from long-run cointegration relationship and log-return in cash market. The φ_2 is significant only for the case of wheat, namely, there is a positive relationship between short-run deviation and futures log-return.

The ARCH and GARCH effects are significant for all five commodities except for the case of live cattle. The estimated coefficient of ARCH and GARCH terms for live cattle are insignificant on conditional variance, but the GARCH effect is significant on conditional covariance. All coefficients of ARCH and GARCH terms are positive in this QGARCH model. In the cases of significant coefficients, the summation of ARCH and GARCH effects are close to unity which reflects a high volatility clustering, but it is not true for the case of live hog. The sum of the two effects for live hog is lower than 0.1 and this small value describes a low volatility clustering which will dies out rapidly. The asymmetric effect of information presents for all cases except for live hog. There are 3 (wheat, soybean and live hog) out of 5 cases holding significant leverage effect. Among the 3 cases, the coefficient d_{11} for wheat is negative which means the leverage effect poses an impact reduction on conditional variance in regression of log-return in cash market. For the cases of wheat and live hog, all coefficients of the asymmetric information effect are positive; and the asymmetric impact produces some extra effect on conditional variance. The same with the result of prior five GARCH models, the interaction between cash and futures markets is evident according to significant coefficients in conditional covariance equation. The QGARCH model is capable of incorporating leverage effect for commodities if there has some leverage effect.

Analysis of estimated OHR and residuals on QGARCH models (normal distribution)

The table 5-22 shows the outcome of basic statistic of estimated OHR from diagonal bivariate QGARCH (1, 1) model. The average estimated OHR for storable commodities maintains in the same range between 0.59 and 0.86. For non-storable products, the average OHR for live hog boosts from 0.08 for other five GARCH models to 0.133 for the QGARCH model. The J-B test shows that all estimated OHR series do not follow normal distribution. Furthermore, significant skewness indicates the OHR series distributes with longer tail than normal distribution on left-hand side and right-hand side for storable and non-storable commodities respectively. All estimated OHR series are either leptokurtic or platykurtic. The apparent higher kurtosis of non-storable commodities than those of storable agricultural goods implies that many more extreme estimated hedge ratio present for non-storable than storable commodities in the full time period. And a certain large amount of extreme value lies on tail for non-storable goods and results in a quite fatter tail than normal distribution does. The estimated OHR series for non-storable products presents more evident non-normality and fatter tail and higher peak than normal distribution does.

With the same method as before, the DF and ADF unit roots tests on the estimated OHR are employed with result in table 5-23. In one-unit root test, the test statistic for the OHR series for all five commodities is larger than critical value, and consequently, the null hypothesis is rejected. With similar outcome, the test of two-unit roots gets rejection of null hypothesis based on the larger test statistic. The two tests support the stationarity of estimated OHR from the diagonal bivariate QGARCH (1, 1) model for all five cases.

Similarly, the L-B autocorrelation test on estimated OHR series is transfer to test residual series, and the result associate with White tests is presented in table 5-24. Only one residual series for coffee is serial correlated, yet result for all other series fails to reject the null of no autocorrelation in the residual series for all five commodities. Generally speaking, all residual series are lack of serial correlation. In the White and further ADF tests, significant yet weak heteroscedasticity presents in residuals for all commodities except for live hog. From the prospective of economics, the insignificant heteroscedastic residual series has no significant effect on validity of this model and the ARCH term adequately incorporates all ARCH effect in the estimated OHR series and the order of ARCH term is sufficient in this bivariate QGARCH (1, 1) model for this study.

Comparison of Estimated OHR Series

We analyse the estimated OHR series from six diagonal bivariate GARCH models for the whole sample in this section. According to the specific characters of different GARCH models, the estimated OHRs through these models show similarity with minor distinctions in terms of estimated coefficient, basic statistic, graphs, correlation, and stationarity.

Judging from the estimated coefficients of parameters, the ARCH and GARCH effects are significant from six GARCH models for all five cases. In particular, the asymmetric GARCH and QGARCH model find out the adverse leverage effect in more than half cases. In other words, positive news has great effect than negative news on conditional (co)variance for agricultural commodity when the residuals follow normal distribution.

In the basic statistics, some special phenomenon attracts our attention. Estimated through six bivariate GARCH models, the average OHR has a range from 0.54 to 0.86 for storable commodities, and from 0.064 to 0.133 for non-storable commodities. The OHR for live hog exceeds 0.1 only for the models of GARCH-X and QGARCH, but the BEKK-GARCH-X and GARCH-GJR models which capture cointegration and information asymmetry do not produce larger than 0.1 OHR in average. In other words, considering either cointegration or leverage effect is not necessarily leading to larger OHR than standard GARCH model. All estimated OHR series are proved to be non-normally distributed. In terms of skewness, the OHR series for coffee are symmetric based on insignificant test statistic for GARCH-GJR and QGARCH models, yet symmetric OHR series for live hog is estimated by GARCH-X model. The three models result in symmetric non-normally distributed OHR for coffee and live hog.

Judging from graphs of estimated OHR, different models produce dissimilar graphs. Taking the best estimated OHR of wheat and live cattle represents a general OHR pattern for storable and non-storable commodities based on six GARCH models.

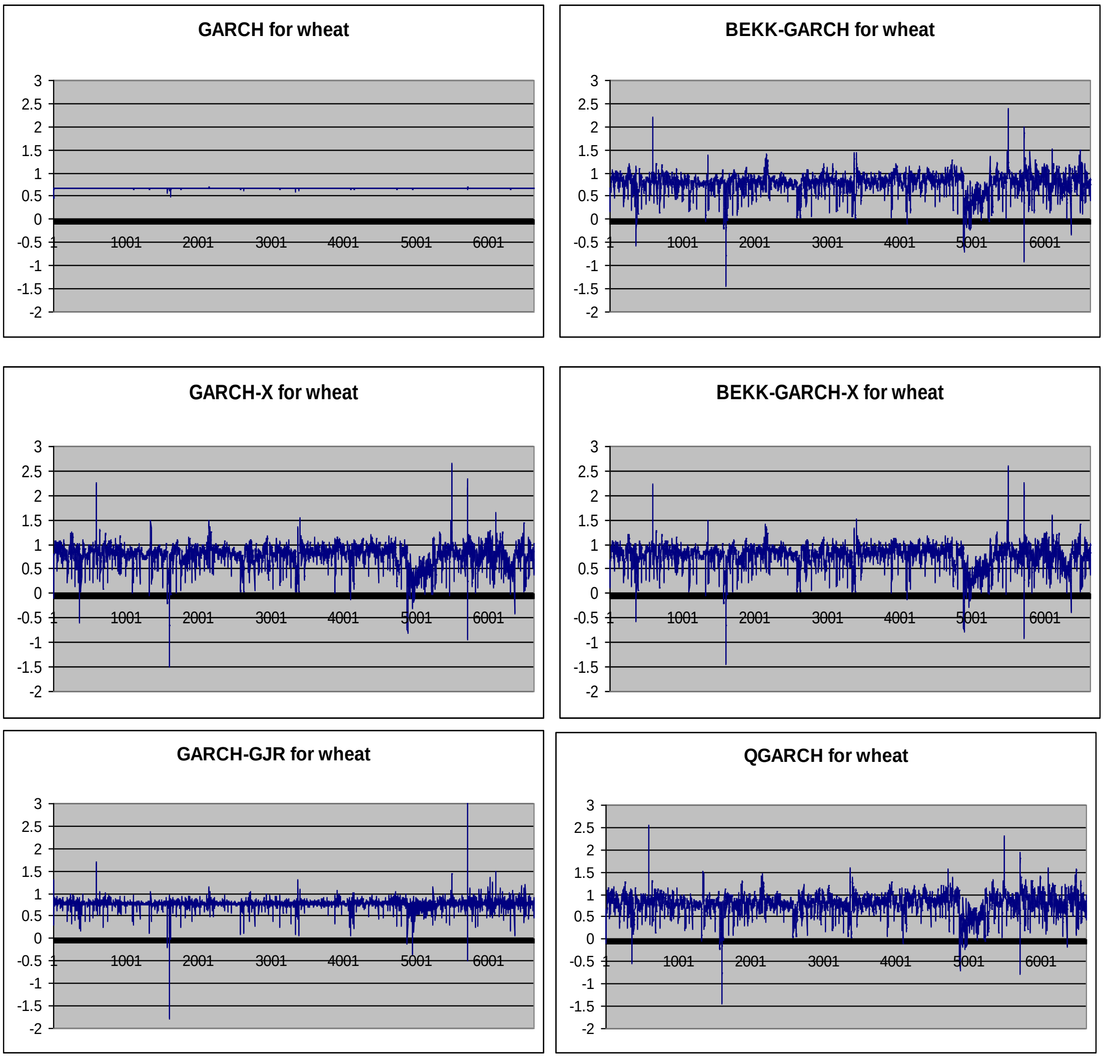


Figure 5-1 Patterns of estimated OHR series with normal distribution for wheat

As it shows in Figure 5-1, the standard GARCH yields a quite flat OHR series between 0.5 and 1 for wheat. The rest five estimated OHR patterns from other five GARCH models show the similar path of movement with much higher volatility than the OHR from standard GARCH model does. The highest and lowest estimated OHR for wheat from BEKK, GARCH-X, BEKK-X and QGARCH models are around 2.5 and -1.5, yet they are approaching 3 and -2 from GARCH-GJR model.

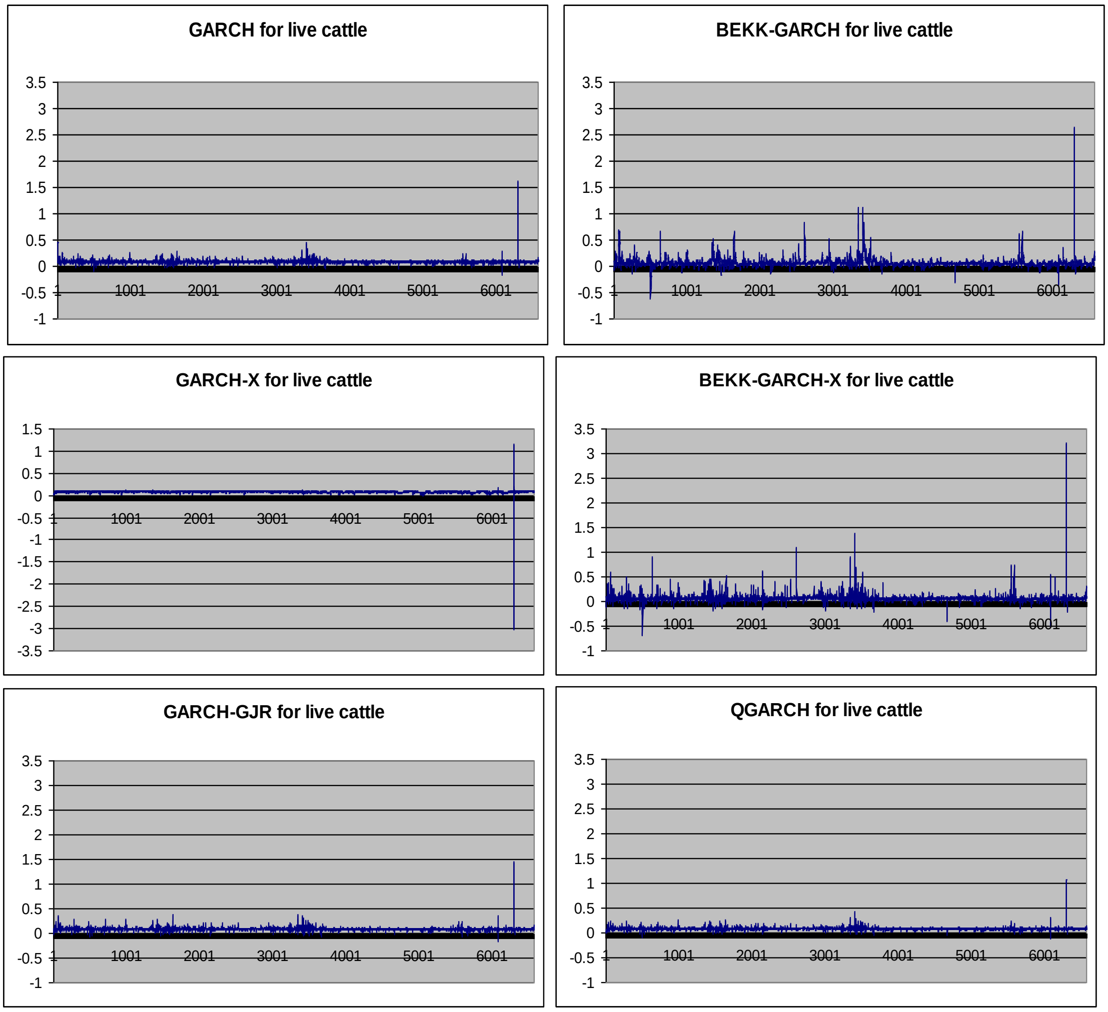


Figure 5-2 Patterns of estimated OHR series with normal distribution for live cattle

For the case of non-storable live cattle in Figure 5-2, the patterns of estimated OHR are fairly similar with varied peak and valley. The GARCH-X model produces low volatility OHR series except one break point in late 2005 and early 2006 at which the lowest OHR is lower than -3. However, the BEKK-GARCH-X obtains a highest OHR of 3.2 at the nearly the same time as GARCH-X reaching the bottom.

All estimated OHR series are found to be stationary and the autocorrelation and heteroscedasticity are removed from residual serial for all GARCH models with normally distributed residuals in full sample. In other words, the GARCH models we employ in this study are adequate and valid for the five commodities. Hence the forecasting of OHR based on these GARCH models is meaningful and well-founded, and it is engaging to compare the prediction power among the six GARCH models.

5.3 One-year Forecast

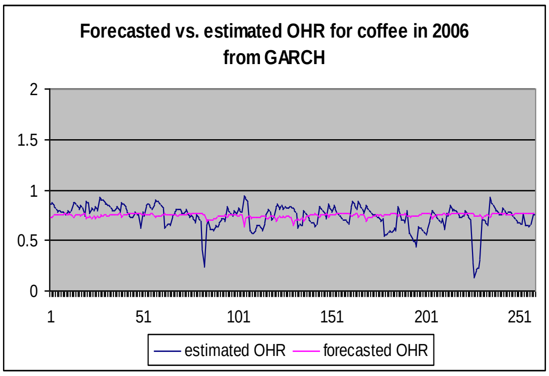
Based on estimated OHR, the out-of-sample one-year OHR and return can be predicted.

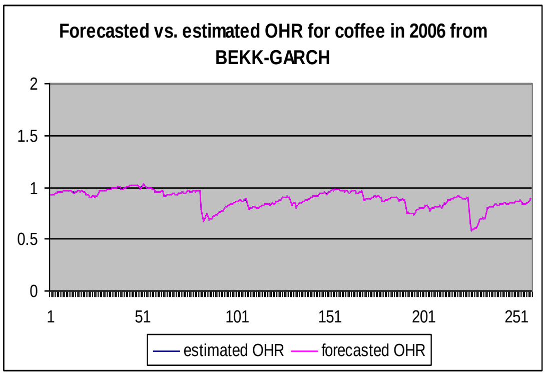
Additionally, by comparing estimated and forecasted OHR, actual return and forecasted return, the forecast error and accuracy of six GARCH model are evaluated using diverse measures.

5.3.1 Forecasting of OHR

The one-step ahead OHR forecasting through the six GARCH models is carried out on 1-year time horizon. As described earlier, the coefficients of each variable in GARCH models for forecasting are obtained from estimation in recursive method. Taking one-year forecasting for storable products for instance, the data from 01/01/1980 to 23/03/2005 are used to estimate coefficients which will be applied as coefficients of GARCH models for one-step ahead out-of-sample OHR prediction for 24/03/2005 to 23/03/2006. The effects of historical data on OHR prediction rely on feather of GARCH models. For example, the standard GARCH model incorporates an equal weight of effect for every return, but GARHC-GJR model puts more weight on negative return. The special properties of the six GARCH models result in somehow different OHR forecasting.

In one-year prediction, we apply the one-step ahead out-of-sample forecasts of OHR by six GARCH models for the period of 24/03/2005 to 23/03/2006 and 15/01/2007 to 14/01/2008 for storable and non-storable commodities. From the graphs of forecasted OHR vs. the estimated OHR series, we can get the preliminary impression visually of forecast power for six GARCH models. The closer the two series are, the better the forecast is. We show the pattern of forecasted and estimated OHR series for coffee and live cattle in Figures 5-3 and 5-4, and graphs of OHR movement for other three commodities will be presented in appendix. The blue line in figures represents the pattern of estimated OHR and the red line stands for the forecasted OHR for one-year forecast.





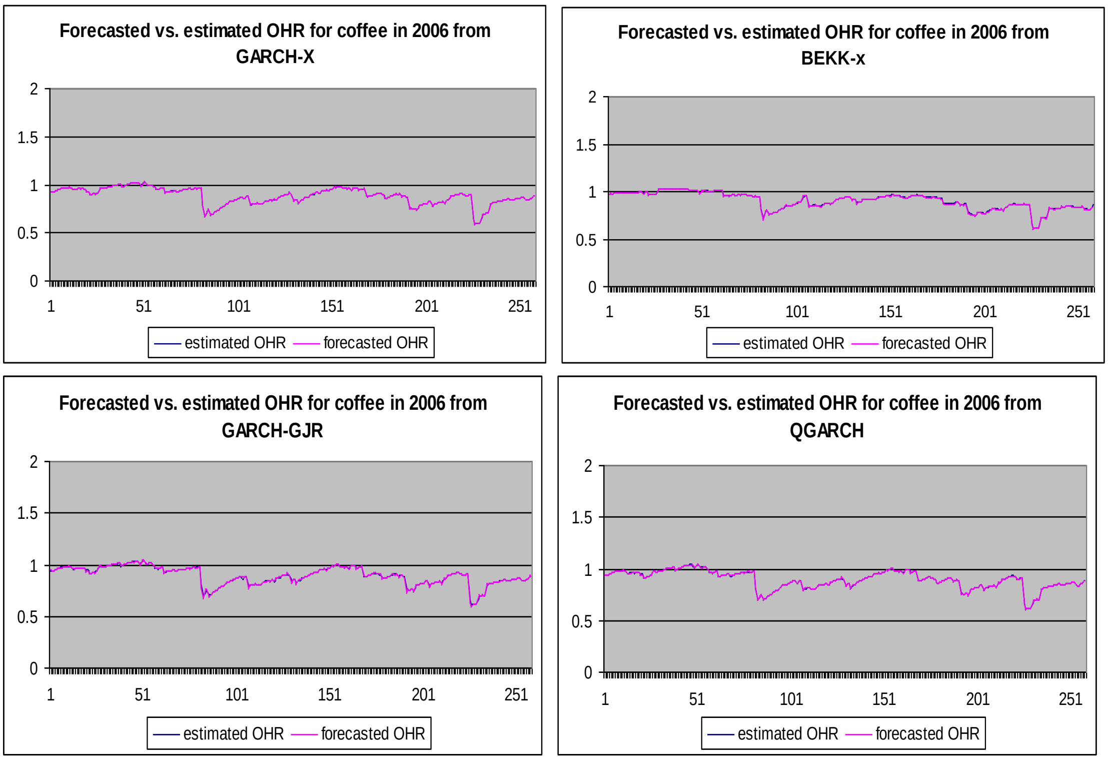
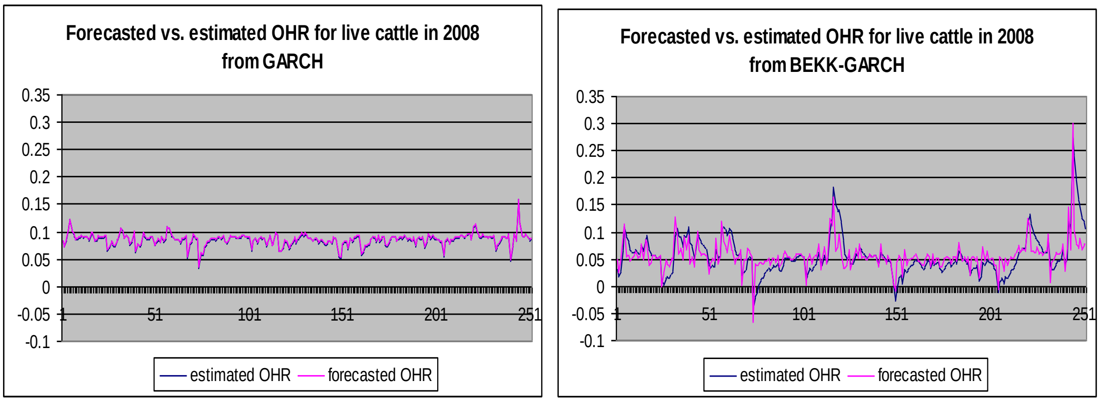


Figure 5-3 One-year forecasted OHR vs. estimated OHR with normal distribution for coffee

For the case of coffee, the estimated and forecasted OHR from GARCH model deviate apart in most of time. The estimated OHR series is much more volatile than the forecasted OHR which is fairly flat with low volatility. The forecasted OHR series from the other five GARCH models fits the estimated OHR quite well and they are almost overlapped and it is hard to tell the difference between estimated and forecasted OHR series. The graphs show an accurate prediction of OHR from all six GARCH model except for the standard GARCH model.



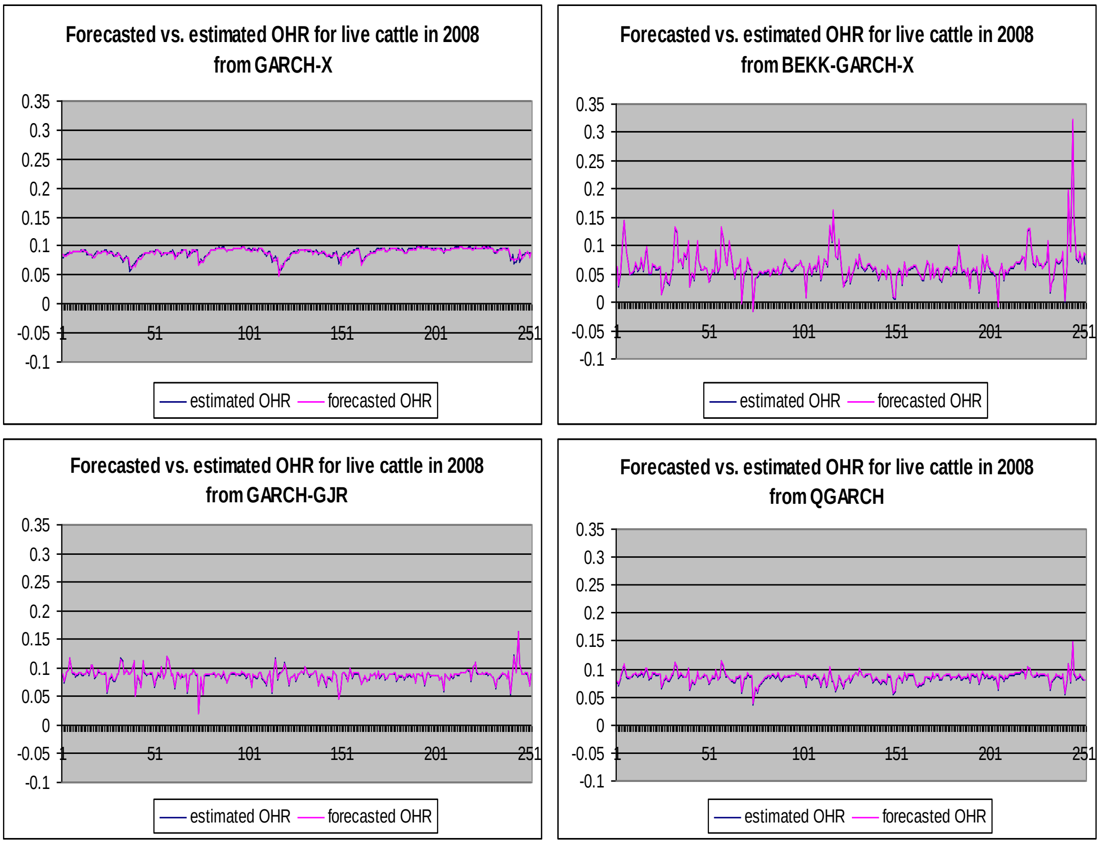


Figure 5-4 One-year forecasted OHR vs. estimated OHR with normal distribution for live cattle

For the non-storable case of live cattle, the sudden jump of OHR in late 2007 and early 2008 is probably due to the effect of more stable log-return in futures market in that period. The BEKK-GARCH model yields moderate forecast on OHR. For other five models, the forecasts look perfectly accurate.

The above two cases demonstrate that the forecasting ability of the six GARCH models we applied varies case by case, while they are powerful in OHR forecast in terms of visible graphs. However, it is too quick to determine the most powerful forecasting GARCH model among the six of them GARCH simply judging from the patterns of estimated and forecasted OHR for coffee and live cattle, and further tests about forecast error and accuracy are needed for a more comprehensive comparison.

5.3.2 Forecasting of Return

Although the estimation of hedge ratio is a moderate proxy of the actual hedge ratio generated by the GARCH-type model, it is not an appropriate scale to measure a hedge ratio series forecasted with time variation. As a result, evaluation of forecast accuracy based on conditional estimated and forecasted hedge ratio by the same approach cannot provide compelling evidence. Given the lack of benchmark for the actual hedge ratio, a logical extension is to examine the deviation of out-of-sample return away from the actual return to assess predictive performance.

5.3.2.1 Forecasts of Return

The 1-year out-of-sample prediction of return is based on the 1-year forecasted OHR. The forecasted return R_t of the portfolio is obtained from the following formula:

$$R_t = R_t^c - \beta_t R_t^f \tag{5-2}$$

where R_t^c and R_t^f are log returns in cash and futures market at time t; and the β_t is the forecasted hedge ratio at time t. Comparing the out-of-sample forecasted return of the portfolio with the actual return observed is the way for accuracy test.

5.3.2.2 Forecast Error Tests

We test predictive power of six GARCH models with normally distributed residuals on return forecasts in 1-year forecast by measuring forecast error with three evaluations in this section. The MAE, MSE and Theil's U methods are applied for forecast error test.

The forecasted return is obtained from the equation (5-2) when the out-of-sample forecasted OHR is known. In the tests of forecast error, the estimated and forecasted returns are compared, and the smaller forecast error indicates a better prediction. The results of forecast error tests for coffee, wheat, soybean, live cattle and live hog from 3 evaluations are reported in tables 5-25, 5-26, 5-27, 5-28 and 5-29.

We present the outcome of three evaluations on 1-year forecasted return for in table 5-25. Forecasted errors of six GARCH models in each evaluation method are close to each other with small discrepancy. Under the MAE statistic, the BEKK model yields the smallest forecast error, and the BEKK-X has a slightly larger error as the second best model in the ranking. The standard GARCH is inferior to others with the highest error. Again, the BEKK and BEKK-X models outperform with low forecast error 0.000007 and 0.000008 with MSE method. In accordance with the MAE test, the standard GARCH model stays in the bottom of ranking. The result of Theil's U statistic has a favour of BEKK model and the BEKK-X model follows with the second smallest forecast error. Without surprise, the standard GARCH once again produces much higher forecast error than others in this test. Generally speaking, for the 1-year return forecast for coffee, the BEKK and BEKK-X models are superior to other GARCH models in terms of forecasting performance. Nevertheless, the standard GARCH model is the weakest model among the six GARCH models. The forecasting accuracy of other three GARCH models is moderate.

We report the forecast error of each model on return prediction for wheat under three evaluations in table 5-26 and the result does not indicate a similar result with that of coffee. With MAE statistic, errors for all GARCH models are less than 0.003 except for standard GARCH model with error of 0.0035. The BEKK-X offers the smallest forecast error, followed by BEKK and GJR models. The GJR is the most accurate forecasting model under MSE test, and QGARCH, BEKK, and BEKK-X models have almost the same predictive power with delicate difference in errors. The outcome from Theil's U method presents another situation that the QGARCH outperforms, but the GJR produces the poorest forecasting performance. Overall, the QGARCH, GJR and BEKK-X models perform best in 1-year forecast with nearly equivalent forecasting ability on return prediction for the case of wheat.

The table 5-27 presents the result of forecast error tests on 1-year return forecast for soybean. The GARCH-X is the most accurate model in six in MAE evaluation, and even the second best GJR model has 10 times larger error than GARCH-X does. The largest error is from the standard GARCH in this measure. The result from MSE method confirms the same finding that the GARCH-X and GJR are best two forecasting models, while the standard GARCH is the weakest model with lowest predictive power. The situation is slightly changes in Theil's U test in which the GARCH-X remains the first in the ranking, but the second place is taken by BEKK model with GJR ranked the 5th. The standard GARCH keeps staying in the bottom of ranking for all three evaluations. Judging from the error tests, the GARCH-X is the best forecasting model, and the GJR model is also a relatively accurate forecasting model for 1-year return forecast in this case.

Three evaluation methods did not reach a consistent conclusion. The result of forecast error tests on forecasted return for three storable commodities does not show any preference about the single 'most powerful predictive model'.

For the 1-year forecasted return of live cattle, the result of forecast error tests is reported in table 5-28. The outcome of MAE method reflects a slightly better forecast of GJR model than GARCH-X, and they are top two 'best forecasting models' among six GARCH models. However, the BEKK-type models do not work well under this measure. In the MSE statistic, all forecast errors are very low. The standard GARCH, GARCH-X, GJR and QGARCH produce exactly the same forecast errors 0.000075. Relatively, forecast errors from BEKK and BEKK-X models are higher than 0.000075. The lowest forecast error from the Theil's U evaluation is made from GARCH-X model and is followed by GJR model. The guaranteed positivity of covariance matrix of BEKK and BEKK-X models does not help in forecasting since the BEKK-type model yields the largest forecast error for the 1-year return forecast for live cattle. From the three methods, the GJR and GARCH-X are superior forecasting models in this case.

The three forecast error tests make a compatible result for live hog which is implied from the outcome in table 5-29. The asymmetric QGARCH model provides the smallest forecast error in six GARCH models for all 3 evaluations. However, the GARCH-X has the poorest forecasting performance with highest errors under all measures in this case. The result for forecast error tests is mixed for three storable products that forecasting ability of GARCH models depends on commodity. However, for the two non-storable commodities, asymmetric models (GJR and QGARCH) have great predictive power for 1-year return forecast in terms of forecast error.

5.3.2.3 Modified Diebold Mariano Test

The MDM test is proposed by Harvey and Leybourne (1997) to test forecast accuracy between two forecasting methods. The modified method based on the Diebold Mariano (DM) test is managed to compare autocorrelated forecasts, and is still valid when the forecast is biased and the forecasted series is non-normally distributed.

The MDM evaluation is employed to test forecast accuracy of forecasted return from six GARCH models. We measure the deviation of forecasted returns from two GARCH models away from actual return with MDM test, and a smaller deviation indicates a more accurate forecast. This method makes the comparison between any two models more straightforward. For each commodity, we analyze the out-of-sample predictive power of six GARCH model on return forecasting by comparing any couple of them for 1-year forecast. The results of MDM test are presented in table 5-30, 5-31, 5-32, 5-33 and 5-34 for coffee, wheat, soybean, live cattle and live hog respectively. In these tables, the '**<**' represents that the latter model has better forecast than the prior one; correspondingly the '>' implies that the prior model has better forecast than the latter one and the '=' means the two models have equal predictive power.

The table 5-30 reports result of MDM test for the case of coffee from GARCH models with normally distributed residuals. In this test, the MSE and MAE approaches are two methods applied to measure relative errors. The first column in this table shows the two comparing models. The symbols in the other two columns imply the superior forecasting ability of either of them in the two comparing models. For instance, the first comparison is between standard GARCH and BEKK models and the result supports the superiority of BEKK model in both MSE and MAE measurements.

Coincidently, the test results with MAE and MSE methods of MDM are exactly the same in this case. The standard GARCH model always under-performs others in the comparisons between standard GARCH and other models. However, the BEKK model performs better than others for both measures. The GARCH-X model provides the worst forecast in all GARCH models except for standard GARCH. The forecasting performance of BEKK-X, GJR-GARCH and QGARCH is different on forecasting return in which the BEKK-X has smaller forecast error than GJR-GARCH, but holds larger error than QGARCH. Furthermore, the forecasting ability of GJR and QGARCH models is not significantly different from each other. In sum, the BEKK model has outstanding forecast, but the standard GARCH performs worst in all models for coffee for the one year forecast with normal distributed residuals.

For the case of wheat, the MSE and MAE tests provide different results of the one-year forecast of return for normally distributed residuals. Judging from the result in table 5-31, the standard GARCH model never outperforms others with MAE measure, while it has equal accuracy with GARCH-X model with MSE measure. The forecasting of BEKK is only worse than GJR model for MSE approach, while the BEKK and GJR have equal forecasting performance in MAE method. The BEKK either performs better or equivalently when comparing the BEKK with others relying on the measure. The BEKK-X, GJR and QGARCH produce smaller forecast errors than GARCH-X model does. Among the three models, BEKK-X is not better than GJR, but is not weaker than QGARCH. For the comparison between GJR and QGARCH models, the GJR is not worse than QGARCH for both measures, and this relationship also can be derived out from comparisons between BEKK-X and GJR, and between BEKK-X and QGARCH. Overall, the GJR yields a notable accurate forecast and the BEKK has remarkable predictive power on forecasting 1-year return forecast in 2006 for wheat under the MDM test.

The table 5-32 reports the result of the MDM test on forecasted return on 1-year forecast horizon for soybean. Similar to the result for coffee and wheat, the standard GARCH is the weakest forecasting model in six GARCH models and the BEKK performs better than the BEKK-X but not better than the GARCH-X model. Additionally, there are no significant differences between BEKK and GARCH-GJR, and between BEKK and QGARCH models in terms of predictive power. Specifically, the GARCH-X produces the most accurate forecast among all six GARCH models with MAE measure, yet its performance equals to that of BEKK and QGARCH under MSE method. The BEKK-X yields larger forecast error than GJR does for MAE method, but they are equal with MSE method. The GJR and QGARCH are detected with the same degree of forecast accuracy of prediction of return. In general, the GARCH-X model outperforms and has great forecasting power with normally distributed residuals for one year forecast for soybean.

Summary 1 the percentage of dominance of one model over others for three storable commodities (coffee, wheat and soybean) on 1-year return forecast with normal distributed residuals based on the MDM test

Model	1-year forecast of return from MDM	
	MSE	MAE
GARCH	0	0
BEKK	60	66
GARCH-X	27	48
BEKK-X	48	48
GJR	48	48
QGARCH	39	33

Note: this summary reports the percentage of dominance of every single GARCH model when they compare with others. For instance, the standard GARCH compares with other five GARCH models, the number of out-performance of standard GARCH for the cases of three commodities divided by 15 is the percentage of dominance of standard GARCH model.

Even though the BEKK model overall outperforms other GARCH models, the standard GARCH model is never better than others for storable commodities on 1-year return forecast based on the output from the summary 1. No consistent conclusion is reached about the most accurate GARCH models since the BEKK, the GJR and the GARCH-X models outperform for the case of coffee, wheat and soybean, respectively. Based on summary 1, we find that the BEKK-type models are generally the best forecasting GARCH models in 1-year forecast of return with normally distributed residuals for these three storable agricultural commodities.

We present the output of the MDM test on one-year forecasted error for non-storable live cattle in table 5-33. The MSE and MAE methods share a unique result for the case of live cattle. The forecasting performance of standard GARCH model is superior to those of BEKK and BEKK-X models and is equally good to GARCH-X and QGARCH models, although it is worse than GARCH-GJR. The BEKK model underperforms all other models. For the GARCH-X model, its forecast error is larger than that of BEKK-X, while is equal to those of GARCH-GJR and QGARCH models. The asymmetric GJR and QGARCH models provide more accurate forecast than BEKK-X does. Between these two asymmetric models, the GJR outperforms QGARCH. To sum up, the GJR yields best prediction of return in 1-year forecast when the residuals is normally distributed for the case of live cattle.

For the case of live hog, the results in table 5-34 from MSE and MAE methods are consistent for the 1-year return forecast. The standard GARCH has a moderate forecasting power that it is better than BEKK and GARCH-X, but is poorer than other three models. The BEKK has better forecast than GARCH-X and BEKK-X do, but yields worse prediction than GJR and QGARCH do. The GARCH-X provides an overwhelming worst forecasting performance among the six GARCH models. The BEKK-X and GJR are equal good in the return forecast, while the GJR is not as great as QGARCH. Between the BEKK-X and QGARCH models, the latter one is superior to the prior one. In other words, the QGARCH is the most powerful forecasting model when the residuals follow normal distribution for live hog for one-year return forecast.

Summary 2 the percentage of dominance of one model over others for two non-storable commodities (live cattle and live hog) on 1-year return forecast based on MDM test

Model	1-year forecast of return from MDM	
	MSE	MAE
GARCH	39	39
BEKK	9	9
GARCH-X	21	21
BEKK-X	39	39

GJR	69	69
Q	69	69

Note: see note in summary 1.

The summary 2 gives evidence to show the general superiority of asymmetric GARCH models in 1-year return forecast for non-storable products. In these two non-storable commodities, both the asymmetric GJR and QGARCH models which capture the leverage effect of information dominate others. Namely, the asymmetric GARCH model (GJR and QGARCH) have higher predictive power than other four GARCH models for non-storable agricultural products for 1-year return prediction.

5.4 Two-year Forecast

Besides the one-year prediction of OHR, the two-year forecast of OHR with normally distributed residual is also applied in this study. This is because Geppert (1995) and Malliaris and Urrutia (1991) find that the length of forecast horizon does affect the hedging performance of GARCH models, and longer horizon results in lower hedging effectiveness. The two-year forecast is to test whether the predictive power of GARCH models varies depending on the length of forecast horizon on OHR prediction in agricultural and commodities' futures markets.

5.4.1 Forecasting of OHR

In order to avoid data contaminated, the two-year out-of-sample forecast is not overlapping the time period of one-year prediction. In this longer time forecast, the out-of-sample prediction of OHR is from 24/03/2003 to 23/03/2005 for storable commodity and from 15/01/2005 to 14/01/2007 and non-storable products. For the 2-year forecast on OHR, the deviation between patterns of estimated and forecasted OHR series indicates some different conclusions regarding the following graphs. For comparison, the OHR patterns for the cases of coffee and live cattle are presented in figures 5-5 and 5-6.

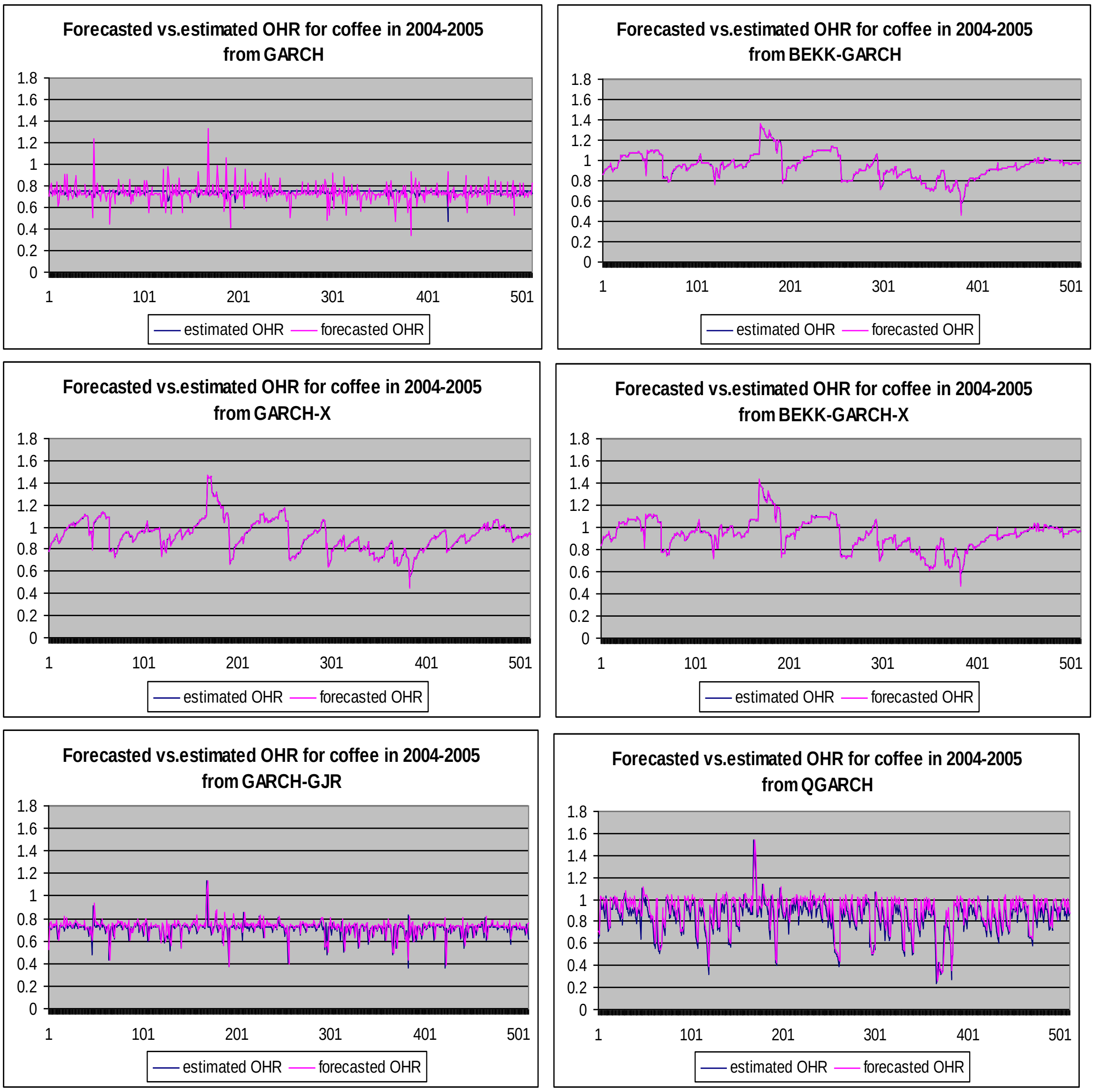


Figure 5-5 Two-year forecasted OHR vs. estimated OHR with normal distribution for coffee

The OHR pattern for coffee from standard GARCH model shows an average OHR around 0.8 and a higher volatility for forecasted OHR than that of estimated OHR. The BEKK, GARCH-X and BEKK-GARCH-X yield similar patterns with higher mean value 0.9. In these patterns, the forecasted and estimated OHR series are compatible with each other quite well and the prior one can be an accurate proxy of the latter one for the two-year out-of-sample forecast. The forecasts from asymmetric GJR and QGARCH models are not as accurate as the BEKK, GARCH-X and BEKK-GARCH-X. Furthermore, the QGARCH model produces higher OHR than other models do in average for the case of coffee.

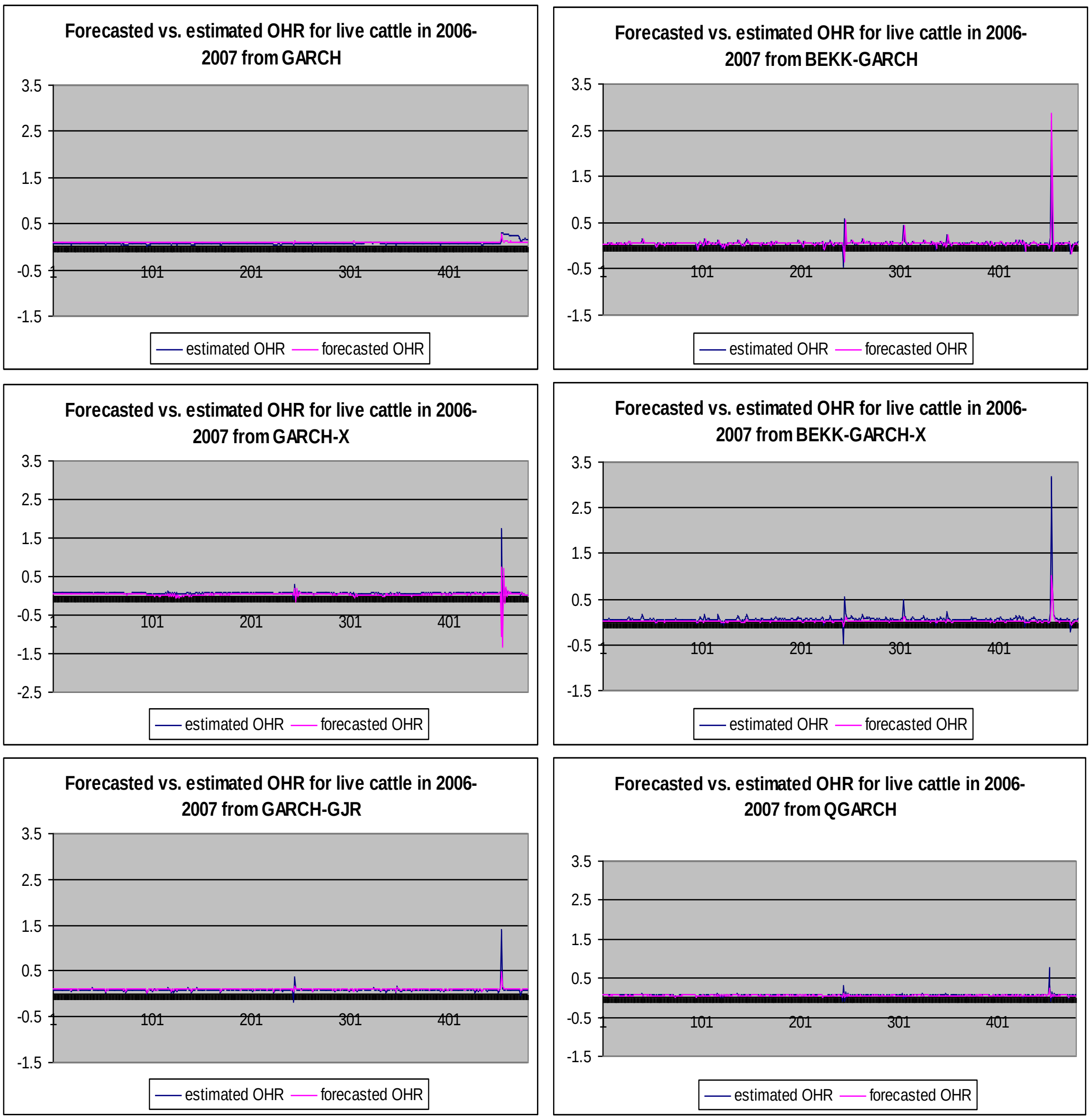


Figure 5-6 Two-year forecasted OHR vs. estimated OHR with normal distribution for live cattle

In the case of live cattle, the influence of the variance reduction of futures log-return is reflected in the early 2006 in Figure 5-6. All patterns indicate extremely low value of estimated and forecasted OHR series. The BEKK-X model tends to provide lower value of forecasted OHR than estimated OHR; however, all others models produce higher forecasted OHR over estimated OHR. The estimated OHR from BEKK has a sudden break point with an OHR as high as 3.2. Almost at the same time point, the GARCH-X forecasts a negative value of 1 as the optimal hedging strategy and this forecasted OHR series apparently does not fit the estimated OHR series for this case.

5.4.2 Forecasts of Return

5.4.2.1 Forecasts of Return

In line with the one-year forecast, the forecasted return for two-year out-of-sample prediction is obtained by using the same portfolio. We test the forecast error and forecast accuracy on forecasted return for two-year time horizon from six GARCH models in the following two sections.

5.4.2.2 Forecast Error Tests

With assumption that the residual series follows normal distribution, we test the forecast error tests on two-year return forecast for coffee, wheat, soybean, live cattle and live hog and present the outputs in table 5-35, 5-36, 5-37, 5-38 and 5-39.

With the result for coffee in table 5-35, the BEKK and BEKK-X models have the lowest forecast errors in MAE test. In contrast, the standard GARCH and GJR models provide the highest errors. The result with MSE statistic is consistent with that from MAE test. In Theil's U measure, the BEKK-X is the most accurate forecasting model with subtly smaller error than BEKK. In this test, the two asymmetric GARCH models have poorest forecasting performance instead of standard GARCH. Based on the above evidence, we indicate that the BEKK and BEKK-X models are most capable forecasting models in 2-year return forecast for coffee.

In the case of wheat, the output in table 5-36 indicates that the 2-year forecasted return from QGARCH firmly close to actual return and the QGARCH model yields the most accurate forecast in six GARCH models in MAE test. When the forecast error is tested with MSE method, the QGARHC model gives the smallest error and maintains the first place in the ranking, although forecast errors of all six GARCH models are small. In the Theil's U statistic, the best forecasting model QGARCH in MAE and MSE tests is not the optimal one anymore. Instead, the GARCH-X model that does not perform well in the previous two evaluation methods outperforms other GARCH models with Theil's U method. Roughly speaking, the QGARCH model is the most accurate forecasting model in this case.

We report the result of forecast error tests on two-year forecasted return for soybean in table 5-37. Under the MAE test, only the GJR model produces forecast error that is lower than 0.004. And the BEKK has the second smallest forecast error in this evaluation. For the MSE statistic, the BEKK is the most accurate forecasting model with lowest forecast error and the GJR ranks third in this case. On the other hand, the BEKK and GJR models rank the first and second in the Theil's U test. In these three evaluation methods, the BEKK outperforms in 2 out of 3 cases and the GJR performs best in the third case. In other words, the BEKK and GJR models are best two forecasting models in six GARCH models on long-term return prediction for soybean.

For the three storable commodities, the BEKK model has good forecasting ability on the 2-year return forecast for coffee and soybean. If the following MDM test confirms its superiority, there will be sufficient evidence to recommend the BEKK model for the long-term return forecast.

We test the forecast error of forecasted return from six GARCH models from 15/01/2005 to 14/01/2007 for live cattle and the report the result in table 5-38. In each evaluate method, forecast errors from different GARCH models close to each other with delicate difference. For the MAE test, the standard GARCH model is slightly more accurate than others with the smallest forecast error, and the GJR follows. We get a similar finding in MSE statistic in which the standard GARCH and GJR are almost equally best forecasting models with fairly equivalent forecast errors. The Theil's U produces higher forecast error for all models. However, the difference of forecast errors between the standard GARCH and GJR models are rather small. Overall, the standard GARCH and GJR models are powerful on return forecast for two-year time horizon for live cattle provided that the residual series is normally distributed.

In the case of live hog, we report the forecast error tests on forecasted return from five GARCH models are employed and the output in table 5-39. The QGARCH model has convergence problem for long-term OHR forecast, and thus the forecast error test on forecasted return from the QGARCH model is not available. In all three evaluations, the GJR model yields the lowest errors in all five GARCH models, even though with delicately lower errors than others. On the other hand, the GARCH-X is called the poorest forecasting model with subtly higher errors for three evaluate methods in this case. In general, the GJR model is superior to other four GARCH models for longer time return forecast for live hog.

It is easy to find that the GJR model perform quite well on the 2-year return forecast for both live cattle and live hog. A further test is required on the superiority of GJR model on return prediction for non-storable agricultural commodities.

5.4.2.3 Modified Diebold Mariano Test

To further test the forecast accuracy of six GARCH models on two-year OHR forecast, the Modified Diebold Mariano (MDM) accuracy test is applied on forecasted return for five agricultural products. Every two GARCH models are compared in terms of deviation of forecasted return from actual return in this method, and the superiority of either of them is implied from symbols ‘<’, ‘>’ and ‘=’. All results are reported in table 5-40, 5-41, 5-42, 5-43 and 5-44 for five commodities.

The MDM test with MSE and MAE methods on forecasted return produces highly similar result in table 5-40 for the case of coffee. The standard GARCH under-performs against all other GARCH models except for GJR which is the poorest forecasting model in this case. The BEKK outperforms all other models with highest forecasting accuracy. The GARCH-X, BEKK-X and QGARCH models have moderate predictive power. For the 2-year return forecast, the BEKK model has most powerful forecasting ability in all six models for coffee.

For the case of wheat, the output of MDM test from MSE and MAE statistic is somehow different in table 5-41. Combining the result from the two statistics, the standard GARCH model underperforms the BEKK and GARCH models, but it is not poorer than rest of them. The performance of BEKK is equal or better than GARCH-X, BEKK-X and GJR models depending on the measure in MDM test. The QGARCH is even more accurate than the BEKK in comparison. The GARCH-X model does not provide worse return forecast than BEKK-X and GJR models, but performs worse than QGARCH does. In the last 3 comparisons, the asymmetric GJR and QGARCH are better than BEKK-X in return forecast. To sum up, the QGARCH model has the most powerful long-term forecasting ability on return prediction in the six GARCH models for wheat.

The result of MDM test on forecasted return reported in table 5-42 is for the case of soybean. The MDM based on MSE and MAE reaches some different results in some extent. The standard GARCH is not better than BEKK, but is equally good to GARCH-X and QGARCH models in terms of forecasting accuracy. In the comparison between GARCH and

BEKK-X, the MSE prefers the latter model, but the MAE has no preference. The reverse outcome shows up in the comparison between the GARCH and GJR models. The BEKK has greater forecasting performance than GARCH-X, BEKK-X and QGARCH models do, yet it is not better than GJR model. The GARCH-X model never outperforms others for this case. There are some mixed results for the last three comparisons which do not affect the general conclusion. In sum, the BEKK and GJR models are the most accurate forecasting models in all six models for 2-year return forecast for soybean.

Summary 3 The percentage of dominance of one model over others for three storable commodities (coffee, wheat and soybean) on 2-year return forecast with normal distributed residuals based on MDM test

Model	2-year forecast of return from MDM	
	MSE	MAE
GARCH	12	27
BEKK	72	87
GARCH-X	27	27
BEKK-X	48	33
GJR	6	39
QGARCH	39	48

Note: see note in summary 1.

As reported in summary 3, the BEKK dominates other models. The outperformance of BEKK model for storable commodities is more apparent for the 2-year return forecast than that on 1-year short term return prediction.

The results of MDM test with MSE and MAE statistic in table 5-43 are consistent with each other on forecasted return for the case of live cattle. The standard GARCH model overall outperform all other five GARCH models. This result approves the best performance of standard GARCH on 2-year return forecast for live cattle. The rest comparisons demonstrate the dominance of the standard GARCH model. In other words, we conclude that the standard GARCH has the greatest predictive power for long-term return prediction for live cattle.

As reported in table 5-44, the output from MDM test for live hog is less than other four commodities. This is because the QGARCH model is not capable of predicting the two-year of OHR in terms of convergence, and the comparisons between forecasted return from QGARCH are not reliable. The MSE and MAE produce the same result. The standard

GARCH outperforms GARCH-X model, but it is not better than any other GARCH models. The BEKK has better return forecast than GARCH-X does. Both the BEKK-X and GJR models are more accurate than GARCH-X, and the GJR is relatively better than BEKK-X. In sum, the most accurate forecasting model for 2-year return prediction with normally distributed residuals for live hog is the GJR model.

Summary 4 The percentage of outperformance of one model over another for two non-storable commodities (live cattle and live hog) on 2-year return forecast based on MDM test

Model	2-year forecast of return from MDM	
	MSE	MAE
GARCH	60	60
BEKK	39	39
GARCH-X	9	9
BEKK-X	30	30
GJR	81	81
QGARCH	30	30

Note: see note in summary 1.

The summary 4 based on the MDM test suggests that the GJR-GARCH model generally provides the most accuracy prediction, and the standard GARCH is the second best forecast model for non-storable commodities on the 2-year return forecast.

In addition, for the categories of storable and non-storable agricultural products, the conclusion is mixed depending on the specific commodity.

5.5 Findings

In this chapter, we confirm the validity of the six GARCH model for in-sample OHR estimation in agricultural and commodities markets when the residual series follows normal distribution, and the reliability of these models for further out-of-sample forecast. Meanwhile, the adverse leverage effect is found in both 1-year and 2-year estimation of OHR for the five agricultural commodities. The predictive power of six GARCH models on one-year and two-year out-of-sample OHR and return forecast is using MAE, MSE, Theil's U and Modified Diebold Mariano tests when the residual series is normally distributed. The forecast accuracy test on the comparison between estimated and forecasted OHR is not appropriate since the estimated OHR is not an accurate proxy of real OHR. As an alternative, the tests on forecasted return and deviation between forecasted and real return are more reliable on determining the best forecasting model in terms of forecast accuracy.

Using the MAE, MSE and Theil's U tests, the BEKK is the most powerful forecasting model among six for coffee for both short-term and long-term return predictions. Additionally, the BEKK-X model performs equivalently with BEKK for the 2-year long-term forecasting. Asymmetric GJR and QGARCH models have high predictive powerful for wheat, live cattle and live hog on both horizons. On the 1-year and 2-year out-of-sample forecast horizons, the GARCH and BEKK models provide the best forecasts respectively for the case of soybean. Furthermore, the BEKK outperforms in three out of six cases for three storable commodities on short- and long-term return predictions.

The result from MDM evaluation method is similar to the result presented above. The BEKK dominates the others for storable commodities. However, the exemption appears in the long-term forecast for live cattle in which the GJR yields the best forecast with the first three tests, while the standard GARCH model performs best with MDM test. This special result does not influence the dominance of asymmetric GARCH model in terms of percentage of dominance for non-storable agricultural product. For the case of soybean, the asymmetric GJR has equally dramatic performance with BEKK model in the long-term forecast in the MDM test. The mix results are in line with the theory of Chen et al. (2003) that the length of hedging horizons might have effect on forecast accuracy for various forecasting methods.

Associate the results from the two types of evaluations; it is hard to draw a general conclusion upon the best out-of-sample forecasting model on OHR forecast. Different GARCH models outperform for coffee, wheat and soybean. However, the asymmetric GJR and QGARCH models are most powerful forecasting models for non-storable commodities (live cattle and live hog). Notwithstanding, according to the summary of dominance percentage of each GARCH model, we state that the BEKK and asymmetric GARCH models have higher possibility to be the most powerful forecasting models for storable and non-storable commodities respectively with normally distributed residuals in this study, although the forecasting ability of the six GARCH models mainly depends on the commodity and horizon of forecasting.

For the long-term 1-year or 2-year investor in agricultural market, the BEKK and asymmetric GJR and QGARCH models are recommended to forecast the optimal hedge ratio for hedging risk for coffee, wheat and soybean. While in the case of livestock, we suggest to applying the GJR and QGARCH models for hedge ratio prediction when the normal distribution is used.

Part B

5.6 Introduction

The comparison of forecasting power among six GARCH models are based on student's *t* distributed residual term in this part, which is parallel with Part A.

In theory, the student's *t* distribution is supposed to be a more proper distribution for residuals for the samples in this study. When the residual term is captured by student's *t* distribution, the fatter or thinner tail, higher or lower peak than normal distribution of pattern of economic time series can be explained more completely. We are aiming to investigate which GARCH model outperforms for each case in this different situation, whether the accuracy of OHR prediction from the six GARCH models becomes better than that of normal distribution, and if there is any ranking change of forecasting power in six GARCH models due to the two different distributions of the residual term. This empirical study will provide evidence and give answers.

The structure of this chapter is similar to Part A which consists of 5 sections. In section 5.7, the interpretation of estimator of each parameter provides more detail to know the features of six GARCH models and the commodities. The sections 5.8 and 5.9 describe one-year and two-year forecasts of OHR and return from various perspectives, i.e. pattern of estimated and forecasted OHR; forecast error and accuracy tests for forecasted return. The comparison of forecast power of GARCH models and its result will be presented in these two sections. Apart from that, an analysis of dominance of each GARCH model over others is reported as summary in order to enhance the generalization of this study. In the last but not the least section 5.10, we draw a conclusion about the forecasting ability of six GARCH models on OHR prediction in agricultural and commodities' futures markets when the residual series follows student's *t* distribution. We compare the results between normal and student's *t* distributed residuals in the last section of this part.

5.7 Estimated OHR

In the short-term one-year forecast of OHR, the OHR is estimated for full sample from 01/01/1980 to 23/03/2006 and 01/01/1980 to 14/01/2008 for storable and non-storable agricultural commodities. The estimated OHR is analysed to confirm the sufficiency and reliability of six GARCH model employed for the five agricultural commodities in this study. Estimated coefficient of variables in both mean equations and GARCH models are reported in table 5-45, 5-49, 5-53, 5-57 and 5-61 for bivariate standard GARCH (1, 1), BEKK-GARCH (1, 1), GARCH-X (1, 1), BEKK-GARCH-X (1, 1), GARCH-GJR (1, 1) and QGARCH (1, 1), when the residual series is conditionally normal distributed for five commodities.

1. Diagonal bivariate standard GARCH (1, 1) model

Estimated parameters

The result in table 5-45 is for diagonal bivariate GARCH (1, 1) model. The error-correction term in mean equation is quite explanatory with evidence of significant coefficients. The φ_1 for five commodities are significant and negative which indicates that the short-run deviation from the long-term cointegration relationship between cash and futures log returns has a negative effect on log-returns in cash markets. However, the φ_2 implies that the short-run deviation has positive influence for wheat, live cattle and live hog, but it does not explain the behavior of log-returns in futures market for the case of coffee and soybean.

All coefficients of ARCH and GARCH terms ($a_{11}, a_{12}, a_{22}, b_{11}, b_{12}, b_{22}$) are positive and significant, and it makes a naturally positive covariance matrix. The ARCH and GARCH effects are obvious in cash and futures log-return series for the five products. The mutual effect between log-returns in cash and futures markets are demonstrated according to the significant a_{12} and b_{12}. Furthermore, the summation of coefficients of ARCH effect and corresponding GARCH effect is moderately 1 for all cases. Which means the volatility clustering of log-return series is persistent and will not die out until a long time period for all five agricultural products when the OHR is estimated from the bivariate diagonal GARCH (1, 1) model with student's t distributed residuals.

Analysis of estimated OHR and residuals from standard GARCH models (student's t distribution)

In this part, we analyze the properties of estimated OHR from diagonal bivariate GARCH (1, 1) model when residual term follows student's t distribution. The result of basic statistic for estimated OHR is represented in table 5-46. The average OHR for wheat and soybean is between 0.85 and 0.90. Among the three storable commodities, the moderate OHR for coffee is the lowest. As expected, the average OHRs for non-storable products are as low as 0.05 and 0.149. From the variance column, we can see that the deviations of OHR for wheat and live cattle away from their average OHR are the smallest. The significant value of kurtosis and skewness tests implies fatter tail for all five products with longer tails on the right-hand side for coffee, live cattle and live hog, yet on the left-hand side for the rest two commodities. The large t-statistic from Jarque-Bera test reveals that all estimated OHR series do not follow normal distribution.

Through the one- and two- DF and ADF unit roots test, the output in table 5-47 confirms the stationarity of estimated OHR series from standard GARCH (1, 1). The statistic value for all commodities is significantly larger than the critical value of the DF/ADF tests at 1% significant level. This implies that the estimated OHR series for full-sample for all cases from the GARCH model is stationary.

The L-B autocorrelation test of residual series is a substantial signal for testing the adequacy of ARCH order in the GARCH model. If the autocorrelation exists, the order is not sufficient to capture all ARCH effect in the estimated OHR, and vice versa. In table 5-48, all residual series do not hold autocorrelation at 5% significant level. In the White test, the heteroscedasticity is not totally removed from residuals for wheat. The ADF stationarity test is applied to these two residual series which are proved to be stationary (the result is not presented). In other words, the stationary residual series with weak heteroscedasticity do not require high order model. In sum, all autocorrelation and heteroscedasticity are removed from residual series in full sample and the order of GARCH (1, 1) model is sufficient in this study.

2. Diagonal bivariate BEKK-GARCH (1, 1) model

Estimated coefficient

The estimated coefficients for each variable from both mean equation and bivariate diagonal BEKK-GARCH model are reported in table 5-49. The φ_1 and φ_2 of error-correction terms for each commodities are roughly significant. It shows a negative relationship between the short-term deviation and log-return in cash and futures markets for coffee, wheat and soybean, yet positive relationship for the other two cases in cash market. Additionally, the insignificant coefficients of φ_2 for live cattle and live hog reveal that the error-correction term is not substantial for describing the long-term cointegration for non-storable commodity in futures market based on this model.

The BEKK model ensures the positivity of covariance matrix with positive variables, but it is hard to judge the significance of coefficients of ARCH and GARCH terms $a_{11}^2, a_{22}^2, a_{11}a_{22}, b_{11}^2, b_{22}^2, b_{11}b_{22}$ providing the value of each single variable. Nevertheless, we can determine that the joint effect of ARCH and GARCH terms are apparent since the summation of the ARCH and corresponding GARCH impacts is fairly unity for all five agricultural products. From another viewpoint, the unity of summation of ARCH and GARCH effects implies that the volatility clustering will die out gradually. The $a_{11}a_{22}$ demonstrates the existence of interaction between cash and futures markets.

Analysis of estimated OHR and residuals from BEKK-GARCH models (student's t distribution)

The basic analysis of estimated OHR series from BEKK-GARCH model for coffee, wheat, soybean, live cattle and live hog is reported in table 5-50, 5-51 and 5-52. The average estimated OHR for coffee is 0.58, yet the OHRs are higher for the cases of wheat and soybean. The estimated OHR for live cattle and live hog is low which means the investor can hold futures contract with 5.7% and 12.4% amount of cash position to hedge in average. The deviation of OHR at each time point apart from the average OHR is fairly reasonable for all five commodities. None of them are normally distributed with significant t-statistic from Jarque-Bera test. The significant kurtosis and skewness are detected in each estimated OHR series. The OHR of coffee has a thinner and longer tail on left hand side with lower peak than normal distribution does. In contrast, the live cattle and live hog yield fatter and right-hand longer tailed OHR series with higher peak. For other two products, the OHR series are non-normally distributed with fatter tail and longer tail on left-hand side and lower peak.

The DF/ADF unit root tests with null hypothesis of one/two unit roots is applied for estimated OHR series for all five agricultural commodities. All t-statistic are significantly larger than critical value at 5% significant level. In other words, the null hypothesis is rejected at 5% significant level with absence of unit root. All estimated OHR series for the BEKK-GARCH model are stationary.

The L-B serial correlation and White tests for residual series from the BEKK-GARCH model shows evidence of free of autocorrelation at 5% significant level in table 5-52. The tests on squared residuals roughly fail to reject the null hypothesis that no serial correlation in the residual series for all except for the case of coffee. Judging from these results, we can conclude that roughly all residual series from BEKK-GARCH model are serial correlation free. There are some significant remaining heteroscedasticity in 3 series. Nevertheless, the ADF test demonstrates the harmlessness of the weak heteroscedasticity on challenging the sufficiency of order of ARCH. Consequently, we state that the order of ARCH term in BEKK-GARCH (1, 1) model with student's t distributed residuals is sufficient to capture all ARCH effects.

3. Diagonal bivariate GARCH-X model

Estimated coefficient

The diagonal bivariate GARCH (1, 1)-X model with student's t distributed residuals captures long-run cointegration between log-return in cash and futures markets. It is theoretically more efficient that that with normal distribution. The estimated coefficient for both mean equation and GARCH-X is reported in table 5-53.

The φ_1 of mean equation are significant and negative for all five products, namely, the short-run deviation of long run cointegration is negatively affect the log-return in cash market. On the other hand, the φ_2 are positive and significant for non-storable live cattle and live hog indicating a positive relationship between short-run deviation and log-return in futures markets. The insignificance of φ_2 for coffee, wheat and soybean implies the low explanatory of short-run deviation on futures log-return for storable commodity.

All coefficients of ARCH and GARCH terms in the GARCH-X model are positive and significant. Namely, the ARCH and GARCH effects are significant for all five agricultural products. The coefficients of error correction terms in GARCH-X model d_{11}, d_{22}, d_{12} are significant in most cases. Specifically, the significant d_{11} and d_{22} suggest that the error correction is critically in explaining some characters of log-return in cash and futures market for the five commodities. In addition, it is useful for explaining the interaction between log-returns in cash and futures markets for coffee and live cattle regarding the significant d_{12}. The sum of coefficients of ARCH and corresponding GARCH terms are close to unity for all commodities except for wheat. In other words, the volatility clustering is more evident and dies out more slowly for coffee, soybean, live cattle and live hog than that of wheat.

Analysis of estimated OHR and residuals from GARCH-X models (student's t distribution)

As reported in table 5-54, the average estimated OHR from the bivariate GARCH-X (1, 1) model for wheat and soybean are the highest. The OHRs for non-storable products are less than 0.19. The estimated OHR for live cattle yields the lowest average estimated OHR. The kurtosis, skewness and J-B tests confirm the non-normality of estimated OHR series for all five commodities, and a fatter/thinner tail, longer tail on left/right hand side for the different product. Providing the information about non-normality, the assumption of student's t distribution residuals is more empirically practical.

The output of one and two unit roots DF and ADF tests on estimated OHR from the bivariate GARCH-X (1, 1) model is reported in table 5-55. All t-statistic value is significantly larger than critical value. Namely, we fail to accept the null hypothesis of existence of unit root in estimated OHR series at 5% significant level. In other words, the five estimated OHR series are stationary when the residual series follows student's t distribution.

The table 5-56 presents the output of L-B autocorrelation and White heteroscedasticity tests on estimated OHR from GARCH-X (1, 1) model. There are 2 (coffee and wheat) out of 10 series have serial autocorrelation. We draw a conclusion that the residual series from GARCH-X model for most products are free of autocorrelation. Significant test statistic from White test fails to reject the null of homoskedasticity for all commodities except for coffee and wheat. Yet, the two heteroscedastic residual series are stationary, namely it is economically weak heteroscedasticity and is negligible. The order of ARCH term in the GARCH-X (1, 1) model with student's t distributed residuals is sufficient to capture all ARCH effect for the five agricultural commodities.

4. Diagonal bivariate BEKK-GARCH-X model

Estimated coefficient

The BEKK-GARCH-X (1, 1) model is an alternative of VECH-GARCH-X (1, 1) and it ensures the positivity of variance matrix and reduces the number of parameters. The estimated coefficient of the diagonal bivariate BEKK-GARCH-X (1, 1) model is presented in table 5-57.

The φ_1 and φ_2 are roughly significant for all five commodities. The significant φ_1 and φ_2 are all negative and thus the log-return in cash and futures markets decreases as the short-run deviation of long term cointegration. In accordance with BEKK-X model with normally distributed residuals, the coefficients for ARCH and GARCH terms in the BEKK-X with student's t distributed residuals are a_{11}^2, $a_{11}a_{22}$, a_{22}^2 and $b_{11}^2, b_{11}b_{22}, b_{22}^2$. The evidence from significant a_{11}, a_{22} b_{11} and b_{22} is not sufficient to determine the significance of ARCH and GARCH effects. The joint effect of ARCH and GARCH is around 1 for all commodities which indicates that the volatility clustering will gradually decrease to none for all five agricultural products. The d_{11}, d_{22} and d_{12} whose square value measures cointegration are not significant for all cases. However, it confirms the existence of cointegration and does not affect the significance of coefficients of cointegration term. The BEKK-GARCH (1, 1) model with student's t distributed residuals is a strong candidate for competing the best forecasting model in six GARCH models in agricultural futures market.

Analysis of estimated OHR and residuals from BEKK-GARCH-X models (student's t distribution)

Estimated from the bivariate diagonal BEKK-GARCH-X (1, 1) model with student's t distributed residuals, the OHR series for the five agricultural commodities do not follow normal distribution with evidence of significant kurtosis, skewness and rejection of J-B normality test. All estimated OHR series has either fatter or thinner tail, either longer tail on left or right hand side with result reported in table 5-58. The average estimated OHR for storable commodities ranges from 0.58 to 0.88, and 0.064 and 0.14 respectively for live cattle and live hog. The lowest deviation of estimated OHR from the mean OHR is for the case of live cattle.

The OHR series estimated from BEKK-GARCH-X (1, 1) model based on student's t distributed residuals is demonstrated to be stationary for all five agricultural products based on the result in table 5-59. Since the null hypothesis that there are one or two unit roots in the estimated OHR series from DF and ADF tests is rejected at 5% significance level. The free of unit root in the OHR series helps to demonstrate their stationarity.

The output of L-B serial correlation and White tests on residuals from the BEKK-GARCH-X (1, 1) model is reported in table 5-60. With the presence of autocorrelation in 2 out of 10 cases, it is more reasonable to accept the null hypothesis that the residual series do not have serial correlation for almost all commodities. This evidence demonstrates the adequacy of order of ARCH term in the BEKK-GARCH-X model. The result of White test here is similar to that of GARCH-X model in which 2 residual series are heteroscedastic for coffee and wheat and they are diagnosed as stationary series. In other words, the order of ARCH model is reasonable in the case of free of autocorrelation and weak heteroscedasticity in general.

5. Diagonal bivariate GARCH-GJR model

Estimated coefficients

The GARCH-GJR model that measures leverage effect by employing a dummy variable is applied to in-sample estimate OHR based on student's *t* distributed residuals for five agricultural commodities. The estimated coefficients for each variable are reported in table 5-61.

The φ_1 is significant and negative for all five products which indicates a negative relationship between short-term deviation from the long-run cointegration and cash log-return. On the other hand, the φ_2 is significant for the case of soybean and two non-storable commodities, while insignificant for coffee and wheat. In futures market, has the effect of short-run deviation on futures log-return depends on the commodity. The ARCH and GARCH terms have significant coefficients for all commodities. In other words, the ARCH and GARCH effects are evident. The γ_1 and γ_3 who evaluate the effect of asymmetric information on conditional variance are significant for all five products except for live cattle. It does not necessary mean there is no leverage effect for live cattle, the possible fact is that the asymmetric effects of news presents for live cattle, but not large enough to reach the 10% significance level. The signal of γ_1 and γ_3 implies that the bad news has larger effect on conditional variance for most cases, but a reverse smaller effect on conditional variance of futures log-return shows up for three storable commodities and live hog. In sum, the diagonal bivariate GARCH-GJR (1, 1) model is doable for completely describing the characters of log-return in cash and futures markets and their relationship.

Analysis of estimated OHR from GARCH-GJR models (student's t distribution)

The basic statistics and stationary tests on estimated OHR and autocorrelation and heteroscedasticity tests on residual series from GJR model are carried out with all result reported in table 5-62, 5-63 and 5-64.

The GJR (1, 1) model who takes the leverage effect into account yields relatively higher estimated OHR for all five agricultural products than those from other models who ignore the asymmetric effect of good and bad information. The deviation of estimated OHR from its average OHR for live cattle is the lowest one. Significant kurtosis and skewness present in estimated OHR for all commodities except for the case of wheat. Namely the estimated OHR for wheat follows a normal-likelihood distribution with the rejection of normality from the J-B test.

At 5% significance level, the null hypothesis of presence of unit root from DF and ADF 1- and 2- unit root tests is rejected for all five estimated OHR series. As expected, the estimated OHR series are stationary.

The results of L-B correlation and White tests are reported in table 5-64. It shows that there is serial correlation in 2 residual series for coffee and 1 for wheat at 5% significant level. In other words, 2/3 residual series are free of autocorrelation and this main trend generally supports the sufficiency of order of ARCH in the GARCH-GJR (1, 1) model. From another perspective, only one residual series for wheat is weak heteroscedastic. As stated earlier, weak heteroscedasticity is economically negligible. Consequently, the order of ARCH effect is sufficient for OHR estimation in this study.

6. Diagonal bivariate QGARCH model

Estimated coefficients

The QGARCH model employs a term of residuals $u_{i,t-1}$ to capture the leverage effect. The coefficients d_{11} and d_{22} of $u_{i,t-1}$ can be both negative and positive which is managed to describe a special case that good news sometimes has more effect on conditional variance than bad news does. The table 5-65 presents the estimated coefficients of variables from the diagonal bivariate QGARCH (1, 1) model.

At 5% significance level, the significant negative φ_1 for all five commodities suggest that the increase of φ_1 will lead to a decrease of log-return in cash markets. While three out of five cases have insignificant φ_2, this implies that the short-run deviation of long-run cointegration between log returns in cash and futures markets does not substantially affect the log-return in futures market for coffee, soybean and live hog. The ARCH and GARCH effects are eventful based on significant coefficients of ARCH and GARCH terms. The sum of coefficients of ARCH and corresponding GARCH terms is close to 1 as expected, and thus the volatility tends to diminish gradually. Subject to d_{11} and d_{22}, the significance of all coefficients demonstrates the presence of leverage effect in both cash and futures commodity's and agricultural markets. Even though, taking into account the asymmetric effect of positive and negative information on conditional variance is essential for sure, and the student's t based QGARCH model is highly qualified for OHR estimation in this study.

Analysis of estimated OHR and residuals from QGARCH models (student's t distribution)

The basic analysis of estimated OHR series for five commodities is reported in table 5-66. The average estimated OHR for storable products has a higher and narrower range from 0.71 to 0.89. The mean OHR for non-storable commodities is in the similar range as those from previous GARCH models. The deviation of OHR from its average estimated OHR is relatively lower for non-storable goods than that of storable products. The features of significant kurtosis, skewness and non-normality are proved. All estimated OHR series hold fatter tail than normal distribution does. The wheat and soybean have longer tailed estimated OHR on left-hand side, yet the others produce the right-hand longer tails.

The estimated OHR series from all five agricultural products are stationary given the evidence from DF and ADF tests. In the 1- and 2- unit roots tests, the null hypothesis of existence of unit roots in estimated OHR series is rejected at 1% significance level based on the result in table 5-67.

The L-B test of autocorrelation and White heteroscedasticity test on residual series are applied with output presented in table 5-68. With the presence of autocorrelation in 1 residual series for wheat, overall, the residual series are free of serial correlation.

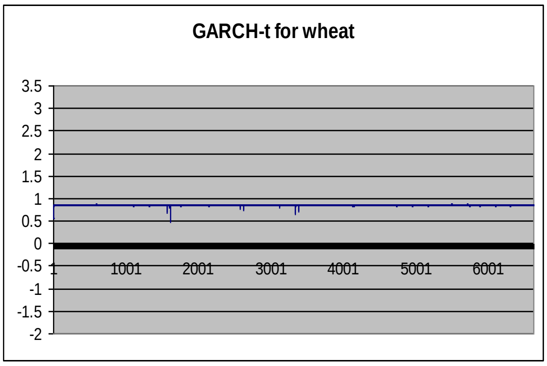
Generally speaking, the estimated OHR series from QGARCH (1, 1) model for all five commodities does not have autocorrelation. The White test finds heteroscedastic residual series for wheat at 5% significant level. However, based on ADF stationary test, we find that the heteroscedasticity of them are not persistent enough to affect the sufficiency of order of ARCH term in this model. As a result, the order of ARCH term in the QGARCH model is said to be sufficient in explaining the ARCH effect on log-return in cash and futures markets.

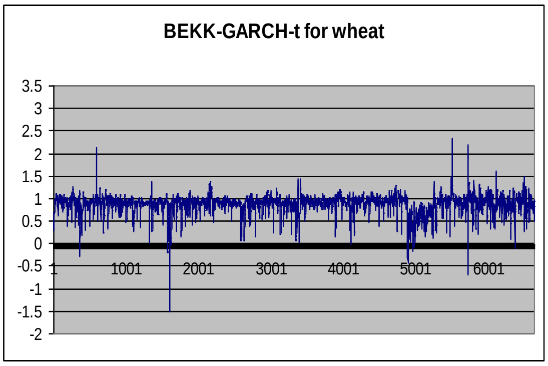
Comparison of estimated OHR

As expected, roughly all ARCH and GARCH effect are significant with evidence from the estimated coefficients of six GARCH models. Negative coefficients of dummy variable for GJR model and leverage effect term for QGARCH model indicates that the effect of positive shock takes more weight than negative shock for 3 and 4 cases out of 5 respectively. In general, the adverse leverage effect is more widely found for agricultural products.

When the residual series is student's t distributed, the estimated OHR from asymmetric GARCH-GJR and QGARCH models for coffee increases to 0.70 which is higher than 0.50 from other 4 GARCH models. Furthermore, the estimated OHR for live hog from all GARCH models are higher than 0.12. The wheat has a non-normally distributed OHR series from GJR model with symmetric tail according to insignificant kurtosis and skewness.

For comparison, the patterns of estimated OHR series for wheat and live cattle represent the general movement of estimated OHR for storable and non-storable products in Figure 5-7 and Figure 5-8.





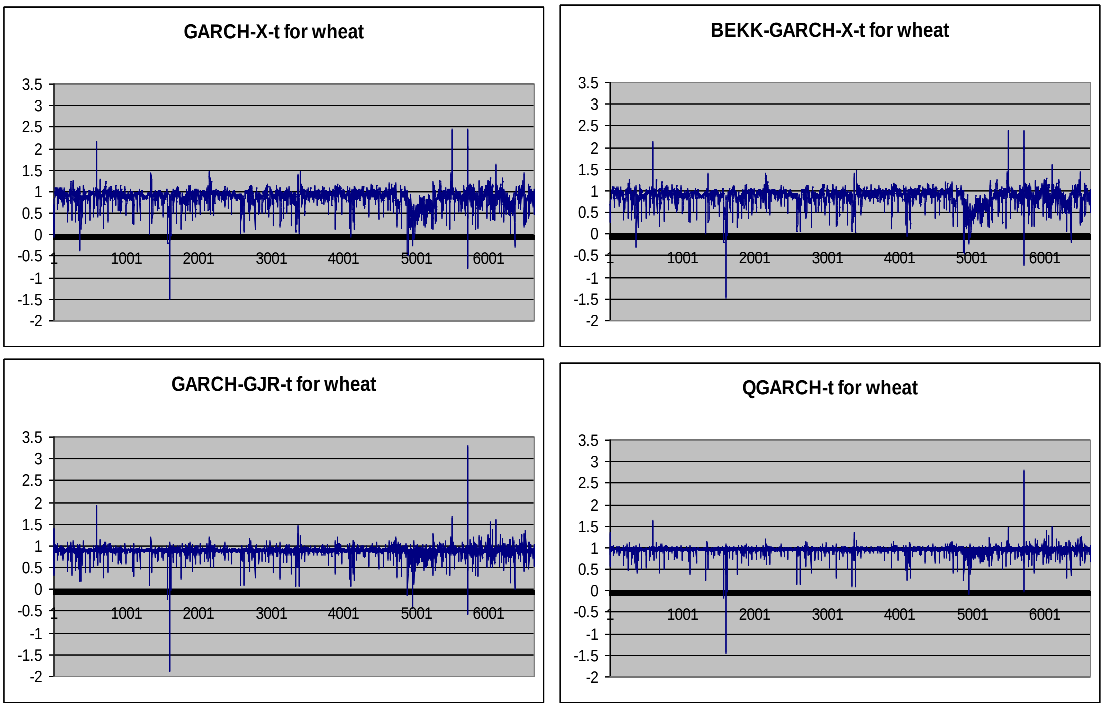
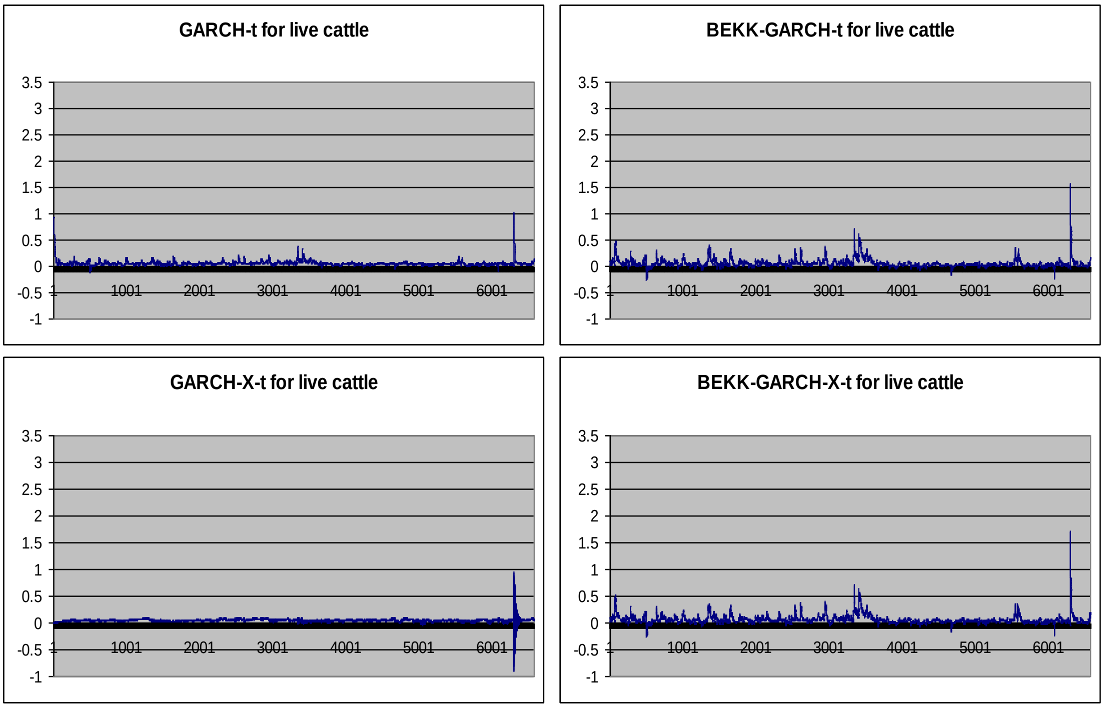
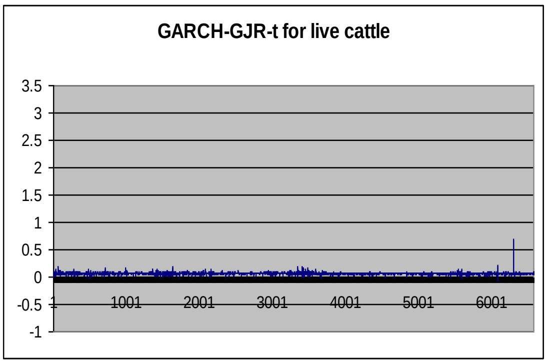


Figure 5-7 Patterns of estimated OHR series with student's t distribution for wheat

The student's t distributed residuals based GARCH models produce highly similar patterns of estimated OHR for wheat with those from normally distributed residuals. The standard GARCH model yields a quite flat OHR series around 0.85, but the others provides much more volatile estimated OHR series in which the GJR model holds the highest volatility.





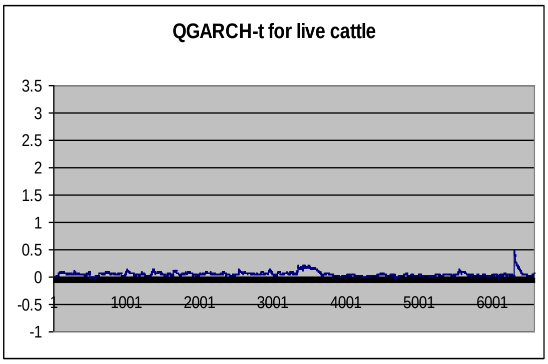


Figure 5-8 Patterns of estimated OHR series with student's t distribution for live cattle

For the case of live cattle, the estimated OHR are more stable with lower OHR, volatility and less break point. Although the OHR from GARCH-X makes the lowest OHR, the peak point is less than 1. Instead, the BEKK and BEKK-X models hold the highest OHR that exceeds 1.5.

The validity of six GARCH models with student's t distributed residuals on OHR estimation for all five agricultural products is demonstrated given that the estimated OHR series are stationary without autocorrelation, and hence the order of ARCH is adequate in the GARCH models. The forecasting power of the six GARCH model with different distributed residuals is an interesting issue to investigate.

5.8 One-year Forecast

The sufficiency of six GARCH model based on the evidence from estimated OHR indicates the following OHR and return prediction is valid. Similarly, four evaluation methods are applied for testing forecast error and accuracy of six GARCH models.

5.8.1 Forecasting of OHR

With the same time period for the one-step ahead forecasting (from 24/03/2005 to 23/03/2006 and from 15/01/2007 to 14/01/2008 for storable and non-storable commodities), the 1–year OHR prediction from six GARCH models with student's t distributed residuals shows some different character and features with that from GARCH models based on normally distributed residuals.

For comparison, we present the patterns of one-year forecasted and estimated OHR for coffee and live cattle for six GARCH models based on student's *t* distributed residuals in the Figure 5-9 and 5-10 respectively. More detail about patterns of other three agricultural products will be reported in appendix. The blue and red series denote the pattern of estimated and forecasted OHR for one-year forecast, respectively.

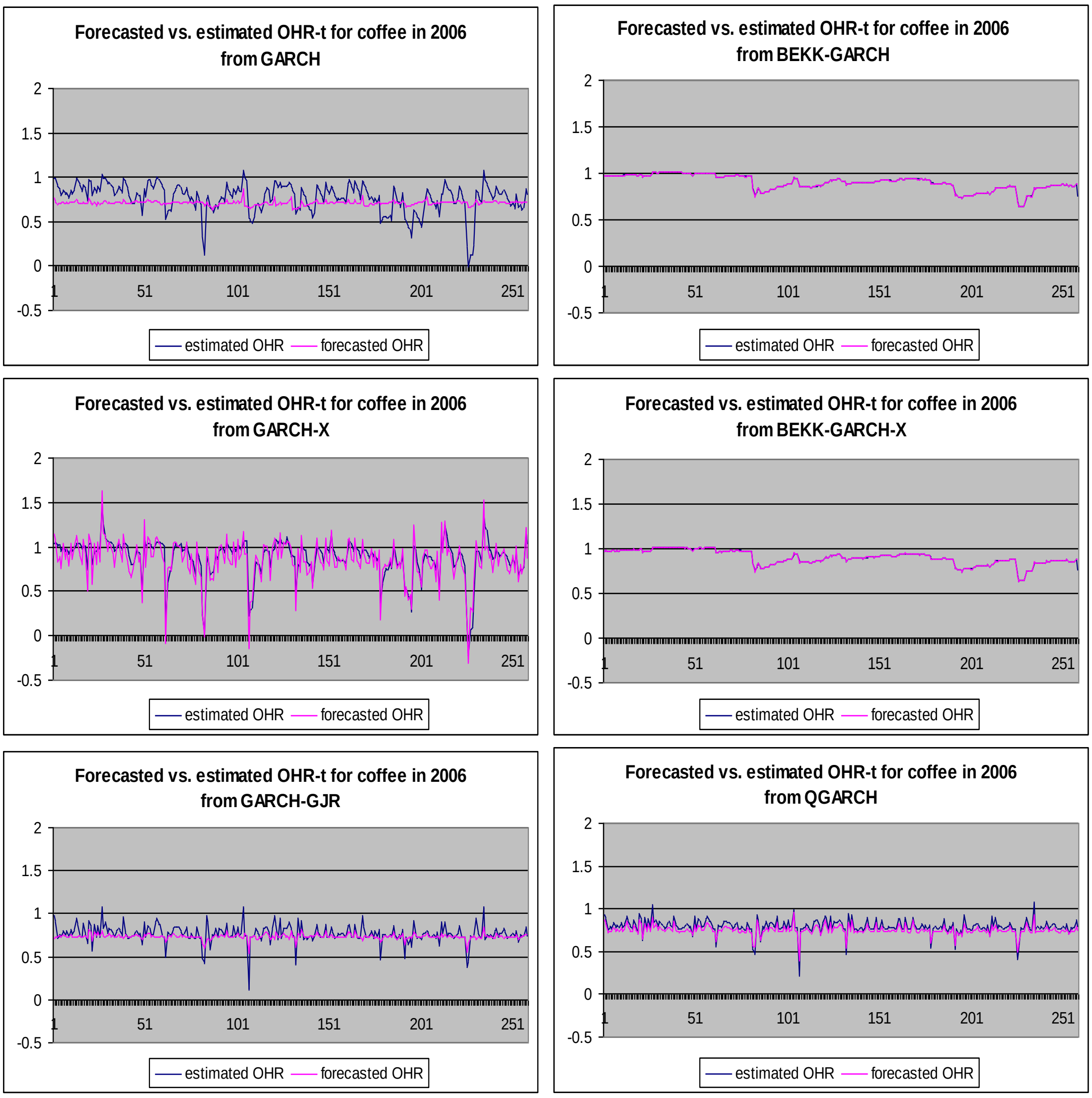


Figure 5-9 One-year forecasted OHR vs. estimated OHR with student's t distribution for coffee

With student's t distributed residuals, the forecasted OHR series for coffee from the BEKK and BEKK-X models have lowest deviation from their estimated OHR with range of 0.55 to 1.10. This phenomenon for the one-year forecasted and estimated OHR for coffee is similar with that based on normal distributed residuals. The standard GARCH and GARCH-X based forecasted OHR series show high volatility ranging from 1.60 to -0.10 and they move apart from the estimated OHR. Judging from the pattern, the asymmetric GJR and QGARCH models produce the moderately accurate out-of-sample one-step ahead forecasted OHR for coffee.

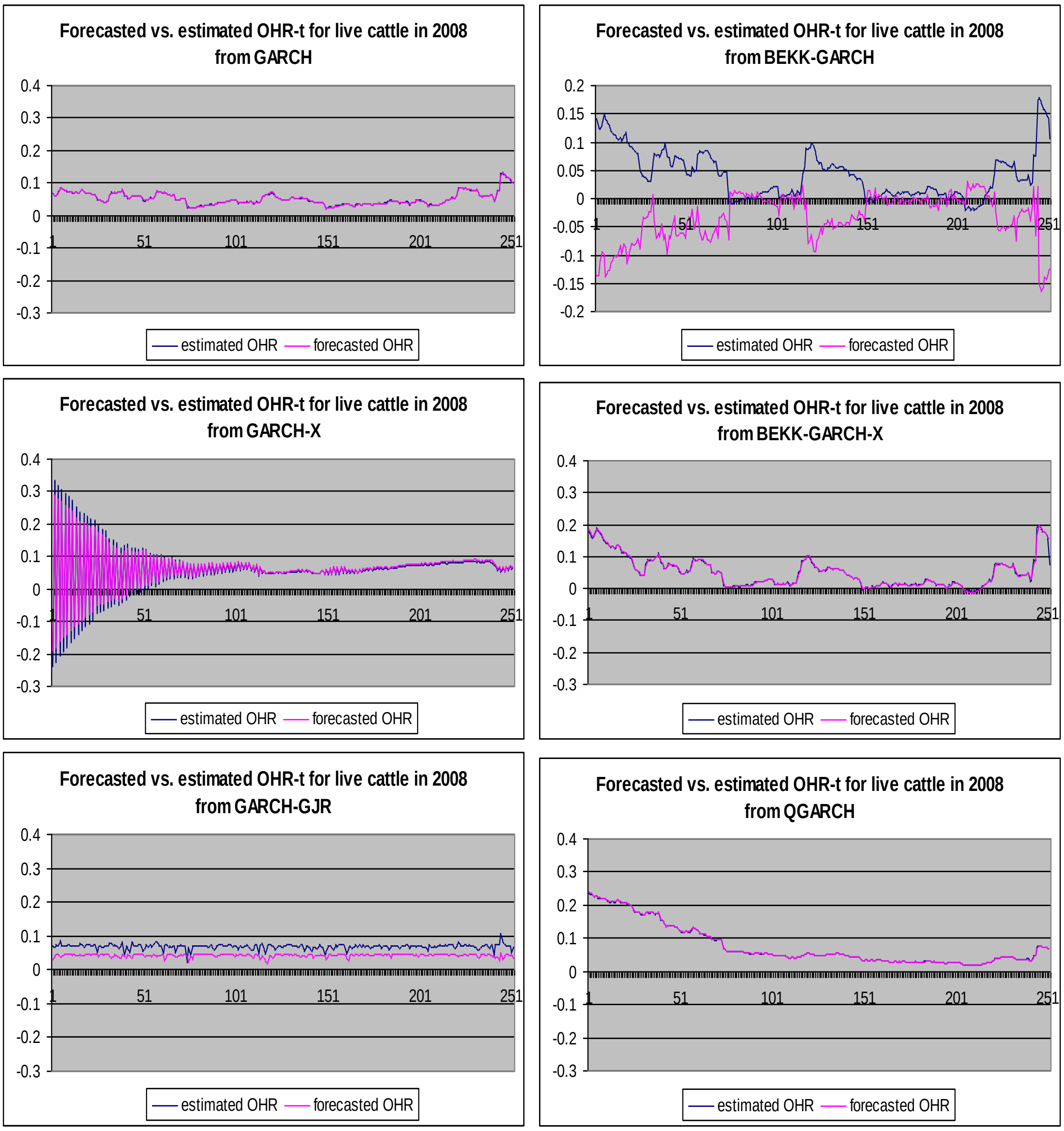


Figure 5-10 One-year forecasted OHR vs. estimated OHR with student's t distribution for live cattle

The student's t distributed residuals based OHR from six GARCH model have more evident features than that with normally distributed residuals. The high rate of overlapping of estimated and forecasted OHR from standard GARCH, BEKK-X and QGARCH models demonstrate their remarkable forecasting power. The BEKK model is in another extreme situation in which the forecasted OHR is almost absolutely opposite to the estimated OHR, and they are nearly symmetric around horizontal axis. The inaccurate one-day ahead out-of-sample OHR forecast is totally out of expectation. The reason is that we relax the positivity constrains on parameters for more general implement. A negative coefficient appears in BEKK model with student's t distribution in OHR estimation for live cattle; consequently, a negative impact of last covariance on the current covariance occurs. A further influence is reflected on the opposite forecasted OHR to estimated OHR. The forecasted and estimated OHR series from GARCH-X model present sharply volatile patterns and dramatic decreasing volatility of OHR ranging between 0.3 and -0.2 to those between 0.06 and 0.08. How close the forecasted OHR to estimated OHR is can be more visible providing a larger scale. The forecasted OHR from GJR model is parallel to the estimated OHR with deviation of 0.3 or so.

5.8.2 Forecasts of Return

5.8.2.1 Forecasts of Return

To test the forecast error of the one-step ahead 1-year forecasted return from six GARCH models with student's t distributed residuals, the MAE, MSE, Theil's U and Modified Diebold Mariano tests are applied.

5.8.2.2 Forecast Error Tests

The output of MAE, MSE and Theil's U tests on out-of-sample 1-year forecasted return for coffee, wheat, soybean, live cattle and live hog are presented in table 5-69, 5-70, 5-71, 5-72, and 5-73, respectively.

In the tests on the 1-year out-of-sample forecasted return for coffee with student's t distributed residuals, the output in table 5-69 implies that the BEKK and BEKK-X models are outstanding with the lowest forecast errors in all of the three evaluations. This finding is in line with that from GARCH models with normally distributed residuals. The forecasting performance of GARCH models based on the two distributions is not necessarily changed for the case of coffee.

For the case of wheat, the forecast errors of GARCH models on 1-year return prediction are smaller than that for coffee under the same evaluation with evidence from table 5-70. The results from the three evaluations reach the unique conclusion that the QGARCH model is the most accurate forecasting model among the six GARCH models without any debate. The GJR, BEKK and standard GARCH model perform well on the return prediction for wheat. On the other hand, the BEKK-X model is the worst one in this case.

The result for storable soybean is reported in table 5-71. The forecast errors of six GARCH models are in a small range without evident difference. Relatively, the BEKK model has smallest error in both MAE and Theil's U tests. In terms of MSE method, the standard

GARCH, BEKK, GARCH-X and BEKK-X produce the same forecast error 0.000002. Overall, the BEKK model outperforms for the case of soybean, despite of the superior forecasting performance of the GARCH-X for soybean when the residual series follows normal distribution.

For the storable agricultural commodities, the BEKK is the best forecasting model on 1-year return for coffee no matter the distribution of residuals. However, different GARCH models out-perform for wheat and soybean depending on the residual's distribution.

For the case of non-storable live cattle, the BEKK model yields the lowest error on return prediction in MAE test with the second lowest error from the QGARCH model regarding the result in table 5-72. The MSE ranks the QGARCH model in the first place, but the BEKK is ranked in the bottom. In this method, the GARCH-X also provides decent 1-year forecasting on return. In addition, the GARCH-X is the most powerful forecasting model under the Theil's U test, and the QGARCH is the third best predictive model. In general, the predictive power of QGARCH and GARCH-X models is neck-and–neck. In this case, we conclude that the QGARCH and GARCH-X models are equally good on 1-year return prediction.

For the live hog, the asymmetric QGARCH outperforms again for non-storable commodity as reported in table 5-73. The forecast errors from QGARCH model on 1-year return prediction are the lowest and the standard GARCH model ranks second after the QGARCH model for the three measures.

In the case of student's *t* distributed residuals, the asymmetric QGARCH model shows a supreme predictive power on 1-year out-of-sample return forecast for non-storable commodity based on the evidence from live cattle and live hog.

5.8.2.3 Modified Diebold Mariano Test

The MDM test on forecasted return from six GARCH models with student's t distributed residuals measures their forecast accuracy by directly comparing any two of them. The output of comparison from MDM test is reported in table 5-74, 5-75, 5-76, 5-77 and 5-78 correspondingly for coffee, wheat, soybean, live cattle and live hog.

For the case of coffee, the result in table 5-74 shows that the BEKK and BEKK-X models are equally best in all six models with student's *t* distributed residuals, yet the standard

GARCH never outperforms any other GARCH models with both the MSE and MAE measures. The GARCH-X is either better than GJR and QGARCH or equivalent to them depending on the measure.

As presented in table 5-75, the forecasted return from standard GARCH model is better than GARCH-X and BEKK-X models for wheat which have worse forecasting performance than those of BEKK, GJR and QGARCH models. The GJR model has great predictive power and moderate forecasting ability for standard GARCH and BEKK models. However, a clear out-performance of the QGARCH model is found according to the superior forecast accuracy on the 1-year return forecast for wheat.

The result of MDM test on the 1-year forecasted return in table 5-76 is for soybean. The standard GARCH shows a moderate forecasting power. The BEKK model is not worse than others. The forecasting performance of GARCH-X and BEKK-X model is almost the same which outperform asymmetric GARCH models. The comparison between the two asymmetric GARCH models presents an equivalent predictive power. Based on the evidence, the BEKK is roughly the most powerful forecasting model on the 1-year out-of-sample return forecast among the six GARCH models for soybean.

Summary 5 the percentage of dominance of one model over others for three storable commodities (coffee, wheat and soybean) on 1-year return forecast with student's t distributed residuals based on MDM test

Model	1-year forecast of return from MDM	
	MSE	MAE
GARCH	20	20
BEKK	60	60
GARCH-X	20	47
BEKK-X	33	40
GJR	27	27
QGARCH	47	47

Note: this summary of MDM test reports the percentage of dominance of every single GARCH model when they compare with others. For instance, the standard GARCH compares with other five GARCH models, the number of out-performance of standard GARCH for the cases of three commodities divided by 15 is the percentage of dominance of standard GARCH model. Here the residual series is student's t distributed.

When the 1-year returns are forecasted using six GARCH models with student's *t* distributed residuals, the summary 5 shows that the BEKK model generally maintains the first place in the ranking of forecasting power for the three storable commodities. The QGARCH model follows. Nevertheless, the standard GARCH model is the worst forecasting model among the six models.

The result in table 5-77 for non-storable live cattle is quite different from that of storable commodity. The BEKK model never outperforms others under both MSE and MAE measures. The GARCH-X model is more accurate than all others except for QGARCH model on the 1-year return prediction. The BEKK-X is only better than BEKK and GJR models. It turns out that the BEKK, BEKK-X and asymmetric GJR models are ranked in the bottom. Among the six GARCH models, the asymmetric QGARCH model has outstanding forecasting performance in the 1-year return prediction for live cattle with student's t distributed residuals.

With the output in table 5-78, the standard GARCH model only under-performs the QGARCH model on the 1-year return forecast for the case of live hog. The BEKK and BEKK-X models hold mediate forecasting ability. The QGARCH performs best in the return prediction for live hog overall. On the other hand, the GARCH-X model is worse than all others in all comparisons.

Summary 6 the percentage of dominance of one model over others for two non-storable commodities (live cattle and live hog) on 1-year return forecast with student's t distributed residuals based on MDM test

Model	1-year forecast of return from MDM	
	MSE	MAE
GARCH	60	60
BEKK	20	20
GARCH-X	30	30
BEKK-X	50	50
GJR	20	20
Q	90	90

Note: the number of out-performance of a single GARCH model divided by 10 is the dominance percentage of this model over other GARCH models for non-storable live cattle and live hog. Here the residual series is student's t distributed.

In the summary 6, the QGARCH model beats almost all other models for non-storable agricultural products with 90% dominance in terms of forecasting accuracy based on the evidence from live cattle and live hog. Namely, the QGARCH has incredible predictive power for the 1-year return forecast when the residual series follows student's t distribution. The most powerful BEKK model for non-storable commodity does not work well for the case of non-storable agricultural products with the lowest dominance percentage in the 1-year return prediction.

5.9 Two-year Forecast

The forecasting power of six GARCH models with student's t distribution on longer time 2-year OHR and return prediction is tested in the same way. The effect of length of forecast horizon on predictive power of GARCH models will be clearly judged from their performance.

5.9.1 Forecasting of OHR

For the reason of comparison, the pattern of 2-year forecasted versus estimated OHR from six GARCH models for coffee and live cattle in Figures 11 and 12 will tell us a different story.

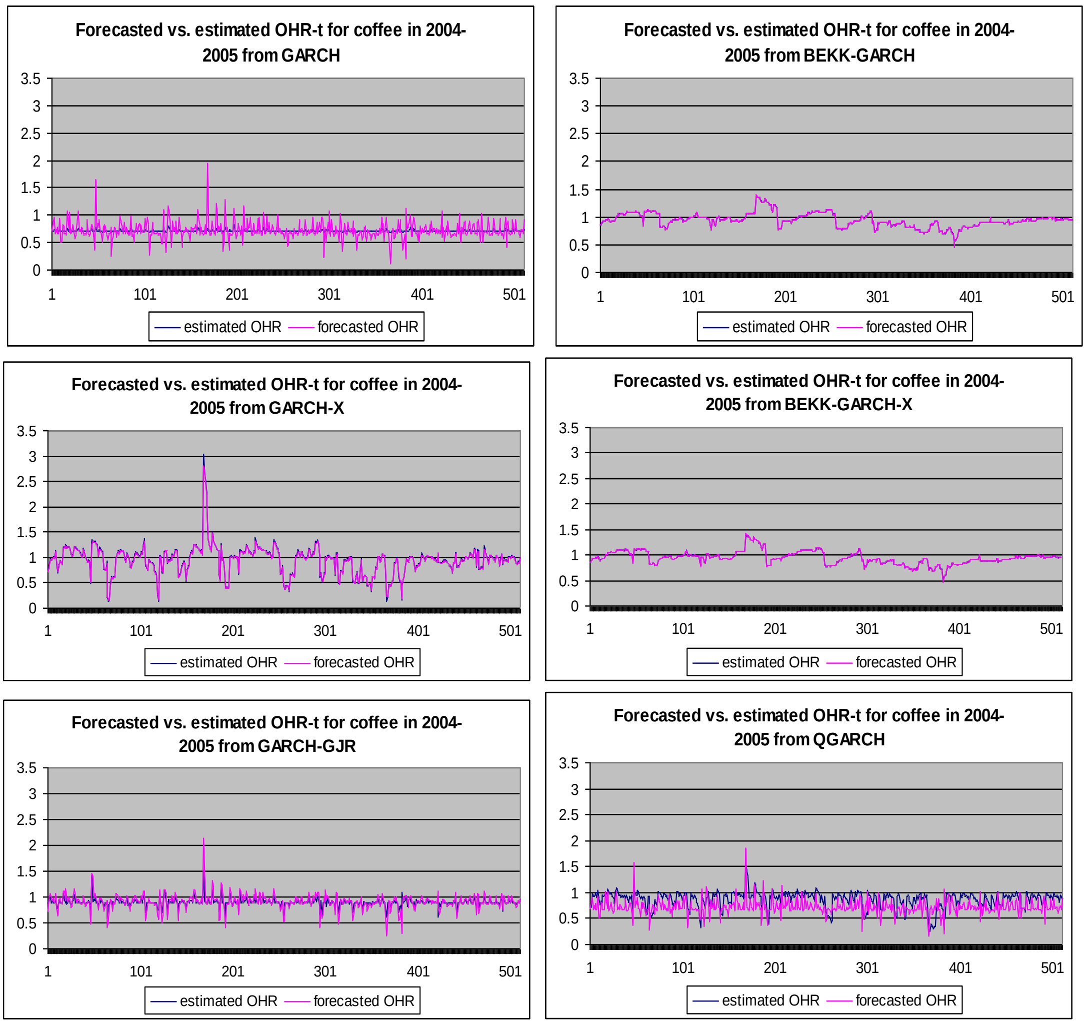


Figure 5-11 Two-year forecasted OHR vs. estimated OHR with student's t distribution for coffee

The standard GARCH model produces a similar pattern as that from GARCH-X, GJR and QGARCH, with higher or lower volatility and deviation of forecasted OHR from estimated OHR. Among the four GARCH models, the GARCH-X has the most volatile OHR and the highest forecasted and estimated OHR are more than 3. The most inaccurate model is QGARCH in which the forecasted OHR departures away from the estimated OHR. The noticeably large amount of overlapped forecasted OHR to estimated OHR from BEKK and BEKK-X models implies that the two BEKK-type models are extremely accurate.

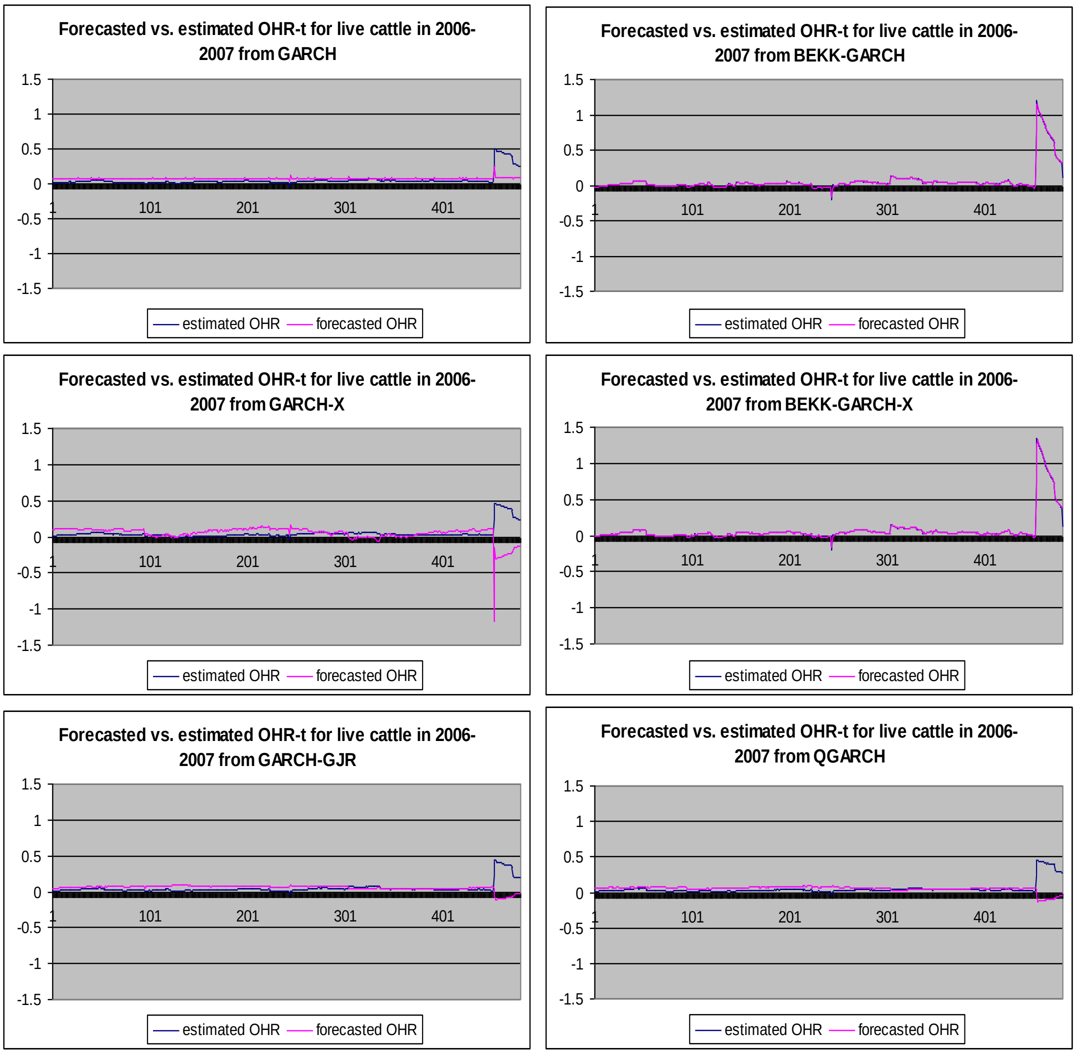


Figure 5-12 Two-year forecasted OHR vs. estimated OHR with student's t distribution for live cattle

In the case of live cattle, its spot price exceeds 100 US cent/pound since May of 2004, while the price of futures maintains as low as 65 US cent/pound. The large spread eventually been reflected on the sharp increase of hedge ratio in late 2007 in Figure 12. The BEKK and BEKK-X again are accurate 2-year out-of-sample forecasting on OHR for live cattle. The rest four GARCH models cannot sense the sudden change on estimated OHR in the last 2 months, and the 1-day-ahead out-of-sample forecasted OHR move to a reverse direction of estimated OHR, particularly evidence for that from GARCH-X model. From this perspective, the GARCH-X model does not forecast accurately in this case.

5.9.2 Forecasts of Return

5.9.2.1 Forecasts of Return

When the residual series follows the student’s t distribution, the forecasting ability of six GARCH models on the two-year long-term return forecast is investigated in this section. We analyse the forecast error and accuracy based on the results from four evaluations.

5.9.2.2 Forecast Error Rests

In the two-year forecast, the MAE, MSE and Theil’s U methods are applied to evaluate the forecasting error on return prediction from six GARCH models with student’s t distributed residuals. The corresponding output for coffee, wheat, soybean, live cattle and live hog is presented in table 5-79, 5-80, 5-81, 5-82 and 5-83.

The output of MAE and MSE test in table 5-79 on the six GARCH models is consistent with each other in which the BEKK and BEKK-X are the most accurate forecasting models, while the standard GARCH and QGARCH models make the largest errors and certainly they are the worst models in the six GARCH models.

In these three tests, result in table 5-80 indicates that the GJR model outperforms other four GARCH models on the longer time return forecasting with the lowest forecast error for wheat. As an opposite, the standard GARCH model produces the highest error under all three evaluations. The forecasting performance of BEKK model is fairly good.

In table 5-81, the BEKK model shows the best forecasting power in six GARCH model on the two-year return prediction for the storable soybean. The lowest errors under the MSE and Theil’s U methods are from BEKK model, although the GJR outperforms with MAE test.

The result for the three storable commodities is similar with that of 2-year return forecast with normally distributed residuals. The BEKK-type model is useful for coffee and soybean. The asymmetric GJR and QGARCH models outperforms for wheat respectively for the normally and student’s t distributed residuals.

The standard GARCH model yields relatively smaller forecast errors than others for all the three evaluation methods on the two-year return prediction for the non-storable live cattle. In table 5-82, the GARCH-X is the second best forecasting model with Theil's U test; however, it produces the highest errors under both MAE and MSE methods. The rest four GARCH models hold similar predictive power providing slightly different errors for the case of live cattle.

In the case of live hog in table 5-83, the standard GARCH which is the most powerful forecasting model on 2-year return prediction for live cattle provides high errors than most GARCH models and is nearly the worst model. The asymmetric QGARCH and GJR are on the top two in the ranking of forecasting power for all three evaluation methods.

The live cattle prefers standard GARCH model for 2-year return forecast instead of asymmetric GARCH model providing student's t distributed residuals. Nevertheless, the asymmetric GARCH model (GJR or QGARCH) is picked as the best forecasting model for live hog on the long-term return prediction.

5.9.2.3 Modified Diebold Mariano Test

The MDM test of forecast accuracy test is also applied on 2-year forecasted return when the residual series follows student's t distribution. The simple interpretation of this test makes the comparison more meaningful. Once again, five tables 5-84, 5-85, 5-86, 5-87 and 5-88 report the result of comparison for five commodities respectively.

The table 5-84 presents the output of MDM test for the case of coffee. The standard GARCH model is never better than others with MSE method, but the MAE report the outperformance of standard GARCH over QGARCH model. That is the only difference the MAE and MSE approaches make in this case. The BEKK and BEKK-X have almost equal best forecasting power providing all winning in the comparisons with all other four models. The QGARCH is the worst model for the coffee. The BEKK-X has better performance with student's t distributed residuals than that with normally distributed residuals with evidence from MDM test.

Without the GARCH-X who is discarded due to convergence problem, the table 5-85 of result for wheat is even easier to interpret. The standard GARCH consistently underperform others for the case of wheat. Once again, the BEKK model shows the most accurate forecast of the 2-year return. The asymmetric GJR has a decent predictive power and ranks second after the BEKK model. However, another asymmetric GARCH model, the QGARCH does not work well who is only more accurate than standard GARCH.

The result in table 5-86 for soybean indicates the forecasting power of BEKK and asymmetric GJR models for the long-time return prediction. Overall, the BEKK and GJR models are never worse than other four models; nevertheless, the former one is better than the latter one under MSE method with a contrary result with MAE test. In other words, it is hard to tell if the BEKK or GJR holds the superiority based on the output. In all, the least thing we can confirm is that both the BEKK and GJR have notable predictive power in those six GARCH model for the case of soybean.

Summary 7 the percentage of out-performance of one model over others for three storable commodities (coffee, wheat and soybean) on 2-year return forecast based on MDM test

Model	2-year forecast of return from MDM	
	MSE	MAE
GARCH	0	7
BEKK	67	73
GARCH-X	20	20
BEKK-X	40	47
GJR	40	67
QGARCH	20	13

Note: see note in summary 5.

Not much difference is found for the case of 2-year return forecast with student's t distributed residuals for three storable agricultural products as reported in summary 7. The BEKK provides the most accurate forecasts among six GARCH models for coffee, wheat and soybean, while the GJR presents equal forecasting power as BEKK for the case of soybean.

The standard GARCH model forecasts well with pronounced performance on the 2-year return prediction for non-storable live cattle as presented in table 5-87. The MAE method presents three "better" for the former model when the MSE approach judges them as equal forecasting models for the comparisons between BEKK and GARCH-X, between BEKK and two asymmetric GARCH models. Additionally, the BEKK-X is better than GARCH-X from MAE while equivalent predictive ability under MSE method. The asymmetric GJR and QGARCH models underperform standard GARCH and BEKK-X models. The equivalency of standard GARCH and BEKK-X from both MSE and MAE methods indicates the equivalently best forecasting power of these two models for live cattle.

With the output from table 5-88, the standard GARCH and BEKK models have moderate forecasting power in the case of live hog. The GJR outperforms other models except for QGARCH model. Put it this way, the QGARCH is best model and the GJR is ranked as the second best forecasting model. The GJR and QGARCH models are superior for the case of live cattle and live hog respectively. The conclusion is in line with the result from those three forecast error tests for live cattle and live hog.

Summary 8 the percentage of out-performance of one model over another for two non-storable commodities (live cattle and live hog) on 2-year return forecast based on MDM test

Model	2-year forecast of return from MDM	
	MSE	MAE
GARCH	40	40
BEKK	20	30
GARCH-X	0	0
BEKK-X	40	70
GJR	60	60
QGARCH	60	60

Note: see note in summary 6.

The GJR and QGARCH models outperform other four symmetric GARCH models with an exception that the BEKK-X model performs better than asymmetric GARCH models with MAE statistic. In general, the asymmetric GJR and QGARCH models perform best in the 2-year return forecast with MDM evaluations.

5.10 Findings

Similarly, four evaluation methods are applied for testing forecast error and accuracy on short- and long-term return prediction when residual series follows student's t distribution in this chapter. Using the student's *t* distribution, the asymmetric information effect that positive news has greater influence than negative news on conditional (co)variance matrix on OHR estimation in the agricultural and commodities' futures markets. The result from the out-of-sample forecasting indicates the dominance of the BEKK type models and asymmetric QGARCH models on both 1-year and 2-year return prediction.

Using the MAE, MSE and Theil's U tests, both the BEKK and BEKK-X models provide the most accurate return forecast for coffee on two forecast horizons. For another two storable commodities wheat and soybean, the QGARCH and BEKK are strongest forecasting models for short-term prediction, but the GJR model is preferred for the 2-year forecast. The QGARCH model yields the most robust return prediction for non-storable products, though the standard GARCH model outperforms others for long-term forecast for live cattle.

The result of 1-year return forecast from MDM test is consistent with the other three evaluation methods that the BEKK is the best forecasting model but the standard GARCH turns out to be the worst model for storable commodity. In addition, the QGARCH dominates others five models for non-storable agricultural products. In the 2-year forecasting, three storable goods consistently approve the superiority of BEKK model on the return prediction in which the wheat has a favour of BEKK model instead of asymmetric GJR and the forecasting performance of BEKK is equally good as GJR model for soybean. The standard GARCH models show some strength on the long-term forecast for live cattle again.

Generally, the BEKK and asymmetric GARCH models can be recommended for storable commodity, and the asymmetric QGARCH has the best forecasting performance for non-storable products for both short-term and long-term return prediction. From another perspective, the summary 7 and 8 who analyzes the percentage dominance of each GARCH model statistically highlights the forecast accuracy of GJR for non-storable commodity. In other words, the asymmetric GJR and QGARCH models have high potential to accurately forecast hedge ratio on both short- and long-term horizons. Certainly, the standard GARCH model deserves a consideration for 2-year long-term forecast for live cattle.

Part C

In order to explore the benefits for investor and traders in agricultural and commodities' futures market, further tests are conducted to check for the improvement in investment performance from dynamic hedging strategy when transaction costs are taken into account. As described, one of the main functions of hedging is to avoid unfavorable loss from price uncertainty. Hence the hedging benefits practitioner through risk reduction rather than increasing utility (profit) (Koziol, 1990).

5.11 Hedging Performance with Transaction Costs

We conduct further tests to check the improvement in investment performance of dynamic hedging strategies when transaction costs are taken into account.

In our study, it is difficult to find the actual transaction fees for the five agricultural products. The main reason is that the futures prices of these five commodities are from three different future exchanges which charge different transaction rates in terms of the trading fees, traders' categories, volume discount policies, etc. Thus, we estimate the return and risk at five different rates of (potential) transaction costs which results in return reduction of portfolio at the rate of 'f', which ranges from 0.01% up to 1%. Though Li (2010) stated that the transaction costs may prevent hedgers from continuous position adjustment, Kroner and Sultan (1993b), Park and Switzer (1995), Haigh and Holt (2002) indicated that the adjustment occurs only if the gain exceeds costs in a transaction. In our study, we also assume that the hedger only adjusts futures position when the potential gain in utility (return) can cover the transaction costs for adjustment.

When a futures position is readjusted, the return of portfolio is equal to:

$$R_t = R_t^c - h_t \cdot R_t^f - f \quad (5\text{-}3)$$

where R_t is the return based on one unit of cash position, R_t^c and R_t^f are returns in cash and futures market respectively and f is the rate of reduction in returns of portfolio caused by transaction cost. We set the f within the range as above when the futures position is rebalanced. If the futures position is not rebalanced, the return to portfolio is $R_t = R_t^c - h_t \cdot R_t^f$ and h_t is the hedge ratio from the last readjustment (Lence, 1995). For an unhedged position, the return of portfolio is $R_t = R_t^c$ which ignores the futures return.

We compare average returns, average risks and variance reductions from dynamic hedge strategies to those of unhedged position when transaction costs are included and excluded. For the sake of conciseness, we present results for two commodities, coffee and soybean, on 1- and 2- forecasting with both normal and student's *t* distributions.

Table 5-89 presents the results from 1-year out-of-sample forecast for coffee under normal distribution. The number in the bracket shows the number of readjustments times required to minimize risks. Larger numbers of readjustments means higher transaction costs. In the 1-year forecast, the unhedged position has an average return of -3.2028e-04 and an average risk of 0.02346. For dynamic hedge excluding transaction costs, the forecasted returns are higher and risks are lower than those of unhedged position independent of the forecasting model applied. The GARCH type models dramatically reduce the variance of the portfolios. The lowest variance reduction to unhedged position is 408.5% by GARCH models, and the BEKK model reduces the variance by 496.5%. In terms of the return of portfolio, all returns from all six models are higher than unhedged strategy except the case using the GARCH models when transaction costs does not take more than 0.05% of return reduction. All returns are lower than the unhedged whenever the transaction costs increase to 0.25%, 0.50% and 1%. As expected, when the futures position is readjusted more often, lower revenue rises up due to the increased transaction costs.

From the student's *t* distribution models, the results of 1-year forecast for coffee in table 5-90 are very similar to those from normal distribution. When transaction costs are negligible, investors can make significantly more profits from dynamic hedge strategies than those from an unhedged position. Dynamic hedging reduces investment risks, and it also makes a profit for investors when the rate of transaction costs is less than 0.05%. The returns from dynamic hedging forecasted by BEKK and BEKK-X models are higher than those of unhedged position when the rate of transaction costs is 0.05%. Let's recall that the BEKK-family models outperform others on 1-year OHR forecast for coffee regardless of the distribution. The BEKK-family models have advantages over others in the terms of making profit and minimizing the risks of portfolio.

For the two-year out-of-sample forecast, the unhedged average return and risk for coffee are 0.00180 and 0.02575 respectively as reported in Table 5-91 and Table 5-92. With normal distribution, all average returns from six GARCH models are lower than 0.00180, while they actually produce lower risks than unhedged position with variance reduction ranging from 237.1% to 306.8%. The GARCH, BEKK and BEKK-X yield lowest risks than the rest of the six models. From the student's *t* distribution models, the number of the long-term hedge readjustments increases to 255. There is no improvement on return, yet the risk of portfolio is reduced up to 441.2%. For both distributions, we also find that the dynamic hedging strategy does not benefit investors with extra return using 2-year out-of-sample forecast even when the transaction costs are not taken into account.

Generally speaking, the dynamic hedge from GARCH model produces higher returns and lower risks for investors provided that the transaction costs are less than 0.05% of total returns in 1-year out-of-sample forecast for coffee, especially when they are estimated via BEKK and BEKK-X models. In the case of coffee, the short-term 1-year forecast is more beneficial than 2-year forecast for investors in terms of the profits since rebalance in the long-term hedge will decrease the returns of portfolio.

Results of return, risk and variance reduction for soybean are presented in Table 5-93, Table 5-94, Table 5-95 and Table 5-96. For 1-year out-of-sample forecast with normal distribution in Table 5-93, an unhedged position makes return of -3.7579e-04 and risk of 0.01677, while dynamic hedge tends to produce extra profits, lower risks and significant variance reduction by up to 472.0% if the transaction costs take 0.01% and less. Furthermore, the GJR and GARCH-X models provide the lowest risk for soybean. With student's *t* distribution (in Table 5-94), the dynamic hedging outperforms unhedged position in terms of return, risk and variance reduction with up to 0.01% transaction costs and the BEKK model is the best in reducing variance of portfolio. A dynamic hedge with GARCH models without transaction costs is better than an unhedged position in terms of return and risk from. Generally, the time-varying hedging strategy for soybean is superior to an unhedged position with a certain level of transaction costs on 1-year forecast.

In a 2-year forecast, the results of these two distributions are consistent and a dynamic hedge with all GARCH models reduces risk along with decreasing in the return of portfolio. With result in Table 5-95 under normal distribution, all models reduce risk of portfolio by up to 172.9% from GJR independent on the rate of transaction costs. With transaction costs excluded, the QGARCH model yields the highest return than any other models when under normal distribution. Using student's *t* distribution models (in Table 5-96), all the six GARCH models reduce risks and the GJR model provides the lowest variance. The finding on return and risk test coincides with the outperformance of GJR for soybean for the case of 2-year forecast. In practice, the dynamic 2-year hedge does not contribute to return, while it reduces the risk of portfolio for all models for soybean regardless of the forecast horizon and distribution.

In general the other commodities provide similar results. The returns from dynamic hedging strategies are generally higher with lower risk than that of an unhedged position for the 1-year forecast, and the returns drop down and risk decreases less for a long-term 2-year forecast. It is known that a hedge using futures contracts may either increase or decrease the return of a portfolio for investors (Hull, 2009). Additionally, a hedge may become less effective as the forecast horizon increases (Geppert, 1995). Alternatively, the low return and hedging effectiveness will fall for longer out-of-sample forecast horizons when we omit the transaction costs.

Based on the empirical study, we find that higher transaction costs will result in lower returns for investors and they also reduce the opportunity to rebalance the futures position in agricultural and futures markets. The dynamic hedge strategy benefits investors with extra returns and low risk on 1-year forecast, while it provides lower profit with low risk on 2-year forecast, when the transaction costs are low. For low risk-averse investors, the short 1-year dynamic hedge improves the performance of portfolio dramatically and yields higher returns over an unhedged strategy. For median risk-averse investors, they can hedge both/either over short-term and/or long-term. For investors with high risk-aversion, it is still worth hedging long term (2 years) even when the rate of transaction costs is high, provided with a proper forecasting model.

5.12 Summary and Conclusion

We summarise the discrepancy of forecasting performance of six GARCH models with normally and student's *t* distributed residuals. The model (s) that outperforms in each case is reported in summary 9. Specifically, this summary lists out the superior forecasting model in the condition of two forecast horizons, two different distributions under two categories of evaluations for five commodities.

Summary 9 of MAE, MSE, Theil' U and MDM tests on forecasted return of portfolio

Evaluation	Horizon	1-year		2-year	
	Goods \ Distribution	Normal	Student's *t*	Normal	Student's *t*
MAE MSE Theil's U	Coffee	BEKK	BEKK,BEKK-X	BEKK,BEKK-X	BEKK,BEKK-X
	Wheat	GJR, Q	Q	Q	GJR
	Soybean	X	BEKK	BEKK	GJR

	Live cattle	GJR, X	Q, X	GJR	GARCH
	Live hog	Q	Q	GJR	Q
MDM TEST	Coffee	BEKK	BEKK,BEKK-X	BEKK	BEKK,BEKK-X
	Wheat	GJR	Q	Q	BEKK
	Soybean	X	BEKK	BEKK, GJR	BEKK, GJR
	Live cattle	GJR	Q	GARCH	GARCH,BEKK-X
	Live hog	Q	Q	GJR	Q

According to the summary 9, we carry out a cross comparison of forecasting power of six GARCH models. For the case of coffee, the BEKK dominates all the time, while the student's *t* distribution highlights the outperformance of both the BEKK and BEKK-X models. The asymmetric GJR and QGARCH models provide the most accurate forecast for storable commodity wheat on both horizons that approves the superiority of BEKK only in the case of 2-year forecast with student's distribution under MDM test. Coincidently, the MDM test prefers the BEKK and GJR models for soybean on long forecast horizon for both distributions. However, different GARCH models outperform in other situation for soybean. Roughly, the BEKK-type model and asymmetric GARCH models yield the best forecasts for both normal and student's *t* distributions on both horizons with exception of soybean using normal distribution in 1-year forecast, while the standard GARCH model never outperform others for storable commodity.

The situation for non-storable commodity is slightly more complicated. For 1-year forecast, the GJR and QGARCH perform best for normal and student's t distribution for live cattle, yet the 3 forecast error tests rank the GARCH-X as most powerful forecasting model as the same as GJR and QGARCH models, respectively. The QGARCH is supreme for the case of live hog. On the long-term 2-year horizon, the GJR with normally distributed residuals is superior for both live cattle and live hog under the MAE, MSE and Theil's U tests, but the standard GARCH yields better forecast with MDM test. Using the student's t distribution, the standard GARCH which is the weakest forecasting model for storable commodity dominates other five GARCH models for non-storable live cattle during both horizons; additionally, the MDM evaluates that the BEKK-X perform equally well as standard GARCH for live cattle. The asymmetric GARCH models is meant to forecast for live hog for both distributions, both horizons and two types of evaluation methods, while they work well for one-year forecast with alternative GARCH-X model; furthermore, the outperformance of standard GARCH model on 2-year forecast takes the first ranking of asymmetric GARCH model in some extent for live cattle. Interestingly, the BEKK is the worst forecasting model for the non-storable goods in any condition.

The above results cannot find out a single supreme model with the most powerful forecasting ability in predicting daily dynamic hedge ratio in agricultural and commodity futures markets. The forecasting power of GARCH models depending on the commodity, the residual distributions, the length of forecast horizons and method for evaluations. The finding backs up the claim of Poon and Granger (2003).

Summary 10 percentage of dominance for each model in all conditions

Horizon	1-year				2-year			
Distribution / models	Normal		Student's *t*		Normal		Student's *t*	
Storable commodity								
GARCH	0	0	20	20	12	27	0	7
BEKK	60	66	60	60	72	87	67	73
GARCH-X	27	48	20	47	27	27	20	20
BEKK-X	48	48	33	40	48	33	40	47
GJR	48	48	27	27	6	39	40	67
QGARCH	39	33	47	47	39	48	20	13
Non-storable commodity								
GARCH	39	39	60	60	60	60	40	40
BEKK	9	9	20	20	39	39	20	30
GARCH-X	21	21	30	30	9	9	0	0
BEKK-X	39	39	50	50	30	30	40	70
GJR	69	69	20	20	81	81	60	60
QGARCH	69	69	90	90	30	30	60	60

Nevertheless, according to the summary 10 about the dominance of GARCH models for storable and non-storable commodity in each condition, the superiority of BEKK and asymmetric GARCH models is found. As a more general parameterization of VECH GARCH, the BEKK-type GARCH models guarantee the positivity of conditional variance. This character improves the statistical meaning.

Recall the graphs in figures 5-3, 5-4, 5-5, 5-6, 5-9, 5-10, 5-11 and 5-12, the forecasted and estimated hedge ratio for storable commodities is mostly positive with very few negative estimates, while that of non-storable products closes to zero and moves above and below zero quite frequently. It is known that the BEKK has a disadvantage that the positivity of conditional covariance assumption is easily violated by some data (Ledoit et al., 2003).

Regarding mathematical interpretation, the conditional covariance for non-storable produces violates the positivity assumption of BEKK-type model, but storable commodities generally obey this rule. This phenomenon is inherent to our finding that BEKK-type GARCH models outperform others for 3 storable produces, but they do not work well for non-storable goods.

The asymmetric GARCH models are more capable of interpreting leverage effect compared to other GARCH models, and they are more accurate forecasting model providing the presence of asymmetric effect of good and bad news on conditional variance for all commodities in this study.

Thereby we suggest that the BEKK and asymmetric GARCH models are generally the most powerful forecasting models among the six GARCH models for storable commodity and it is worth applying the later type model (asymmetric GARCH model) to forecast time-varying hedge ratio for non-storable agricultural commodity for 1- and 2-year prediction. On the other hand, the worst forecasting performance of standard GARCH model for the case of storable products demonstrates its disability on the OHR prediction. In addition, the BEKK and GARCH-X models are not appropriate on forecasting the 1-year and 2-year forecast respectively for non-storable agricultural commodity.

When transaction costs are considered, the dynamic hedge strategy benefits investors with excess return and low risk on 1-year forecast, yet it provides lower profit with low risk on 2-year forecast. If appropriate forecasting models are selected, low risk-averse investor will gain from short 1-year dynamic hedge; median risk-averse investor can profit from both/either short-term and/or long-term hedging; investor with high risk-aversion get benefits from both short-term and long term hedging even if the rate of transaction costs is high.

Table 5-1 Estimated coefficients from diagonal bivariate GARCH (1, 1) model with normal distribution for full sample

$$vech(H_t) = C + A_1 vech(u_{t-1})^2 + B_1 vech(H_{t-1})$$

$$H_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{12} & 0 \\ 0 & 0 & a_{22} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{12} & 0 \\ 0 & 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

product / variables	coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	-3.93e-04*** (-1.7199)	2.7186e-06 (0.0282)	-3.0559e-05 (-0.5834)	8.1935e-04 (1.5377)	1.0771e-03* (7.6602)
φ_1	-0.0262* (-3.5346)	-0.0137* (-4.7831)	-0.0586* (-8.5288)	-0.0051** (-2.3029)	-0.0312* (-17.9697)
μ_2	-1.2425e-04 (-0.4547)	-1.0431e-05 (-0.10564)	4.1674e-05 (0.6306)	0.0001 (0.9404)	5.3843e-05 (0.2167)
φ_2	-3.3225e-03 (-0.6037)	0.0101* (4.4011)	0.0250* (4.3086)	0.0053* (3.8096)	3.1933e-03 (1.2880)

c_{11}	6.9228e-06 (2.5469)	2.6977e-04* (19.4245)	2.1061e-04* (21.1039)	4.725e-05* (8.1461)	1.4211e-06* (5.2347)
a_{11}	0.1881* (4.6584)	1.0138e-03* (215.4001)	9.8075e-04* (35.9686)	1.1619 (1.3889)	0.1975* (29.0249)
b_{11}	0.8262* (0.0000)	0.1000* (25209.5729	0.1000* (7173.1991)	0.0431 (0.8939)	0.8392* (192.0103)
c_{22}	2.4026e-05* (5.1766)	2.3306e-04* (15.4810)	1.8854e-04* (22.5635)	0.0001 (1.2251)	1.9818e-06* (11.7056)
a_{22}	0.1995* (10.5923)	1.0132e-03* (214.9912)	9.6440e-04* (42.9313)	0.0566 (1.4070)	2.5148e-03* (16.8871)
b_{22}	0.7771* (43.8295)	0.1000* (25208.8185)	0.1000* (7135.7231)	0.56434*** (1.7056)	0.9928* (1908.3039)
c_{12}	1.3164e-05* (3.9555)	1.5586e-04* (29.4687)	1.5433e-04* (25.6542)	0.0000 (2.5091)	4.0945e-06* (4.8143)
a_{12}	0.1991* (6.6979)	1.0000e-03* (15384.9252)	-5.4269e-03 (-1.6347)	0.0355 (1.4118)	0.0407* (5.6921)
b_{12}	0.7537* (35.1749)	0.1000* (28344.9237)	0.1004* (60.8746)	0.2757 (1.3027)	0.8071* (36.4565)
LLF	34981.57344	36954.02684	40057.20480	41712.57867	36133.73123

Note: *t*-statistic of coefficients in parentheses
‘*’ represents that it shows significance at 1% significant level
‘**’ represents that it shows significance at 5% significant level
‘***’ represents that it shows significance at 10% significant level
‘LLF’ is the value of log-likelihood function

Table 5-2 Basic statistics for estimated OHR from standard GARCH model with normal distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.542288	0.084746	0.063255	0.227465*	57.72400*
wheat	0.668650	0.000029	1090.360*	-31.15796*	3289557*
Soybean	0.813090	0.000255	198.8022*	-11.16328*	1110077*
Live cattle	0.085901	0.001160	795.8147*	20.16696*	1737112*
Live hog	0.080913	0.007488	71.82127*	4.293682*	1542143*

Note: ‘*’ represents that it shows significance at 1% significant level
‘**’ represents that it shows significance at 5% significant level
‘***’ represents that it shows significance at 10% significant level

Table 5-3 Stationary test for estimated OHR from standard GARCH model with normal distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-15.71068	-12.15783	-9.623920	-8.37686
	2	-90.17303	-50.53262	-41.79082	-35.5911
Wheat	1	-55.96862	-54.49108	-35.38047	-28.0677
	2	-113.7797	-66.32210	-55.61671	-46.9503
Soybean	1	-56.46840	-34.06714	-21.32611	-16.3461
	2	-126.0854	-71.18384	-50.63725	-44.9588
Live cattle	1	-47.12348	-35.39892	-27.77978	-23.3522
	2	-97.02196	-62.48444	-48.77069	-40.5040
Live hog	1	-23.35715	-21.88594	-19.91380	-17.4171
	2	-83.53678	-50.22969	-40.23642	-33.5082

Table 5-4 Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from GARCH model with normal distribution

test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	22.5584 (0.4868)	6.7309 (0.4574)	0.0221 (0.8818)	1.3206 (0.2505)
Wheat	524.699 (0.0000)	28.9276 (0.1827)	391.58 (0.0000)	9.3852 (0.0022)
Soybean	1686.00 (0.0000)	1632.53 (0.0000)	567.84 (0.0000)	116.49 (0.0000)
Live cattle	1.2617 (1.0000)	7.0927 (0.9994)	0.0091 (0.9239)	0.4131 (0.5204)
Live hog	3.8706 (0.9999)	9.3425 (0.9947)	1.0985 (0.2946)	0.0695 (0.7920)

Note: The Ljung-Box test is applied to test serial correlation with the joint null hypothesis of correlation coefficient are not significant from zero. The u_{t1} and u_{t2} are residuals from linear regression of $R_c^t = C + u_{t1}$ and $R_f^t = C + u_{t2}$; and the u_{t1}^2 / h_t and u_{t2}^2 / h_t are squared residuals. The p-values for Ljung-Box correlation test are in parentheses. The white statistic tests the reminding heteroscedasticity of residuals after estimating OHR with this model.

Table 5-5 Estimated coefficients from bivariate DBEKK-GARCH (1, 1) model with normal distribution for full sample

$$H_t = CC' + A_1(u_{t-1}u'_{t-1})A'_1 + B_1 H_{t-1} B'_1$$
$$h_{11,t} = c_{11}^2 + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1}$$
$$h_{12,t} = c_{11}c_{12} + a_{11}a_{22}u_{1,t-1}u_{2,t-1} + b_{11}b_{22}h_{12,t-1}$$
$$h_{22,t} = c_{12}^2 + c_{22}^2 + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1}$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	-2.8861e-04* (-2.2986	-0.0001 (-0.60074)	0.0001 (0.30834)	0.0006*** (1.68724)	0.0001 (-0.43281)
φ_1	-0.0129 (-1.6038)	-0.0301* (-3.14371)	-0.1572* (-7.91942)	0.2556* (8.12562)	0.4640* (10.1979)
μ_2	-0.0003 (-1.36729)	-0.0005* (-3.71630)	0.0000 (-0.00508)	0.0001 (0.48688)	0.0000 (0.12053)
φ_2	-0.0589* (-6.82470)	-0.0498* (-5.39292)	-0.1259* (-8.59913)	0.0200 (1.28854)	-0.0002 (-0.01601)

c_{11}	0.0007* (67.7620)	0.0036* (48.1931)	0.0030* (3.63191)	0.0033** (2.74987)	0.0009* (4.30190)
a_{11}	0.2117* (117.577)	0.3499* (93.7629)	0.3606* (6.17532)	0.8375 (2.40955)	0.4276* (14.6878)
b_{11}	0.9787* (3240.50)	0.9244* (0791.63)	0.9178* (31.3745)	0.7599* (6.70681)	-0.9205* (-91.2178)
c_{22}	0.0018* (61.6604)	0.0024* (16.5852)	-0.0010* (-2.89417)	-0.0005** (-1.91862)	0.0001 (0.01562)
a_{22}	0.2197* (87.1162)	0.4582* (121.439)	0.2825* (9.19872)	0.0675* (6.97227)	0.0488* (5.43500)
b_{22}	0.9728* (1730.97)	0.8285* (230.724)	0.9503* (94.0474)	0.9962* (966.143)	-0.9973* (-1349.47)
c_{12}	0.0007* (7.86007)	0.0057* (34.0609)	0.0018* (4.18889)	0.0004* (3.01821)	0.0011* (4.34179)
LLF	35554.97414	38574.06018	42289.39506	41828.53441	36398.64173

Note: see note for table 5-1

Table 5-6 Basic statistics for estimated OHR from BEKK-GARCH model with normal distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.600272	0.157965	-1.518144*	-0.068903**	635.8320*
wheat	0.778052	0.051849	6.785356*	-1.221845*	14344.67*
Soybean	0.852143	0.044588	10.34668*	-0.607820*	30103.92*
Live cattle	0.064247	0.007880	194.3759*	9.491724*	1043512*
Live hog	0.071866	0.005467	37.83134*	3.894555*	439731.9*

Note: see note of table 5-2

Table 5-7 Stationary test for estimated OHR from BEKK-GARCH model with normal distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-3.315000	-3.293290	-3.080570	-3.096590
	2	-82.65826	-42.68924	-32.98751	-27.61856
Wheat	1	-23.03803	-20.31117	-18.02713	-15.84106
	2	-91.14223	-49.26445	-39.16327	-35.78696
Soybean	1	-13.90363	-12.76612	-12.05907	-10.90183
	2	-85.74390	-44.60225	-36.57555	-32.43126
Live cattle	1	-26.86104	-23.14103	-21.52010	-18.98824
	2	-88.23126	-52.18766	-41.63245	-36.33680
Live hog	1	-14.16898	-14.10162	-13.44343	-12.85278
	2	-81.40454	-46.29437	-35.90887	-29.75429

Table 5-8 Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from BEKK-GARCH model with normal distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	15.4601 (0.2795)	57.0200 (0.0001)	9.6366 (0.0020)	23.905 (0.0000)
Wheat	31.0632 (0.1212)	13.9469 (0.9284)	5.2771 (0.0216)	0.0005 (0.9825)
Soybean	12.9174 (0.9538)	12.4732 (0.8644)	0.5794 (0.4466)	0.0536 (0.8170)
Live cattle	0.2976 (1.0000)	6.4509 (0.9997)	0.0161 (0.8989)	0.8905 (0.3453)
Live hog	28.1749 (0.2092	9.2309 (0.9951)	17.696 (0.0001)	0.0813 (0.7754)

Note: see note for table 5-4.

Table 5-9 Estimated coefficients from diagonal bivariate GARCH (1, 1)-X model with normal distribution for full sample

$$vech(H_t) = C + A_1 vech(u_{t-1})^2 + B_1 vech(H_{t-1}) + D_1 vech(z_{t-1})^2$$
$$h_{11,t} = c_{11} + a_{11}u_{1,t-1}^2 + b_{11}h_{11,t-1} + d_{11}z_{t-1}^2$$
$$h_{12,t} = c_{12} + a_{12}u_{1,t-1}u_{2,t-1} + b_{12}h_{12,t-1} + d_{12}z_{t-1}^2$$
$$h_{22,t} = c_{22} + a_{22}u_{2,t-1}^2 + b_{22}h_{22,t-1} + d_{22}z_{t-1}^2$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	-8.213e-06 (-0.0457)	0.0001 (0.3667)	4.341e-05 (0.1020)	0.0008 (1.5778)	1.077e-03* (7.7805)
φ_1	-0.0164* (-4.7354)	-0.0081** (-2.0247)	-0.0265** (-2.1418)	-0.0060** (-3.0493)	-0.0319* (-20.9255)
μ_2	1.650e-04 (0.6137)	-0.0004 (-1.3593)	-1.790e-04 (-0.4801)	0.0001 (1.0250)	1.048 (0.4150)

φ_2	6.788e-03** (2.1260)	0.0058 (1.5161)	9.789e-03 (1.0672)	0.0051* (3.3692)	3.691e-03 (1.0100)
c_{11}	5.212r-07 (1.6232)	0.0001*** (1.8854)	9.884e-06 (1.4645)	0.0001* (8.3674)	1.604e-06* (4.7308)
a_{11}	0.0666* (5.4889)	0.1421* (3.8504)	0.1331* (2.6332)	1.1477 (1.4516)	0.1992* (28.6448)
b_{11}	0.9351* (87.6362)	0.7690* (9.6390)	0.8321* (11.6527)	0.0492 (0.9667)	0.8392* (189.8666)
d_{11}	1.37e-04*** (1.9293)	0.0011*** (1.9557)	5.869e-04 (0.5011)	-0.0006* (-4.7801)	-4.21e-05*** (-2.5746)
c_{22}	3.617e-06* (2.7993)	0.0001* (2.7515)	1.121e-05 (1.4463)	0.0001 (1.5234)	1.579e-06* (15.3003)
a_{22}	0.0737* (6.0736)	0.2448* (3.7904)	0.1463** (2.2104)	0.0403 (1.1134)	9.474e-04* (10.0670)
b_{22}	0.9209* (74.9235)	0.5358* (4.4371)	0.8027* (8.4961)	0.6932** (3.2234)	0.9946* (3076.4276)
d_{22}	1.607e-04** (1.9846)	0.0022*** (1.8170)	1.191e-03 (0.7537)	-0.0002* (-4.4629)	2.281e-05* (15.4980)
c_{12}	6.791e-07** (1.9920)	0.0001* (2.5779)	1.033e-05 (1.1060)	0.0001* (6.5494)	5.403e-08 (1.4247)
a_{12}	0.0593* (6.9038)	0.1899* (4.0584)	0.1268*** (1.7762)	0.0046 (0.2059)	3.554e-03** (3.2757)
b_{12}	0.9353* (101.5241)	0.6439* (6.9702)	0.8134* (6.8688)	-0.3825* (-3.3576)	0.9944* (512.4629)
d_{12}	1.394e-04** (2.1287)	0.0001 (0.0907)	1.116e-03 (0.7010)	-0.0002** (-2.0887)	-4.802e-07 (-0.2548)
LLF	35782.73697	38786.92016	42440.10853	41789.01318	36197.74464

Note: see note for table 5-1

Table 5-10 Basic statistics for estimated OHR from diagonal bivariate GARCH (1, 1)-X model with normal distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.596126	0.139146	-1.490057*	-0.05064***	610.3290*
wheat	0.773308	0.053629	6.741123*	-1.044601*	13736.49*
Soybean	0.854933	0.040931	8.559593*	-1.185123*	21880.65*
Live cattle	0.082777	0.001834	4339.275*	-56.49537*	5154877*
Live hog	0.112727	0.005494	-1.227714*	-0.042533	446.4040*

Table 5-11 Stationary test for estimated OHR from diagonal bivariate GARCH (1, 1)-X model with normal distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)

Coffee	1	-4.406710	-4.305260	-4.064460	-3.979350
	2	-80.75996	-42.10455	-32.79895	-27.87831
Wheat	1	-25.29099	-20.58869	-16.40461	-13.82619
	2	-92.15673	-52.44717	-42.10928	-37.52871
Soybean	1	-16.67581	-15.37342	-14.27506	-12.67764
	2	-86.97670	-44.07507	-36.77408	-33.35880
Live cattle	1	-102.0246	-36.63957	-25.99219	-21.04599
	2	-173.4215	-73.39273	-53.32577	-43.70049
Live hog	1	-2.221540	-2.410820	-2.540330	-2.547740
	2	-75.98554	-4.862450	-30.43546	-24.78845

Table 5-12 Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from diagonal bivariate GARCH (1, 1)-X model with normal distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	26.6821 (0.2697)	36.9086 (0.0332)	6.3146 (0.0120)	10.0020 (0.0016)
Wheat	31.1107 (0.1201)	11.1821 (0.9813)	4.7085 (0.0300)	1.5681 (0.2105)
Soybean	14.0547 (0.9253)	23.3956 (0.1368)	0.0741 (0.7855)	5.1821 (0.0228)
Live cattle	1.0761 (1.0000)	6.2203 (0.9998)	0.00518 (0.9426)	0.14189 (0.7064)
Live hog	3.9411 (0.9999)	11.6051 (0.9761)	1.11913 (0.2901)	0.0066 (0.9354)

Table 5-13 Estimated coefficients from bivariate diagonal BEKK-GARCH-X (1, 1) model with normal distribution for full sample

$$H_t = CC' + A_1(u_{t-1}u'_{t-1})A'_1 + B_1 H_{t-1} B'_1 + D_1 Z_{t-1}^2 D'_1$$
$$h_{11,t} = c_{11}^2 + a_{11}^2 u_{11,t-1}^2 + b_{11}^2 h_{11,t-1} + d_{11}^2 z_{t-1}^2$$
$$h_{12,t} = c_{11}c_{12} + a_{11}a_{22}u_{1,t-1}u_{2,t-1} + b_{11}b_{22}h_{12,t-1} + d_{11}d_{22}z_{t-1}^2$$
$$h_{22,t} = c_{12}^2 + c_{22}^2 + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} + (d_{12}^2 + d_{22}^2) z_{t-1}^2$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	0.0001 (0.1262)	0.0001 (0.6672)	0.0002 (0.9916)	0.0008*** (1.7894)	0.0011* (3.5784)
φ_1	-0.0178* (-9.0815)	-0.0082* (-2.9998)	-0.0289* (-2.6735)	-0.0050* (-2.8474)	-0.0309 (0.0000)
μ_2	0.0003 (1.5400)	-0.0004* (-2.9460)	-0.0001 (-0.2052)	0.0001 (0.4002)	0.0001 (0.2846)
φ_2	0.0070*** (2.7455)	0.0056** (2.3440)	0.0075 (1.1753)	0.0050* (2.8415)	0.0040 (1.1518)

c_{11}	0.0006* (50.7246)	0.0051* (57.6055)	0.0031** (2.2669)	0.0068* (20.7336)	0.0012** (2.0945)
a_{11}	0.2293* (114.0424)	0.3728* (71.7465)	0.3629* (3.6269)	1.0783* (2.9382)	0.4431* (16.3437)
b_{11}	0.9742* (2622.9490)	0.8799* (402.8297)	0.9151* (16.9403)	0.2166** (2.1879)	0.9163* (68.1300)
d_{11}	0.0097* (25.7782)	0.0329* (15.9341)	0.0181 (0.5427)	0.0001 (6.5e-04)	-0.0001 (-4.0090)
c_{22}	0.0020* (59.1597)	0.0025* (15.4384)	0.0011* (2.7039)	0.0001 (0.0025)	0.0131 (1.4568)
a_{22}	0.2358* (84.0631)	0.4910* (128.7683)	0.2872* (5.7976)	0.0701* (8.3560)	0.1333* (2.6614)
b_{22}	0.9674* (1399.4713)	0.7436* (133.6714)	0.9486* (56.5358)	0.9953* (788.8047)	0.2538 (0.9393)
d_{22}	0.0079* (8.0896)	0.0465* (22.6801)	-0.0001 (-0.00342)	0.0001 (9.5e-04)	0.0001 (9.1e-06)
c_{12}	0.0007* (5.7094)	0.0071* (37.5394)	0.0018* (2.6899)	0.0008* (4.2252)	0.0150 (1.5765)
d_{12}	0.0070* (4.3005)	0.0014 (0.7895)	0.0134 (0.5925)	0.0001 (1.9e-04)	-0.0001 (-4.0e-05)
LLF	35645.84836	38789.35169	42243.12565	41748.69248	36086.43656

Note: see note for table 5-1.

Table 5-14 Basic statistics for estimated OHR from bivariate diagonal BEKK-GARCH-X (1, 1) model with normal distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.593322	0.153227	-1.519957*	-0.055741**	635.548*
wheat	0.773202	0.052342	6.450982*	-1.086091*	12778.40*
Soybean	0.845497	0.050105	10.49322*	-0.645976*	31004.11*
Live cattle	0.062124	0.006228	563.1826*	16.92351*	8708711*
Live hog	0.066490	0.003241	130.2296*	5.343088*	5032541*

Table 5-15 Stationary test for estimated OHR from bivariate diagonal BEKK-GARCH-X (1, 1) model with normal distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-3.816210	-3.746070	-3.478220	-3.492630

	2	-82.42875	-43.25735	-33.32222	-27.94605
Wheat	1	-25.02949	-20.47984	-16.42373	-13.91222
	2	-92.81157	-53.08718	-42.57982	-37.94158
Soybean	1	-13.58759	-12.63811	-11.86069	-10.68071
	2	-84.87704	-44.57171	-36.58463	-32.36416
Live cattle	1	-55.54475	-34.60459	-26.97017	-22.05501
	2	-115.29360	-67.66726	-51.30998	-43.18350
Live hog	1	-55.81548	-35.10071	-26.57071	-19.99605
	2	-115.68300	-69.14085	-52.39065	-43.93620

Table 5-16 Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from bivariate diagonal BEKK-GARCH-X (1, 1) model with normal distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	31.8998 (0.1022)	46.4084 (0.0027)	9.8329 (0.0017)	18.5071 (0.0001)
Wheat	8.3217 (0.9978)	11.5321 (09771)	0.0757 (0.7832)	1.4947 (0.2215)
Soybean	13.6822 (0.9356)	13.4620 (0.8141)	1.6697 (0.1963)	0.0759 (0.7829)
Live cattle	1.2725 (1.0000)	6.6502 (0.9996)	0.0084 (0.9269)	0.8725 (0.3503)
Live hog	3.8624 (0.9999)	7.2291 (0.9993)	1.1065 (0.2928)	0.0121 (0.9125)

Note: see note for table 5-4.

Table 5-17 Estimated coefficients from bivariate diagonal GARCH-GJR (1, 1) model with normal distribution for full sample

$$\sigma_t^2 = c + \sum_{i=1,j=1}^{2}(a_{ij} + \gamma_i I_{t-1})u_{t-1}^2 + \sum_{i=1,j=1}^{2} b_{ij}\sigma_{t-1}^2$$

$$H_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} + \gamma_1 I_{t-1} & 0 & 0 \\ 0 & a_{12} & 0 \\ 0 & 0 & a_{22} + \gamma_2 I_{t-1} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{12} & 0 \\ 0 & 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	8.0e-05 (0.4270)	2.6e-04 (1.5035)	1.9e-04 (0.9793)	-0.0001 (-0.3323)	1.1e-03* (3.5304)
φ_1	-0.0219* (-2.8714)	-0.0111** (-1.9798)	-0.0234** (-2.2798)	-0.0054* (-3.6680)	-0.0308* (-8.4729)
μ_2	2.3e-04 (1.0160)	-2.2e-05 (-0.1014)	-3.5e-05 (-0.1884)	-0.0001 (-0.5184)	7.9e-05 (0.3455)

φ_2	3.4e-03 (0.5203)	5.9e-03 (1.1172)	0.0149 (1.6251)	0.0052* (3.8838)	4.1e-03*** (1.9337)
$(d_0)\ c_{11}$	7.0e-07 (2.3476)	2.3e-04* (21.623)	6.9e-06* (3.3829)	0.0001* (10.529)	1.6e-06 (1.1949)
$(d_1)\ a_{11}$	0.0640* (8.0973)	0.1000* (9036.9)	0.1166* (5.5692)	0.0915* (2.3000)	0.0208* (3.9949)
$(d_2)\ b_{11}$	0.9400* (128.55)	0.0550* (9089.7)	0.8664* (35.309)	0.0503* (180.96)	0.8378* (32.044)
$(d_3)\ \gamma_1$	1.2e-04 (0.0149)	0.0771** (1.9856)	-0.0130 (-0.8029)	1.8808 (1.3089)	-8.2e-03 (-0.1812)
$(e_0)\ c_{22}$	3.5e-06* (3.2857)	2.0e-04* (23.3091)	8.1e-06* (4.9138)	0.0001* (18.090)	2.6e-04* (14.046)
$(e_1)\ a_{22}$	0.0694* (7.2384)	0.1000* (9676.150)	0.1315* (6.9963)	0.0415 (1.0860)	8.5e-03 (0.5415)
$(e_2)\ b_{22}$	0.9264* (101.16)	0.0550* (8730.3)	0.8468* (37.596)	0.0503* (163.98)	0.3764* (27.635)
$(e_3)\ \gamma_2$	3.1e-03 (0.3629)	-5.0e-03 (-0.1406)	-0.0281** (-2.3267)	0.0761 (1.3652)	3.0e-03 (0.1094)
$(f_0)\ c_{12}$	8.9e-07* (2.8248)	1.5e-04* (28.4390)	6.8e-06* (4.2623)	0.0001* (7.8081)	2.7e-05* (6.3033)
$(f_1)\ a_{12}$	0.0576* (9.2974)	0.1000* (21637)	0.1005* (6.1338)	0.0398** (2.4378)	0.0406** (2.2512)
$(f_2)\ b_{12}$	0.9397* (141.99)	0.0550* (20063)	0.8611* (39.900)	0.0499* (383.31)	-0.0751* (-16.089)
LLF	41985.9346 4	37879.75536	42480.75843	41985.93464	36080.74487

Note : see note for table 5-1.

Table 5-18 Basic statistics for estimated OHR from bivariate diagonal GARCH-GJR (1, 1) model with normal distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.596691	0.146784	-1.492316*	-0.049181	612.0120*
wheat	0.769345	0.009209	142.4433*	-2.808400*	5603691*
Soybean	0.847880	0.045118	7.325646*	-1.129000*	16299.57*
Live cattle	0.087623	0.001108	717.7426*	18.88783*	1413281*
Live hog	0.063160	0.001459	99.71006*	4.008756*	2949379*

Table 5-19 Stationary test for estimated OHR from bivariate diagonal GARCH-GJR (1, 1) model with normal distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)

Coffee	1	-4.195550	-4.114660	-3.893160	-3.831810
	2	-80.56198	-41.98765	-32.65763	-27.73527
Wheat	1	-70.80494	-36.18859	-26.55998	-22.04252
	2	-135.22524	-69.27170	-51.85872	-43.37593
Soybean	1	-13.86778	-13.27336	-12.56807	-11.31220
	2	-84.42869	-42.63828	-35.21898	-31.84448
Live cattle	1	-60.04101	-37.55633	-28.67534	-24.14954
	2	-113.13234	-66.99474	-50.63068	-42.11926
Live hog	1	-80.85013	-40.77439	-29.61966	-22.83525
	2	-144.09904	-73.69251	-53.28370	-46.26016

Table 5-20 Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from bivariate diagonal GARCH-GJR (1, 1) model with normal distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	26.0354 (0.2992)	39.6361 (0.0169)	5.7221 (0.0167)	11.8858 (0.0006)
Wheat	169.550 (0.0000)	27.0482 (0.2539)	5.6812 (0.0171)	3.7651 (0.0523)
Soybean	12.9796 (0.9524)	19.5672 (0.4210)	1.5919 (0.2071)	2.2496 (0.1336)
Live cattle	5.5680 (0.9999)	10.0155 (0.9912)	2.0416 (0.1530)	0.9904 (0.3196)
Live hog	3.8695 (0.9999)	7.3355 (0.9992)	1.1156 (0.2909)	0.0137 (0.9067)

Note: see note for table 5-4.

Table 5-21 Estimated coefficients from bivariate diagonal QGARCH (1, 1) model with normal distribution for full sample

$$vech(H_t) = C + A_i vech(u_{t-1}^2) + B_i vech(H_{t-1}) + D_i vech(u_{t-1})$$

$$h_{11,t} = c_{11} + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1} + d_{11} u_{1,t-1}$$

$$h_{12,t} = c_{12} + a_{12} u_{1,t-1} u_{2,t-1} + b_{12} h_{12,t-1}$$

$$h_{22,t} = c_{22} + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} + d_{22} u_{2,t-1}$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	9.1e-05 (0.4631)	-8.4e-05 (-0.3733)	-9.0e-05 (-0.3300)	0.0004* (2.8003)	-2.0e-04 (-0.9548)
φ_1	-0.0205* (-2.8.54)	-0.0126*** (-1.6833)	-0.0322** (-2.2268)	-0.0083* (-4.5107)	-0.0255* (0.0000)
μ_2	2.3e-04 (0.8239)	-3.1e-04 (-1.4026)	-2.0e-04 (-0.8585)	0.0001 (0.0040)	-7.7e-05 (-0.3638)

φ_2	4.3e-03 (0.7358)	0.0115*** (1.8178)	5.9e-03 (0.4980)	0.0042 (0.4165)	3.4e-03 (1.3764)
$(d_0)\ c_{11}$	6.6e-07* 2.9238	1.5e-05* (3.1792)	1.3e-05** (2.4637)	0.0001** (2.0939)	3.6e-04* (8.5058)
$(d_1)\ a_{11}$	0.0640* (7.4979)	0.1278* (4.8651)	0.1565* (6.0938)	1.0018 (0.9129)	0.0446* (60.7121)
$(d_2)\ b_{11}$	0.9403* (129.65)	0.8456* (27.402)	0.8003* (19.700)	0.0597 (0.1604)	0.0492* (449.42)
$(d_3)\ d_{11}$	9.6e-05 (1.1803)	-6.8e-04* (-2.6026)	-4.0e-04 (-0.9285)	-0.0076 (-1.5537)	5.8e-05* (7.6498)
$(e_0)\ c_{22}$	3.6e-06* (3.0368)	4.0e-05* (4.3487)	1.3e-05* (4.9548)	0.0001 (0.4139)	4.0e-04* (13.856)
$(e_1)\ a_{22}$	0.0717* (6.5175)	0.1962* (4.2379)	0.1535* (15.278)	0.0413 (0.7681)	0.0447* (62.357)
$(e_2)\ b_{22}$	0.9257* (85.213)	0.6858* (12.220)	0.7924* (43.903)	0.6663 (0.9413)	0.0492* (448.66)
$(e_3)\ d_{22}$	-7.3e-05 (-0.4873)	1.1e-03* (3.1460)	3.3e-04*** (1.8583)	-0.0003 (-0.6986)	6.2e-05* (7.3942)
$(f_0)\ c_{12}$	8.7e-07* (2.9772)	2.1e-05* (44.060)	1.3e-05* (3.0284)	0.0001** (2.1663)	5.2e-05* (7.4209)
$(f_1)\ a_{12}$	0.0581* (8.2276)	0.1596* (4.8979)	0.1454* (17.780)	0.0325 (0.1674)	0.0517* (223.05)
$(f_2)\ b_{12}$	0.9395* (132.34)	0.7638* (18.588)	0.7845* (22.490)	0.0366** (2.3484)	0.0500* (13301)
LLF	35747.06805	38661.27019	42447.53809	41868.68418	35039.36581

Note: see note for table 5-1.

Table 5-22 Basic statistics for estimated OHR from bivariate diagonal QGARCH (1, 1) model with normal distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.596427	0.147750	-1.491056*	-0.047940	610.8510*
wheat	0.783781	0.057898	5.517680*	-0.910293*	9310.541*
Soybean	0.859968	0.044930	8.819705*	-1.150972*	23046.02*
Live cattle	0.083895	0.000750	506.0844*	14.74067*	7030834*
Live hog	0.132519	0.001845	143.8644*	5.251258*	6132943*

Note: see note for table 5-2.

Table 5-23 Stationary test for estimated OHR from bivariate diagonal QGARCH (1, 1) model with normal distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-4.211520	-4.124560	-3.904720	-3.840530
	2	-80.57313	-42.02610	-32.66515	-27.78138
Wheat	1	-22.84747	-20.22363	-17.64993	-15.63917
	2	-89.17773	-48.14376	-39.37247	-35.29907
Soybean	1	-18.11483	-16.18978	-14.82472	-12.95079
	2	-88.00918	-45.15704	-37.69801	-34.05315
Live cattle	1	-60.32692	-35.99523	-27.94102	-23.45790
	2	-117.73951	-66.28624	-49.58167	-41.16622
Live hog	1	-69.78328	-38.98669	-29.17189	-22.44652
	2	-130.04995	-70.83572	-52.90420	-44.80955

Table 5-24 Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from bivariate diagonal QGARCH (1, 1) model with normal distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	29.3589 (0.1687)	39.3860 (0.0180)	6.0609 (0.0138)	11.6829 (0.0006)
Wheat	31.1964 (0.1180)	14.0747 (0.9247)	6.2442 (0.0125)	0.0392 (0.8431)
Soybean	18.4628 (0.7319)	27.6366 (0.0907)	0.1752 (0.6755)	5.4142 (0.0120)
Live cattle	1.4301 (1.000)	6.9412 (0.9999)	0.0899 (0.7643)	0.1978 (0.6565)
Live hog	2.3765 (0.4976)	7.6043 (0.9989)	10.3627 (0.0013)	0.3026 (0.5822)

Note: see note in table 5-4.

Table 5-25 Forecast error test on 1-year forecasted return for coffee in 2006

Forecast error of return for coffee			
2006-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00465195	0.000041	0.97510
BEKK	0.00160528	0.000007	0.37614
GARCH-X	0.00220389	0.000013	0.51841
BEKK-X	0.00177040	0.000008	0.43387
GARCH-GJR	0.00205206	0.000012	0.49014
QGARCH	0.00205103	0.000012	0.48918

Note: 2006-N means that the OHR forecast is based on normal distribution for one-year prediction in 2006.

Table 5-26 Forecast error test on 1-year forecasted return for wheat in 2006

Forecast error of return for wheat			
2006-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00351069	0.000023	0.55765
BEKK	0.00263084	0.000017	0.52104
GARCH-X	0.00293144	0.000022	0.49365
BEKK-X	0.00258049	0.000017	0.48988
GARCH-GJR	0.00261779	0.000014	0.69674
QGARCH	0.00299338	0.000015	0.38633

Note: see note for table 5-25.

Table 5-27 Forecast error test on 1-year forecasted return for soybean in 2006

Forecast error of return for soybean			
2006-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00204558	0.000008	1.17735
BEKK	0.00120369	0.000004	0.22150
GARCH-X	0.00093627	0.000003	0.14833
BEKK-X	0.00126829	0.000005	0.31490
GARCH-GJR	0.00106406	0.000003	0.44346
QGARCH	0.00118671	0.000004	0.35242

Note: see note for table 5-25.

Table 5-28 Forecast error test on 1-year forecasted return for live cattle in 2008

Forecast error of return for live cattle			
2006-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00609517	0.000075	1.53620
BEKK	0.00629081	0.000079	1.60862
GARCH-X	0.00608626	0.000075	1.52182
BEKK-X	0.00624049	0.000078	1.59445
GARCH-GJR	0.00608472	0.000075	1.53405
QGARCH	0.00609663	0.000075	1.53588

Note: 2008-N means that the OHR forecast is based on normal distribution for one-year prediction in 2008.

Table 5-29 Forecast error test on 1-year forecasted return for live hog in 2008

Forecast error of return for live hog			
2006-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00919207	0.000268	0.78710
BEKK	0.00934552	0.000276	0.80080
GARCH-X	0.00949203	0.000285	0.81509
BEKK-X	0.00910446	0.000261	0.76559
GARCH-GJR	0.00910821	0.000262	0.77760
QGARCH	0.00852617	0.000229	0.72239

Note: see note in table 5-28.

Table 5-30 MDM test of forecasted return from six GARCH models with normally distributed residuals for coffee from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for coffee		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	<	<
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	>	>

BEKK vs. QGARCH	>	>
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	>	>
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	=	=

Note: ‘<’ represents that the latter model has better forecast than the prior one
‘>’ represents that the prior model has better forecast than the latter one
‘=’ represents that the two models have equal predictive power

Table 5-31 MDM test of forecasted error from six GARCH models with normal distribution for wheat from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for wheat		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	=	<
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	=	=
BEKK vs. GARCH-GJR	<	=
BEKK vs. QGARCH	=	>
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	<	=
BEKK-X vs. GARCH-GJR	<	=
BEKK-X vs. QGARCH	=	>
GARCH-GJR vs. QGARCH	=	>

Note: see note for table 5-30.

Table 5-32 MDM test of forecasted return from GARCH models with normal distribution for soybean from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for soybean		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	<	<
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	=	<
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	=	=
BEKK vs. QGARCH	=	=

GARCH-X vs. BEKK-X	>	>
GARCH-X vs. GARCH-GJR	=	>
GARCH-X vs. QGARCH	>	>
BEKK-X vs. GARCH-GJR	=	<
BEKK-X vs. QGARCH	>	=
GARCH-GJR vs. QGARCH	=	=

Note: see note for table 5-30.

Table 5-33 MDM test of forecasted return from six GARCH models with normal distribution for live cattle from 15/01/2007 to 14/01/2008

(M)DM test of forecasted return for live cattle		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	>	>
GARCH vs. GARCH-X	=	=
GARCH vs. BEKK-X	>	>
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	=	=
BEKK vs. GARCH-X	<	<
BEKK vs. BEKK-X	<	<
BEKK vs. GARCH-GJR	<	<
BEKK vs. QGARCH	<	<
GARCH-X vs. BEKK-X	>	>
GARCH-X vs. GARCH-GJR	=	=
GARCH-X vs. QGARCH	=	=
BEKK-X vs. GARCH-GJR	<	<
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	>	>

Note: see note for table 5-30.

Table 5-34 MDM test of forecasted return from six GARCH models with normal distribution for live hog from 15/01/2007 to 14/01/2008

(M)DM test of forecasted return for live hog		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	>	>
GARCH vs. X	>	>
GARCH vs. BEKK-X	<	<
GARCH vs. GJR	<	<
GARCH vs. QGARCH	<	<
BEKK vs. X	>	>
BEKK vs. BEKK-X	<	<
BEKK vs. GJR	<	<

BEKK vs. QGARCH	<	<
X vs. BEKK-X	<	<
X vs. GJR	<	<
X vs. QGARCH	<	<
BEKK-X vs. GJR	=	=
BEKK-X vs. QGARCH	<	<
GJR vs. QGARCH	<	<

Note: see note for table 5-30.

Table 5-35 Forecast error test on 2-year forecasted return for coffee in 2004-2005

Forecast error of return for coffee			
2004-2005-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00387036	0.000031	2.13834
BEKK	0.00168371	0.000010	2.10741
GARCH-X	0.00214089	0.000015	2.26416
BEKK-X	0.00180862	0.000011	2.03210
GARCH-GJR	0.00437994	0.000038	2.45961
QGARCH	0.00211404	0.000019	2.67699

Note: 2004-2005-N means that the OHR forecast is based on normally distributed residuals for two-year prediction in 2004-2005.

Table 5-36 Forecast error test on 2-year forecasted return for wheat in 2004-2005

Forecast error of return for wheat			
2004-2005-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00408781	0.000052	0.52248
BEKK	0.00370553	0.000045	0.47366
GARCH-X	0.00425549	0.000049	0.35688
BEKK-X	0.00526222	0.000066	0.61217
GARCH-GJR	0.00485352	0.000044	0.63147
QGARCH	0.00359048	0.000042	0.52681

Note: see note for table 5-35.

Table 5-37 Forecast error test on 2-year forecasted return for soybean in 2004-2005

Forecast error of return for soybean			
2004-2005-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00471120	0.000076	0.38548
BEKK	0.00434657	0.000055	0.29326
GARCH-X	0.00469721	0.000073	0.40628
BEKK-X	0.00458678	0.000058	0.32768
GARCH-GJR	0.00345021	0.000070	0.29297
QGARCH	0.00473419	0.000073	0.33289

Note: see note for table 5-35.

Table 5-38 Forecast error test on 2-year forecasted return for live cattle in 2006-2007

Forecast error of return for live cattle			
2006-2007-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.00613793	0.000084	0.49452
BEKK	0.00650794	0.000095	0.56340
GARCH-X	0.00662816	0.000098	0.51184
BEKK-X	0.00674707	0.000102	0.54817
GARCH-GJR	0.00616012	0.000085	0.49414
QGARCH	0.00644704	0.000093	0.51543

Note: 2006-2007-N means that the OHR forecast is based on normal distribution for two-year prediction in 2006-2007.

Table 5-39 Forecast error test on 2-year forecasted return for live hog in 2006-2007

Forecast error of return for live hog			
2006-2007-N	**MAE**	**MSE**	**Theil's U**
GARCH	0.01073423	0.000323	3.28201
BEKK	0.01061650	0.000316	3.33904
GARCH-X	0.01081173	0.000328	3.28583
BEKK-X	0.01026813	0.000296	3.10389
GARCH-GJR	0.01019829	0.000292	3.07023

Note: The QGARCH model cannot reach convergence on long-term OHR forecast for the case of live hog, and hence the forecasted return from OHR is not obtainable. Consequently, the forecast error test based on the QGARCH model does not exist.

Table 5-40 The MDM test of forecasted return from six GARCH models with normal distribution

for coffee from 24/03/2003 to 23/03/2005

(M)DM test of forecasted return for coffee		
distribution	Normal	
Measurement Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	<	<
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	>	>
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	>	>
BEKK vs. QGARCH	>	>
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	>	>
GARCH-X vs. QGARCH	>	=
BEKK-X vs. GARCH-GJR	>	>
BEKK-X vs. QGARCH	>	>
GARCH-GJR vs. QGARCH	<	<

Note: ‘**<**’ represents that the latter model has better forecast than the prior one
‘**>**’ represents that the prior model has better forecast than the latter one
‘**=**’ represents that the two models have equal predictive power

Table 5-41 The MDM test of forecasted return from six GARCH models with normal distribution for soybean from 24/03/2003 to 23/03/2005

(M)DM test of forecasted return for wheat		
distribution	Normal	
Measurement Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	=	>
GARCH vs. BEKK-X	>	>
GARCH vs. GARCH-GJR	=	>
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	=	>
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	=	>

BEKK vs. QGARCH	<	<
GARCH-X vs. BEKK-X	>	>
GARCH-X vs. GARCH-GJR	=	>
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	<	<
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	=	<

Note: see note in table 5-40.

Table 5-42 The MDM test of forecasted return from six GARCH models with normal distribution for soybean from 24/03/2003 to 23/03/2005

(M)DM test of forecasted return for soybean		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	=	=
GARCH vs. BEKK-X	<	=
GARCH vs. GARCH-GJR	=	<
GARCH vs. QGARCH	=	=
BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	=	<
BEKK vs. QGARCH	>	>
GARCH-X vs. BEKK-X	<	=
GARCH-X vs. GARCH-GJR	=	<
GARCH-X vs. QGARCH	=	=
BEKK-X vs. GARCH-GJR	=	<
BEKK-X vs. QGARCH	>	>
GARCH-GJR vs. QGARCH	=	>

Note: see note in table 5-40.

Table 5-43 The MDM test of forecasted return from six GARCH models with normal distribution for live cattle from 15/01/2005 to 14/01/2007

(M)DM test of forecasted return for live cattle		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	>	>
GARCH vs. GARCH-X	>	>
GARCH vs. BEKK-X	>	>
GARCH vs. GARCH-GJR	>	>
GARCH vs. QGARCH	>	>
BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	<	<

BEKK vs. QGARCH	<	<
GARCH-X vs. BEKK-X	>	>
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	<	<
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	>	>

Note: see note in table 5-40.

Table 5-44 The MDM test of forecasted return from six GARCH models with normal distribution for live hog from 15/01/2005 to 14/01/2007

(M)DM test of forecasted return for live hog		
distribution	Normal	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	>	>
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	<	<
BEKK vs. GARCH-GJR	<	<
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	<	<
BEKK-X vs. GARCH-GJR	<	<

Note: The QGARCH has convergence problem on the long-term OHR forecast, and hence the forecasted return from the QGARCH model is not available.

Table 5-45 Estimated coefficients from diagonal bivariate GARCH (1, 1) model with student's t distribution for full sample

$$vech(H_t) = C + A_1 vech(u_{t-1})^2 + B_1 vech(H_{t-1})$$

$$H_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{12} & 0 \\ 0 & 0 & a_{22} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{12} & 0 \\ 0 & 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

product / variables	coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	8.7e-06 (0.1020)	-1.3e-05 (-0.2790)	2.7e-04*** (1.9485)	1.7e-04** (2.4643)	1.2e-03* (7.3551)
φ_1	-7.1e-03* (-2.9068)	-6.3e-03* (-5.3243)	-0.0261* (-4.6003)	-3.1e-03* (-4.4346)	-0.0332* (-17.760)

μ_2	2.2e-04*** (1.8531)	-1.5e-04* (-3.0731)	4.1e-06 (0.0301)	3.5e-04* (2.8677)	4.5e-04** (2.4507)
φ_2	4.0e-03 (1.4478)	2.5e-03** (2.4957)	-1.9e-03 (-0.3464)	2.4e-03** (2.0001)	4.6e-03** (2.5532)
c_{11}	3.0e-06*** (1.8883)	2.1e-04* (45.508)	1.2e-05* (16.437)	1.5e-06* (14.976)	4.6e-06* (6.3152)
a_{11}	0.2569* (8.3792)	1.7e-03* (76.800)	0.1228* (23.696)	0.0661* (19.365)	0.2035* (16.284)
b_{11}	0.7235* (30.920)	0.1005* (6653.1)	0.8154* (140.43)	0.8726* (172.67)	0.8069* (83.332)
c_{22}	1.5e-05* (4.2888)	1.8e-04* (47.269)	1.3e-05* (15.912)	9.3e-07* (5.8684)	7.3e-07* (5.9227)
a_{22}	0.2188* (14.486)	1.7e-03* (76.598)	0.1400* (23.521)	0.0129* (9.5201)	3.0e-03* (10.683)
b_{22}	0.7327* (39.245)	0.1005* (6647.3)	0.7870* (109.57)	0.9744* (375.46)	0.9916* (1329.4)
c_{12}	1.9e-06** (2.5105)	1.5e-04* (43.601)	1.2e-05* (17.066)	3.6e-07* (3.8463)	4.4e-06* (5.3372)
a_{12}	0.2332* (11.515)	4.5e-04* (26.844)	0.1275* (25.142)	0.0128* (3.1224)	0.0307* (5.6134)
b_{12}	0.6914* (38.190)	0.0994* (5051.8)	0.8020* (135.02)	0.9027* (84.507)	0.8180* (37.883)
LLF	36775.67777	39523.99017	43940.20549	44681.12625	38083.69344

Note: *t*-statistic of coefficients in parentheses
'*' represents that it shows significance at 1% significant level
'**' represents that it shows significance at 5% significant level
'***' represents that it shows significance at 10% significant level
'LLF' is the value of log-likelihood function

Table 5-46 Basic statistics for estimated OHR from diagonal bivariate GARCH (1, 1) model with student's t distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.521004	0.137520	0.190756**	0.366137*	156.683*
wheat	0.862594	0.000100	888.5171*	-27.46950*	218559375*
Soybean	0.887959	0.044798	9.948756*	-1.239786*	29159.323*
Live cattle	0.056128	0.002634	126.5239*	9.139427*	4471013*
Live hog	0.149471	0.015908	106.6495*	5.890277*	3393406*

Note: see note of table 5-2.

Table 5-47 Stationary test for estimated OHR from diagonal bivariate GARCH (1, 1) model with student's t distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-18.26702	-13.45682	-10.16863	-8.54640
	2	-92.52662	-52.77133	-44.04587	-37.58027
Wheat	1	-57.18140	-48.22266	-32.75261	-26.61579
	2	-116.0570	-65.80026	-52.78599	-45.64985
Soybean	1	-17.13707	-15.74124	-14.54735	-12.83480
	2	-87.17568	-44.33958	-37.17941	-33.68395
Live cattle	1	-18.13089	-18.91103	-17.95321	-16.79592
	2	-68.67817	-41.40035	-32.76549	-28.27657
Live hog	1	-20.85328	-19.84432	-18.08492	-15.87200
	2	-80.76764	-49.28020	-39.83211	-33.45326

Note: Critical value for DF/ADF test: 1%=-3.434, 5%=-2.863, 10%=-2.567

Table 5-48 The Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from diagonal bivariate GARCH (1, 1) model with student's t distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	14.8996 (0.7821)	15.5636 (0.0294)	0.6299 (0.4274)	4.9105 (0.0267)
Wheat	519.988 (0.0000)	27.9587 (0.2173)	378.227 (0.0000)	8.5652 (0.0034)
Soybean	16.6904 (0.8242)	26.9158 (0.1066)	0.0598 (0.8068)	4.2260 (0.0398)

Live cattle	0.0913 (1.0000)	13.4162 (0.9424)	0.0504 (0.8223)	0.0036 (0.9525)
Live hog	2.6342 (0.9999)	12.6617 (0.9589)	0.5749 (0.4483)	0.0032 (0.9551)

Note: the Ljung-Box test is applied to test serial correlation with the joint null hypothesis of correlation coefficient are not significant from zero. The u_{t1} and u_{t2} are residuals from linear regression of $R_c^t = C + u_{t1}$ and $R_f^t = C + u_{t2}$; and the u_{t1}^2/h_t and u_{t2}^2/h_t are squared residuals. The p-values for Ljung-Box correlation test are in parentheses. The white statistic tests the reminding heteroscedasticity of residuals after estimating OHR with this model.

Table 5-49 Estimated coefficients from bivariate DBEKK-GARCH (1, 1) model with student's t distribution for full sample

$$H_t = CC' + A_1(u_{t-1}u'_{t-1})A'_1 + B_1 H_{t-1} B'_1$$
$$h_{11,t} = c_{11}^2 + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1}$$
$$h_{12,t} = c_{11}c_{12} + a_{11}a_{22}u_{1,t-1}u_{2,t-1} + b_{11}b_{22}h_{12,t-1}$$
$$h_{22,t} = c_{12}^2 + c_{22}^2 + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1}$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	-0.0001 (-0.6269)	-0.0001 (-0.3624)	0.0001 (0.7594)	0.0001 (1.3751)	-0.0001 (-0.5533)
φ_1	-0.0202** (-2.2971)	-0.0161 (-1.6297)	-0.1401* (-15.962)	0.1614* (12.281)	0.4253* (20.716)

μ_2	0.0001 (0.4555)	-0.0003*** (-1.7477)	-0.0001 (-0.1475)	0.0003** (2.4338)	0.0004*** (1.8171)
φ_2	-0.0449* (-5.0213)	-0.0176*** (-1.6623)	-0.1153* (-13.120)	0.0205 (1.5776)	0.0020 (0.1893)
c_{11}	0.0001 (1.4861)	0.0045* (35.149)	0.0029* (10.1305)	0.0012* (7.8799)	0.0011* (9.1051)
a_{11}	0.1971* (66.043)	0.9077* (268.04)	0.3109* (14.181)	0.2543* (15.7341)	0.4196* (15.994)
b_{11}	0.9740* (1559.6)	0.3123* (43.385)	0.9257* (86.203)	-0.9351* (-96.287)	-0.9167* (-96.577)
c_{22}	0.0008* (2.6000)	0.0015* (6.0529)	0.0009* (5.3941)	-0.0006* (-5.1782)	0.0001 (0.0218)
a_{22}	0.2008* (50.909)	0.3678* (42.230)	0.3533* (14.622)	0.0890* (12.227)	0.0564* (15.575)
b_{22}	0.9737* (1032.7)	0.8314* (42.230)	0.8993* (57.569)	-0.9930* (-821.80)	-0.9958* (-1344.1)
c_{12}	0.0004 (0.5497)	0.0060* (28.807)	0.0033* (8.7453)	0.0001* (3.2794)	0.0008* (6.5549)
LLF	37697.81812	40295.23271	43990.17610	44761.85578	38253.16250

Note: see note for table 5-45.

Table 5-50 Basic statistics for estimated OHR from bivariate DBEKK-GARCH (1, 1) model with student's t distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.581421	0.160734	-1.501020*	-0.114075*	630.736*
wheat	0.873973	0.042113	9.762161*	-1.661494*	29328.278*
Soybean	0.892745	0.038619	9.433394*	-1.163973*	26186.490*
Live cattle	0.057219	0.006810	61.024643*	5.512453*	1052079.938*
Live hog	0.123660	0.021281	65.866544*	5.239993*	1311116.287*

Note: see note of table 5-2.

Table 5-51 Stationary test for estimated OHR from bivariate DBEKK-GARCH (1, 1) model with student's t distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-3.57588	-3.51244	-3.26367	-3.26719
	2	-82.9988	-43.0585	-33.2426	-27.9116
Wheat	1	-23.5100	-20.5287	-18.2077	-15.9586
	2	-91.5773	-49.6764	-39.3257	-35.8520
Soybean	1	-16.1729	-15.0452	-14.1638	-12.6660
	2	-87.9965	-44.2657	-36.7967	-33.4038
Live cattle	1	-13.0926	-13.4685	-13.7254	-12.9259

	2	-78.0164	-44.4156	-34.7820	-30.0267
Live hog	1	-14.3373	-14.5461	-14.0022	-13.3923
	2	-79.2827	-46.0101	-35.8658	-30.1130

Note: Critical value for DF/ADF test: 1%=-3.434, 5%=-2.863, 10%=-2.567

Table 5-52 The Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from bivariate DBEKK-GARCH (1, 1) model with student's t distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	19.4020 (0.6777)	56.1577 (0.0001)	1.2079 (0.2717)	21.9701 (0.0001)
Wheat	32.0079 (0.0999)	10.7065 (0.9860)	10.325 (0.0013)	0.5317 (0.4659)
Soybean	11.7248 (0.9745)	22.0109 (0.2837)	1.8675 (0.1718)	1.7301 (0.1884)
Live cattle	0.1305 (1.0000)	10.4268 (0.9883)	0.0888 (0.7657)	0.0650 (0.7988)
Live hog	22.3263 (0.5006)	13.2855 (0.9455)	13.3728 (0.0002)	0.0001 (0.9908)

Note: see note in table 5-48.

Table 5-53 Estimated coefficients from diagonal bivariate GARCH (1, 1)-X model with student's t distribution for full sample

$$vech(H_t) = C + A_1 vech(u_{t-1})^2 + B_1 vech(H_{t-1}) + D_1 vech(z_{t-1})^2$$
$$h_{11,t} = c_{11} + a_{11}u_{1,t-1}^2 + b_{11}h_{11,t-1} + d_{11}z_{t-1}^2$$
$$h_{12,t} = c_{12} + a_{12}u_{1,t-1}u_{2,t-1} + b_{12}h_{12,t-1} + d_{12}z_{t-1}^2$$
$$h_{22,t} = c_{22} + a_{22}u_{2,t-1}^2 + b_{22}h_{22,t-1} + d_{22}z_{t-1}^2$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog

μ_1	8.8e-05 (0.9678)	0.0001 (0.1819)	2.7e-04*** (1.9502)	1.8e-04* (2.7058)	1.2e-03* (7.5436)
φ_1	-0.0118* (-5.6871)	-0.0087* (-3.8093)	-0.0263* (-4.4185)	-3.1e-03* (-4.8995)	-0.0337* (-18.034)
μ_2	2.7e-04** (1.9708)	-0.0003** (-1.9706)	5.5e-06 (0.0408)	3.5e-04* (3.1155)	4.9e-04* (2.6860)
φ_2	-8.8e-05 (-0.0350)	0.0005 (0.2001)	-2.0e-03 (-0.3482)	2.2e-03** (2.0755)	5.4e-03* (2.6111)
c_{11}	4.3e-06* (4.6055)	0.0001*** (1.6711)	1.2e-05* (14.897)	2.0e-06* (5.7545)	3.4e-06* (5.0030)
a_{11}	0.5968* (71.829)	0.1164* (3.4474)	0.1258* (23.464)	0.0660* (7.4861)	0.1954* (16.685)
b_{11}	0.5193* (73.343)	0.6957* (5.0574)	0.8072* (131.95)	0.8666* (58.678)	0.8193* (92.415)
d_{11}	1.8e-03* (9.5243)	0.0011 (1.5124)	1.4e-03** (2.0712)	-2.3e-05* (-3.5316)	-3.3e-06 (-0.0841)
c_{22}	2.8e-05* (8.8427)	0.0001* (3.0613)	1.3e-05* (14.726)	1.0e-06* (3.2436)	6.1e-07* (6.6422)
a_{22}	0.5735* (70.708)	0.1692* (4.5100)	0.1421* (23.428)	0.0126* (9.0900)	1.9e-03* (8.6450)
b_{22}	0.5062* (67.626)	0.5058* (3.9614)	0.7816* (106.39)	0.9744* (302.17)	0.9934* (1777.8)
d_{22}	1.7d-03* (11.643)	0.0011** (2.3218)	1.2e-03*** (1.7097)	-3.7e-06 (-0.6163)	8.7e-06* (5.1889)
c_{12}	4.7e-06* (205.23)	0.0001** (2.4252)	1.2e-05* (15.5577)	1.1e-05* (6.9018)	4.0e-08 (1.1074)
a_{12}	0.5823* (31166)	0.1369* (4.1782)	0.1301* (24.9463)	-1.4e-03 (-0.7698)	3.8e-03* (3.3817)
b_{12}	0.5149* (33166)	0.6046* (4.8054)	0.7954* (129.05)	-0.9638* (-49.228)	0.9935* (491.98)
d_{12}	1.5e-03* (43.157)	-0.0001 (0.8840)	1.0e-03 (1.6014)	-1.4e-04*** (-1.7464)	-2.3e-07 (-0.1222)
LLF	36875.27900	40402.70076	43948.04475	44685.12154	38132.07092

Note: see note for table 5-45.

Table 5-54 Basic statistics for estimated OHR from diagonal bivariate GARCH (1, 1)-X model with student's t distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.539106	0.204618	1.312852*	0.301344*	571.004*
wheat	0.867192	0.039044	10.83621*	-1.655230*	35347.116*
Soybean	0.889864	0.042991	9.837006*	-1.181204*	28388.683*
Live cattle	0.048606	0.002280	172.1147*	0.082966*	8104498.0*
Live hog	0.180712	0.016807	-0.611303*	0.152095*	137.41921*

Note: see note of table 5-2.

Table 5-55 Stationary test for estimated OHR from diagonal bivariate GARCH (1, 1)-X model

with student's t distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-23.7206	-16.0043	-11.3158	-9.31244
	2	-96.0563	-56.8428	-46.7521	-39.2581
Wheat	1	-28.5648	-22.3697	-17.7293	-14.8537
	2	-94.6830	-53.9073	-43.0414	-38.2381
Soybean	1	-17.6634	-16.1709	-14.9189	-13.1461
	2	-87.4332	-44.5360	-37.3666	-33.9049
Live cattle	1	-209.980	-11.1725	-8.52415	-7.68438
	2	-817.792	-68.3048	-44.6146	-35.4652
Live hog	1	-2.57432	-2.82284	-2.99430	-2.97512
	2	-72.2744	-40.7837	-30.1629	-24.9257

Note: Critical value for DF/ADF test: 1%=-3.434, 5%=-2.863, 10%=-2.567

Table 5-56 The Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from diagonal bivariate GARCH (1, 1)-X model with student's t distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	19.3422 (0.6820)	111.803 (0.0000)	2.2576 (0.1329)	14.3604 (0.0002)
Wheat	44.4337 (0.0047)	8.0431 (0.9983)	9.6916 (0.0019)	0.0486 (0.8255)
Soybean	17.3603 (0.7912)	28.9112 (0.0674)	0.0135 (0.9074)	4.8994 (0.0269)
Live cattle	0.0913 (1.0000)	13.1537 (0.9486)	0.0509 (0.8215)	0.0015 (0.9689)
Live hog	2.9826 (0.9999)	13.3896 (0.9430)	0.7150 (0.3978)	0.0071 (0.9328)

Note: see note for table 5-48.

Table 5-57 Estimated coefficients from bivariate diagonal BEKK-GARCH-X (1, 1) model with student's t distribution for full sample

$$H_t = CC' + A_1(u_{t-1}u'_{t-1})A'_1 + B_1 H_{t-1} B'_1 + D_1 Z_{t-1}^2 D'_1$$
$$h_{11,t} = c_{11}^2 + a_{11}^2 u_{11,t-1}^2 + b_{11}^2 h_{11,t-1} + d_{11}^2 z_{t-1}^2$$
$$h_{12,t} = c_{11}c_{12} + a_{11}a_{22}u_{1,t-1}u_{2,t-1} + b_{11}b_{22}h_{12,t-1} + d_{11}d_{22}z_{t-1}^2$$
$$h_{22,t} = c_{12}^2 + c_{22}^2 + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} + (d_{12}^2 + d_{22}^2) z_{t-1}^2$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog

μ_1	0.0001** (2.2346)	0.0001 (0.2183)	0.0003** (2.2375)	0.0002* (2.5643)	0.0011* (5.5102)
φ_1	-0.0088* (-6.1424)	-0.0090* (-3.1545)	-0.0275* (-5.3538)	-0.0031* (-5.6653)	-0.0332 (-13.281)
μ_2	0.0003* (3.2527)	-0.0003*** (-1.6568)	-0.0001 (-0.0661)	0.0003* (2.6824)	0.0005* (2.5722)
φ_2	0.0038* (2.3100)	-0.0006 (0.2220)	-0.0022 (-0.4508)	0.0025*** (1.7795)	0.0047* (3.0439)
(vc11) c_{11}	0.0001 (0.2403)	0.0060* (33.839)	0.0030* (10.924)	0.0012* (10.386)	0.0020* (8.9078)
(VB11) a_{11}	0.1976* (8.1499)	0.3260* (38.039)	0.3190* (14.280)	0.2531* (14.7219)	0.4238* (15.157)
(VA11) b_{11}	0.9738* (160.26)	0.8559* (136.69)	0.9204* (83.830)	0.9352* (118.50)	0.9084* (78.494)
d_{11}	0.0025 (0.7519)	0.0304* (9.4253)	0.0283* (2.9128)	0.0001 (0.1111)	0.0084* (3.1609)
(vc22) c_{22}	0.0006 (0.3751)	0.0017* (5.2143)	0.0009* (5.88110	0.0006* (4.7472)	-0.0001 (-0.0115)
(VB22) a_{22}	0.2025* (8.2118)	0.3927* (39.599)	0.3582* (14.769)	0.0900* (16.355)	0.0496* (10.576)
(VA22) b_{22}	0.9733* (153.82)	0.7364* (66.632)	0.8963* (58.746)	0.9929* (960.64)	0.9962* (1483.0)
d_{22}	-0.0001 (-0.0050)	0.0308* (7.3863)	0.0200* (3.2730)	-0.0005 (-0.2228)	0.0001 (0.0118)
(vc12) c_{12}	0.0006 (0.3958)	0.0075* (28.462)	0.0033* (8.8485)	0.0001* (-3.4396)	0.0008* (7.3083)
d_{12}	0.0040 (1.4062)	-0.0003 (-0.0409)	0.0229*** (1.8491)	-0.0005 (-0.2195)	0.0028* (4.2378)
LLF	37724.37248	40405.15448	43932.75068	44688.17122	38107.67408

Note: see note for table 5-45.

Table 5-58 Basic statistics for estimated OHR from bivariate diagonal BEKK-GARCH-X (1, 1) model with student's t distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.581489	0.159719	-1.512973*	-0.115620*	640.983*
wheat	0.867140	0.038618	10.39713*	-1.659138*	32849.867*
Soybean	0.886552	0.042892	9.509225*	-1.159426*	26573.203*
Live cattle	0.064348	0.007774	78.83799*	6.369228*	1744831.7*
Live hog	0.143037	0.014148	58.75159*	4.518248*	1041471.8*

Note: see note of table 5-2.

Table 5-59 Stationary test for estimated OHR from bivariate diagonal BEKK-GARCH-X (1, 1) model with student's t distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-3.605510	-3.552110	-3.300240	-3.309380
	2	-82.44769	-43.12228	-33.19641	-27.91067
Wheat	1	-26.58361	-21.28389	-17.18509	-14.47845
	2	-94.42433	-53.58279	-42.83360	-38.23621
Soybean	1	-15.93133	-15.04718	-14.03101	-12.48829
	2	-86.80486	-44.32818	-36.98463	-33.63566
Live cattle	1	-12.63860	-13.63997	-13.88815	-13.12435
	2	-75.21349	-43.85920	-34.71731	-29.81775
Live hog	1	-14.29119	-14.67787	-14.29495	-13.29590
	2	-78.14003	-46.00102	-36.37124	-30.36332

Note: Critical value for DF/ADF test: 1%=-3.434, 5%=-2.863, 10%=-2.567

Table 5-60 The Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from diagonal BEKK-GARCH-X (1, 1) model with student's t distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	18.2658 (0.7429)	55.4528 (0.0002)	1.2812 (0.2577)	21.3898 (0.0001)
Wheat	151.593 (0.0000)	7.9841 (0.9984)	105.902 (0.0000)	0.0007 (0.9796)
Soybean	13.4701 (0.9410)	24.0580 (0.1939)	3.1280 (0.0770)	3.0738 (0.0796)
Live cattle	0.0920 (1.0000)	10.5080 (0.9877)	0.0521 (0.8195)	0.0783 (0.7796)
Live hog	2.8894 (0.9999)	14.1486 (0.9225)	0.8197 (0.3653)	0.0005 (0.9823)

Note: see note for table 5-48.

Table 5-61 Estimated coefficients from bivariate diagonal GARCH-GJR (1, 1) model with student's t distribution for full sample

$$\sigma_t^2 = c + \sum_{i=1,j=1}^{2}(a_{ij} + \gamma_i I_{t-1})u_{t-1}^2 + \sum_{i=1,j=1}^{2} b_{ij}\sigma_{t-1}^2$$

$$H_t = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} + \begin{bmatrix} a_{11}+\gamma_1 I_{t-1} & 0 & 0 \\ 0 & a_{12} & 0 \\ 0 & 0 & a_{22}+\gamma_2 I_{t-1} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{12} & 0 \\ 0 & 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}$$

product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog
μ_1	-3.3e-05 (-0.1978)	6.2e-05 (0.4074)	4.4e-04* (13.095)	9.1e-05 (1.5072)	1.1e-03* (6.9093)
φ_1	-0.0107* (-4.1793)	-0.0112* (-5.1977)	-0.0309* (-8.9029)	-3.4e-03* (-5.4871)	-0.0339* (-17.747)
μ_2	3.0e-04 (1.6164)	-1.3e-04 (-0.9639)	2.0e-04* (6.1508)	3.3e-04* (3.0649)	5.0e-04* (2.7144)
φ_2	3.1e-03 (1.1304)	-2.5e-03 (-1.1523)	-6.1e-03** (-1.9587)	2.6e-03** (2.3944)	4.3e-03** (2.3954)
$(d_0)\ c_{11}$	2.5e-04* (24.881)	1.9e-04* (36.650)	1.2e-05* (17.479)	2.9e-05* (22.856)	3.4e-06* (5.5581)
$(d_1)\,a_{11}$	0.2053* (441.33)	0.1003* (1691.8)	0.1175* (8.6966)	0.1232* (4.0152)	0.1680* (12.879)
$(d_2)\,b_{11}$	5.2e-03* (333.52)	0.0502* (1209.1)	0.8098* (55.077)	0.0478* (328.76)	0.8238* (94.435)
$(d_3)\,\gamma_1$	7.5e-03* (1441.2)	0.0388* (2.6696)	0.0161* (0.0007)	2.2e-03 (0.0554)	0.0397** (2.4579)
$(e_0)\,c_{22}$	3.0e-04* (36.072)	1.7e-04* (37.681)	1.4e-05* (3.3985)	8.9e-05* (27.632)	7.6e-07* (5.8338)
$(e_1)\,a_{22}$	0.2051* (452.36)	0.1003* (1627.6)	0.1491* (12.109)	0.0550** (2.0024)	2.1e-03* (5.3756)
$(e_2)\,b_{22}$	5.2e-03* (336.05)	0.0502* (1215.2)	0.7854* (70.704)	0.0476* (298.82)	0.9915* (1276.7)
$(e_3)\,\gamma_2$	-0.0240* (-13.537)	-2.7e-03 (-0.3277)	-0.0172* (-3.1116)	0.0498 (1.0556)	1.6e-03* (3.0007)
$(f_0)\,c_{12}$	2.2e-04* (462.14)	1.5e-04* (33.741)	1.2e-05* (103.66)	6.5e-06* (8.2006)	4.6e-08 (1.2839)
$(f_1)\,a_{12}$	0.1990* (462.14)	0.0999* (4134.9)	0.1291* (10.6186)	0.0161 (1.4029)	4.2e-03* (3.5345)
$(f_2)\,b_{12}$	4.9e-03* (414.45)	0.0499* (3405.7)	0.7984* (70.000)	0.0508* (893.23)	0.0029* (468.06)
LLF	35570.81685	39931.08290	43946.61250	44386.90968	38129.14834

Note: see note for table 5-45.

Table 5-62 Basic statistics for estimated OHR from bivariate diagonal GARCH-GJR (1, 1) model with student's t distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.720784	0.027160	27.045556*	0.734416*	200736.7*
wheat	0.885137	0.011867	121.48291	-3.319316	4082315 *
Soybean	0.890730	0.045578	9.918846*	-1.178045*	28828.90*
Live cattle	0.068865	0.000287	478.8451*	13.28310*	6292372*
Live hog	0.182456	0.016610	-0.11373***	0.258508*	82.60051*

Note: see note for table 5-2.

Table 5-63 Stationary test for estimated OHR from bivariate diagonal GARCH-GJR (1, 1) model with student's t distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-64.25785	-27.09843	-18.66617	-14.83528
	2	-136.6073	-68.21613	-52.52602	-42.71468
Wheat	1	-70.14377	-36.45876	-26.77550	-22.35832
	2	-133.8558	-69.00796	-51.83007	-43.26080
Soybean	1	-17.36479	-15.93083	-14.68863	-12.92057
	2	-87.10607	-44.41946	-37.25454	-33.80349
Live cattle	1	-62.42860	-37.82929	-28.74070	-24.20235
	2	-116.8596	-67.55901	-51.17530	-42.30591
Live hog	1	-3.04554	-3.36107	-3.55629	-3.54721
	2	-75.57055	-40.82875	-30.14020	-25.13816

Note: Critical value for DF/ADF test: 1%=-3.434, 5%=-2.863, 10%=-2.567

Table 5-64 The Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from bivariate diagonal GARCH-GJR (1, 1) model with student's t distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	551.321 (0.0000)	620.254 (0.0000)	0.3460 (0.5564)	0.8564 (0.3547)
Wheat	170.921 (0.0000)	24.9281 (0.3540)	5.3589 (0.0206)	2.7708 (0.0960)
Soybean	17.1120 (0.8037)	24.1931 (0.1142)	0.0062 (0.9377)	4.0983 (0.0429)
Live cattle	0.0708 (1.0000)	9.8816 (0.9920)	0.0288 (0.8653)	1.7364 (0.1876)
Live hog	2.8317 (0.9999)	13.1873 (0.9478)	0.6413 (0.4233)	0.0003 (0.9852)

Note: see note for table 5-48.

Table 5-65 Estimated coefficients from bivariate diagonal QGARCH (1, 1) model with student's t distribution for full sample

$$vech(H_t) = C + A_i vech(u_{t-1}^2) + B_i vech(H_{t-1}) + D_i vech(u_{t-1})$$

$$h_{11,t} = c_{11} + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1} + d_{11} u_{1,t-1}$$

$$h_{12,t} = c_{12} + a_{12} u_{1,t-1} u_{2,t-1} + b_{12} h_{12,t-1}$$

$$h_{22,t} = c_{22} + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} + d_{22} u_{2,t-1}$$

Product / variables	Coffee	Wheat	Soybean	Live cattle	Live hog

μ_1	7.4e-05 (1.0025)	-1.1e-05* (-19.962)	2.4e-04 (1.5073)	1.9e-04* (2.6554)	1.1e-03* (0.0000)
φ_1	-6.9e-03* (-3.1541)	-3.5e-04* (-13.422)	-0.0282* (-3.7316)	-3.4e-03* (-5.0004)	-0.0341* (-15.215)
μ_2	1.4e-04 (0.8999)	-5.0e-05* (-130.37)	2.0e-05 (0.1340)	3.2e-04* (2.5984)	3.7e-04* (3.0703)
φ_2	4.7e-03 (1.6096)	3.3e-04* (18.688)	-3.5e-03 (-0.5155)	2.5e-03** (2.2583)	1.5e-03 (0.9436)
$(d_0)\ c_{11}$	9.9e-07** (2.1537)	3.3e-04* (393.03)	1.2e-05* (3.1178)	1.6e-068 (15.171)	5.4e-06* (5.6539)
$(d_1)\,a_{11}$	0.0986* (5.9413)	0.1002* (10709)	0.1262* (4.6101)	0.0652* (19.117)	0.2024* (8.3668)
$(d_2)\,b_{11}$	0.8114* (42.169)	0.03808 (31610)	0.8091* (20.117)	0.8724* (172.81)	0.8037* (39.619)
$(d_3)\,d_{11}$	2.2e-04 (1.1289)	3.9e-06* (23.532)	-1.5e-04** (-2.2956)	-1.1e-04* (-3.6436)	-4.4e-04* (-3.7364)
$(e_0)\,c_{22}$	1.3e-05* (4.5590)	2.8e-04* (346.86)	1.4e-05* (4.2869)	8.2e-078 (5.9267)	4.0e-07** (2.2398)
$(e_1)\,a_{22}$	0.1627* (9.7276)	0.1002* (10663)	0.1408* (6.1757)	0.01148 (8.9899)	2.0e-03* (5.0522)
$(e_2)\,b_{22}$	0.7819* (37.194)	0.0380* (31596)	0.7841* (24.756)	0.9769* (418.29)	0.9943* (849.49)
$(e_3)\,d_{22}$	-7.6e-04* (-2.7861)	2.9e-06* (23.830)	1.6e-04*** (1.6588)	-1.4e-04* (-4.1897)	-2.3e-04* (-5.8384)
$(f_0)\,c_{12}$	-1.6e-06** (2.4784)	2.7e-04* (464.78)	1.2e-05* (3.7772)	5.8e-08 (1.5298)	3.0e-05* (12.514)
$(f_1)\,a_{12}$	0.1149* (7.5069)	0.1000* (49053)	0.1295* (5.3922)	6.0e-03** (2.2465)	0.02918 (4.3182)
$(f_2)\,b_{12}$	0.7706* (40.641)	0.0379* (10082)	0.7974* (22.952)	0.9764* (96.9062)	-0.0533* (-384.08)
LLF	29995.23857	39632.19927	43946.85716	44695.86894	38104.77121

Note see note for table 45.

Table 5-66 Basic statistics for estimated OHR from bivariate diagonal QGARCH (1, 1) model with student's t distribution

GARCH	mean	variance	kurtosis	skewness	Jarque-Bera
coffee	0.716458	0.026926	27.430448*	0.700346*	206420.35*
wheat	0.949184	0.007029	166.27446*	-5.241485*	7655173.8*
Soybean	0.890168	0.045221	9.822220*	-1.209441*	29382.949*
Live cattle	0.055738	0.001848	17.197753*	2.978045*	90621.117*
Live hog	0.133458	0.004491	122.91700*	3.677729*	4469203.5*

Note: see note of table5-2.

Table 5-67 Stationary test for estimated OHR from bivariate diagonal QGARCH (1, 1) model with student's t distribution

Test / Commodity	Number of unit roots	DF test	ADF(lags=3)	ADF(lags=6)	ADF(lags=9)
Coffee	1	-13.71077	-10.98206	-8.62915	-7.31688
	2	-86.61627	-51.25549	-41.26928	-36.35300
Wheat	1	-70.41498	-36.90448	-27.44931	-23.04616
	2	-133.2285	-68.69261	-51.67880	-43.41380
Soybean	1	-17.42633	-15.92820	-14.70831	-12.92501
	2	-87.29126	-44.52086	-37.22563	-33.78432
Live cattle	1	-6.370790	-7.224490	-7.470090	-7.165510
	2	-68.76601	-39.63029	-39.55132	-26.11706
Live hog	1	-52.28276	-23.23068	-15.22852	-11.21611
	2	-138.01995	-71.12877	-52.67573	-44.56817

Note: Critical value for DF/ADF test: 1%=-3.434, 5%=-2.863, 10%=-2.567

Table 5-68 The Ljung-Box correlation and White heteroscedasticity tests for residuals after estimating OHR from bivariate diagonal QGARCH (1, 1) model with student's t distribution

Test / commodity	Ljung-Box Q test		White heteroscedasiticity	
	u_{t1}^2/h_t	u_{t2}^2/h_t	u_{t1}^2/h_t	u_{t2}^2/h_t
Coffee	10.9855 (0.9466)	12.0243 (0.0998)	0.0037 (0.9514)	0.0033 (0.9542)
Wheat	234.896 (0.0000)	28.1892 (0.2094)	37.6030 (0.0000)	5.0700 (0.0243)
Soybean	17.6526 (0.7761)	27.8484 (0.0864)	0.0161 (0.8990)	4.3017 (0.0381)
Live cattle	0.0933 (1.0000)	12.9935 (0.9521)	0.0537 (0.8168)	0.0185 (0.8917)
Live hog	2.4223 (0.9999)	14.9008 (0.8982)	0.5278 (0.4675)	0.0133 (0.9082)

Note: see note for table 5-48.

Table 5-69 Forecast error test on 1-year forecasted return for coffee in 2006

Forecast error of return for coffee			
2006-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00975636	0.000208	2.47771
BEKK	0.00772309	0.000182	1.82180
GARCH-X	0.00842883	0.000203	2.36511
BEKK-X	0.00771350	0.000183	1.82526
GARCH-GJR	0.00941974	0.000200	2.36746
QGARCH	0.00924566	0.000197	2.38602

Note: see note for table 5-25.

Table 5-70 Forecast error test on 1-year forecasted return for wheat in 2006

Forecast error of return for wheat			
2006-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00184559	0.000009	0.37626
BEKK	0.00180001	0.000009	0.35928
GARCH-X	0.00197447	0.000012	0.40880
BEKK-X	0.00221919	0.000016	0.39057
GARCH-GJR	0.00170918	0.000008	0.62578
QGARCH	0.00080619	0.000001	0.17998

Note: see note for table 5-25.

Table 5-71 Forecast error test on 1-year forecasted return for soybean in 2006

Forecast error of return for soybean			
2006-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00083103	0.000002	0.33313
BEKK	0.00077752	0.000002	0.27611
GARCH-X	0.00078876	0.000002	0.28827
BEKK-X	0.00080451	0.000002	0.44256
GARCH-GJR	0.00096563	0.000003	0.29067
QGARCH	0.00089951	0.000003	0.34328

Note: see note for table 5-25.

Table 5-72 Forecast error test on 1-year forecasted return for live cattle in 2008

Forecast error of return for live cattle			
2006-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00632000	0.000081	1.62736
BEKK	0.00589464	0.000096	1.67909
GARCH-X	0.00623809	0.000079	1.56305
BEKK-X	0.00732132	0.000081	1.66651
GARCH-GJR	0.00638630	0.000082	1.60227
QGARCH	0.00613510	0.000077	1.61877

Note: see note for table 5-25.

Table 5-73 Forecast error test on 1-year forecasted return for live hog in 2008

Forecast error of return for live hog			
2006-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00843285	0.000223	0.71491
BEKK	0.00913300	0.000263	0.78043
GARCH-X	0.00941428	0.000281	0.81104
BEKK-X	0.00872953	0.000240	0.73590
GARCH-GJR	0.00939004	0.000279	0.80986
QGARCH	0.00830406	0.000215	0.69841

Note: see note for table 5-25.

Table 5-74 The MDM test of forecasted return from six GARCH models with student's t distribution for coffee from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for coffee		
distribution	Student's *t*	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	=	<
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	=	>

BEKK vs. BEKK-X	=	=
BEKK vs. GARCH-GJR	>	>
BEKK vs. QGARCH	>	>
GARCH-X vs. BEKK-X	=	<
GARCH-X vs. GARCH-GJR	=	>
GARCH-X vs. QGARCH	=	>
BEKK-X vs. GARCH-GJR	>	>
BEKK-X vs. QGARCH	>	>
GARCH-GJR vs. QGARCH	<	<

Note: ‘<’ represents that the latter model has better forecast than the prior one
‘>’ represents that the prior model has better forecast than the latter one
‘=’ represents that the two models have equal predictive power

Table 5-75 The MDM test of forecasted return from six GARCH models with student’s t distribution for wheat from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for wheat		
distribution	Student’s *t*	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	=	=
GARCH vs. GARCH-X	>	=
GARCH vs. BEKK-X	>	>
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	=	=
BEKK vs. QGARCH	<	<
GARCH-X vs. BEKK-X	>	>
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	<	<
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	<	<

Note: see note for table 5-74.

Table 5-76 The MDM test of forecasted return from six GARCH models with student’s t distribution for soybean from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for soybean		
distribution	Student’s *t*	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	<	<
GARCH vs. BEKK-X	<	=
GARCH vs. GARCH-GJR	=	>
GARCH vs. QGARCH	>	>
BEKK vs. GARCH-X	=	=

BEKK vs. BEKK-X	>	=
BEKK vs. GARCH-GJR	>	>
BEKK vs. QGARCH	>	>
GARCH-X vs. BEKK-X	=	=
GARCH-X vs. GARCH-GJR	=	>
GARCH-X vs. QGARCH	>	>
BEKK-X vs. GARCH-GJR	=	>
BEKK-X vs. QGARCH	>	>
GARCH-GJR vs. QGARCH	=	=

Note: see note for table 5-74.

Table 5-77 The MDM test of forecasted return from six GARCH models with student's t distribution for live cattle from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for live cattle		
distribution	Student's *t*	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	>	>
GARCH vs. GARCH-X	=	<
GARCH vs. BEKK-X	=	=
GARCH vs. GARCH-GJR	>	>
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	<	<
BEKK vs. BEKK-X	<	<
BEKK vs. GARCH-GJR	<	<
BEKK vs. QGARCH	<	<
GARCH-X vs. BEKK-X	>	>
GARCH-X vs. GARCH-GJR	>	>
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	>	>
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	<	<

Note: see note for table 5-74.

Table 5-78 The MDM test of forecasted return from six GARCH models with student's t distribution for live hog from 24/03/2005 to 23/03/2006

(M)DM test of forecasted return for live hog		
distribution	Student's *t*	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	>	>
GARCH vs. GARCH-X	>	>
GARCH vs. BEKK-X	>	>
GARCH vs. GARCH-GJR	>	>
GARCH vs. QGARCH	<	<
BEKK vs. GARCH-X	>	>

BEKK vs. BEKK-X	<	<
BEKK vs. GARCH-GJR	>	>
BEKK vs. QGARCH	<	<
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	>	>
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	<	<

Note: see note for table 5-74.

Table 5-79 Forecast error test on 2-year forecasted return for coffee in 2004-2005

Forecast error of return for coffee			
2004-2005-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00384626	0.000037	1.16391
BEKK	0.00174876	0.000010	1.91038
GARCH-X	0.00283722	0.000032	0.56441
BEKK-X	0.00175792	0.000010	1.85396
GARCH-GJR	0.00215180	0.000022	0.41405
QGARCH	0.00390974	0.000038	1.47750

Note: 2004-2005-T means that the OHR forecast is based on student's t distribution for two-year prediction in 2004-2005.

Table 5-80 Forecast error test on 2-year forecasted return for wheat in 2004-2005

Forecast error of return for wheat			
2004-2005-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00316407	0.000040	0.45033
BEKK	0.00277728	0.000031	0.40487
BEKK-X	0.00291709	0.000034	0.42567
GARCH-GJR	0.00207157	0.000011	0.27122
QGARCH	0.00282611	0.000032	0.42988

Note: see note for table 5-79; additionally, the GARCH-X model cannot reach convergence for the case of wheat, and thus there is no output for GARCH-X model.

Table 5-81 Forecast error test on 2-year forecasted return for soybean in 2004-2005

Forecast error of return for soybean			
2004-2005-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00413635	0.000068	0.29208
BEKK	0.00402091	0.000062	0.27744
GARCH-X	0.00418689	0.000065	0.31020
BEKK-X	0.00415329	0.000064	0.30166
GARCH-GJR	0.00370366	0.000070	0.34970
QGARCH	0.00424311	0.000066	0.27221

Note: see note for table 5-79.

Table 5-82 Forecast error test on 2-year forecasted return for live cattle in 2006-2007

Forecast error of return for live cattle			
2006-2007-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.00631774	0.000089	0.50907
BEKK	0.00640080	0.000095	0.53359
GARCH-X	0.00657540	0.000097	0.50916
BEKK-X	0.00632725	0.000093	0.52986
GARCH-GJR	0.00647540	0.000093	0.52394
QGARCH	0.00650396	0.000094	0.52208

Note: 2006-2007-T means that the OHR forecast is based on student's t distribution for two-year prediction in 2006-2007.

Table 5-83 Forecast error test on 2-year forecasted return for live hog in 2006-2007

Forecast error of return for live hog			
2006-2007-T	**MAE**	**MSE**	**Theil's U**
GARCH	0.01060874	0.000314	3.29306
BEKK	0.01031265	0.000296	3.39084
GARCH-X	0.01062552	0.000316	3.29649
BEKK-X	0.00993482	0.000274	3.22206
GARCH-GJR	0.00943846	0.000248	2.81585
QGARCH	0.00746218	0.000154	2.15872

Note: see note for table 5-82.

Table 5-84 The MDM test of forecasted return from six GARCH models with student's t distribution for coffee from 24/03/2003 to 23/03/2005

(M)DM test of forecasted return for coffee		
distribution	Student's t	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	<	<
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	=	>

BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	=	=
BEKK vs. GARCH-GJR	>	>
BEKK vs. QGARCH	>	>
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	>	>
BEKK-X vs. GARCH-GJR	>	>
BEKK-X vs. QGARCH	>	>
GARCH-GJR vs. QGARCH	>	>

Note: '<' represents that the latter model has better forecast than the prior one
'>' represents that the prior model has better forecast than the latter one
'=' represents that the two models have equal predictive power

Table 5-85 The MDM test of forecasted return from six GARCH models with student's t distribution for soybean from 24/03/2003 to 23/03/2005

(M)DM test of forecasted return for wheat		
distribution	Student's t	
Measurement Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	<	<
BEKK vs. BEKK-X	>	>
BEKK vs. GARCH-GJR	>	>
BEKK vs. QGARCH	=	>
BEKK-X vs. GARCH-GJR	<	<
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	>	>

Note: see note for table 5-84. The GARCH-X model cannot reach convergence for the case of wheat, and thus there is no output for GARCH-X model.

Table 5-86 The MDM test of forecasted return from six GARCH models with student's t distribution for soybean from 24/03/2003 to 23/03/2005

(M)DM test of forecasted return for soybean		
distribution	Student's t	
Measurement Models	MSE	MAE
GARCH vs. BEKK	=	=
GARCH vs. GARCH-X	=	=
GARCH vs. BEKK-X	=	=
GARCH vs. GARCH-GJR	=	<
GARCH vs. QGARCH	=	=

BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	=	>
BEKK vs. GARCH-GJR	>	<
BEKK vs. QGARCH	>	>
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	=	<
GARCH-X vs. QGARCH	=	>
BEKK-X vs. GARCH-GJR	=	<
BEKK-X vs. QGARCH	>	>
GARCH-GJR vs. QGARCH	=	>

Note: see note for table 5-84.

Table 5-87 The MDM test of forecasted return from six GARCH models with student's t distribution for live cattle from 15/01/2005 to 14/01/2007

(M)DM test of forecasted return for live cattle		
distribution	Student's t	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	=	=
GARCH vs. GARCH-X	>	>
GARCH vs. BEKK-X	=	=
GARCH vs. GARCH-GJR	>	>
GARCH vs. QGARCH	>	>
BEKK vs. GARCH-X	=	>
BEKK vs. BEKK-X	<	<
BEKK vs. GARCH-GJR	=	=
BEKK vs. QGARCH	=	=
GARCH-X vs. BEKK-X	=	<
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	=	>
BEKK-X vs. QGARCH	=	>
GARCH-GJR vs. QGARCH	>	>

Note: see note for table 5-84.

Table 5-88 The MDM test of forecasted return from six GARCH models with student's *t* distribution for live hog from 15/01/2005 to 14/01/2007

(M)DM test of forecasted return for live hog		
distribution	Student's t	
Measurement / Models	MSE	MAE
GARCH vs. BEKK	<	<
GARCH vs. GARCH-X	>	>
GARCH vs. BEKK-X	<	<
GARCH vs. GARCH-GJR	<	<
GARCH vs. QGARCH	<	<

BEKK vs. GARCH-X	>	>
BEKK vs. BEKK-X	<	<
BEKK vs. GARCH-GJR	<	<
BEKK vs. QGARCH	<	<
GARCH-X vs. BEKK-X	<	<
GARCH-X vs. GARCH-GJR	<	<
GARCH-X vs. QGARCH	<	<
BEKK-X vs. GARCH-GJR	<	<
BEKK-X vs. QGARCH	<	<
GARCH-GJR vs. QGARCH	<	<

Note: see note for table 5-84.

Table 5-89 The 1-year out-of-sample forecast for coffee with normal distribution (return, risk, variance comparison)

1-year forecast for coffee with normal distribution							
Hedging strategy		**Unhedged position**					
Return		-3.20285e-04					
risk		0.02346					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	1.1683e-05(128)	-3.8676e-04 (128)	-8.8482e-04(128)	-8.8482e-04(128)	-0.0099(128)	1.1129e-04 (128)
	Risk	0.01040	0.01040	0.01040	0.01040	0.01040	0.01040
	V. R.	-408.8%	-408.8%	-408.9%	-409.1%	-408.5%	-408.7%
BEKK	Return	1.6537e-04(122)	-2.3307e-04 (122)	-7.3113e-04(122)	-0.0047 (122)	-0.0097(122)	2.6498e-04 (122)
	Risk	0.00961	0.00961	0.00961	0.00961	0.00961	0.00961
	V. R.	-495.6%	-495.7%	-495.9%	-496.5%	-496.2%	-495.6%
GARCH-X	Return	1.4872e-04(128)	-2.4972e-04 (128)	-7.4777e-04(128)	-0.0047 (128)	-0.0097(128)	2.4834e-04 (128)
	Risk	0.00973	0.00973	0.00973	0.00973	0.00973	0.00974
	V. R.	-480.9%	-481.0%	-481.1%	481.7%	-481.2%	-480.9%
BEKK-X	Return	1.8758e-04(124)	-2.1086e-04 (124)	-7.0892e-04(124)	-0.0047 (124)	-0.0097(124)	2.8719e-04 (124)
	Risk	0.00965	0.00965	0.00965	0.00964	0.00964	0.00965
	V. R.	-491.4%	-491.5%	-491.7%	-492.3%	-491.9%	-491.4%
GJR	Return	1.5693e-04(124)	-2.4151e-04 (124)	-7.3957e-04(124)	-0.0047 (124)	-0.0097(124)	2.5654e-04 (124)
	Risk	0.00970	0.00970	0.00970	0.00970	0.00970	0.00970
	V. R.	-484.6%	-484.7%	-484.8%	-485.4%	-485.0%	-484.6%
QGARCH	Return	1.7796e-04(125)	-2.2048e-04 (125)	-7.1854e-04(125)	-0.0047 (125)	-0.0097(125)	2.7757e-04 (125)
	Risk	0.00969	0.00969	0.00969	0.00968	0.00969	0.00969
	V. R.	-486.5%	-486.6%	-486.7%	487.3%	-486.9%	-486.4%

Note: all ‘return’ and ‘risk’ are average log-return and average risk (standard deviation of average log-return) of the portfolio, and the number of rebalance in bracket; the ‘ V.R.’ represents the variance reduction of portfolio from GARCH-type models to unhedged position; the rates of ‘transaction costs’ are rates of return reduction of portfolio.

Table 5-90 The 1-year out-of-sample forecast for coffee with student's t distribution (return, risk, variance comparison)

1-year forecast for coffee with normal distribution							
Hedging strategy		**Unhedged position**					
Return		-3.20285e-04					
risk		0.02346					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	-1.7319e-05(128)	-4.1576e-04 (128)	-9.1382e-04(128)	-0.0049(128)	-0.0099 (128)	8.2292e-05 (128)
	Risk	0.01088	0.01088	0.01088	0.01088	0.01088	0.01088
	V. R.	-364.8%	-364.8%	-364.9%	-365.1%	-364.6%	-364.7%
BEKK	Return	1.8796e-04(125)	-2.1049e-04 (125)	-7.0854e-04(125)	-0.0047(125)	-0.0097 (125)	2.8757e-04 (125)
	Risk	0.00966	0.00966	0.00965	0.00965	0.00965	0.00966
	V. R.	**-490.5%**	-490.6%	-490.7%	-491.3%	-491.0%	-490.4%
GARCH-X	Return	7.27723e-05(130)	-3.2567e-04 (130)	-8.2373e-04(130)	-0.0048(130)	-0.0098(130)	1.7238e-04(130)
	Risk	0.01093	0.01093	0.01093	0.01093	0.01093	0.01094
	V. R.	-360.4%	-360.5%	-360.6%	-361.1%	-361.0%	-360.4%
BEKK-X	Return	1.9231e-04(124)	-2.0613e-04 (124)	-7.0419e-04(124)	-0.0047 (124)	-0.0097 (124)	2.9192e-04 (124)
	Risk	0.00966	0.00966	0.00966	0.00965	0.00966	0.00966
	V. R.	-489.7%	-489.8%	-489.9%	-490.6%	-490.2%	-489.7%
GJR	Return	-2.0907e-05(130)	-4.1935e-04 (130)	-9.1741e-04(130)	-0.0049 (130)	-0.0099 (130)	7.8704e-05 (130)
	Risk	0.01057	0.01057	0.01057	0.01057	0.01058	0.01057
	V. R.	-392.4%	-392.4%	-392.5%	-392.7%	-392.1%	-392.4%
QGARCH	Return	-5.7751e-05(127)	-4.5620e-04 (127)	-9.5425e-04(127)	-0.0049 (127)	-0.0099 (127)	4.1860e-05 (127)
	Risk	0.01050	0.01049	0.01049	0.01049	0.01049	0.01050
	V. R.	-399.8%	-399.8%	-399.9%	-400.2%	-399.8%	-399.7%

Note: see note in table 5-89.

Table 5-91 The 2-year out-of-sample forecast for coffee with normal distribution (return, risk, variance comparison)

2-year forecast for coffee with normal distribution							
Hedging strategy		**Unhedged position**					
Return		0.00180					
risk		0.02575					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	5.1448e-04(254)	1.1604e-04 (254)	-3.8200e-04(254)	-0.0044 (254)	-0.0094 (254)	6.1409e-04 (254)
	Risk	0.01280	0.01280	0.01280	0.01279	0.01279	0.01280
	V. R.	-304.3%	-304.4%	-304.5%	-305.1%	-305.4%	-304.3%
BEKK	Return	4.2401e-04(258)	2.5662e-05 (258)	-4.7238e-04(258)	-0.0045 (258)	-0.0094(258)	5.2371e-04 (258)
	Risk	0.01290	0.01290	0.01290	0.01290	0.01290	0.01290
	V. R.	**-298.2%**	-298.2%	-298.2%	-298.3%	-298.1%	-298.2%
GARCH-X	Return	5.0647e-04(258)	1.0803e-04 (258)	-3.9001e-04(258)	-0.0044 (258)	-0.0094 (258)	6.0608e-04 (258)
	Risk	0.01296	0.01296	0.01296	0.01295	0.01296	0.01296
	V. R.	-294.7%	-294.7%	-294.7%	-295.0%	-294.8%	-294.6%
BEKK-X	Return	3.9913e-04(257)	6.9341e-07 (257)	-4.9735e-04(257)	-0.0045 (257)	-0.0095 (257)	4.9874e-04 (257)
	Risk	0.01277	0.01277	0.01277	0.01276	0.01277	0.01277
	V. R.	-306.5%	-306.6%	-306.6%	-306.8%	-306.6%	-306.5%
GJR	Return	6.9500e-04(251)	2.9656e-04 (251)	-2.0148e-04(251)	-0.0042 (251)	-0.0092 (251)	7.9461e-04 (251)
	Risk	0.01402	0.01402	0.01402	0.01402	0.01402	0.01402
	V. R.	-237.1%	-237.2%	-237.2%	-237.3%	-237.2%	-237.1%
QGARCH	Return	5.0404e-04(270)	1.0560e-04 (270)	-3.9244e-04(270)	-0.0044(270)	-0.0094 (270)	6.0365e-04 (270)
	Risk	0.01339	0.01339	0.01339	0.01339	0.01340	0.01339
	V. R.	-269.9%	-269.9%	-269.9%	-269.8%	-269.3%	-269.9%

Note: see note in table 5-89.

Table 5-92 The 2-year out-of-sample forecast for coffee with student's t distribution (return, risk, variance comparison)

2-year forecast for coffee with normal distribution							
Hedging strategy		**Unhedged position**					
Return		0.00180					
risk		0.02575					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	2.5929e-04(255)	-1.3915e-04 (255)	-6.3719e-04(255)	-0.0046(255)	-0.0096(255)	3.5889e-04 (255)
	Risk	0.01141	0.01141	0.01141	0.01139	0.01139	0.01141
	V. R.	-409.3%	-409.4%	-409.6%	-410.5%	-411.0%	-409.2%
BEKK	Return	4.1744e-04(259)	1.9004e-05 (259)	-4.7904e-04(259)	-0.0045 (259)	-0.0094(259)	5.1705e-04 (259)
	Risk	0.01286	0.01285	0.01285	0.01285	0.01286	0.01286
	V. R.	**-301.1%**	-301.1%	-301.2%	-301.3%	-301.1%	-301.1%
GARCH-X	Return	3.1022e-04(252)	-8.8212e-05 (252)	-5.8626e-04(252)	-0.0046(252)	-0.0096 (252)	4.0983e-04 (252)
	Risk	0.01128	0.01127	0.01127	0.01127	0.01127	0.01128
	V. R.	-421.3%	-421.4%	-421.5%	-421.9%	-421.7%	-421.3%
BEKK-X	Return	4.0787e-04(259)	9.4372e-06 (259)	9.4372e-06 (259)	-0.0045 (259)	-0.0095 (259)	5.0748e-04 (259)
	Risk	0.01283	0.01283	0.01283	0.01282	0.01283	0.01283
	V. R.	-302.8%	-302.8%	-302.8%	-303.0%	-302.7%	-302.7%
GJR	Return	1.5511e-04(258)	-2.4332e-04 (258)	-7.4137e-04(258)	-0.0047(258)	-0.0097(258)	2.5472e-04 (258)
	Risk	0.01107	0.01107	0.01107	0.01107	0.01108	0.01107
	V. R.	-441.2%	-441.2%	-441.2%	-440.0%	-440.0%	-441.2%
QGARCH	Return	3.33205e-04(255)	-6.5229e-05 (255)	-5.6327e-04(255)	-0.0046(255)	-0.0095 (255)	4.3281e-04 (255)
	Risk	0.01163	0.01163	0.01163	0.01162	0.01161	0.01163
	V. R.	-389.9%	-390.0%	-390.1%	-391.0%	-391.4%	-389.9%

Note: see note in table 5-89.

Table 5-93 The 1-year out-of-sample forecast for soybean with normal distribution (return, risk, variance comparison)

1-year forecast for soybean with normal distribution							
Hedging strategy		**Unhedged position**					
Return		-3.75794e-04					
risk		0.01677					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	-8.0921e-05(130)	-4.7939e-04 (130)	-9.7747e-04(131)	-0.0050 (131)	-0.0099(131)	1.8696e-05 (130)
	Risk	0.00750	0.00750	0.00750	0.00751	0.00753	0.00750
	V. R.	-399.4%	-399.4%	-399.3%	-398.2%	-395.2%	-399.4%
BEKK	Return	-8.4882e-05(129)	-4.8335e-04 (129)	-9.8143e-04(129)	-0.0050 (128)	-0.0100(128)	1.4735e-05 (129)
	Risk	0.00732	0.00732	0.00732	0.00733	0.00735	0.00732
	V. R.	-424.0%	-424.0%	-424.0%	-422.9%	-420.0%	-424.1%
GARCH-X	Return	-1.4863e-04(132)	-5.4710e-04 (132)	-0.0011 (132)	-0.0050 (131)	-0.0100(131)	-4.9014e-05(132)
	Risk	0.00709	0.00709	0.00709	0.00710	0.00712	0.00709
	V. R.	-459.2%	-459.2%	-459.1%	-457.8%	-454.3%	-459.2%
BEKK-X	Return	-1.2752e-04(130)	-5.2598e-04 (130)	-0.0010 (130)	-0.0050 (129)	-0.0100(129)	-2.7898e-05(130)
	Risk	0.00738	0.00738	0.00738	0.00739	0.00741	0.00738
	V. R.	-416.4%	-416.3%	-416.3%	-415.2%	-412.3%	-416.4%
GJR	Return	-1.5114e-05(133)	-4.1358e-04 (133)	-9.1167e-04(133)	-0.0049 (132)	-0.0099(132)	8.4503e-05 (133)
	Risk	0.00701	0.00701	0.00701	0.00702	0.00704	0.00701
	V. R.	-472.0%	-472.0%	-471.9%	-470.7%	-467.2%	-472.0%
QGARCH	Return	-1.8052e-04(131)	-5.7899e-04 (131)	-0.0011 (131)	-0.0051 (130)	-0.0100(130)	-8.0903e-05(131)
	Risk	0.00734	0.00734	0.00734	0.00735	0.00737	0.00734
	V. R.	-421.8%	-421.8%	-421.8%	-420.7%	-417.6%	-421.8%

Note: see note in table 5-89.

Table 5-94 The 1-year out-of-sample forecast for soybean with student's t distribution (return, risk, variance comparison)

1-year forecast for soybean with normal distribution							
Hedging strategy		**Unhedged position**					
Return		-3.75794e-04					
risk		0.01677					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	-1.1914e-04(133)	-5.1761e-04 (133)	-0.0010 (133)	-0.0050 (132)	-0.0100 (132)	-1.9523e-05 (133)
	Risk	0.00708	0.00708	0.00708	0.00709	0.00711	0.00708
	V. R.	-460.3%	-460.4%	-460.2%	-459.1%	-455.8%	-460.3%
BEKK	Return	-6.5387e-05(131)	-4.6385e-04 (131)	-9.6194e-04(131)	-0.0050 (130)	-0.0099 (130)	3.4230e-05 (131)
	Risk	0.00700	0.00700	0.00700	0.00701	0.00703	0.00700
	V. R.	-473.7%	-473.7%	-473.7%	-472.5%	-469.1%	-473.7%
GARCH-X	Return	-1.0937e-04(131)	-5.0784e-04 (131)	-0.0010 (131)	-0.0050 (130)	-0.0100 (130)	-9.7523e-06 (131)
	Risk	0.00707	0.00707	0.00707	0.00707	0.00710	0.00707
	V. R.	-463.0%	-463.0%	-463.0%	-461.8%	-458.4%	-463.0%
BEKK-X	Return	-1.0087e-04(131)	-4.9934e-04 (131)	-9.9742e-04(131)	-0.0050 (130)	-0.0100 (130)	-1.2530e-06 (131)
	Risk	0.00703	0.00703	0.00703	0.00704	0.00706	0.00703
	V. R.	-468.6%	-468.6%	-468.5%	-467.4%	-464.0%	-468.6%
GJR	Return	2.2688e-05(134)	-3.7578e-04 (134)	-8.7386e-04(134)	-0.0049 (133)	-0.0098 (133)	1.2231e-04 (134)
	Risk	0.00718	0.00718	0.00718	0.00718	0.00720	0.00718
	V. R.	-446.0%	-446.0%	-445.9%	-444.8%	-441.7%	-446.0%
QGARCH	Return	-1.3627e-04(133)	-5.3474e-04 (133)	-0.0010 (133)	-0.0050 (132)	-0.0100 (132)	-3.6653e-05 (133)
	Risk	0.00715	0.00715	0.00715	0.00716	0.00718	0.00715
	V. R.	-450.0%	-450.0%	-449.9%	-448.8%	-445.7%	-450.0%

Note: see note in table 5-89.

Table 5-95 The 2-year out-of-sample forecast for soybean with normal distribution (return, risk, variance comparison)

2-year forecast for soybean with normal distribution							
Hedging strategy		**Unhedged position**					
Return		1.23704e-04					
risk		0.02019					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	-1.4577e-04(243)	-5.4499e-04 (243)	-0.0010 (243)	-0.0050 (243)	-0.0100 (243)	-4.5958e-05 (243)
	Risk	0.01395	0.01395	0.01395	0.01395	0.01395	0.01395
	V. R.	-109.5%	-109.5%	-109.5%	-109.6%	-109.5%	-109.5%
BEKK	Return	-1.3010e-04(242)	-5.2933e-04 (242)	-0.0010 (242)	-0.0050 (242)	-0.0100 (242)	-3.0291e-05 (242)
	Risk	0.01380	0.01380	0.01380	0.01380	0.01380	0.01380
	V. R.	-114.0%	-114.0%	-114.0%	-114.0%	-114.0%	-114.0%
GARCH-X	Return	-1.6026e-04(247)	-5.5949e-04 (247)	-0.0011 (247)	-0.0051 (247)	-0.0100 (247)	-6.0452e-05 (247)
	Risk	0.01364	0.01364	0.01364	0.01363	0.01364	0.01364
	V. R.	-119.2%	-119.3%	-119.3%	-119.3%	-119.3%	-119.2%
BEKK-X	Return	-1.3420e-05(246)	-4.1265e-04 (246)	-9.1168e-04(246)	-0.0049 (246)	-0.0099 (246)	8.6388e-05 (246)
	Risk	0.01387	0.01387	0.01387	0.01387	0.01387	0.01387
	V. R.	-111.8%	-111.8%	-111.8%	-111.9%	-111.8%	-111.8%
GJR	Return	-0.0011 (246)	-0.0015 (246)	-0.0020 (246)	-0.0060(246)	-0.0110 (246)	-9.9816e-04 (246)
	Risk	0.01222	0.01222	0.01222	0.01222	0.01222	0.01222
	V. R.	-172.9%	-172.9%	-172.9%	-173.0%	-172.9%	-172.9%
QGARCH	Return	6.8530e-05(252)	-3.3070e-04 (252)	-8.297e-04(252)	-0.0048 (252)	-0.0098 (252)	1.6834e-04 (252)
	Risk	0.01340	0.01340	0.01340	0.01340	0.01340	0.01340
	V. R.	-126.9%	-126.9%	-126.9%	-127.0%	-126.9%	-126.9%

Note: see note in table 5-89.

Table 5-96 The 2-year out-of-sample forecast for soybean with student's t distribution (return, risk, variance comparison)

2-year forecast for soybean with student's t distribution							
Hedging strategy		**Unhedged position**					
Return		1.23704e-04					
risk		0.02019					
Hedging strategy		**Dynamic hedge**					
Models	Transaction costs	0.01%	0.05%	0.25%	0.5%	1%	Without TCs
GARCH	Return	-6.7790e-05(249)	-4.6702e-04 (249)	-9.6605e-04(249)	-0.0050 (249)	-0.0100 (249)	3.2017e-05 (249)
	Risk	0.01494	0.01494	0.01494	0.01494	0.01494	0.01494
	V. R.	-82.66%	-82.67%	-82.68%	-82.73%	-82.72%	-82.66%
BEKK	Return	-2.9237e-04(247)	-6.9159e-04 (247)	-0.0012 (247)	-0.0052(247)	-0.0102 (247)	-1.9256e-04(247)
	Risk	0.01341	0.01341	0.01341	0.01341	0.01341	0.01341
	V. R.	-126.6%	-126.6%	-126.6%	-126.7%	-126.6%	-126.6%
GARCH-X	Return	-1.8492e-04(248)	-5.8415e-04 (248)	-0.0011 (248)	-0.0051 (248)	-0.0101(248)	-8.5112e-05 (248)
	Risk	0.01348	0.01348	0.01348	0.01348	0.01348	0.01348
	V. R.	-124.3%	-124.3%	-124.3%	-124.3%	-124.3%	-124.3%
BEKK-X	Return	-2.0045e-04(248)	-5.9968e-04 (248)	-0.0011 (248)	-0.0051 (248)	-0.0101 (248)	-1.0064e-04 (248)
	Risk	0.01348	0.01348	0.01347	0.01347	0.01348	0.01348
	V. R.	-124.5%	-124.5%	-124.5%	-124.6%	-124.5%	-124.5%
GJR	Return	-4.9314e-04(241)	-8.9236e-04 (241)	-0.0014 (241)	-0.0054 (241)	-0.0104 (241)	-3.9333e-04 (241)
	Risk	0.01224	0.01224	0.01224	0.01223	0.01223	0.01224
	V. R.	-172.3%	-172.3%	-172.3%	-172.4%	-172.4%	-172.3%
QGARCH	Return	-6.4213e-05(248)	-4.6344e-04 (248)	-9.6248e-04(248)	-0.0050 (248)	-0.0100 (248)	3.5594e-05 (248)
	Risk	0.01345	0.01345	0.01345	0.01345	0.01345	0.01345
	V. R.	-125.3%	-125.3%	-125.3%	-125.4%	-125.3%	-125.3%

Note: see note in table 5-89.

6 Conclusions

Forecasting of hedge ratio is important and it helps investors apply appropriate hedging strategies and technique to minimize price risk and protect from unacceptable loss. Diverse forecasting models have been proposed for predicting optimal hedge ratio in futures markets, such as the ECM (error correction model), CI (cointegration), EWMA (exponentially-weighted-moving average) and GARCH family models.

Following a few prominent studies in agricultural markets, such as Baillie and Myers (1991) , Roh et al. (1995), Tse et al. (2002) and Choudhry (2003), we provide a study of estimation and forecasting daily dynamic optimal hedge ratio (OHR) in agricultural and commodities' futures markets using six GARCH models and then comparing the forecasting power of these models. To our knowledge no previous study provides an empirical forecast of the OHR in agricultural futures markets. Thus, this thesis extends the literature on OHR beyond estimation of the relationship between the cash and futures prices and the estimation of OHR in agricultural futures markets. In particular, we investigate the predictive power of the models (standard GARCH, BEKK, GARCH-X, BEKK-GARCH-X, GARCH-GJR and QGARCH models) on OHR forecast for five agricultural commodities (three storable commodities; coffee, wheat, soybean, and two non-storable commodities; live cattle and live hog), and we apply non-overlapping 1-year and 2-year out-of-sample OHR forecasting based on normal and student's *t* distributed residuals. Furthermore, the forecast error and accuracy of the six GARCH models are evaluated using fours norms of the error measurements (MAE, MSE, Theil's U and Modified Diebold Mariano tests). The comparative view on the forecasting ability of the six GARCH models based on agricultural OHR forecast contributes towards the existing literature.

In general, we focus on investigating the predictive power of GARCH models on OHR forecast taking into account the potential effects of residuals distribution, forecast time horizon, storability of product, and evaluation measure. To the best of our knowledge, no previous study provides a comparison of the forecasting power of the GARCH models based on the prediction of OHR in agricultural futures market. In addition, this study is different from that of Choudhry (2009) in terms of several aspects, such as research objective and models.

6.1 Findings and Conclusions

The major findings are divided into two categories, the results from in-sample estimation and those from out-of-sample forecasting.

In-sample estimation

The first research question is addressed in the in-sample estimation. First, we estimate the in-sample OHR by means of the six GARCH models. For both normal and student's *t* distributions, the ARCH and GARCH effects are found to be significant, and volatility clustering die out gradually for all cases on both forecasting time horizons in general. This result provides answer to the first research question of this thesis. The estimated OHR series do not follow normal distribution and generally have higher or lower peaks than those of normal distribution with asymmetric tails. Furthermore, all the estimated OHR series are stationary and out of auto-correlation based on the Dickey-Fuller and Augmented Dickey-Fuller unit root tests and the L-B (Liung-Box) test. Consequently, the estimated OHR series are free of serial correlations demonstrates the adequacy of the same order as ARCH terms in the six GARCH models for these five agricultural commodities. This evidence consolidates the appropriateness and validity of the six diagonal bivariate GARCH models in OHR estimation and further out-of-sample forecasting.

Out-of-sample forecasting

In the dynamic estimation and forecasting of hedge ratios, since the actual hedge ratio is not available and the estimated hedge ratio is not an appropriate proxy of actual hedge ratio, we need to deal with benchmark missing of real hedge ratio. To solve this issue, we forecast the returns based on the forecasted hedge ratios, and then compare the forecasted return with real return to test the accuracy of the models.

Our study shows that both of distributions of the residuals and forecasting time horizon significantly affect the forecasting performance of the six GARCH models. With residuals normally distributed, the BEKK model provides the best return prediction for both forecasting horizons for coffee among the six models, while for wheat the GJR and QGARCH provide the best forecasting over 1-year and 2-year forecasting horizons correspondingly. For soybean, the GARCH-X model outperforms for 1-year, yet the BEKK is the most accurate forecasting model on the long-term 2-year prediction. The asymmetric GJR and QGARCH provide the best forecasting performance for both the non-storable products except for the condition of 2-year forecast with MDM test. Moreover, the BEKK and asymmetric GARCH models are superior to other four GARCH models. When the residual series is student's *t* distributed, the forecasting performance of BEKK-X model dominates other models for those of the coffee and soybean. The GJR and QGARCH dominates the rest of models for wheat, live cattle and live hog, while the standard GARCH has better prediction than asymmetric GARCH models on 2-year forecast for live cattle. Generally, the BEKK-type GARCH and asymmetric models have best predictive power for the 1- and 2-year OHR forecast.

Overall, even though the forecasting power of a GARCH model depends on the commodity types, the residual distribution, the length of forecast horizons and method for error evaluations, Summary 9 that contains the output from four evaluation methods of forecast error and accuracy suggests that the BEKK type model works well for coffee and soybean, while the GARCH-X outperforms in short-term 1-year forecast with normal distributed residuals for soybean. In particular, the two asymmetric GARCH models dominate other models including the BEKK type models for the storable wheat regardless of the distributions of the residuals. The forecasting of the returns from asymmetric GARCH models fits the real return well for non-storable commodities. Furthermore, the standard GARCH model is highly accurate for the forecasting of live cattle on 2-year out-of-sample. The second research question of this thesis is addressed by these results.

From another point view, the analysis of each GARCH model in summary 10 provides a more general view of the outstanding GARCH model for both storable and non-storable agricultural commodities. Specifically, for storable commodities, the BEKK and asymmetric GARCH models are the best forecasting models among these six GARCH models. In the case of non-storable commodities, the asymmetric GARCH models are strongly recommended to out-of-sample forecast the OHR regardless of the distributions of residuals and forecast horizons. However, the standard GARCH is not capable of predicting the time-varying OHR for storable commodities as indicated by the poorest forecast, while it is powerful for 2-year forecast for live cattle with student's *t* distribution. More interestingly, the BEKK model which outperforms for storable products provides the most inaccurate 1-year forecasted return for non-storable commodities. Over a longer forecasting horizon, the GARCH-X model underperforms all other GARCH models for non-storable live cattle and live hog. The answer to research question 3 is provided by these results.

According to summary 9 and 10, we find that the effect of an evaluation method of determining the predictive power of these six GARCH models is not significant to the major findings, while the time horizon of the forecast and distribution of residuals influence the forecasting accuracy of models. Alternatively, from this empirical study, general conclusion is reached that the BEKK, asymmetric GJR and QGARCH models are recommended for OHR forecast on both 1- and 2-year horizons for normal and student's *t* distributions respectively for storable products and the asymmetric model for non-storable commodities in agricultural and commodities' futures markets. The predictive power of GARCH models relies on the residual distribution, the commodity and also the forecast horizon, consistent with the result those from Poon and Granger (2003) and Chen et.al (2003). Thus, results in this thesis adequately provide satisfactory answer to the research questions.

6.2 Contributions and Implications

We forecast the optimal hedge ratio in agricultural and commodities' futures markets which take up to 11 percent of futures trading in recent decades but is overlooked by most scholars. Our study on agricultural and commodities' futures markets makes the research on hedge ratio forecasting in futures market more comprehensive and diverse.

Provided that the estimated OHR series from all six GARCH models for all five commodities are stationary, the forecasting of OHR is possible and the prediction of return based on forecasted OHR is reliable under the EMH in this study. The forecasting power of six different GARCH models are compared pairwise for 1-year and 2-year hedge ratio and return prediction with both normal and student's *t* distributions for five different commodities (coffee, wheat, soybean, live cattle and live hog). Four evaluation methods (2 categories) are employed to measure the forecast error and accuracy of a model. We also study the potential effects of forecast horizon combined with the choice of distributed of residuals and measures on a model's predictive ability.

Additionally, we find that the predictive power of six GARCH models in agricultural and commodities' markets demonstrate certain unique features. Firstly, the forecasting performance of six GARCH models is different for storable and non-storable agricultural commodities. The BEKK-type models and asymmetric GARCH models outperform for the case of storable products, while asymmetric GARCH models dominate for non-storable commodities regardless of the forecast horizon and residual distribution. We document that the assured positivity of coefficients of conditional variance of BEKK-type model for storable commodity and the capture of asymmetric effect of information of asymmetric GARCH models contribute to the literature by providing more accurate OHR and return forecast in the agricultural and commodities' markets. Secondly, the leverage effect on conditional variance is detected, and the reverse leverage effect is widely found in the agricultural markets. The phenomenon that positive news has a larger shock on conditional variance than negative news does in agricultural markets verifies the superiority of asymmetric GJR and QGARCH models in certain extent. Moreover, asymmetric GARCH models capture leverage effect on log-return in cash and futures markets for agricultural commodities.

The accurate forecast of daily dynamic OHR by either BEKK or asymmetric GARCH models is helpful for 1-year and 2-year long-term investments in agricultural and commodities' futures markets where the asymmetric GJR and QGARCH models on hedge ratio prediction dominate other four GARCH models for non-storable commodities. A proper choice of forecasting model benefits to predict the price risk in the future. Based on an accurately forecasted OHR, investors could choose to hold the optimal quantity of futures contract at expiration to minimize risks. The hedging position can be readjusted according to the forecasted time-varying OHR before expiration. Moreover, the investors can determine appropriate hedging strategies and portfolio management for risk reduction and transference, and also prepare the capitals needed for hedging.

When transaction costs are considered, the dynamic hedge strategy benefits investors with excess return and low risk on 1-year forecast, yet it provides lower profit with low risk on 2-year forecast. As Geppert (1995) stated, a hedge may become less effective as the forecast horizon increases (Geppert, 1995). If appropriate forecasting models are selected, low risk-averse investor would gain from short 1-year dynamic hedge; median risk-averse investor can profit from both/either short-term and/or long-term hedging;

investor with high risk-aversion can benefit from both short-term and long term hedging even if the rate of transaction costs is high.

6.3 Further research

The forecasting of OHR is really an interesting and dynamic topic with many implications for the investor. Technically, there are several other different estimation methods which could also be used for forecasting purpose, such as the dynamic conditional correlation (DCC) GARCH and the Copula-based GARCH models. Furthermore, the forecasting of hedge ratio could be carried on for the short-term time horizon, for instance, 1-month, 3-month and 6-month, to benefit short-term investors. If more agricultural products are empirically tested, the OHR prediction will potentially boost the future trading on hedging agricultural and commodities' products.

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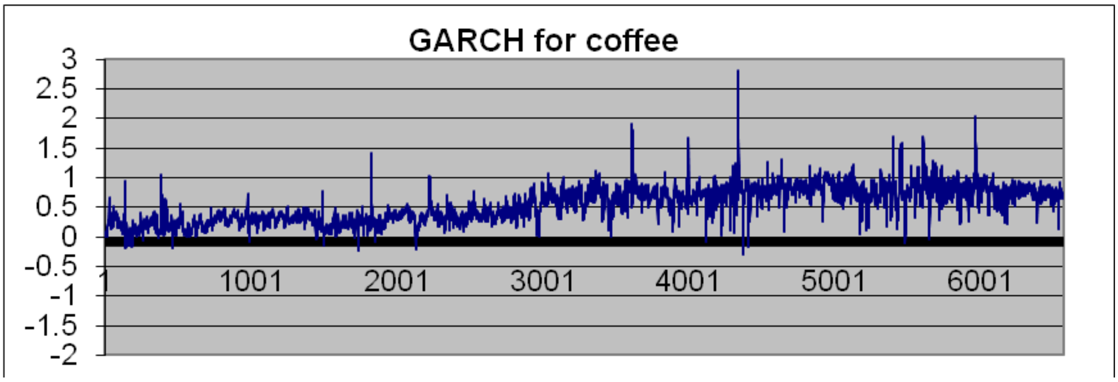
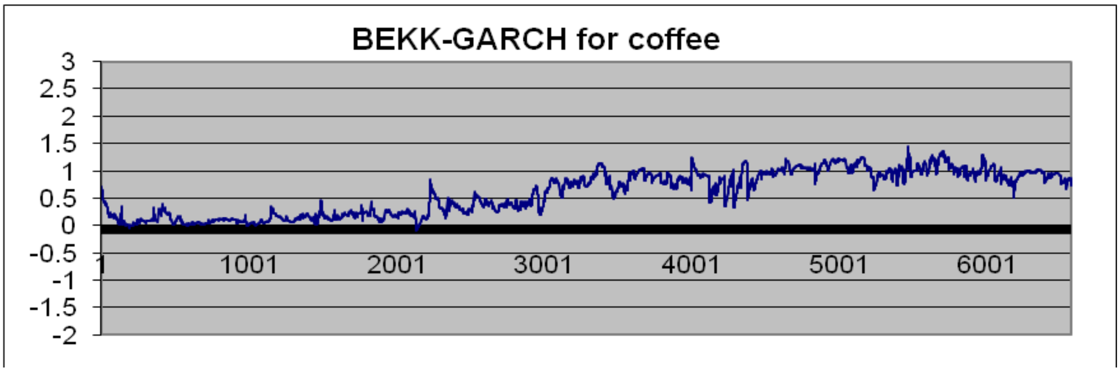
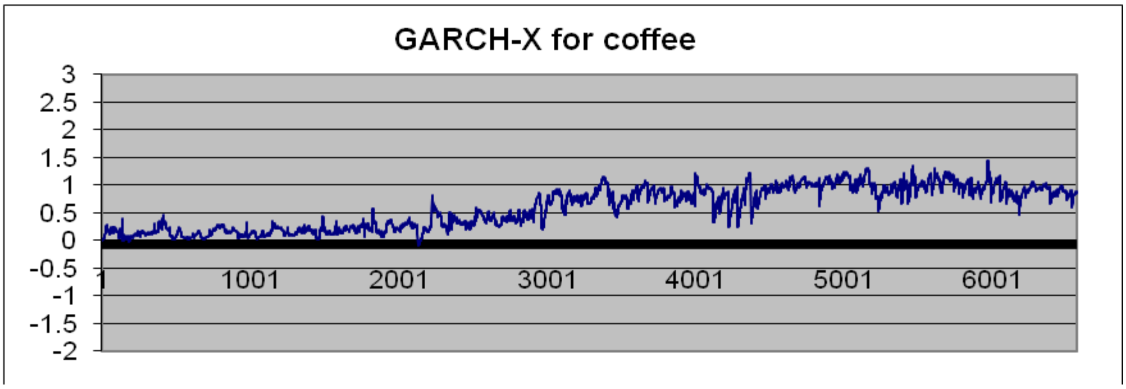
Appendix

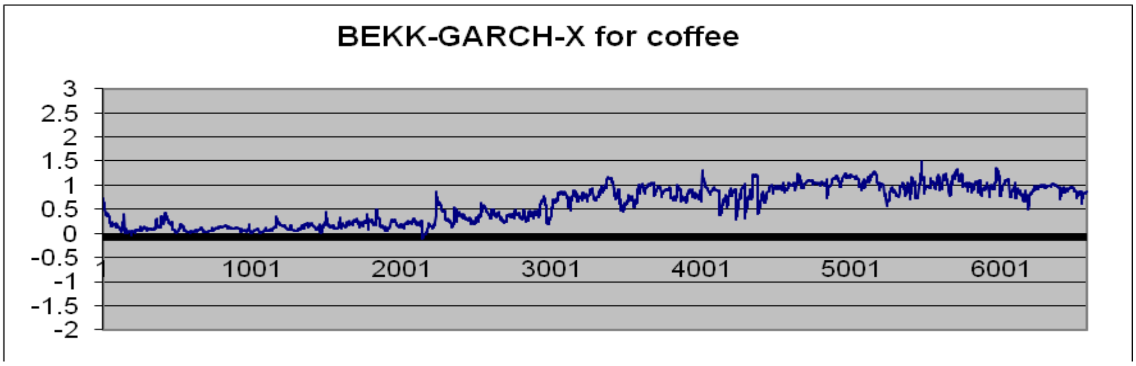
Part 1: Graphs for estimated OHR with normally distributed residuals in full sample for coffee, soybean and live hog

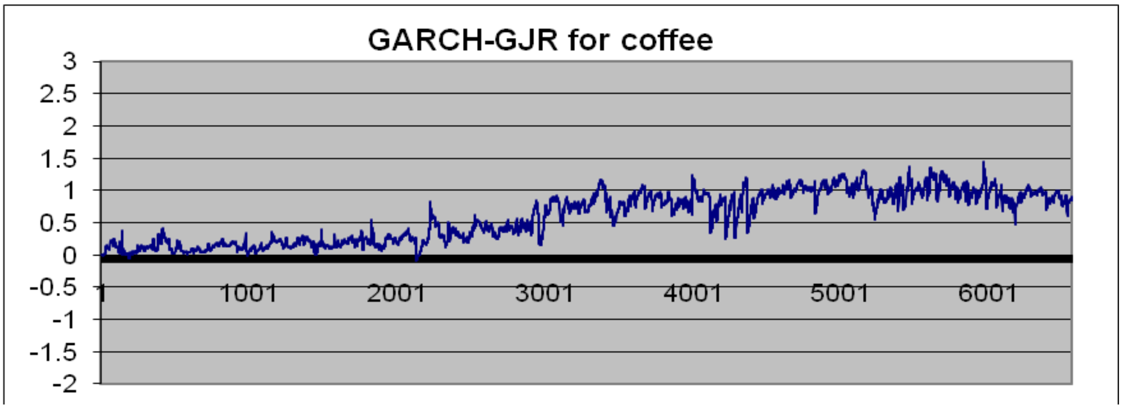
1. **coffee**

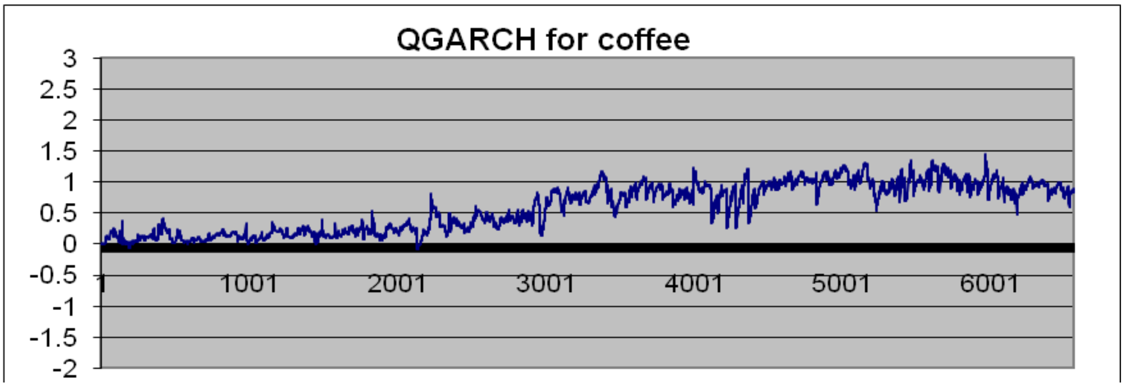




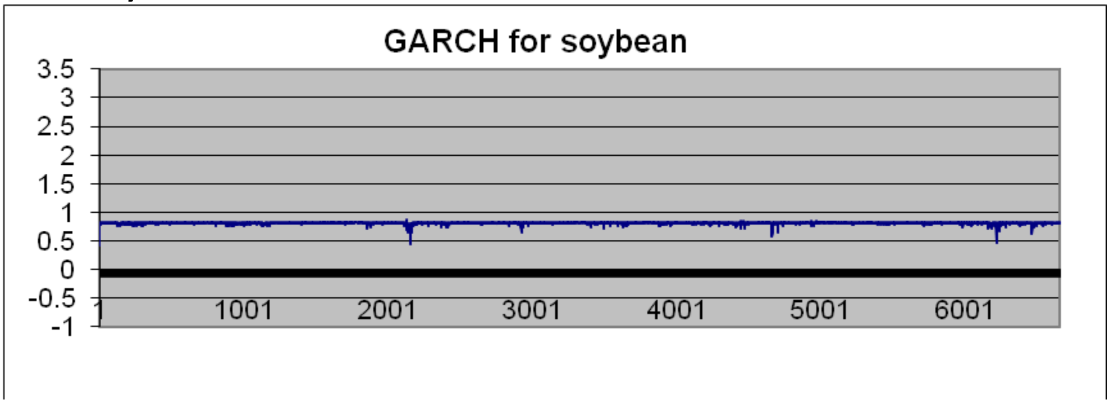


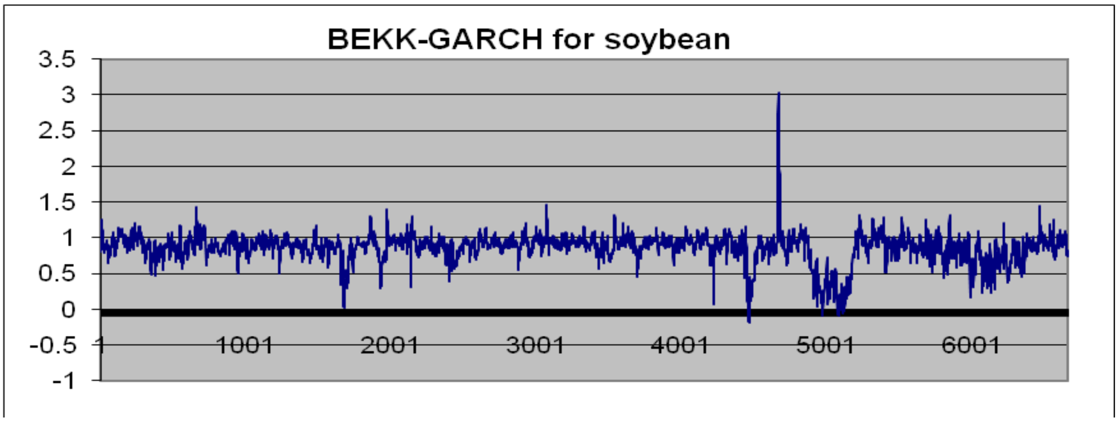


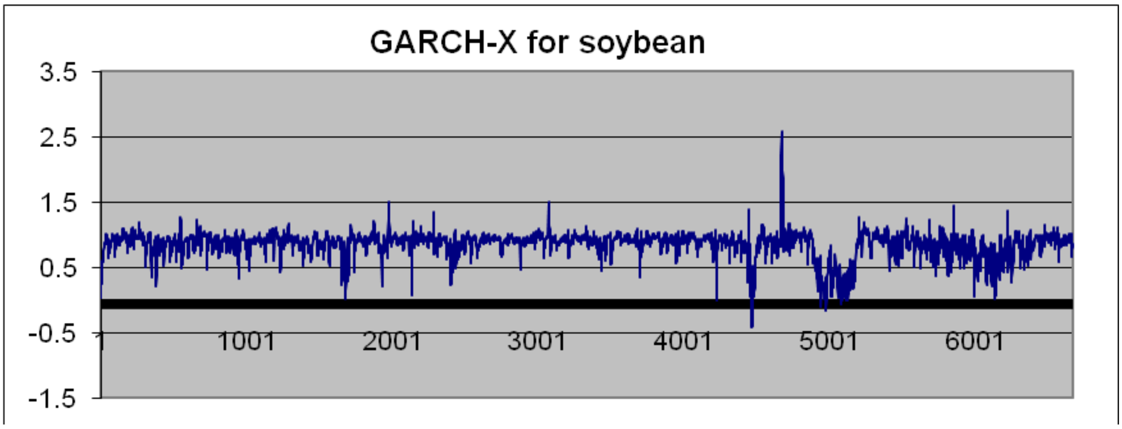

GARCH-GJR for coffee
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1
1001
2001
3001
4001
5001
6001

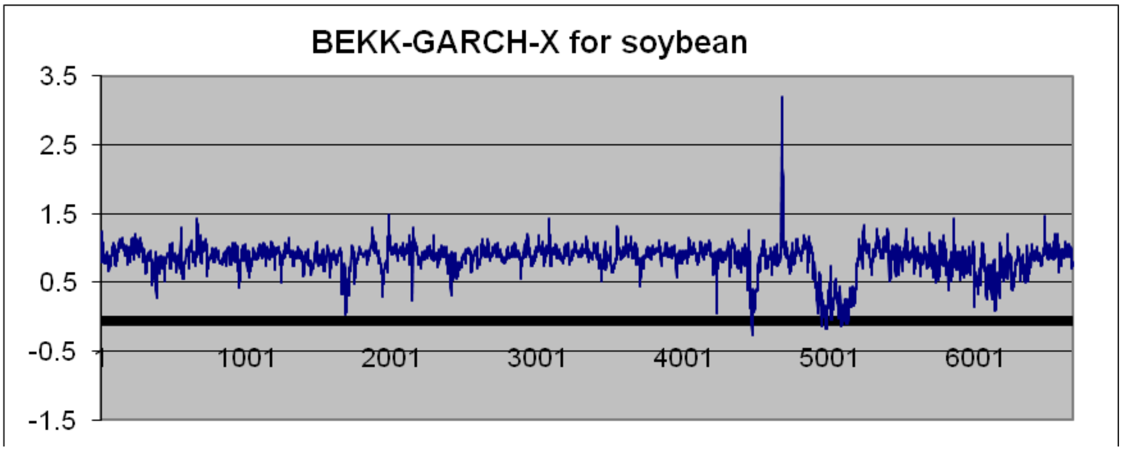

QGARCH for coffee
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1
1001
2001
3001
4001
5001
6001

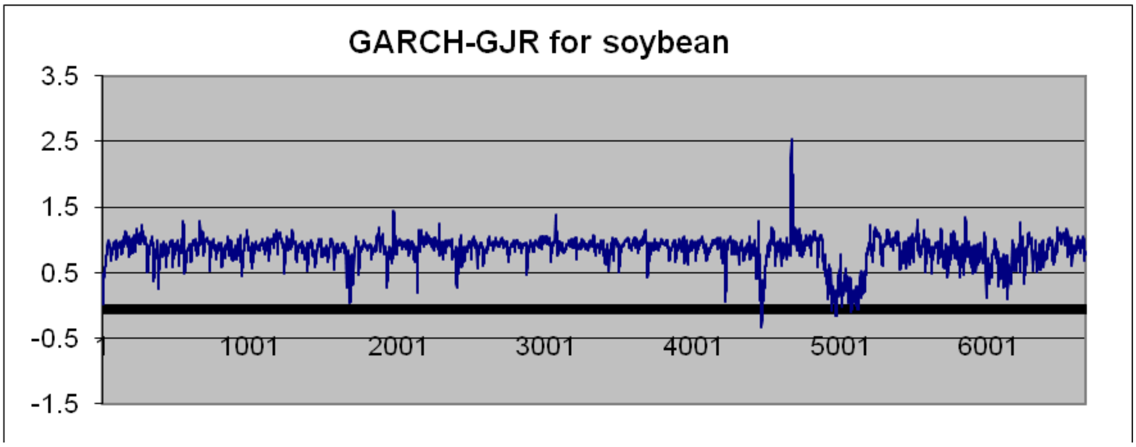
2. soybean

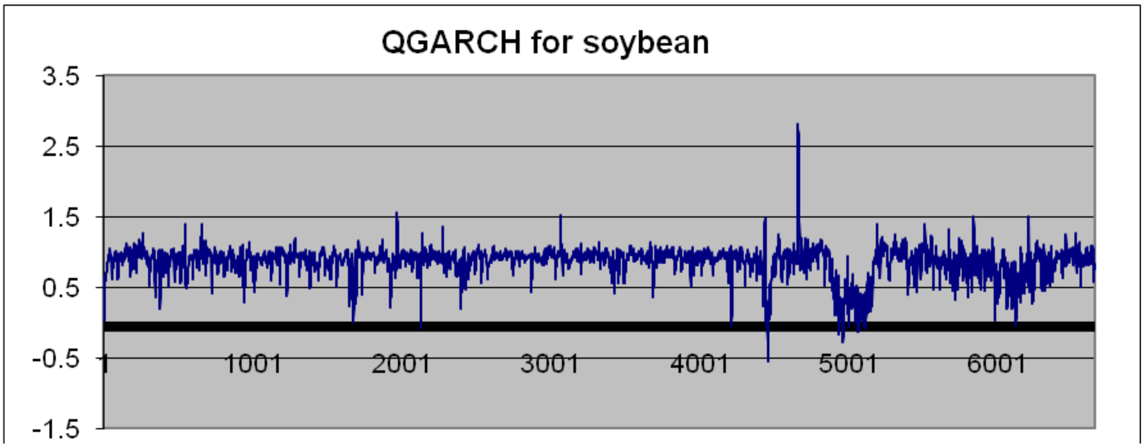

GARCH for soybean
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001


BEKK-GARCH for soybean
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001

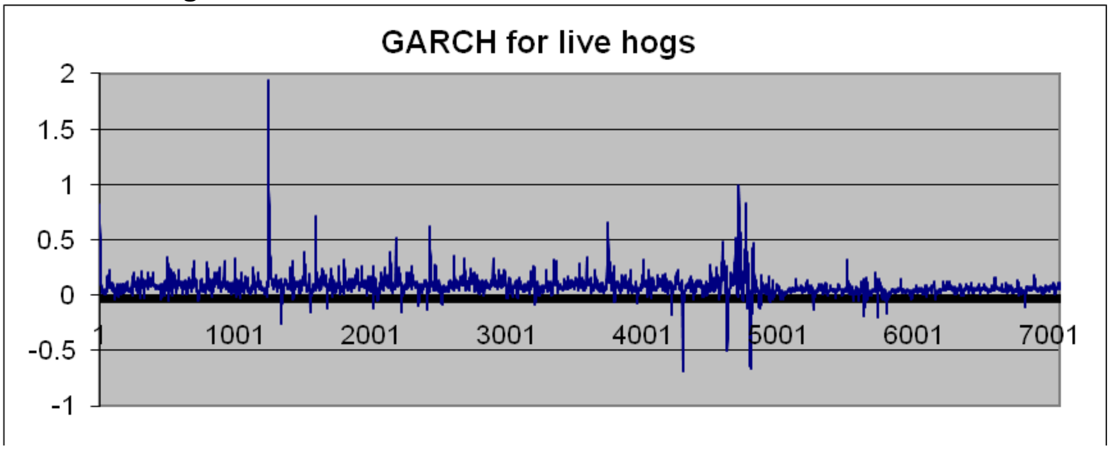

GARCH-X for soybean
3.5
2.5
1.5
0.5
-0.5
-1.5
1
1001
2001
3001
4001
5001
6001

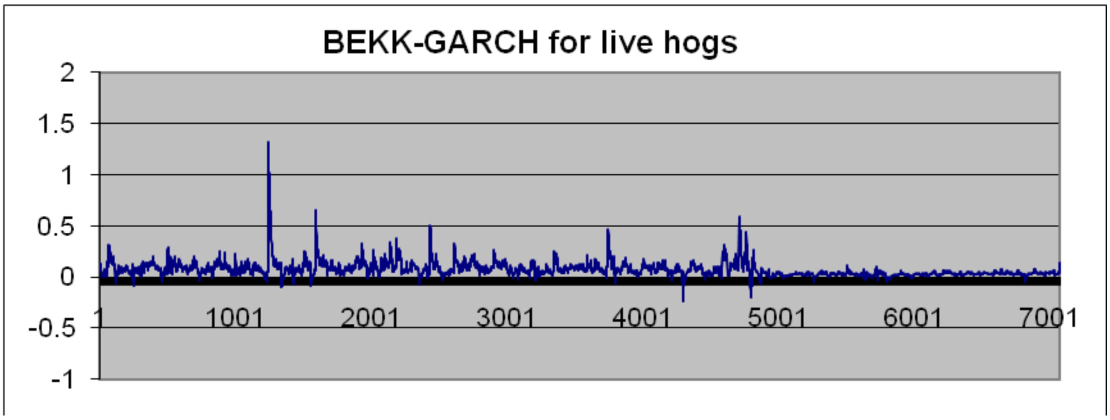

BEKK-GARCH-X for soybean
3.5
2.5
1.5
0.5
-0.5
-1.5
1
1001
2001
3001
4001
5001
6001

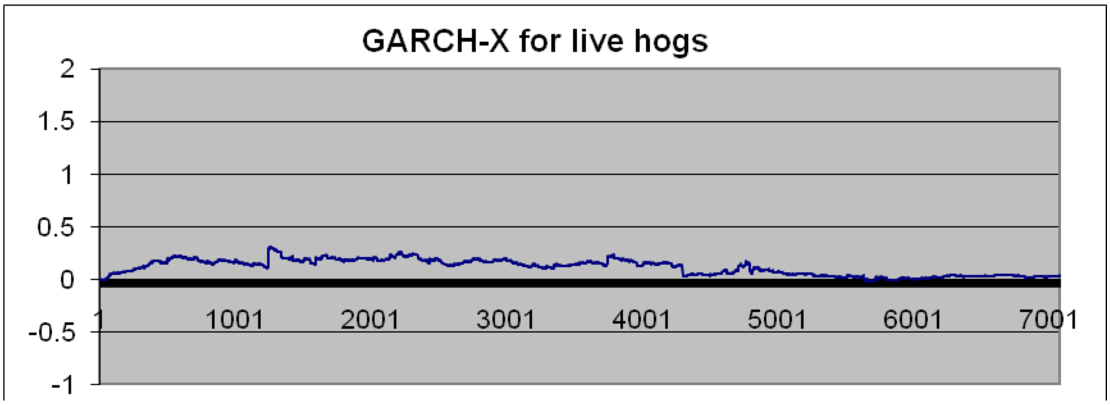

GARCH-GJR for soybean
3.5
2.5
1.5
0.5
-0.5
-1.5
1
1001
2001
3001
4001
5001
6001

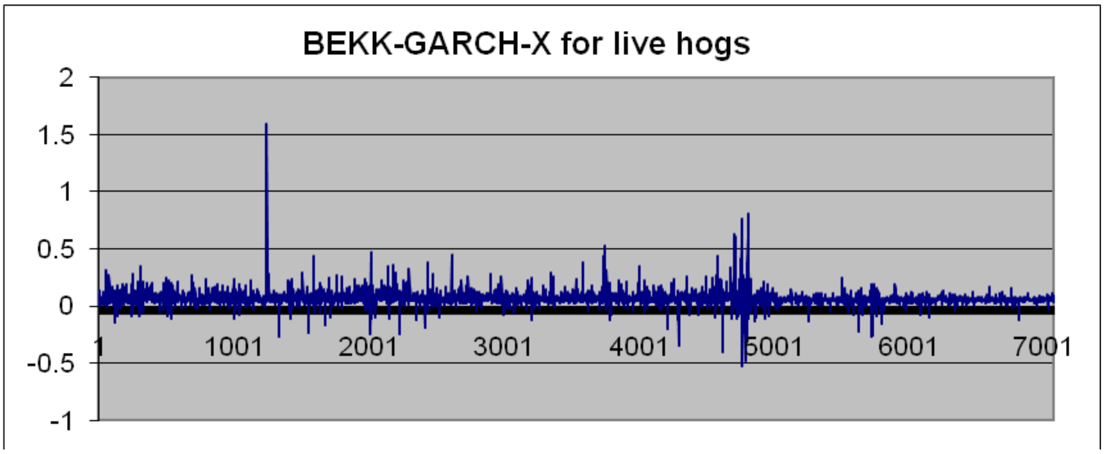

QGARCH for soybean
3.5
2.5
1.5
0.5
-0.5
-1.5
1
1001
2001
3001
4001
5001
6001

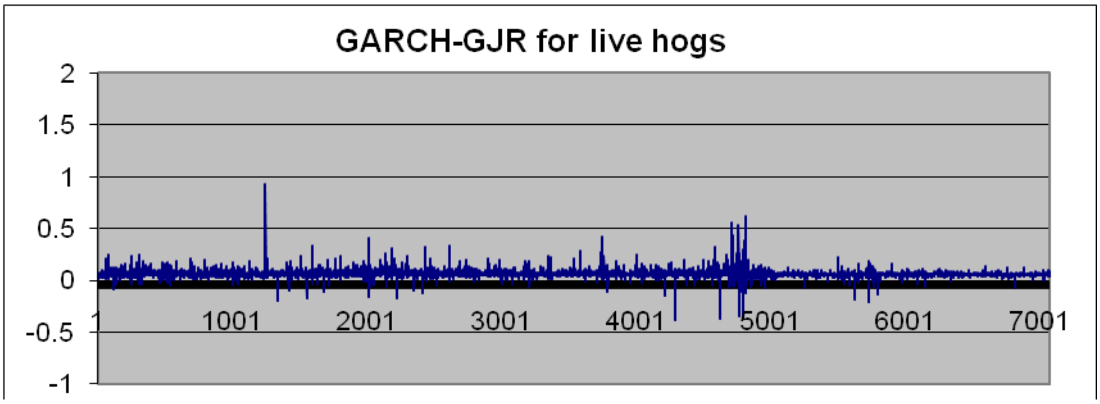
3. live hog

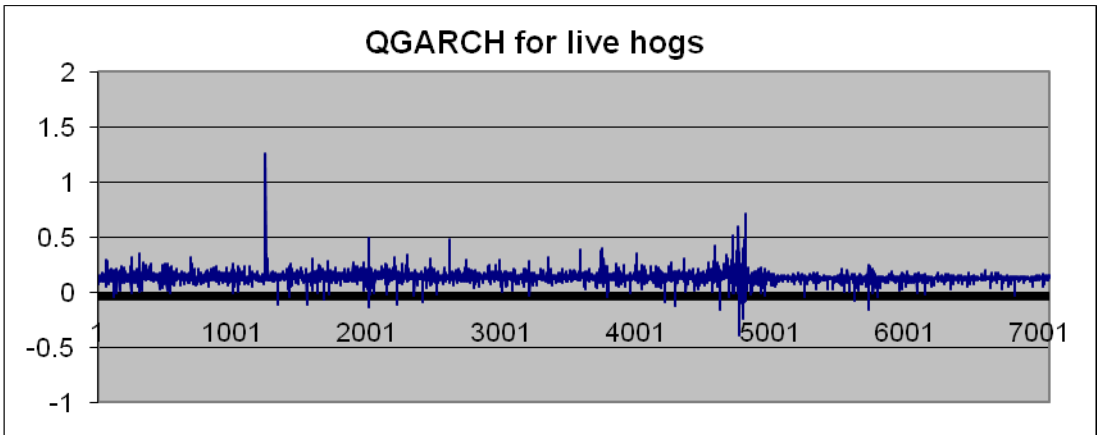

GARCH for live hogs
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001
7001


BEKK-GARCH for live hogs
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001
7001


GARCH-X for live hogs
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001
7001

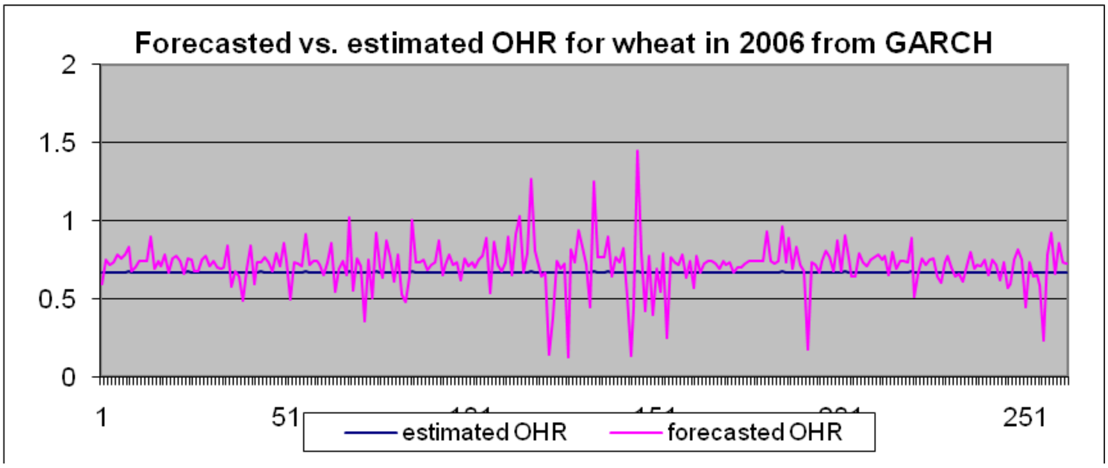

BEKK-GARCH-X for live hogs
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001
7001

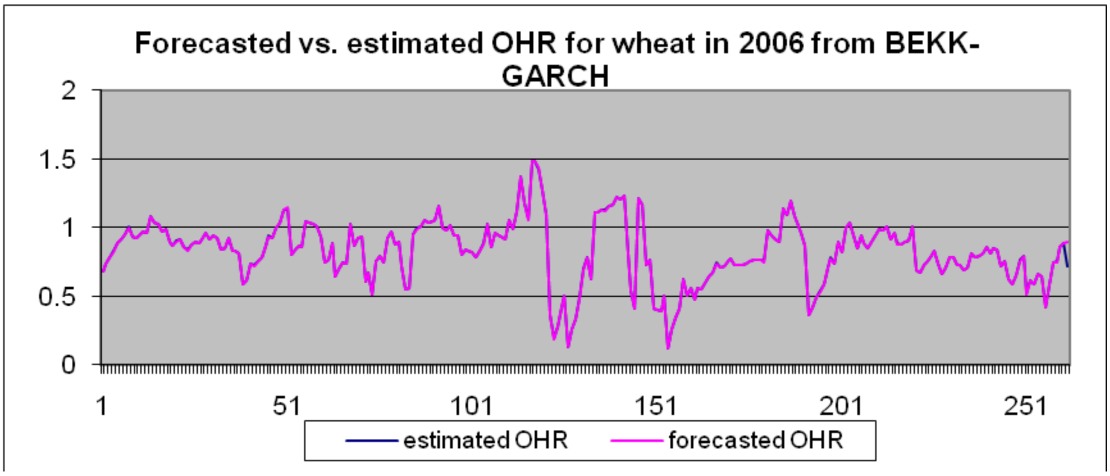

GARCH-GJR for live hogs
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001
7001

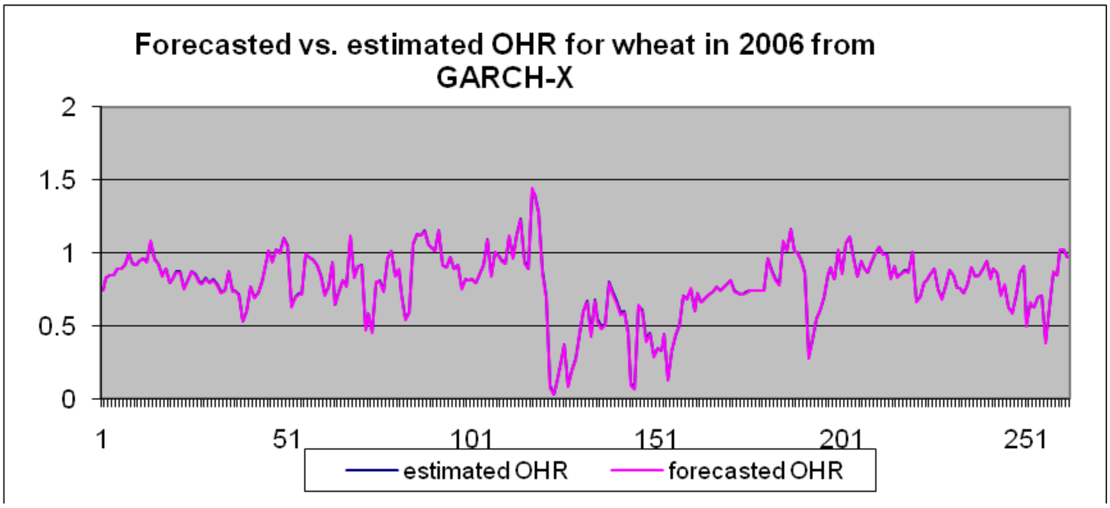

QGARCH for live hogs
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001
7001

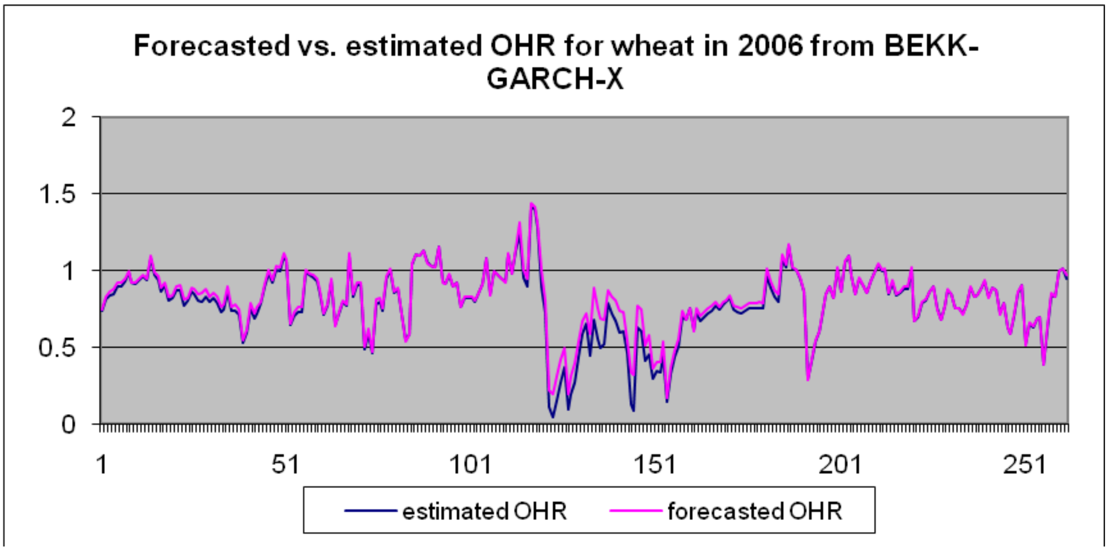
Part 2: 1-year estimated vs. forecasted OHR with normally distributed residuals for wheat, soybean and live hog

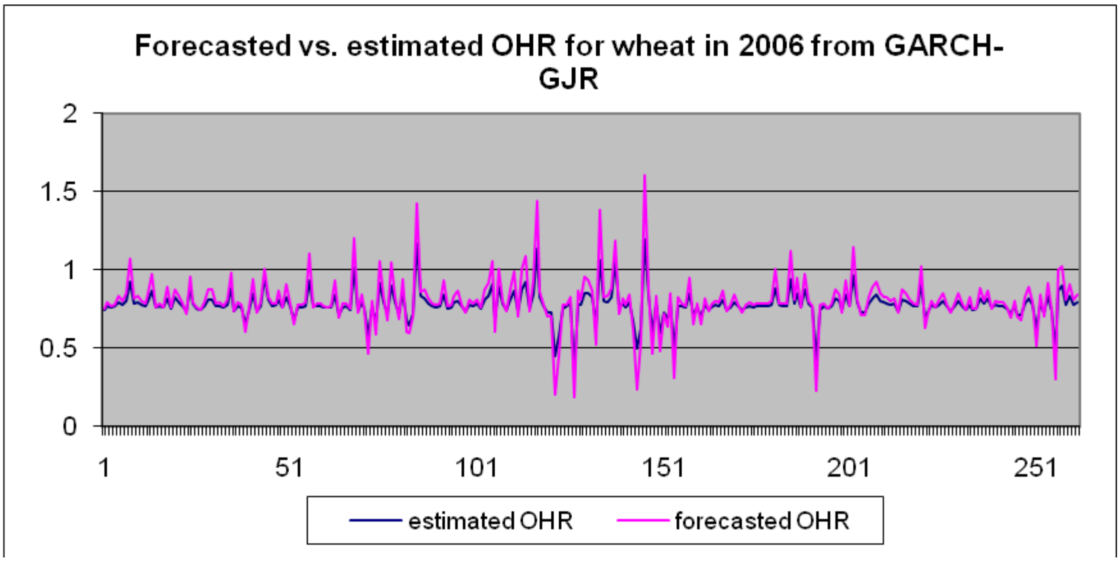
1. wheat

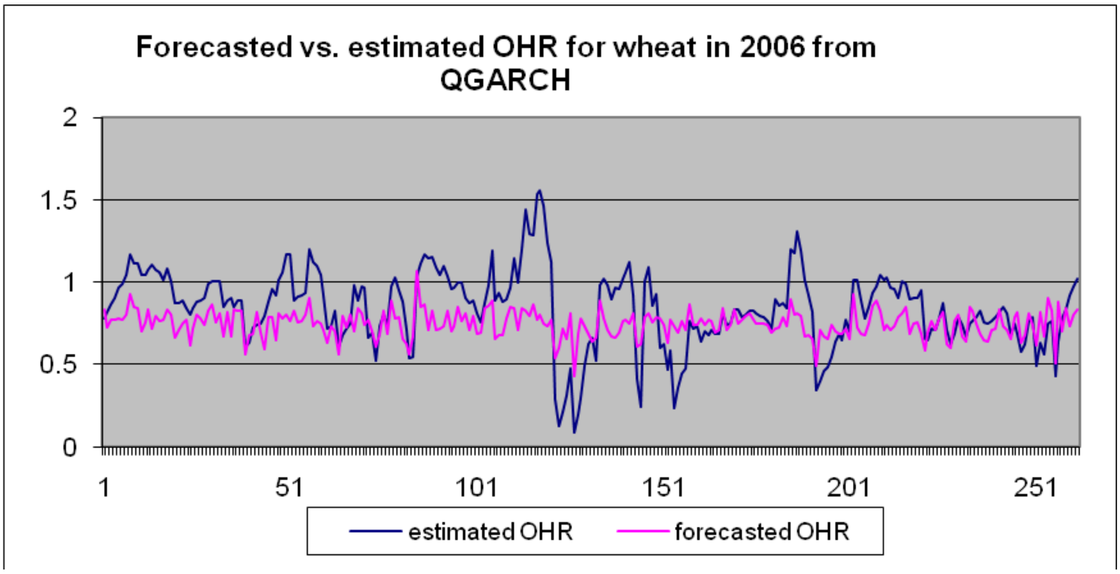




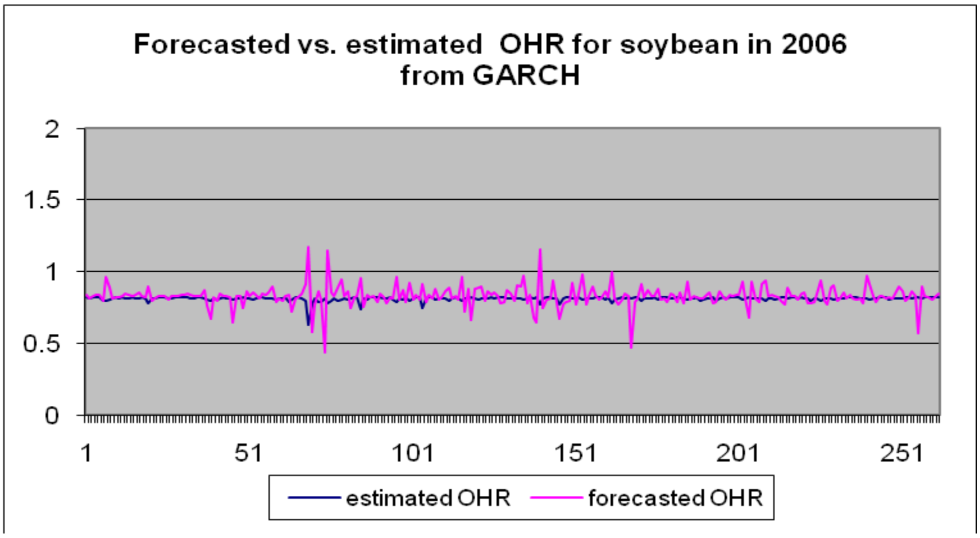


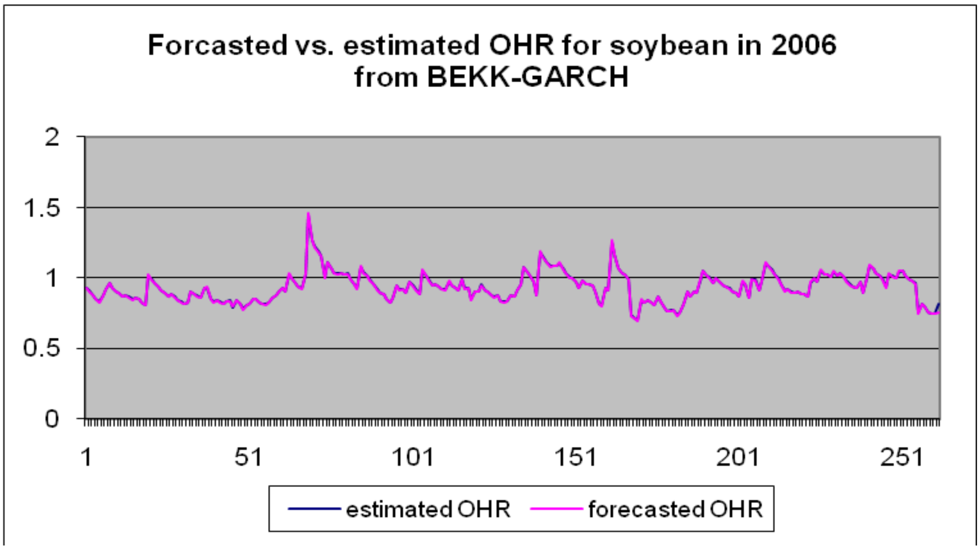

Forecasted vs. estimated OHR for wheat in 2006 from BEKK-GARCH-X
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

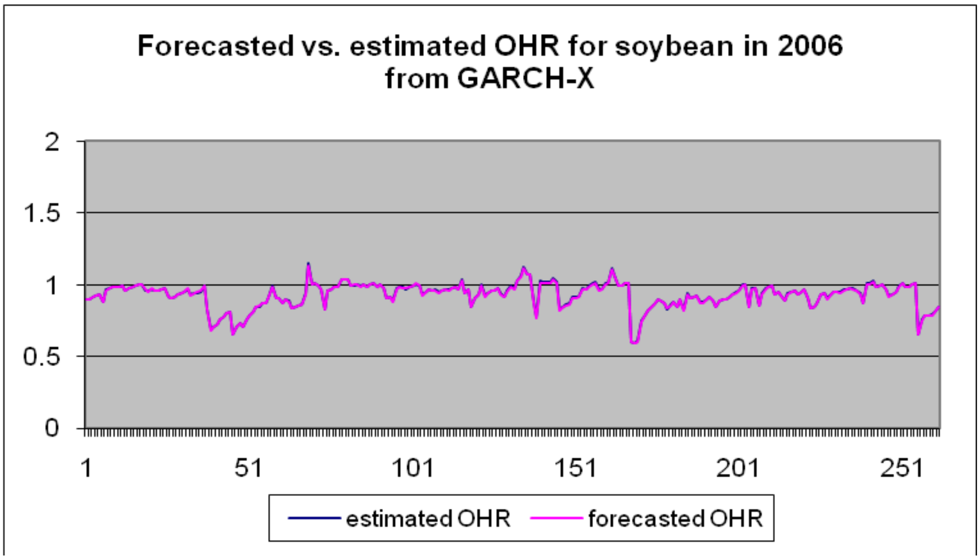

Forecasted vs. estimated OHR for wheat in 2006 from GARCH-GJR
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

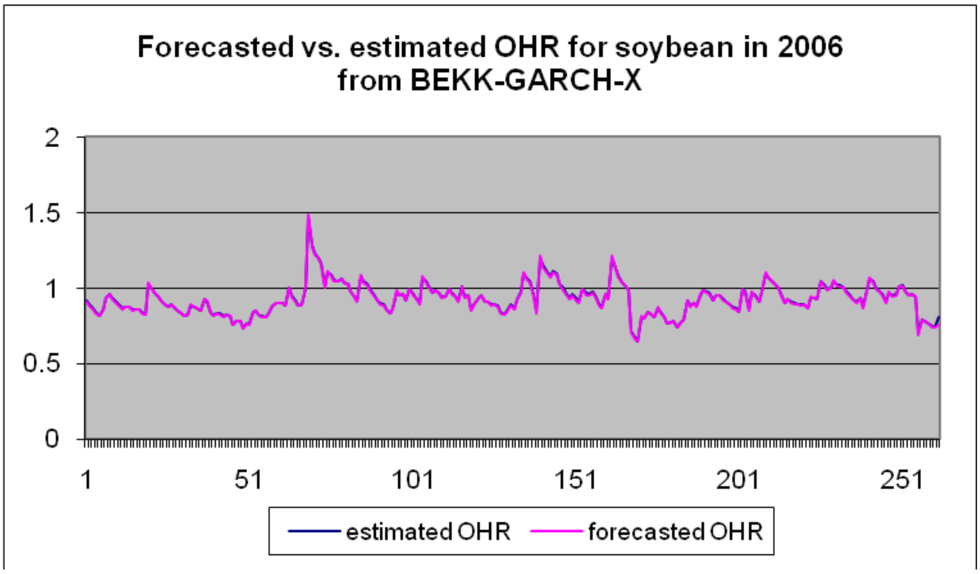

Forecasted vs. estimated OHR for wheat in 2006 from QGARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

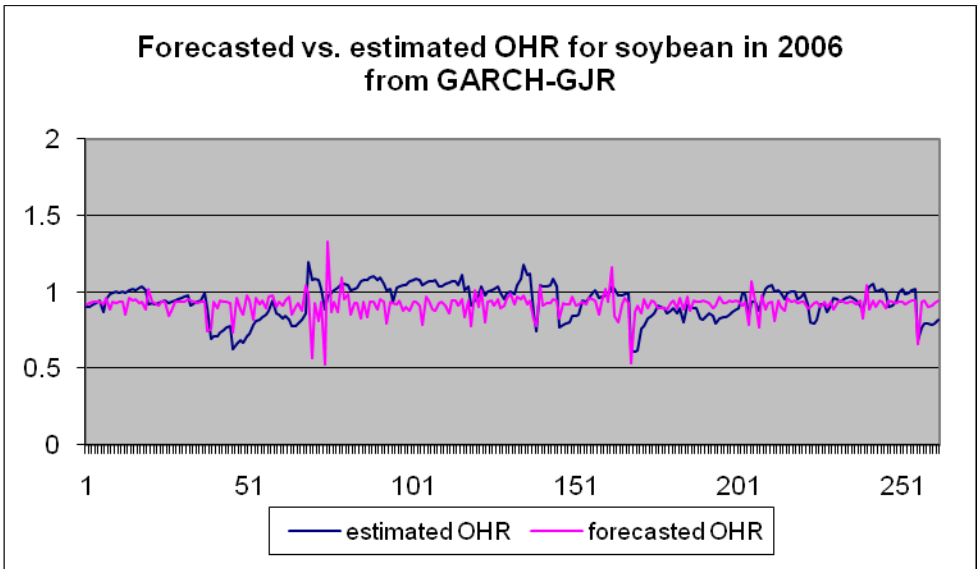
2. soybean

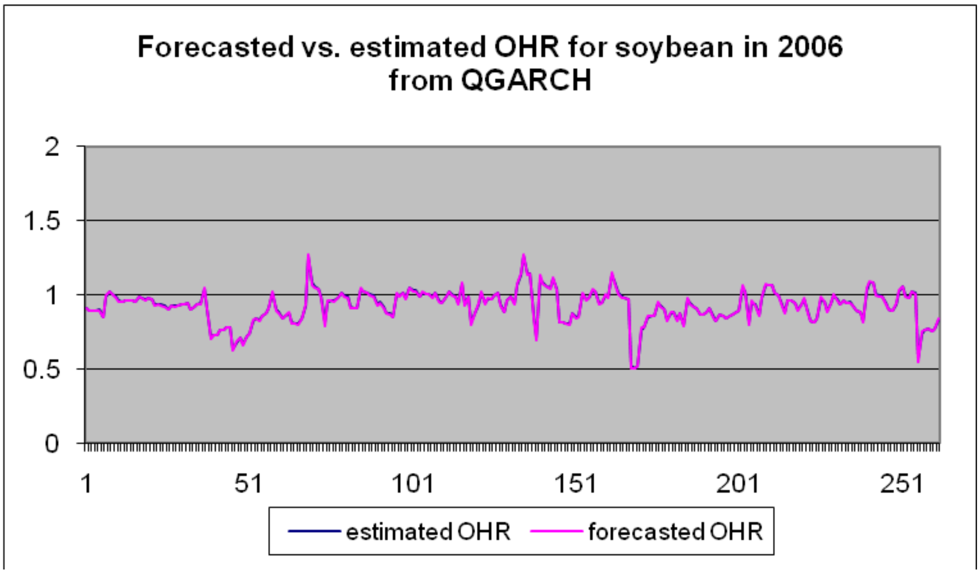

Forecasted vs. estimated OHR for soybean in 2006 from GARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR


Forcasted vs. estimated OHR for soybean in 2006 from BEKK-GARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

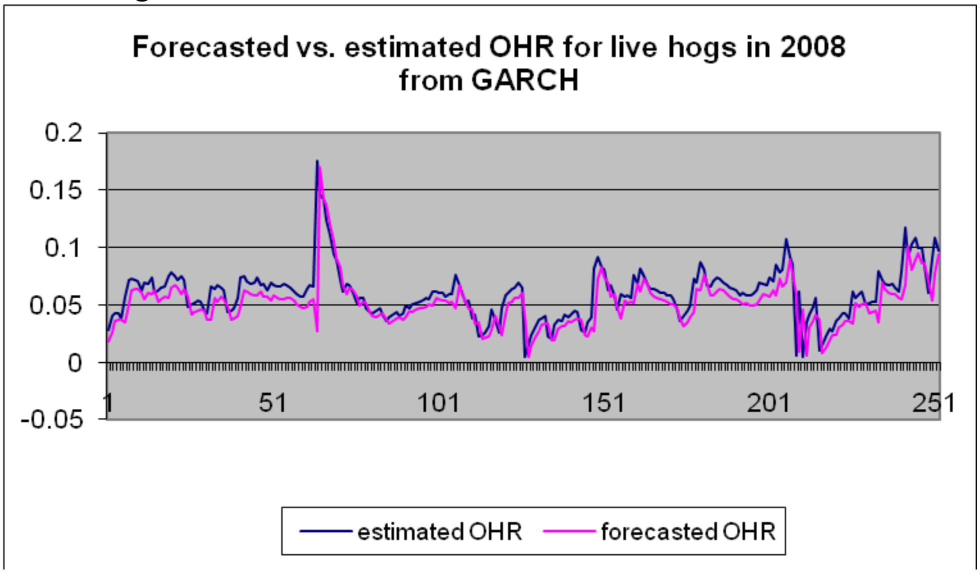

Forecasted vs. estimated OHR for soybean in 2006 from GARCH-X
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

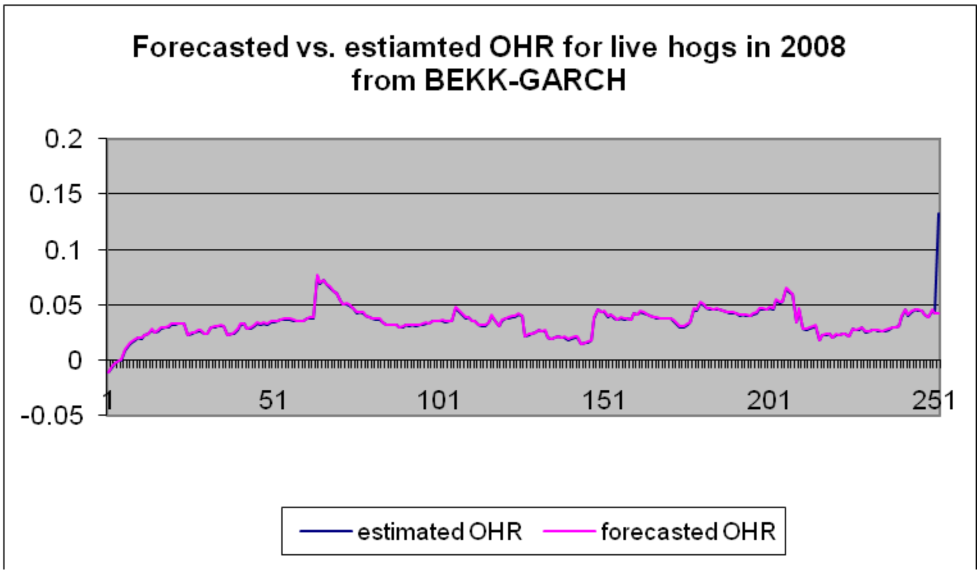

Forecasted vs. estimated OHR for soybean in 2006
from BEKK-GARCH-X
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

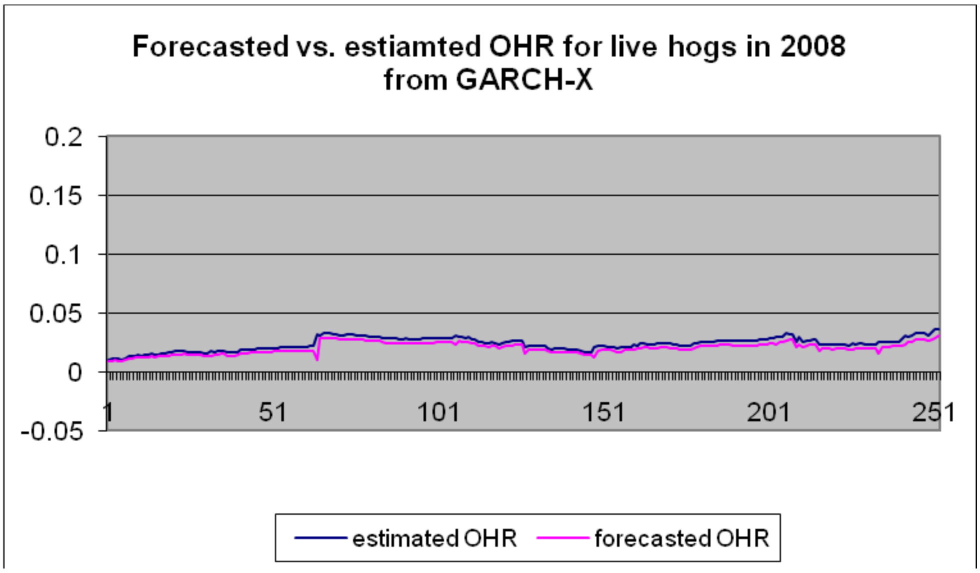

Forecasted vs. estimated OHR for soybean in 2006
from GARCH-GJR
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

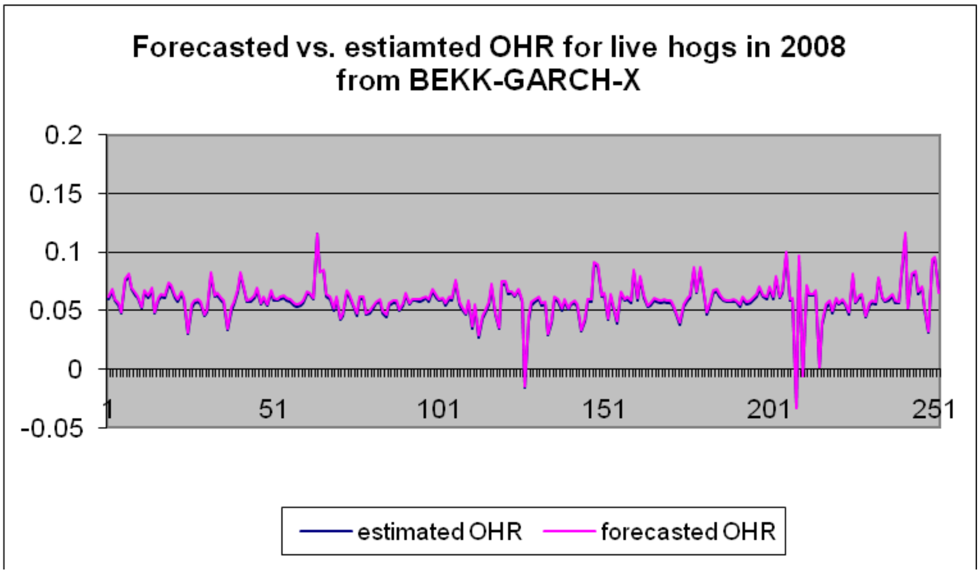

Forecasted vs. estimated OHR for soybean in 2006
from QGARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

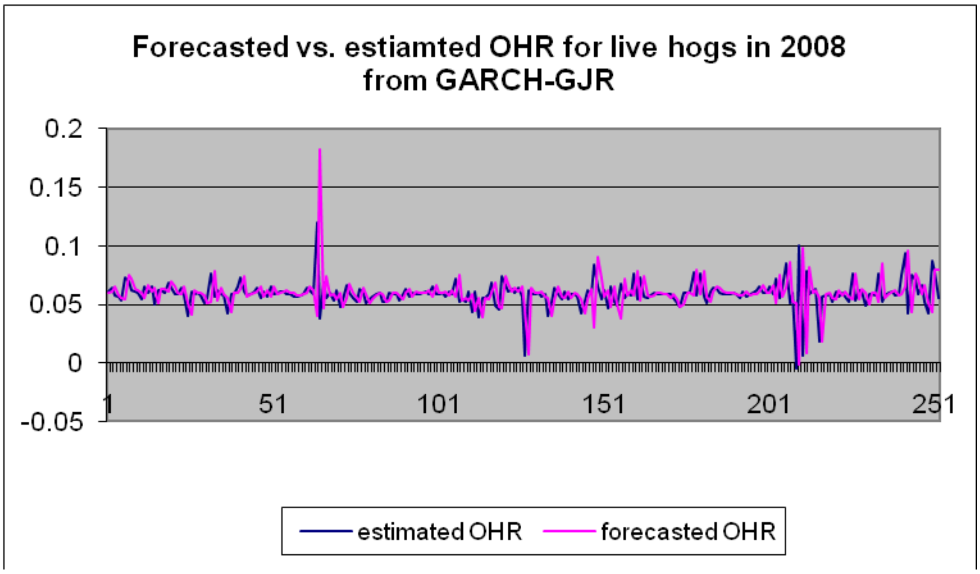
3. live hog

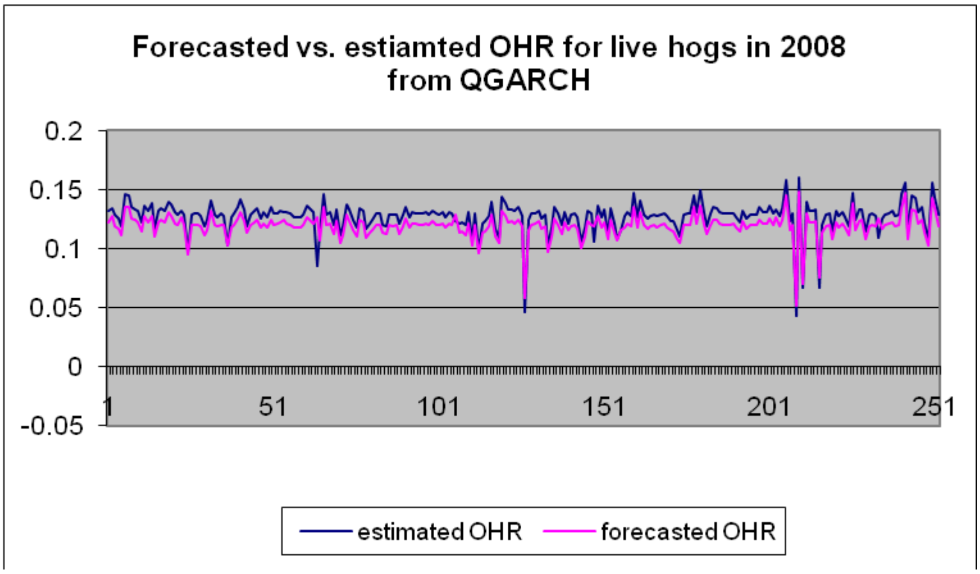

Forecasted vs. estimated OHR for live hogs in 2008
from GARCH
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR


Forecasted vs. estiamted OHR for live hogs in 2008
from BEKK-GARCH
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR


Forecasted vs. estiamted OHR for live hogs in 2008
from GARCH-X
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR

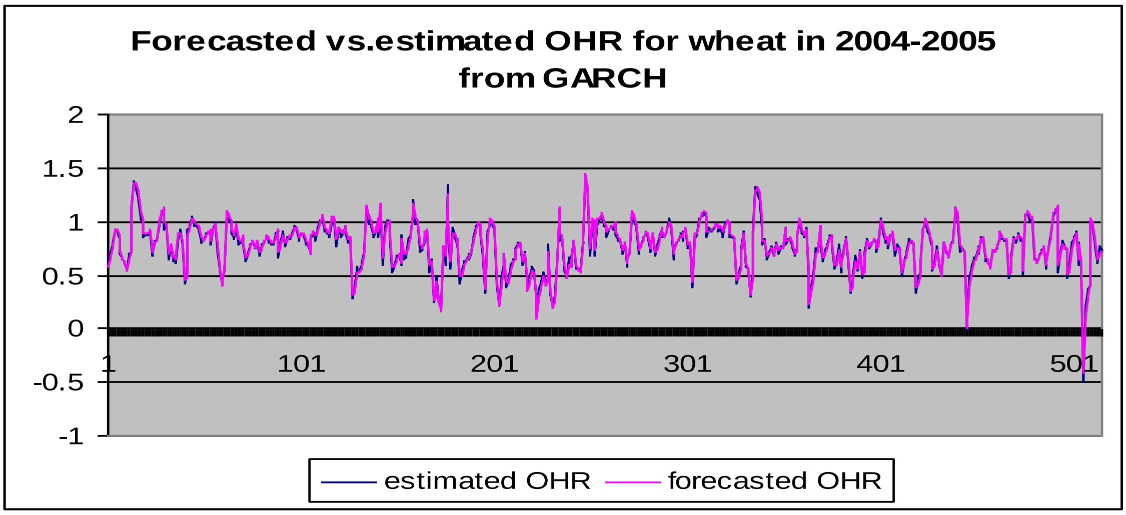

Forecasted vs. estiamted OHR for live hogs in 2008
from BEKK-GARCH-X
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR

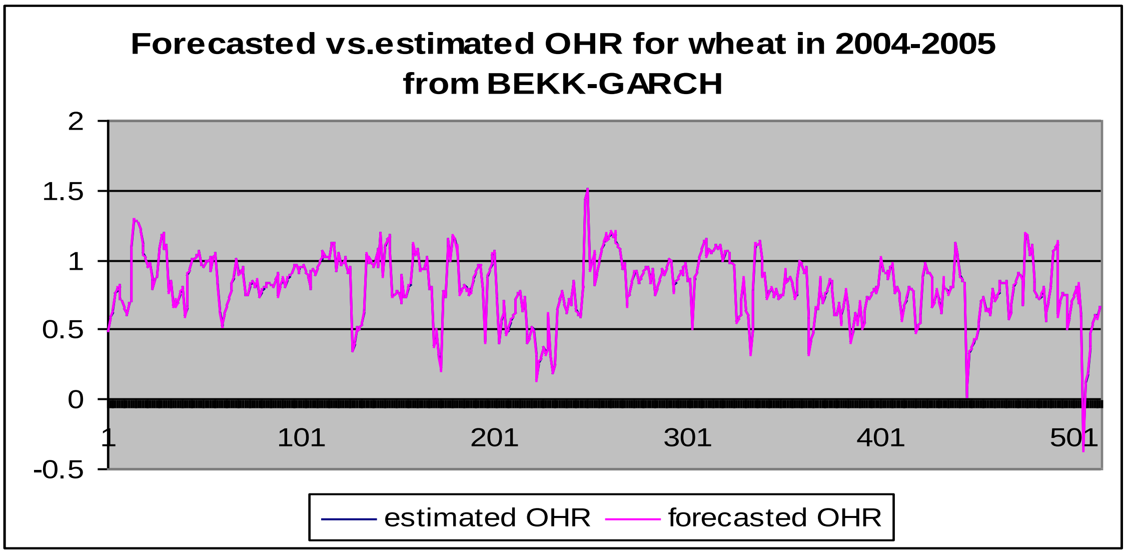

Forecasted vs. estiamted OHR for live hogs in 2008
from GARCH-GJR
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR

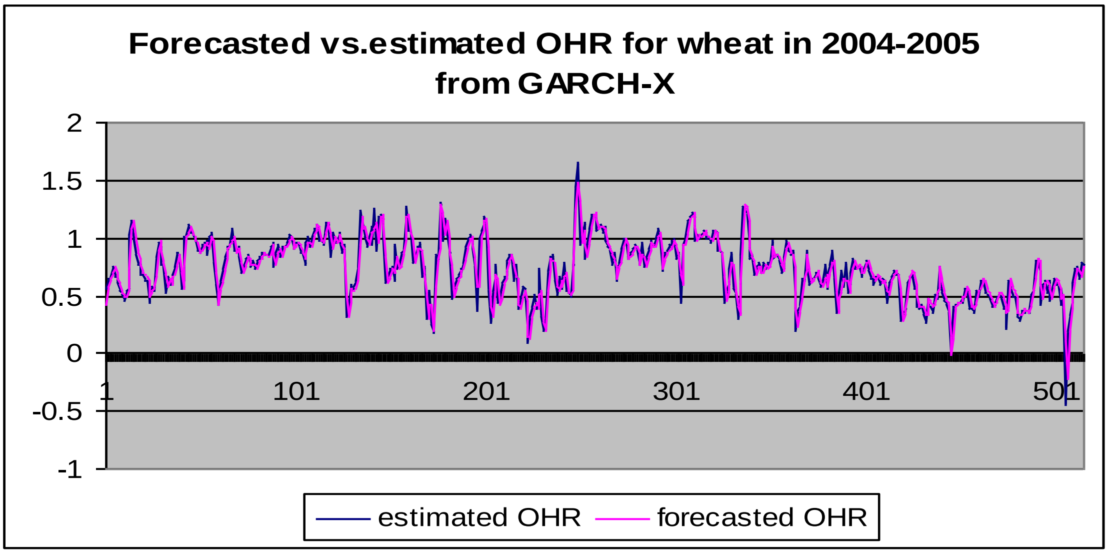

Forecasted vs. estiamted OHR for live hogs in 2008
from QGARCH
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR

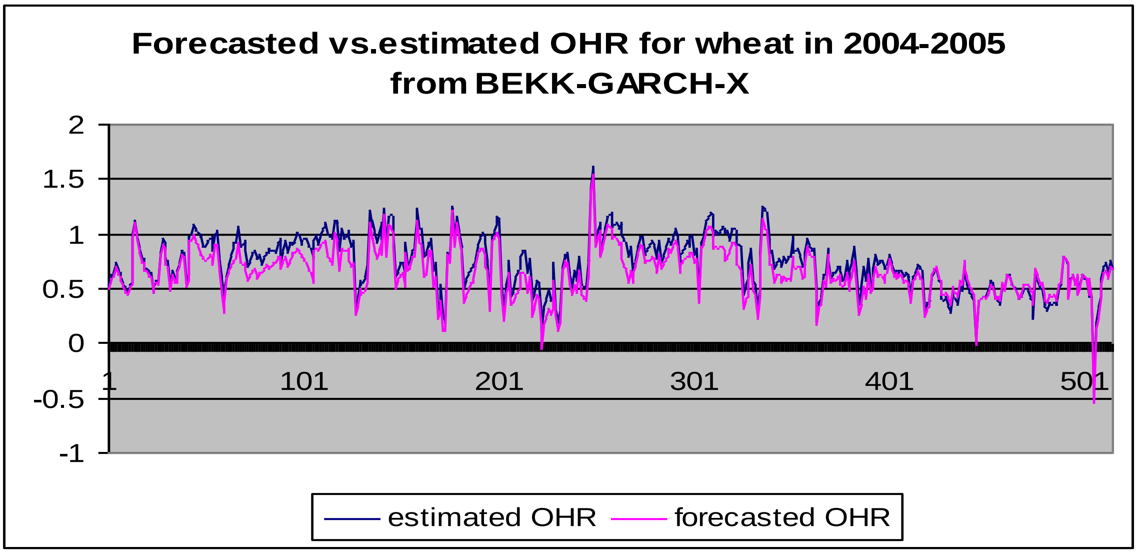
Part 3: 2-year estimated vs. forecasted OHR with normally distributed residuals for wheat, soybean and live hog

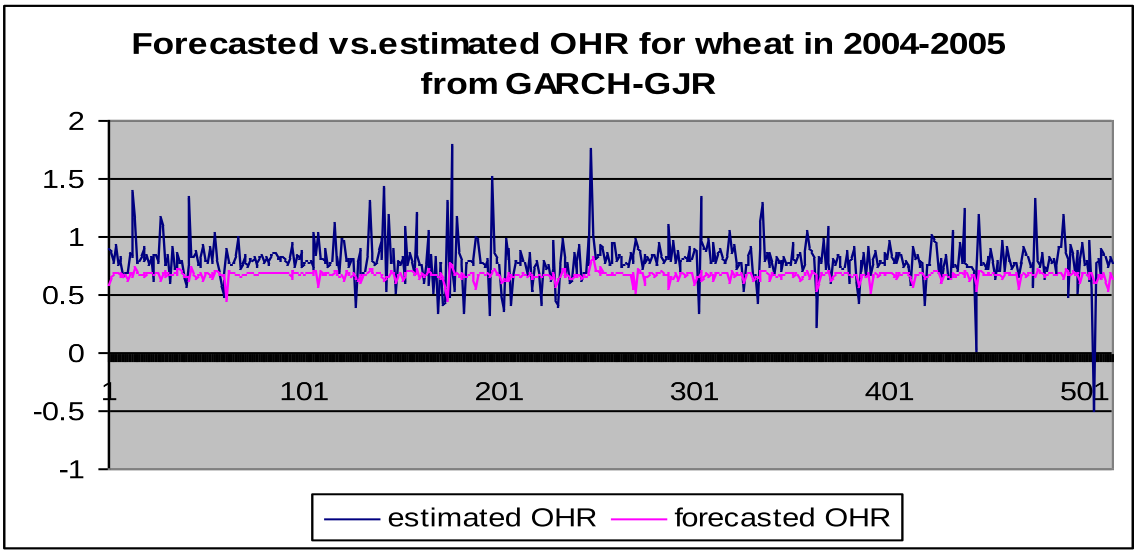
1. wheat

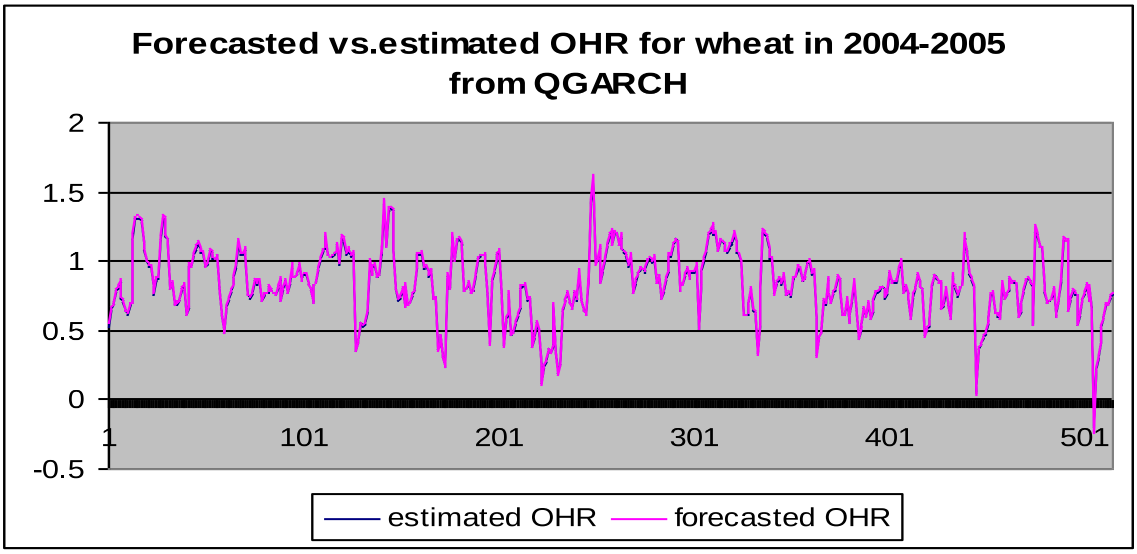




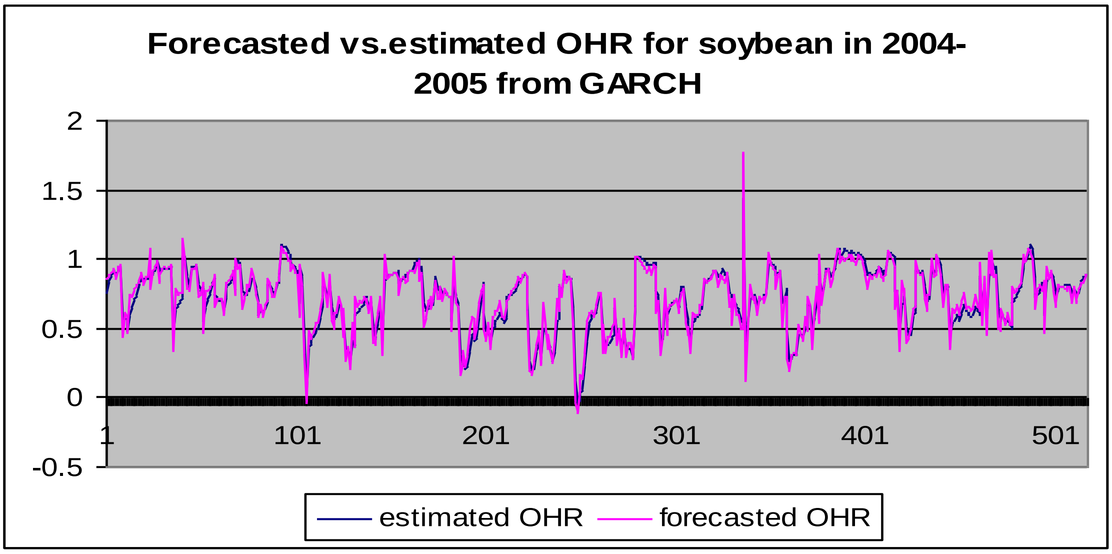


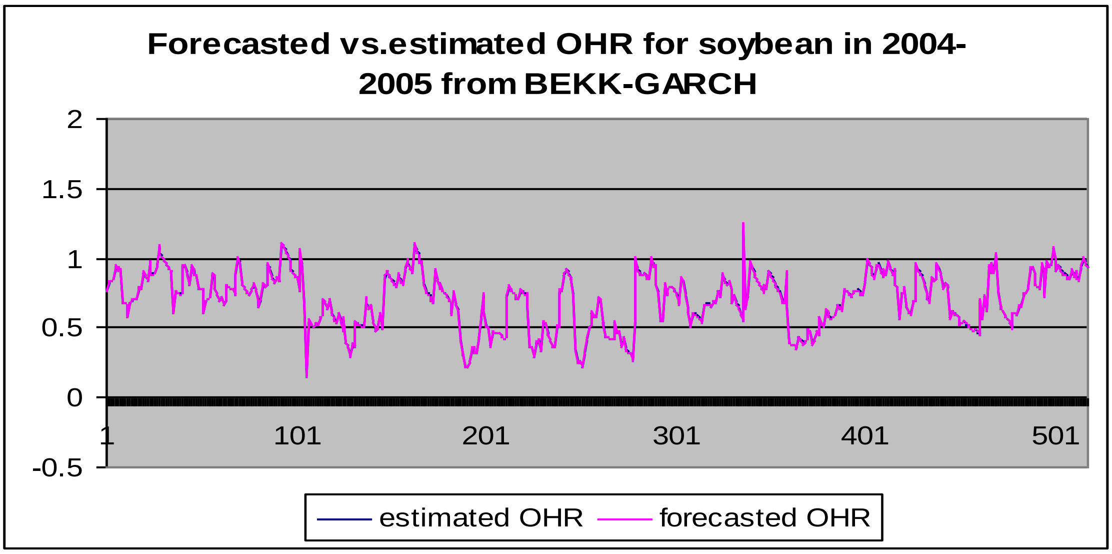

Forecasted vs.estimated OHR for wheat in 2004-2005
from BEKK-GARCH-X
2
1.5
1
0.5
0
-0.5
-1
1
101
201
301
401
501
estimated OHR
forecasted OHR

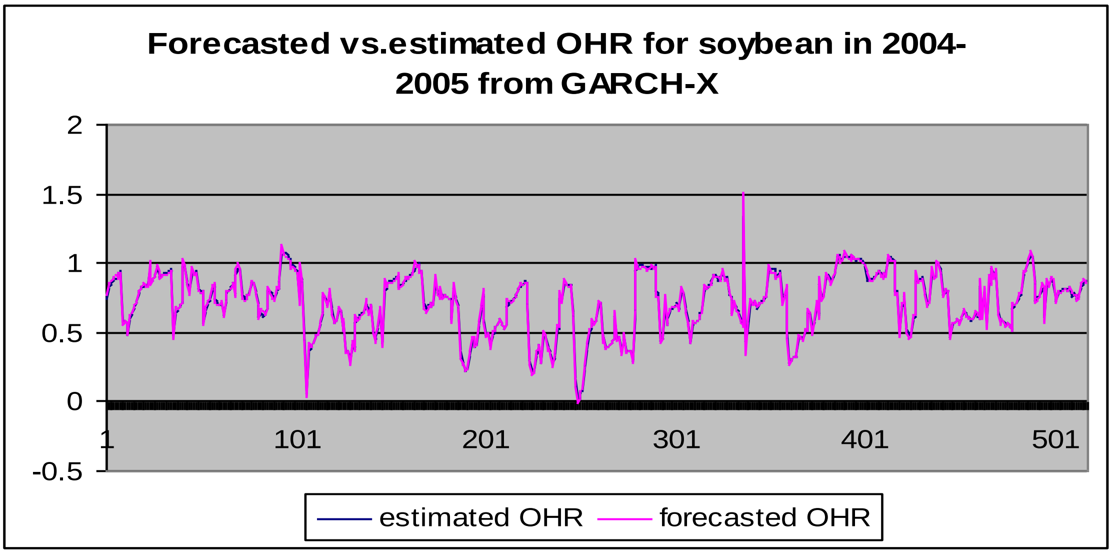

Forecasted vs.estimated OHR for wheat in 2004-2005
from GARCH-GJR
2
1.5
1
0.5
0
-0.5
-1
1
101
201
301
401
501
estimated OHR
forecasted OHR

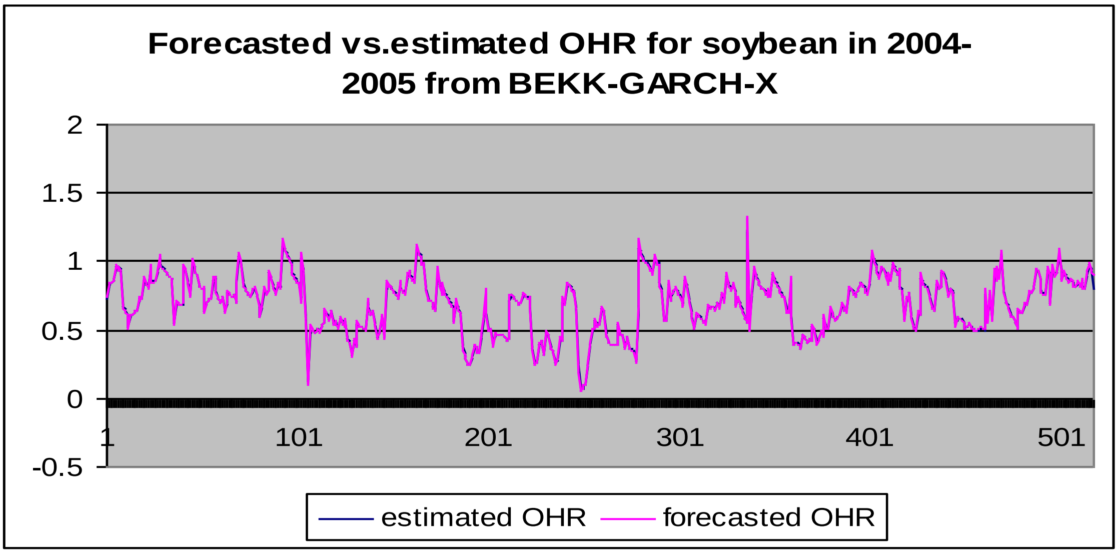

Forecasted vs.estimated OHR for wheat in 2004-2005
from QGARCH
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

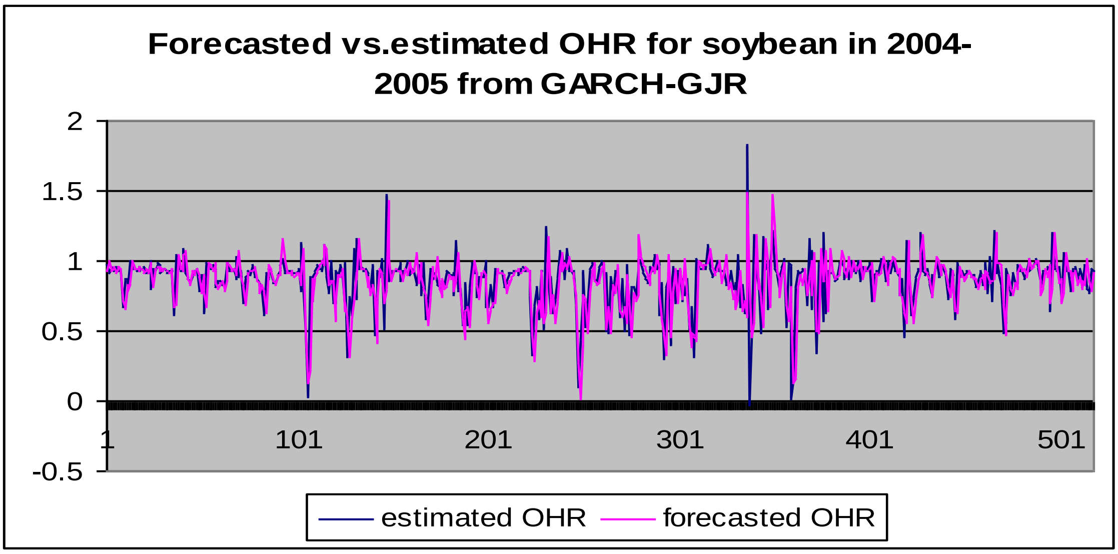
2. soybean

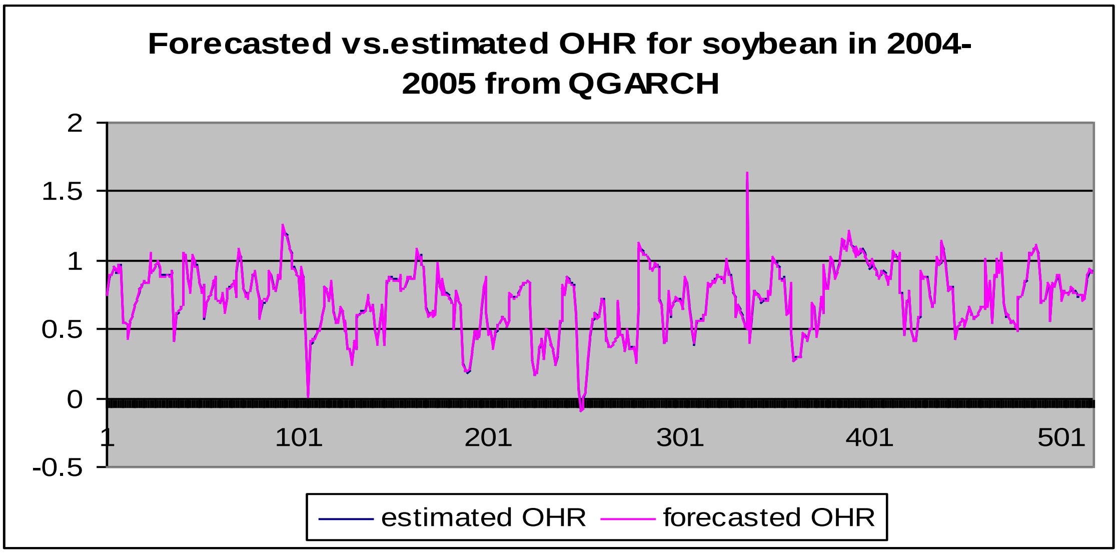

Forecasted vs.estimated OHR for soybean in 2004-2005 from GARCH
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR


Forecasted vs.estimated OHR for soybean in 2004-2005 from BEKK-GARCH
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

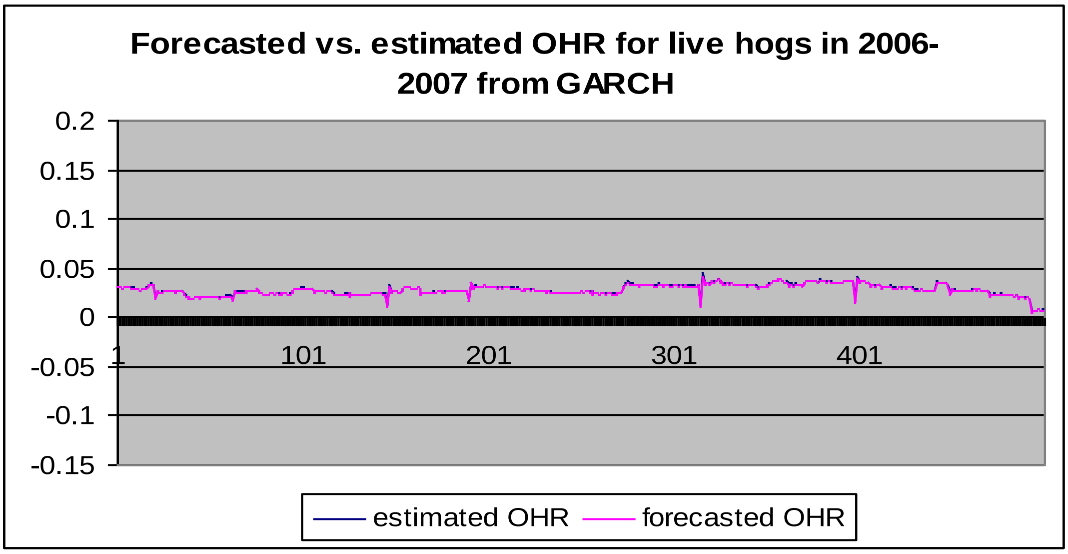

Forecasted vs.estimated OHR for soybean in 2004-2005 from GARCH-X
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

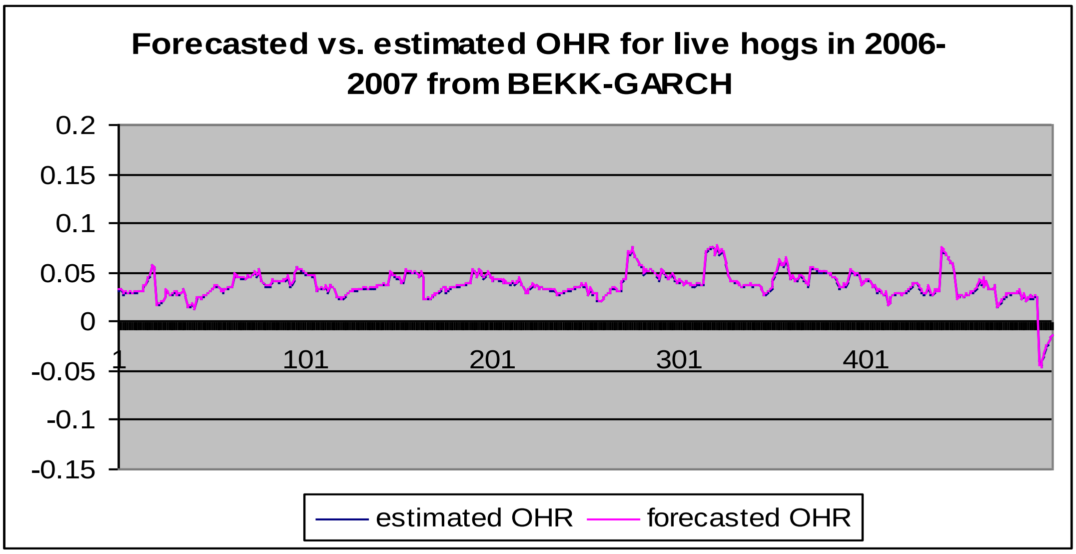

Forecasted vs.estimated OHR for soybean in 2004-2005 from BEKK-GARCH-X
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

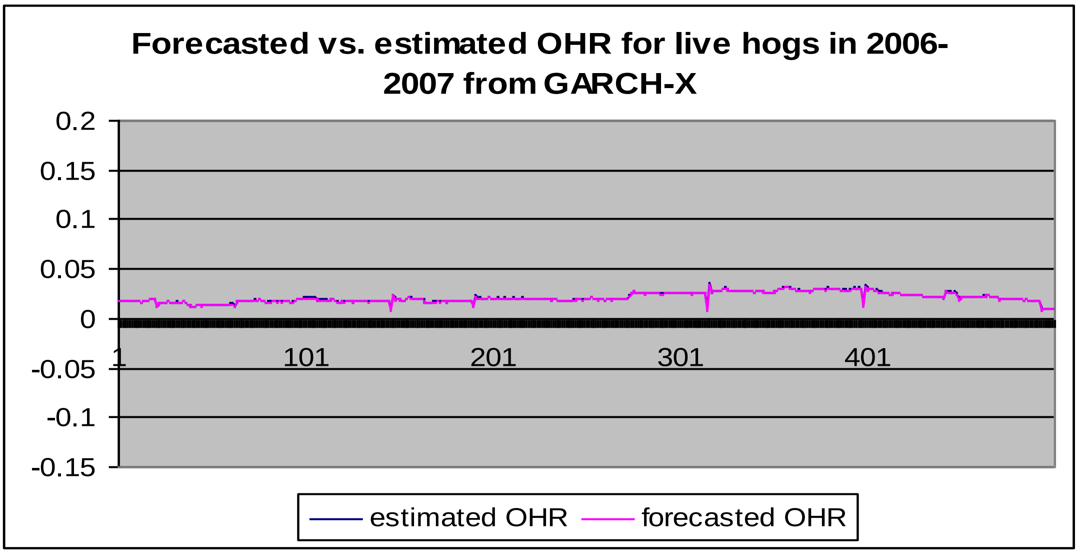

Forecasted vs.estimated OHR for soybean in 2004-2005 from GARCH-GJR
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

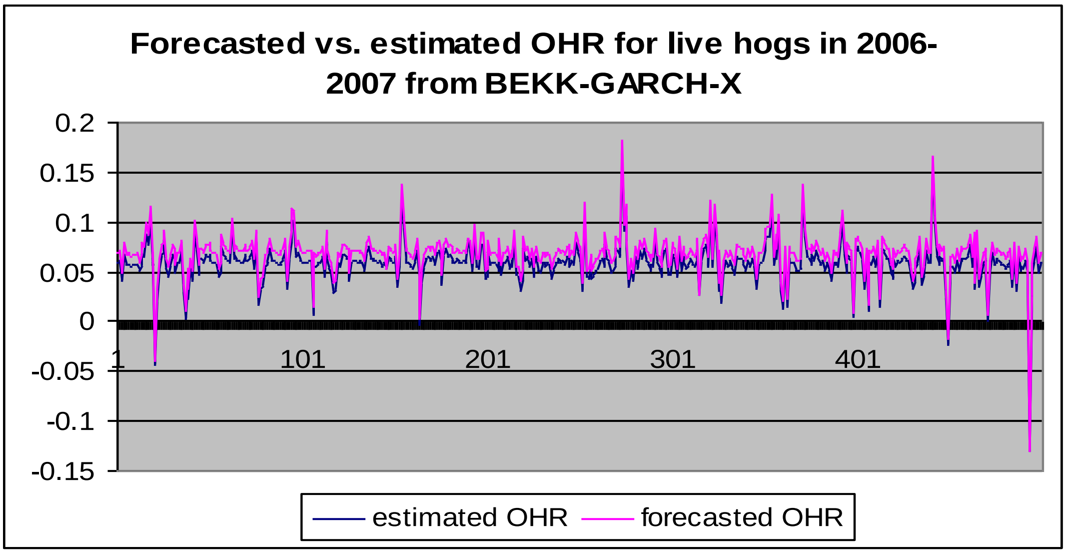

Forecasted vs.estimated OHR for soybean in 2004-2005 from QGARCH
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

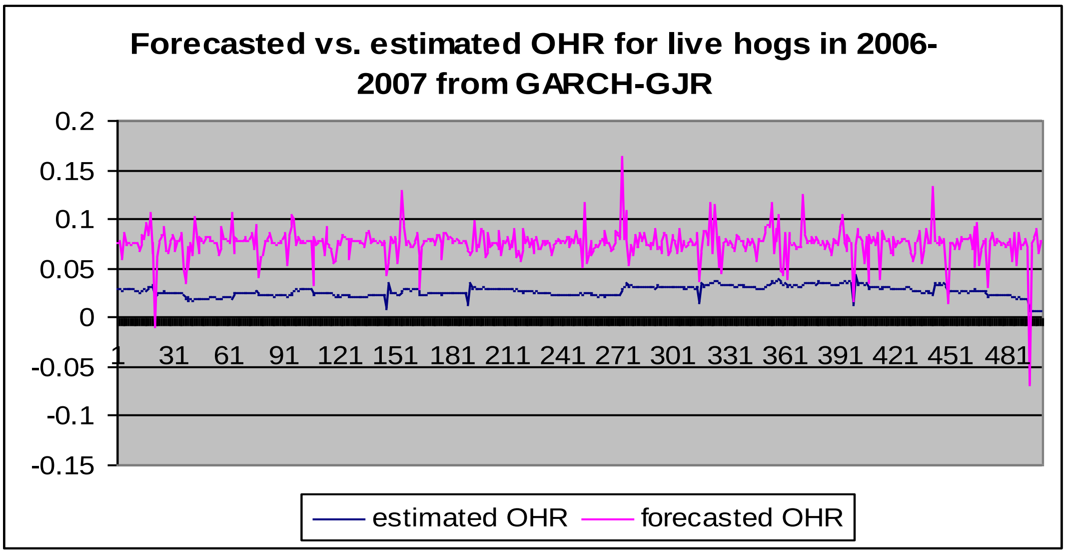
3. live hog


Forecasted vs. estimated OHR for live hogs in 2006-2007 from GARCH
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR for live hogs in 2006-2007 from BEKK-GARCH
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR

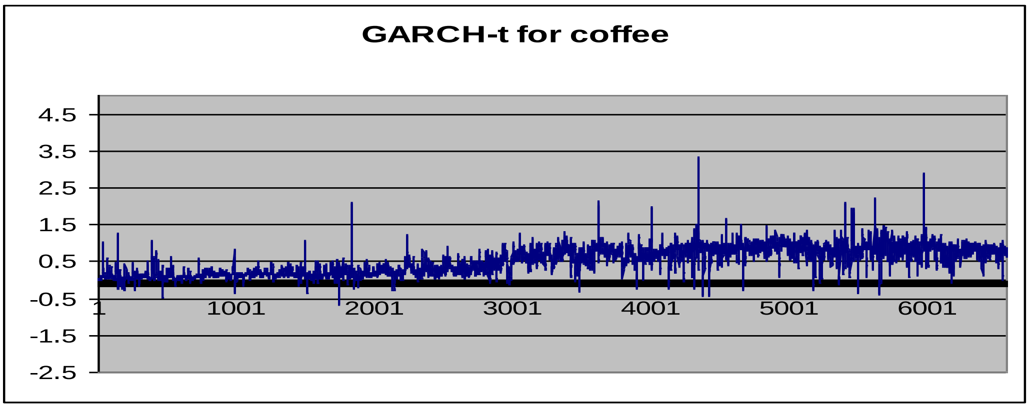

Forecasted vs. estimated OHR for live hogs in 2006-2007 from GARCH-X
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR

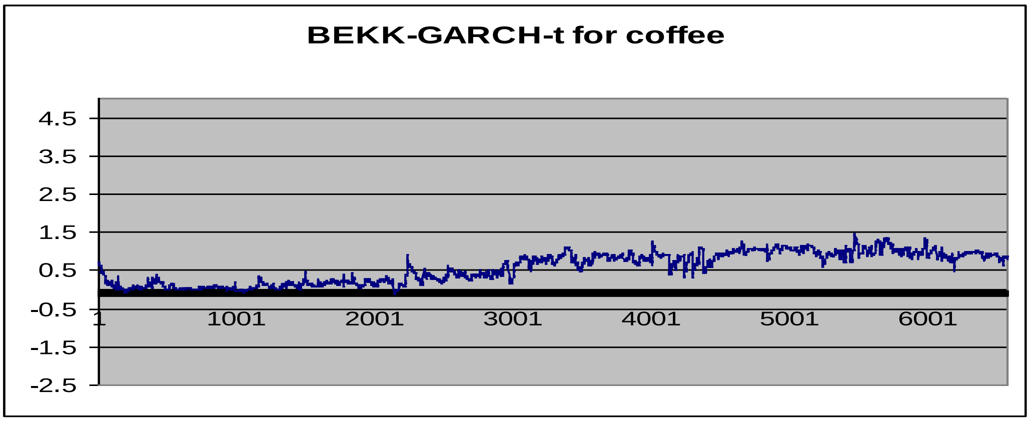

Forecasted vs. estimated OHR for live hogs in 2006-2007 from BEKK-GARCH-X
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR

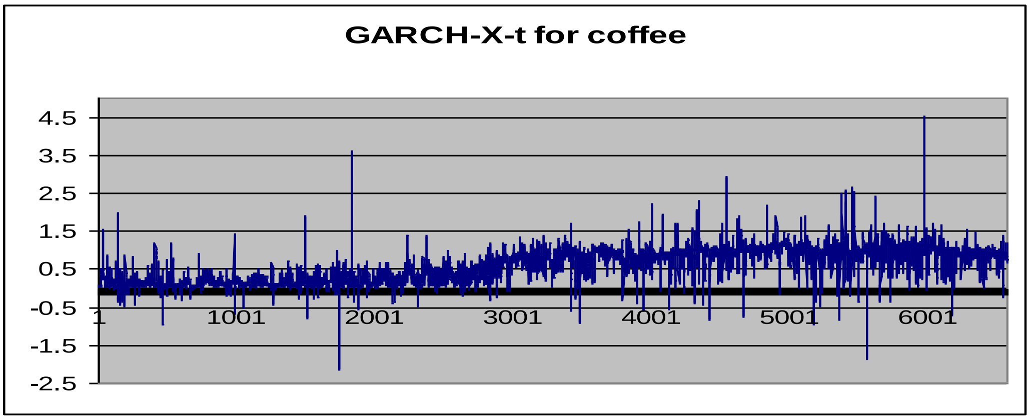

Forecasted vs. estimated OHR for live hogs in 2006-2007 from GARCH-GJR
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1 31 61 91 121 151 181 211 241 271 301 331 361 391 421 451 481
estimated OHR
forecasted OHR

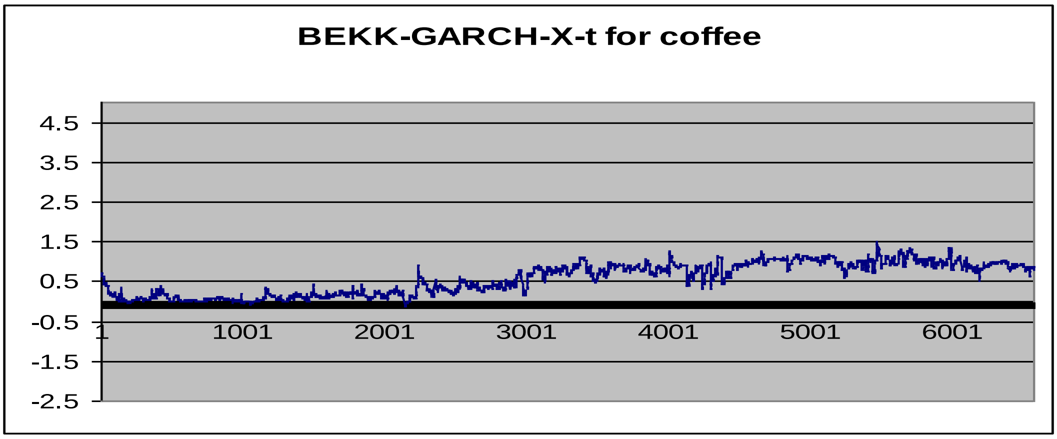
Part 4: Graphs for estimated OHR with student's *t* distributed residuals in full sample for coffee, soybean and live hog

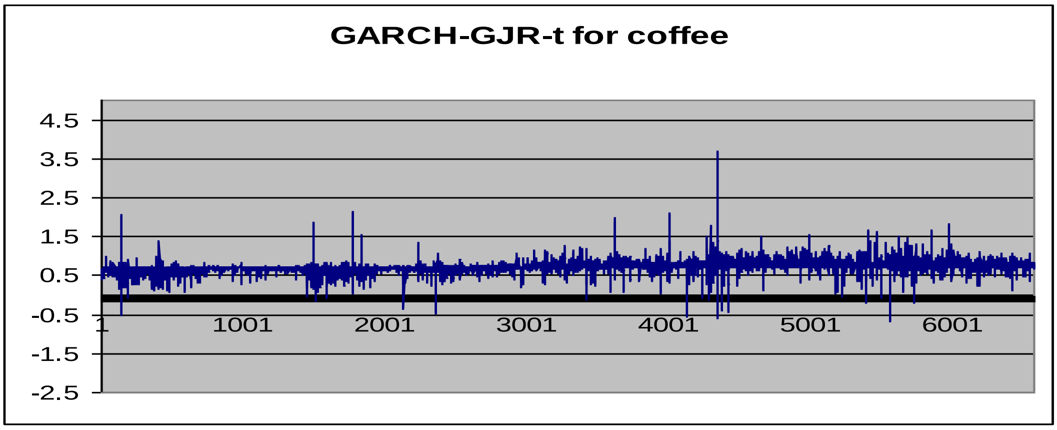
1. **coffee**

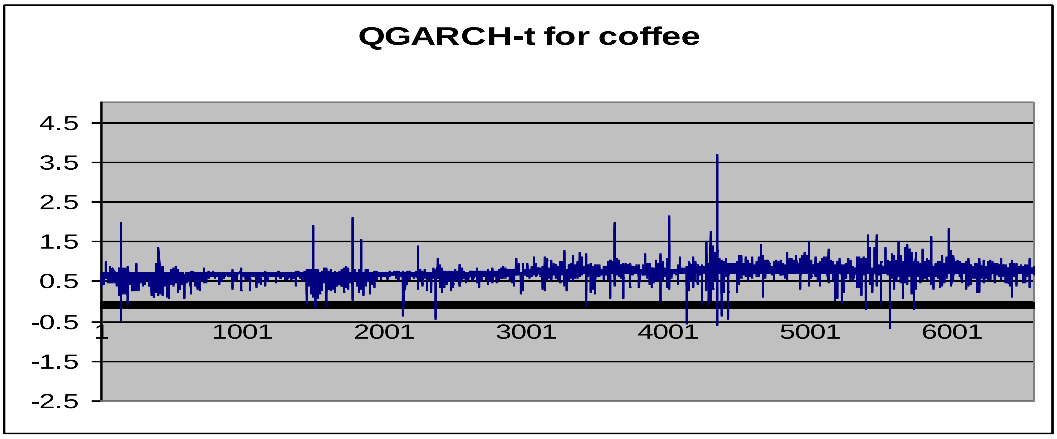




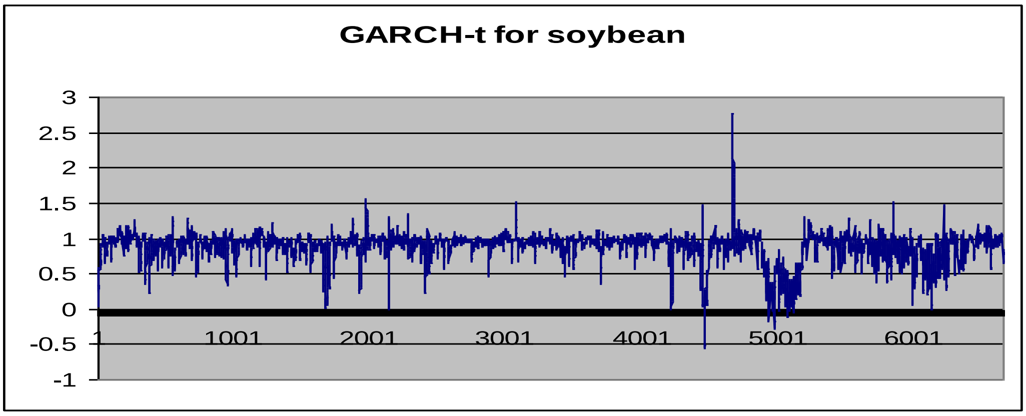


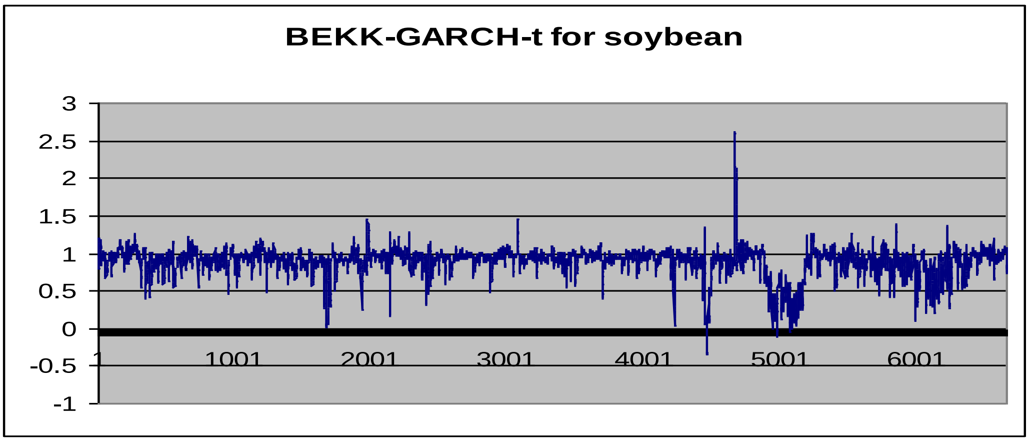


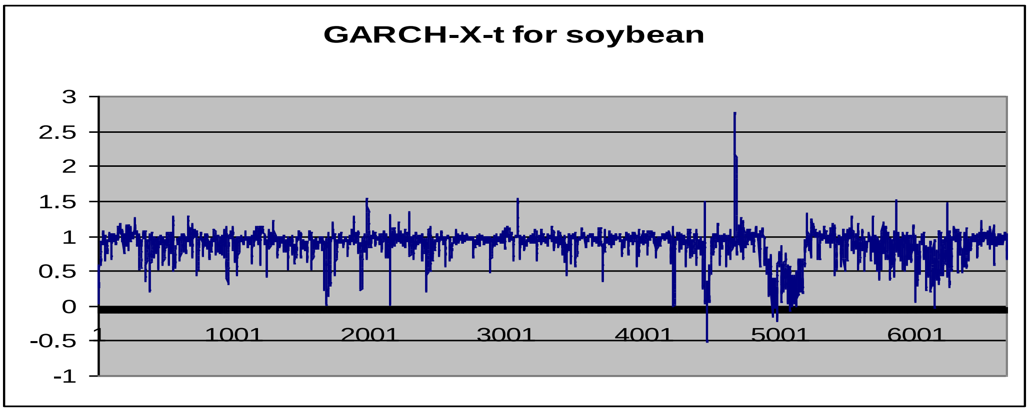

GARCH-GJR-t for coffee
4.5
3.5
2.5
1.5
0.5
-0.5
-1.5
-2.5
1
1001
2001
3001
4001
5001
6001

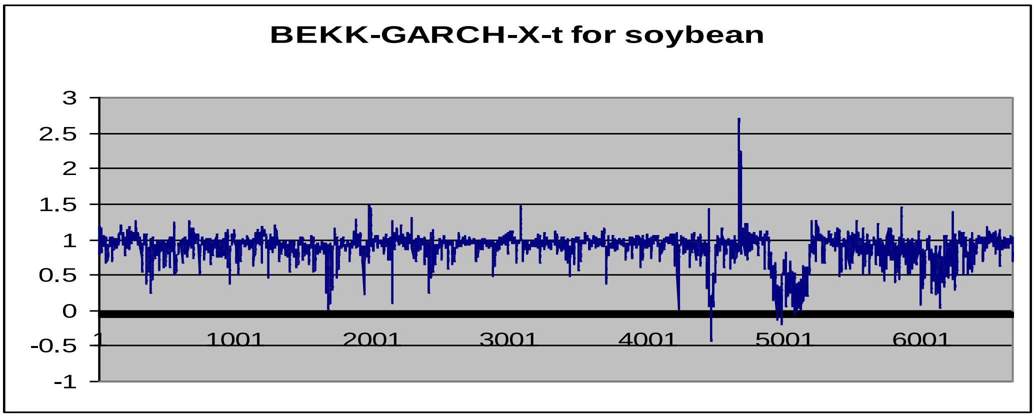

QGARCH-t for coffee
4.5
3.5
2.5
1.5
0.5
-0.5
-1.5
-2.5
1
1001
2001
3001
4001
5001
6001

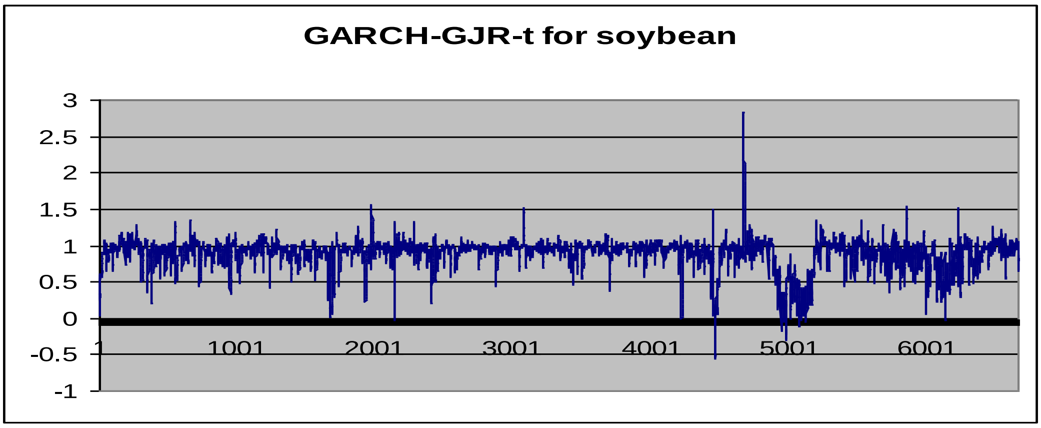
2. soybean

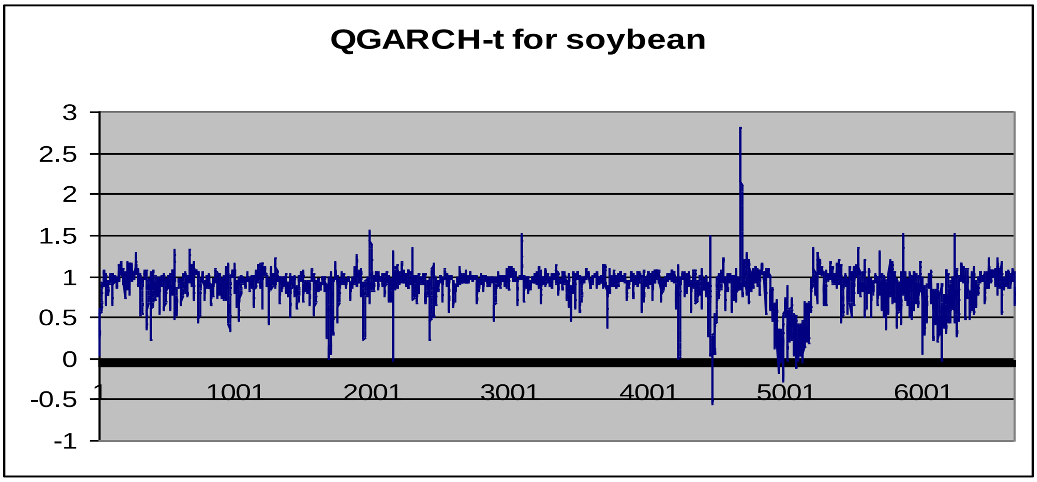

GARCH-t for soybean
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001


BEKK-GARCH-t for soybean
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001

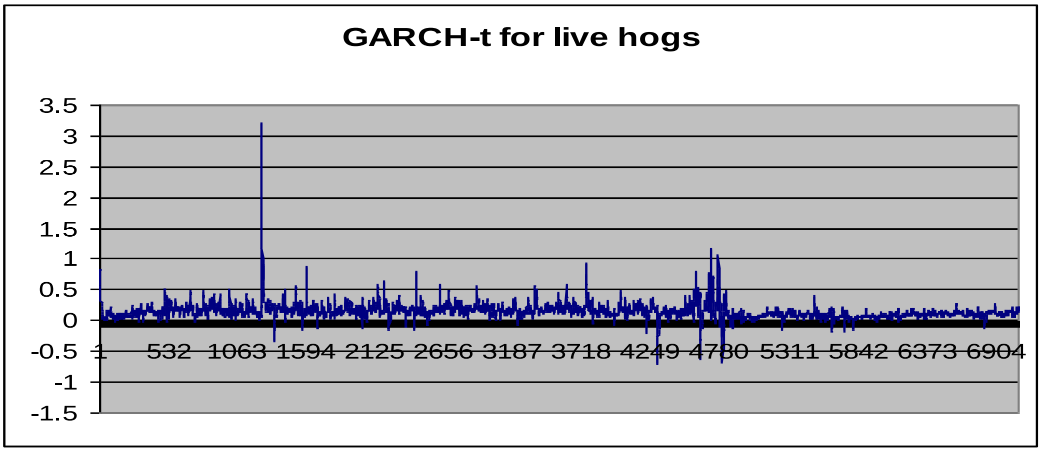

GARCH-X-t for soybean
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001

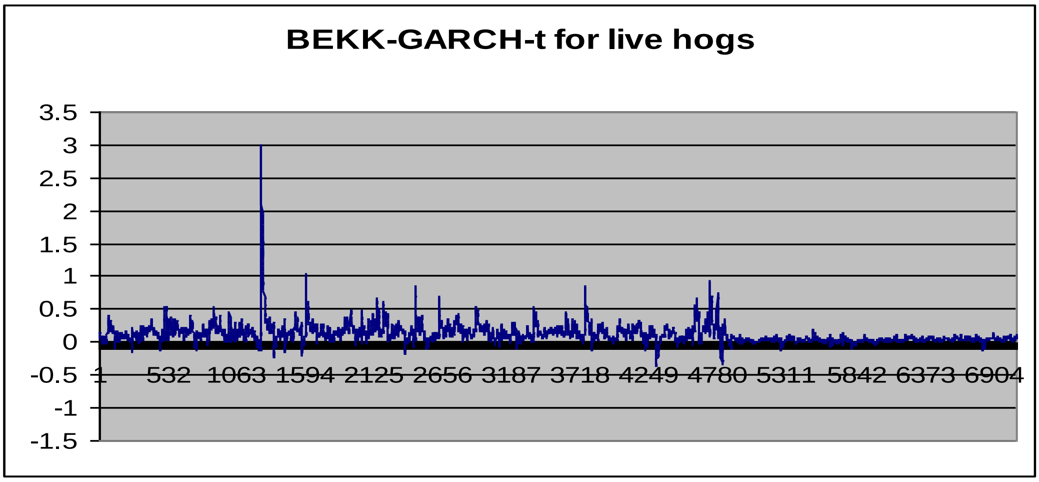

BEKK-GARCH-X-t for soybean
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001

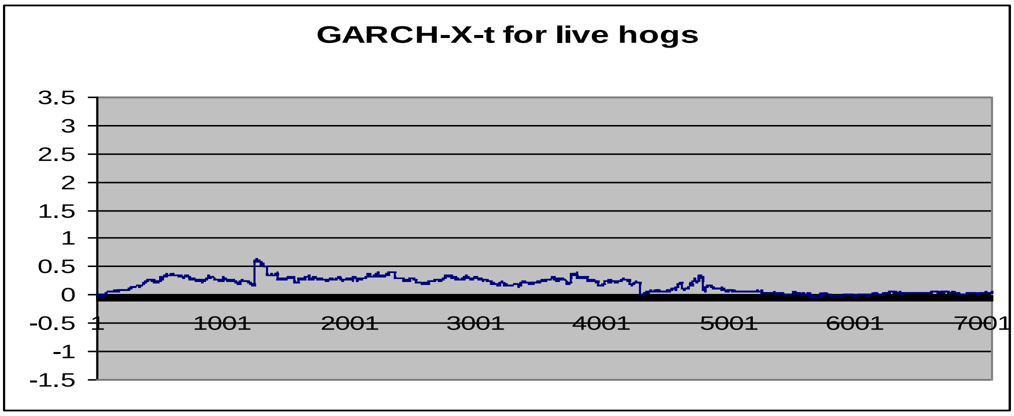

GARCH-GJR-t for soybean
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001

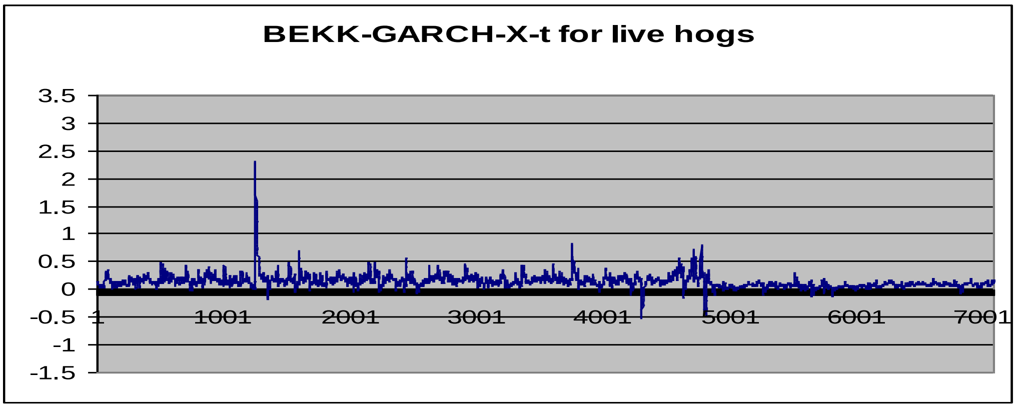

QGARCH-t for soybean
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
1001
2001
3001
4001
5001
6001

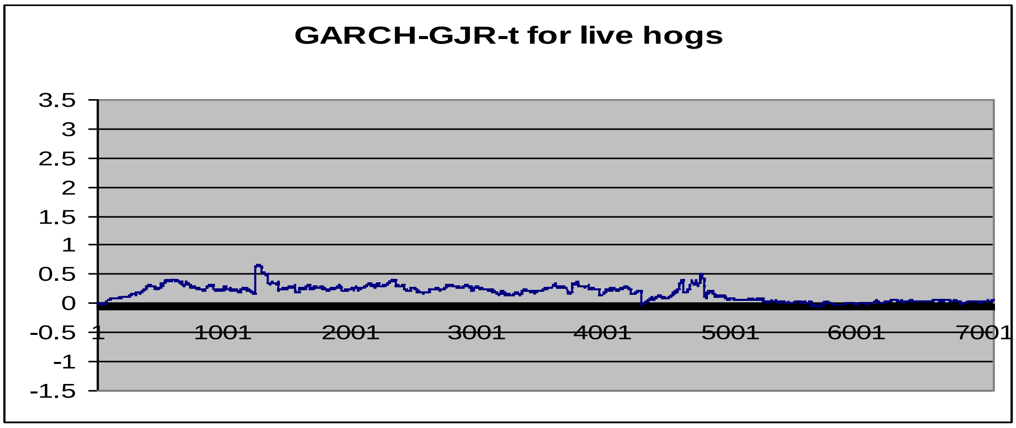
3. live hog

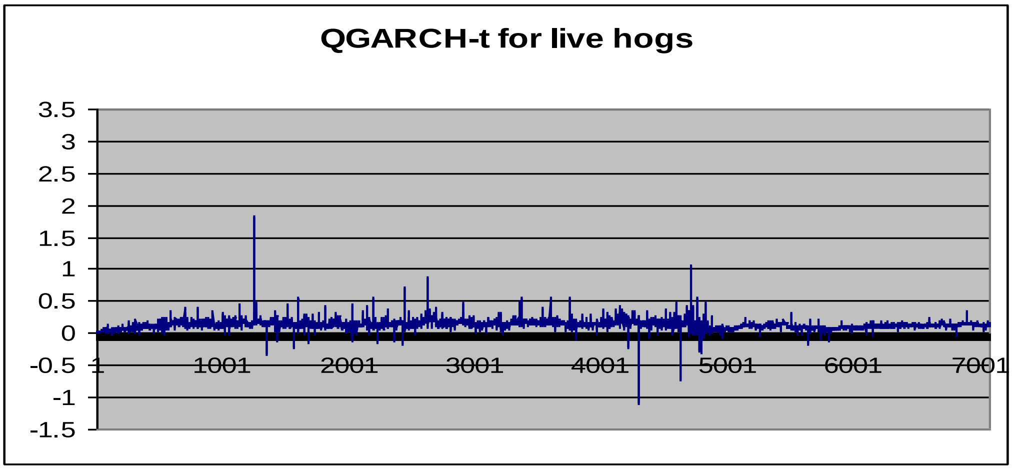

GARCH-t for live hogs
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
1 532 1063 1594 2125 2656 3187 3718 4249 4780 5311 5842 6373 6904


BEKK-GARCH-t for live hogs
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
1 532 1063 1594 2125 2656 3187 3718 4249 4780 5311 5842 6373 6904


GARCH-X-t for live hogs
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
1 1001 2001 3001 4001 5001 6001 7001

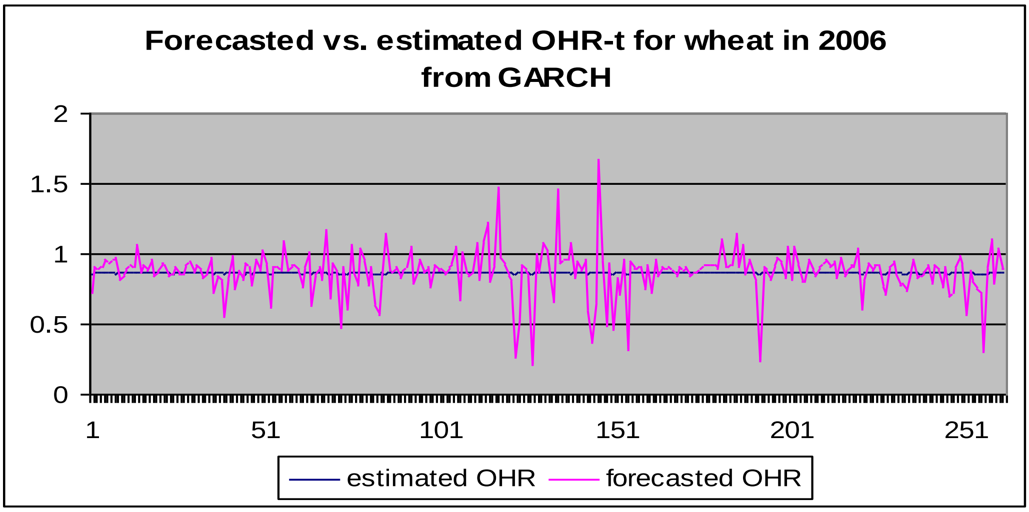

BEKK-GARCH-X-t for live hogs
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
1 1001 2001 3001 4001 5001 6001 7001

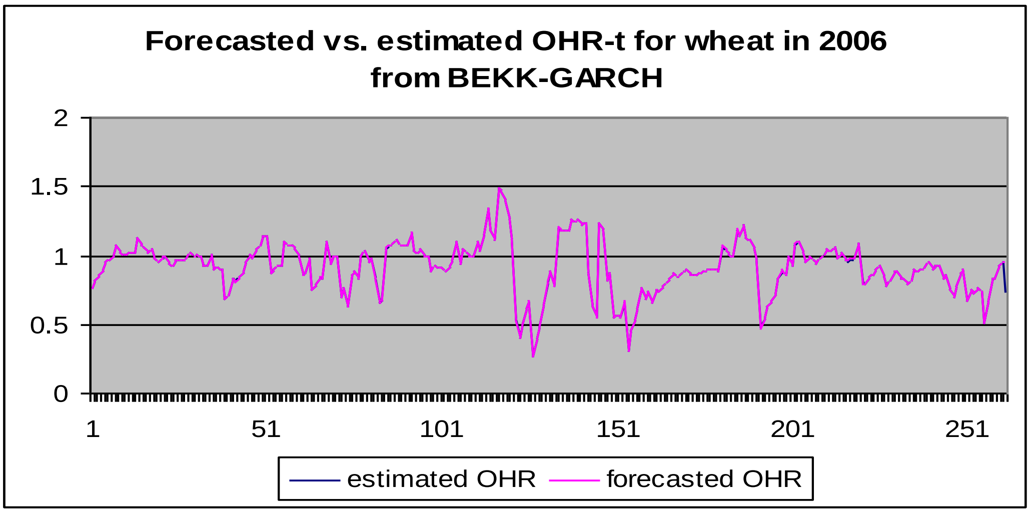


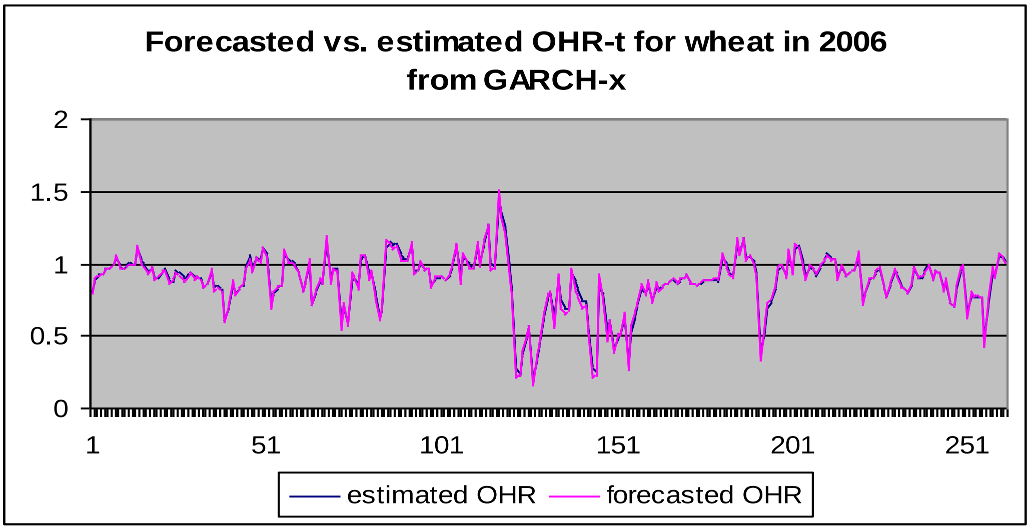


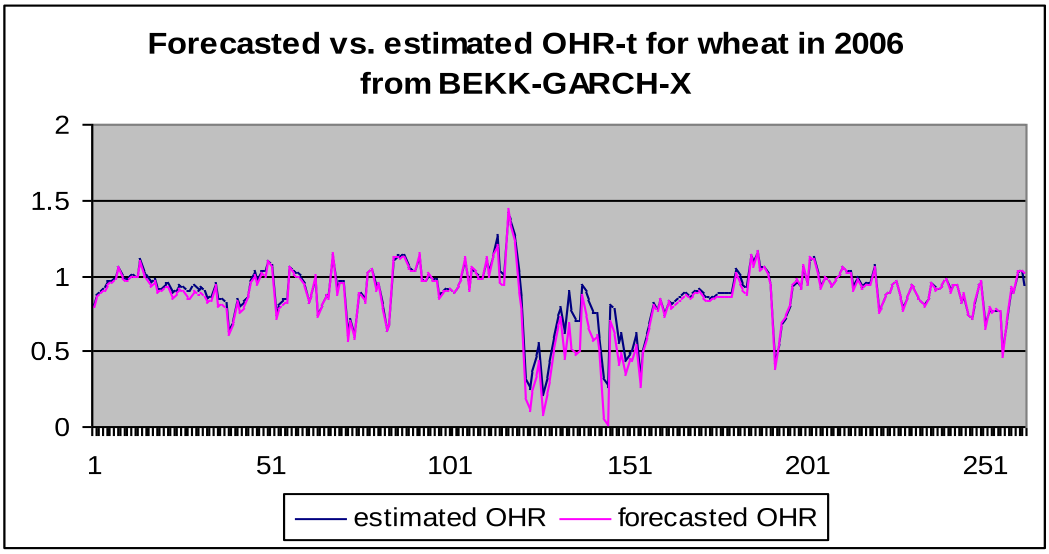
Part 5: 1-year estimated vs. forecasted OHR with student's t distributed residuals for wheat, soybean and live hog

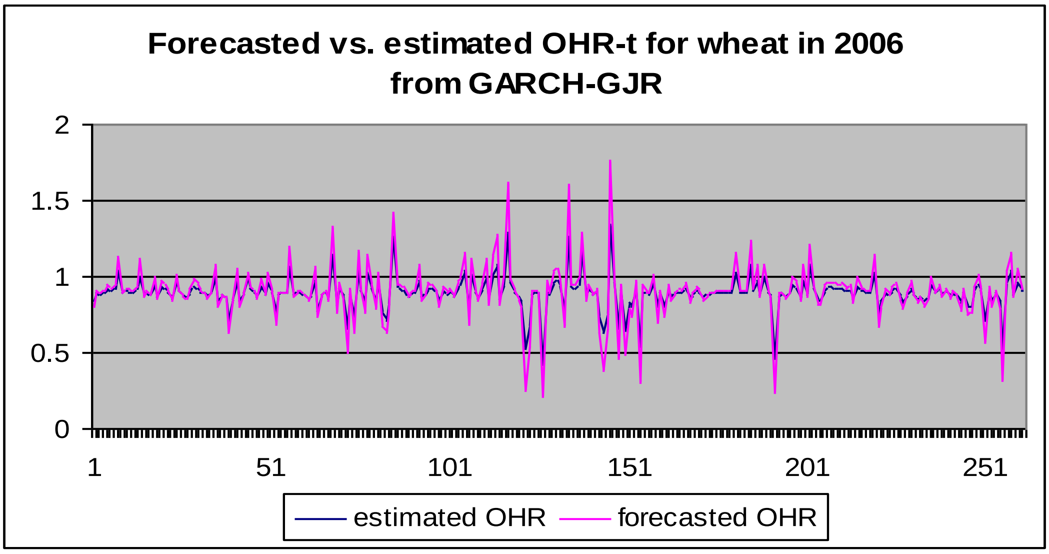
1. **wheat**

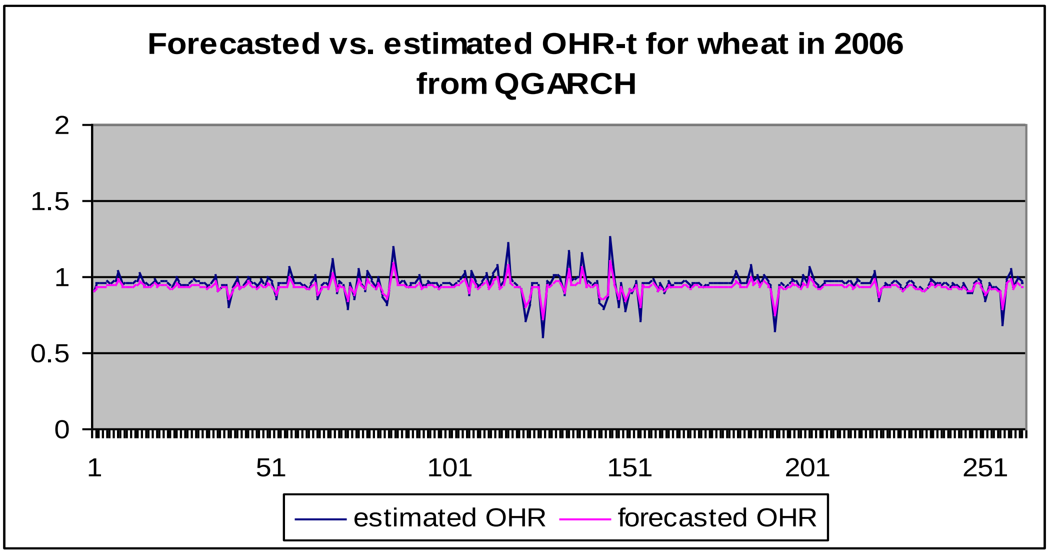



Forecasted vs. estimated OHR-t for wheat in 2006
from BEKK-GARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

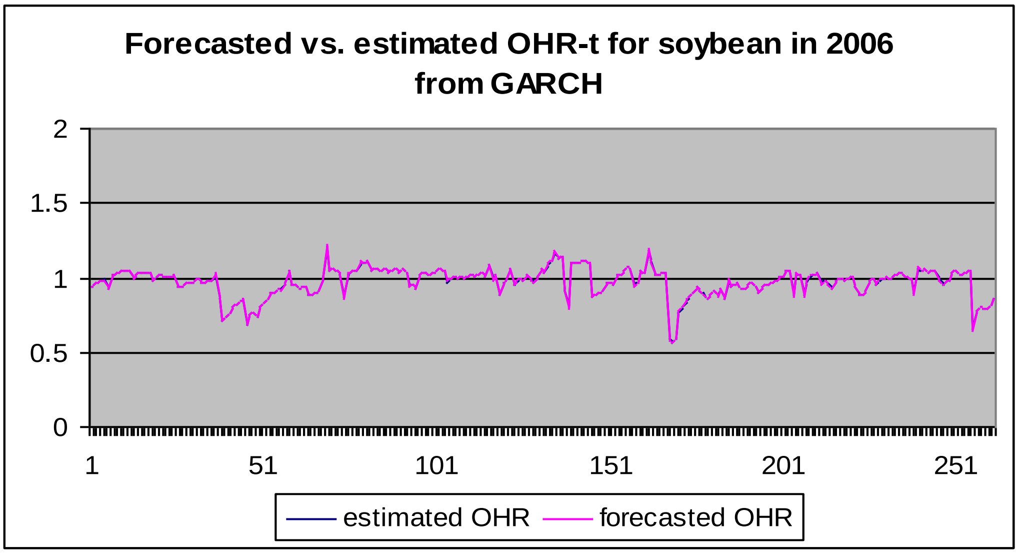

Forecasted vs. estimated OHR-t for wheat in 2006
from GARCH-x
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

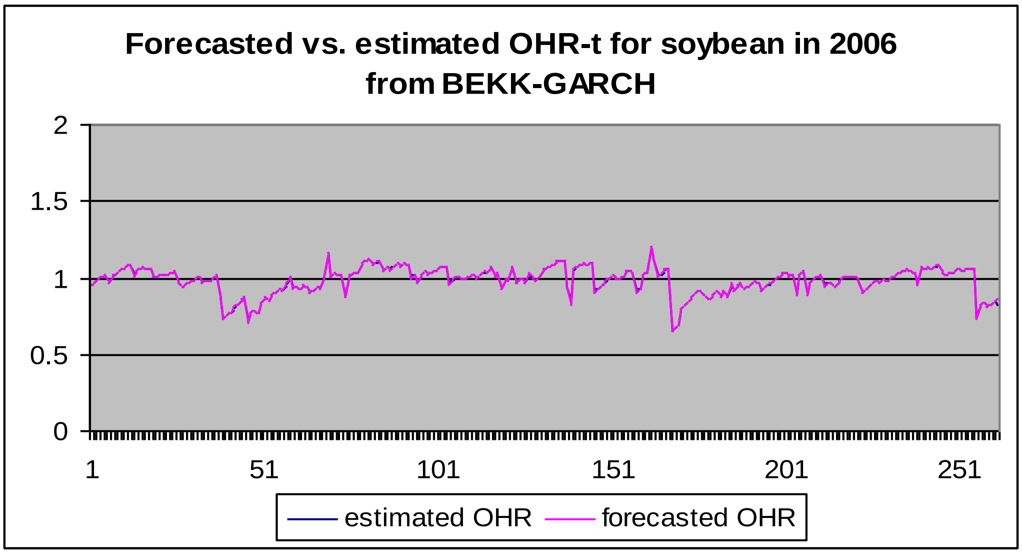

Forecasted vs. estimated OHR-t for wheat in 2006
from BEKK-GARCH-X
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

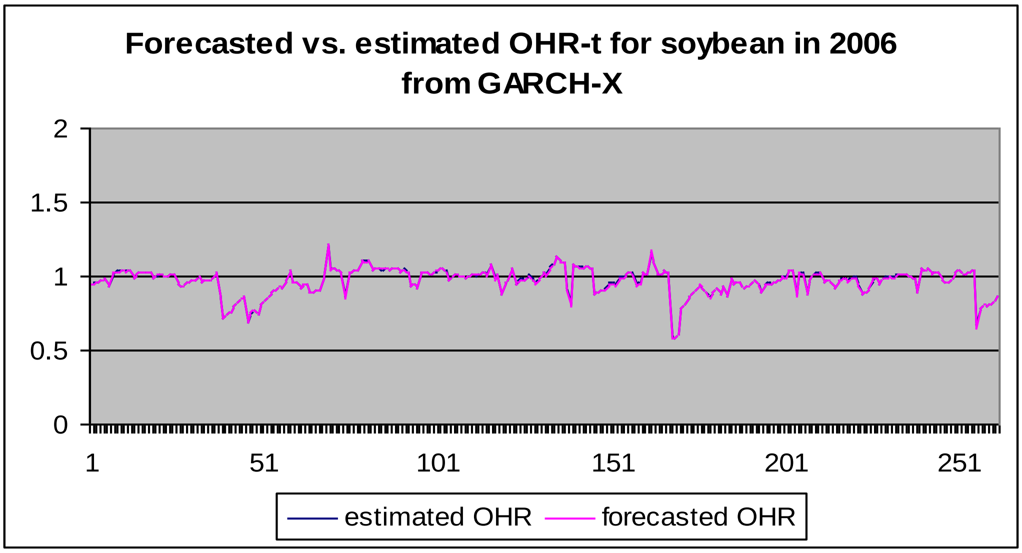

Forecasted vs. estimated OHR-t for wheat in 2006 from GARCH-GJR
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

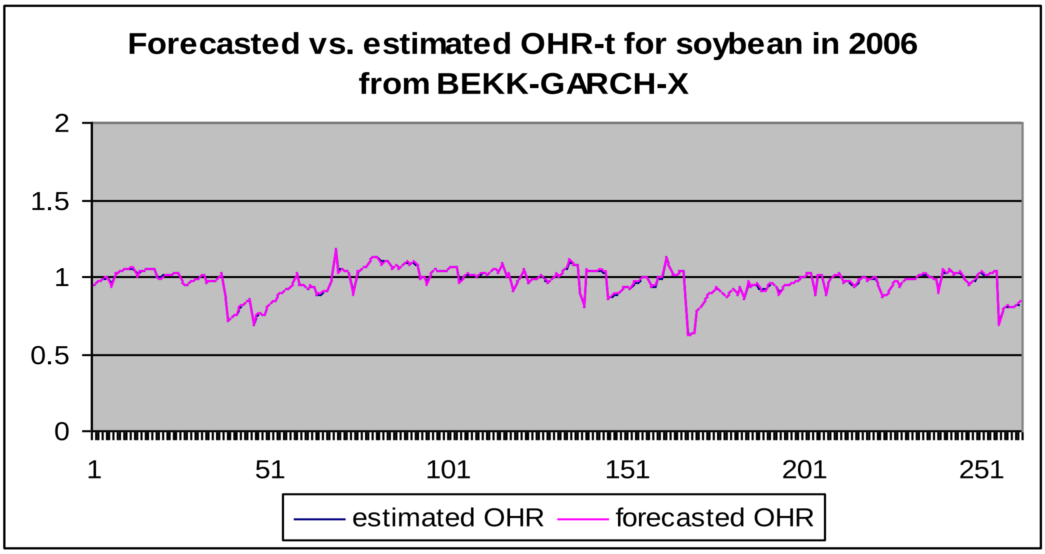

Forecasted vs. estimated OHR-t for wheat in 2006 from QGARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

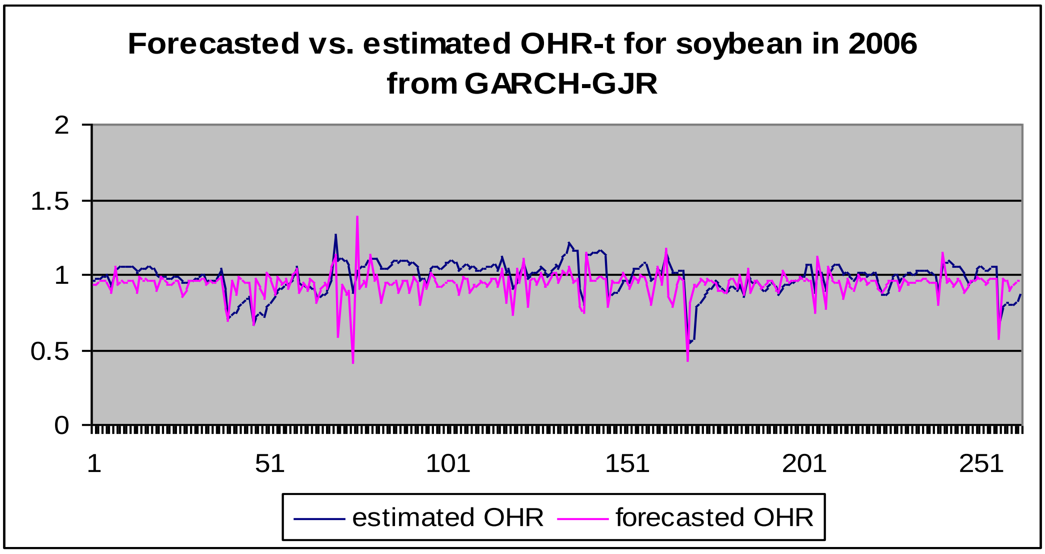
2. soybean

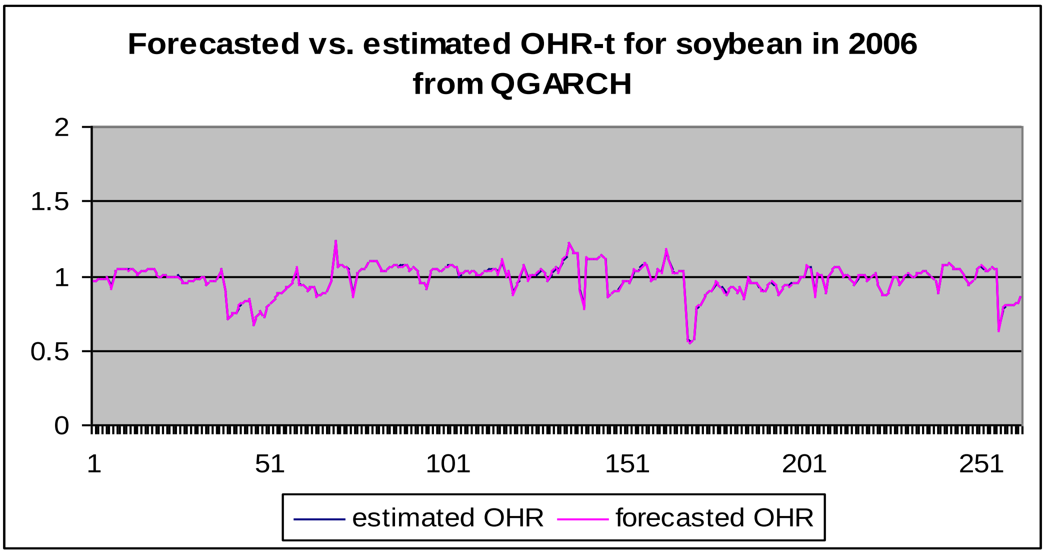

Forecasted vs. estimated OHR-t for soybean in 2006 from GARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for soybean in 2006 from BEKK-GARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

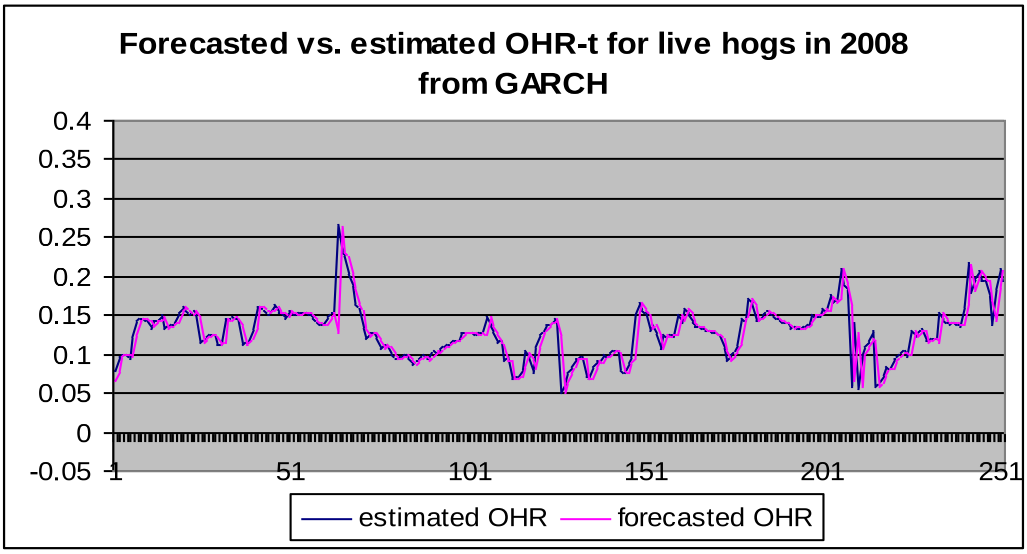

Forecasted vs. estimated OHR-t for soybean in 2006 from GARCH-X
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

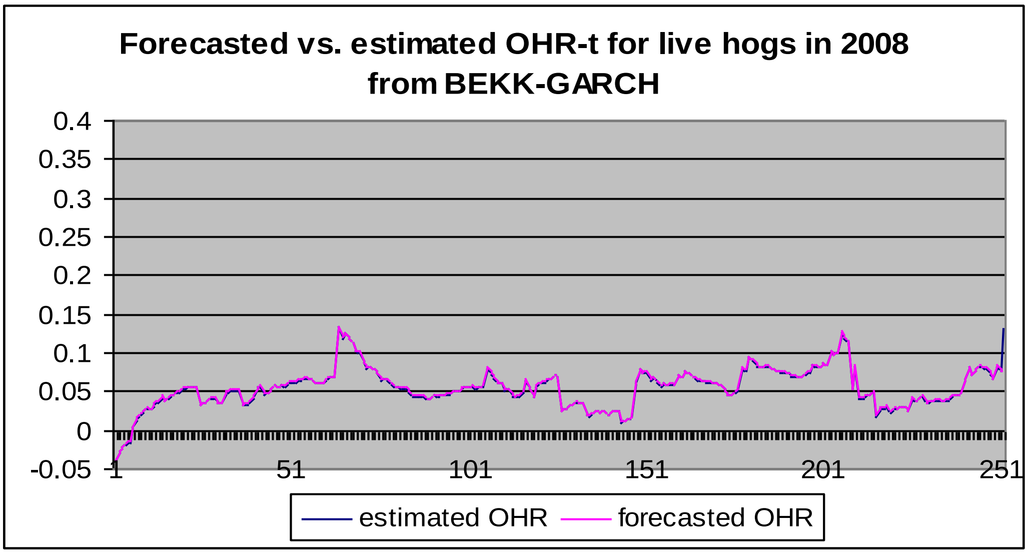

Forecasted vs. estimated OHR-t for soybean in 2006 from BEKK-GARCH-X
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

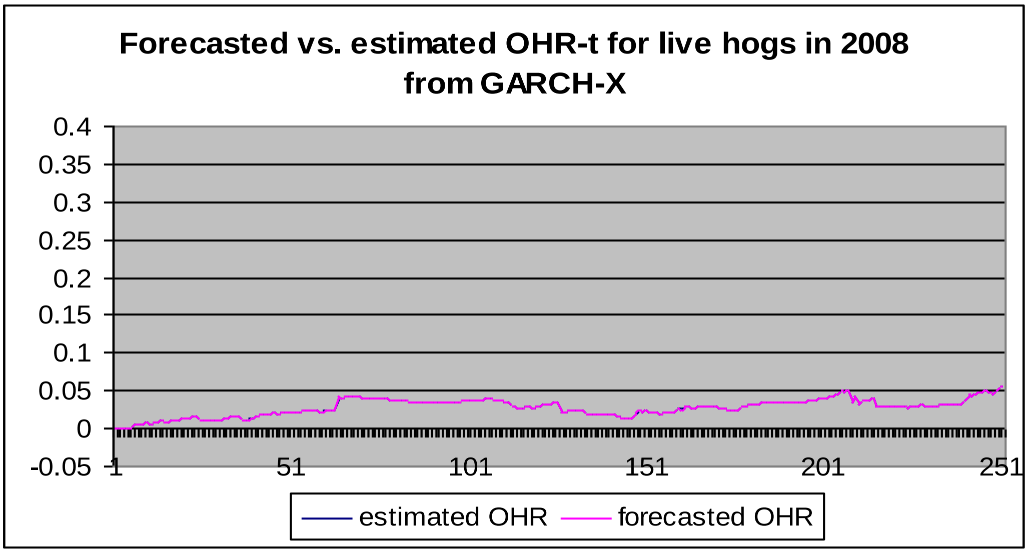

Forecasted vs. estimated OHR-t for soybean in 2006 from GARCH-GJR
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

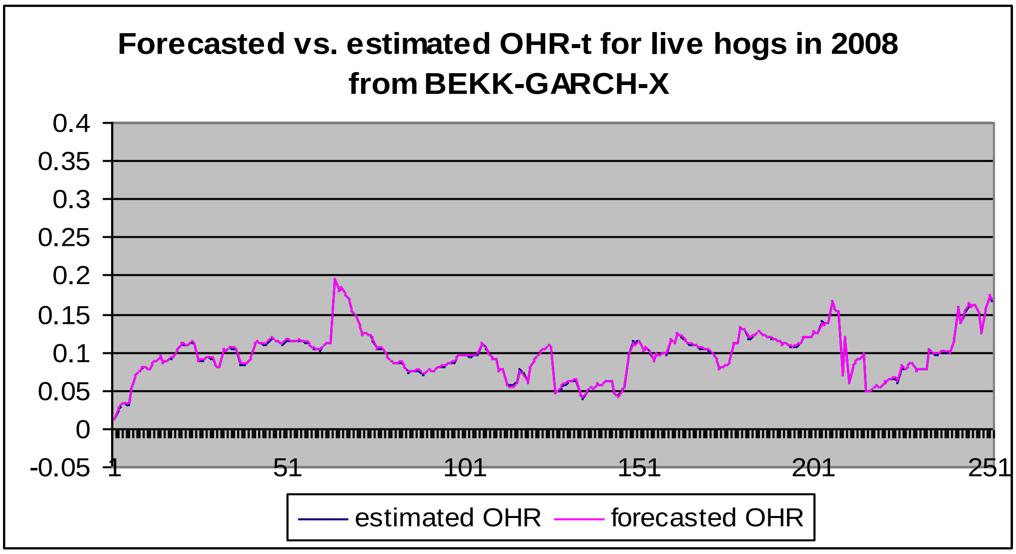

Forecasted vs. estimated OHR-t for soybean in 2006 from QGARCH
2
1.5
1
0.5
0
1
51
101
151
201
251
estimated OHR
forecasted OHR

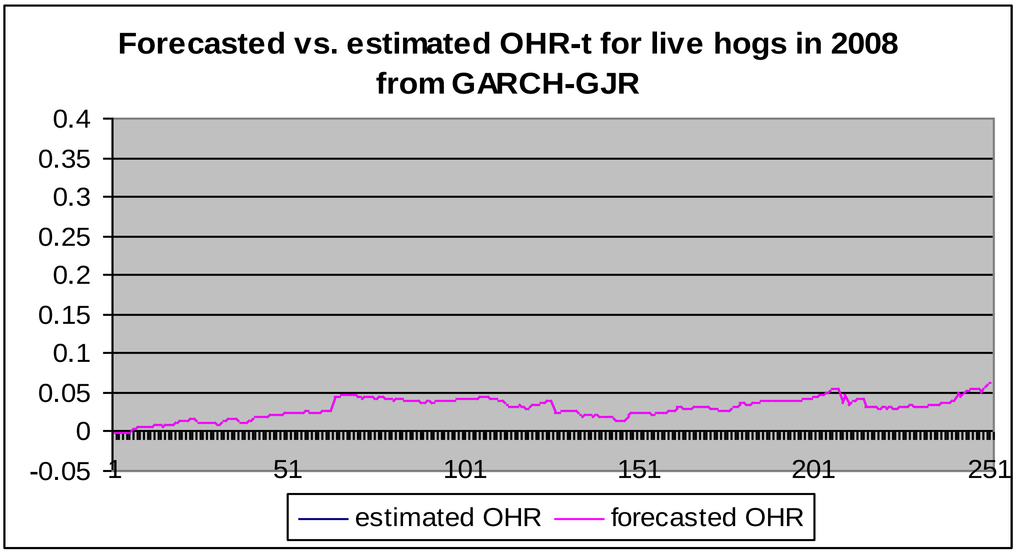
3. live hog

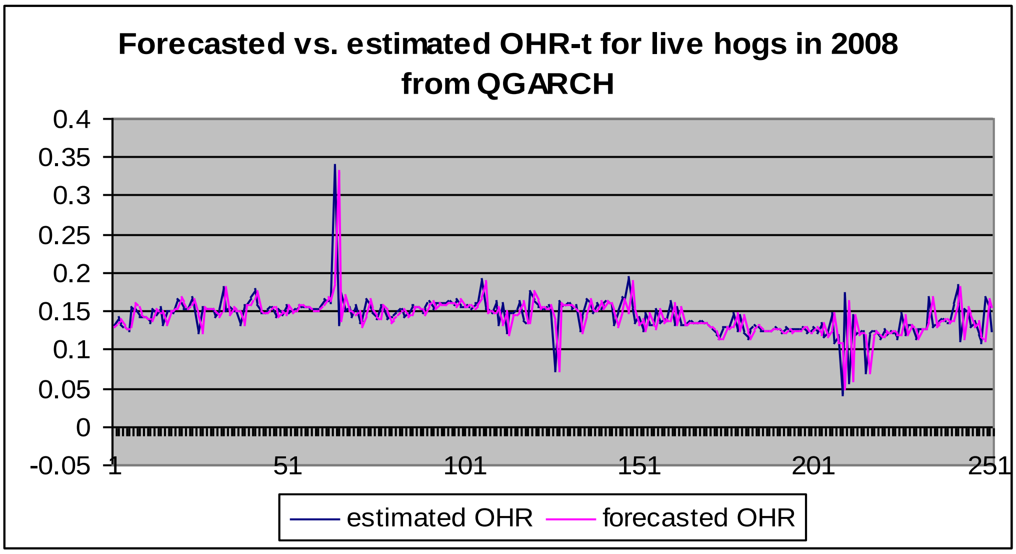

Forecasted vs. estimated OHR-t for live hogs in 2008 from GARCH
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for live hogs in 2008
from BEKK-GARCH
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for live hogs in 2008
from GARCH-X
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR

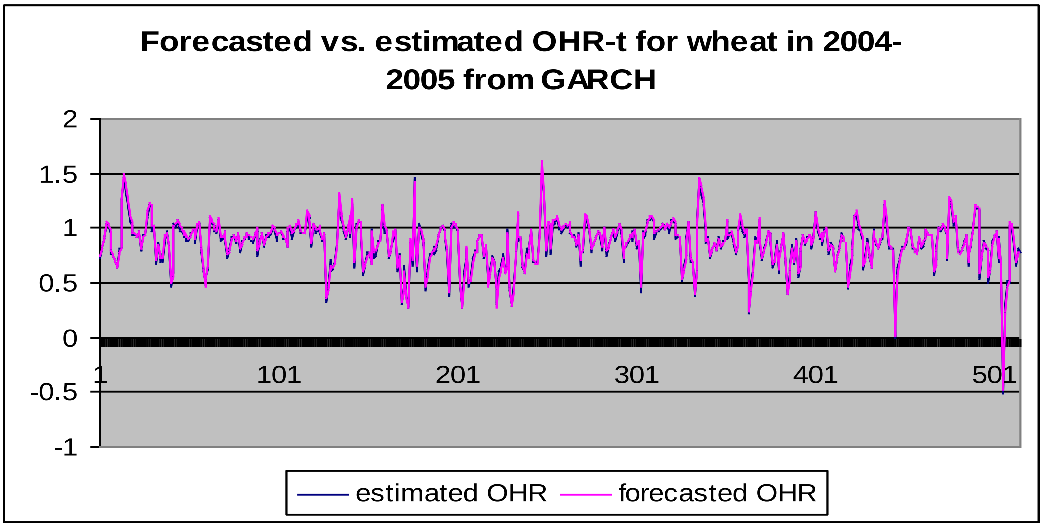

Forecasted vs. estimated OHR-t for live hogs in 2008
from BEKK-GARCH-X
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
1
51
101
151
201
251
estimated OHR
forecasted OHR

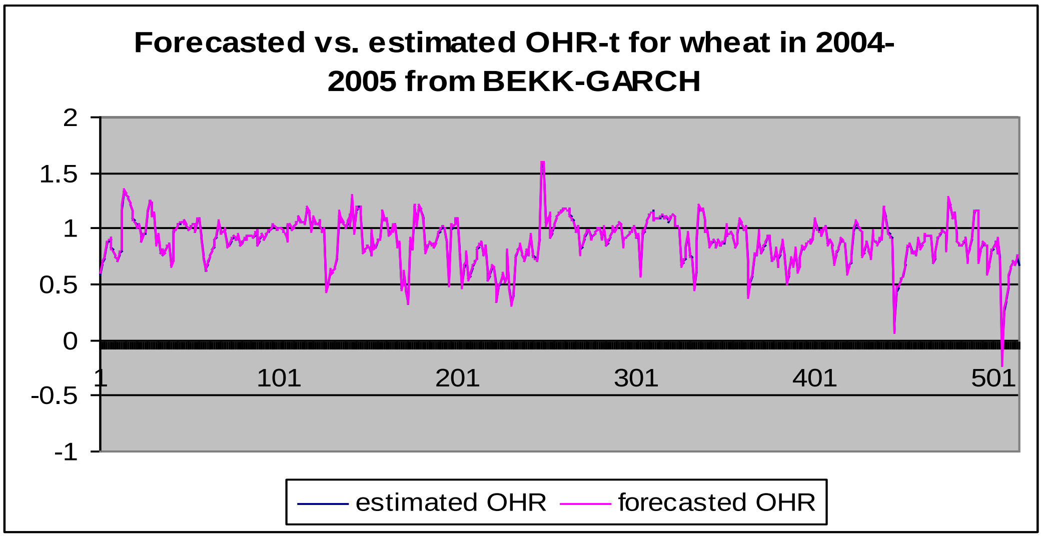


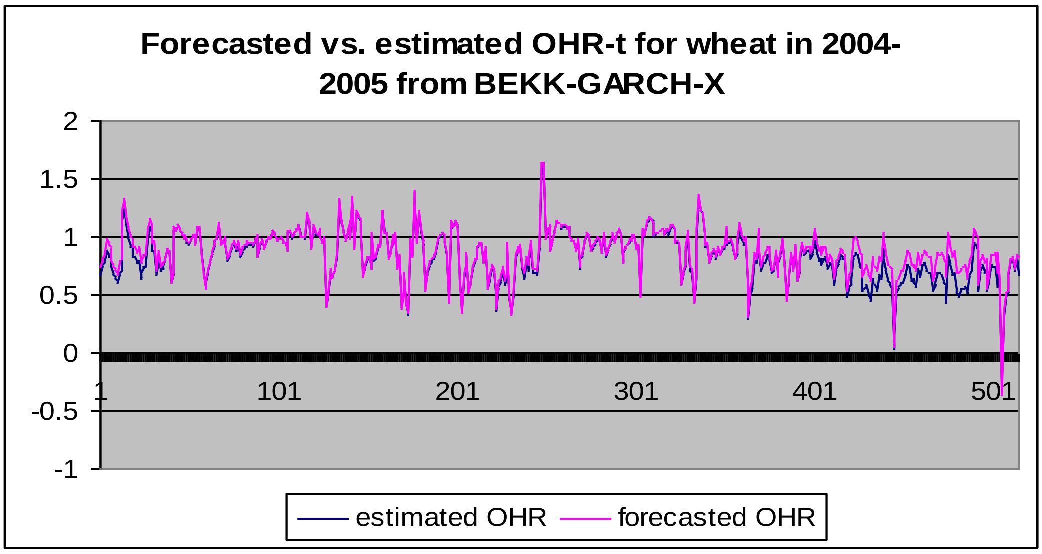


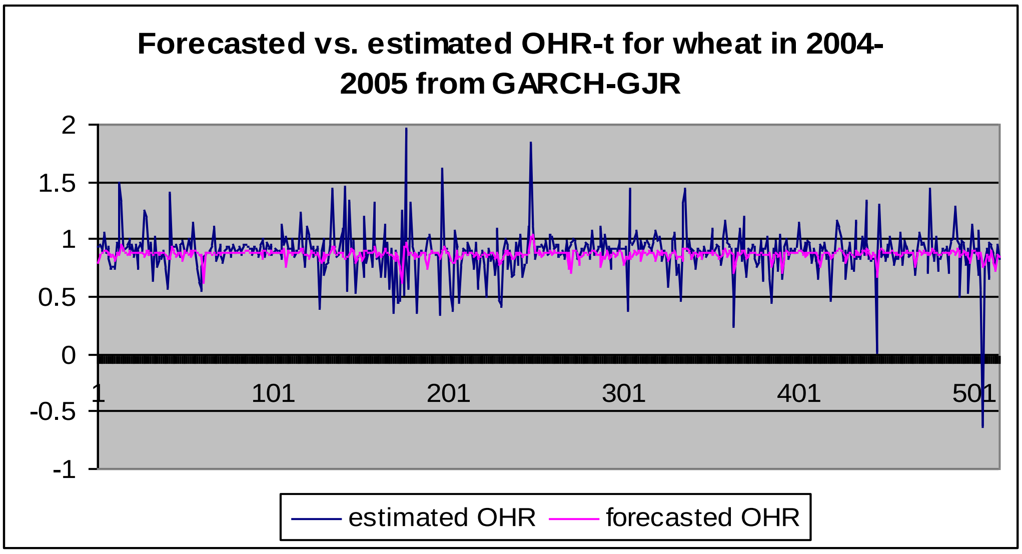
Part 6: 2-year estimated vs. forecasted OHR with student's t distributed residuals for wheat, soybean and live hog

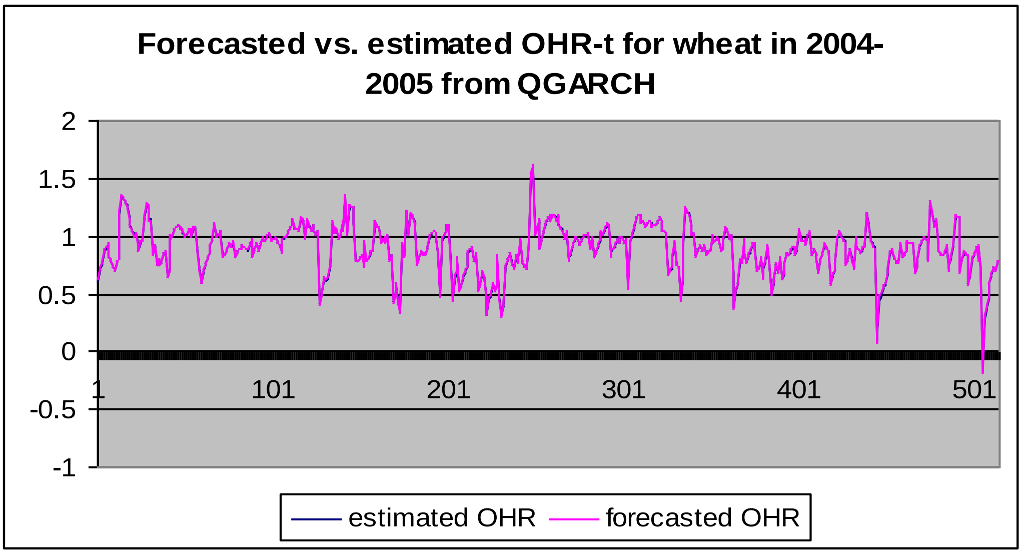
1. **Wheat**


Forecasted vs. estimated OHR-t for wheat in 2004-2005 from GARCH
2
1.5
1
0.5
0
-0.5
-1
1
101
201
301
401
501
estimated OHR
forecasted OHR

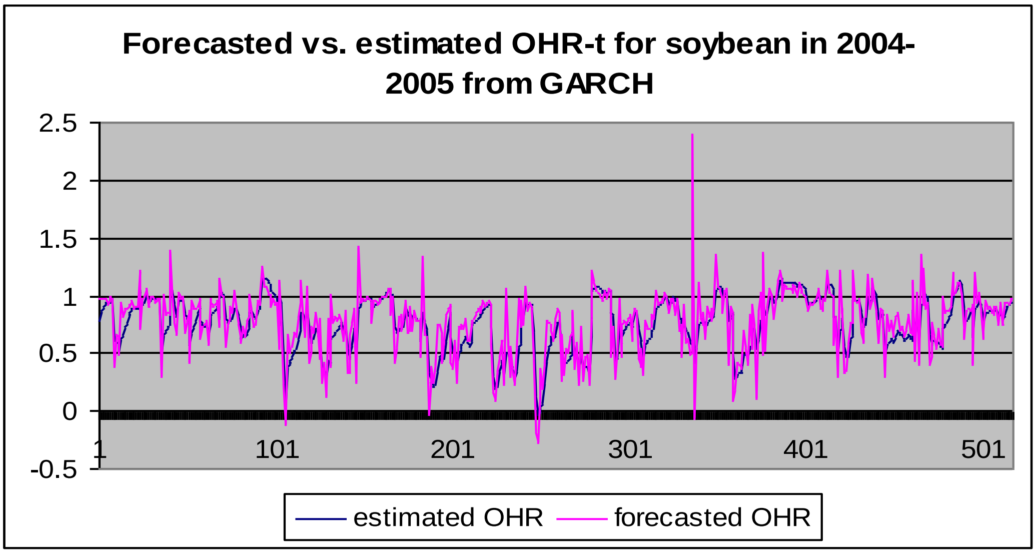

Forecasted vs. estimated OHR-t for wheat in 2004-2005 from BEKK-GARCH
2
1.5
1
0.5
0
-0.5
-1
1
101
201
301
401
501
estimated OHR
forecasted OHR

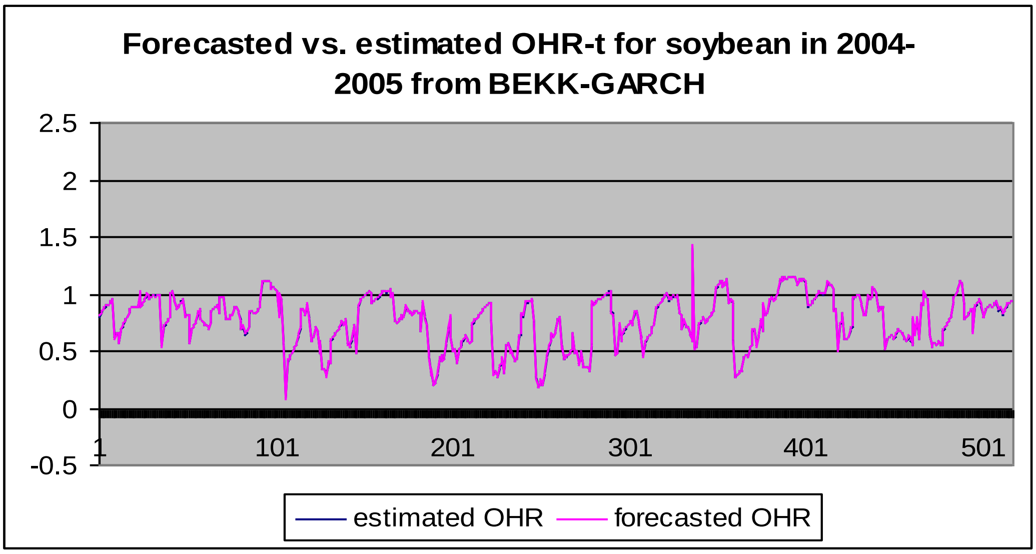

Forecasted vs. estimated OHR-t for wheat in 2004-2005 from BEKK-GARCH-X
2
1.5
1
0.5
0
-0.5
-1
1
101
201
301
401
501
estimated OHR
forecasted OHR

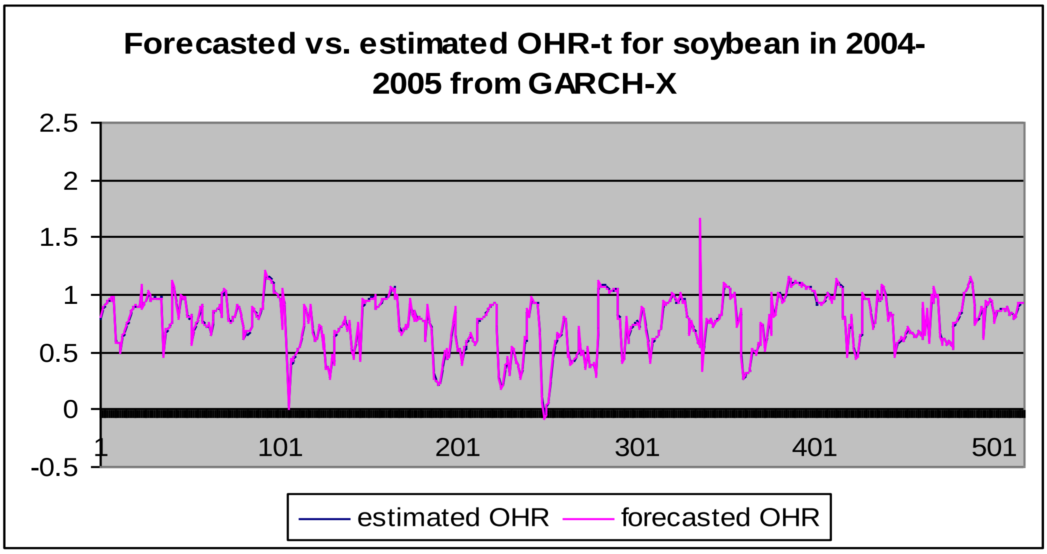

Forecasted vs. estimated OHR-t for wheat in 2004-2005 from GARCH-GJR
2
1.5
1
0.5
0
-0.5
-1
1
101
201
301
401
501
estimated OHR
forecasted OHR

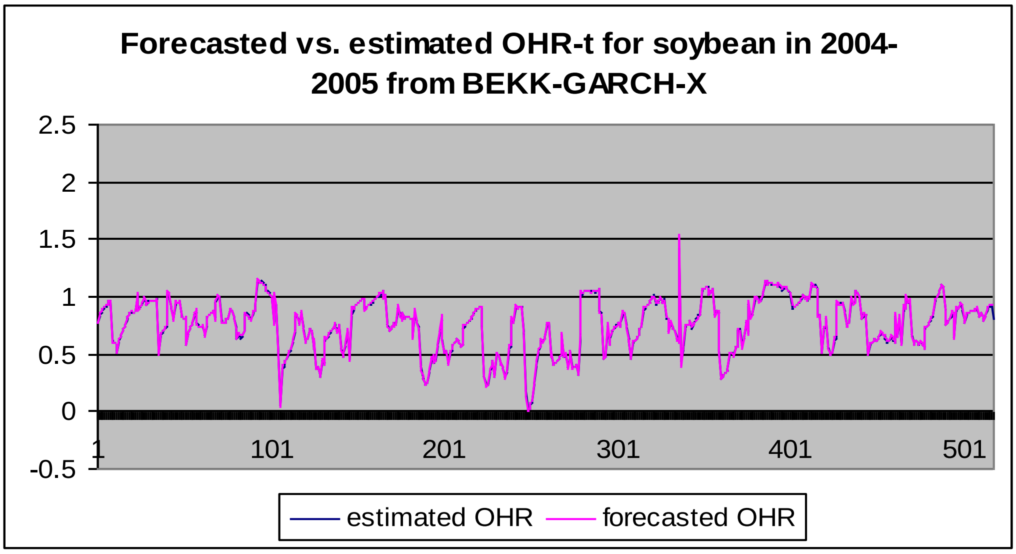

Forecasted vs. estimated OHR-t for wheat in 2004-2005 from QGARCH
2
1.5
1
0.5
0
-0.5
-1
1
101
201
301
401
501
estimated OHR
forecasted OHR

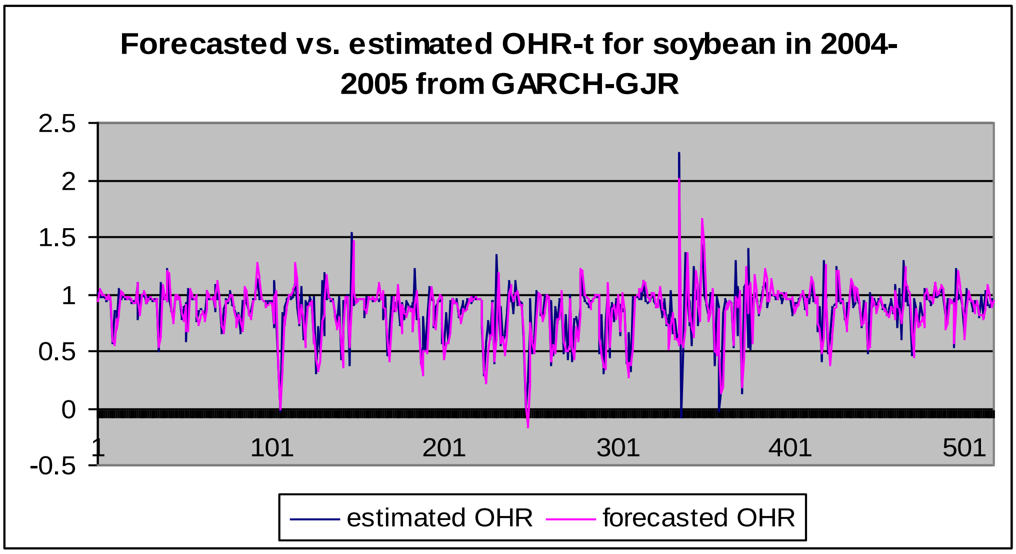
2. soybean

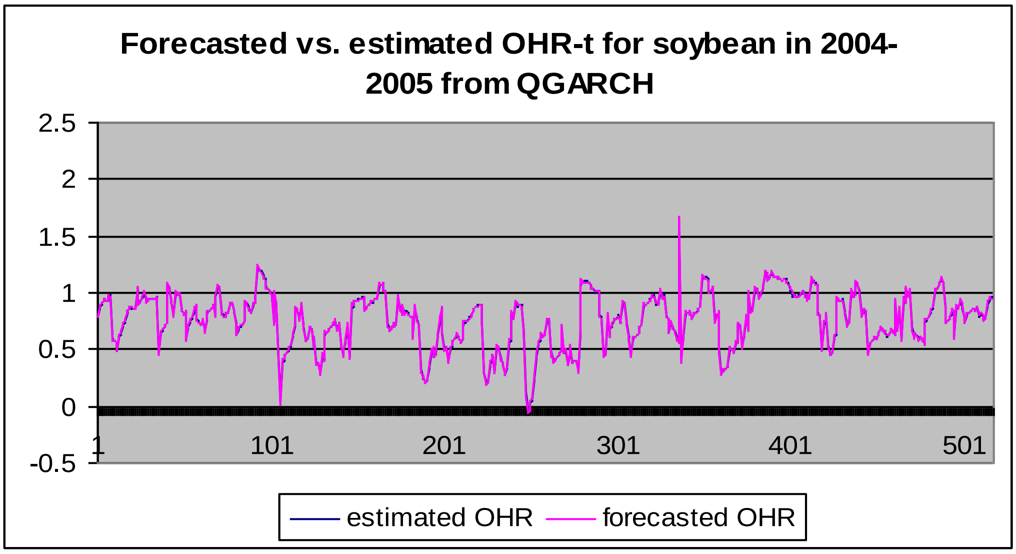

Forecasted vs. estimated OHR-t for soybean in 2004-2005 from GARCH
2.5
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for soybean in 2004-2005 from BEKK-GARCH
2.5
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

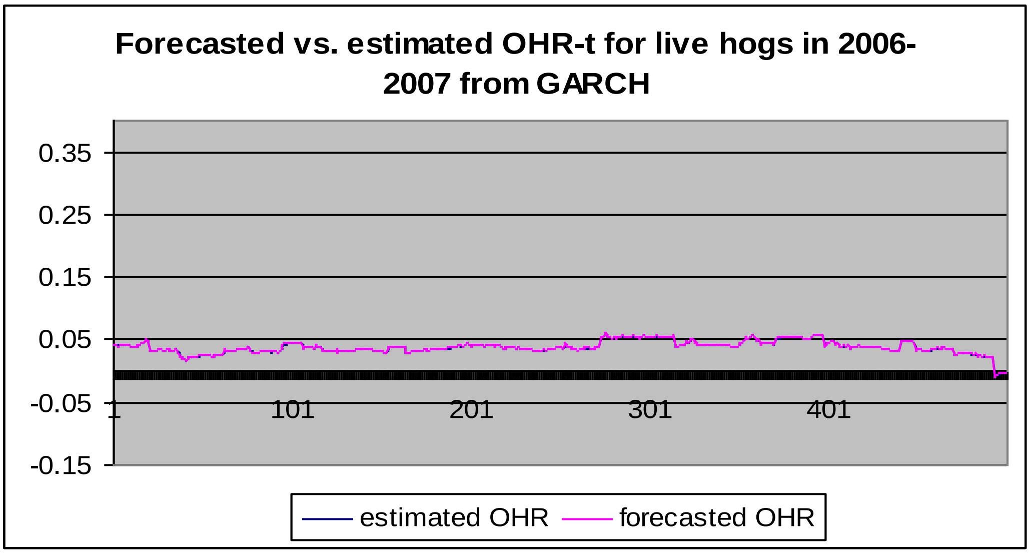

Forecasted vs. estimated OHR-t for soybean in 2004-2005 from GARCH-X
2.5
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

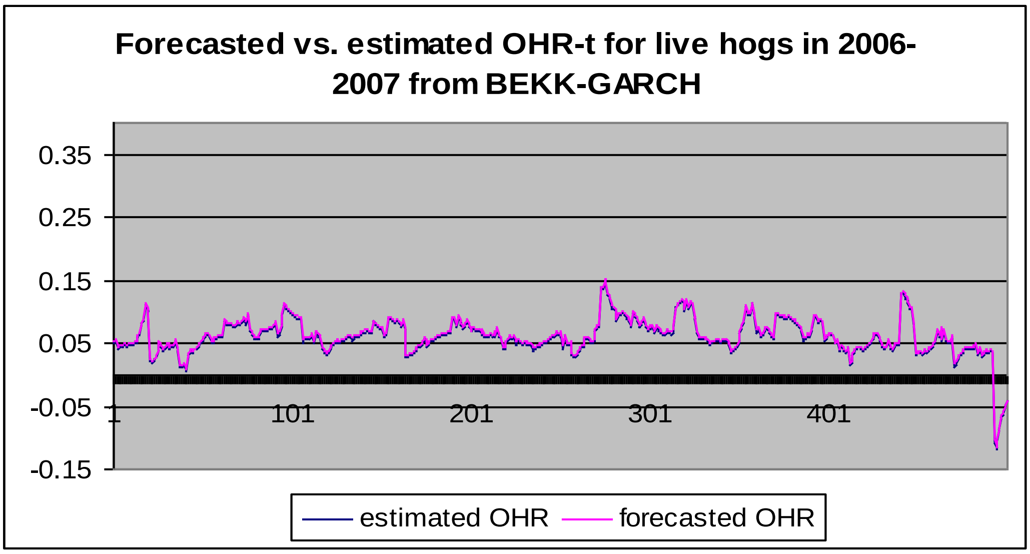

Forecasted vs. estimated OHR-t for soybean in 2004-2005 from BEKK-GARCH-X
2.5
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

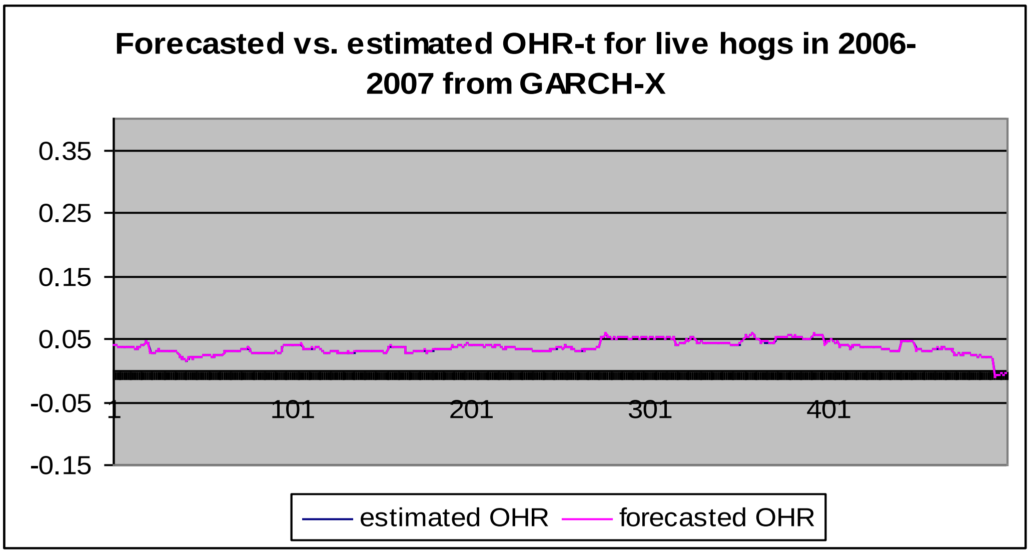

Forecasted vs. estimated OHR-t for soybean in 2004-2005 from GARCH-GJR
2.5
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

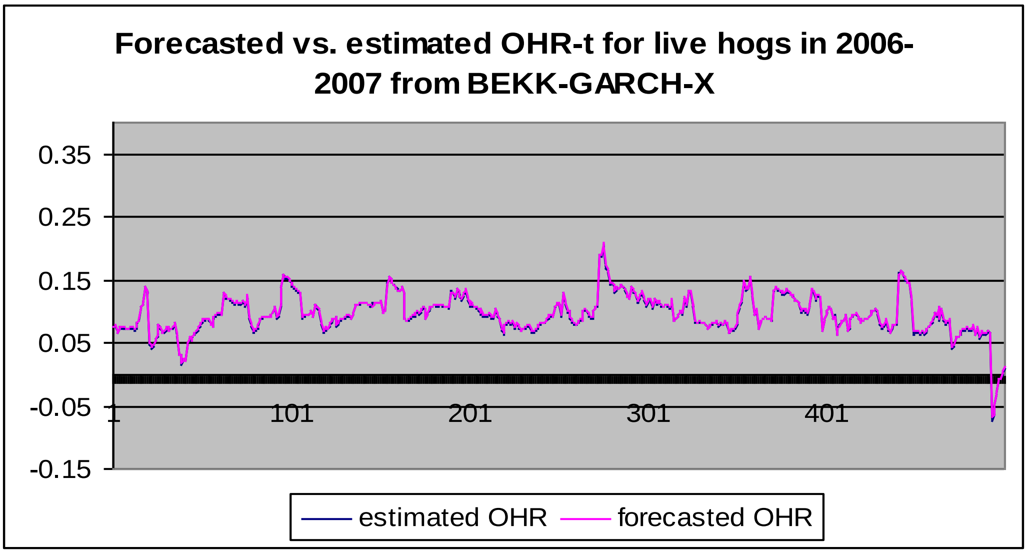

Forecasted vs. estimated OHR-t for soybean in 2004-2005 from QGARCH
2.5
2
1.5
1
0.5
0
-0.5
1
101
201
301
401
501
estimated OHR
forecasted OHR

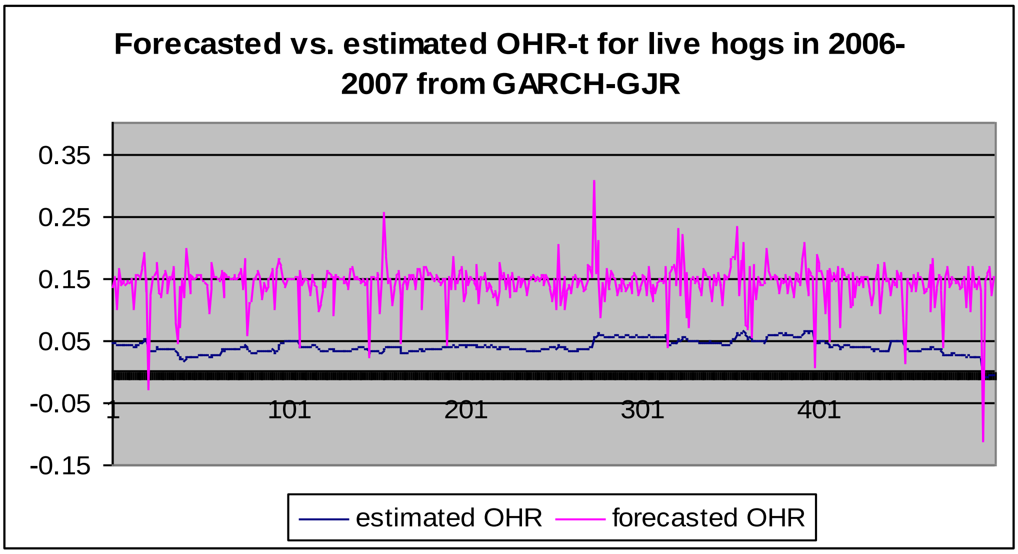
3. Live hog

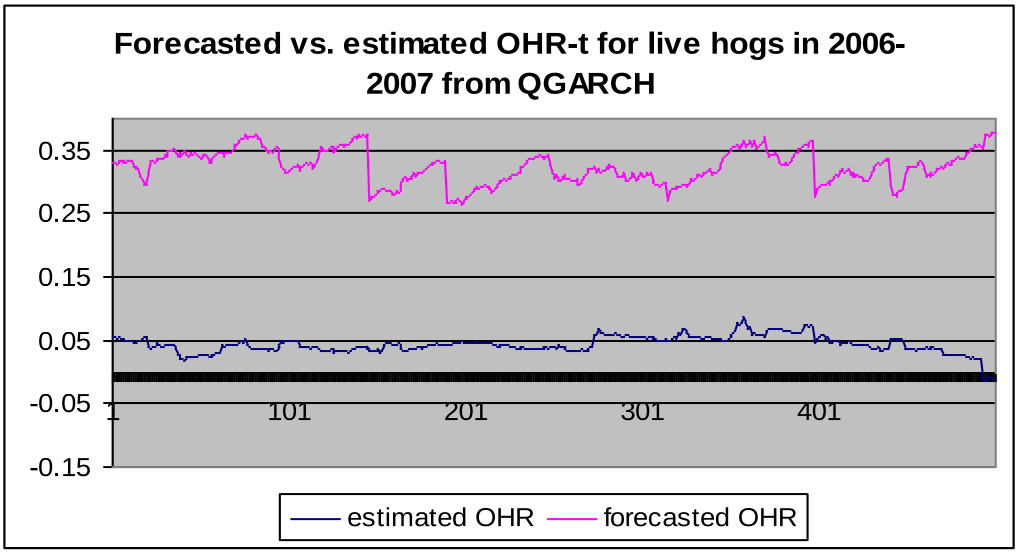

Forecasted vs. estimated OHR-t for live hogs in 2006-2007 from GARCH
0.35
0.25
0.15
0.05
-0.05
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for live hogs in 2006-2007 from BEKK-GARCH
0.35
0.25
0.15
0.05
-0.05
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for live hogs in 2006-2007 from GARCH-X
0.35
0.25
0.15
0.05
-0.05
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for live hogs in 2006-2007 from BEKK-GARCH-X
0.35
0.25
0.15
0.05
-0.05
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for live hogs in 2006-2007 from GARCH-GJR
0.35
0.25
0.15
0.05
-0.05
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR


Forecasted vs. estimated OHR-t for live hogs in 2006-2007 from QGARCH
0.35
0.25
0.15
0.05
-0.05
-0.15
1
101
201
301
401
estimated OHR
forecasted OHR