POSITIVE REALNESS AND LYAPUNOV FUNCTIONS FOR SWITCHED BEHAVIORS

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Abstract. We define a switched behavioral system as a finite bank of linear differential behaviors, together with some "gluing conditions" relating the system variables and their derivatives before and after each switching instant. The behaviors of the switched system do not necessarily share the same state space, differently from the classical state-space setting. We present a sufficient condition for the existence of a switched Lyapunov function for two scalar switched behavioral systems, and formulas to compute it. Instrumental in our results is the notion of positive-realness of a rational function associated with the polynomials describing the system.

Key words. Behavioral system theory; quadratic differential forms; switched behavior; Lyapunov stability.

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1. Introduction. In [4] we defined a switched behavioral system as a finite bank of linear differential behaviors, together with some "gluing conditions" relating the system variables and their derivatives before and after each switching instant. The behaviors of the switched system do not necessarily share the same state space, differently from the classical state-space setting. We also proved that the existence of a common Lyapunov function guarantees stability, and we gave a sufficient condition for the existence of such a functional for two behaviors with equal state-space dimension. Instrumental in our results was the notion of positive-realness (see [7]), which is also used, albeit only implicitly, in some classical work on switched systems, see [5].

In this paper we extend the results of [4] to scalar switched behaviors of different order. After a brief review of [4], in section 3 we present a sufficient condition for the existence of a switched Lyapunov function for two scalar switched behavioral systems. The conclusions are gathered in section 4. The notation and some basic notions of behavioral system theory are summarized in an Appendix at the end of the paper.

2. Background material. A *switched behavior* is a set of trajectories produced by a *switching structure*.

Definition 2.1. A *switching structure* Σ is a quadruple $\Sigma = \{\mathcal{P}, \mathcal{F}, \mathcal{S}, \mathcal{G}\}$ where:

- $\mathcal{P} = \{1, \dots, N\} \subset \mathbb{N}$ is the *set of indices*;
- $\mathcal{F} = (\mathcal{B}_1, \dots, \mathcal{B}_N)$, with $\mathcal{B}_j \in \mathfrak{L}^{\mathtt{w}}$ for $j \in \mathcal{P}$, is the *bank of behaviors*;
- $\mathcal{S} = \{s : \mathbb{R} \to \mathcal{P} \; : \; s$ is piecewise constant and right-continuous$\}$ is the set of admissible *switching signals*; and
- $\mathcal{G} = \{((k,\ell), G^+_{k,\ell}(\xi), G^-_{k,\ell}(\xi)) \mid (G^+_{k,\ell}(\xi), G^-_{k,\ell}(\xi)) \in (\mathbb{R}[\xi]^{\mathtt{g}_{k,\ell} \times \mathtt{w}})^2$ and $(k,\ell) \in \mathcal{P} \times \mathcal{P}, \; k \neq \ell\}$ is the set of *gluing conditions*.

For a given $s \in \mathcal{S}$, the set of *switching instants* with respect to s is $\mathbb{T}_s := \{t \in \mathbb{R} \mid \lim_{\tau \nearrow t} s(\tau) \neq s(t)\} = \{t_1, t_2, \dots\}$ where $t_i < t_{i+1}$. In the following, we define $f(t^-) := \lim_{\tau \nearrow t} f(\tau)$ and $f(t^+) := \lim_{\tau \searrow t} f(\tau)$.

Definition 2.2. Let $\Sigma = (\mathcal{P}, \mathcal{F}, \mathcal{S}, \mathcal{G})$ be a switching structure. For a given $s \in \mathcal{S}$, the *s-switched behavior $\mathcal{B}^s$ with respect to* Σ is the set of trajectories satisfying the following conditions:

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1. for all $t_i, t_{i+1} \in \mathbb{T}_s$, there exists $\mathcal{B}_k$, $k \in \mathcal{P}$ such that $w_{|[t_i,t_{i+1})} \in \mathcal{B}_{k|[t_i,t_{i+1})}$;
2. w satisfies the gluing conditions $\mathcal{G}$ at the switching instants:

$$(G^{+}_{s(t_{i-1}),s(t_i)}(\frac{d}{dt}))w(t_i^{+}) = (G^{-}_{s(t_{i-1}),s(t_i)}(\frac{d}{dt}))w(t_i^{-}) \text{ for each } t_i \in \mathbb{T}_s.$$

The *switched behavior* $\mathcal{B}^{\Sigma}$ *of* Σ is defined by $\mathcal{B}^{\Sigma} := \bigcup_{s\in\mathcal{S}} \mathcal{B}^{s}$. A switching structure Σ is *stable* if $\lim_{t\to\infty} w(t) = 0$ for all $w \in \mathcal{B}^{\Sigma}$. In [4] we carried out a Lyapunov stability analysis of switched structures using the calculus of *quadratic differential forms* (*QDFs*, see [6]), which we now introduce.

Let $\mathbb{R}_s^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta] := \{\Phi(\zeta,\eta) \in \mathbb{R}^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta] \ : \ \Phi(\zeta,\eta) = \Phi(\eta,\zeta)^{\top}\}$ denote the set of *symmetric* real two-variable $\mathtt{w}\times\mathtt{w}$ polynomial matrices. $\Phi \in \mathbb{R}_s^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$ has *order* L if $\Phi(\zeta,\eta) = \sum_{k,\ell=0}^{L} \Phi_{k,\ell}\zeta^k\eta^\ell$ where $\Phi_{k,L} = \Phi_{L,k} \neq 0$ for some k. The QDF Q_Φ associated with $\Phi \in \mathbb{R}_s^{\mathtt{w}\times\mathtt{w}}[\zeta,\eta]$ is defined as

$$\begin{aligned} Q_\Phi : \mathcal{C}^\infty(\mathbb{R},\mathbb{R}^{\mathtt{w}}) &\longrightarrow \mathcal{C}^\infty(\mathbb{R},\mathbb{R}) \\ w &\mapsto Q_\Phi(w) = \sum_{k,\ell}(\frac{d^k}{dt^k}w)^{\top}\Phi_{k,\ell}\frac{d^\ell}{dt^\ell}w \ . \end{aligned}$$

The *order* of a quadratic differential form Q_Φ is the order of the associated matrix $\Phi(\zeta,\eta)$. Note that $\Phi(\zeta,\eta)$ can be written as $\Phi(\zeta,\eta) = S_L^{\mathtt{w}}(\zeta)^{\top}\widetilde{\Phi}\ S_L^{\mathtt{w}}(\eta)$, where L is the order of $\Phi(\zeta,\eta)$, $S_L^{\mathtt{w}}(\xi)^{\top} := \begin{bmatrix} I_{\mathtt{w}} & \zeta I_{\mathtt{w}} & \cdots \xi^L I_{\mathtt{w}}\end{bmatrix}$, and $\widetilde{\Phi} \in \mathbb{R}^{L\mathtt{w}\times L\mathtt{w}}$ is the *coefficient matrix* of Φ.

We say that a QDF Q_Φ is *nonnegative along* $\mathcal{B}$, denoted $Q_\Phi \overset{\mathcal{B}}{\geq} 0$, if $(Q_\Phi(w))(t) \geq 0$ for all $w \in \mathcal{B}$ and $t \in \mathbb{R}$. If a QDF Q_Φ is nonnegative for every trajectory in $\mathcal{C}^\infty(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ we write $Q_\Phi \geq 0$ and say that Φ (or Q_Φ) is *nonnegative definite*. Note that Φ is nonnegative definite if and only if $\widetilde{\Phi} \geq 0$. We say that Q_Φ is *positive* along $\mathcal{B}$, denoted by $Q_\Phi \overset{\mathcal{B}}{>} 0$, if $Q_\Phi \overset{\mathcal{B}}{\geq} 0$ and $Q_\Phi(w) \equiv 0$ with $w \in \mathcal{B}$ implies that $w \equiv 0$. A QDF is *positive definite* if it is positive along $\mathcal{C}^\infty(\mathbb{R},\mathbb{R}^{\mathtt{w}})$; this happens if and only if $\widetilde{\Phi} > 0$. We define $Q_\Phi \overset{\mathcal{B}}{<} 0$, $\Phi < 0$, etc. accordingly.

A *Lyapunov function* for a behavior $\mathcal{B} \in \mathfrak{L}^{\mathtt{w}}$ is defined as a QDF Q_Φ such that $Q_\Phi \overset{\mathcal{B}}{\geq} 0$ and $\frac{d}{dt}Q_\Phi \overset{\mathcal{B}}{<} 0$, where $\frac{d}{dt}Q_\Phi$ denotes the QDF that maps $w \in \mathcal{C}^\infty(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ to $\frac{d}{dt}(Q_\Phi w)$. We call Q_Φ a *common Lyapunov function* for $\mathcal{F} = (\mathcal{B}_1,\mathcal{B}_2,\ldots,\mathcal{B}_N)$ if it is a Lyapunov function for every $\mathcal{B}_j$, $j = 1,\ldots,N$. The main result of [4] is the following (see Theorems 9 and 10, and Corollary 12 therein).

THEOREM 2.3. *Let* $\Sigma = \left(\{1,2\}, \left(\ker\ p_1(\frac{d}{dt}), \ker\ p_2(\frac{d}{dt})\right), \mathcal{S}, \mathcal{G}\right)$ *be a switching structure where* p_1, p_2 *are Hurwitz polynomials of degree* n *and*

$$\mathcal{G} = \left\{ \left((k,\ell), \begin{bmatrix} 1 \\ \vdots \\ \xi^{n-1} \end{bmatrix}, \begin{bmatrix} 1 \\ \vdots \\ \xi^{n-1} \end{bmatrix} \right), k \neq \ell \in \mathcal{P} \right\}. \tag{2.1}$$

If $\frac{p_2}{p_1}$ *is strictly positive real, then there exists a common Lyapunov function for* $\mathcal{F}$*, and consequently* Σ *is stable. If* $\frac{p_2}{p_1}$ *is strictly positive real, a common Lyapunov function can be computed as follows. Define* $\Phi(\zeta,\eta) := p_1(\zeta)p_2(\eta) + p_1(\eta)p_2(\zeta)$*; then there exists* $f \in \mathbb{R}[\xi]$ *anti-Hurwitz such that* $f(\xi)f(-\xi) := p_1(\xi)p_2(-\xi) + p_1(-\xi)p_2(\xi)$*. Let*

$$V(\zeta,\eta) := \frac{\Phi(\zeta,\eta) - f(\zeta)f(\eta)}{\zeta+\eta} \ ; \tag{2.2}$$

then $V \in \mathbb{R}_s[\zeta,\eta]$*, and* Q_V *is a common Lyapunov function of order* $n-1$ *for* Σ*.*

3. Main result. It is a standard result in the theory of positive-real functions (see Th. 5.10 p. 92 of [7]) that if $\frac{p_2}{p_1}$ is positive real and $\deg(p_1) \neq \deg(p_2)$, then $\deg(p_2) = \deg(p_1) - 1$. We assume that this is the case, and moreover we assume that p_1 and p_2 are monic, and we set $p_2(\xi) =: \sum_{j=0}^{n-1} p_{2,j}\xi^j$. Given $\mathcal{B}_i := \ker\ p(\frac{d}{dt})$, $i = 1, 2$, we define a switched behavior as in Def. 2.2, with gluing conditions:

$$\begin{aligned}
\left(G_{2,1}^+(\xi), G_{2,1}^-(\xi)\right) &:= \left(\begin{bmatrix} 1 \\ \vdots \\ \xi^{n-2} \\ \xi^{n-1} \end{bmatrix}, \begin{bmatrix} 1 \\ \vdots \\ \xi^{n-2} \\ -p_{2,0} - \ldots - p_{2,n-2}\xi^{n-2} \end{bmatrix}\right) \\
\left(G_{1,2}^+(\xi), G_{1,2}^-(\xi)\right) &:= \left(\begin{bmatrix} 1 \\ \vdots \\ \xi^{n-2} \end{bmatrix}, \begin{bmatrix} 1 \\ \vdots \\ \xi^{n-2} \end{bmatrix}\right) .
\end{aligned} \tag{3.1}$$

For a switch from $\mathcal{B}_2$ to $\mathcal{B}_1$, in order to obtain the "initial conditions" uniquely by specifying $w \in \mathcal{B}_1$, the $(n-1)$-th derivative of w after the switching instant needs to be defined; the gluing conditions specify that this is done compatibly with the fact that since $w \in \mathcal{B}_2$ before the switch, $\frac{d^{n-1}}{dt^{n-1}}w = -p_{2,0}w - \ldots - p_{2,n-2}\frac{d^{n-2}}{dt^{n-2}}w$. For a switch from $\mathcal{B}_1$ to $\mathcal{B}_2$, we project the vector of derivatives characteristic of a trajectory $w \in \mathcal{B}_1$ down onto the smaller vector of derivatives characteristic of a trajectory $w \in \mathcal{B}_2$. Note that these gluing conditions guarantee that the corresponding trajectory is as smooth as possible.

In the following we consider the two-variable polynomial $V(\zeta, \eta)$ defined by (2.2) together with the associated polynomial

$$V'(\zeta, \eta) := V(\zeta, \eta) \bmod p_2 , \tag{3.2}$$

the *canonical representative* of $V(\zeta, \eta)$ modulo ker $p_2\left(\frac{d}{dt}\right)$, see p. 1716 of [6]. Note that since $\deg(p_1) = n$ and $\deg(p_2) = n - 1$, the highest power of ζ and η in $V(\zeta, \eta)$ is $n - 1$, and in $V'(\zeta, \eta)$ it is $n - 2$. We now define a functional Q_Λ by

$$Q_\Lambda := \begin{cases} Q_V \text{ if } \mathcal{B}_1 \text{ is active;} \\ Q_{V'} \text{ if } \mathcal{B}_2 \text{ is active .} \end{cases} \tag{3.3}$$

In the following we show that Q_Λ is a switched Lyapunov function (see section III.B of [2]), i.e. Q_V and $Q_{V'}$ are Lyapunov functions for $\mathcal{B}$ and $\mathcal{B}'$, respectively, and moreover Q_Λ does not increase along the trajectories of $\mathcal{B}^\Sigma$; we begin with the following result.

LEMMA 3.1. *Define V, V' by (2.2), (3.2); then $Q_V, Q_{V'} \geq 0$ and $\frac{d}{dt}Q_V \overset{\mathcal{B}_1}{<} 0$, $\frac{d}{dt}Q_{V'} \overset{\mathcal{B}_2}{<} 0$.*

Proof. The first part of the claim follows from the same argument used in the proof of Theorem 10 of [4]. To prove the second part, use the calculus of QDFs to verify that the derivative of $Q_{\Psi'}$ along $\mathcal{B}_2$ is induced by the two-variable polynomial $-f'(\zeta)f'(\eta)$, with $f' := f \bmod p_2$, the remainder in the Euclidean division of f by p_2. Note that $GCD(f', p_2) = 1$, since if f' and p_2 would have a common root, then also f and p_2 would have the same root, and since one is anti-Hurwitz and the other Hurwitz, this is impossible. This yields the claim. □

We now show that in a switch from $\mathcal{B}_1$ to $\mathcal{B}_2$ the value of Q_Λ does not increase.

LEMMA 3.2. *Let $w \in \mathcal{B}^\Sigma$, with $w_{|_{[t_{i-1},t_i)}} \in \mathcal{B}_1{}_{|_{[t_{i-1},t_i)}}$, and $w_{|_{[t_i,t_{i+1})}} \in \mathcal{B}_2{}_{|_{[t_i,t_{i+1})}}$. Then $Q_V(w)(t_i^-) \geq Q_{V'}(w)(t_i)$.*

Proof. Define $v := \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, where $v_1^\top := \begin{bmatrix} w & \frac{d}{dt}w & \dots & \frac{d^{n-2}}{dt^{n-2}}w \end{bmatrix}^\top$ and $v_2 := \frac{d^{n-1}}{dt^{n-1}}w$. If $w \in \mathcal{B}_1$, then $Q_V(w)(t) = v(t)^\top \widetilde{V} v(t)$. Moreover, from the gluing conditions it follows that if $w \in \mathcal{B}_2$, $Q_{V'}(w)(t) = v_1(t)^\top \begin{bmatrix} I_{n-1} & \mathfrak{p}_2^\top \end{bmatrix} \widetilde{V} \begin{bmatrix} I_{n-1} \\ \mathfrak{p}_2 \end{bmatrix} v_1(t)$, where

$$\mathfrak{p}_2 := \begin{bmatrix} -p_{2,0} & -p_{2,1} & \cdots & -p_{2,n-2} \end{bmatrix} \in \mathbb{R}^{1\times(n-1)} . \tag{3.4}$$

Now partition $\widetilde{V}$ accordingly with the partition of v; since Q_V is continuous, it holds that $\lim_{t \nearrow t_i} Q_V(w)(t_i) - Q_{V'}(w)(t_i) = Q_V(w)(t_i) - Q_{V'}(w)(t_i)$ equals

$$\begin{bmatrix} v_1(t_i)^\top & v_2(t_i)^\top \end{bmatrix} \underbrace{\begin{bmatrix} -\mathfrak{p}_2^\top \widetilde{V}_{22}\mathfrak{p}_2 - \mathfrak{p}_2^\top \widetilde{V}_{12}^\top - \widetilde{V}_{12}\mathfrak{p}_2 & \widetilde{V}_{12} \\ \widetilde{V}_{12}^\top & \widetilde{V}_{22} \end{bmatrix}}_{=:\widetilde{\Delta V}} \begin{bmatrix} v_1(t_i) \\ v_2(t_i) \end{bmatrix} .$$

Since $\begin{bmatrix} v_1(t_i) \\ v_2(t_i) \end{bmatrix}$ is arbitrary, we need to prove that $\widetilde{\Delta V} \geq 0$. Note that

$$\begin{bmatrix} I_{n-1} & \mathfrak{p}_2^\top \\ 0 & 1 \end{bmatrix} \widetilde{\Delta V} \begin{bmatrix} I_{n-1} & 0 \\ \mathfrak{p}_2 & 1 \end{bmatrix} = \begin{bmatrix} 0_{(n-1)\times(n-1)} & \widetilde{V}_{12} + \mathfrak{p}_2^\top \widetilde{V}_{22} \\ \widetilde{V}_{12}^\top + \widetilde{V}_{22}\mathfrak{p}_2 & \widetilde{V}_{22} \end{bmatrix} . \tag{3.5}$$

It follows from (2.2) that $\widetilde{V}_{22} = 1 > 0$; taking the Schur complement of $\widetilde{V}_{22}$ in (3.5) yields $\begin{bmatrix} -\left(\widetilde{V}_{12} + \mathfrak{p}_2^\top \widetilde{V}_{22}\right) \widetilde{V}_{22}^{-1} \left(\widetilde{V}_{12}^\top + \widetilde{V}_{22}\mathfrak{p}_2\right) & 0_{(n-1)\times 1} \\ 0_{1\times(n-1)} & \widetilde{V}_{22} \end{bmatrix}$. To conclude the proof, multiply both sides of (2.2) by $\zeta + \eta$, and equate the highest powers of ζ and η on the left- and on the right-hand side. It follows that the last row of $\widetilde{V}$ equals $\mathfrak{p}_2$, and consequently $\widetilde{V}_{12}^\top = \mathfrak{p}_2$; consequently $\widetilde{V}_{12}^\top + \widetilde{V}_{22}\mathfrak{p}_2 = 0$. The claim is proved. ☐

Finally, we prove that in a switch from $\mathcal{B}_2$ to $\mathcal{B}_1$ the value of Q_Λ remains the same.

Lemma 3.3. *Let $w \in \mathcal{B}^\Sigma$, with $w_{|_{[t_{i-1},t_i)}} \in \mathcal{B}_2{}_{|_{[t_{i-1},t_i)}}$, and $w_{|_{[t_i,t_{i+1})}} \in \mathcal{B}_1{}_{|_{[t_i,t_{i+1})}}$. Then $Q_{V'}(w)(t_i^+) = Q_V(w)(t_i)$.*

Proof. Since $Q_{V'}$ is continuous, $\lim_{t \nearrow t_i} Q_{V'}(w)(t_i) = Q_{V'}(w)(t_i)$. Moreover, $Q_{V'}(w)(t_i) = v_1(t_i)^\top \widetilde{V'} v_1(t_i) = v_1(t_i)^\top \begin{bmatrix} I_{n-1} & \mathfrak{p}_2^\top \end{bmatrix} \widetilde{V} \begin{bmatrix} I_{n-1} \\ \mathfrak{p}_2 \end{bmatrix} v_1(t_i)$, with $\mathfrak{p}_2$ defined by (3.4). Due to the definition of gluing conditions, the last expression equals the value of Q_V after the switch. This concludes the proof. ☐

We can now prove the main result of this section.

Theorem 3.4. *Let $\Sigma = \left(\{1,2\}, \left(\ker\ p_1(\frac{d}{dt}), \ker\ p_2(\frac{d}{dt})\right), \mathcal{S}, \mathcal{G}\right)$ be a switching structure where p_1, p_2 are Hurwitz polynomials with $\deg(p_1) =: n$, $\deg(p_2) = n-1$, and the gluing conditions are as in (3.1). If $\frac{p_2}{p_1}$ is strictly positive real, then the functional Q_Λ defined as in (3.3) is a switched Lyapunov function for the switched system $\mathcal{B}^\Sigma$, and consequently $\mathcal{B}^\Sigma$ is asymptotically stable.*

Proof. It is easy to see that Q_Λ is continuous everywhere except (possibly) at the switching times, and non-increasing from one switching time to the next (Lemmas 3.1, 3.2 and 3.3). Moreover, the value of Q_Λ in an interval $[t_i, t_{i+1})$ is bounded from above by its value at t_i. We can now apply Th. 4.1 of [8] to conclude the validity of the claim of the Theorem. ☐

4. Conclusions. Together with [4], the results presented in this paper show that for behavioral switched systems consisting of two behaviors ker $p_1\left(\frac{d}{dt}\right)$ and ker $p_1\left(\frac{d}{dt}\right)$ with gluing conditions such as those considered in Theorems 2.3 and 3.4, the positive-realness of $\frac{p_2}{p_1}$ is a sufficient condition for the stability of the switched behavior. Moreover, in this paper we have also shown how a switched Lyapunov function can be constructed from p_1 and p_2.

Notation and review of behavioral system theory. The space of real vectors with $\mathtt{n}$ components is denoted by $\mathbb{R}^{\mathtt{n}}$, and the space of $\mathtt{n} \times \mathtt{m}$ real matrices by $\mathbb{R}^{\mathtt{m}\times\mathtt{n}}$. The ring of polynomials with real coefficients in the indeterminate ξ is denoted by $\mathbb{R}[\xi]$; the ring of two-variable polynomials with real coefficients in the indeterminates ζ and η is denoted by $\mathbb{R}[\zeta,\eta]$. $\mathbb{R}^{\mathtt{n}\times\mathtt{m}}[\xi]$ is the space of $\mathtt{n} \times \mathtt{m}$ polynomial matrices in ξ, and the space of $\mathtt{n} \times \mathtt{m}$ polynomial matrices in ζ and η is denoted by $\mathbb{R}^{\mathtt{n}\times\mathtt{m}}[\zeta,\eta]$.

$\mathcal{C}^\infty(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ is the set of infinitely-differentiable (smooth) functions from $\mathbb{R}$ to $\mathbb{R}^{\mathtt{w}}$. We call $\mathcal{B} \subseteq \mathcal{C}^\infty(\mathbb{R},\mathbb{R}^{\mathtt{w}})$ a *linear time-invariant differential behavior* if $\mathcal{B}$ is the set of solutions of a finite system of constant-coefficient differential equations, i.e., if there exists a polynomial matrix $R \in \mathbb{R}^{\mathtt{g}\times\mathtt{w}}[\xi]$ such that $\mathcal{B} = \{w \in \mathcal{C}^\infty(\mathbb{R},\mathbb{R}^{\mathtt{w}}) \mid R(\frac{d}{dt})w = 0\} = \ker\ R(\frac{d}{dt})$. If $\mathcal{B}$ is represented by $R(\frac{d}{dt})w = 0$, then we call R a kernel representation of $\mathcal{B}$. We denote with $\mathfrak{L}^{\mathtt{w}}$ the set of all linear time-invariant differential behaviors with $\mathtt{w}$ variables.

A polynomial $p \in \mathbb{R}[\xi]$ is *Hurwitz* if its roots are all in the open left half-plane. A rational function g is *strictly positive real* if: g has no poles s with $\Re(s) \geq 0$; $\Re(g(j\omega)) > 0$ for all $\omega \geq 0$; and $g(\infty) > 0$, or $\lim_{\omega\to\infty} \omega^2\Re(g(j\omega)) > 0$.

REFERENCES

[1] R. Frasca, M.K. Çamlibel, I. Cem Göknar, L. Iannelli, and F. Vasca, *Linear Passive Networks With Ideal Switches: Consistent Initial Conditions and State Discontinuities*, IEEE Trans. Circ. Syst. I, vol. 57, no. 12, pp. 3138–3151, 2011.

[2] H. Lin, and P.J. Antsaklis, *Stability and Stabilizability of Switched Linear Systems: A Survey of Recent Results*, IEEE Trans. Aut. Contr., vol. 54, no. 2, pp. 308–322, 2009.

[3] J. W. Polderman and J. C. Willems, *Introduction to Mathematical System Theory: A Behavioral Approach*, Springer-Verlag, Berlin, 1997.

[4] P. Rocha, J.C. Willems, P. Rapisarda and D. Napp, *On the stability of switched behavioral systems*, Proc. 50th IEEE Conf. on Decision and Control & 10th European Control Conf., Orlando, Florida, pp. 1534-1538, 2011.

[5] R.N. Shorten and K.S. Narendra, *On Common Quadratic Lyapunov Functions for Pairs of Stable LTI Systems Whose System Matrices Are in Companion Form*, IEEE Trans. Aut. Contr., vol. 48, no. 4, pp. 618–621, 2003.

[6] J.C. Willems and H.L. Trentelman, *On quadratic differential forms*, SIAM J. Contr. Opt., 36(5), 1998, pp. 1703–1749.

[7] O. Wing, *Classical Circuit Theory*, Springer-Verlag, New-York, 2008.

[8] H. Ye, A.N. Michel, and L. Hou, *Stability theory for hybrid dynamical systems*, IEEE Trans. Aut. Contr., vol. 43, no. 4, pp. 461–474, 1998.