Available online at www.sciencedirect.com
ScienceDirect
Procedia Engineering 00 (2012) 000–000
Procedia Engineering
www.elsevier.com/locate/procedia

Proc. Eurosensors XXVI, September 9-12, 2012, Kraków, Poland

Direct Solution of the Rayleigh Integral to Obtain the Radiation Pattern of an Annular Array Ultrasonic Transducer

Y. Qian*, N. R. Harris

School of Electronics and Computer Science, University of Southampton, University Road, Southampton, SO17 1BJ, UK

Abstract

The Spatial Impulse Response (SIR) method has difficulty numerically evaluating the imaging pattern of annular ultrasonic transducer arrays. An alternative algorithm based on the direct solution of the Rayleigh integral has been developed, and this successfully evaluates the annular array pattern with good efficiency and accuracy. Furthermore, in combination with FEA this method gives good results when compared to practical results.

Keywords: Rayleigh integral; SIR; Imaging; Numerical evaluation; ultrasonic; array

1. Introduction

Annular array ultrasonic transducers have been used in the high frequency (>30MHz) medical imaging field recently and offer very fine imaging resolution. The SIR (Spatial Impulse Response) method is usually used to evaluate the imaging pattern to aid the transducer design, and a programme (Field II) is available from the Technical University of Denmark [1, 2] that uses this technique as an example.

However, although Field II shows good efficiency on the imaging evaluation of single-element or linear array transducers, it meets difficulties in analyzing annular structures. The axi-symmetric geometry of annular arrays leads to complicated Hankel transforms or Bessel functions during SIR calculations, instead of more familiar Fourier transforms [3]. Thus we report on an alternative method by directly solving the Rayleigh integral in the time domain. It avoids the Hankel transform required by the SIR

* Corresponding author: Yichen Qian; Tel.: +447429335230
E-mail: ycq07r@gmail.com.

method, and thus reduces the calculation difficulties in annular structures. The imaging pattern of an annular array can be successfully evaluated by using this method, with good accuracy and efficiency. Furthermore, a combined method using the direct Rayleigh solution and FEA (Finite Element Analysis) has been developed to aid evaluation of practical structures with a much reduced computing time (more than 48hours) compared with using FEA on its own.

2. Direct solution of the Rayleigh integral

The Rayleigh integral (R-I) is a well-established estimator to evaluate the acoustic diffraction produced by an ideal pistonic radiator. Figure 1(a) illustrates the method for obtaining the pressure at point *P* in the focal plane by using this integral, where *dS* is the radiating source element located in the XY plane with its polar coordinate of (r, φ); and *R* is the distance from the source *dS* to the point *P*. The line drawn from the point on the Z-axis at the focal plane is described as a focal line. It can be used to represent the whole focal plane due to the axis-symmetric condition for annular arrays.

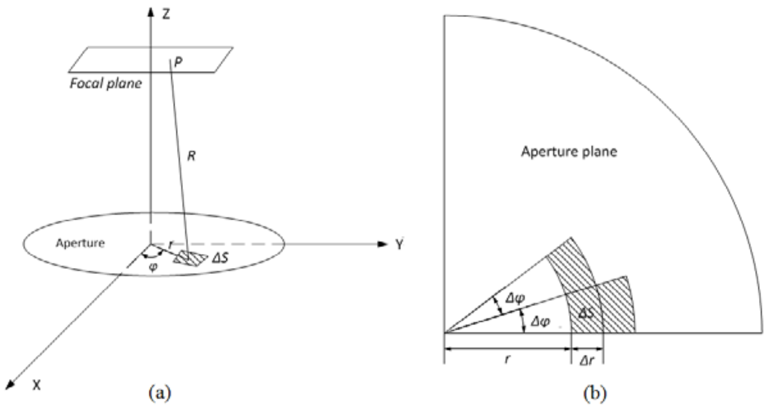

Figure 1 Radiating source with arbitrary shape for Rayleigh integral

The pressure *p* at the point P is a time (*t*) -dependent function related to its position represented by vector $\vec{r_f}$, and can be expressed by Equation (1). Here c is the sound speed, and *g(t)* represents the emitted pressure pulse for every radiating source *dS* and is related to its position represented by vector $\vec{r}$.

$$p(\vec{r_f}, t) = \rho_0 \cdot \int_S \frac{g(\vec{r}, t - R/c)}{2\pi R} dS \quad ; \qquad (1)$$

Instead of using Fourier or Hankel transforms as in the SIR method, Equation (1) is solved directly in the time domain by a discretization process. The radiation aperture is divided into many tiny elements. If the element area *ΔS* is small enough, it represents the source element *dS* in the Rayleigh integral. This allows the integral to be solved directly and numerically by a summation process.

Polar coordinates are then introduced for the annular array geometry; three of these tiny elements are magnified to be clearly seen in Figure 1(b) (shaded region). The increment of radius and azimuth angle is set to be Δr and $\Delta\varphi$ for the element, respectively. Equation (1) is thus transferred to a discrete Rayleigh integral expressed by Equation (2). The distance *R* in Equation (2) is related to the cylindrical coordinate (r_f, 0, F) of vector $\vec{r_f}$ at the focal plane and is given by Equation (3).

$$p(\vec{r_f}, t) = \rho_0 \sum_{N_{ap}, n_\varphi} \frac{g(t - R/c)}{2\pi R} \cdot r_{(N_{ap})} \Delta r \Delta\varphi \qquad (2)$$

$$R = \sqrt{\left(r_f - r_{N_{th}} \cdot cos\ (\varphi_{N_{th}})\right)^2 + \left(r_{n_r}\, sin(\varphi_{N_{th}})\right)^2 + F^2} \qquad (3)$$

Here the subscripts N_{ap} and N_{th} denote the number of points along the radial and azimuth directions in the aperture plane, respectively.

It should be noted that for an annular array to focus, the time delay t_d for each element is required to follow the focusing rule [3]. It means t_d is a function of the radius r for a certain element ΔS. $t_d(r)$ must also be involved in the function g(t), which is now expressed as $g(t\text{-}t_d(r_{(Nth)}))$. In addition, the function $g(t)$ is assumed to be a sinusoidal Gaussian-modulated signal (or Gauss pulse) due to its similar shape to the real pulse emitted from an array element.

By using Matlab (computing software), an algorithm based on Equation (2) has been developed. A 30MHz, 5 element, 1mm diameter annular array was chosen as an example for the numerical evaluation, as its design can be referenced to previous publications [4, 5]. The accuracy and imaging results of this direct method are given in the next section.

3. Accuracy and Results

The accuracy of this direct R-I method is obviously dependent on the value of N_{ap} and N_{th} given in Equation (2), but large values, although offering improved accuracy, would reduce the computing speed. A study of the compromise between speed and accuracy was implemented. Figure 2 shows the imaging pattern along the focal plane for the 30MHz array with different N_{ap} and N_{th}. It can be found that N_{ap} of 128 and N_{th} of 512 is sufficient to show accurate results, and this also offers a quite acceptable computing time.

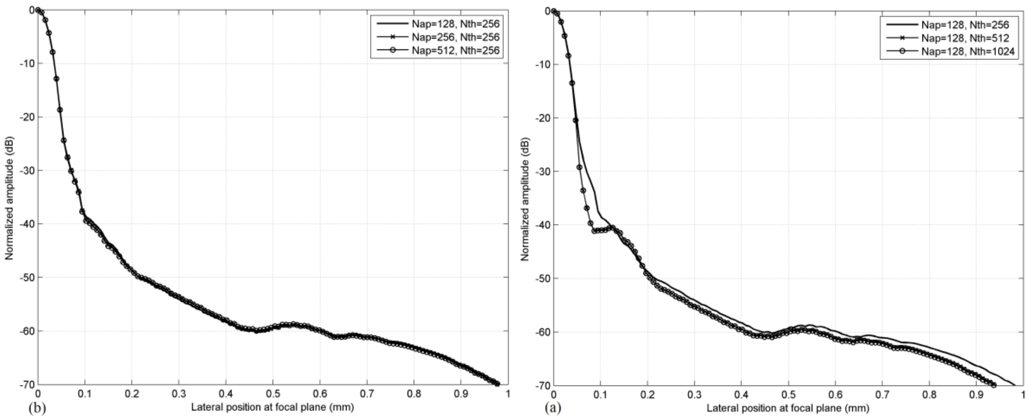

Figure 2 Imaging pattern by the direct method with (a) various N_{ap} but N_{th}=256, and (b) N_{ap}=128 but various N_{th}

However, the function $g(t)$ used in this direct method is the same for every radiating source ΔS, since the array aperture is assumed to be an ideal piston. However, a real emitted pulse has extra factors such as cross coupling from adjacent elements. To allow for these practical discrepancies, FEA is combined with the direct Rayleigh solution, as FEA allows such real imperfections to be modelled. FEA is used to get the pulse responses at the element surface; this is an equivalent to a g(r,t) function for the array aperture. All the data are then imported into the Matlab-coded algorithm to compute the imaging pattern. This combined method is found to show good agreement to the pure FEA response, as illustrated in Figure 3. The response calculated using direct Rayleigh method is also displayed in the figure, and the difference between the two only becomes apparent after a lateral position around 0.1mm onwards.

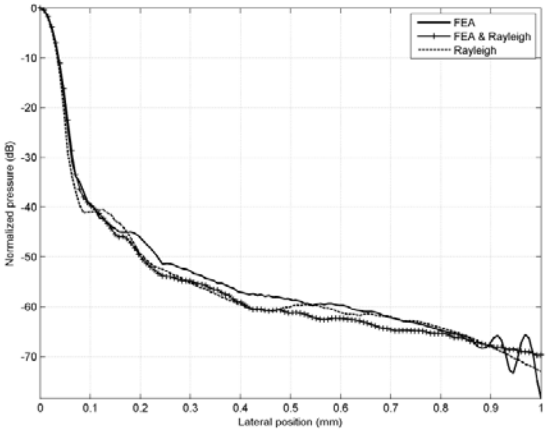

Figure 3 Imaging pattern of the 1mm array along focal line by using different method

More importantly, the combined method not only provides reliable results compared to the practical response, but also reduce the huge amount of computing time if FEA is used for imaging evaluation. FEA needs to model a large volume of fluid medium to simulate the propagation of pressure waves, and therefore this increases the model size and is time-costly. However, the combined method only needs the emitted pulse from the array aperture, which avoids the simulation of wave propagation in FEA. This allows the combined method to save more than 48-hour computing time compared to FEA method only. Thus the efficiency is significantly improved, while the imaging responses are comparable to the FEA results.

4. Conclusion

The direct solution of the Rayleigh integral avoids the Hankel transform calculation in the SIR method for annular arrays. It successfully evaluates the imaging pattern numerically with good efficiency. By combining FEA into this method, the results are more comparable to the practical (FEA) response, and the combined method significantly reduces the computing time compared to using only FEA.

References

1. Jensen, J.A. and N.B. Svendsen, *Calculation of pressure fields from arbitrarily shaped, apodized, and excited ultrasound transducers.* Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on, 1992. **39**(2): p. 262-267.
2. Jensen, J.A., *A new calculation procedure for spatial impulse responses in ultrasound*. Vol. 105. 1999: ASA. 3266-3274.
3. Szabo, T.L., *Diagnostic Ultrasound Imaging: Inside Out*. 2004: Elsevier Academic Press.
4. Snook, K.A., et al., *High-frequency ultrasound annular-array imaging. Part I: Array design and fabrication.* Ieee Transactions on Ultrasonics Ferroelectrics and Frequency Control, 2006. **53**(2): p. 300-308.
5. Wo-Hsing, C., et al. *Design and development of a 30 MHz six-channel annular array ultrasound backscatter microscope*. in *Proceedings of 2002 IEEE International Ultrasonics Symposium*. 2002. Munich, Germany: Ieee.