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Southampton High Energy Physics

On discrete flavour symmetries, neutrino mass and mixing

Iain Ker Cooper

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ABSTRACT

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by Iain Ker Cooper

Neutrino mixing is a thriving area of particle physics research, with the recent discovery of non-zero θ_{13} inspiring a large amount of research into the field. This thesis presents two models which aim to explain the observed neutrino mixing patterns in the context of Grand Unified Theories, which also output quark masses and mixings.

A model predicting Tri-Bimaximal mixing is presented which combines a previously published SU(5) model with an A_4 family symmetry. Extra adjoint fermionic matter is present as prescribed by the original Unified model, and this provides 2 seesaw particles; however they are constrained to give the same contribution to neutrino mixing once the flavour symmetry is imposed. This motivates the addition of an extra field in order to obtain two non-zero neutrino masses. This model has the desirable property of having a diagonal Majorana sector, something which is normally assumed in such models.

In order to explain the discovery of non-zero θ_{13}, a second model is presented which produces Tri-Maximal mixing, a perturbed version of Tri-Bimaximal mixing which retains the solar prediction whilst changing the atmospheric and reactor predictions. This is also performed in a unified context and therefore charged lepton corrections to mixing are related to the Cabibbo angle in a new way via a sum rule.

Finally the impact of flavour symmetries on leptogenesis is discussed; it is mentioned that models which predict neutrino mixing can very often lead to 0 leptogenesis and therefore no baryon asymmetry in the Universe. However this conclusion is drawn without considering the difference in scales between flavour symmetry breaking and leptogenesis. When this is taken into account it is shown in the context of two simple models that successful leptogenesis can be achieved.

Contents

List of Figures

Declaration of Authorship

I, Iain Cooper, declare that this thesis, entitled 'Discrete flavour symmetries, neutrino mass and mixing' and the work presented in it are my own and was done wholly while in candidature for a research degree at this university. The material presented in Chapters 2, 3 and the Appendicies is not original and main sources of information have been referenced therein. Where the published work of others has been consulted or quoted from this is always clearly attributed and all main sources of help have been acknowledged.

The original work presented was conducted by myself in collaboration with my PhD supervisor Steve King and our colleague Christoph Luhn. The calculations in Chapter 4 (based on work published in [1]) were performed by myself, with the exception of the anomaly cancellation conditions placed upon the flavons which was checked independently by me, and the presentation of the original GUT model in Section 4.1. Similarly in Chapter 5 (based on work published in [2]), the formulation of the model was my own work. The calculations in Section 5.3 were performed by Steve King and Chrisotph Luhn and were checked by me. Finally, in Chapter 6 (based on work published in [3]), the first 4 Sections are background and are therefore not original work. The major calculation of the CP violating parameters and the baryon asymmetry were performed by me using Mathematica.

Signed:

Date:

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Abbreviations and Conventions

AF	Altarelli-Feruglio
CKM	Cabibbo-Kobayashi-Maskawa
CN	Canonical Normalisation
CP	Charge-Parity
CSD	Constrained Sequential Dominance
EW	ElectroWeak
EWSB	ElectroWeak Symmetry Breaking
FCNCs	Flavour Changing Neutral Currents
FD	Form Dominance
GJ	Georgi-Jarlskog
GIM	Glashow-Iliopoulos-Maiani
GUT	Grand Unified Theory
GWS	Glashow-Weinberg-Salam
HO	Higher Order
IH	Inverted Hierarchy
IMB	Irvine-Michigan-Brookhaven
LEP	Large Electron-Positron (collider)
LH	Left-Handed
LHC	Large Hadron Collider
LO	Leading-Order
LSP	Lightest Supersymmetric Particle
NH	Normal Hierarchy
NLO	Next-to-Leading-Order
PMNS	Pontecorvo-Maki-Nakagawa-Sakata

QCD	Quantum ChromoDynamics
QED	Quantum ElectroDynamics
QFT	Quantum Field Theory
RG	Renormalisation Group
RGEs	Renormalisation Group Equations
RH	Right-Handed
TB	Tri-Bimaximal
TM	Tri-Maximal
VEV	Vacuum Expectation Value

BSM	Beyond the Standard Model
SM	Standard Model (of particle physics)
SUSY	SUperSYmmetry
MSSM	Minimal SSM

The natural system of units is used throughout where what is written is what is meant multiplied by factors of c and $\hbar$ until it has the dimensions displayed. The Lagrangian density is consistently referred to as the Lagrangian. Mass matrices are given in the left-right convention such that two fields ψ_L and ψ_R have a Dirac mass $m_{LR}\psi_L\psi_R$; related to this is the convention that heavy masses are denoted with a capital M, whereas light masses are denoted with a lower case m. The conventions for spinor notation are found in appendix A. The metric is given by
$\mathrm{diag}\,(1,-1,-1,-1)$.

Chapter 1

Introduction

The SM of particle physics describes the properties and interactions of all directly observed matter in the Universe. With the recent discovery of “a neutral boson with a measured mass of $126.0 \pm 0.4(\text{stat}) \pm 0.4(\text{sys})$ GeV” [4] , it seems that the last piece of the SM puzzle, the Higgs boson, has been discovered. Nevertheless, there are observed phenomena that the SM fails to explain; solutions to these problems require new, BSM physics.

A particular aspect of BSM physics is studied in this thesis, namely the generation of observed neutrino masses and mixings. Since the 60’s when Ray Davis conducted the Homestake experiment, data from neutrino observation had been inconsistent with theory. In these early cases, the neutrinos came from the Sun and experiment observed $\sim \frac{1}{3}$ of the predicted neutrino flux [5]. Solar models were well trusted and tested in other experiments so the conclusion was that there was some missing ingredient in neutrino theory which overestimated the flux. This was known as the solar neutrino problem. A solution was proposed by Pontecorvo [6] and refined in the case of solar neutrinos by Mikheyev, Smirnov and Wolfenstein [7]: neutrinos can oscillate or change flavour during propagation. In the case of the solar neutrino problem, electron neutrinos produced in the Sun’s core can change to muon or tauon neutrinos between creation and detection. Since the Homestake experiment was only configured to detect electron neutrinos, there would be an observable deficit. Super-Kamiokande [8] in Japan lent credence to this hypothesis with the observation of a muon neutrino deficit from cosmic rays interacting with the atmosphere - the atmospheric neutrino anomaly. The first experimental evidence for neutrino mixing came from SNO [9] which measured both the electron neutrino flux and the total neutrino flux from the Sun. Its measurement of the former agreed with Homestake while the latter agreed with the predicted neutrino flux. This implied that the electron neutrinos from the sun were indeed oscillating into muon and tauon neutrinos before detection. Further experiments [10–17] have measured these oscillation phenomena more and more accurately and future experiments are being planned to measure previously inaccessible parameters.

This thesis studies how the observed pattern of neutrino mixing can be explained by imposing some extra symmetry on the Lagrangian of the theory. A wide variety of symmetries have been studied for this purpose [18] but the present work focuses on what appears to be the smallest group available: A_4, the group of even permutations on four elements. This group, and those which contain it, have garnered much attention [19] since the first successful model was built with it [20]. The work presented in this thesis is more ambitious than simply reproducing neutrino mixing patterns however: it also attempts to explain why mixing in the charged lepton sector is so much bigger than mixing in the quark sector. This is achieved by constructing GUT models and using the discrete symmetry to produce both mixing sectors.

One of the consequences of imposing a discrete symmetry on the Lagrangian is also studied in this thesis: the effects of constraining Yukawa couplings on leptogenesis. Leptogenesis attempts to generate the baryon asymmetry of the Universe using the decays of right handed neutrinos introduced to explain why SM neutrinos have such a small mass. One of the parameters of leptogenesis, which encodes CP violation in such decays, depends on the Yukawa couplings. It turns out that constraining these Yukawa couplings in such a way as to explain neutrino mixing will very often lead to the CP violating parameter being 0 and by extension, no baryon asymmetry. A method for avoiding this conclusion is studied and applied to two well known models of neutrino mixing.

The rest of the thesis is organised as follows. Chapter 2 presents a brief overview of the SM in its current state, from the Lagrangian terms to EW symmetry breaking ($SU(3)_c$ interactions are mostly suppressed in this discussion since they are not relevant for the work presented). Important BSM concepts are also introduced: neutrino mass, GUTs and SUSY. Chapter 3 then looks more closely at the PMNS matrix and how the parameters compare to experimental data. A well studied mixing scheme, TB mixing is introduced and it is shown how such a scheme can be related to a symmetry of the Lagrangian. The prototype A_4 model is discussed along with an extension to account for recent observations of non-zero θ_{13} and parameters describing deviations from the TB scheme are presented. Chapter 4 presents an original model studied in [1], which combines a GUT with A_4 to predict TB mixing. This model has several interesting features, most notably a naturally diagonal Majorana sector, the significance of which is also discussed. Next, Chapter 5 presents work published in [2] which combines an extension to the prototype A_4 model with a GUT. This model accommodates non-zero θ_{13} and also gives rise to new sum rules between the neutrino parameters and the Cabibbo angle (the largest parameter in the quark mixing matrix). Chapter 6 then presents work published in [3] dealing with the consequences of discrete family symmetries for leptogenesis. The common problem of family symmetry models producing 0 leptogenesis is addressed by noting the difference in energy scales between the breaking of a family symmetry and the onset of leptogenesis. This means parameters should be evolved between the scales before calculations are performed. Two example models are tested and both obtain successful leptogenesis, reproducing the observed baryon asymmetry of the Universe for a finite region of parameter space. Finally, Chapter 7 concludes the thesis and two Appendices discuss spinor formalism (A) and D-term vacuum alignment (B).

Chapter 2

The Standard Model and beyond

2.1 The Standard Model of Particle Physics

The SM of particle physics provides a description of three of the fundamental forces of nature: the EM force, the weak nuclear force and the strong nuclear force. It does not include the fourth, gravity, as it is not currently known how to provide a QFT description of general relativity. Since gravitational effects are only expected to become important at energy scales around the Planck Mass $M_P \sim 1.2209 \times 10^{19}\ GeV$, one can use the SM as a starting point for describing physics below this scale. In fact, the SM is expected to be valid only up to around the TeV scale (see the SUSY part of Section 2.2) and in this sense, it should only be viewed as an effective theory. The current Section gives a brief overview of the SM and its limitations and then several important BSM concepts are introduced, namely neutrino mass, GUTs and SUSY. In writing this Section, the following sources were consulted: [21], [22], [23], [24] and [25].

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$Q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$(u^c)^i_L$	$\overline{3}$	1	$-\frac{2}{3}$
$(d^c)^i_L$	$\overline{3}$	1	$\frac{1}{3}$
$L^i_L = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$(e^c)^i_L$	1	1	1
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	1	2	$\frac{1}{2}$

Table 2.1: Matter and Higgs content of the SM. The index i runs from $1-3$, reflecting the fact that each matter field comes in three flavours, identical except for their mass.

2.1.1 Gauge symmetry and particle content

The SM is a very successful description of particle physics, describing the properties and interactions of matter and gauge boson fields remarkably successfully. It is based on the local gauge symmetry $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$; here the $SU(3)_c$ symmetry describes QCD, the theory of coloured interactions involving quarks and gluons; and $SU(2)_L \otimes U(1)_Y$ describes the EW interactions of the fermions with the massive gauge bosons (as well as the Higgs boson) and the photon. The field content of the SM can be found in Table 2.1, where all fields are LH in anticipation of GUT building later on in this thesis (spinor conventions can be found in Appendix A). The Lagrangian of the SM encodes all the processes and interactions that the matter in Table 2.1 undergoes; it can be presented as

$$\mathscr{L} = \mathscr{L}_{Gauge} + \mathscr{L}_{Matter} + \mathscr{L}_{Yukawa} + \mathscr{L}_{Higgs}. \tag{2.1}$$

The gauge portion contains the kinetic and self-interaction terms of the gauge bosons; these are of the form

$$-\frac{1}{4}A^{a\mu\nu}A^a_{\mu\nu} \quad \text{with} \quad A^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A_\mu + gf^{abc}A^b_\mu A^c_\nu, \tag{2.2}$$

with one copy of the above term for each simple subgroup of the SM; the gauge fields A^a_μ are in the adjoint representation of the group, and so there are $8+3+1$ gauge degrees of freedom. The f^{abc} are the structure constants for the relevant subgroup. For the rest of this thesis, $SU(3)_c$ interactions will not be considered (beyond its existence in unified theories), so unless specified only $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge bosons will be considered. The matter portion of (2.1) contains kinetic and gauge interaction terms for the five matter fields in Table 2.1, of the form

$$i\overline{\psi}^i \not{D} \psi^i. \tag{2.3}$$

Here, the familiar Feynman slash notation is used, and

$$D_\mu = \partial_\mu - \frac{ig_2\sigma_a}{2} W^a_\mu - \frac{ig_1}{2} B_\mu, \tag{2.4}$$

is the covariant derivative for $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ with weak coupling g_2 and hypercharge coupling g_1. The W^a_μ and B_μ are gauge fields, but do not yet represent the physical gauge boson states. So far, the SM symmetry has forbidden fermion mass terms and this means that there is a rather large accidental symmetry: each of the five matter fields can undergo a U(3) rotation on its flavour index, leaving these terms unchanged. This means that there is an accidental global $\mathrm{U}(3)^5$ symmetry in this part of the Lagrangian. This symmetry is broken by the Yukawa sector which is presented explicitly as

$$\mathscr{L}_{Yukawa} = -Y_u^{ij} \overline{Q}^i_L \epsilon H^* u^j_R - Y_d^{ij} \overline{Q}^i_L H d^j_R - Y_e^{ij} \overline{L}^i_L H e^j_R + h.c. \tag{2.5}$$

where $\epsilon = i\sigma^2$ is the totally antisymmetric tensor, required to maintain Lorentz invariance. The presence of these terms breaks most of the accidental symmetry from $\mathrm{U}(3)^5 \to \mathrm{U}(1)_B \otimes \mathrm{U}(1)_L$, corresponding to baryon and lepton number symmetries respectively. It turns out that $\mathrm{U}(1)_B \otimes \mathrm{U}(1)_L$ suffers from dangerous quantum effects, known as anomalies, that introduce serious problems with the theory; these will be briefly discussed in Section 2.1.4. However, $\mathrm{U}(1)_{B-L}$ does not suffer from anomalies and so this is still used as a constraining symmetry of the

SM. The Yukawa terms describe matter interactions with the Higgs field, which give rise to fermion masses once EWSB takes place and the Higgs obtains its VEV. This process is dictated by

$$\mathscr{L}_{Higgs} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - V\left(H\right). \tag{2.6}$$

At the time of writing, the LHC at CERN has recently published data showing discovery of a boson with the same behaviour as the Higgs, with a mass of $\sim 125\ GeV$, and with a statistical significance of $\sim 5\sigma$ [4, 26]. This result completes experimental observation of the SM particles.

2.1.2 The Higgs potential and Lagrangian masses

The term $V\left(H\right)$ in Eq. (2.6) controls the breaking $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \rightarrow \mathrm{U}(1)_Q$ which allows the fermions and gauge bosons to attain masses. The resultant unbroken symmetry describes electromagnetic interactions and its generator, the electric charge, is built from the broken EW ones by

$$Q = \tau_3 + Y. \tag{2.7}$$

where the $\tau_i = \frac{\sigma_i}{2}$. Explicitly, the potential is

$$V\left(H\right) = -m^2 H^{\dagger}H + \lambda\left(H^{\dagger}H\right)^2, \tag{2.8}$$

where $m^2, \lambda > 0$. These bounds ensure, respectively, a non-zero VEV for the Higgs, and a potential which is bounded from below preventing an infinite cascade of decays. Minimising this potential one obtains the Higgs VEV

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \tag{2.9}$$

with

$$v = \sqrt{\frac{m^2}{\lambda}}. \tag{2.10}$$

Below the EW scale the Yukawa Lagrangian, Eq. (2.5), becomes

$$\mathscr{L}_M = -m_u^{ij}\overline{u}_L^i u_R^j - m_d^{ij}\overline{d}_L^i d_R^j - m_e^{ij}\overline{e}_L^i e_R^j + h.c. \tag{2.11}$$

with

$$m_\alpha^{ij} = Y_\alpha^{ij}\frac{v}{\sqrt{2}}. \tag{2.12}$$

To study how the Higgs VEV affects the gauge bosons, it is convenient to transform to the unitary gauge where the Higgs doublet is real and has no charged component. Then expanding about the vacuum state gives

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\sigma \end{pmatrix}, \tag{2.13}$$

where σ is the physical Higgs boson. Inserting this into Eq. (2.6) and keeping only mass terms then gives

$$\mathscr{L}_{Higgs}^{mass} = \frac{v^2}{8}\left(g_2^2\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right) + \left(g_1 B_\mu - g_2 W_\mu^3\right)^2\right) - \lambda v^2\sigma^2. \tag{2.14}$$

The fields $W_\mu^{1,2}$ are not eigenstates of the charge operator Q defined in Eq. (2.7) since $[Q, \tau_{1,2}] = i\tau_{2,1}$. However it is possible to define $W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$ with corresponding SU(2) raising and lowering operators $\tau_\pm = \frac{1}{\sqrt{2}}\left(\tau_1 \pm i\tau_2\right)$. These operators satisfy

$$[Q, \tau_\pm] = \pm\tau_\pm, \tag{2.15}$$

meaning the fields have charges $\pm e$. The fields W_μ^3 and B_μ require a more careful treatment since although they are both neutral eigenstates of Q, they are mixed in Eq. (2.14); indeed the term involving these fields may be written as

$$\begin{pmatrix} B_\mu & W_\mu^3 \end{pmatrix}\begin{pmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{pmatrix}\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}. \tag{2.16}$$

In order to obtain the mass states, this term must be diagonalised which can be done by defining the rotation matrix

$$\begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} = \begin{pmatrix} \frac{g_2}{\sqrt{g_1^2+g_2^2}} & -\frac{g_1}{\sqrt{g_1^2+g_2^2}} \\ \frac{g_1}{\sqrt{g_1^2+g_2^2}} & \frac{g_2}{\sqrt{g_1^2+g_2^2}} \end{pmatrix}, \tag{2.17}$$

where θ_W is known as the Weinberg angle. Applying this rotation to the fields gives the diagonalised system

$$\begin{pmatrix} A_\mu & Z_\mu^0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & g_1^2+g_2^2 \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix}, \tag{2.18}$$

where the familiar photon $A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$ and neutral Z-boson $Z_\mu^0 = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3$ fields are defined. If one expands Eq. (2.3) using the physical fields, the coupling between A_μ and the fermions (defined to be the electric charge) can be read off as

$$e = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}. \tag{2.19}$$

Inserting the physical fields into Eq. (2.14) then gives

$$\mathscr{L}_{Higgs}^{mass} = \frac{v^2}{8}\left(2g_2^2 W_\mu^+ W_\mu^- + \left(g_1^2+g_2^2\right)\left(Z_\mu^0\right)^2\right) - \lambda v^2 \sigma^2, \tag{2.20}$$

allowing the masses to be read off:

$$M_W = \frac{1}{2} v g_2 = \frac{ev}{2\sin\theta_W}, \tag{2.21}$$

$$M_Z = \frac{1}{2} v \sqrt{g_1^2+g_2^2} = \frac{ev}{2\sin\theta_W\cos\theta_W}, \tag{2.22}$$

$$M_A = 0, \tag{2.23}$$

$$M_\sigma = \sqrt{2\lambda} v. \tag{2.24}$$

It is instructive to ask what has happened in terms of degrees of freedom during this breaking process. In the EW symmetric phase, there is a massless vector boson triplet, a massless vector boson singlet and a massless complex scalar doublet, giving $6+2+4=12$ degrees of freedom. After EW symmetry breaking, there are three massive vector bosons, one massless vector boson and one massive scalar boson giving $9+2+1=12$ again. What has happened is that three of the massless degrees of freedom in the Higgs doublet H have been "eaten" by the gauge bosons, becoming their longitudinal degrees of freedom. These massless degrees of freedom are known as Goldstone bosons and theory dictates that there exists one Goldstone boson per broken symmetry generator; thus in the case of the breaking $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{U}(1)_Q$, there are $4-1=3$ Goldstone bosons as expected. Equations (2.21) and (2.22) lead to the relation

$$\cos\theta_W = \frac{M_W}{M_Z}, \tag{2.25}$$

which itself gives the prediction that

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1. \tag{2.26}$$

This is very accurately measured and so is a benchmark for BSM physics to conform to.[1]

2.1.3 Quark mixing: the CKM matrix

In the previous subsection it was shown how the Higgs mechanism leads to fermion mass matrices, in particular Eq. (2.11) was presented. As in the case of the B_μ, W_μ^3 sector, the actual fermion masses are the eigenvalues of this matrix and so a diagonalisation needs to be performed. This can be done by applying rotations to each of the fermion fields in the SM; in the absence of right handed neutrinos (which will be introduced in Section 2.2), the lepton rotations have no effect on the rest of the Lagrangian. The quark rotations are more interesting in this framework

[1]Although it has now been superseded by the EW precision parameters S, T and U.

however; denoting the rotations by

$$
\begin{aligned}
u'_L &= V_{u_L} u_L, \\
u'_R &= V_{u_R} u_R, \\
d'_L &= V_{d_L} d_L, \\
d'_R &= V_{d_R} d_R,
\end{aligned}
\tag{2.27}
$$

puts the Lagrangian in the mass basis, where the mass matrices are diagonal:

$$m_u^{diag} = (V_{u_L})^\dagger m_u V_{u_R} = \mathrm{diag}\,(m_u, m_c, m_t)\,, \tag{2.28}$$

$$m_d^{diag} = (V_{d_L})^\dagger m_d V_{d_R} = \mathrm{diag}\,(m_d, m_s, m_b)\,. \tag{2.29}$$

All the Lagrangian terms involving quark fields will undergo this rotation, however only the quark coupling to the $W_\mu^\pm$ is affected

$$\overline{u}_L \gamma^\mu W_\mu^\pm d_L + h.c. \to \overline{u}_L \gamma^\mu W_\mu^\pm U_{CKM} d_L + h.c. \tag{2.30}$$

with

$$U_{CKM} = (V_{u_L})^\dagger V_{d_L}. \tag{2.31}$$

This matrix describes interactions, mediated by the $W_\mu^\pm$, which change the flavour of the quark fields. It can be seen that the matrix arises due to the difference in rotation between two fields which exist in the same SU$(2)_L$ doublet before EWSB. This means that only terms which depend on the doublet structure will be affected by this rotation; the A_μ and the Z_μ^0, which have couplings proportional to $\tau_3 + Y$, are left invariant and therefore transitions changing flavour mediated by these neutral gauge bosons do not exist within the SM. The statement that there are no FCNCs in the SM arises from the GIM mechanism and is very well observed in experiment.

The CKM matrix is a unitary 3×3 matrix and therefore has 9 parameters, however only 4 of these are physical. The reason for this is that one is free to perform individual phase redefinitions on each of the 6 quark fields involved in the definition of the CKM matrix. One of these phases cannot be removed since U_{CKM} is invariant under a global phase redefinition, so one can choose 5 of the quark phases to cancel 5 of the CKM parameters, leaving the 4 physical parameters. These correspond to 3 mixing angles and 1 complex phase; a popular parameterisation using these parameters is

$$U_{CKM} = \begin{pmatrix} c_{12}c_{23} & s_{12}c_{13} & s_{13}e^{-i\delta^q} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta^q} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta^q} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta^q} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta^q} & c_{23}c_{13} \end{pmatrix}, \tag{2.32}$$

where $c_{ij} = \cos\theta^q_{ij}$, $s_{ij} = \sin\theta^q_{ij}$ and δ^q is the phase (the superscript ^{q} is to differentiate the parameters from those in the lepton sector that are introduced in Section 3). Experimentally the CKM matrix is observed to be close to diagonal with the biggest angle, $\theta^q_{12} = \theta_C$ known as the Cabbibo angle, being roughly 13°.

2.1.4 Anomalies

Of importance for the consistency of a QFT are anomalies: the quantum violation of a classical symmetry. A very brief overview of this topic follows. Classically, conserved currents are associated to a symmetry principle by Noether's theorem. In particular, vector and axial currents are classically conserved (in the massless limit)

$$\begin{aligned} j^\mu &= \overline{\psi}\gamma^\mu\psi, \qquad & j^{\mu 5} &= \overline{\psi}\gamma^\mu\gamma^5\psi, \\ \partial_\mu j^\mu &= 0, \qquad & \partial_\mu j^{\mu 5} &= 0. \end{aligned} \tag{2.33}$$

whereas at the quantum level, the axial current diverges as

$$\partial_\mu j^{\mu 5} \propto \mathrm{Tr}\left(T^a\left\{T^b, T^c\right\}\right), \tag{2.34}$$

where the T^a are the normalized generators of the relevant gauge group. This current can be used to construct triangle diagrams which violate Ward identities and therefore gauge invariance. Explicitly, these diagrams can provide the photon with a divergent mass (or equivalently, longitudinal and time-like degrees of

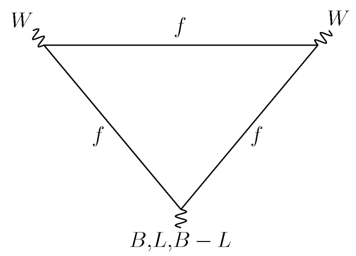

Figure 2.1: A triangle diagram combining 2 $\mathrm{SU}(2)_L$ vertices and one B, L, or $B-L$ vertex. Individually B and L give non-zero contributions from this diagram, but the combination $B-L$ gives no contribution.

freedom), and therefore should be cancelled in order to have a consistent theory. There are a large number of possible triangle diagrams that arise in the SM which, after calculation, are indeed 0 under three assumptions:

- the hypercharges of the fields are assigned as in Table 2.1,
- there are complete generations of matter, i.e. if there are u-, c- and t-type quarks then there must also be e-, μ- and τ- type leptons,
- there are three copies of quark doublet for every lepton doublet, i.e. there are three colours of quark.

The first assumption finds no explanation within the SM, but GUTs can provide an answer as described in subsection 2.2.2; the third arises from assigning quarks to an $\mathrm{SU}(3)_c$ gauge theory. The second, in combination with LEP data for the Z^0 decay widths, constrains the number of light generations to be 3 [27]. As an example of anomalies in the SM, consider Fig. 2.1, a triangle diagram with 2 $\mathrm{SU}(2)_L$ vertices and one B, L, or $B-L$ vertex. The contribution of this diagram to the anomaly is given by

$$\mathrm{Tr}\left(T^{B,\,L,\,B-L}\left\{\tau^b,\tau^c\right\}\right)=\frac{1}{2}\delta^{bc}\sum_{f_L}Q_{f_L}^{B,\,L,\,B-L}, \tag{2.35}$$

which evaluate to

$$\begin{aligned}&\frac{1}{2}\delta^{bc}\times 3\times 3\times\frac{1}{3},\\&\frac{1}{2}\delta^{bc}\times 3\times 1,\\&\frac{1}{2}\delta^{bc}\left(3\times 3\times\frac{1}{3}-3\times 1\right)=0,\end{aligned} \tag{2.36}$$

respectively. This demonstrates that while B and L are individually anomalous, $B - L$ is not.

2.2 Beyond the Standard Model

2.2.1 Neutrino mass and the seesaw mechanism

As can be seen in the previous Sections, the SM does not admit mass for the neutrinos. This is because a Majorana mass term (the charge conjugation matrix C is defined in Appendix A)

$$-\frac{1}{2}m_\nu^{ij}\left(\nu_L^{iT}C\nu_L^j + h.c.\right), \tag{2.37}$$

breaks the $B - L$ symmetry and is therefore forbidden. Experimental observation of neutrino oscillation indicate that the neutrino cannot be massless however; therefore the SM needs to be extended in some way as to provide neutrinos with mass. Furthermore several constraints, such as the non-observation of $0\nu\beta\beta$, exist to bound the sum of the neutrino masses at $\lesssim 1\ eV$. This means that the neutrino mass scale is a factor $\sim 10^{-6}$ smaller than the electron and so the extension to the SM should also explain this ratio ideally without inserting such a factor arbitrarily. The key is to recall that the SM is an effective theory, valid up to a particular energy scale Λ:

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{1}{\Lambda}\mathscr{L}_5 + \frac{1}{\Lambda^2}\mathscr{L}_6 + \dots \tag{2.38}$$

At energies below Λ, the higher dimensional terms $\mathscr{L}_5$, $\mathscr{L}_6$ etc are suppressed by powers of Λ. Using the field content and gauge symmetries of the SM, the only allowed dimension-5 term is [28] (the notation Y_ν^{ij} is in anticipation of this leading to neutrino mass terms)

$$\mathscr{L}_5 = \frac{Y_\nu^{ij}}{\Lambda}L_L^{iT}\epsilon\phi C\phi^T\epsilon L_L^j + h.c. \tag{2.39}$$

known as the Weinberg dimension-5 operator. When the Higgs field obtains its VEV, this becomes a Majorana mass for the neutrino field

$$\mathscr{L}_{Maj} = -\frac{Y_\nu^{ij}}{2}\frac{v^2}{\Lambda}\nu_L^{iT} C \nu_L^j + h.c. \tag{2.40}$$

which is suppressed by the scale at which new physics enters. This term still violates the $B - L$ symmetry, but only at a high scale, meaning it remains an approximate symmetry at low energies. The issue is now to explain the origin of the Weinberg operator. The most common explanation is to extend the SM by introducing a RH neutrino N_R (completing the pairs of LH and RH fields) which has SM charges $(\mathrm{SU}(3)_c, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y} = (1,1)_0$, i.e. it is a singlet. This is known as the Type I seesaw mechanism [29] and gives rise to the new Lagrangian terms

$$\mathscr{L}_N = -Y_\nu^{ij}\overline{L}_L^i \epsilon \phi^* N_R^j - \frac{1}{2} N_R^{iT} M_R^{ij} C N_R^j + h.c. \tag{2.41}$$

where there is no constraint on the size of M_R. If it is taken to be 0, ν_L and N_R pair up to form a Dirac neutrino with mass $\sim v$; however, if $M_R \gg v$ the N_R can be integrated out of the Lagrangian using its equation of motion. This results in an effective Majorana mass for the ν_L

$$\mathscr{L}_{TypeI} = -\frac{1}{2} m_\nu^{ij} \nu_L^{iT} C \nu_L^j + h.c. \tag{2.42}$$

with

$$m_\nu \sim -v^2 Y_\nu M_R^{-1} Y_\nu^T. \tag{2.43}$$

Schematically this can be represented by the Feynman diagram in Fig. 2.2. Taking the Yukawa couplings to be ~ 1, the Higgs VEV to be 246 GeV and the neutrino mass scale to be 0.1 eV, one obtains a RH neutrino mass scale of around 10^{14} GeV. Looking more closely at the Dirac vertex in Fig. 2.2, since L_L and ϕ are $\mathrm{SU}(2)_L$ doublets, the internal field can be either **1** or **3** under $\mathrm{SU}(2)_L$. The former case is the above Type I seesaw [29], whilst the latter is the Type III seesaw [30]. There is also the Type II [31] seesaw where a new Higgs triplet couples to two L_L, but this is fundamentally different from the other two and is not discussed further.

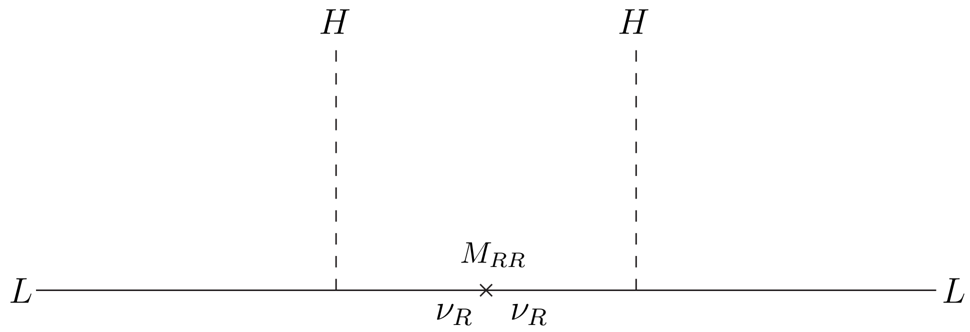

Figure 2.2: Schematic diagram of the type I seesaw mechanism.

Knowledge of neutrino masses is fairly limited: the major observable effect is neutrino oscillation, whose probability is dependent on the squared mass splittings rather than the absolute masses. Current global fits [32, 33] give these mass splittings to be (using extreme 1σ ranges)

$$7.32 < \Delta m^2_{sol} = m_2^2 - m_1^2 \left(\frac{eV^2}{10^{-5}}\right) < 7.81, \tag{2.44}$$

$$2.37 < \Delta m^2_{atm} = m_3^2 - m_1^2 \left(\frac{eV^2}{10^{-3}}\right) < 2.61 \quad (NH), \tag{2.45}$$

$$-2.53 < \Delta m^2_{atm} = m_3^2 - m_1^2 \left(\frac{eV^2}{10^{-3}}\right) < -2.30 \quad (IH). \tag{2.46}$$

This shows that while one of the signs of the splittings is known, the other is not and therefore the neutrino spectrum could be one of the two shown in Fig. 2.3. Furthermore, the absolute mass scale is not currently known, although cosmological bounds can be placed on the sum of the neutrino masses, presently $\sim 1eV$ (this is a difficult parameter to place bounds on, see discussion and references in [34]).

The introduction of right handed neutrinos and, in particular, the Yukawa term in Eq. (2.41) introduces a mixing matrix for the lepton sector, analogously to the CKM matrix. It is known as the PMNS matrix and is defined in terms of charged lepton and neutrino diagonalisation matrices as

$$U_{PMNS} = V_{e_L} V^\dagger_{\nu_L}. \tag{2.47}$$

The major difference between the PMNS matrix and the CKM matrix is the number of parameters: whereas in the CKM case, 5 of the phases could be removed,

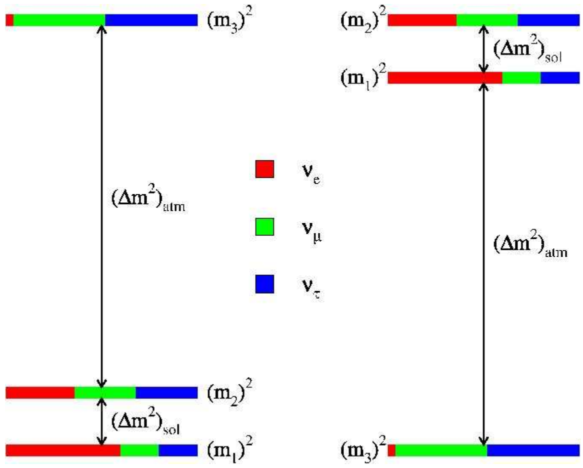

Figure 2.3: The present status of neutrino mass measurements. The colours also represent the approximate flavour content in each mass eigenstate (taken from [35]).

in the PMNS case only 3 may be removed. This difference is because of the (assumed) Majorana nature of the neutrino and therefore less freedom to redefine fields in order to remove phases. If one were to use the neutrino fields to remove PMNS phases, then the Majorana mass matrix of Eq. (2.43) would pick up unremovable phases. Therefore one cannot absorb PMNS phases into the Majorana neutrino fields (apart from an overall phase) and so the PMNS matrix has 3 angles and 3 phases. It can be parameterised in the same way as the CKM matrix in Eq. (2.32) but right multiplied by a diagonal matrix containing 2 phases.

It was stated above that the Majorana nature of neutrinos is assumed. Observation of a process known as neutrinoless double beta decay will confirm that neutrinos are indeed Majorana since the process cannot happen otherwise. The current status and experimental progress on neutrinoless double beta decay can be found in [36] and references therein. Experimentally measured values for the mixing parameters (excluding Majorana phases) will be introduced and discussed in the next Chapter.

2.2.2 Grand Unification

The gauge group of the SM, $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, has three factors and therefore three gauge couplings. Although these are sometimes referred to as constants, in fact they run with energy scale. This will be used in Chapter 6, but

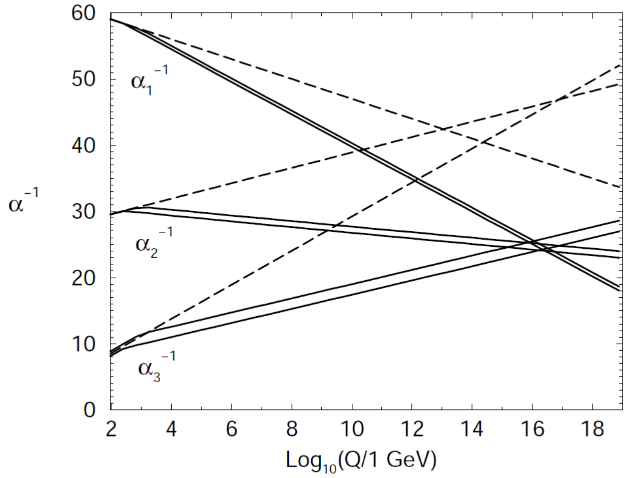

Figure 2.4: Running of the inverse fine structure constants with energy scale. The dotted lines correspond SM couplings and the solid to MSSM couplings (taken from [37]).

for now it suffices to observe that the couplings appear to converge as the energy scale is increased, as in Fig. 2.4 (dotted lines - here the running of the inverse fine structure constants $\alpha_i = \frac{g_i^2}{4\pi}$ are plotted such that the plot is linear). Although the convergence is not exact in the SM, this tendency is enough to suggest that the 3 couplings could unify to one at some high scale denoted M_{GUT}. This would correspond to the SM gauge group being embedded in some larger group with only one factor; the smallest group that can achieve this is SU(5) [38], generated by $\frac{\lambda_{1-24}}{2}$. The fields of the SM are grouped together in larger multiplets and the embedding is defined by the decomposition of SU(5) representations under the SM. In particular [39]

$$\begin{aligned} \overline{\mathbf{5}} &\to \left(\overline{3},1\right)_{\frac{1}{3}} + (1,2)_{-\frac{1}{2}}\,, \\ \mathbf{10} &\to (3,2)_{\frac{1}{6}} + \left(\overline{3},1\right)_{-\frac{2}{3}} + (1,1)_1\,. \end{aligned} \tag{2.48}$$

This shows that the matter content of the SM can be contained in the combination $\overline{\mathbf{5}}_i + \mathbf{10}_i$ with i being the generation index. One generation is then written as

$$\overline{\mathbf{5}} = \psi_j = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu^e \end{pmatrix}_L , \tag{2.49}$$

$$\mathbf{10} = \psi^{jk} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -(e^c) \\ d_1 & d_2 & d_3 & (e^c) & 0 \end{pmatrix}_L . \tag{2.50}$$

Here, the numerical indices represent the three distinct colour charges of SU(3)$_c$ (recall that the quark fields are triplets under SU(3)$_c$). Calculating the anomaly coefficient, Eq. (2.34), for these two representations shows that they cancel each other and the SU(5) theory is anomaly free. The **5** representation contains the charges corresponding to the Higgs doublet, along with a colour triplet in the field

$$H_{\mathbf{5}} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{pmatrix} , \tag{2.51}$$

whilst the GUT symmetry is broken by a Higgs in the adjoint representation $H_{\mathbf{24}} = \phi^a \frac{\lambda^a}{2}$ (here, λ^a are the SU(5) generators). The existence of the SM Higgs doublet in a representation which also contains a colour triplet is a common problem with GUTs - while the SM state should be light (around the TeV scale), the coloured state which could lead to proton decay should be heavy in order to suppress such decays. This is known as the doublet-triplet splitting problem; there are several generic solutions to this problem [40], generally involving introducing extra large Higgs representations, which give the triplet Higgs a GUT scale mass while keeping the SM Higgs at the EW scale.

The general Yukawa Lagrangian with the above field content is (denoting $\overline{\mathbf{5}}$ by F and $\mathbf{10}$ by T)

$$\mathscr{L} = (Y_d)_{ij}\, F_i T_j H_{\overline{\mathbf{5}}} + (Y_u)_{ij}\, T_i T_j H_{\mathbf{5}}, \tag{2.52}$$

which demonstrates a problem with the unification: the mass matrices of the down quark and charged leptons are the same at the GUT scale. This prediction fails when the parameters are evolved to the EW scale and compared with experiment [41] and so extra matter can be included in the model in order to fix this. Introducing a $H_{\overline{\mathbf{45}}}$ gives the correct high energy mass (GJ) relations [42]; the Yukawa sector is now (inserting Greek SU(5) indicies explicitly)

$$\begin{aligned} \mathscr{L} = &(F_\alpha)_i \left(T^{\alpha\beta}\right)_j (Y_{d1})_{ij} (H_{\overline{\mathbf{5}}})_\beta + (F_\alpha)_i \left(T^{\beta\gamma}\right)_j (Y_{d2})_{ij} (H_{\overline{\mathbf{45}}})^{\alpha}_{\beta\gamma} \\ &+ (Y_u)_{ij} \left(T^{\alpha\beta}\right)_i \left(T^{\gamma\delta}\right)_j (H_{\mathbf{5}})^{\epsilon}\, \epsilon_{\alpha\beta\gamma\delta\epsilon}. \end{aligned} \tag{2.53}$$

with $\epsilon_{\alpha\beta\gamma\delta\epsilon}$ the totally antisymmetric rank 5 tensor. The new term gives contributions to the mass of the charged leptons and down type quarks once the Higgs fields obtain their VEVs (note that the $H_{\mathbf{45}}$ satisfies ${H_{\mathbf{45}}}^{\alpha\beta}_{\gamma} = -\,{H_{\mathbf{45}}}^{\beta\alpha}_{\gamma}$ and ${H_{\mathbf{45}}}^{\alpha\beta}_{\alpha} = 0$)

$$\langle H_{\mathbf{5}}\rangle^\alpha = \begin{cases} v_5 & \text{for} \quad \alpha = 5, \\ 0 & \text{for} \quad \alpha \neq 5. \end{cases} \tag{2.54}$$

$$\langle H_{\mathbf{45}}\rangle^{\alpha\beta}_{\gamma} = \begin{cases} v_{45} & \text{for} \quad \beta = 5,\ \alpha = \gamma = 1-3, \\ -3v_{45} & \text{for} \quad \beta = 5,\ \alpha = \gamma = 4, \\ -v_{45} & \text{for} \quad \alpha = 5,\ \beta = \gamma = 1-3, \\ 3v_{45} & \text{for} \quad \alpha = 5,\ \beta = \gamma = 4, \\ 0 & \text{otherwise}. \end{cases} \tag{2.55}$$

The factors of three arise from the tracelessness and antisymmetry of the $H_{\mathbf{45}}$ and the fact that the VEV of the $H_{\mathbf{45}}$ leaves $SU(3)_c$ unbroken:
$\langle H_{\mathbf{45}}\rangle^{\alpha 5}_{\alpha} = \langle H_{\mathbf{45}}\rangle^{15}_{1} + \langle H_{\mathbf{45}}\rangle^{25}_{2} + \langle H_{\mathbf{45}}\rangle^{35}_{3} + \langle H_{\mathbf{45}}\rangle^{45}_{4} + \langle H_{\mathbf{45}}\rangle^{55}_{5} = 3v_{45} + \langle H_{\mathbf{45}}\rangle^{45}_{4} = 0.$

Using (2.54) and (2.55) (2.53) can be expanded to find the mass matrices

$$m_d = Y_{d1} v_5^* + 2 Y_{d2} v_{45}^* \quad \text{and} \quad m_e = \left(Y_{d1}\right)^T v_5^* - 6 \left(Y_{d2}\right)^T v_{45}^*. \tag{2.56}$$

If the $Y_{d1,2}$can be constrained such that[2]

$$Y_{d1} \sim \begin{pmatrix} 0 & A & 0 \\ B & 0 & 0 \\ 0 & 0 & D \end{pmatrix} \quad , \quad Y_{d2} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{C}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{2.57}$$

then the form of the mass matrices is fixed

$$m_d \sim \begin{pmatrix} 0 & A & 0 \\ B & C & 0 \\ 0 & 0 & D \end{pmatrix} , \quad m_e \sim \begin{pmatrix} 0 & B & 0 \\ A & -3C & 0 \\ 0 & 0 & D \end{pmatrix} . \tag{2.58}$$

These yield GUT scale mass relations of [42]

$$\frac{m_d}{3 m_e} = 1, \tag{2.59}$$

$$\frac{3 m_s}{m_\mu} = 1, \tag{2.60}$$

$$\frac{m_b}{m_\tau} = 1, \tag{2.61}$$

which are much closer to the data than those from minimal SU(5) [41].

Unifying the SM fields in this manner has some appealing properties beyond unifying the fundamental forces; a particularly interesting property is explaining the quantization of hypercharge and therefore electric charge. This simply arises from the fact that, since Y is a generator of SU(5), its action on any representation should sum to 0, leading to (see e.g. [43])

$$Y\left(d^c\right) = -\frac{1}{3} Y\left(L\right). \tag{2.62}$$

This argument explains the somewhat arbitrary looking hypercharge assignments

[2]This constraining can be enforced using discrete symmetries which is the method used in the rest of this thesis.

found in Table 2.1. Connected to this is the fact that in order to identify the common GUT coupling (which is denoted by g_5) with the SM couplings g_1, g_2 (and g_3), one needs to perform normalisation correctly. The covariant derivative in SU(5) is (denoting gauge boson fields by $\hat{A}_\mu$) $\partial_\mu - ig_5\hat{A}_\mu = \partial_\mu - i\frac{g_5}{2}W^i_\mu\sigma^i - i\frac{g_5}{2}B_\mu\lambda_{24} + \ldots$; in an $SU(2)_L \otimes U(1)_Y$ theory, the covariant derivative is $\partial_\mu - i\frac{g_2}{2}W^i_\mu\sigma^i - i\frac{g_1}{2}B_\mu Y$. In order for the couplings to be identified, λ_{24} must be rewritten in terms of the hypercharge matrix Y: $\lambda_{24} = \sqrt{\frac{3}{5}}Y$. This allows for the identification (where the g_3 identification has been made for completeness)

$$g_{2,(3)} = g_5 \quad \text{and} \quad g_1 = \sqrt{\frac{3}{5}}g_5, \tag{2.63}$$

ensuring that the hypercharges are defined consistently with Table 2.1.

As appealing as this theory is, it has already been excluded by several experiments which search for decays of the proton. The reason is that the predicted proton decay lifetime is given by [44]

$$\tau_p = \frac{1}{\alpha_5^2}\frac{M_{GUT}^4}{m_p^5}. \tag{2.64}$$

α_5 is the SU(5) fine structure constant and M_{GUT} is determined by where the gauge couplings meet, $\sim 10^{15}$ GeV in the SM; using this then gives a predicted proton decay lifetime of $\sim 10^{30}$ years. The IMB has put a lower limit on the proton decay lifetime at $\geq 10^{32}$ years, whilst more recently Super-Kamiokande has strengthened this limit to $\geq 10^{33}$ years [45]. Still, the simplicity of this theory motivates further study and in the next subsection, the predicted value of τ_p will be increased.

2.2.3 SUSY and the hierarchy problem

The SM with a Higgs boson is very robust from an experimental point of view, however it has one major theoretical flaw known as the hierarchy problem. This arises from the fact that any complex scalar in a QFT will receive dangerous contributions to its mass beyond tree level, since it is not protected by any symmetry (in other words, no symmetry is restored by setting the Higgs mass to 0). Schematically, consider the first diagram in Fig. 2.5, where the scalar mass term is

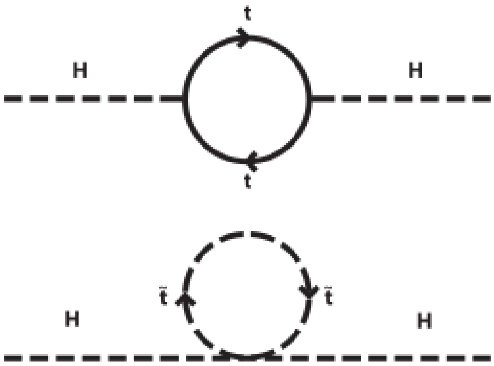

Figure 2.5: Dominant one-loop corrections to the Higgs mass from top and stop loops.

corrected by a fermion in the loop. This correction will generically have the form

$$-\lambda_f^2 \left(a\Lambda_{UV}^2 + bm_F^2 + ... \right), \tag{2.65}$$

where a is some order 1 coefficient and b is at most logarithmically divergent; both of these parameters are renormalisation scheme dependent. This immediately looks like a problem, since even if no new matter existed between the EW and the Planck scale, the theory is still an effective one and so Λ_{UV} would be the cutoff of the theory, M_P. Therefore in order that the Higgs mass parameter is $\sim (125\ GeV)^2$, one needs to tune the bare Higgs mass and the one-loop corrections to around one part in 10^{30}. It is of course possible here to exploit the fact that the theory is renormalisable and so use counterterms to cancel the quadratic divergences at all orders. One now has to consider the parts proportional to the mass of the particle in the loop; this will be dominated by the highest m_F in the theory which could very well be around M_P itself. This part of the correction can be removed by a counterterm at the current order, however new corrections of this sort will be regenerated at the next order. In order to cancel these terms to all orders, one needs to retune the counterterms *at every order* in order to prevent the mass receiving a large correction $\propto m_F$. Unless one is willing to accept these large fine tunings, a solution must be sought.

Consider introducing a scalar field with mass m_S and allowing this to couple to the

Higgs as well, as in the second diagram of Fig. 2.5. This contributes

$$\lambda_S \left(a\Lambda_{UV}^2 + b' m_S^2 + ... \right), \tag{2.66}$$

to the Higgs mass parameter and these quadratic divergences will therefore cancel with the fermion ones if $\lambda_S = \lambda_F^2$. Furthermore if $m_F = m_S$, then the logarithmic divergences will also cancel each other out; even if the masses are not exactly equal, so long as their difference is not too large, the logarithmic divergences will not be too damaging for the theory.

Such a situation arises in the form of SUSY, an extension of the Poincaré algebra to include anticommuting operators Q which act on fermions to produce bosons and vice versa:

$$\begin{aligned} Q|\text{Boson}\rangle = |\text{Fermion}\rangle, \qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle, \\ \left\{Q, Q^\dagger\right\} = P^\mu, \\ \{Q, Q\} = \left\{Q^\dagger, Q^\dagger\right\} = 0, \\ [P^\mu, Q] = \left[P^\mu, Q^\dagger\right] = 0, \end{aligned} \tag{2.67}$$

where P^μ is the four-momentum generator of spacetime translations. Spinor indices (necessary since the Q are fermionic operators) have been suppressed in the above for simplicity. The action of these new generators Q doubles the spectrum of the SM, by introducing a bosonic partner for every fermion and vice versa. The combined contributions of these partners cancels one another out when calculating the correction to the Higgs mass. The MSSM is the minimal version of SUSY, adding only the required superpartners to each SM field. The extra content can be found in Table 2.2; notice that there are now two Higgs doublets as opposed to the one in the SM. This is required for several reasons, the simplest of which is anomaly cancellation: when the SM Higgs acquires its fermionic superpartner, the corresponding hypercharge will contribute to the anomaly and spoil the cancellation. A multiplet with opposing hypercharge must be introduced in order to restore this cancellation. Each partner and superpartner reside in a multiplet called a superfield; superfields which contain chiral fermions are called chiral superfields,

Field	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$
$\widetilde{Q}^i_L = \begin{pmatrix} \widetilde{u}^i_L \\ \widetilde{d}^i_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$(\widetilde{u}^c)^i_R$	$\overline{3}$	1	$-\frac{2}{3}$
$\left(\widetilde{d}^c\right)^i_R$	$\overline{3}$	1	$\frac{1}{3}$
$\widetilde{L}^i_L = \begin{pmatrix} \widetilde{\nu}^i_L \\ \widetilde{e}^i_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$(\widetilde{e}^c)^i_R$	1	1	1
$\widetilde{H}_u = \begin{pmatrix} \widetilde{H}^+_u \\ \widetilde{H}^0_u \end{pmatrix}$	1	2	$\frac{1}{2}$
$\widetilde{H}_d = \begin{pmatrix} \widetilde{H}^0_d \\ \widetilde{H}^-_d \end{pmatrix}$	1	2	$-\frac{1}{2}$

Table 2.2: Matter superpartners of the MSSM. The index i runs from $1-3$, reflecting the fact that each matter field comes in three flavours, identical except for their mass.

whereas those containing the gauge bosons and fermion partners are called vector superfields.

With respect to naming individual superpartners: a scalar partner to an SM fermion is prefixed with an "s", such that the partner of a fermion is called a sfermion (and explicitly, the partner of an electron is called a selectron); a fermionic partner to a SM scalar is appended with "-ino" (explicitly, the partner of a photon is called a photino). Along with these fermion/scalar partners, each superfield must also contain a non-propagating auxiliary field F which is a complex scalar with mass dimension 2: this is required in order to close the SUSY algebra off-shell.

Invariance under SUSY is very restrictive when trying to construct Lagrangians and in fact all the interactions of matter fields can be described by the Superpotential which is a polynomial in the scalar components of a superfield. The general superpotential is given by

$$W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k, \tag{2.68}$$

where objects of rank > 1 are symmetric in all indicies. Invariance under SUSY transformations also requires that W be holomorphic (simply put, it cannot contain both ϕ and ϕ^*); this is more motivation for the addition of a second Higgs doublet. In the SM, H interacted with d-type quarks and charged leptons while ϵH^* interacted with u-type quarks. In W, this is not possible due to the holomorphic property, necessitating the introduction of a Higgs field in the conjugate representation under the SM group; the notations H_u and H_d correspond to the quark sector with which they interact. A general Lagrangian can then be constructed as (here the σ^μ are the 2×2 identity for $\mu = 0$ and the Pauli matrices for $\mu = 1 - 3$)

$$\mathscr{L}_{SUSY} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \overline{\sigma}^\mu \partial_\mu \psi_i - \frac{1}{2}\left(W^{ij}\psi_i\psi_j + W^*_{ij}\psi^{\dagger i}\psi^{\dagger j}\right) - W^i W^*_i, \quad (2.69)$$

where the indicies i, j on the W indicate a partial derivative of W with respect to $\phi_{i,j}$:

$$W_{i,j} = \frac{\partial W}{\partial \phi_{i,j}}. \quad (2.70)$$

From this Lagrangian, it can be seen that W has mass dimension 3 and encapsulates all the interactions of matter fields with one another. The gauge part of the Lagrangian can be constructed similarly using fields from a vector supermultiplet which contains the SM gauge bosons, their fermion superpartners the gauginos and a real bosonic auxiliary field D^a. This is analogous to the F_i field in the chiral supermultiplet; it doesn't propagate and has mass dimension 2. Both of these auxiliary fields can be re-expressed in terms of the scalar fields ϕ using their equations of motion

$$F_i = -W^*_i, \quad D_a = -g\left(\phi^* T_a \phi\right), \quad (2.71)$$

where g and T_a are the relevant gauge couplings and normalised generators. Using these equations one can write down the scalar potential of the theory

$$V\left(\phi, \phi^*\right) = F^{*i}F_i + \frac{1}{2}\sum_a D^a D^a = W^*_i W^i + \frac{1}{2}\sum_a g_a^2 \left(\phi^* T^a \phi\right)^2. \quad (2.72)$$

As opposed to the Higgs potential of the SM, the parameters in this potential are defined by the SUSY interactions (Yukawa couplings, mass terms and gauge couplings). Since experiment has not observed the numerous superpartners predicted by exact SUSY, it can be inferred that these superpartners are heavier than their SM counterparts, and therefore SUSY is a broken symmetry. This means that SUSY breaking vacua can be found by looking for models with $F_i \neq 0$ and/or $D^a \neq 0$. It must be kept in mind however that this SUSY breaking can not be too large, otherwise the hierarchy problem will be reintroduced again.

The introduction of a second Higgs doublet complicates the SUSY Higgs potential somewhat; the most general potential is now

$$\begin{aligned} V_H = &\left(|\mu|^2 + m_{H_u}^2\right)\left(\left|H_u^0\right|^2 + \left|H_u^+\right|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(\left|H_d^0\right|^2 + \left|H_d^-\right|^2\right) \\ &[b\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + h.c.] + \frac{1}{8}\left(g_2^2 + g_1^2\right)\left(\left|H_u^0\right|^2 + \left|H_u^+\right|^2\right. \\ &\left. - \left|H_d^0\right|^2 - \left|H_d^-\right|^2\right)^2 + \frac{1}{2} g_2^2 \left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2, \end{aligned} \tag{2.73}$$

where b, m_{H_u} and m_{H_d} are SUSY breaking parameters. As before, one of the VEVs can be rotated away; choosing $\langle H_u^+ \rangle = 0$ then implies that $\langle H_d^- \rangle = 0$ meaning that $U(1)_Q$ is still unbroken in SUSY. This leaves the simplified potential

$$\begin{aligned} V_H = &\left(|\mu|^2 + m_{H_u}^2\right)\left(\left|H_u^0\right|^2\right) + \left(|\mu|^2 + m_{H_d}^2\right)\left(\left|H_d^0\right|^2\right) - b\left(H_u^0 H_d^0 + h.c.\right) \\ &\frac{1}{8}\left(g_2^2 + g_1^2\right)\left(\left|H_u^0\right|^2 - \left|H_0^d\right|^2\right)^2. \end{aligned} \tag{2.74}$$

Denoting the VEVs of the two fields by $v_u = \langle H_u^0 \rangle$ and $v_d = \langle H_d^0 \rangle$, the SM Higgs VEV is related to these two by

$$v_u^2 + v_d^2 = \frac{v^2}{2} = (174\ GeV)^2. \tag{2.75}$$

It is popular to express the ratio between the two VEVs as

$$\tan\beta = \frac{v_u}{v_d}, \tag{2.76}$$

which means that

$$v_u = v\sin\beta, \quad v_d = v\cos\beta. \tag{2.77}$$

The parameter μ in the above is a source of concern for physicists: in order for the quadratic Higgs terms in (2.74) to obtain VEVs around the EW scale without large cancellations, $|\mu|^2 \sim m_{H_u}^2 \sim m_{H_d}^2$. Whilst μ is a SUSY preserving parameter, the other two break SUSY and so can in principle be much larger. This is known as the μ-problem of SUSY. There are several solutions (see, for example [47], [48], [49], [50] or [51]), mostly involving forbidding this term but allowing a term coupling the two Higgs doublets to a singlet; this singlet can then obtain a VEV which becomes the μ parameter. The size of the VEV is naturally related to the EW scale by the SUSY breaking procedure.

The final consideration that needs to be made when constructing SUSY models is the fact that B- and L-violating terms are allowed in the superpotential at the renormalisable level:

$$W_{\Delta L=1} = \frac{1}{2}\lambda^{ijk} L_i L_j \overline{e}_k + \lambda'^{\,ijk} L_i Q_j \overline{d}_k + \mu'^{\,i} L_i H_u, \tag{2.78}$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''^{\,ijk} \overline{u}_i \overline{d}_j \overline{d}_k. \tag{2.79}$$

These operators can lead to very rapid proton decay, violating the experimental bounds from IMB and Super-Kamiokande. Such terms can generically be forbidden by imposing a discrete symmetry known as an R symmetry, where a field's charge defined by

$$P_R = (-1)^{3(B-L)+2s}. \tag{2.80}$$

This definition implies that SM fields have an R-charge of $+1$ whilst the superpartners have -1, meaning that at a vertex there must be an even number of superpartners in order for R-symmetry to be conserved. One can conclude from this that the LSP will be stable and therefore, if electrically neutral, a promising candidate for dark matter that makes up $\sim 25\%$ of the matter in the Universe [46].

Referring back to Fig. 2.4, it seems that the unification of gauge couplings in SUSY scenarios is better than in the SM; furthermore, it occurs at a higher energy, at around 2×10^{16} GeV [52]. Using this value to estimate the proton decay lifetime instead of the SM value gives $\tau_p \sim 10^{35}$ years which is much more promising when compared to experiment than the SM value.

Chapter 3

Discrete symmetry and neutrino mixing

One of the outstanding issues that the SM leaves unanswered is that of flavour: why are there three copies of SM generations, with the specific mass ratios and mixing patterns observed? A particularly interesting question is the origin of the large differences between CKM and PMNS parameters. The CKM matrix is observed to be close to the identity; in the well known Wolfenstein parameterisation [53]

$$U_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ \lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3\left(1-\rho-i\eta\right) & -A\lambda^2 & 1 \end{pmatrix} + O\left(\lambda^4\right), \tag{3.1}$$

where $\lambda = \sin\theta_C \sim 0.22$ controls the magnitude of the entries. In contrast, the PMNS matrix is observed to have two large mixing angles, as can be seen from the latest global fits [32, 33] in Table 3.1. The recent measurement of a non-zero θ_{13} in early 2012 represented a big step in neutrino physics as until then, most attempts to explain mixing patterns focused on predicting $\theta_{13} = 0$. Recalling that the PMNS

Parameter	Extreme range
$\sin^2\theta_{12}$	$0.291 - 0.335$
$\sin^2\theta_{23}$	$0.365 - 0.57$
Daya Bay $\sin^2 2\theta_{13}$	$0.078 - 0.100$
RENO $\sin^2 2\theta_{13}$	$0.100 - 0.126$
Double Chooz $\sin^2 2\theta_{13}$	$0.079 - 0.139$

Table 3.1: Experimentally measured mixing angles, from combined global fits [32, 33]. Values for θ_{12} and θ_{23} are obtained by combining extreme 1 σ ranges from the two fits; values for θ_{13} are simply taken from the most recent observations [15–17].

matrix may be parameterised by

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{23} & s_{12}c_{13} & s_{13}\mathrm{e}^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{3.2}$$

it can be seen that $\theta_{13} = 0$ would preclude CP violation in neutrino mixing; therefore the observation of relatively large θ_{13} is welcome from the point of view of experimental searches.

3.1 Tri-Bimaximal Mixing

Before this observation was made, experimental data was consistent with $\theta_{23} = \frac{\pi}{4}$ (maximal mixing in the atmospheric sector) and $\theta_{13} = 0$ (0 reactor angle). Then an interesting and still experimentally viable case of neutrino mixing could be obtained by taking $s_{12} = \frac{1}{\sqrt{3}}$; these three conditions are known collectively as the TB mixing scheme [54] and the mixing matrix becomes (up to phases)

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.3}$$

The existence of a maximal and a minimal angle, leading to the uniform structure of the TBM matrix appears indicative of some symmetry in the Lagrangian, broken at a high energy but leaving some observable remnant. In order to study this it is helpful to work in a basis where the charged lepton mass matrix is diagonal. Since $U_{PMNS} = V_{e_L} V^\dagger_{\nu_L}$, in such a basis (where $V_{e_L} = 1$) the PMNS matrix diagonalises the effective neutrino mass matrix according to

$$m_\nu = U^*_{PMNS} m^{diag}_\nu U^{-1}_{PMNS}. \tag{3.4}$$

Using Eq. (3.4) with $U_{PMNS} = U_{TB}$ one finds

$$\begin{aligned} m_\nu &= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \\ &= \frac{m_1}{6}A + \frac{m_2}{3}B + \frac{m_3}{2}C, \end{aligned} \tag{3.5}$$

where m_i are the eigenvalues of m_ν. These eigenvalues have corresponding (normalised) eigenvectors $\frac{1}{\sqrt{6}}(-2,1,1)$, $\frac{1}{\sqrt{3}}(1,1,1)$ and $\frac{1}{\sqrt{2}}(0,1,-1)$; if it can be argued that some symmetry requires the seesaw Lagrangian to be $\propto \nu_L^T \phi \phi^T \nu_L$ then ensuring the field ϕ obtains a VEV in the direction of one of these eigenvectors will go some way to producing the TB mixing pattern.

The most general symmetry of m_ν will be represented by a unitary matrix W such that $W^* m_\nu W^\dagger = m_\nu$ (since the neutrino mass term must remain invariant when W is applied: $\nu^T m_\nu \nu \;\rightarrow\; \nu^T W^T W^* m_\nu W^\dagger W \nu$). This means that W should satisfy $W^*A = AW$, $W^*B = BW$ and $W^*C = CW$; inserting a generic matrix

$$W = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \tag{3.6}$$

gives, in general

$$W = \begin{pmatrix} a & b & b \\ b & c & a+b-c \\ b & a+b-c & c \end{pmatrix}. \tag{3.7}$$

Further requiring that $\det(W) = 1$ then gives four possibilities for W:

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & 1 \\ -2 & 1 & -2 \end{pmatrix}. \quad (3.8)$$

Notice that assigning label U to the second of these and S to the third, then the fourth is given by SU. These 4 matrices are in fact the four elements of the Klein 4 group, $K_4 \cong Z_2^S \otimes Z_2^U$ were the superscripts denote the generator of the Z_2 factor. Since this is guided by experimental data, the conclusion is that the low energy symmetry of the neutrino sector is K_4. In a similar manner the most general symmetry of the lepton sector can be found:

$$T = \begin{pmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_3} \end{pmatrix}. \quad (3.9)$$

Since in the current basis the lepton mass matrix is diagonal and non-degenerate, $\delta_i \neq \delta_j$; further restricting attention to $\det(T) = 1$ then leads to

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad (3.10)$$

where $\omega = \exp\left(\frac{2\pi i}{3}\right)$. Therefore it seems that groups generated by S, T and/or U should be considered when searching for symmetries to impose on the Lagrangian.

3.1.1 The alternating group on four elements: A4

Guided by work in the previous Section, the extra symmetry chosen here to reproduce this mixing pattern is A4, the group of even permutations on four elements (or alternatively, the group of symmetries of the tetrahedron). Detailed information about this group may be found in, for example, [55]; here, it suffices to state that A4 can be generated by two elements S and T such that

$S^2 = (ST)^3 = T^3 = 1$. It has three inequivalent 1-dimensional representations

$$\begin{aligned} &\mathbf{1} \quad S = 1 \;\; T = 1, \\ &\mathbf{1}' \quad S = 1 \;\; T = \exp\left(\frac{4\pi i}{3}\right) = \omega^2, \\ &\mathbf{1}'' \quad S = 1 \;\; T = \exp\left(\frac{2\pi i}{3}\right) = \omega, \end{aligned} \tag{3.11}$$

and a 3-dimensional representation which is basis dependent. The following will be referred to as the **T diagonal** basis:

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}. \tag{3.12}$$

Note that these correspond to the matrices (3.10) and one of (3.8) discussed in Section 3.1. Equations (3.11) and (3.12) can be used to show how to multiply triplets correctly, in a basis dependent manner. The group character table [55] shows that $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3} \oplus \mathbf{3}$; taking two triplets $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ this multiplication rule can be decomposed into combinations of triplet components a_i and b_i. For instance, (3.11) encodes the fact that the representation $\mathbf{1}$ stays invariant under the actions of both S and T; the combination which satisfies this condition is $(a_1b_1 + a_2b_3 + a_3b_2)$. In a similar manner the other decompositions may be constructed to find

$$\begin{aligned} \mathbf{1} \;\; &= (ab) = (a_1b_1 + a_2b_3 + a_3b_2), \\ \mathbf{1}' \;\; &= (ab)' = (a_3b_3 + a_1b_2 + a_2b_1), \\ \mathbf{1}'' &= (ab)'' = (a_2b_2 + a_1b_3 + a_3b_1), \\ \mathbf{3} \;\; &= (ab)_S = \frac{1}{3}(2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_1b_2 - a_2b_1, 2a_2b_2 - a_1b_3 - a_3b_1), \\ \mathbf{3} \;\; &= (ab)_A = \frac{1}{2}(a_2b_3 - a_3b_2, a_1b_2 - a_2b_1, a_1b_3 - a_3b_1), \end{aligned} \tag{3.13}$$

where the subscripts S and A mean, respectively, symmetric and antisymmetric under index permutation. The first equality in each line of the above also serves to define a notation used throughout this thesis: (ab) means the portion of the product $\mathbf{3} \otimes \mathbf{3}$ which transforms as $\mathbf{1}$; $(ab)''$ means the portion transforming as $\mathbf{1}''$ etc. In future chapters, the notation $(ab)_{\mathbf{3}}$ will be used in a similar manner. Singlets may be multiplied as follows: $\mathbf{1} \otimes \mathbf{1} = \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}$, $\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}''$ and $\mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}'$.[1]

The group A4 has two subgroups, one generated by S (and is one of the Z_2 factors of K_4) and one by T, which correspond to the low energy neutrino and charged lepton symmetries respectively. Breaking A4 by letting a scalar triplet φ obtain a VEV in a particular direction can constrain the form of the relevant mass matrices and so reproduce the TB mixing pattern. The two relevant VEV directions are

$$\langle \varphi_S \rangle = (1, 1, 1), \tag{3.14}$$

which is invariant under S, and

$$\langle \varphi_T \rangle = (1, 0, 0), \tag{3.15}$$

which is invariant under T.

A second useful basis of A4 is found by applying the transformation $V^\dagger G_i V$ to all group elements G_i, where V is defined to be

$$V = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}. \tag{3.16}$$

This results in the three dimensional generators

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3.17}$$

and will therefore be referred to as the **S diagonal** basis. The decomposition of

[1] An easy way to remember this is that when multiplying singlets, add the primes, mod 3.

triplets in the product $\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3} \oplus \mathbf{3}$ also changes as follows:

$$\begin{aligned}
\mathbf{1} \;\; &= (ab) = (a_1 b_1 + a_2 b_2 + a_3 b_3), \\
\mathbf{1}' \;&= (ab)' = (a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3), \\
\mathbf{1}'' &= (ab)'' = (a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3), \\
\mathbf{3} \;\; &= (ab)_{3_1} = (a_2 b_3, a_3 b_1, a_1 b_2), \\
\mathbf{3} \;\; &= (ab)_{3_2} = (a_3 b_2, a_1 b_3, a_2 b_1),
\end{aligned} \tag{3.18}$$

and the triplet alignments preserving the Z_2 and Z_3 subgroups swap to become

$$\langle \varphi_S \rangle = (1, 0, 0), \tag{3.19}$$

which is invariant under S, and

$$\langle \varphi_T \rangle = (1, 1, 1), \tag{3.20}$$

which is invariant under T. This basis is particularly important for a group of models classified as indirect [56]; since S in Eq. (3.17) is not part of the K_4 neutrino symmetry,[2] it is not clear that such a basis will give the required neutrino mixing. However, the approaches taken in building a direct or indirect model are rather different. In a direct model one chooses flavons such as (3.14) and (3.15) such that the resulting Lagrangian terms preserve some subgroup of the flavour symmetry at low energies. Since the subgroup in the neutrino sector is generated by the S of the observed K_4 symmetry, TB mixing is expected to be recovered. In an indirect model, flavon VEVs are instead chosen to be aligned with eigenvectors of (3.5); therefore they will break the entire symmetry group (indeed the group's only purpose here is to realise such VEV alignments). The model is constructed in such a way that what remains at low energies are outer products of the flavons, such as to exactly reproduce (3.5). By construction these preserve the low energy K_4 symmetry generators, but these generators are not part of the original group; the

[2]It should be emphasised that this S is a part of a K_4 symmetry but not the one inferred from experiment.

Field	ν^c	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\widetilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A4	3	3	1	$1''$	$1'$	1	3	3	1	1	3	3	1
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	2	2	2
Z_3	ω^2	ω	ω^2	ω^2	ω^2	1	1	ω^2	ω^2	ω^2	1	ω^2	ω^2

Table 3.2: Superfields and their transformations for the AF model

K_4 symmetry is said to have arisen accidentally.

3.1.2 An A4 model with Type I seesaw: the Altarelli-Feruglio model

This Section follows closely work done in [20] and is constructed in the T-diagonal basis; as such it is a direct model. Note that this is presented in a SUSY framework for two main reasons. The first is to take advantage of SUSY as a cure for the hierarchy problem and the provision of a natural LSP. The second is more technical: the scalar potential of SUSY is naturally more constrained than the SM scalar potential (including extra flavons) due to R-parity. This means that the minimization of such a potential in order to obtain the desired vacuum alignments is significantly less complicated and requires fewer assumptions . The relevant superfield content here is displayed in Table 3.2 where A4 assignments are also given. Two extra symmetries have been imposed, which will be explained imminently; the Table also includes A4 singlets ξ, $\widetilde{\xi}$ and triplets φ_0^T, φ_0^S and ξ_0, which play a role in the vacuum alignment of the φ fields. Under these symmetries, the superpotential of the theory is composed of two parts: $w = w_l + w_d$ where w_l is the lepton sector and w_d is the driving sector, which is where the vacuum alignment in Eqs. (3.14) and (3.15) is constrained. Focusing first on the lepton sector, the superpotential is

$$\begin{aligned} w_l = y_e e^c(\varphi_T l) + y_\mu \mu^c(\varphi_T l)' + y_\tau \tau^c(\varphi_T l)'' + y(\nu^c l) + (y_1\xi + \widetilde{y}_1\widetilde{\xi})(\nu^c\nu^c) \\ + y_2(\varphi_S \nu^c\nu^c), \end{aligned} \quad (3.21)$$

where Higgses and powers of the cutoff scale Λ are suppressed. It can be seen that the extra Z_3 symmetry prevents the interchange of the fields φ_T and φ_S, meaning that the structures of the neutrino and charged lepton mass matrices arise from independent sets of fields. The extra $U(1)_R$, known as R-symmetry, gives rise to the familiar R-parity of SUSY[3], preventing unwanted B- and L- violating decays and keeping the lightest SUSY particle stable. After EW and A4 symmetry breaking (where Higgs obtain VEVs $v_{u,d}$), the flavon fields obtain the VEVs

$$\begin{aligned}\langle\varphi_S\rangle &= (v_S, v_S, v_S),\\ \langle\varphi_T\rangle &= (v_T, 0, 0),\\ \langle\xi\rangle &= u.\end{aligned} \tag{3.22}$$

Using the A4 decompositions from Eq. (3.13), the lowest order mass terms which result are (including Higgs VEVs and factors of Λ):

$$\begin{aligned}\mathscr{L}_m = v_d\frac{v_T}{\Lambda}(y_e e^c e + y_\mu \mu^c \mu + y_\tau \tau^c \tau) + y v_u(\nu_e^c \nu_e + \nu_\mu^c \nu_\tau + \nu_\tau^c \nu_\mu) + y_1 u(\nu_e^c \nu_e^c + 2\nu_\mu^c \nu_\tau^c)\\ + y_2\frac{2v_S}{3}(\nu_e^c\nu_e^c + \nu_\mu^c\nu_\mu^c + \nu_\tau^c\nu_\tau^c - \nu_e^c\nu_\mu^c - \nu_\mu^c\nu_\tau^c - \nu_\tau^c\nu_e^c) + h.c.\end{aligned} \tag{3.23}$$

Inspection of the first term in this equation then leads to the charged lepton mass matrix

$$m_l = v_d\frac{v_T}{\Lambda}\begin{pmatrix} y_e & 0 & 0\\ 0 & y_\mu & 0\\ 0 & 0 & y_\tau\end{pmatrix}. \tag{3.24}$$

There are three terms remaining, which give rise to ν masses: the second term gives the Dirac mass matrix m_ν^D while the remaining two terms give the right handed Majorana mass matrix M_ν:

$$m_D = y v_u\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, \qquad M_R = \begin{pmatrix} A+\frac{2B}{3} & -\frac{B}{3} & -\frac{B}{3}\\ -\frac{B}{3} & \frac{2B}{3} & A-\frac{B}{3}\\ -\frac{B}{3} & A-\frac{B}{3} & \frac{2B}{3}\end{pmatrix} u, \tag{3.25}$$

where $A = 2y_1$ and $B = 2y_2\frac{v_S}{u}$. The matrix M_R is diagonalised by the TB mixing

[3]Specifically R-parity is a discrete Z_2 subgroup of $U(1)_R$, where the transformation parameter θ is chosen to take the value π.

matrix, Eq. (3.3) to give

$$U_{TB}^T M_R U_{TB} = \begin{pmatrix} A+B & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & -A+B \end{pmatrix} u. \tag{3.26}$$

In order to apply the seesaw formula, (2.43), M_R^{-1} is needed:

$$M_R^{-1} = \frac{1}{3A(A+B)u} \begin{pmatrix} 3A+B & B & B \\ B & \frac{2AB+B^2}{B-A} & \frac{B^2-AB-3A^2}{B-A} \\ B & \frac{B^2-AB-3A^2}{B-A} & \frac{2AB+B^2}{B-A} \end{pmatrix}. \tag{3.27}$$

Application of Eq. (2.43) then gives the effective LH Majorana mass matrix

$$m_\nu = \frac{y^2 v_u^2}{3A(A+B)u} \begin{pmatrix} 3A+B & B & B \\ B & \frac{2AB+B^2}{B-A} & \frac{B^2-AB-3A^2}{B-A} \\ B & \frac{B^2-AB-3A^2}{B-A} & \frac{2AB+B^2}{B-A} \end{pmatrix}, \tag{3.28}$$

and thus diagonalising this using (3.3) gives the light neutrino masses[4]

$$m_1 = \frac{y^2}{(A+B)} \frac{v_u^2}{u}, \quad m_2 = \frac{y^2}{A} \frac{v_u^2}{u}, \quad m_3 = \frac{y^2}{(-A+B)} \frac{v_u^2}{u}. \tag{3.29}$$

Both normal ($m_3 \gg m_1$) and inverted ($m_1 \gg m_3$) hierarchies can be obtained, depending on the relative phase between A and B.

Charged lepton mass hierarchy is also obtainable by imposing an extra $U(1)_F$ symmetry upon only the RH charged leptons: $e^c \sim 3-4$, $\mu^c \sim 2$ and $\tau^c \sim 0$. Then introducing an extra field $\theta \sim -1$ which obtains a VEV $\frac{\langle\theta\rangle}{\Lambda} = \lambda < 1$ naturally gives the required hierarchy by ensuring w_l is invariant under this new symmetry. This general idea is known as the Froggatt-Nielsen [57] mechanism and variants will be used later on in this thesis.

The second part of the superpotential, w_d, contains the driving fields φ_0^T, φ_0^S and

[4]This can be seen easily since if a square matrix A has eigenvalues λ_i, then A^{-1} has eigenvalues $\frac{1}{\lambda_i}$.

ξ_0; since they have R-parity 2 the driving terms are linear in these fields

$$w_d = M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) + g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \widetilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) \\ + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \widetilde{\xi} + g_6 \xi_0 \widetilde{\xi}^2. \tag{3.30}$$

Note that in the above, since up until now there has been no distinction made between ξ and $\widetilde{\xi}$, $\widetilde{\xi}$ is defined to be the combination of ξ and $\widetilde{\xi}$ that couples to $(\varphi_0^S \varphi_S)$. In order to fix the VEVs of the flavon fields φ_S and φ_T, they must minimise the scalar potential $\sum_i \left| \frac{\partial w}{\partial \phi_i} \right|^2 + m_i^2 \left| \phi_i^2 \right| + \ldots$; minimisation is performed without soft SUSY breaking terms (i.e. in the SUSY limit) and these are accounted for subsequently, since SUSY breaking occurs at a scale much below the seesaw scale. Thus minimisation of the potential amounts to finding solutions to[5] $\partial w_d / \partial \phi_{0i} = 0$. From (3.30) this gives 7 equations

$$\frac{\partial w_d}{\partial \varphi_{01}^T} = M \varphi_{T_1} + \frac{2g}{3}[\varphi_{T_1}^2 - \varphi_{T_2}\varphi_{T_3}] = 0, \tag{3.31}$$

$$\frac{\partial w_d}{\partial \varphi_{02}^T} = M \varphi_{T_3} + \frac{2g}{3}[\varphi_{T_2}^2 - \varphi_{T_3}\varphi_{T_1}] = 0, \tag{3.32}$$

$$\frac{\partial w_d}{\partial \varphi_{03}^T} = M \varphi_{T_2} + \frac{2g}{3}[\varphi_{T_3}^2 - \varphi_{T_1}\varphi_{T_2}] = 0, \tag{3.33}$$

$$\frac{\partial w_d}{\partial \varphi_{01}^S} = g_2 \widetilde{\xi} \varphi_{S_1} + \frac{2g_1}{3}[\varphi_{S_1}^2 - \varphi_{S_2}\varphi_{S_3}] = 0, \tag{3.34}$$

$$\frac{\partial w_d}{\partial \varphi_{02}^S} = g_2 \widetilde{\xi} \varphi_{S_3} + \frac{2g_1}{3}[\varphi_{S_2}^2 - \varphi_{S_1}\varphi_{S_3}] = 0, \tag{3.35}$$

$$\frac{\partial w_d}{\partial \varphi_{03}^S} = g_2 \widetilde{\xi} \varphi_{S_2} + \frac{2g_1}{3}[\varphi_{S_3}^2 - \varphi_{S_1}\varphi_{S_2}] = 0, \tag{3.36}$$

$$\frac{\partial w_d}{\partial \xi_0} = g_4 \xi^2 + g_5 \xi \widetilde{\xi} + g_6 \widetilde{\xi}^2 + g_3[\varphi_{S_1}^2 + 2\varphi_{S_2}\varphi_{S_3}] = 0. \tag{3.37}$$

Equations (3.31)-(3.33) can be solved by setting any two of the $\varphi_{T_i} = 0$, however the choices $i = 1, 2$ or $i = 1, 3$ give the trivial solution $\langle \varphi_T \rangle = (0, 0, 0)$; choosing $i = 2, 3$ then leads to

$$\langle \varphi_T \rangle = (v_T, 0, 0) \quad \text{with} \quad v_T = -\frac{3M}{2g}, \tag{3.38}$$

which is in the direction of (3.15). Turning to Eqs. (3.34)-(3.37), the trivial

[5] Differentiating with respect to flavon fields will produce terms $\propto$ a driving field, and so give zero when the fields obtain their VEVs.

solution $\langle\xi\rangle = \langle\varphi_S\rangle = 0$ is inevitable with only one singlet flavon. Thus including both singlets but choosing $m^2_{\tilde{\xi}} > 0 \Rightarrow \langle\tilde{\xi}\rangle = 0$ leads to the solution

$$\begin{aligned}
\langle\tilde{\xi}\rangle &= 0, \\
\langle\xi\rangle &= u, \\
\langle\varphi_S\rangle &= (v_S, v_S, v_S) \quad \text{with} \quad v_S^2 = -\frac{g_4}{3g_3}u^2,
\end{aligned} \tag{3.39}$$

which is consistent with Eqs. (3.14) and (3.22). Choosing positive SUSY breaking masses for the driving fields then ensures they obtain zero VEV.

3.2 Deviations from TBM

As can be seen from the neutrino data in Table 3.1, although TBM is a reasonable approximation to data, it should only be taken as a starting point to describing the observed mixing. To this end it is useful to introduce three parameters defining deviations from TBM [58]:

$$s_{13} = \frac{r}{\sqrt{2}}, \tag{3.40}$$

$$s_{12} = \frac{1}{\sqrt{3}}(1+s), \tag{3.41}$$

$$s_{23} = \frac{1}{\sqrt{2}}(1+a). \tag{3.42}$$

These are defined for the full PMNS matrix, but can also be defined for individual sectors by simply adding a superscript l or ν as appropriate. Using these parameters, the PMNS matrix may be expanded and to first order is given as

$$U_{\text{PMNS}} \approx \begin{pmatrix} \frac{2}{\sqrt{6}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}, \tag{3.43}$$

up to Majorana phases. This is analogous to the Wolfenstein parameterisation which is an expansion of the CKM matrix away from unity. Using the data provided in Table 3.1, these deviation parameters can be constrained to lie in extreme 1 σ ranges too (here, the range of r is simply the extreme range given by the three experiments)

$$0.199 < r < 0.269, \quad -0.066 < s < 0.003, \quad -0.118 < a < 0.068. \tag{3.44}$$

Whilst the ranges for s and a still include 0, the range for r is rather a long way from 0, indicating that TBM is indeed experimentally disfavoured without any modification. Nevertheless, in the next Chapter a model predicting TBM is studied since TBM is still a reasonable first approximation to the data and with some modification can be used as a starting point for many models.

3.2.1 Extending the AF model to account for non-zero θ_{13}

Instead of TBM, schemes such as TM mixing remain viable [59]:

$$U_{TM} = \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\theta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\theta e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta e^{-i\rho} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{6}}\sin\theta e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta e^{-i\rho} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{6}}\sin\theta e^{i\rho} \end{pmatrix}. \tag{3.45}$$

Here $\frac{2}{\sqrt{6}}\sin\theta = \sin\theta_{13}$ and ρ is related to the Dirac phase. It is possible to extend the AF model above by adding flavons in the $\mathbf{1}'$ and $\mathbf{1}''$ representations of A_4 which reproduces this pattern [60]:

$$W_{\mathbf{1}'+\mathbf{1}''} = \left(y_2'\xi' + y_2''\xi''\right) NN. \tag{3.46}$$

Flavons in these representations explicitly break the U generator of K_4 and have been shown to lead to non-zero θ_{13} [61, 62]. These extensions lead to the mass matrices

$$m_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v_u \ , \tag{3.47}$$

and

$$M_R = \left[A \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + B \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + C' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + C'' \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right], \tag{3.48}$$

with $A = 2y_1 \langle \varphi_S \rangle$, $B = 2y_2 \langle \xi \rangle$, $C' = 2y_2' \langle \xi' \rangle$ and $C'' = 2y_2'' \langle \xi'' \rangle$. The above matrix may be rewritten as a sum of two matrices, one of which preserves TB mixing and one which violates it:

$$M_R = M_R^{TB} + \Delta M_R, \tag{3.49}$$

$$M_R^{TB} = A \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + B \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \tag{3.50}$$

$$\Delta M_R = \Delta \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}. \tag{3.51}$$

Here $\Delta = \frac{1}{2}(C'' - C')$ and $\gamma = \frac{1}{2}(C' + C'')$. Since experimentally the mixing is still close to TB mixing, the model requires $|\Delta| \ll |A|, |B|$, whereas no such constraint applies to γ. This observation allows one to diagonalise M_R perturbatively, such that one ends up with $U_{TM} = U_{TB} + \Delta U$; performing this procedure gives the lepton mixing matrix arising from the A4 model

$$U_{TM} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \end{pmatrix}. \tag{3.52}$$

The complex parameter α_{13} is the only combination of input parameters (i.e. A, B, γ, Δ) which appears and is given by [60]

$$\alpha_{13} = \frac{\sqrt{3}}{2} \left(\mathrm{Re}\frac{\Delta}{(A-\gamma)} + \mathrm{Im}\frac{\Delta}{(A-\gamma)} \frac{\mathrm{Im}\frac{B}{A-\gamma}}{\mathrm{Re}\frac{B}{A-\gamma}} - i \frac{\mathrm{Im}\frac{\Delta}{(A-\gamma)}}{\mathrm{Re}\frac{B}{A-\gamma}} \right). \tag{3.53}$$

A comparison of Eqns. (3.52) with (3.43) then allows one to write α_{13} in terms of the TB deviation parameters

$$s \approx 0, \quad a \approx \frac{\mathrm{Re}\,(\alpha_{13})}{\sqrt{3}}, \quad r\cos\delta \approx -\frac{2}{\sqrt{3}}\mathrm{Re}\,(\alpha_{13})\,, \quad \delta \approx \arg\,(\alpha_{13}) + \pi. \qquad (3.54)$$

Chapter 4

SUSY $SU(5)$ with singlet plus adjoint matter and A_4 family symmetry

This chapter presents a model combining several of the elements introduced previously and which is published in [1]. The aim of the model is to combine the framework of SUSY $SU(5)$ with a family symmetry predicting TBM and a seesaw mechanism. The choice of seesaw matter or Higgs is very *ad hoc* since the $SU(5)$ theory does not specify the nature of this extra matter and only requires that it be anomaly-free. A popular choice is to add three RH neutrinos which arise from singlet $SU(5)$ representations. However the number of singlets is not predicted in $SU(5)$, and it is possible to add just a single RH neutrino to describe the atmospheric mass scale [63]. In order to describe both atmospheric and solar neutrino mass scales with two large mixing angles using the type I seesaw mechanism two RH neutrinos are sufficient [64]. However, within $SU(5)$ GUTs, there are other possibilities.

It has been pointed out that, in (SUSY) $SU(5)$ GUTs, non-fundamental matter multiplets have decompositions which include both fermion singlets and fermion triplets suitable for the type I and III seesaw mechanism, the smallest such example

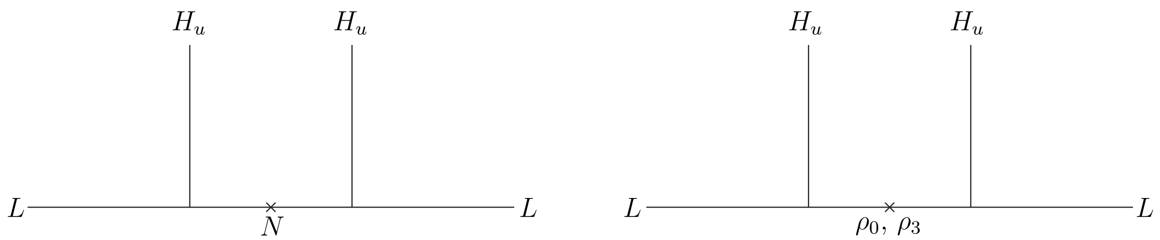

Figure 4.1: Schematic diagrams of the type I (left) and combined type I + type III (right) seesaw mechanisms present in the model. The seesaw messenger states are N and the ρ_0, ρ_3 components of $\psi_{\mathbf{24}}$. L is the $SU(2)_L$ doublet contained in the $\overline{\mathbf{5}}$ of $SU(5)$.

being the adjoint **24** representation [65–67]. The decomposition of a matter **24** under the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ involves an $SU(2)_L$ singlet $\rho_0 = (\mathbf{1}, \mathbf{1})_0$ as well as a triplet $\rho_3 = (\mathbf{1}, \mathbf{3})_0$, thus leading to a combination of a type I seesaw with a type III seesaw [30]. However, assuming the simplest Higgs sector, the ρ_0 and ρ_3 are constrained by $SU(5)$ to give equal contributions to the neutrino mass matrix, up to an overall constant, resulting in a rank one neutrino mass matrix and only one non-zero neutrino mass. This problem may be addressed by allowing additional couplings to a Higgs **45** [67], but here a different possibility is considered.

Instead, one can introduce a single RH neutrino singlet superfield N plus one adjoint matter superfield $\psi_{\mathbf{24}}$ below the GUT scale. The model combines a type I seesaw mechanism from the single RH neutrino N below the GUT scale [63] with a type I plus type III seesaw mechanism from the ρ_0 and ρ_3 components contained in a single adjoint matter superfield $\psi_{\mathbf{24}}$ below the GUT scale [67]. The seesaw mechanism in the model therefore results from three distinct diagrams as shown in Fig. 4.1. Instead of using an adjoint Higgs representation $H_{\mathbf{24}}$ to spontaneously break $SU(5)$ to the SM gauge group, the assumption that the GUT group is broken by geometrical effects in extra dimensions is made. However the theory here is formulated in four dimensions and can then subsequently be uplifted to a higher dimensional setting (as in, for example, [68]). The absence of $H_{\mathbf{24}}$ is crucial in forbidding the mixing between the RH neutrino N and $\psi_{\mathbf{24}}$, leading to no mass mixing between N and ρ_0 and hence a diagonal heavy Majorana sector as required by CSD [69].

The first part of this Chapter introduces the relevant GUT without a flavour symmetry and it is demonstrated that this cannot be simply augmented by a discrete symmetry in order to predict TBM. Instead it needs a small adjustment which is explained in the second part of the Chapter; this is then uplifted to a flavour model and the results presented in the remainder of the Chapter.

4.1 An SU(5) model with Type III seesaw

This Section is based on work from [66] and [67]. In these papers it was shown that the simplest SU(5) GUT, which fails to unify the fundamental forces in satisfactory manner, can have its unification properties improved with the addition of an extra matter **24** to the particles listed in Section 2.2.2. Under the gauge group $\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$, **24** decomposes as $\psi_{\mathbf{24}} = (8,1)_0 \oplus (1,3)_0 \oplus (3,2)_{-5/6} \oplus (\overline{3},2)_{5/6} \oplus (1,1)_0 = (\rho_8, \rho_3, \rho_{(3,2)}, \rho_{(\overline{3},2)}, \rho_0)$, which contains the quantum numbers of both types I and III seesaw particles.[1] Two seesaw particles makes it possible to predict two massive neutrinos with the addition of only one SU(5) superfield (also note that since the adjoint is a real representation, no extra anomalies are introduced here).

The introduction of this new superfield gives rise to the superpotential for neutrino mass

$$w_\nu = c_i F_i \psi_{\mathbf{24}} H_{\mathbf{5}} + p_i F_i \psi_{\mathbf{24}} H_{\mathbf{45}}, \tag{4.1}$$

and this means that the seesaw mechanism has contributions from both the $\mathbf{H_5}$ and the $\mathbf{H_{45}}$. The $\psi_{\mathbf{24}}$ field can be represented as a 5×5 matrix using $\psi_{\mathbf{24}} = \rho_a T^a$ where the T^a are the generators of SU(5)[2] [70] and the ρ_a are related to the fields contained in the $\psi_{\mathbf{24}}$. Using this decomposition along with Eqns. (2.54) and (2.55),

[1]Note that this is the main motivation for the study undertaken in the current Chapter, as a SUSY version of this model is used which has less need for improved unification.

[2]Normalised so that $\mathrm{Tr}\,\{T_a T_b\} = \frac{\delta_{ab}}{2}$.

the superpotential may be expanded:

$$\begin{aligned}w_\nu = c_i &\left(\ldots - \nu_i \left(-\sqrt{\frac{3}{5}}\frac{\rho_0}{2} - \frac{\rho_3^0}{2} \right) \right) v_5 \\ - p_i &\left(\ldots - \nu_i \left(\frac{1}{2}(\rho_3^c - \rho_3^c) + \frac{1}{2}\left(\frac{\rho_8^c}{\sqrt{3}} + \frac{\rho_8^c}{\sqrt{3}} - \frac{2\rho_8^c}{\sqrt{3}} \right) \right.\right. \\ &\left.\left. +3\sqrt{\frac{3}{5}}\frac{\rho_0}{3} - 3\left(\frac{\rho_3^0}{2} - \sqrt{\frac{3}{5}}\frac{\rho_0}{2} \right) \right) \right) v_{45}.\end{aligned} \tag{4.2}$$

In the above, the ρ_3^0 is the neutral component of the ρ_3 corresponding to the diagonal generator of $SU(2)_L$ and the $\rho_{3,8}^c$ are the fields contained in ρ_8 corresponding to diagonal generators of $SU(3)_c$. The $\ldots$ represent interactions between the $\psi_{\mathbf{24}}$ and non-ν fields, and the cancellation of interactions between ν and the coloured fields $\rho_{3,\,8}^c$ has been explicitly demonstrated . Rearranging the result into seesaw interaction terms gives

$$w_\nu = \frac{1}{2}\left(c_i v_5 - 3p_i v_{45}\right)\nu_i \rho_3^0 + \frac{\sqrt{15}}{2}\left(\frac{c_i v_5}{5} + p_i v_{45}\right)\nu_i \rho_0. \tag{4.3}$$

Application of the seesaw mechanism, Eq. (2.43) to integrate out the ρ fields then results in

$$m_{ij}^\nu = \frac{a_i a_j}{M_{\rho_3}} + \frac{b_i b_j}{M_{\rho_0}}, \tag{4.4}$$

with

$$a_i = \frac{1}{2}\left(c_i v_5 - 3p_i v_{45}\right) \quad \text{and} \quad b_i = \frac{\sqrt{15}}{2}\left(\frac{c_i v_5}{5} + p_i v_{45}\right). \tag{4.5}$$

It is important to note that the $H_{\mathbf{45}}$ is crucial to a satisfactory model of neutrino mass; if it were not present, then $a_i \propto b_i$ and so the mass matrix M_{ij}^ν would have rank one $\Rightarrow$ the model would only predict one massive neutrino.

The fields in the $\psi_{\mathbf{24}}$ get masses from their interactions with the $H_{\mathbf{24}}$

$$w_\psi = M_\Sigma \mathrm{Tr}\left(H_{\mathbf{24}}^2\right) + \lambda_\Sigma \mathrm{Tr}\left(H_{\mathbf{24}}^3\right) + M\mathrm{Tr}\left({\psi_{\mathbf{24}}}^2\right) + \lambda \mathrm{Tr}\left({\psi_{\mathbf{24}}}^2 H_{\mathbf{24}}\right), \tag{4.6}$$

which gives

$$\begin{aligned} M_{\rho_0} &= M - \frac{2M_\Sigma \lambda}{3\lambda_\Sigma}, \\ M_{\rho_3} &= M - \frac{2M_\Sigma \lambda}{\lambda_\Sigma}, \\ M_{\rho_8} &= M + \frac{4M_\Sigma \lambda}{3\lambda_\Sigma}, \\ M_{\rho_{(3,2)}} &= M - \frac{M_\Sigma \lambda}{3\lambda_\Sigma}, \\ M_{\rho_{(\overline{3},2)}} &= M - \frac{M_\Sigma \lambda}{3\lambda_\Sigma}, \end{aligned} \tag{4.7}$$

once the $H_{\mathbf{24}}$ obtains its VEV, $\langle H_{\mathbf{24}} \rangle = \frac{2M_\Sigma}{3\lambda_\Sigma}\,\mathrm{diag}(2,2,2,-3,-3)$ (calculated using the first two terms of w_ψ). For instance, inserting the decomposition of the $\psi_{\mathbf{24}}$ into (4.6) and extracting the ρ_0 term gives

$$\begin{aligned} w_{\rho_0} &= \frac{1}{4} \times \frac{3}{5}\left(M\left(\frac{4}{9}+\frac{4}{9}+\frac{4}{9}+2\right) + \lambda\left(\frac{8}{9}+\frac{8}{9}+\frac{8}{9}-6\right)\frac{2M_\Sigma}{3\lambda_\Sigma}\right)(\rho_0)^2, \\ &= \frac{1}{2}\left(M - \frac{2M_\Sigma\lambda}{3\lambda_\Sigma}\right)(\rho_0)^2, \end{aligned} \tag{4.8}$$

giving $M_{\rho_0} = M - \frac{2M_\Sigma \lambda}{3\lambda_\Sigma}$ as required.

In order to extend this model to predict lepton mixings, the F_i fields containing the neutrinos will be combined into a triplet of $A4$, meaning the neutrino Yukawa superpotential must be augmented by triplet flavons as in the AF model in Chapter 3

$$w_\nu = c(\varphi_S F)\psi_{\mathbf{24}} H_{\mathbf{5}} + p(\varphi_S F)\psi_{\mathbf{24}} H_{\overline{\mathbf{45}}}. \tag{4.9}$$

Unfortunately this assignment leads to a prediction of only one massive neutrino. Expanding (4.9) gives

$$\begin{aligned} w_\nu = c v_5 v_S(\nu_e + \nu_\mu + \nu_\tau)&\left(\frac{\rho_3^0}{2} + \sqrt{\frac{3}{5}}\frac{\rho_0}{2}\right) \\ + 3p v_{45} v_S(\nu_e + \nu_\mu + \nu_\tau)&\left(\frac{\rho_3^0}{2} - \sqrt{\frac{3}{5}}\frac{5\rho_0}{6}\right). \end{aligned} \tag{4.10}$$

This will lead to a mass matrix with all entries proportional which, while part of the TB mixing structure (3.5), is of rank 1 and thus has only one non-zero eigenvalue.

To try and generate a more realistic phenomenology, the model can be extended with another flavon φ_{23}, whose VEV is proportional to the third eigenvector of (3.5)

$$\langle \varphi_{23} \rangle = v_{23}(0, 1, -1). \tag{4.11}$$

The superpotential then becomes

$$w_\nu = c(\varphi_S F)\psi_{\mathbf{24}} H_{\mathbf{5}} + p(\varphi_{23} F)\psi_{\mathbf{24}} H_{\overline{\mathbf{45}}}, \tag{4.12}$$

leading to (for simplicity, the contribution from the ρ_0 can be ignored)

$$m_{LL} \sim a \otimes a^T \quad \text{with} \quad a = \varphi_S + \varphi_{23}, \tag{4.13}$$

$$\Rightarrow m_{LL} \sim \varphi_S \varphi_S^T + \varphi_{23}\varphi_{23}^T + \varphi_S \varphi_{23}^T + \varphi_{23}\varphi_S^T. \tag{4.14}$$

The cross terms here are not contained in (3.5) and so spoil the TB mixing pattern. Reintroducing ρ_0 will simply add an extra multiplicative factor, keeping the structure the same and so not changing the conclusion. An extra ingredient is required in order to uplift this to a flavour model, which is introduced in the next Section.

4.2 SUSY $SU(5)$ with singlet and adjoint matter

This Section presents a SUSY $SU(5)$ GUT with one single RH neutrino arising from a singlet representation N below the GUT scale plus one extra adjoint matter representation $\psi_{\mathbf{24}}$ with mass also below the GUT scale. The matter contained in the $\psi_{\mathbf{24}}$ is degenerate thus avoiding problems with gauge coupling unification. The model represents a new way to achieve a hierarchical neutrino mass spectrum arising from a type I plus type III seesaw mechanism, as is now discussed.

The superpotential describing the neutrino sector takes the form

$$W = c_i F_i \psi_{\mathbf{24}} H_{\mathbf{5}} + p_i F_i N H_{\mathbf{5}} + \frac{1}{2} M_N N N + \frac{1}{2} M \,\mathrm{Tr}\,(\psi_{\mathbf{24}}{}^2). \tag{4.15}$$

The seesaw diagrams illustrated in Fig. 4.1 then yield the light neutrino mass matrix,

$$m_\nu^{ij} = c_i c_j v_u^2 \left(\frac{1}{4M_{\rho_3}} + \frac{3}{20M_{\rho_0}} \right) + \frac{p_i p_j}{M_N} v_u^2 \,. \tag{4.16}$$

Here v_u is the VEV of the Minimal Supersymmetric Standard Model (MSSM) Higgs field H_u which corresponds to the $SU(2)_L$ doublet within the $SU(5)$ Higgs $H_{\mathbf{5}}$. As can be seen from Eq. (4.15), the Majorana masses for the seesaw messengers ρ_0 and ρ_3 are identical, i.e. $M_{\rho_0} = M_{\rho_3} = M$, while N has an independent mass M_N. Note that there is no adjoint Higgs $H_{\mathbf{24}}$ which would break the degeneracy of the components in the $\psi_{\mathbf{24}}$ and, more importantly, allow a mixing term $N\psi_{\mathbf{24}}H_{\mathbf{24}}$ leading to a mass mixing between N and ρ_0. Note also that c_i and p_i are independent dimensionless coefficients (where i and j are family indices); this independence is crucial to obtaining a rank two mass matrix and thus two non-zero neutrino masses.

As c_i and p_i are uncorrelated parameters, Eq. (4.16) does not in general conform to the TB structure of the neutrino mass matrix. It is the aim of this Chapter to obtain TB neutrino mixing as a consequence of a discrete family symmetry in this type of model. To this end, in the next Section, the adjoint SUSY $SU(5)$ model is augmented with the tetrahedral family symmetry A_4.

4.3 SUSY $A_4 \times SU(5)$ with singlet and adjoint matter

In this Section the model in Eq. (4.15) is uplifted to include a tetrahedral family symmetry. The S-diagonal basis of [71] is used (see Chapter 3), in which two A_4 triplets $a = (a_1, a_2, a_3)^T$ and $b = (b_1, b_2, b_3)^T$ give a singlet through the combination $a_1 b_1 + a_2 b_2 + a_3 b_3$. As before the three families of $\overline{\mathbf{5}}$s are unified into an A_4 triplet $F \sim \mathbf{3}$, and in order for Eq. (4.15) to remain invariant, flavons φ_i are introduced to break the A_4 symmetry and generate the Yukawa couplings.

Table 4.1 shows the chiral superfields present in the model. As mentioned above, the three $\overline{\mathbf{5}}$s of $SU(5)$ are embedded in a triplet of A_4, while the three $\mathbf{10}$s are singlets. The $\psi_{\mathbf{24}}$ is an A_4 singlet as is the RH neutrino N. The Higgs sector

Field	$\psi_{\mathbf{24}}$	N	F	T_1	T_2	T_3	$H_{\mathbf{5}}$	$H_{\overline{\mathbf{5}}}$	$H_{\overline{\mathbf{45}}}$	φ_{123}	φ_{23}	φ_3	ξ	ξ'	φ_1
$SU(5)$	$\mathbf{24}$	$\mathbf{1}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
$U(1)_R$	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
$U(1)$	-1	2	0	4	1	0	0	0	2	1	-2	0	-1	-4	q_1
Z_2^1	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$+$
Z_2^2	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$+$

Table 4.1: Matter, Higgs and flavon chiral superfields in the model. The $U(1)$ charge q_1 can take any value which prevents φ_1 from significantly interacting with the other fields of the model, for instance $q_1 = -\frac{126}{24}$ as discussed below.

consists of fundamental Higgs fields $H_{\mathbf{5}}$ and $H_{\overline{\mathbf{5}}}$; introducing another Higgs in the $\overline{\mathbf{45}}$ representation, $H_{\overline{\mathbf{45}}}$, enables the implementation of the GJ mechanism [42] to obtain the well known GUT scale mass relations from Eq. (2.61).

The $U(1)_R$ is the familiar R-symmetry; it is essential in forbidding F-term contributions to the flavon superpotential which otherwise could dominate the relevant D-term operators used for obtaining the desired vacuum alignment (see Appendix B and the discussion in [72] and [73]). The $U(1)$ and the two Z_2 symmetries constrain the structure of the Yukawa matrices in the quark and charged lepton sectors. The standard MSSM μ-term[3] $\mu H_u H_d$ is forbidden by the first of the Z_2 symmetries as well as by $U(1)_R$, allowing for a natural solution to the μ-problem of the MSSM using a GUT singlet from the hidden sector of Supergravity theories [51].

The flavon fields φ_i, ξ and ξ' break the A_4 symmetry and constrain the form of the lepton and down quark Yukawa matrices. The vacuum alignments of the triplet flavon VEVs assumed in this model are displayed in Table 4.2. They are achieved using the D-term vacuum alignment mechanism discussed recently in [73]. This mechanism is ideally suited for models such as this in which the flavons are used to generate the neutrino flavour symmetry as an indirect result of the A_4 symmetry as discussed in [56]. Moreover, the D-term vacuum alignment mechanism does not involve the introduction of extra "driving fields" in the superpotential and does not impose any restrictions on the model other than the requirement that higher order

[3]Where H_u is the SM doublet of $H_{\mathbf{5}}$; and H_d is a linear combination of the SM doublets in $H_{\overline{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$.

Flavon VEV	VEV alignment
$\langle\varphi_1\rangle$	$(1,0,0)^T$
$\langle\varphi_3\rangle$	$(0,0,1)^T$
$\langle\varphi_{23}\rangle$	$\frac{1}{\sqrt{2}}(0,1,-1)^T$
$\langle\varphi_{123}\rangle$	$\frac{1}{\sqrt{3}}(1,1,1)^T$

Table 4.2: The vacuum alignments of the triplet flavons used in the model. Without loss of generality, the alignments are given without phases; the relative sign between $\langle\varphi_{23}\rangle_2$ and $\langle\varphi_{23}\rangle_3$ is relevant, though the actual position of the minus sign is mere convention.

terms in the flavon potential do not spoil the vacuum alignment arising from the D-terms. This has been demonstrated to arise in a fairly generic way in [73] providing that the model also respects a $U(1)_R$ symmetry and involves no superfields with $R = 2$ which, like driving fields, could appear linearly in the superpotential and lead to large terms in the flavon potential. The present model involves only fields with $R = 0, 1$ and so the D-term flavon potential will not receive large corrections from the superpotential. Since the D-term vacuum alignment mechanism is generic and does not provide any other restrictions on the model than those stated, the operation of this mechanism is assumed, leading to the stated alignments for $\varphi_{123}, \varphi_{23}, \varphi_3, \varphi_1$.

In order to avoid the massless Goldstone boson associated with the spontaneously broken $U(1)$ symmetry, it is assumed to be gauged.[4] In addition to the particle content specified in Table 4.1 extra matter is needed to cancel the respective gauge anomalies. The cubic $SU(5)$ anomaly requires the introduction of a Higgs field $H_{\mathbf{45}}$ whose $U(1)$ charge is determined by the mixed $SU(5) - SU(5) - U(1)$ anomaly to be $q(H_{\mathbf{45}}) = -\frac{53}{24}$. Then the cubic $U(1)$ anomaly can be removed in many ways; for example, choosing $q_1 = -\frac{126}{24}$ requires that three extra $A_4 \times SU(5)$ singlets are added with $U(1)$ charges $\frac{5}{24}, \frac{25}{24}, \frac{51}{24}$. Assuming that $H_{\mathbf{45}}$ has the same Z_2 charges as $H_{\overline{\mathbf{45}}}$ while the three extra $A_4 \times SU(5)$ singlets are neutral under both Z_2 symmetries, that these additional fields lead to only negligible contributions to the fermion mass matrices discussed below, provided they get VEVs of order $\epsilon\Lambda$ or smaller, see Eq. (4.21).

[4] If it were not gauged, Goldstone boson masses could arise from explicit $U(1)$ breaking in the hidden sector which could generate soft SUSY breaking terms involving only flavon fields where such terms explicitly violate the $U(1)$. However such terms could jeopardise the D-term alignment mechanism so here a gauged $U(1)$ is preferred to avoid any potential problems.

4.3.1 Allowed terms

The neutrino sector is composed of Dirac and Majorana mass terms which take the form in the superpotential:

$$W_\nu = \frac{\varphi_{123}}{\Lambda} cF\psi_{\mathbf{24}} H_{\mathbf{5}} + \frac{\varphi_{23}}{\Lambda} pFNH_{\mathbf{5}} + \frac{\varphi_{23}^2}{2\Lambda} y_N NN + \frac{\xi^4}{2\Lambda^3} y'_N NN + \frac{\varphi_{123}^2}{2\Lambda} y \mathrm{Tr}\left(\psi_{\mathbf{24}}{}^2\right), \quad (4.17)$$

with Λ a heavy mass scale and c, p, y_N, y'_N, y dimensionless coupling constants. When the flavons get their VEVs the superpotential in Eq. (4.17) reproduces that in Eq. (4.15) but with constrained couplings c_i and p_i leading to TB mixing.

The superpotential terms of the down quark and charged lepton sector are given as follows

$$W_d \sim \frac{\varphi_{23}\xi^2}{\Lambda_d^3} T_1 F H_{\overline{\mathbf{5}}} + \frac{\varphi_{123}\xi^2}{\Lambda_d^3} T_2 F H_{\overline{\mathbf{5}}} + \frac{\varphi_{23}\xi}{\Lambda_d^2} T_2 F H_{\overline{\mathbf{45}}} + \frac{\varphi_3}{\Lambda_d} T_3 F H_{\overline{\mathbf{5}}}, \quad (4.18)$$

where Λ_d is the relevant messenger mass. The flavon ξ plays a role similar to a Froggatt-Nielsen field [57], except that it is not the sole contributor to the generated mass hierarchy, here combined as it is with the triplet flavons.

Finally the up quark sector Yukawa superpotential terms take the form

$$\begin{aligned} W_u \sim\; & \frac{(\xi')^2}{\Lambda_u^2} T_1 T_1 H_{\mathbf{5}} + \left(\frac{\varphi_{23}^2\xi}{\Lambda_u^3} + \frac{\xi^5}{\Lambda_u^5}\right)(T_1T_2 + T_2T_1)H_{\mathbf{5}} \\ & + \frac{\varphi_{23}\varphi_3\xi^2}{\Lambda_u^4}(T_1T_3 + T_3T_1)H_{\mathbf{5}} + \frac{\xi^2}{\Lambda_u^2} T_2T_2H_{\mathbf{5}} + \frac{\varphi_{123}\varphi_3\xi^2}{\Lambda_u^4}(T_2T_3 + T_3T_2)H_{\mathbf{5}} \quad (4.19) \\ & + T_3T_3H_{\mathbf{5}}. \end{aligned}$$

It should be mentioned that the messenger mass in this sector, Λ_u, may in principle be different from that in the down quark sector. The field ξ' is introduced specifically to generate the T_1T_1 term to the required order.

4.3.2 Fermion mass matrices

After spontaneous breakdown of the A_4 family symmetry by the flavon VEVs, the superpotential terms of Eqs. (4.17), (4.18) and (4.19) predict mass matrices for the respective sectors. In the following, order one coefficients in the quark and charged lepton sectors are omitted (including flavon VEV normalisation factors). Regarding the scale of the flavon VEVs, an expansion parameter is defined

$$\eta_i = \frac{\langle|\varphi_i|\rangle}{\Lambda}, \tag{4.20}$$

where $\varphi_i = \varphi_{123}$, φ_{23}, φ_3, ξ or ξ'. In order to obtain the hierarchical structure of the quark and charged lepton mass matrices the assumption[5]

$$\eta_{123}, \eta_{23}, \eta_{\xi'} = \epsilon^2 \quad \text{and} \quad \eta_3, \eta_\xi = \epsilon, \tag{4.21}$$

is made, where the numerical values for ϵ depend on the messenger scale of the relevant sector. The superpotential terms of the quark and charged lepton sectors are given up to and including $\mathcal{O}(\epsilon^5)$.

In the Higgs sector, it is not the $H_{\mathbf{5}}$, $H_{\overline{\mathbf{5}}}$ or $H_{\overline{\mathbf{45}}}$ which get VEVs but their SM doublet components. These are the two MSSM doublets H_u (corresponding to $H_{\mathbf{5}}$) and H_d (corresponding to a linear combination of $H_{\overline{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$); they originate below the GUT scale and remain massless down to the EW scale. The non-MSSM states all acquire GUT scale masses, including the linear combination of $H_{\overline{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$ orthogonal to H_d. EW symmetry is broken after the light MSSM doublets $H_{u,d}$ acquire VEVs $v_{u,d}$ and they then generate the fermion masses.

4.3.3 Neutrino sector

In this model the light neutrino masses arise from a combination of type I and type III seesaw. Due to the absence of a $H_{\mathbf{24}}$ the heavy seesaw messenger particles N and ρ_0 do not mix as can be seen from Eq. (4.17). Thus the 2×2 Majorana mass matrix of the heavy RH $SU(2)_L$ singlets is automatically diagonal. Furthermore, the seesaw messenger responsible for the type III contribution, ρ_3, cannot mix with N as they furnish different $SU(2)_L$ representations. A very generic method for obtaining neutrino masses and mixings is to enforce a scheme known as

[5]It is possible to have a hierarchy in the flavon VEVs since the scales at which their mass terms are driven negative can vary [73].

CSD. In CSD, a heavy neutrino mass hierarchy is assumed as well as specific relations between parameters of the Dirac mass matrix. The origin of these relationships in this Chapter is the flavour symmetry, as is the case in many models predicting TB mixing. However, in CSD the (approximate) diagonal nature of the seesaw particles is usually a necessary extra assumption which often lacks a fundamental explanation. In the current adjoint model, however, it is directly built into the theory by not including $H_{\mathbf{24}}$. Therefore the model represents a very natural realisation of CSD.

In the Dirac neutrino sector of Eq. (4.17), the spontaneous breaking of the A_4 family symmetry by the flavon VEVs $\langle\varphi_{123}\rangle$ and $\langle\varphi_{23}\rangle$ gives

$$\mathscr{L}_\nu = \frac{c\eta_{123}v_u}{\sqrt{3}}(\nu_e+\nu_\mu+\nu_\tau)\left(\frac{\rho_3^0}{2}-\sqrt{\frac{3}{20}}\,\rho_0\right)-\frac{p\eta_{23}v_u}{\sqrt{2}}(\nu_\mu-\nu_\tau)N\ +\ h.c.\ , \quad (4.22)$$

where the numerical factors of ρ_3^0 and ρ_0 are determined from the normalised $SU(5)$ generators in the adjoint representation [70]. Upon application of the seesaw formula of Eq. (4.16) the effective LH Majorana neutrino mass matrix is found to be

$$m_\nu = \frac{2c^2v_u^2}{15y\Lambda}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\frac{p^2v_u^2}{2(y_N+y'_N\eta_\xi^4/\eta_{23}^2)\Lambda}\begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix}. \quad (4.23)$$

Since any matrix diagonalisable by Eq. (3.3) may be written as[6] $m_1\varphi'_1\,(\varphi'_1)^T/|\varphi'_1|^2+m_2\varphi_{123}(\varphi_{123})^T/|\varphi_{123}|^2+m_3\varphi_{23}(\varphi_{23})^T/|\varphi_{23}|^2$ [56], the masses may be read off as

$$m_\nu^{\rm diag} = \begin{pmatrix}0&0&0\\0&m_2&0\\0&0&m_3\end{pmatrix}, \quad \text{with} \quad m_2=\frac{2c^2v_u^2}{5y\Lambda}\ , \quad m_3=\frac{p^2v_u^2}{(y_N+y'_N\eta_\xi^4/\eta_{23}^2)\Lambda}. \quad (4.24)$$

Hence the model predicts one massless left-handed neutrino and thus a hierarchical neutrino mass spectrum.

[6] $\varphi'_1 \propto \frac{1}{\sqrt{6}}(-2,1,1)^T$.

4.3.4 Down quark and charged lepton sector

In the down quark and charged lepton sector, the superpotential of Eq. (4.18) predicts a mass matrix of the form (with messenger mass Λ_d in η_i)

$$\begin{pmatrix} 0 & \eta_{23}\eta_\xi^2 & -\eta_{23}\eta_\xi^2 \\ \eta_{123}\eta_\xi^2 & \eta_{123}\eta_\xi^2 + k_f\eta_{23}\eta_\xi & \eta_{123}\eta_\xi^2 - k_f\eta_{23}\eta_\xi \\ 0 & 0 & \eta_3 \end{pmatrix} v_d, \tag{4.25}$$

where k_f is the GJ factor (in the case that $f = e$, the mass matrix must also be transposed):

$$k_f = \begin{cases} 1 & \text{for} \quad f = d, \\ -3 & \text{for} \quad f = e. \end{cases} \tag{4.26}$$

Inserting the ϵ suppressions of the flavon VEVs from Eq. (5.6) the down quark mass matrix becomes

$$m_d \sim \begin{pmatrix} 0 & \epsilon^3 & -\epsilon^3 \\ \epsilon^3 & \epsilon^2 & -\epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \epsilon\, v_d, \tag{4.27}$$

whilst the charged lepton mass matrix reads

$$m_e \sim \begin{pmatrix} 0 & \epsilon^3 & 0 \\ \epsilon^3 & -3\epsilon^2 & 0 \\ -\epsilon^3 & 3\epsilon^2 & 1 \end{pmatrix} \epsilon\, v_d. \tag{4.28}$$

Here the further assumption the numerical value $\epsilon \sim 0.15$ is made. Upon diagonalisation, these give mass ratios of $\epsilon^4 : \epsilon^2 : 1$ for the down quarks and $\frac{\epsilon^4}{3} : 3\epsilon^2 : 1$ for the charged leptons. These ratios are in good agreement with quark and lepton data and also predict GUT scale mass relations of $m_e \sim \frac{m_d}{3}$, $m_\mu \sim 3m_s$ and $m_\tau \sim m_b$ as desired. In the low quark angle approximation, left-handed down quark mixing angles $\theta^d_{12} \sim \epsilon$, $\theta^d_{13} \sim \epsilon^3$ and $\theta^d_{23} \sim \epsilon^2$ are also predicted in agreement with data (assuming an approximately diagonal up sector which is obtained in the next Section). The corresponding charged lepton mixing angles are $\theta^e_{12} \sim \frac{\epsilon}{3}$, $\theta^e_{13} \sim 0$ and $\theta^e_{23} \sim 0$.

The PMNS matrix is not of exact TB form but receives small corrections from charged lepton mixing. In particular, the reactor angle deviates from zero by $\theta_{13} \sim \frac{1}{\sqrt{2}}\frac{\epsilon}{3}$ [74]. Furthermore, since $\theta^e_{13} \sim \theta^e_{23} \sim 0$, two sum rules for lepton mixing are respected [74, 75]. Expressed in terms of the TB deviation parameters in Eq. (3.42), the sum rules read $s = r\cos\delta$ and $a = -r^2/4$ [76], with δ being the leptonic Dirac CP phase.

4.3.5 Up quark sector

Eq. (4.19) may be expanded after A_4 symmetry breaking and is responsible for up quark masses:

$$\begin{pmatrix} \eta^2_{\xi'} & \eta^2_{23}\eta_\xi + \eta^5_\xi & -\eta_{23}\eta_3\eta^2_\xi \\ \eta^2_{23}\eta_\xi + \eta^5_\xi & \eta^2_\xi & \eta_{123}\eta_3\eta^2_\xi \\ -\eta_{23}\eta_3\eta^2_\xi & \eta_{123}\eta_3\eta^2_\xi & 1 \end{pmatrix} v_u. \tag{4.29}$$

Taking the VEV hierarchy as in Eq. (5.6), but now adopting the messenger scale $\Lambda_u \approx 3\Lambda_d$, gives a mass matrix with an expansion parameter $\bar{\epsilon} \sim 0.05$,

$$m_u \sim \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^5 & -\bar{\epsilon}^5 \\ \bar{\epsilon}^5 & \bar{\epsilon}^2 & \bar{\epsilon}^5 \\ -\bar{\epsilon}^5 & \bar{\epsilon}^5 & 1 \end{pmatrix} v_u. \tag{4.30}$$

and an up quark mass hierarchy $\bar{\epsilon}^4 : \bar{\epsilon}^2 : 1$. As the mass matrix of Eq. (4.30) is diagonal to a good approximation, the up quark mixing is negligible. An important consequence of this observation is that the CKM mixing arises predominantly from the down quark sector, with the Cabibbo angle being $\theta_C \sim \theta^d_{12} \sim \epsilon$.

4.4 Conclusions

In conclusion, minimal (SUSY) $SU(5)$ represents an attractive route to unification, but the Weinberg operator cannot account for neutrino mass and mixing, and the seesaw mechanisms all require extra matter or Higgs below the GUT scale. An appealing possibility, considered here, is to extend SUSY $SU(5)$ by assuming a single RH neutrino singlet and an adjoint matter representation below the GUT scale, including an A_4 family symmetry as well as a gauged anomaly-free $U(1)$. Hierarchical neutrino masses result from a combined type I and type III seesaw mechanism, and TB mixing arises indirectly from the A_4 family symmetry.

One attractive feature of this scheme is that the mixing between the single RH neutrino and the matter in the adjoint can be forbidden by not including the $H_{\mathbf{24}}$, leading to a diagonal heavy Majorana sector as required by CSD. The flavon vacuum alignments arise from the elegant SUSY D-term mechanism. The model also reproduces a realistic description of quark and charged lepton masses and quark mixings, including the GJ relations.

Corrections to TB mixing in the lepton sector come solely from the 1-2 mixing of the left-handed charged leptons, resulting in a PMNS matrix with two angles within the experimentally allowed limits (recall that $\theta_{13} = 0$ is now experimentally disfavoured). In particular the model respects the sum rules $s = r \cos \delta$ and $a = -r^2/4$ with $r = \theta_C/3$.

Chapter 5

$A_4 \times SU(5)$ SUSY GUT of Flavour with Trimaximal Neutrino Mixing

As mentioned in Chapter 3 the Daya Bay and RENO collaborations have published results confirming the discovery of a sizeable reactor angle θ_{13} [16, 17] in the range $7.95^\circ \lesssim \theta_{13} \lesssim 10.8^\circ$ (combining statistical and systematic errors in quadrature for each experiment separately and using the extreme 1σ bounds). This confirms the previous indications from T2K [13], MINOS [14], DOUBLE CHOOZ [15] and the global fits based on several experiments [32, 33].

The measured reactor angle $\theta_{13} \sim 9^\circ$ clearly rules out the hypothesis of exact TB mixing [54]. However, in the framework of SUSY GUTs of Flavour [77] (i.e. with a Family Symmetry [18] implemented) it is already known that TB mixing cannot be exact. As an example consider the model in the previous Chapter: TB mixing is realised exactly in the neutrino sector, but observable lepton mixing is subject to charged lepton (CL) corrections (due to the fact that $U_{\mathrm{PMNS}} = V_e V_\nu^\dagger$). There are also RG corrections, not to mention other corrections due to CN (for a unified discussion of all three corrections see e.g. [75] and references therein). Therefore, in the framework of SUSY GUTs of Flavour, the question of whether TB mixing may be maintained in the neutrino sector is a quantitative one: can the above CL, RG and CN corrections be sufficiently large to account for the observed reactor angle? The answer is yes in some cases (see e.g. [78]), but no in many other cases. For example, in models based on the GJ mechanism [42], where the CL corrections are less than or about 3°, and where the RG and CN corrections are less than or about 1° (which is the case for hierarchical neutrinos), it would be difficult to account for a reactor angle $\theta_{13} \sim 9°$. For this reason, there is a good motivation to consider other patterns of neutrino mixing beyond TB mixing, and many alternative proposals [19] have indeed been put forward to account for a non-zero θ_{13}. On the other hand, since the solar and atmospheric mixing angles remain consistent with TB mixing, there is also a good motivation to maintain these successful predictions of TB mixing.

In a SUSY GUT of Flavour, the Family Symmetry is responsible for determining the neutrino mixing pattern, which then gets corrected by CL, RG and CN contributions to yield the observed lepton mixing angles. The question is what is the underlying neutrino mixing pattern? To go beyond TB neutrino mixing, there are many possibilities. One simple scheme is the TM mixing pattern [59]:

$$U^{\nu\dagger}_{\mathrm{TM}} = P' \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\vartheta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\vartheta\, e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\vartheta - \frac{1}{\sqrt{2}}\sin\vartheta\, e^{-i\rho} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\cos\vartheta - \frac{1}{\sqrt{6}}\sin\vartheta\, e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\vartheta + \frac{1}{\sqrt{2}}\sin\vartheta\, e^{-i\rho} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\cos\vartheta - \frac{1}{\sqrt{6}}\sin\vartheta\, e^{i\rho} \end{pmatrix} P\,, \qquad (5.1)$$

where $\frac{2}{\sqrt{6}}\sin\vartheta = \sin\theta^{\nu}_{13}$, P' is a diagonal phase matrix required to put $U_{\mathrm{PMNS}} = U^{e}U^{\nu\dagger}_{\mathrm{TM}}$ into the PDG convention [79], and $P = \mathrm{diag}(1, e^{i\frac{\alpha_2}{2}}, e^{i\frac{\alpha_3}{2}})$ contains the usual Majorana phases. In particular TM mixing approximately predicts TB neutrino mixing for the solar neutrino mixing angle $\theta^{\nu}_{12} \approx 35°$ as the correction due to a non-zero but relatively small reactor angle is of second order. However it is emphasised again that, in a SUSY GUT of Flavour, TM mixing refers to the neutrino mixing angles only, and the physical lepton mixing angles will involve additional CL, RG and CN corrections. Nevertheless, TM neutrino mixing could provide a better starting point than TB neutrino mixing, given that $\theta_{13} \sim 9°$, and this provides the motivation for the approach followed in this Chapter.

Recently, an A_4 model of TM neutrino mixing was discussed in [60]. In the original A_4 models of TB mixing Higgs fields or flavon fields transforming under A_4 as $\mathbf{3}$ and $\mathbf{1}$ but not $\mathbf{1}'$ or $\mathbf{1}''$ were used to break the family symmetry and to lead to TB mixing. However, as discussed above, exact TB mixing is no longer consistent with data; a non-zero θ_{13} must be accommodated, and the chain of logic to achieve this is as follows. In the presentation of Section 3.1.1, A_4 has two generators S and T . In addition, the neutrino sector of the AF model respects an accidental U symmetry which enforces $\theta_{13} = 0$ (as well as $\theta_{23} = \frac{\pi}{4}$) [56, 80]. This can be broken by including flavons transforming as $\mathbf{1}'$ or $\mathbf{1}''$ [61], and in particular it was noted that they lead to TM mixing [62], allowing a non-zero θ_{13}. In [60] the vacuum alignment of the AF A_4 family symmetry model [20], including additional flavons in the $\mathbf{1}'$ and/or $\mathbf{1}''$ representations, was studied and it was shown that it leads to TM neutrino mixing.

In this Chapter it will be shown how such a model with TM neutrino mixing may arise from a SUSY GUT based on $SU(5)$, leading to the sum rule bounds $|s| \leq \frac{\theta_C}{3}$ and $|a| \leq \frac{1}{2}(r + \frac{\theta_C}{3})|\cos\delta|$, up to RG and CN corrections, where r, s, a are the TB deviation parameters, δ is the CP violating oscillation phase, and θ_C is the Cabibbo angle. Although the model is formulated at the GUT scale, the details of its breaking are not discussed, since the results rely mainly on the assumption of a GJ factor of -3, rather than the full details of the underlying GUT breaking mechanism. As such, the GJ mechanism can be realised in various contexts. One possibility to break the GUT, mentioned previously, is to rely on geometrical effects in extra dimensions, which are known to provide an elegant solution to the doublet-triplet splitting problem. In such a GUT breaking scenario, any 4 dimensional model (like the one presented here) would have to be uplifted to a higher dimensional setting. This could be achieved along the lines of, e.g., [68]. Alternatively, the GUT could be broken spontaneously using large Higgs representations. In that case, the existence of a family symmetry typically requires the introduction of more GUT Higgses than would be necessary without a family symmetry, see for instance [81], entailing a rather intricate Higgs sector. With the main focus being on the quark and lepton sector, any detailed discussion of the (geometrical or spontaneous) GUT breaking is, however, beyond the scope of this

Field	N	F	T_1	T_2	T_3	$H_{\mathbf{5}}$	$H_{\overline{\mathbf{5}}}$	$H_{\overline{\mathbf{45}}}$
$SU(5)$	$\mathbf{1}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{10}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$
$U(1)_R$	1	1	1	1	1	0	0	0
$U(1)$	1	-1	3	3	0	0	-1	-2
Z_2	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$-$
Z_3	ω	ω^2	ω^2	1	1	1	ω	ω
Z_5	ρ	ρ^4	1	1	1	1	ρ	ρ

Table 5.1: Matter and Higgs chiral superfields in the model.

Thesis.

The work in this Chapter is based on a paper published in [2]. The rest of the Chapter is organised as follows. In Section 5.1 the model is introduced, presenting field content, charges, flavon alignments and LO superpotential terms. Section 5.2 then presents the mass matrices and mixing angles for neutrinos, quarks and charged leptons arising from the LO superpotential. The effect of the non-trivial charged lepton corrections (due to the grand unified setup) on the physical lepton mixing angles is discussed in Section 5.3. The discussion of the vacuum alignment and the NLO terms is presented in Sections 5.4 and 5.5, respectively. The conclusion can be found in Section 5.6.

5.1 The model

The transformation properties of the $SU(5)$ matter and Higgs multiplets are shown in Table 5.1. N and F furnish the triplet representation of A_4, thus unifying the three families of leptons, while the three families of the T_i transform in the three distinct one-dimensional representations of A_4. The Higgs sector again contains the $H_{\overline{\mathbf{45}}}$ in order to implement the GJ mechanism [42].[1]

The full set of flavon fields is shown in Table 5.2. The fields φ_S and ξ^i are responsible for the flavour structure of the neutrino sector, while the flavons φ_T and θ^i control the quark and charged lepton sector. The vacuum structure is obtained

[1]As before, the standard MSSM μ-term $\mu H_u H_d$ is forbidden by the A_4, $U(1)$, Z_3 and Z_5 symmetries as well as $U(1)_R$, allowing for a natural solution to the μ-problem of the MSSM using a GUT singlet from the hidden sector of Supergravity theories [51].

Field	φ_S	ξ	ξ'	ξ''	φ_T	θ	θ'	θ''	$\widetilde{\theta}'$	σ
$SU(5)$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
A_4	**3**	**1**	$\mathbf{1}'$	$\mathbf{1}''$	**3**	**1**	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{1}'$	**1**
$U(1)_R$	0	0	0	0	0	0	0	0	0	0
$U(1)$	-2	-2	-2	-2	2	-1	-1	-1	-5	2
Z_2	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$+$
Z_3	ω	ω	ω	ω	1	ω	ω^2	ω^2	ω^2	1
Z_5	ρ^3	ρ^3	ρ^3	ρ^3	1	1	1	1	1	1

Table 5.2: Flavon chiral superfields in the model.

via the standard F-term alignment mechanism [20] where the F-terms of the driving fields (presented in Section 5.4) are set to zero, thus giving rise to constraints which in turn fix the flavon alignments. As shown in Section 5.4, one obtains the following triplet flavon alignments,[2]

$$\langle\varphi_T\rangle \propto \begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad \langle\varphi_S\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix}. \tag{5.2}$$

Since F-term alignment is being used in this Chapter the U(1) symmetry does not need to be gauged, as the Goldstone bosons are free to obtain soft SUSY breaking masses without fear of jeapordising the alignment mechanism. The model is constructed in the T-diagonal basis of 3.

The $U(1)_R$ again represents an R-symmetry; the $U(1)$ and the three Z_N shaping symmetries constrain the structure of the Yukawa matrices in the quark and charged lepton sectors. Specifically, the Z_5 prevents the neutrino flavons (φ_S and ξ^i) from appearing in the quark and charged lepton Yukawa couplings.

In the neutrino sector, the A_4 family symmetry is broken by the flavon fields φ_S and ξ^i, thereby leading to a TM mixing pattern as observed in [60]. In the quark and charged lepton sector the A_4 symmetry is broken differently by virtue of the flavon fields φ_T and θ^i. Due to the $SU(5)$ structure, the form of the charged lepton and down quark Yukawa matrices is intimately related, leading to a non-trivial LH

[2]The auxiliary flavon field σ is introduced for the purpose of achieving the alignment of the $U(1)$ charged flavon field φ_T.

charged lepton mixing which combines with the TM structure of the neutrino mixing to give the physical PMNS mixing.

5.1.1 Allowed terms

The neutrino sector is composed of Dirac and Majorana mass terms which take the leading order form in the superpotential,

$$W_\nu = yFNH_{\mathbf{5}} + \left(y_1\varphi_S + y_2\xi + y_2'\xi' + y_2''\xi''\right) NN \ , \tag{5.3}$$

with $y,\ y_1,\ y_2,\ y_2',\ y_2''$ being dimensionless couplings.

The leading order superpotential terms of the down quark and charged lepton sector are given as follows

$$\begin{aligned} W_d \ \sim\ & \left(\frac{\theta^2\theta''}{\Lambda_d^4}\left(F\varphi_T\right)' + \frac{\theta^2\theta'}{\Lambda_d^4}\left(F\varphi_T\right)''\right) H_{\overline{\mathbf{5}}}T_1 + \frac{\sigma\theta\theta'\left(\theta''\right)^2}{\Lambda_d^6}\left(F\varphi_T\right) H_{\overline{\mathbf{45}}}T_1 \\ +\ & \frac{\left(\theta'\right)^2\theta''}{\Lambda_d^4}\left(F\varphi_T\right) H_{\overline{\mathbf{5}}}T_2 + \left(\frac{\theta\theta''}{\Lambda_d^3}\left(F\varphi_T\right)' + \frac{\theta\theta'}{\Lambda_d^3}\left(F\varphi_T\right)''\right) H_{\overline{\mathbf{45}}}T_2 \\ +\ & \left(\frac{\sigma^2\theta^2\left(\theta'\right)^2}{\Lambda_d^7}\left(F\varphi_T\right) + \frac{1}{\Lambda_d}\left(\left(F\varphi_T\right)''\right)\right) H_{\overline{\mathbf{5}}}T_3 + \left(\frac{\sigma^2\theta^3}{\Lambda_d^6}\left(F\varphi_T\right)'\right) H_{\overline{\mathbf{45}}}T_3 \ , \end{aligned} \tag{5.4}$$

where Λ_d is the relevant messenger mass. Note that for some entries of the down quark Yukawa matrix, there are several different operators of the same order; here an example is chosen for illustrative purposes. The flavons θ^i again play a role similar to a Froggatt-Nielsen field [57].

Finally the leading order up quark sector Yukawa superpotential terms take the form

$$\begin{aligned} W_u \ \sim\ & \frac{\theta^4\left(\theta'\right)^2}{\Lambda_u^6}T_1T_1H_{\mathbf{5}} + \left(\frac{\theta^2\left(\theta'\right)^2\left(\theta''\right)^2}{\Lambda_u^6} + \frac{\sigma\theta\left(\theta'\right)^2\widetilde{\theta}'}{\Lambda_u^5}\right)(T_1T_2+T_2T_1)H_{\mathbf{5}} \\ +\ & \frac{\theta^2\theta'}{\Lambda_u^3}(T_1T_3+T_3T_1)H_{\mathbf{5}} + \frac{\theta\widetilde{\theta}'}{\Lambda_u^2}T_2T_2H_{\mathbf{5}} \\ +\ & \frac{\theta'\left(\theta''\right)^2}{\Lambda_u^3}(T_2T_3+T_3T_2)H_{\mathbf{5}} + T_3T_3H_{\mathbf{5}}. \end{aligned} \tag{5.5}$$

As before the messenger mass in this sector, Λ_u, may in principle be different from that in the down quark sector. The field $\widetilde{\theta}'$ is introduced specifically to generate the T_2T_2 term to the required order.

Examples of the many subleading higher order operators allowed by the symmetries of the model are listed in Section 5.5.[3] As their contribution to the mass matrices is negligible, they do not induce physically relevant modifications of the LO picture.

5.2 Fermion mass matrices

After spontaneous breakdown of the A_4 family symmetry by the flavon VEVs, the superpotential terms of Eqs. (5.3)-(5.5) predict mass matrices for the respective sectors. In the following, order one coefficients in the quark and charged lepton sectors are omitted (including flavon VEV normalisation factors). Regarding the scale of the flavon VEVs the expansion parameter η_i from Eq. (4.20) is again used, where φ_i=φ_T, θ^i or σ. In order to get the hierarchical structure of the quark and charged lepton mass matrices the suppressions

$$\eta_{\widetilde{\theta}'} = \epsilon^2 \quad \text{and} \quad \eta_{\text{others}} = \epsilon, \tag{5.6}$$

are assumed, where the numerical values for ϵ depend on the messenger scale of the relevant sector. This hierarchy is justified in Section 5.4, where the driving superpotential is studied. LO operators for each entry in the mass matrices are presented; NLO operators can be found in Section 5.5.

5.2.1 Neutrino sector

Eq. (5.3) gives Dirac and Majorana mass matrices

$$m_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} y v_u \ , \tag{5.7}$$

[3] It is emphasised that the full NLO spectrum has been studied, however only example terms are presented since there are too many to include all of them.

and

$$M_R = \left[A \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + B \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + C' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + C'' \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right], \qquad (5.8)$$

with $A = 2y_1 \langle \varphi_S \rangle$, $B = 2y_2 \langle \xi \rangle$, $C' = 2y_2' \langle \xi' \rangle$ and $C'' = 2y_2'' \langle \xi'' \rangle$. As shown in Chapter 3, the standard type I seesaw formula then yields a light neutrino mass matrix of TM structure, and hence a neutrino mixing matrix of the form as given in Eq. (5.1). The relationships between the given parameters and θ_{13} are given in Chapter 3; note however that in the limit that $C' = C''$, exact TB mixing is recovered.

5.2.2 Down quark and charged lepton sector

In the down quark and charged lepton sector, the superpotential of Eq. (5.4) predicts a mass matrix of the form (with messenger mass Λ_d in η_i)

$$\begin{pmatrix} k_f \eta_\sigma \eta_\theta \eta_{\theta'} \eta_{\theta''}^2 & \eta_\theta^2 \eta_{\theta''} & \eta_\theta^2 \eta_{\theta'} \\ \eta_{\theta'}^2 \eta_{\theta''} & k_f \eta_\theta \eta_{\theta''} & k_f \eta_\theta \eta_{\theta'} \\ \eta_\sigma^2 \eta_\theta^2 \eta_{\theta'}^2 & k_f \eta_\sigma^2 \eta_\theta^3 & 1 \end{pmatrix} \eta_T v_d \ , \qquad (5.9)$$

where this matrix has to be transposed for the charged leptons. k_f is the familiar GJ factor. Inserting the ϵ suppressions of the flavon VEVs from Eq. (5.6) the down quark mass matrix becomes

$$m_d \sim \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^6 & \epsilon^5 & 1 \end{pmatrix} \epsilon \, v_d, \qquad (5.10)$$

whilst the charged lepton mass matrix reads

$$m_e \sim \begin{pmatrix} -3\epsilon^5 & \epsilon^3 & \epsilon^6 \\ \epsilon^3 & -3\epsilon^2 & -3\epsilon^5 \\ \epsilon^3 & -3\epsilon^2 & 1 \end{pmatrix} \epsilon\, v_d. \tag{5.11}$$

Again the numerical value $\epsilon \sim 0.15$ is assumed. Upon diagonalisation, these give mass ratios of $\epsilon^4 : \epsilon^2 : 1$ for the down-type quarks and $\frac{\epsilon^4}{3} : 3\epsilon^2 : 1$ for the charged leptons. These ratios are in good agreement with quark and lepton data and also predict the GJ GUT scale mass relations of Eq. (2.61) as desired. In the low quark angle approximation, the LH down quark mixing angles $\theta^d_{12} \sim \epsilon$, $\theta^d_{13} \sim \epsilon^3$ and $\theta^d_{23} \sim \epsilon^2$ are also predicted in agreement with data (assuming an approximately diagonal up quark sector which we obtain in the next subsection). The corresponding charged lepton mixing angles are $\theta^e_{12} \sim \frac{\epsilon}{3}$, $\theta^e_{13} \sim \epsilon^6$ and $\theta^e_{23} \sim 3\epsilon^5$. Therefore, the only significant charged lepton correction to the TM mixing of the neutrino sector originates from $\theta^e_{12} \sim \frac{\theta_C}{3}$, where θ_C denotes the Cabibbo angle.

5.2.3 Up quark sector

Eq. (5.5) may be expanded after A_4 symmetry breaking and is responsible for up-type quark masses

$$\begin{pmatrix} \eta_\theta^4\eta_{\theta'}^2 & \eta_\theta^2\eta_{\theta'}^2\eta_{\theta''}^2 + \eta_\sigma\eta_\theta\eta_{\theta'}^2\eta_{\widetilde{\theta}'} & \eta_\theta^2\eta_{\theta'} \\ \eta_\theta^2\eta_{\theta'}^2\eta_{\theta''}^2 + \eta_\sigma\eta_\theta\eta_{\theta'}^2\eta_{\widetilde{\theta}'} & \eta_\theta\eta_{\widetilde{\theta}'} & \eta_{\theta'}\eta_{\theta''}^2 \\ \eta_\theta^2\eta_{\theta'} & \eta_{\theta'}\eta_{\theta''}^2 & 1 \end{pmatrix} v_u \ . \tag{5.12}$$

Taking the VEV hierarchy as in Eq. (5.6), but now adopting the messenger scale $\Lambda_u \approx \frac{3}{2}\Lambda_d$, a mass matrix with an expansion parameter $\overline{\epsilon} \sim 0.1$ is obtained,

$$m_u \sim \begin{pmatrix} \overline{\epsilon}^6 & \overline{\epsilon}^6 & \overline{\epsilon}^3 \\ \overline{\epsilon}^6 & \overline{\epsilon}^3 & \overline{\epsilon}^3 \\ \overline{\epsilon}^3 & \overline{\epsilon}^3 & 1 \end{pmatrix} v_u \ . \tag{5.13}$$

and an up-type quark mass hierarchy $\bar{\epsilon}^6 : \bar{\epsilon}^3 : 1$. This matrix gives mixing angles of $\theta^u_{12} \sim \theta^u_{13} \sim \theta^u_{23} \sim \bar{\epsilon}^3$. This means that the CKM mixing matrix is dominated by down quark mixing, except that there may be a contribution to $\theta^{\rm CKM}_{13}$ from the up quark sector which is almost as significant as the contribution coming from the down-type quarks. The Cabibbo angle is still approximately $\theta_C \sim \theta^d_{12} \sim \epsilon$.

5.3 Charged lepton corrections to lepton mixing

The previous Sections present mixing angles which rotate the charged leptons and neutrino fields between the mass and flavour bases, however these individual rotations are not what experiments observe. It is the combination of the two mixing matrices that appears in the EW coupling to the W boson, giving the physical mixing matrix, as in Chapter 2

$$U_{\rm PMNS} = U_{e_L} U^\dagger_{\nu_L}. \tag{5.14}$$

While the neutrino sector predicts exact TM mixing, the effect of the charged lepton corrections generates an experimentally detectable deviation from this in the physical parameters. In this Section RG and CN corrections are ignored and the CL corrections are studied.

There are (at least) two popular ways to parameterise the PMNS matrix; firstly one can write $U_{\rm PMNS} = U_{23} U_{13} U_{12}$ with [82]

$$U_{12} = \begin{pmatrix} c_{12} & s_{12}\exp(-i\delta_{12}) & 0 \\ -s_{12}\exp(i\delta_{12}) & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5.15}$$

$$U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}\exp(-i\delta_{13}) \\ 0 & 1 & 0 \\ -s_{13}\exp(i\delta_{13}) & 0 & c_{13} \end{pmatrix}, \tag{5.16}$$

$$U_{23}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\exp\left(-i\delta_{23}\right)\\ 0 & -s_{23}\exp\left(i\delta_{23}\right) & c_{23}\end{pmatrix}. \tag{5.17}$$

Individual rotation matrices U_{e_L} and $U^{\dagger}_{\nu_L}$ are parameterised in the same way with relevant superscripts. The second parameterisation is that used by the PDG [79] and is as in Chapter 3, with a Dirac phase δ and Majorana phases α_2 and α_3; this is constructed as $U^{\mathrm{PDG}}_{\mathrm{PMNS}} = R_{23}U^{\mathrm{PDG}}_{13}R_{12}P$ where the R_{ij} are standard orthogonal rotations, $U^{\mathrm{PDG}}_{13} = U_{13}\left(\delta_{13}=\delta\right)$ and $P = \mathrm{diag}(1, e^{i\frac{\alpha_2}{2}}, e^{i\frac{\alpha_3}{2}})$. A comparison of the two parameterisations, after performing a global phase redefinition to absorb remaining unphysical phases and obtain consistency with the convention stated in the introduction, shows that [69]

$$\delta = \delta_{13} - \delta_{23} - \delta_{12}, \tag{5.18}$$

$$\alpha_2 = -2\delta_{12}, \tag{5.19}$$

$$\alpha_3 = -2\left(\delta_{12}+\delta_{23}\right). \tag{5.20}$$

It is possible to write the parameters of U_{PMNS} in terms of the neutrino mixing parameters, with perturbative corrections from the charged lepton sector as follows [69] (neglecting θ^e_{13} and θ^e_{23} as they are small),[4]

$$s_{23}\exp\left(-i\delta_{23}\right) \approx s^{\nu}_{23}\exp\left(-i\delta^{\nu}_{23}\right), \tag{5.21}$$

$$s_{13}\exp\left(-i\delta_{13}\right) \approx \theta^{\nu}_{13}\exp\left(-i\delta^{\nu}_{13}\right) - \theta^{e}_{12}s^{\nu}_{23}\exp\left(-i\left(\delta^{\nu}_{23}+\delta^{e}_{12}\right)\right), \tag{5.22}$$

$$s_{12}\exp\left(-i\delta_{12}\right) \approx s^{\nu}_{12}\exp\left(-i\delta^{\nu}_{12}\right) - \theta^{e}_{12}c^{\nu}_{23}c^{\nu}_{12}\exp\left(-i\delta^{e}_{12}\right). \tag{5.23}$$

The dominance of the first term in Eq. (5.23) allows for the approximation $\delta_{12} \approx \delta^{\nu}_{12}$, while Eq. (5.21) gives directly $\delta_{23} \approx \delta^{\nu}_{23}$. The phase δ_{13} requires a more careful treatment, since the first term of Eq. (5.22) is larger but not dominant

[4] In order to derive these equations consistently to first order, the Majorana phases from Eqs. (5.18)-(5.20) must be redefined by a correction of order θ^{ν}_{13}; this is however only a subtlety in the derivation and therefore this redefinition is not explicitly demonstrated.

enough to drop the second term. If one assumes that $\frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}}$ is small,[5] then

$$\begin{aligned}\tan\delta_{13} &\approx \frac{\left(\sin\delta^\nu_{13} - \frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}}\sin(\delta^\nu_{23}+\delta^e_{12})\right)\left(\cos\delta^\nu_{13} + \frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}}\cos(\delta^\nu_{23}+\delta^e_{12})\right)}{\cos^2\delta^\nu_{13}} \\ &\approx \tan\delta^\nu_{13}\left(1+\frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}}k\right),\end{aligned} \tag{5.24}$$

with $k = \frac{\cos\left(\delta^\nu_{23}+\delta^e_{12}\right)}{\cos\delta^\nu_{13}} - \frac{\sin\left(\delta^\nu_{23}+\delta^e_{12}\right)}{\sin\delta^\nu_{13}}$. The expectation is that $\delta_{13} = \delta^\nu_{13} + \Delta\delta_{13}$ where the correction is small; this allows for the approximation
$\tan(\theta+\Delta\theta) \approx \tan\theta + \frac{\Delta\theta}{\cos^2\theta}$ and therefore

$$\Delta\delta_{13} \approx \frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}}k\sin\delta^\nu_{13}\cos\delta^\nu_{13}. \tag{5.25}$$

This leads to an analytic form for δ_{13}

$$\delta_{13} \approx \delta^\nu_{13} - \frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}}\sin(\delta^\nu_{23} - \delta^\nu_{13} + \delta^e_{12}) \ . \tag{5.26}$$

Using Eq. (5.18) allows the physical Dirac oscillation phase to be approximated by

$$\delta \ \approx \ \delta^\nu_{13} - \delta^\nu_{23} - \delta^\nu_{12} - \frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}}\sin(\delta^\nu_{23} - \delta^\nu_{13} + \delta^e_{12}) \ . \tag{5.27}$$

Turning to the resulting mixing angles, experimentally the TM mixing of the neutrino sector must necessarily be a small deviation from TB mixing. Therefore the results may be expressed using the *neutrino* TB deviation parameters [58],

$$\sin\theta^\nu_{12} = \frac{1}{\sqrt{3}}(1+s^\nu)\ , \quad \sin\theta^\nu_{23} = \frac{1}{\sqrt{2}}(1+a^\nu)\ , \quad \sin\theta^\nu_{13} = \frac{r^\nu}{\sqrt{2}}\ , \tag{5.28}$$

where here these parameters refer only to the neutrino sector. In terms of angles and phases, using Eqs. (5.21)-(5.23) (see, e.g. [75] for a discussion of this procedure), the TB deviation parameters for the complete lepton mixing can be written in terms of the TB deviations parameters in the neutrino sector and the

[5]Using $\theta^e_{12} \sim \frac{\theta_C}{3}$, $s^\nu_{23} \sim \frac{1}{\sqrt{2}}$ and $\theta^\nu_{13} \sim 0.15$ gives a numerical value of $\frac{\theta^e_{12}s^\nu_{23}}{\theta^\nu_{13}} \sim \frac{1}{3}$.

charged lepton corrections as,

$$a \approx a^{\nu}, \tag{5.29}$$

$$r \approx \left| r^{\nu} \exp\left(-i\delta_{13}^{\nu}\right) - \theta_{12}^{e} \exp\left(-i\left(\delta_{23}^{\nu} + \delta_{12}^{e}\right)\right)\right| , \tag{5.30}$$

$$s \approx s^{\nu} - \theta_{12}^{e} \cos\left(\delta_{12}^{\nu} - \delta_{12}^{e}\right). \tag{5.31}$$

With the neutrino mixing being of TM form as given in Eq. (5.1), the deviation parameters of the neutrino sector can be shown to satisfy, see [58, 60, 83], $s^{\nu} = 0$ and $a^{\nu} \approx -\frac{r^{\nu}}{2} \cos\delta^{\nu}$. Using this and the fact that $\theta_{12}^{e} \sim \frac{\theta_C}{3}$ and Eq. (5.27), the above equations for the TB deviation parameters may be further simplified to first order as

$$a \approx -\frac{r^{\nu}}{2} \cos\delta, \tag{5.32}$$

$$r \approx r^{\nu} - \frac{\theta_C}{3} \cos\left(\delta_{23}^{\nu} - \delta_{13}^{\nu} + \delta_{12}^{e}\right), \tag{5.33}$$

$$s \approx -\frac{\theta_C}{3} \cos\left(\delta_{12}^{\nu} - \delta_{12}^{e}\right), \tag{5.34}$$

again assuming that $\frac{\theta_{12}^{e} s_{23}^{\nu}}{\theta_{13}^{\nu}} \sim \frac{\theta_C}{3r^{\nu}}$ is small. In the limit that charged lepton corrections are switched off, the above results reduce to the usual TM sum rules [58, 60, 83], $s \approx 0$ and $a \approx -\frac{r}{2} \cos\delta$. In the limit that the neutrino mixing angle θ_{13}^{ν} is switched off the above results reduce to the usual TB sum rules [74], $s \approx r \cos\delta$ where $r \approx \theta_C/3$ and $\delta \approx \delta_{12}^{e} - \delta_{12}^{\nu}$.

The results in Eqs. (5.32)-(5.34) imply the relatively simple sum rule bounds:

$$|s| \leq \frac{\theta_C}{3}, \tag{5.35}$$

$$|a| \leq \frac{1}{2}\left(r + \frac{\theta_C}{3}\right)|\cos\delta|, \tag{5.36}$$

where, again, r, s, a are the tri-bimaximal deviation parameters, in particular $r \approx \sqrt{2}\theta_{13}$, δ is the CP violating oscillation phase, and θ_C is the Cabibbo angle. These bounds do not include RG and CN corrections, which however are expected to be rather small for the case of hierarchical neutrino masses. For example,

Field	φ_T^0	φ_S^0	ξ^0	A''	B	C
$SU(5)$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_R$	2	2	2	2	2	2
$U(1)$	-4	4	4	3	3	6
Z_2	$+$	$+$	$+$	$-$	$+$	$+$
Z_3	1	ω	ω	ω	1	1
Z_5	1	ρ^4	ρ^4	1	1	1

Table 5.3: Driving fields in the model.

assuming $\theta_{13} \sim 9^\circ$ gives $r \approx 0.22$,[6] and using $\theta_C/3 \approx 0.075$ these bounds become $|s| \leq 0.075$ and $|a| < 0.15|\cos\delta|$. The present approximate limits from the global fit $|a| < 0.118$, $-0.066 < s < 0.003$ quoted in Eq. (3.44) are nicely consistent with these sum rule bounds.

5.4 Vacuum alignment

In order that the flavon fields obtain the alignment presented in Eq. (5.2), their potential must be minimised in the correct way. The method of [60] is followed very closely, which employs F-term alignment as described in Chapter 3; the driving fields can be found in Table 5.3. The leading order contributions to the driving superpotential aligning the flavon triplets are:

$$\begin{aligned} W_0 = {} & \varphi_T^0 \left(g_1 \sigma \varphi_T + g_2 \varphi_T \varphi_T\right) + \varphi_S^0 \left(g_3 \varphi_S \varphi_S + g_4 \varphi_S \xi + g_4' \varphi_S \xi' + g_4'' \varphi_S \xi''\right) \\ & + \xi^0 \left(g_5 \varphi_S \varphi_S + g_6 \xi\xi + g_7 \xi' \xi''\right). \end{aligned} \tag{5.37}$$

Here, $g_1 \langle\sigma\rangle = M$ which appears in the vacuum alignment of [60]; this is required since φ_T is charged under the auxiliary symmetries and so the original structure $\varphi_T^0 \left(M\varphi_T + \varphi_T\varphi_T\right)$ that drives the φ_T alignment cannot be used. Minimising with respect to φ_T^0 gives

$$\langle\varphi_T\rangle = v_T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_T = -\frac{g_1 \langle\sigma\rangle}{2g_2}. \tag{5.38}$$

[6] Note that in [60], it is demonstrated that $r^\nu \sim \frac{\gamma''-\gamma'}{\beta-\gamma'-\gamma''}$ and so a partial cancellation between γ' and γ'' is required, to the level of $\sim 20\%$.

The conditions from φ_S^0 and ξ^0 are

$$2g_3 \begin{pmatrix} s_1^2 - s_2 s_3 \\ s_2^2 - s_3 s_1 \\ s_3^2 - s_1 s_2 \end{pmatrix} + g_4 u \begin{pmatrix} s_1 \\ s_3 \\ s_2 \end{pmatrix} + g_4' u' \begin{pmatrix} s_3 \\ s_2 \\ s_1 \end{pmatrix} + g_4'' u'' \begin{pmatrix} s_2 \\ s_1 \\ s_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{5.39}$$

$$g_5 \left(s_1^2 + 2 s_2 s_3\right) + g_6 u^2 + g_7 u' u'' = 0. \tag{5.40}$$

Here, $\langle \varphi_{S_i} \rangle = s_i$, $\langle \xi \rangle = u$, $\langle \xi' \rangle = u'$ and $\langle \xi'' \rangle = u''$. The solutions to these equations are

$$\langle \varphi_S \rangle = v_S \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad v_S^2 = -\frac{g_6 u^2 + g_7 u' u''}{3 g_5}, \quad u = -\frac{g_4' u' + g_4'' u''}{g_4}. \tag{5.41}$$

As in [20], the undetermined singlets are assumed to obtain their VEVs as a result of their soft mass parameters m_s^2 (where s stands for singlet) being driven negative in some portion of parameter space.

The remaining flavons obtain the hierarchy in their VEVs through the driving superpotential:

$$\begin{aligned} \widetilde{W}_0 = A'' \left(\frac{\tilde{g}_1}{\Lambda} \theta \left(\theta''\right)^2 + \tilde{g}_2 \sigma \widetilde{\theta}' \right) + B \left(\frac{\tilde{g}_3}{\Lambda} \left(\theta'\right)^3 + \frac{\tilde{g}_4}{\Lambda} \left(\theta''\right)^3 \right) \\ + C \left(\frac{\tilde{g}_5}{\Lambda^4} \theta^6 + \frac{\tilde{g}_6}{\Lambda^4} \left(\theta'\right)^6 + \frac{\tilde{g}_7}{\Lambda^4} \left(\theta''\right)^6 + \frac{\tilde{g}_8}{\Lambda^4} \left(\theta'\right)^3 \left(\theta''\right)^3 \right). \end{aligned} \tag{5.42}$$

Solving the F-flat conditions for B and C ensures that the VEVs of θ, θ' and θ'' are correlated in the desired manner. The condition from A'' then leads to the hierarchy

$$\frac{\langle \widetilde{\theta}' \rangle}{\Lambda} \sim \left(\frac{\langle \theta \rangle}{\Lambda} \right)^2, \tag{5.43}$$

used in Section 5.2, under the assumption that $\langle \sigma \rangle \sim \langle \theta \rangle$.

5.5 Higher order operators

There are many higher order corrections to the mass matrices presented in Section 4.3.2 of this paper; these give negligible contributions to masses and mixings. In Tables 5.4 and 5.5 are given suppressions and examples of the NLO operators for each sector ; it can be seen that none of these will change the LO results significantly (it has been confirmed that the LO structure is not altered by any NLO terms, but there are too many to present here).

5.6 Conclusions

Recently Daya Bay and RENO have measured a sizeable reactor angle $\theta_{13} \sim 9^\circ$ which rules out exact TB lepton mixing. On the other hand, the TB predictions $\sin\theta_{23} = 1/\sqrt{2}$ and $\sin\theta_{12} = 1/\sqrt{3}$ remain in agreement with global fits and continue to provide tantalising hints for an underlying Family Symmetry. For example, an A_4 family symmetry model including additional flavons in the $\mathbf{1}'$ and $\mathbf{1}''$ representations leads to TM neutrino mixing which maintains the prediction $\sin\theta_{12} \approx 1/\sqrt{3}$, at least approximately, while allowing an arbitrarily large reactor angle. Indeed, as discussed in a recent paper [60], the problem in this model is in explaining why the reactor angle should be smaller than the atmospheric or solar angles, which follows from the fact that the additional flavons would be expected to have VEVs of the same order as the other TB flavon VEVs, with all undetermined coefficients being of order unity. However, apart from this drawback, such a model provides a simple example of a Family Symmetry model with a non-zero reactor angle.

This Chapter presents a SUSY GUT of Flavour with a non-zero θ_{13} based on A_4 Family Symmetry with additional flavons in the $\mathbf{1}'$ and $\mathbf{1}''$ representations, and an $SU(5)$ GUT group. The model involves an additional continuous $U(1)$ family symmetry as well as three discrete symmetries designed to control the operator structure of the model. All flavon representations of A_4 are populated, and the main flavon content of the quark sector mirrors that of the neutrino sector. The vacuum alignment is obtained using the conventional F-term mechanism. NLO terms to the mass matrices are negligible, demonstrating the stability of the LO matrix textures. The resulting model exhibits TM mixing in the neutrino sector, with the physical lepton mixing involving charged lepton corrections, which in turn are related to

Term	Contributes to	NLO Example	c.f. LO
$FNH_{\mathbf{5}}$	m_D	$\varphi_T^2\theta^2\theta'\theta'' \sim \epsilon^6$	1
NN	M_R	$\varphi_T^2\theta^2\left(\theta''\right)^2\xi'' \sim \epsilon^7$	ϵ
$F\varphi_T H_{\overline{\mathbf{5}}}T_1$	$(m_d)_{11}$	$\sigma\theta'\left(\theta''\right)^4 \sim \epsilon^7$	ϵ^6
	$(m_d)_{12}$	$\sigma\left(\theta'\right)^2\left(\theta''\right)^3 \sim \epsilon^7$	ϵ^4
	$(m_d)_{13}$	$\sigma\left(\theta''\right)^5 \sim \epsilon^7$	ϵ^4
$F\varphi_T H_{\overline{\mathbf{45}}}T_1$	$(m_d)_{11}$	$\sigma^2\theta^5\theta'' \sim \epsilon^9$	ϵ^6
	$(m_d)_{12}$	$\sigma\theta\left(\theta'\right)^2\theta'' \sim \epsilon^6$	ϵ^4
	$(m_d)_{13}$	$\sigma\theta\left(\theta'\right)^3 \sim \epsilon^6$	ϵ^4
$F\varphi_T H_{\overline{\mathbf{5}}}T_2$	$(m_d)_{21}$	$\sigma\theta^4\theta' \sim \epsilon^7$	ϵ^4
	$(m_d)_{22}$	$\left(\theta'\right)^3 \sim \epsilon^4$	ϵ^3
	$(m_d)_{23}$	$\theta'\left(\theta''\right)^2 \sim \epsilon^4$	ϵ^3
$F\varphi_T H_{\overline{\mathbf{45}}}T_2$	$(m_d)_{21}$	$\sigma^2\theta^3\left(\theta''\right)^3 \sim \epsilon^9$	ϵ^4
	$(m_d)_{22}$	$\sigma^2\theta^3\theta'\left(\theta''\right)^2 \sim \epsilon^9$	ϵ^3
	$(m_d)_{23}$	$\sigma^2\theta^3\left(\theta'\right)^2\theta'' \sim \epsilon^9$	ϵ^3
$F\varphi_T H_{\overline{\mathbf{5}}}T_3$	$(m_d)_{31}$	$\sigma^3\theta'\left(\theta''\right)^5 \sim \epsilon^{10}$	ϵ^7
	$(m_d)_{32}$	$\sigma^2\theta^2\left(\theta''\right)^2 \sim \epsilon^7$	ϵ^6
	$(m_d)_{33}$	$\sigma^2\theta^2\theta'\theta'' \sim \epsilon^7$	ϵ
$F\varphi_T H_{\overline{\mathbf{45}}}T_3$	$(m_d)_{31}$	$\sigma^3\theta\theta'\left(\theta''\right)^3 \sim \epsilon^9$	ϵ^7
	$(m_d)_{32}$	$\sigma^3\theta\left(\theta'\right)^2\left(\theta''\right)^2 \sim \epsilon^9$	ϵ^6
	$(m_d)_{33}$	$\sigma^3\theta\left(\theta''\right)^4 \sim \epsilon^9$	ϵ

Table 5.4: NLO corrections in the model. The first column shows each basic term that exists in the neutrino, down quark (and charged lepton) Yukawa superpotential, as specified in the second column. A collection of flavons is appended to these basic terms to obtain the complete term invariant under the symmetries. In the third column an example of such a collection of flavons is given at NLO as well as the order of its contribution, to be compared to the LO contribution given in the final column. Note that in the terms contributing to M_d, there is a flavon φ_T already present in the basic term. It is furthermore not specified whether the LO term comes from an $H_{\overline{\mathbf{5}}}$ or an $H_{\overline{\mathbf{45}}}$; the reader may refer back to Eq. (4.18) if required.

Term	Contributes to	NLO Example	c.f. LO
$T_1 T_1 H_{\mathbf{5}}$	$(m_u)_{11}$	$\sigma^2 \xi^4 \xi'' \sim \bar{\epsilon}^7$	$\bar{\epsilon}^6$
$T_1 T_2 H_{\mathbf{5}}$	$(m_u)_{12}\,,\ (m_u)_{21}$	$\sigma \theta^6 \theta' \theta'' \sim \bar{\epsilon}^9$	$\bar{\epsilon}^6$
$T_1 T_3 H_{\mathbf{5}}$	$(m_u)_{13}\,,\ (m_u)_{31}$	$\sigma\,(\theta')^3\,(\theta'')^2 \sim \bar{\epsilon}^6$	$\bar{\epsilon}^3$
$T_2 T_2 H_{\mathbf{5}}$	$(m_u)_{22}$	$(\theta')^5\,\theta'' \sim \bar{\epsilon}^6$	$\bar{\epsilon}^3$
$T_2 T_3 H_{\mathbf{5}}$	$(m_u)_{23}\,,\ (m_u)_{32}$	$\sigma \theta^4 \theta'' \sim \bar{\epsilon}^6$	$\bar{\epsilon}^3$
$T_3 T_3 H_{\mathbf{5}}$	$(m_u)_{33}$	$\sigma^2 \theta^2 \theta' \theta'' \sim \bar{\epsilon}^6$	1
φ_T^0	W_0	$\sigma^3 \varphi_T \theta^2 \theta' \theta'' \sim \widetilde{\epsilon}^8$	$\widetilde{\epsilon}^2$
φ_S^0	W_0	$\sigma^2 \varphi_S \theta^2\,(\theta'')^2\,\xi'' \sim \widetilde{\epsilon}^8$	$\widetilde{\epsilon}^2$
ξ^0	W_0	$\sigma^2 \theta^2\,(\theta'')^2\,\xi \xi'' \sim \widetilde{\epsilon}^8$	$\widetilde{\epsilon}^2$
A''	$\widetilde{W}_0$	$\sigma^2 \theta^3\,(\theta')^4 \sim \widetilde{\epsilon}^9$	$\widetilde{\epsilon}^3$
B	$\widetilde{W}_0$	$\sigma^2 \theta^2 \theta'\,(\theta'')^4 \sim \widetilde{\epsilon}^9$	$\widetilde{\epsilon}^3$
C	$\widetilde{W}_0$	$\sigma \theta^4\,(\theta')^2\,(\theta')^2 \sim \widetilde{\epsilon}^9$	$\widetilde{\epsilon}^6$

Table 5.5: NLO corrections in the model. The first column shows each basic term that exists in the up quark Yukawa and vacuum alignment sectors, as specified in the second column. A collection of flavons is appended to these basic terms to obtain the complete term invariant under the symmetries. In the third column an example of such a collection of flavons is given at NLO as well as the order of its contribution, to be compared to the LO contribution given in the final column. The notation $\widetilde{\epsilon}$ is simply used to denote a different sector to ϵ or $\bar{\epsilon}$.

quark mixing angles. In particular, the model involves a GJ relation, leading to bounds on the TB deviation parameters $|s| \leq \frac{\theta_C}{3}$, $|a| \leq \frac{1}{2}(r + \frac{\theta_C}{3})|\cos\delta|$ (up to RG and CN corrections) derived for the first time, which are in good agreement with current global fits. The presence of this GJ factor of -3 is dependent on the $SU(5)$ breaking chain which is not studied here. The considered model shows that it is possible to accommodate $\theta_{13} \sim 9°$, within a SUSY GUT of Flavour which relates quark and lepton masses and mixing angles, while continuing to provide an explanation for the TB nature of the solar and atmospheric lepton mixing angles.

Chapter 6

Renormalisation group improved leptogenesis in family symmetry models

One of the most important and well studied questions in particle physics is why the observable Universe has a tiny but non-zero ratio of baryons to photons without which there would be no stars, planets or life. The measurement of cosmic microwave background anisotropies and the successful prediction of light element abundances from big bang nucleosynthesis, both lead to a consistent value of this ratio at the recombination time when atoms are formed [84],

$$\eta = \frac{n_B}{n_\gamma} \approx 6.2 \times 10^{-10}, \tag{6.1}$$

where n_B and n_γ are baryon and photon number densities respectively.[1] Any theory which successfully produces such a baryon asymmetry must fulfil the famous Sakharov conditions [85] of C and CP violation, B violation and departure from thermal equilibrium. One of the most popular of these is known as leptogenesis [86], which takes advantage of the fact that non-perturbative, $B - L$ conserving, $B + L$ violating sphaleron processes can convert a lepton number

[1]Corresponding to a portion of comoving volume containing 1 photon at temperatures where the RH neutrinos are relativistic.

asymmetry into a B asymmetry. The lepton number asymmetry is obtained from the decays of heavy Majorana neutrinos and so leptogenesis is intimately linked to neutrino mass, mixing and CP violation.

Many models of neutrino mixing (predominantly employing the type I seesaw) exhibit a property known as FD [87], defined by the condition that the columns of the neutrino Yukawa matrix are proportional to columns of the mixing matrix in a particular basis corresponding to diagonal charged lepton and RH neutrino mass matrices. As discussed in several papers [83, 88, 90–94], models with family symmetry typically predict vanishing CP violating lepton asymmetry parameters ϵ and hence zero leptogenesis.[2] As pointed out in [88], this can be understood very simply from the FD property that the columns of the neutrino Yukawa matrix are mutually orthogonal since they are proportional to the columns of the mixing matrix which is unitary.[3] However in family symmetry models the Yukawa matrices are predicted at the scale of family symmetry breaking, which may be close to the GUT scale, and above the mass scale of RH neutrinos. Therefore in such models the Yukawa matrix will be subject to RG running from the family symmetry breaking scale down to the scale of RH neutrino masses relevant for leptogenesis. To illustrate the effects of RG corrections, two specific models involving sizeable neutrino and τ Yukawa couplings and satisfying FD at LO are analysed: the first model [20] reproduces the well studied TB mixing pattern [54]; and the second model [60] reproduces the TM mixing pattern [59] consistent with the results from Daya Bay, RENO and Double Chooz. Both of these models have been briefly introduced in Chapter 3. Although in both models RG running occurs over only one or two orders of magnitude in the energy scale, this will be shown to lead to sufficient violation of FD to allow successful leptogenesis in each case.

One could ask why RG effects should be considered when HO operators in the (TB) A_4 model have been shown to produce a realistic value of η [92]. The answer is that RG effects turn out to be of equal importance to HO operators in determining

[2]For a discussion of how to achieve leptogenesis in the flavour symmetric phase, see e.g. [95].

[3]The vanishing of leptogenesis due to the orthogonality of the columns of the neutrino Yukawa matrix was first observed in the case of hierarchical neutrinos and constrained sequential dominance with TB mixing in [90] and was subsequently generalised to the case of FD with any neutrino mass pattern and any mixing pattern in [88].

leptogenesis and so in general both effects should be considered together. Here the effect of HO operators is dropped for clarity: the effects of RG corrections to leptogenesis are studied in isolation in order to illustrate the magnitude of the effect. Moreover, there are ultraviolet completions of the A_4 model of both TB [96] and TM mixing [60] in which HO operators play a negligible role, and the viability of leptogenesis in such cases then relies exclusively on the effects of RG corrections considered here.

The results in this Chapter show that RG corrections have a large impact on leptogenesis in any family symmetry models involving neutrino Yukawa couplings of order unity. Therefore, when considering leptogenesis in such models, RG corrections should not be ignored even when corrections arising from HO operators are also present. It should be pointed out that the phrases "RG effect" and "RG corrections" are taken to mean those between the family symmetry scale and the leptogenesis scale, and those which help to generate a non-zero η. RG effects in evolving parameters from the leptogenesis scale to the EW scale are well studied (e.g. in [97] or [98]) and are a generic consideration for all models which explain neutrino mixings using a family symmetry broken at high energies. Furthermore, in the A_4 models considered here, such effects are expected to be small.

The rest of the Chapter is organised as follows. Section 6.1 briefly outlines the process of calculating the baryon asymmetry of the universe η arising from leptogenesis. Then in Section 6.2, the idea of FD is recalled and it is shown that the CP violating parameter in leptogenesis is indeed zero under the condition of FD. Section 6.3 presents the relevant parameters of the AF A_4 model of TB neutrino mixing, while Section 6.4 presents the relevant parameters of the A_4 model of TM mixing. In Section 6.5 the RG running of the neutrino Yukawa matrices is analytically estimated in the leading log approximation. Numerical results for the baryon asymmetry of the universe arising from leptogenesis in both TB and TM models are presented in Section 6.6 including contour plots of input parameters reproducing the physical value of η. Section 6.7 concludes the Chapter.

6.1 Leptogenesis

Leptogenesis takes advantage of the heavy RH neutrinos introduced in many models to account for the smallness of the LH neutrino mass. As described in Chapter 2, the addition of these RH neutrino fields N_i introduces two new terms into the superpotential[4]

$$W_\nu = (Y_\nu)_{\alpha i} (l_\alpha \cdot H_u) N_i + \frac{1}{2} N_i (M_R)_{ij} N_j, \tag{6.2}$$

which then lead to an effective light neutrino mass once the heavy degrees of freedom are integrated out. These interactions also fulfil the well known Sakharov conditions [85] required to generate a baryon asymmetry: 1) C and CP violation (coming from the complex Yukawa coupling); 2) B violation (the Majorana mass of Ns violates L; sphalerons convert $\sim \frac{1}{3}$ of this into B violation); 3) Departure from thermal equilibrium (due to out-of-equilibrium decays of the RH neutrinos). The procedure for calculation of this asymmetry is first to calculate the amount of CP violation in the decays of the RH neutrinos. This is then used as an input parameter to find the $B - L$ asymmetry through integration of the Boltzmann equations [99]. These equations take into account the evolution of a $B - L$ asymmetry generated by N decays against the background of N inverse decays partially washing it out. This procedure is not considered in detail in this Chapter since the goal is to generate a non-zero ϵ. Finally, this $B - L$ asymmetry is converted into a B asymmetry using previously calculated results for sphaleron processes [100, 101].

6.1.1 Unflavoured asymmetry

To one-loop order, the CP asymmetry arises from the interference of the diagrams in Fig. 6.1. Using the standard supersymmetric Feynman rules, one can calculate the decay widths for the decay $N_i \to l_\alpha + H_u$, $\Gamma_i = \sum_\alpha \Gamma_{\alpha i}$; these are then used to

[4] Notation has changed slightly here, in line with notation used in leptogenesis studies: the charged lepton flavour index is now an α to distinguish it from the RH neutrino index.

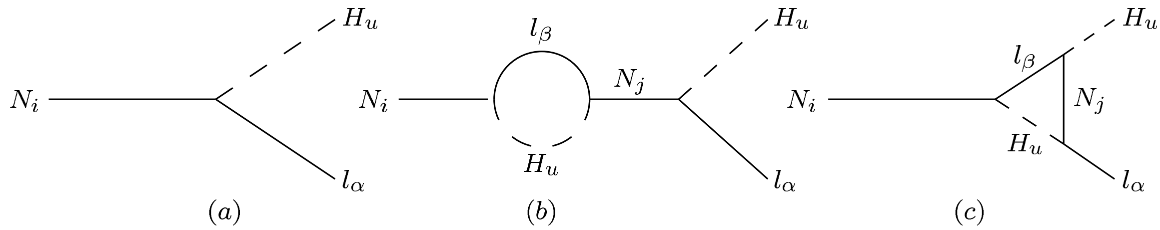

Figure 6.1: Diagrams contributing to the CP violating parameter $\epsilon_{i,\alpha i}$; it is the interference of (a) with (b) and (c) which gives rise to non-zero $\epsilon_{i,\alpha i}$. Lines labelled N can be any one of the seesaw particles.

find the CP asymmetry for N_i by summing over all lepton flavours α [102],

$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma}_i}{\Gamma_i + \overline{\Gamma}_i} = \frac{1}{8\pi \left(Y_\nu^\dagger Y_\nu\right)_{ii}} \sum_{j \neq i} \mathrm{Im}\left(\left(Y_\nu^\dagger Y_\nu\right)^2_{ij}\right) f\left(\frac{M_j^2}{M_i^2}\right). \tag{6.3}$$

Here, M_i are the real mass eigenvalues of M_R, and [88, 90, 103]

$$f(x_{ij}) = f_{ij} = \sqrt{x_{ij}}\left(\frac{2}{1 - x_{ij}} - \ln\left(\frac{1 + x_{ij}}{x_{ij}}\right)\right), \tag{6.4}$$

with $x_{ij} = \frac{M_j^2}{M_i^2}$, is the loop factor. Note that ϵ_i is summed over all flavours of the outgoing lepton and is called the *unflavoured* asymmetry. This formula implicitly assumes that the N_i are not degenerate (since this would lead to an infinite self-energy contribution unless one considers resonance effects); for studies of leptogenesis with nearly degenerate neutrinos, see e.g. [104] or, in the context of Abelian family symmetries, [105].

6.1.2 Flavoured asymmetry

The above discussion and formula for ϵ_i is relevant when the lepton doublets produced are a coherent superposition of the three flavours. This is only the case above a certain energy when the expansion rate of the universe is greater than all charged lepton interaction rates. However, as the universe cools, the τ lepton Yukawa coupling will start to come in to equilibrium at an energy of around [90] $\left(1 + \tan^2\beta\right) \times 10^{12}$ GeV,[5] breaking the coherence of the single state superposition $e + \mu + \tau$ down into two states: the τ and the remaining coherent combination

[5] Here, $\tan\beta$ is the ratio of MSSM Higgs VEVs defined in (2.76).

$e + \mu$. Thus, if the dynamics of leptogenesis occur below this temperature,[6] one should take such differences into account in the calculations. The CP parameter taking into account such flavour effects is [88, 90, 103]

$$\epsilon_{\alpha i} = \frac{1}{8\pi \left(Y_\nu^\dagger Y_\nu\right)_{ii}} \sum_{j \neq 1} \left(\mathrm{Im}\left(Y_{\alpha i}^* Y_{\alpha j} (Y_\nu^\dagger Y_\nu)_{ij}\right) f(x_{ij}) \right. \\ \left. + \mathrm{Im}\left(Y_{\alpha i}^* Y_{\alpha j} (Y_\nu^\dagger Y_\nu)_{ji}\right) g(x_{ij}) \right), \tag{6.5}$$

with $g(x_{ij}) = g_{ij} = \frac{1}{(1-x_{ij})}$ and f_{ij} as above.

6.1.3 Final asymmetry

Ultimately an estimate for the value of the baryon to photon ratio at recombination is desired; this is related to the $B - L$ asymmetry N_{B-L} at the leptogenesis scale by [107]

$$\eta = 0.89 \times 10^{-2} N_{B-L}. \tag{6.6}$$

The numerical coefficient above has two contributions: 1) from the $B - L$ conserving sphaleron processes (which are only $\sim 33\%$ efficient at converting $B - L$ into B); 2) from scaling by photon number density in the relevant comoving volume (recall that the baryon to photon ratio at recombination is calculated). The sphalerons convert part of the L asymmetry into a B asymmetry via a suppressed dimension 18 operator active at the energies considered, $\gg M_{EW}$. The CP asymmetries calculated in the previous Section are then related to N_{B-L} via

$$N_{B-L} = \sum_{\alpha, i} \epsilon_{\alpha i} \kappa_{\alpha i}, \tag{6.7}$$

which defines the efficiency parameters $\kappa_{\alpha i}$; these encode how efficiently the decays of N produce a $B - L$ asymmetry at the leptogenesis scale. In the strong washout regime, the $\kappa_{\alpha i}$ are approximated analytically by (up to superpartner effects which

[6] Strictly speaking the τ interaction rate must be faster than the N inverse decay rate to overcome the Quantum Zeno effect [106], but this is a small effect and beyond the scope of this Thesis.

increase N_{B-L} by a factor of $\sqrt{2}$; see, for example, [107]):

$$\kappa_{\alpha i} \approx \frac{2}{K_{\alpha i} z_B(K_{\alpha i})} \left(1 - \exp\left(-\frac{1}{2} K_{\alpha i} z_B(K_{\alpha i})\right)\right), \tag{6.8}$$

with

$$z_B(K_{\alpha i}) \approx 2 + 4(K_{\alpha i})^{0.13} \exp\left(-\frac{2.5}{K_{\alpha i}}\right), \tag{6.9}$$

the decay parameter

$$K_{\alpha i} = \frac{\widetilde{m}_{\alpha i}}{m^*_{MSSM}}, \tag{6.10}$$

and effective neutrino mass

$$\widetilde{m}_{\alpha i} = \left(Y^\dagger_\nu\right)_{i\alpha} (Y_\nu)_{\alpha i} \frac{v_u^2}{M_i}. \tag{6.11}$$

The $\widetilde{m}_{\alpha i}$ are model specific and are presented below for the model in question (in Table 6.1), while $m^*_{MSSM} = 1.58 \times 10^{-3} \sin^2\beta$ eV [90] is the equilibrium neutrino mass. The main point to address is then the form that the Yukawa matrices take. This is discussed in the context of family symmetries which is the topic of the next Section.

6.2 Form dominance

As studied in previous Chapters, many models invoke the idea that a high energy family symmetry unifying the three flavours is spontaneously broken in a specific way that leaves some imprint in the neutrino sector at low energies. This method introduces relationships between the parameters of Y_ν leading to predictions for $\epsilon_{\alpha i}$ and ϵ_i. It is a striking fact that many of these family symmetry models exhibit FD [87], which constrains the CP violating parameter of leptogenesis to be identically zero [88], as is now discussed. The FD [87] condition is that the columns of Y_ν in Eq. (6.2) are proportional to the columns of U_{PMNS},

$$A_i = \alpha U_{i1}, \quad B_i = \beta U_{i2}, \quad C_i = \gamma U_{i3}, \tag{6.12}$$

where U_{PMNS} is the unitary PMNS matrix. The consequences of such FD on leptogenesis is then very simple to understand: since U_{PMNS} is unitary, the columns of Y_ν must be mutually orthogonal. This means that the contraction $(Y_\nu^\dagger Y_\nu)_{ij}$, with $i \neq j$, appearing in Eqs (6.3) and (6.5) is identically zero and so leptogenesis gives $\eta = 0$. This condition also explains why washout of, for example ϵ_1, due to N_2 and N_3 is not considered: the Dirac matrix describes how to write a RH neutrino as a linear combination of charged leptons. Since FD implies that the columns of the Dirac matrix are orthogonal, it also means that these RH neutrino 'flavour vectors' are orthogonal. Therefore there is no projection of one onto another, meaning that the washout from one will not affect another. FD is only expected to be broken by a small amount in the calculation considered and so any projection of one RH neutrino onto another will be small and is therefore neglected.

The FD condition also greatly simplifies the form of the effective neutrino mass matrix arising from the type I seesaw formula. In terms of parameters in Eq. (6.2), the effective neutrino mass matrix can be written,

$$m_\nu = -v_u^2 Y_\nu M_R^{-1} Y_\nu^T. \tag{6.13}$$

In the basis where the RH neutrinos are diagonal, i.e. that in which $M_R = \mathrm{diag}\,(M_A, M_B, M_C)$, Eq. (6.13) gives

$$m_\nu = -v_u^2 \left(\frac{AA^T}{M_A} + \frac{BB^T}{M_B} + \frac{CC^T}{M_C} \right). \tag{6.14}$$

In the charged lepton diagonal basis, m_ν is diagonalised by $U_{PMNS}^\dagger$. Assuming FD, m_ν is diagonalisable independently of the parameters α, β, γ, and, from (6.14) and (6.12), one finds

$$m_\nu^{diag} = v_u^2 \mathrm{diag} \left(\frac{\alpha^2}{M_A}, \frac{\beta^2}{M_B}, \frac{\gamma^2}{M_C} \right). \tag{6.15}$$

A particularly well studied case is that of TB mixing [54]. However, as emphasised in [83], TB mixing is not linked to FD. Indeed this Chapter considers two A_4 family symmetry models, one with TB mixing and one with TM mixing, where FD is present in both cases, leading to zero leptogenesis at LO, before RG corrections are included.

A useful parameterisation when considering leptogenesis in light of low energy data is due to Casas and Ibarra [89]. It is constructed as follows: in the charged lepton diagonal basis, denote U as both the PMNS matrix and the matrix which diagonalises the effective light neutrino mass matrix

$$U^{\dagger} m_{\nu} U^{*} = D_k. \tag{6.16}$$

One may also define a matrix which diagonalises the heavy right handed neutrino mass matrix

$$U_M^{\dagger} M_R U_M^{*} = D_M. \tag{6.17}$$

These objects can be used to construct a complex orthogonal matrix R [89]

$$R = v_u D^{-1}_{\sqrt{M}} U_M^{\dagger} Y_{\nu}^{T} U^{*} D^{-1}_{\sqrt{k}}, \tag{6.18}$$

which is basis invariant [88]. For fixed choices of U, D_k and D_M, the so called R-matrix parameterises the freedom in Y_{ν}. Using this parameterisation, it is possible to rewrite the unflavoured and flavoured CP asymmetries as follows

$$\epsilon_i = -\frac{3M_i}{16\pi v^2} \frac{\mathrm{Im}\left[\sum_{j\neq i} m_j^2 \left(R_{ij}^{*}\right)^2\right]}{\sum_{j\neq i} m_j \left|R_{ij}\right|^2}, \tag{6.19}$$

$$\epsilon_{\alpha i} = -\frac{3M_i}{16\pi v^2} \frac{\mathrm{Im}\left[\sum_{j\neq i}\sum_k m_j^{\frac{1}{2}} m_k^{\frac{3}{2}} U_{\alpha j}^{*} U_{\alpha k} R_{ij}^{*} R_{ik}^{*}\right]}{\sum_{j\neq i} m_j \left|R_{ij}\right|^2}. \tag{6.20}$$

In order to relate the R-matrix to FD, one can write

$$v_u Y_{\nu} = UD = U D_{\sqrt{k}} R_d D_{\sqrt{M}} \tag{6.21}$$

where D is some real diagonal matrix and has been factored into two matrices defined above and some remainder, all of which are real and diagonal; furthermore the remainder has entries of ± 1. Rearranging for this remainder, one finds

$$R_d = v_u D^{-1}_{\sqrt{M}} Y_\nu^T U^* D^{-1}_{\sqrt{k}} \tag{6.22}$$

which looks remeniscent of Eq. (6.18). In fact, using the basis invariance of the R matrix implies that (6.22) is (6.18) in the diagonal right handed neutrino basis. Therefore FD $\Rightarrow R$ is a real, diagonal matrix with entries of ± 1; thus it can be seen that both the flavoured and unflavoured asymmetries are 0 in FD regimes, and so again it can be seen that leptogenesis appears to be unsuccessful in models which exhibit FD. It is important to note that a merely real R-matrix is not sufficient to fulfill this statement: this will lead to $\epsilon_i = 0$ but not neccesarily $\epsilon_{\alpha i} = 0$ since the matrix U may still have complex entries.

This discussion shows that in order to generate non-zero values of the CP violating parameter, one will need to generate a shift from FD; since this is imposed on the Yukawa matrices at the family symmetry breaking scale, the rest of this Chapter is presented using Eqs (6.3) and (6.5).

6.3 Parameters of the A_4 model of TB mixing

Here the relevant parameters of the AF A_4 model of TB mixing [20] are briefly given; the superpotential is

$$W_\nu = y(lN)H_u + (x_A\xi + \widetilde{x}_A\widetilde{\xi})(NN) + x_B(\varphi_S NN), \tag{6.23}$$

where x_i are constant complex parameters. The charged lepton mass matrix in the basis used in [20] is diagonal so the mixing structure in the neutrino sector will not receive corrections from charged lepton rotations. The TB structure in the neutrino sector arises from the flavon fields obtaining VEVs in particular directions,

$$\langle \varphi_S \rangle = v_s \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \ \langle \xi \rangle = u \quad \text{and} \quad \left\langle \tilde{\xi} \right\rangle = 0, \tag{6.24}$$

where the dynamics responsible for vacuum alignment has been extensively studied (for instance, in [20] for F-term alignment or in [71] for D-term alignment) and briefly discussed in Chapter 3.

The TB structure arises in the Majorana sector of Eq. (6.23), explicitly

$$M_R = \begin{pmatrix} A + \frac{2B}{3} & -\frac{B}{3} & -\frac{B}{3} \\ -\frac{B}{3} & \frac{2B}{3} & A - \frac{B}{3} \\ -\frac{B}{3} & A - \frac{B}{3} & \frac{2B}{3} \end{pmatrix}, \tag{6.25}$$

with $A = 2x_A u$, $B = 2x_B v_s$ being complex parameters with phase $\phi_{a,b}$. For the purposes of leptogenesis it is convenient to rotate the N such that their mass matrix is diagonal. The resulting neutrino Yukawa matrix in the diagonal N basis is then,

$$Y_{TB} = y \begin{pmatrix} \frac{-2}{\sqrt{6}} e^{i\phi_A} & \frac{1}{\sqrt{3}} e^{i\phi_B} & 0 \\ \frac{1}{\sqrt{6}} e^{i\phi_A} & \frac{1}{\sqrt{3}} e^{i\phi_B} & \frac{-1}{\sqrt{2}} e^{i\phi_C} \\ \frac{1}{\sqrt{6}} e^{i\phi_A} & \frac{1}{\sqrt{3}} e^{i\phi_B} & \frac{1}{\sqrt{2}} e^{i\phi_C} \end{pmatrix}. \tag{6.26}$$

One can see explicitly that FD is present in this model, since the columns of Y_{TB} are manifestly proportional to the columns of the TB mixing matrix, and thus it immediately follows that $\epsilon_i = \epsilon_{\alpha i} = 0$ at the scale of A_4 breaking. The phases defined in (6.26) are given as,

$$\phi_A = -\frac{1}{2} \left(\phi_b + \tan^{-1} \left(\frac{-|A| \sin(\phi_b - \phi_a)}{|B| + |A| \cos(\phi_b - \phi_a)} \right) \right), \tag{6.27}$$

$$\phi_B = -\frac{1}{2} \phi_a, \tag{6.28}$$

$$\phi_C = -\frac{1}{2} \left(\phi_b + \tan^{-1} \left(\frac{|A| \sin(\phi_b - \phi_a)}{|B| - |A| \cos(\phi_b - \phi_a)} \right) \right). \tag{6.29}$$

Therefore, there are actually only two phases (ϕ_a and ϕ_b) and two magnitudes ($|A|$

and $|B|$) in the model, although only phase differences appear when considering physical quantities. This means that one phase may be set to zero without loss of generality; here $\phi_a = 0$ is chosen.

In this basis, the Majorana neutrino mass matrix is real and diagonal and is given by

$$M_R^{diag} = \text{diag}\left(M_1, M_2, M_3\right) = \begin{pmatrix} |A+B| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |-A+B| \end{pmatrix}. \tag{6.30}$$

The effective LH neutrino masses are then given by[7]

$$m_i = \frac{y_\beta^2 v^2}{M_i}, \tag{6.31}$$

which incorporates the SUSY parameter $\tan\beta$ introduced in Chapter 2; this can be absorbed into the coupling as

$$y_\beta = y \sin\beta. \tag{6.32}$$

6.4 Parameters of the A_4 model of TM mixing

As discussed in Chapters 3 and 5, models predicting TB mixing are now ruled out. Instead, schemes such as TM mixing remain viable [59]:

$$U_{TM} = \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\theta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\theta e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta e^{-i\rho} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{6}}\sin\theta e^{i\rho} \\ -\frac{1}{\sqrt{6}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta e^{-i\rho} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{6}}\sin\theta e^{i\rho} \end{pmatrix}. \tag{6.33}$$

Here $\frac{2}{\sqrt{6}}\sin\theta = \sin\theta_{13}$ and ρ is related to the Dirac phase. It is possible to minimally extend the AF model above by adding a flavon in the $\mathbf{1'}$ representation

[7]Note that this Chapter considers a normal ordering of light neutrino masses, therefore M_1 is the heaviest RH neutrino mass. This means that ϵ_3 and $\epsilon_{\alpha 3}$ will be dominant contributions to leptogenesis, coming from the lightest RH neutrino. This is simply a notational consideration, and does not affect the physics.

of A_4 which reproduces this pattern [60]:[8]

$$W_{\mathbf{1}'} = x_C \xi' NN, \tag{6.34}$$

with the complex parameter $C = x_C \langle \xi' \rangle$, with phase ϕ_c. It has been shown in [60] that the addition of this flavon doesn't affect the RH neutrino masses to first order, and so the parameters in common with the previous Section will be unaffected. Analogously to Eq. (6.26), in the basis where charged leptons are diagonal and RH neutrinos are real and diagonal, the Yukawa matrix for TM mixing is,

$$Y_{TM} = y \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \\ -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha_{13} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}\alpha_{13}^* \end{pmatrix} \begin{pmatrix} \exp(i\phi_A) & 0 & 0 \\ 0 & \exp(i\phi_B) & 0 \\ 0 & 0 & \exp(i\phi_C) \end{pmatrix}, \tag{6.35}$$

where the $\phi_{A,B,C}$ are as in Eq. (6.27). The columns of this matrix are proportional to columns of U_{TM} and therefore the model respects FD. Therefore, as for the previous model of TB mixing, this model of TM mixing also gives zero leptogenesis and $\eta = 0$, to leading order. The parameter α_{13} measures the deviation from TB mixing and is given by [60]

$$\alpha_{13} = \frac{\sqrt{3}}{2} \left(\mathrm{Re}\frac{C}{2\left(A - \frac{C}{2}\right)} + \mathrm{Im}\frac{C}{2\left(A - \frac{C}{2}\right)} \frac{\mathrm{Im}\frac{B}{A-\frac{C}{2}}}{\mathrm{Re}\frac{B}{A-\frac{C}{2}}} - i\frac{\mathrm{Im}\frac{C}{2\left(A-\frac{C}{2}\right)}}{\mathrm{Re}\frac{B}{A-\frac{C}{2}}} \right). \tag{6.36}$$

Note that this is the same as Eq. (3.53) but with $\gamma'' = 0$.

6.5 Renormalisation group evolution of the Yukawa couplings

In order to generate a non-zero $\epsilon_{\alpha i}$ and ϵ_i, the effects of running the neutrino Yukawa couplings from the scale at which A_4 is broken down to the scale at which leptogenesis takes place are now considered. At one-loop, the RG equation for the neutrino Yukawa couplings in the MSSM above the scale of RH neutrino masses is

[8]For simplicity, only the ξ' flavon is introduced in contrast to Chapters 3 and 5

given by [97, 108],[9]

$$\frac{\mathrm{d}Y_\nu}{\mathrm{d}t} = \frac{1}{16\pi^2}\left[N_l \cdot Y_\nu + Y_\nu \cdot N_\nu + (N_{H_u})\, Y_\nu\right], \tag{6.37}$$

where

$$N_l = Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right) \cdot I_3, \tag{6.38}$$

$$N_\nu = 2Y_\nu^\dagger Y_\nu, \tag{6.39}$$

$$N_{H_u} = 3\mathrm{Tr}\left(Y_u^\dagger Y_u\right) + \mathrm{Tr}\left(Y_\nu^\dagger Y_\nu\right) - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right). \tag{6.40}$$

In these equations, $t = \log\left(\frac{Q_1}{Q_0}\right)$ with Q_1 being the renormalisation scale and Q_0 the family symmetry breaking scale; $Y_{e,u}$ are the charged lepton and up-type quark Yukawa couplings respectively; $g_{1,2}$ are the[10] $U(1)_Y$ and $SU(2)_L$ gauge couplings respectively; and I_3 is the 3×3 identity matrix. Each N_X arises from all one-loop insertions allowed by gauge symmetry on the X-leg of the vertex.

In leading log approximation, taking the continuous derivatives to be approximately equal to a single discrete step, Eq. (6.37) may be approximated as:

$$\frac{\mathrm{d}Y_\nu}{\mathrm{d}t} \approx \frac{\Delta Y_\nu}{\Delta t} = \frac{Y_\nu(Q_0) - Y_\nu(Q_1)}{t(Q_0) - t(Q_1)} \equiv Z, \tag{6.41}$$

yielding the solution,

$$Y_\nu(Q_1) \approx Y_\nu(Q_0) - Z\Delta t. \tag{6.42}$$

As an example, the RG evolution of the TB Yukawa matrix in (6.26) $Y_\nu = Y_{TB}$ is presented (the case of Y_{TM} is completely analogous). Inserting (6.26) into (6.37)

[9] Note that, as has been pointed out before, running from the leptogenesis scale down to M_{EW} is not considered; this which would be necessary if one wanted to consider leptogenesis effects on neutrino mass bounds as studied in [98, 109]. Also in [98] the importance of RG corrections in calculating leptogenesis predictions in the framework of a generic GUT scale theory was emphasised, although specific models were not considered.

[10] Note that g_1 is the GUT normalised hypercharge coupling, related to the standard hypercharge coupling g' by $g_1 = \sqrt{\frac{5}{3}}g'$.

and using the third family approximation then gives

$$\frac{\mathrm{d}Y_{TB}}{\mathrm{d}t} \approx \frac{y}{16\pi^2}\left(\left(J+3\left|y\right|^2\right)\begin{pmatrix} \frac{-2}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & 0 \\ \frac{1}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & \frac{-1}{\sqrt{2}}e^{i\phi_C} \\ \frac{1}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & \frac{1}{\sqrt{2}}e^{i\phi_C} \end{pmatrix} \right. \\ \left. +y_\tau^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{\sqrt{6}}e^{i\phi_A} & \frac{1}{\sqrt{3}}e^{i\phi_B} & \frac{1}{\sqrt{2}}e^{i\phi_C} \end{pmatrix}\right) \equiv Z_{TB}, \tag{6.43}$$

where $J = N_{H_u} - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right)$ and y_τ is the Yukawa coupling of the τ lepton. This shows that the contributions from the charged lepton Yukawa couplings breaks the orthogonality of the columns, appearing as they do in only the third component of each column. In SUSY models y_τ can be related to $\tan\beta$ using $v_d y_\tau = m_\tau$ so a scan over y_τ will correspond to a scan over $\tan\beta$. This is the effect which gives rise to a non-zero CP violating parameter. The leading log solution for the TB case is then given by

$$Y_{TB}(Q_1) \approx Y_{TB}(Q_0) - Z_{TB}\Delta t. \tag{6.44}$$

One must also consider how the charged lepton Yukawa coupling runs; the relevant RGE is [97, 108]

$$\frac{\mathrm{d}Y_e}{\mathrm{d}t} = \frac{1}{16\pi^2}\left[N_l \cdot Y_e + Y_e \cdot N_e + (N_{H_d})\, Y_e\right], \tag{6.45}$$

with N_l as before and

$$N_e = 2Y_e^\dagger Y_e - \frac{6}{5}g_1^2 \cdot I_3, \tag{6.46}$$

$$N_{H_d} = 3\mathrm{Tr}\left(Y_d^\dagger Y_d\right) + \mathrm{Tr}\left(Y_e^\dagger Y_e\right) - \left(\frac{3}{2}g_2^2 + \frac{3}{10}g_1^2\right). \tag{6.47}$$

Here Y_d is the down quark Yukawa coupling matrix. Since Y_ν is unitary for both models, specifically see Eqs. (6.26) and (6.35), there will be no off-diagonal entries in Eq. (6.45). Using again the third family and leading log approximations gives small corrections to y_τ dependent upon y, y_b the bottom quark Yukawa and y_τ at the GUT scale; taking values of $y = 2\sqrt{\pi}$, $y_b = 1$ and $y_\tau = 0.5$ gives a correction of $\sim 10\%$ to the value of y_τ and therefore this effect is neglected (notice that the

	Asymmetry	$\widetilde{m}_{\alpha i}$
$\epsilon_{\alpha 1}$	$\frac{1}{8\pi A^\dagger A}\left[\mathrm{Im}\left(A^*_\alpha B_\alpha\left(A^\dagger B\right)\right) f_{12} + \mathrm{Im}\left(A^*_\alpha B_\alpha\left(B^\dagger A\right)\right) g_{12} + \mathrm{Im}\left(A^*_\alpha C_\alpha\left(A^\dagger C\right)\right) f_{13} + \mathrm{Im}\left(A^*_\alpha C_\alpha\left(C^\dagger A\right)\right) g_{13}\right]$	$\frac{\lvert A_\alpha\rvert^2}{M_1} v_u^2$
$\epsilon_{\alpha 2}$	$\frac{1}{8\pi B^\dagger B}\left[\mathrm{Im}\left(B^*_\alpha A_\alpha\left(B^\dagger A\right)\right) f_{21} + \mathrm{Im}\left(B^*_\alpha A_\alpha\left(A^\dagger B\right)\right) g_{21} + \mathrm{Im}\left(B^*_\alpha C_\alpha\left(B^\dagger C\right)\right) f_{23} + \mathrm{Im}\left(B^*_\alpha C_\alpha\left(C^\dagger B\right)\right) g_{23}\right]$	$\frac{\lvert B_\alpha\rvert^2}{M_2} v_u^2$
$\epsilon_{\alpha 3}$	$\frac{1}{8\pi C^\dagger C}\left[\mathrm{Im}\left(C^*_\alpha A_\alpha\left(C^\dagger A\right)\right) f_{31} + \mathrm{Im}\left(C^*_\alpha A_\alpha\left(A^\dagger C\right)\right) g_{31} + \mathrm{Im}\left(C^*_\alpha B_\alpha\left(C^\dagger B\right)\right) f_{32} + \mathrm{Im}\left(C^*_\alpha B_\alpha\left(B^\dagger C\right)\right) g_{32}\right]$	$\frac{\lvert C_\alpha\rvert^2}{M_3} v_u^2$

Table 6.1: Flavoured asymmetries and washout parameters

chosen values of y and y_τ are at the extreme end of the ranges that are scanned over and so this correction is the largest expected).

6.6 Results

This Section details the results of analyses for both the TB and TM models in leading log approximation. The use of leading log approximation is justified by the small interval of energies over which the running takes place. Since this approximation is used and since the neutrino hierarchy is not very strong (using work in [92] to fix neutrino parameters), threshold effects from successive decoupling of the right handed neutrinos are not considered. Furthermore, the RH neutrino mass matrix will also run in a full calculation and this will in general give rise to off-diagonal entries; thus at each successive stage in decoupling, one should rediagonalise this matrix before proceeding. The prescription for dealing with this is to replace the Yukawa matrix elements $Y_{\alpha i} \to Y'_{\alpha i}\theta_p\left(\ln\mu - \ln M_i\right)$ [97] where the prime denotes the effect of rediagonalisation. This means that after crossing a threshold one of the columns of the Yukawa matrix will be frozen out of the process, remaining in its corrected form. The two remaining columns will be corrected further until the next threshold and so on. The resulting Yukawa matrix is then expected to be further from FD than in the current approximation. This consideration also means that if the heaviest RH neutrino were heavier than the flavour symmetry breaking scale, the 3×2 Yukawa matrix would still run and FD would still be broken. For a detailed analysis of such effects one can consult [97] or [98]. As before, one can represent the Yukawa matrix derived in (6.42) as

$Y_\nu(Q_1) = (A(Q_1), B(Q_1), C(Q_1))$ where $A(Q_1)$, $B(Q_1)$ and $C(Q_1)$ are the RG evolved versions of the column vectors in Section 6.2, which, as clearly seen in (6.43), (6.44) are no longer orthogonal after RG corrections are included. This allows the flavoured asymmetries to be written as in Table 6.1. Using Eq. (6.5) one notices immediately that $\epsilon_{13} = 0$ since $C_1(Q_1) = 0$. To see that the $\epsilon_{\alpha i}$ receive a correction from RG running consider, e.g.

$$A(Q_1)^\dagger B(Q_1) = \left(A(Q_0) - (Z\Delta t)_{\alpha 1}\right)^\dagger \left(B(Q_0) - (Z\Delta t)_{\alpha 2}\right), \tag{6.48}$$

where the leading term on the right-hand side vanishes since FD implies that $A(Q_0)$ and $B(Q_0)$ (and $C(Q_0)$) are orthogonal.

In order to progress further, one will need to insert specific values for the parameters in the matrix, which are model dependent. Here, guided by work presented in [92] the parameters are fixed consistently with experimental data. The leptogenesis scale Q_1 is taken to be approximately the seesaw scale, $Q_1 \sim (1.74^2 y^2) 10^{14}$ GeV (using the basic seesaw formula).[11] This indicates that for small y the two flavour regime is relevant for $\tan\beta > 10$; for larger values of y, $\tan\beta$ needs to be larger to be in the 2 flavour regime. However in the forthcoming plots, parts of the contour existing at large y correspond also to larger y_τ and so sufficiently large $\tan\beta$. The family symmetry scale is around an order of magnitude below the GUT scale, roughly $Q_0 \sim 1.5 \times 10^{15}$ GeV; and $y_t \sim 1$. The asymmetry is calculated for $0 < y < 2\sqrt{\pi}$ (to keep the coupling perturbative) and $0 < y_\tau < 0.5$ (to remain within bounds for $\tan\beta$ [113]).[12]

In the course of this calculation, several complicated inter-dependencies of parameters have been suppressed. The most obvious of these is that, as mentioned above, the two-flavour regime is only valid for a subsection of the ranges scanned over. The two-flavour regime is used for simplicity over the whole range, even areas

[11] This mass scale may look quite large especially when compared to the upper bound on the reheating temperature due to the over-production of late-decaying gravitinos [110]. However, heavy gravitinos with masses $m_{3/2} > 40$ TeV, will decay before nucleosynthesis. Assuming dark matter to have a significant axion/axino component, then allows reheat temperatures to be sufficiently high to produce RH neutrinos of mass $\sim 10^{14}$ GeV, as recently discussed in e.g. [111], [112] (and references therein).

[12] Note that the point where the couplings are 0 is unphysical and no significance should be inferred by this.

where the value of y_τ and hence $\tan\beta$ are such that the unflavoured regime is in fact preferred. So when considering plots in the y-y_τ plane, one should in fact be considering the portion where y and y_τ correspond to values of Q_1 and $\tan\beta$ consistent with the two-flavour regime. This is still a significant region of the plot and contains parts of the η contour required by observation. A second dependency comes from the fact that [92] only fixes RH neutrinos masses up to factors of $\frac{1}{y_\beta^2}$ (namely $M_1 \sim \frac{5\times10^{15}}{y_\beta^2}\ GeV$, $M_2 \sim \frac{3\times10^{15}}{y_\beta^2}\ GeV$ and $M_3 \sim \frac{6\times10^{14}}{y_\beta^2}\ GeV$), so that in some portions of the parameter space considered, the RH neutrino mass may well be above the family symmetry breaking scale, a possibility already mentioned above. The corresponding family symmetry breaking scale could be increased to account for this, since this value is not fixed by anything. These approximations are made in order to demonstrate that the studied effect is enough to generate a non-zero baryon asymmetry, and they should be dealt with more thoroughly if on wanted to perform a precise calculation.

6.6.1 TB mixing

Specialising to the case of RG improved leptogenesis in the TB model, where the TB Yukawa couplings are given in (6.44), repeated below,

$$Y_{TB}(Q_1) \approx Y_{TB}(Q_0) - Z_{TB}\Delta t. \tag{6.49}$$

The results for the flavoured asymmetries versus y and y_τ are presented in Fig. 6.2, in the two-flavour regime. It can be seen that the contributions from $\epsilon_{\alpha 3}$ are the dominant ones, as expected.

Following the procedure set out in Section 6.1.3, the next step is to calculate the baryon to photon ratio η. Fig. 6.3 displays the contour matching the experimentally measured value of 6.2×10^{-10}, along with two others, demonstrating the sensitivity of the required Yukawa couplings to the value of η. This shows that there is a definite range of Yukawa couplings for which a realistic matter-antimatter asymmetry can be obtained purely by considering RG evolution of the neutrino Yukawa matrix, without the need for any extra particles or HO operators to be

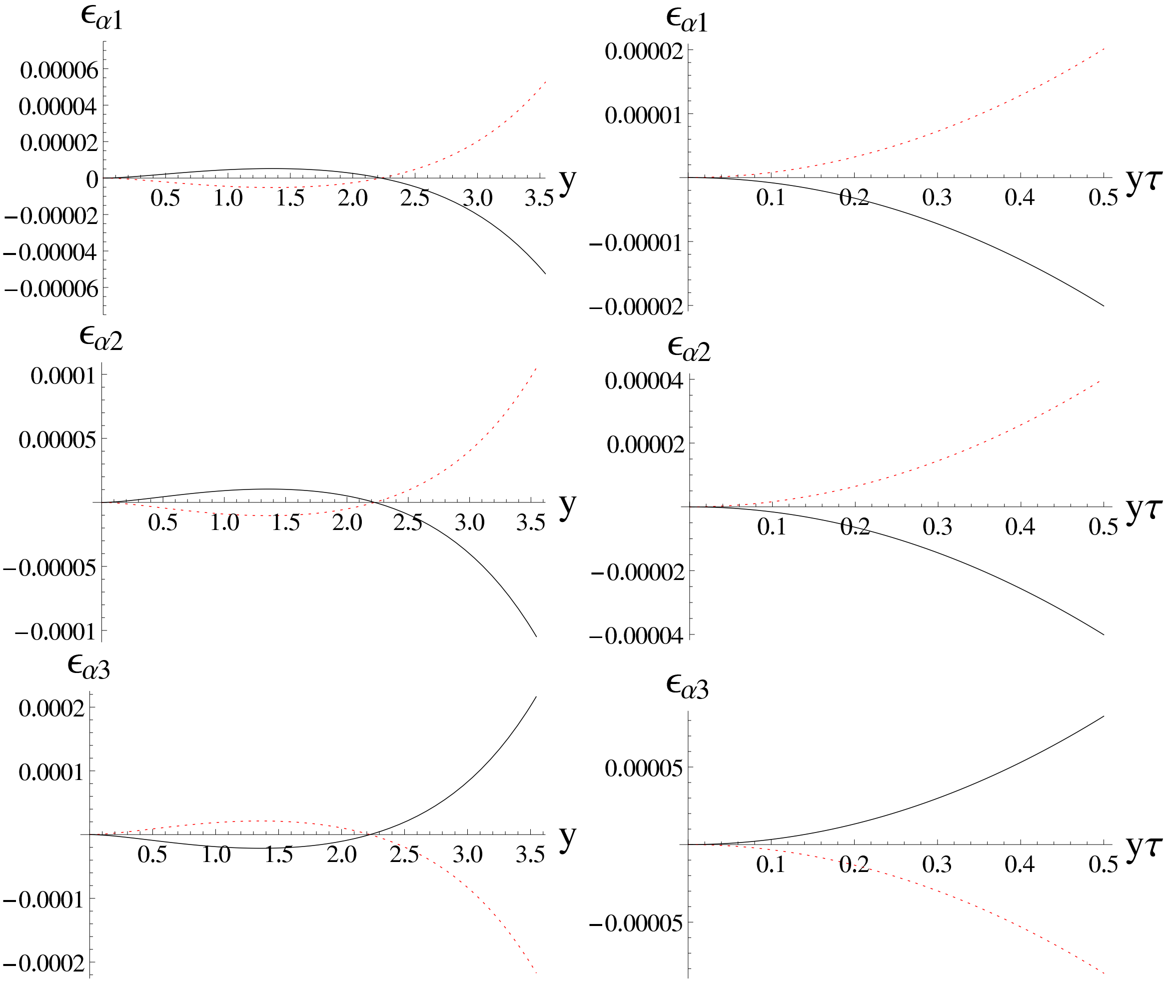

Figure 6.2: Flavoured asymmetries plotted against neutrino Yukawa y and tau lepton Yukawa y_τ in the two flavour regime (i.e. $e-\mu$ and τ) for the TB model. In the y_τ graphs, y is fixed to be 3, while in the y graphs, y_τ is fixed to be 0.5. $\epsilon_{e\mu,i}$ are black solid lines while $\epsilon_{\tau,i}$ are red dashed lines.

considered.

6.6.2 TM mixing

A similar analysis is now performed on the TM model, using the RG improved Yukawa matrix analogous to (6.44), namely,

$$Y_{TM}(Q_1) \approx Y_{TM}(Q_0) - Z_{TM}\Delta t. \tag{6.50}$$

where the high energy Yukawa matrix $Y_{TM}(Q_0)$ is given in (6.35), with Z_{TM} analogous to (6.43) and otherwise assuming similar parameters to the case of TB mixing. However one must choose the new complex parameter C carefully in order to satisfy the relation [60]

$$\frac{\sqrt{6}}{2}\sin\theta_{13} = |\alpha_{13}|\,. \tag{6.51}$$

Flavoured asymmetries are given for $\theta_{13} = 8^\circ$ (consistent with current measurements, see Section 3) and $\phi_c = 0$ in Fig. 6.4. Contours of $\eta = 4.2\times10^{-10},\ 6.2\times10^{-10},\ 8.2\times10^{-10}$ for $\theta_{13} = 8^\circ$ and $\eta = 6.2\times10^{-10}$ for $\theta_{13} = 0.1^\circ,\ 3^\circ,\ 6^\circ,\ 9^\circ,\ 12^\circ$ are also presented; and for each value of θ_{13}, four different choices of phase and modulus of C which satisfy (6.51) are used. These can be seen in Figs 6.5 and 6.6. For small θ_{13} and therefore small C, the results are very similar to those for TB mixing (c.f. Figs 6.3 and 6.6 purple line), which is expected since the only difference between the two models is the presence of the ξ' flavon. For the larger values of θ_{13}, it is clear that changing C has a significant effect as one can see from the variation of contours in Fig. 6.6; for instance the 12° contour for a phase of $\phi_c = 0.91$ rad doesn't show up across the whole displayed plane.

Because experimental (i.e. low energy) input is used here, running between the leptogenesis scale and the EW scale should briefly be mentioned. As with all models of neutrino mixing the obtained high-energy (in this case leptogenesis scale) parameters have to be RG evolved down to the EW scale before being compared with data. However these effects have been well studied, [97, 98] and shown to be possible to control with respect to fitting the data. This discussion also applies for the TB case presented in the previous subsection. Finally, one should not let these considerations detract from the main goal of this Chapter which is to obtain a non-zero value for ϵ in the presence of FD.

6.7 Conclusion

This Chapter investigates RG corrections relevant for leptogenesis in the case of family symmetry models such as the AF A_4 model of TB lepton mixing or its extension to TM mixing. Such corrections are particularly relevant since in large classes of family symmetry models, to LO, the CP violating parameters of leptogenesis would be identically zero at the family symmetry breaking scale, due to the FD property. The third family approximation is used, keeping only the largest Yukawa couplings, subject to the constraint of perturbativity. In addition, the τ Yukawa coupling is related to the SUSY parameter $\tan\beta$, which has had experimental bounds placed upon it.

The results demonstrate that it is possible to obtain the observed value for the baryon asymmetry of the Universe in models with FD by exploiting RG running of the neutrino Yukawa matrix over the small energy interval between the family symmetry breaking scale and the RH neutrino mass scale $\sim 10^{14}$ GeV. Of course, the importance of RG corrections applies more generally than to the particular models considered here for illustrative purposes, and the RH neutrino masses may be lower in some models.

In conclusion, the results in this Chapter show that RG corrections have a large impact on leptogenesis in any family symmetry models involving neutrino and charged lepton Yukawa couplings of order unity, even though the range of RG running between the flavour scale and the leptogenesis scale may be only one or two orders of magnitude in energy. Therefore, when considering leptogenesis in such models, RG corrections should not be ignored, even when corrections arising from HO operators are also present.

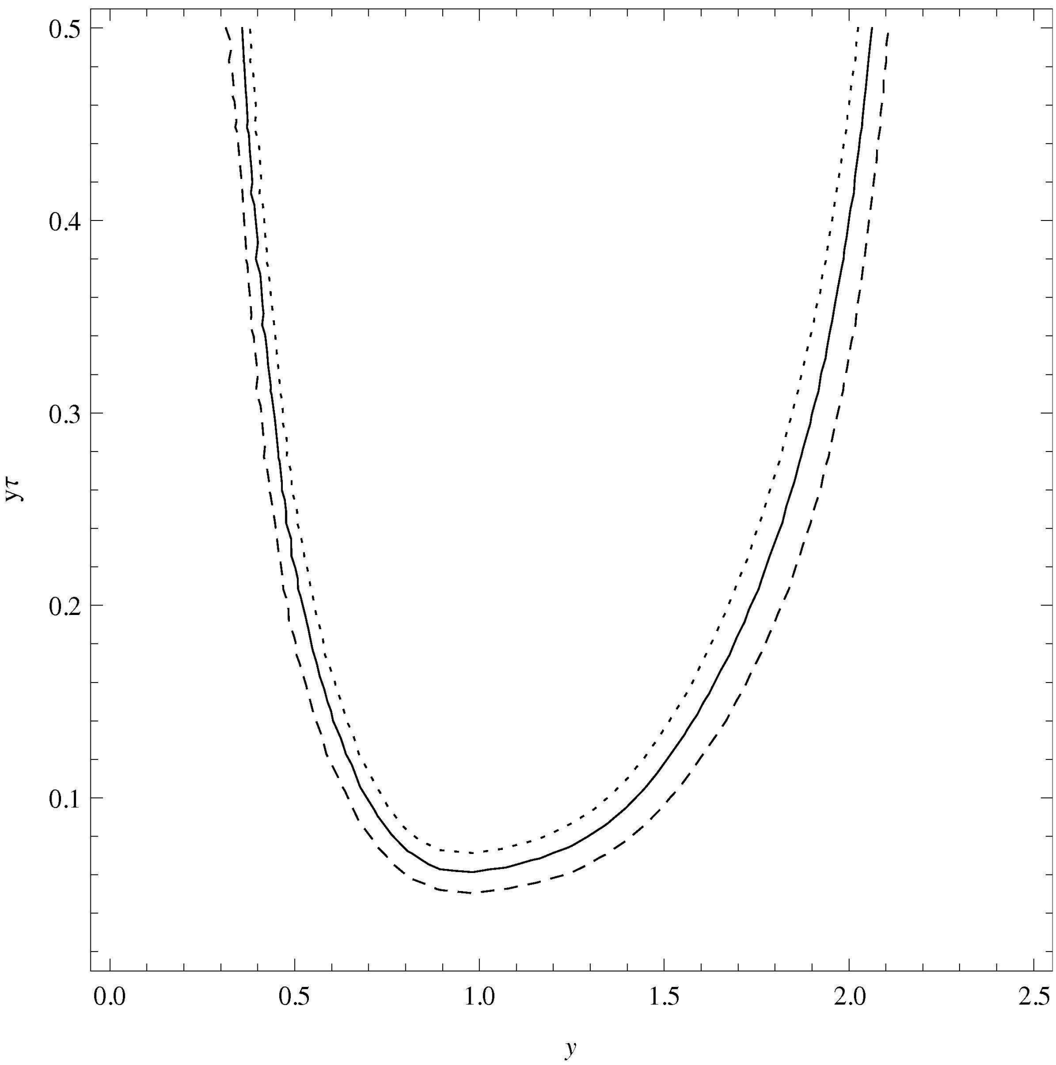

Figure 6.3: A plot showing the contours of the baryon to photon ratio η in the tau Yukawa, y_τ, versus neutrino Yukawa, y, plane. The dotted and dashed lines are $\eta = 8.2 \times 10^{-10}$ and $\eta = 4.2 \times 10^{-10}$ while the solid line is the measured value of $\eta = 6.2 \times 10^{-10}$.

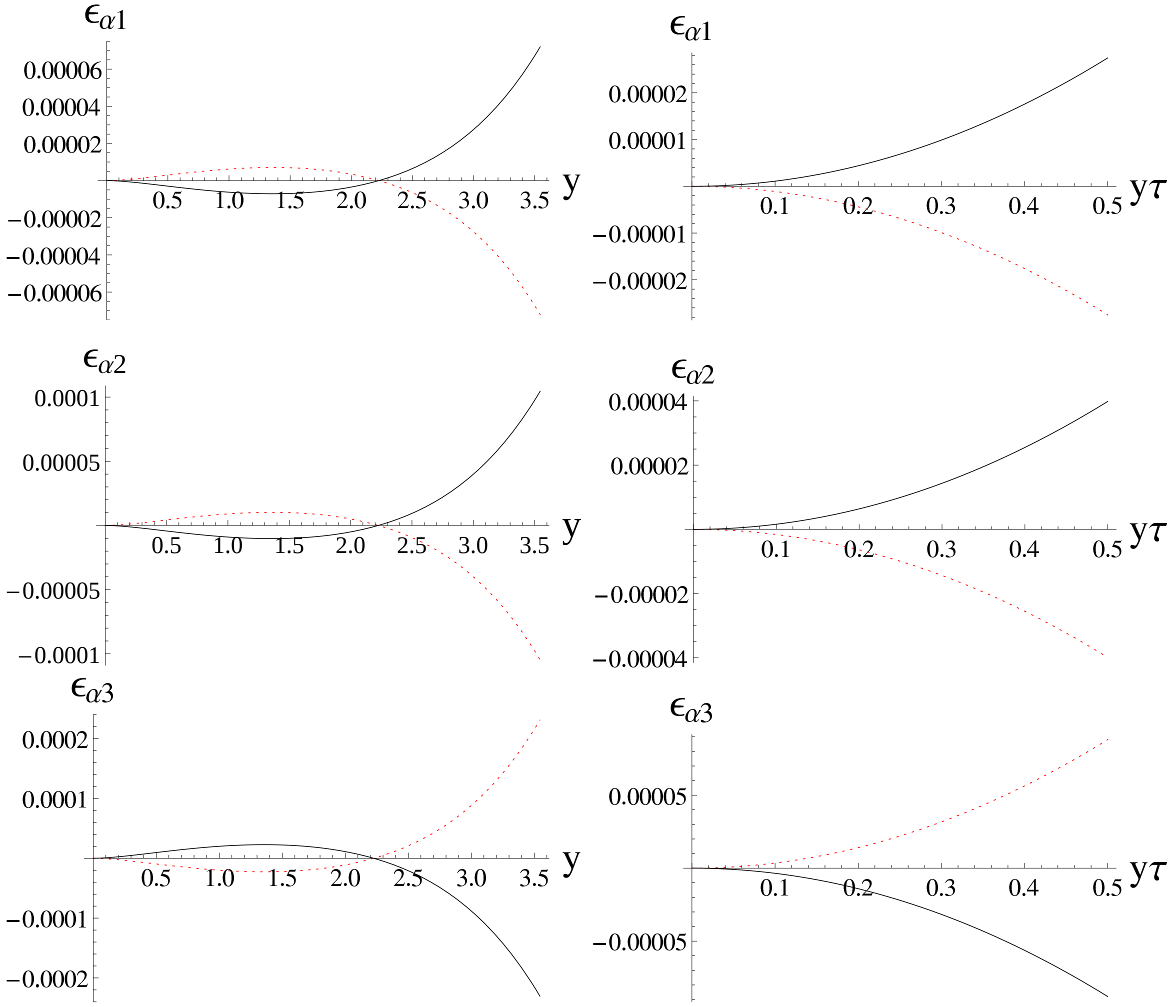

Figure 6.4: Flavoured asymmetries plotted against neutrino Yukawa y and tau lepton Yukawa y_τ in the two flavour regime (i.e. $e-\mu$ and τ) for the TM model with $\theta_{13} = 8^\circ$ and a real parameter $C = x_C\langle\xi'\rangle$. In the y_τ graphs, y is fixed to be 3, while in the y graphs, y_τ is fixed to be 0.5. $\epsilon_{e\mu,i}$ are black solid lines while $\epsilon_{\tau,i}$ are red dashed lines.

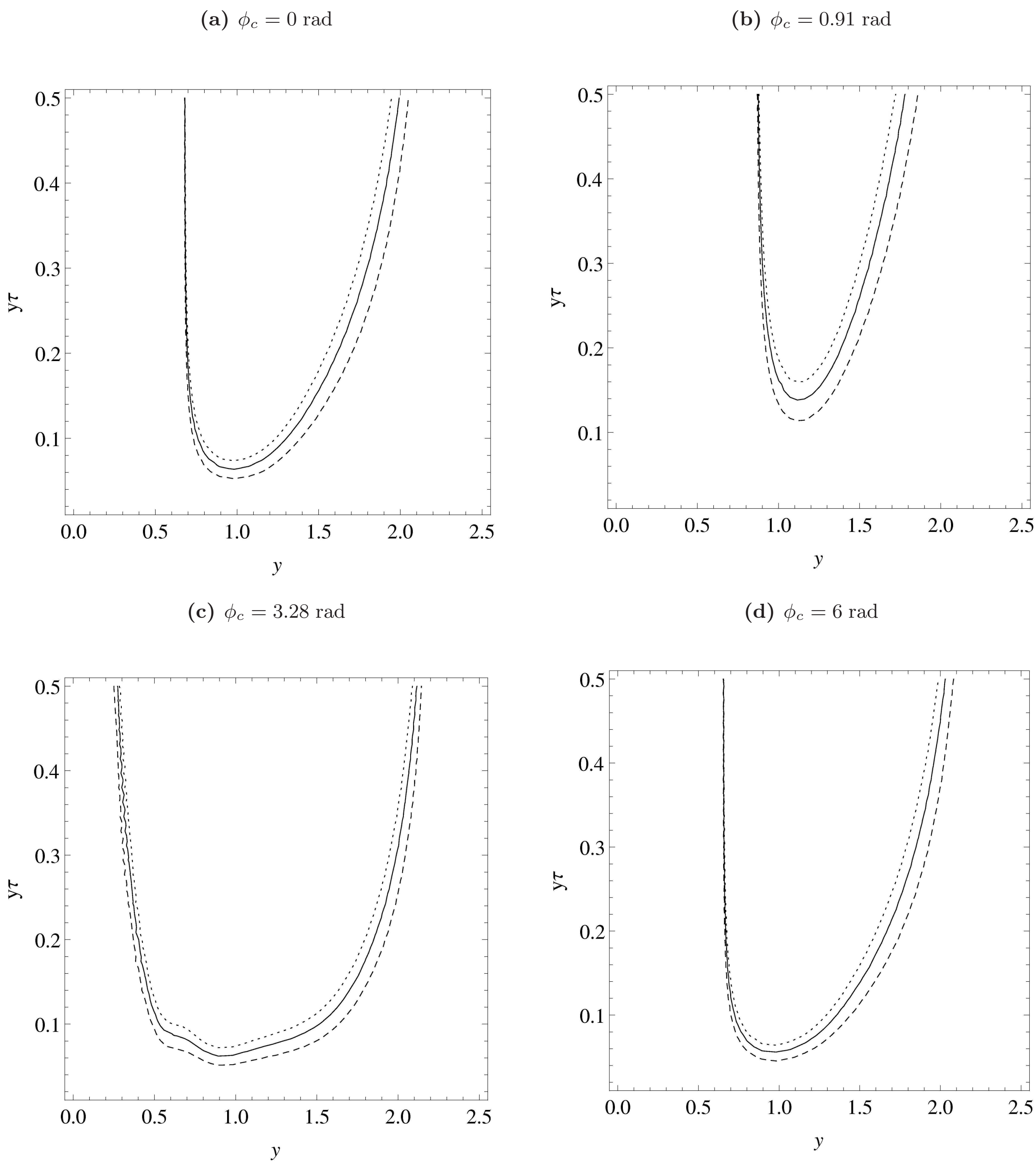

Figure 6.5: These plots show contours of the baryon to photon ratio η from the TM model with $\theta_{13} = 8°$ in the tau Yukawa, y_τ, versus neutrino Yukawa, y, plane. The dotted and dashed lines are $\eta = 8.2 \times 10^{-10}$ and $\eta = 4.2 \times 10^{-10}$ while the solid line is the measured value of $\eta = 6.2 \times 10^{-10}$. Each plot is for a different value of the phase of $C = x_C \langle \xi' \rangle$, given above the relevant panel.

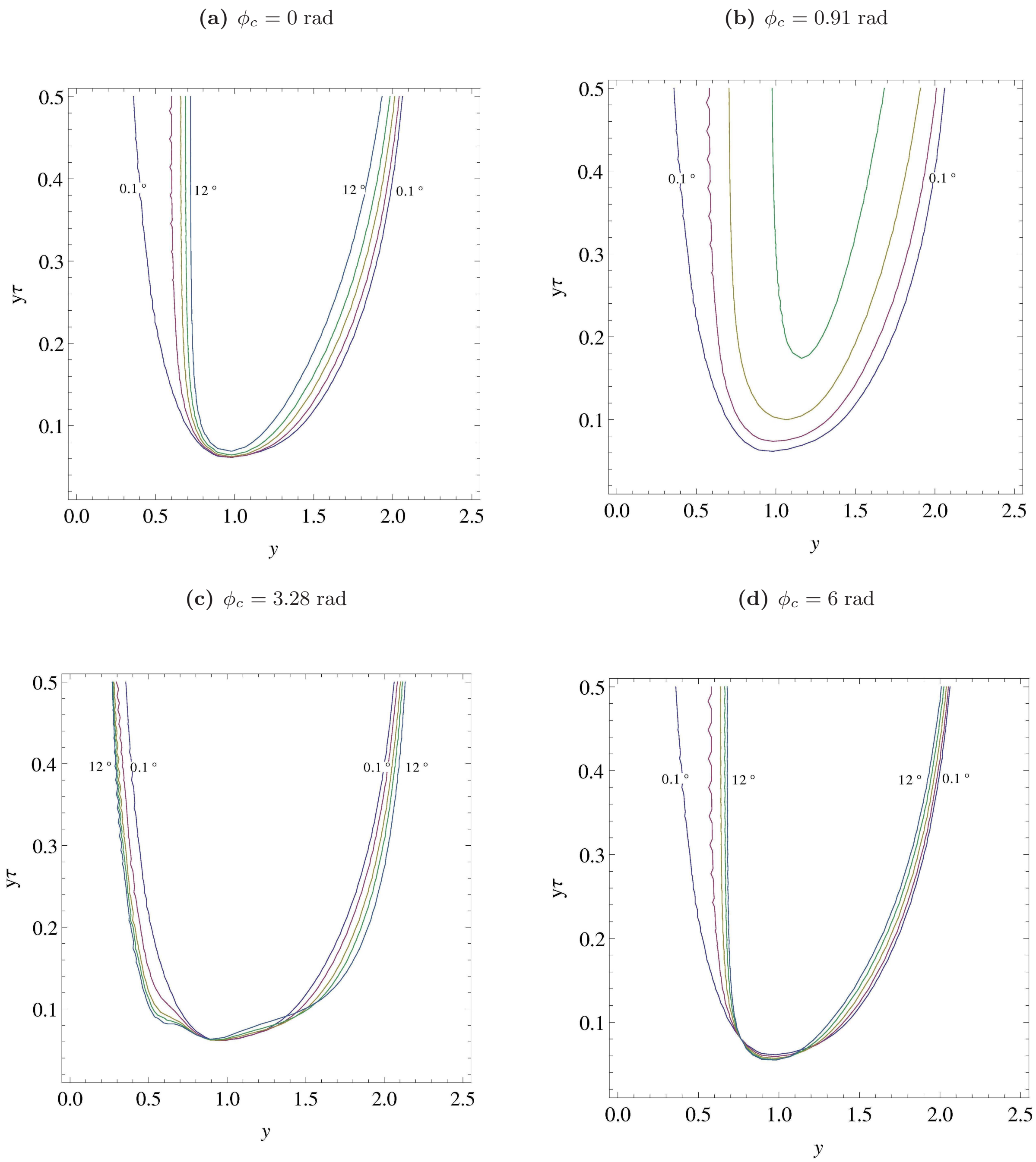

Figure 6.6: These plots show the observed baryon to photon ratio $\eta = 6.2 \times 10^{-10}$ from the TM model with $\theta_{13} = 0.1°, 3°, 6°, 9°, 12°$ (purple, red, yellow, green, blue respectively) in the tau Yukawa, y_τ, versus neutrino Yukawa, y, plane. Each plot is for a different value of the phase of $C = x_C\langle\xi'\rangle$, given above the relevant panel. Note that the $\theta_{13} = 12°$ contour is not possible for $\phi_c = 0.91$ radians.

Chapter 7

Summary and Conclusions

This thesis has presented two models which attempt to explain the observed pattern of neutrino mixing using discrete flavour symmetries. Both models have been constructed in a GUT context in order to attempt to reproduce the observed quark mixing pattern as well. The consequences for leptogenesis are then studied and a solution to a common problem with the combination of flavour symmetries and leptogenesis is discussed.

In Chapter 4, the first model [1] is presented. An SU(5) GUT with extra fermionic field content [67] is extended to introduce a flavour symmetry. The consequence of introducing a flavour symmetry means that the initial matter content is not sufficient to reproduce the neutrino data, so the minimal extension of one RH neutrino is made. Once this is done, a model can be constructed that predicts TB neutrino mixing with corrections from the charged lepton sector. The charged lepton masses are related to the down quark masses by the GUT nature of the model, and the GJ mechanism is utilised in order to obtain more phenomonogically preferred relationships between these parameters. Quark mixings are small and come predominantly from the down sector, while the mass ratios are quite large due to the top mass being renormalisable. An attractive feature of the model is that there is no mixing between the Majorana particles and therefore CSD is obtained without having to assume a diagonal form for this matrix.

In Chapter 5, a second model [2] attempts to uplift a flavour model predicting TM mixing [60] to a GUT. This results in a theory predicting TM neutrino mixing with corrections from the charged lepton sector; the GJ mechanism is again used and this relates the charged lepton corrections to the Cabibbo angle. These corrections are studied in the context of deviations from the TB mixing scheme, and new sum rules are derived which give phenomonologically different predictions to those existing in the literature. The NLO structure of the model is studied and example terms are presented; these are known not to affect the LO model, leaving the predictions unchanged. The vacuum alignment of the model is also briefly studied and the assumed hierarchy of flavon VEVs is motivated.

Chapter 6 studies the effect of flavour symmetry on leptogenesis [3]. The idea of FD is introduced and it is shown that this leads to a CP violating parameter of 0, meaning that there is 0 baryon asymmetry in such models. However, the difference in scales between the breaking of the family symmetry (and consequential constraining of the Yukawa couplings) and the onset of leptogenesis caused by out of equilibrium decays of RH neutrinos is not usually considered. It is demonstrated that taking this scale difference into account by first running the neutrino Yukawas down to the relevant scale can violate FD and generate a non-zero value for the CP violating parameter. This procedure is performed in the context of the AF model predicting TB mixing and the previously studied model of TM mixing [60]. Contours of the baryon asymmetry in Yukawa space are produced, showing that the observed value of the baryon asymmetry of the Universe can be reproduced by a range of values of neutrino and tau Yukawa (which is the dominant parameter in such a calculation). For the TM model, similar contours are produced assuming a range of values for θ_{13} and the same conclusion is reached: leptogenesis can be successful in the context of flavour symmetries by nothing more that RG evolving the relevant parameters to the correct energy scale.

In the future, it would be interesting to study other flavour symmetries in the context of non-zero θ_{13}. Indeed, current work involves obtaining a Golden Ratio mixing pattern incorporating non-zero θ_{13} coming from an A_5 flavour symmetry. A consequence of the method chosen to do this is that it also alters the prediction for θ_{12}, which in the LO Golden Ratio models tends to be too low to fit the data. This work also includes a detailed discussion of the breaking of A_5 down to the low energy K_4 symmetry of the neutrino mixing matrix, something that has not been considered before.

Appendices

Appendix A

Spinor Formalism

This Appendix introduces the spinor formalism used throughout the thesis.

Since the Lorentz group, $\mathrm{SO}(3,1)$ is locally isomorphic to $\mathrm{SU}(2) \otimes \mathrm{SU}(2)$, it can be represented by a pair of numbers representing the spin of each factor: (a, b). The simplest nontrivial representation of this group is therefore $\left(\frac{1}{2}, 0\right)$ (or $\left(0, \frac{1}{2}\right)$) and is known as a Weyl spinor. It has two components and transforms as

$$\chi \to \exp\left(-i\frac{1}{2}\sigma \cdot \theta\right)\chi, \tag{A.1}$$

$$\chi \to \exp\left(-i\frac{1}{2}\sigma \cdot \eta\right)\chi, \tag{A.2}$$

where θ is the angle of rotation, η is the rapidity of the boost (related to velocity by $\beta = \tanh\eta$) and the Pauli matrices are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.3}$$

Using such a spinor, one can construct a mass term which is invariant under Lorentz transformations

$$\mathscr{L} = \frac{1}{2}m\left(\chi^T\epsilon\chi + h.c.\right), \tag{A.4}$$

with $\epsilon = i\sigma^2$. This is a Majorana mass term, the simplest mass term possible. If the Weyl spinor is allowed to transform under some (global or local) symmetry, this mass term is no longer invariant unless the spinor transforms as a real representation; therefore in order to construct a theory with complex spinor representations, a second Weyl spinor must be introduced in the complex conjugate representation. A Dirac mass term is then constructed as

$$\mathscr{L} = m\left(\xi^T \epsilon \chi + h.c.\right), \tag{A.5}$$

where the term is so-called since it can be constructed from a single four-component (Dirac) spinor as follows

$$\mathscr{L} = -m\overline{\psi}\psi = -m \begin{pmatrix} \chi^\dagger & -\xi^T\epsilon \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \chi \\ \epsilon\xi^* \end{pmatrix}. \tag{A.6}$$

Here a specific basis, the Weyl basis, has been chosen for the Dirac gamma matrices:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.7}$$

where each entry is understood to be itself a 2×2 matrix. The advantage of this basis is that the chiral projection operator projects out the upper and lower components of the Dirac spinor

$$P_{L,R}\psi = \frac{1 \mp \gamma_5}{2}\psi = \psi_{L,R}, \tag{A.8}$$

with $\psi_L = (\chi, 0)^T$ and $\psi_R = (0, \epsilon\xi^*)^T$. Since the Dirac spinor transforms as $\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)$ under the Lorentz group, the projection operator is selecting only one or other of these representations. The subscripts L, R then denote the representation of the Lorentz group selected: this is the chirality of the spinor. Setting $\chi = \xi$ in the Dirac spinor gives a four-component Majorana spinor which has a mass term

$$\mathscr{L} = -\frac{1}{2}m\overline{\psi}_M\psi_M, \tag{A.9}$$

reproducing Eqn. (A.4).

In order to simplify GUTs, the convention is to write mass terms with only LH fields (since LH and RH fields are unified into single representations in a GUT); this is done using the charge conjugation matrix

$$C = \begin{pmatrix} -\epsilon & 0 \\ 0 & \epsilon \end{pmatrix} . \tag{A.10}$$

Then it is possible to define the charge conjugated spinor

$$\psi^c = C\gamma^0\psi^* = \begin{pmatrix} \xi \\ \epsilon\chi^* \end{pmatrix} , \tag{A.11}$$

such that

$$(\psi^c)_L = \begin{pmatrix} \xi \\ 0 \end{pmatrix} = (\psi_R)^c . \tag{A.12}$$

This allows Dirac and Majorana masses to be written respectively as

$$\mathscr{L} = -m \left((\psi^c)_L^T C\psi_L + h.c. \right) , \tag{A.13}$$

$$\mathscr{L} = -\frac{1}{2} m \left(\psi_L^T C\psi_L + h.c. \right) . \tag{A.14}$$

Appendix B

D-term alignment

In order to break the A_4 symmetry, the flavon fields need to obtain non-zero VEVs. The direction of these VEVs must in some way be forced into the desired configuration; indirect models can use the method of D-term alignment [72] to achieve this and so work in this Appendix is understood to be in the S diagonal basis of Section 3.1.1. An A_4 triplet flavon φ will in general have a scalar potential of the form [71]

$$V \ni -m_\varphi^2 \varphi^{i\dagger}\varphi_i + \lambda_\varphi(\varphi^{i\dagger}\varphi_i)^2 + \kappa_\varphi \varphi^{i\dagger}\varphi_i\varphi^{i\dagger}\varphi_i + \dots \tag{B.1}$$

where i here is the A_4 index. The first two terms in this equation have an enhanced SO(3) symmetry, which means that their vacuum alignment is not unique; in fact, there is a continuum of possible alignments available. The third term breaks this symmetry and is what is used to ensure the flavons obtain VEVs in the desired directions. The alignment of φ depends on the sign of κ_φ:[1]

$$\langle\varphi\rangle \propto \begin{cases} (0,0,1)^T & \text{for} \quad \kappa_\varphi < 0, \\ (1,1,1)^T & \text{for} \quad \kappa_\varphi > 0. \end{cases}$$

Referring to table 4.2, it is clear that this argument is enough to generate the desired alignments for φ_3 and φ_{123}. In order to obtain the remaining two,

[1]The choice of $(0,0,1)^T$ as opposed to, e.g. $(1,0,0)^T$ simply defines what is meant by the 3-direction of the A_4 triplet.

orthogonality arguments are invoked: arranging for the term $\kappa_{23}|\varphi_{123}^{\dagger} \cdot \varphi_{23}|^2$ to have a positive coefficient forces these flavons to be orthogonal in order to minimise V. In this case, the condition gives that with $\langle\varphi_{23}\rangle \propto (x, y, z)^T$, $x + y + z = 0$ which is not sufficient to define the desired alignment. This motivates the introduction of an extra flavon φ_1 whose only purpose is to impose $x = 0$ through an orthogonality condition between it and φ_{23}. Its charge q_1 is chosen such that it doesn't interact with any other field in the model (and thus, if $\eta_1 \sim \epsilon$ then $q_1 \geq 9$). With this in place, and a further orthogonality condition imposed by the term $\kappa_1|\varphi_3^{\dagger} \cdot \varphi_1|^2$, the desired vacuum alignment is achieved. One should note that the scalar potential (B.1) is invariant under a product of global U(1) symmetries, one for each avon component. For a discussion of the implications of this observation, see [73]. Finally, since all terms used in this alignment are phase-blind each flavon has an undetermined phase associated with it. Additionally, $\langle\varphi_{123}\rangle$ has independent phases for each of its components; the component phases of $\langle\varphi_{23}\rangle$ are related to those of $\langle\varphi_{123}\rangle$ by the orthogonality relation above. However, in terms of obtaining tri-bimaximal mixing, what is important is that the orthogonality conditions $\langle\varphi_{123}^{\dagger}\rangle \cdot \langle\varphi_{23}\rangle = 0$ and $\langle\varphi_1^{\dagger}\rangle \cdot \langle\varphi_{23}\rangle = 0$ are sufficient to generate $\theta_{13}^{\nu} \sim 0$ and $\tan\theta_{12}^{\nu} \sim \frac{1}{\sqrt{2}}$ in accordance with the CSD [69] conditions, regardless of the phases of φ_{123}.

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