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Can Lighthill's Elongated Body Theory Predict Hydrodynamic Forces in Underwater Undulatory Swimming?

Angus P. Webb[a], Christopher W.G. Phillips[a], Dominic A. Hudson[a], Stephen R. Turnock[a]

Froude Building(28),Faculty of Engineering and the Environment, University of Southampton, University Road, Southampton, SO17 1BJ, United Kingdom

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Abstract

Underwater Undulatory Swimming (UUS) is an area of continuing development in elite swimming. The propulsive forces generated during UUS are investigated experimentally, during an over-speed tow, and numerically using Elongated Body Theory (EBT), developed initially for fish locomotion. Two-dimensional kinematic motion data (foot, shank, thigh, torso, upper arm, lower arm, and hand) at 25Hz in the sagittal plane is acquired by manual digitisation of video recorded from a stationary camera during an over-speed active tow and input into an EBT model. Thrust (T) determined from EBT and a semi-empirical passive resistance (R) is used to estimate R-T for comparison with the experimental tow line measurement. The forces predicted from EBT although significantly larger than the experimental measurement indicate that the EBT has the potential, with suitable refinement, to provide detailed insight into the hydrodynamics of UUS. Areas for further refinement are in the use of a three-dimensional correction and that higher resolution motion data for the feet are required.

Keywords: Swimming; resistance; propulsion; underwater undulatory swimming; elongated body theory; elite sport

1. Introduction

Underwater undulatory swimming (UUS), also known as the fifth stroke, is performed after the start and every turn in most swimming races. This method of swimming is faster than surface swimming and therefore rules limit it to the first 15m of each length. In recent years, a competitive advantage has been gained if a swimmer sustains good UUS technique for the full 15m.

[a] A. P. Webb. Tel.: +44 (0)23 8059 6625.
E-mail address: apw105@soton.ac.uk.

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It is important to understand how an effective UUS technique can be developed to increase swimming velocity within the available 15m. In UUS, propulsion is generated from a body movement similar to the carangiform motion of a cetacean [1]. Elongated Body Theory (EBT) [2] assumes that propulsion originates from reactive forces between the surface of the body and the volume of surrounding water. These are inertial forces and occur due to a change in momentum the surrounding water experiences caused by the forced motion of the body surface and a change of momentum generated in the wake. A study comparing the predicted propulsive thrust using this method has found good correlation with experimental data from resistance tests of a robotic fish [3], however no such published study has been performed for human swimming [4]. A swimming model SWUM [5] has been developed to predict forces from a segmented body and used to optimise body motion during UUS. This model predicts the forces on a body segment through components of buoyancy, added mass and drag, however momentum added to the wake is not included. Predicting the reactive forces and hence propulsion of UUS allows quantitative analysis to be performed, for example, variations in stroke frequency, amplitude and range of motion of the swimmer. This process can be used to improve understanding of how humans perform UUS and used as a tool to analyse the technique of a specific swimmer. This method can be used to identify the best from a range of techniques and allow a stroke optimisation to be performed. In this study, an over-speed tow is conducted with a subject performing UUS and the tow force R-T measured [6]. Through analysis of the swimming motion, the thrust is predicted using EBT. This thrust is converted to R-T using a semi-empirical resistance model based on ship hydrodynamics [7] and compared with values determined from experimental testing.

2. Theory and Methodology

2.1. Elongated Body Theory

Elongated Body Theory (EBT), proposed by Sir James Lighthill, assumes that propulsion originates from reactive forces between the surface of the body and the volume of surrounding water. This theory of propulsion is particularly dominant when the cross-sectional area of the body is much smaller in the swimming direction than in the nearly perpendicular direction of the undulatory motion. The formulation of this theory is based on the following three principles and is illustrated in Figure 1;

- The imparted fluid momentum near the fish acts perpendicular to the body longitudinal axis and is expressed as the component of added mass *m* per unit length multiplied by vertical velocity *w*.
- Total thrust acting on the body is the summation of the net rate of change of momentum within the volume of fluid surrounding the body.
- When balancing the momentum it is necessary to account for momentum transfer through the control volume and from the resultant $1/2\, mw^2$ of the pressures generated.

The motion of the body is defined using the axis system in figure 1, where *x* and *z* are global axis coordinates and *a* is the parametric distance from the tip of the caudal fin or feet in the body axis. Using this axis system, the position and orientation of any point along the body at any point in time is described.

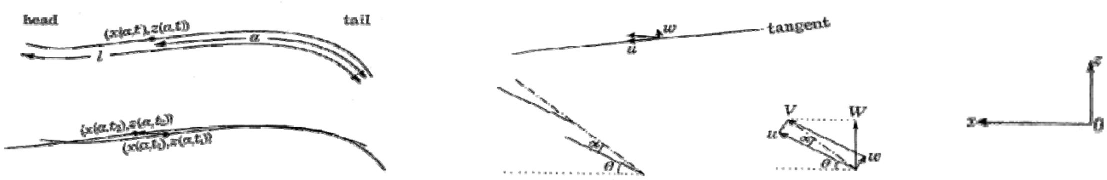

Fig. 1. Axis system used to define the motion for Elongated Body Theory with the origin at the tail/feet

The local body velocity components are described in equations 1 and 2, where partial derivatives of a describe the local slope.

$$u = \frac{\partial x}{\partial t}\frac{\partial x}{\partial a} + \frac{\partial z}{\partial t}\frac{\partial z}{\partial a} \tag{1}$$

$$w = \frac{\partial z}{\partial t}\frac{\partial x}{\partial a} - \frac{\partial x}{\partial t}\frac{\partial z}{\partial a} \tag{2}$$

The added mass per unit length at any point along the body is determined from equation 3, where ρ is the density of the water and s the cross-sectional area perpendicular to the direction of undulation.

$$m = \frac{1}{4}\pi\rho s^2 \tag{3}$$

The total reactive force acting on the body in components of axial thrust (P) and vertical force (Q) are expressed in equation 4, where the component at a=0 is the momentum added to the wake due to vortex shedding and convection at the tail and the integral is the reaction force along the body.

$$\left(P,Q\right) = \left[mw\left(\frac{\partial z}{\partial t}, -\frac{\partial x}{\partial t}\right) - \frac{1}{2}mw^2\left(\frac{\partial x}{\partial a}, \frac{\partial z}{\partial a}\right)\right]_{a=0} - \frac{d}{dt}\int_0^l mw\left(-\frac{\partial z}{\partial a}, \frac{\partial x}{\partial a}\right)da \tag{4}$$

2.2. Kinematic Input Motion

To model the thrust produced from UUS, two-dimensional kinematic motion data in the sagittal plane is acquired by manual digitisation of video recorded from a stationary camera. This analysis provides *z(a,t)* and *x(a,t)* for the following segments along the body;

1. Foot 2. Shank 3. Thigh 4. Torso 5. Upper arm 6. Lower arm 7. Hand

In order to determine the position of each segment, it was assumed that only the coordinates of the segment ends were required. Therefore, and as can be seen in figure 2, seven points along the body were manually digitised providing the motion for UUS.

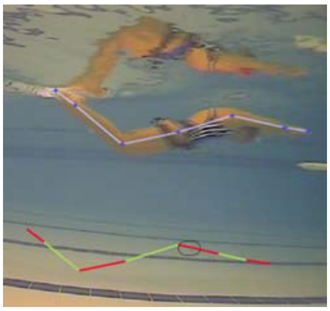

Fig. 2. Manual Digitisation to determine real kinematic data

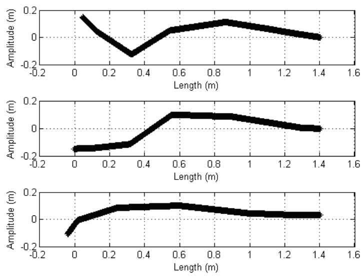

Fig. 3. Three discrete stages of the digitised motion input into elongated body theory

With *z(a,t)* and *x(a,t)* for the end points of each segment, it is necessary to expand this dataset within each segment to a reasonable level of resolution to accurately determine the reaction force along the body. By assuming the segments were straight and rigid, linear interpolation of the segment end data was performed for discrete values of a within each segment.

Digitisation was performed at a frame rate of 25 fps, and therefore provided segment position data for every 0.04 seconds. It was necessary to input this data independent of time, to allow a range of kicking frequencies. Instead of time, phase was used to describe the progression through a kick cycle, were *0* radians represented the beginning and 2π radians represented the end of a kick cycle. For every digitised frame of data, a phase value between 0 and 2π radians is assigned and the position of the swimmer is determined by interpolating the dataset with respect to phase to provide $z(a,t)$ and $x(a,t)$.

Due to the subjective nature of manual digitisation, an error is associated with the position selection by the user. Figure 4 displays how a point on the body progresses through the stroke cycle. It can be clearly seen there is a general trend, however, a noise is present. To eliminate this noise and retain the general trend, a 3rd order polynomial fit has been used.

2.3. Numerical Implementation of EBT

To numerically model UUS using Elongated Body Theory, the variables contained within equation 4 are given discrete values, determined from $z(a,t)$ and $x(a,t)$. Numerical differentiation with respect to t is performed between two time steps, using a single step finite differencing method for all values of a. Differentiation with respect to a is performed at each time step to determine the gradient between each point along the length of the body. The integration detailed in equation 4 is performed using a trapezoidal rule.

2.4. Estimating R-T from EBT

It is not possible in an over-speed tow to measure thrust directly, since the force measured by the tow system is the resistance of the swimmer minus the thrust they are producing (R-T). In a free swimming condition, the instantaneous differences in force result in changes in swimming speed. In an over-speed tow the speed is maintained at a constant value chosen such that the resistance is always greater than thrust and therefore a difference can be measured. In a method developed by Webb [6], the active drag or thrust of the swimmer may be determined by predicting the passive drag of the swimmer for a given tow speed. The active drag of a swimmer during a tow may be expressed as;

$$R_{Active} = \left(R - T\right)_{Measured} + R_{Naked} \tag{5}$$

where (R-T) is measured during the tow and R_{Naked} is the passive resistance of the swimmer at the tow speed. R_{Naked} may be obtained by conducting a passive tow, however it is also possible to predict using the method described by Webb [6]. This method assumes total resistance contains contributions from viscous pressure resistance, wave making resistance and skin friction resistance. In this case the contribution from wave making resistance is neglected as the swimmer is not on the surface and is below the critical depth for wave resistance to occur [8]. The thrust determined from EBT is therefore converted to R-T, using equation 5 to allow comparison with experimental data.

3. Results and Discussion

An active tow of an elite swimmer during UUS was conducted at a tow speed of 2.23 ms^{-1} with a mean kicking frequency of 1.7 Hz. Figure 5 displays the predicted passive resistance against speed [6]. A passive resistance at a tow speed of 2.23 ms^{-1} is therefore 142.9 N.

Figure 6 displays the predicted thrust determined from EBT using the input motion $z(a,t)$ and $x(a,t)$ at a fixed swimming speed of 2.23ms^{-1}. It is clear that many phases of the stroke cycle produce a significant amount of negative thrust. This is expected during the recovery phase of UUS, when the posterior of the legs present a large projected area to the flow, resulting in drag. In an over-speed situation, the swimming speed is fixed and therefore does not decrease during the recovery phase of the stroke. This will increase the magnitude of drag experienced during the recovery since the onset flow velocity and acceleration exposed to the leg posterior will be increased.

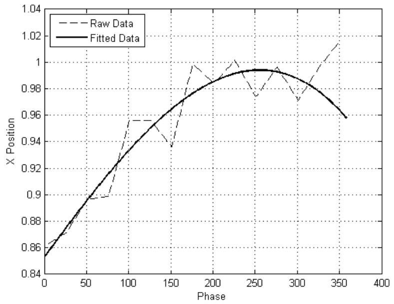

Fig. 4. X Position data for the hip displaying the raw data, containing noise, with the fitted data overlaid, where phase is in degrees and X Position is in meters

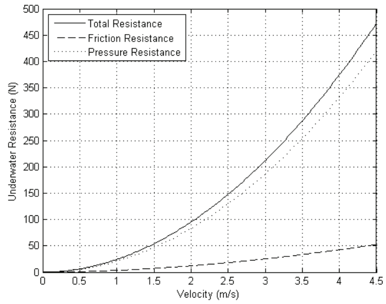

Fig. 5. Passive resistance prediction against speed. A passive resistance of 143 N was determined for a tow speed of 2.23 ms^{-1}

Figure 7 displays the measured R-T for one kicking cycle. During an active over-speed tow, a reduction in R-T represents an increase in thrust. Figure 7 also displays the predicted R-T from equation 4 using the thrust in figure 6 and a passive resistance of 142.9 N. The propulsive range of the experimental measurement is 307 N, whereas the range of the EBT prediction is 605 N. In addition, there are very large amplitude oscillations occurring during the stroke cycle for the EBT prediction, where the oscillations in the experimental measurement are much smaller. This suggests that the generation of forces using EBT model may be missing certain detail present in the real life situation. Due to EBT being a two-dimensional model, the predicted thrust from momentum added to the wake does not take into account three-dimensional effects. Cohen et al [9] has identified large vortex rings, due to the jets created by the toe flick, present in the wake behind a swimmer performing UUS. The actual momentum in a wake containing vortex rings will be different to the predicted momentum from two dimensional vortex shedding theory. Another three dimensional effect not currently accounted for in EBT is the variation of shape along the length of the body. This will affect the added mass calculation, where EBT assumes a constant cross-sectional shape. An improvement would be to use a local added mass function specific to each body segment.

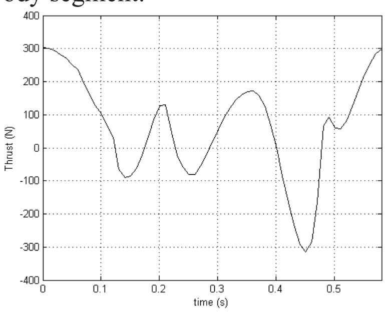

Fig. 6. Predicted thrust from EBT using 2D kinematic input motion determined from manual digitisation in the sagittal plane and a fixed forward velocity of 2.23ms^{-1}

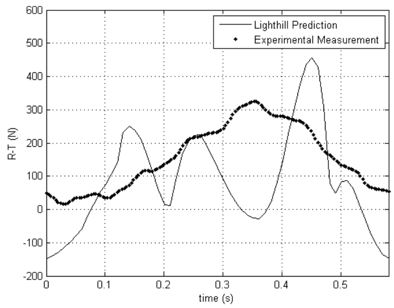

Fig. 7. Experimental R-T measured during one kick cycle for a tow speed of 2.23ms^{-1} and a kicking frequency of 1.7 Hz. The predicted thrust from EBT is deducted from the mean predicted drag to produce a predicted R-T

The accuracy of the EBT prediction is dependent on the quality of the input motion. The input motion defines both the orientation and the acceleration of each segment, which is used to determine the local force components. Comparison of the input motion and the video footage of the swimmer has identified the motion of the feet have not been accurately captured. The motion of the feet is crucial to EBT since both the vortex shedding [10] and a large proportion of the body reaction force are determined at the feet. As shown in figure 2 only two points define the motion of the feet, however analysis of the video has identified the feet are not rigid, and flex similar to a caudal fin. It therefore necessary to increase the number of points defining the feet or incorporate a flexible plate model to define the motion of the feet more realistically.

4. Conclusion

The forces associated with underwater undulatory swimming (UUS) have been studied experimentally and numerically. By assuming the motion of human UUS is similar to that of a carangiform cetacean it is proposed that Elongated Body Theory (EBT) [2] may be used to determine the propulsive forces. This initial work indicates that further improvements to the EBT model are required to model the hydrodynamic forces in UUS.

An active over-speed tow of a human swimmer performing UUS is conducted and R-T is measured. Film footage of the swimmer in the sagittal plane is recorded and manual digitisation is used to provide two-dimensional kinematic data of seven body segments for one kick cycle. The kinematic data is input into an EBT model with a prescribed swimming speed of 2.23ms^{-1} and the thrust for one kick cycle is determined. The thrust determined from EBT is converted to R-T [6] to allow comparison with experimental data. The force range of the predicted propulsion is significantly larger than the experimental measurement and contains large amplitude oscillations throughout the stroke cycle.

The wake momentum predicted by EBT does not account for three dimensional effects, present in human UUS, such as vortex rings and therefore may affect the accuracy of the prediction. Analysis of the foot motion has identified that treating the foot as a single rigid segment provides insufficient detail and will significantly affect the accuracy of the propulsion prediction. A more accurate method of defining the foot motion, using either a larger number of anatomical landmarks or a flexible plate model, is proposed.

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