UNIVERSITY OF Southampton

Southampton Statistical
Sciences Research Institute
& Institute of Sound and
Vibration Research

BAYESIAN WORKSHOP 2012

APPLICATION OF BAYESIAN METHODS IN STRUCTURAL AND VIBRO-ACOUSTICS DYNAMICS

21-22 March 2012, Southampton

University of Southampton and Chilworth Manor, Southampton UK

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PROGRAMME

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BOOK OF ABSTRACTS

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Programme

Wednesday, 21st March

10:00-11:00	Welcome, registration and tea/coffee
11:00-12:00	Plenary session - **Nozer Singpurwalla,** Bayesian inference for adversarial parameters in diagnostic and threat detection (open to S3RI members)
12:00-13:00	Lunch
13:00-13:35	**Peter Jan van Leeuwen,** Bayes in high-dimensions: efficient particle filters
13:35-14:10	**Andrew M. Stuart and Kody J.H. Law,** Data assimilation for fluid mechanics: A comparison of Bayesian and filtering methodologies
14:10-14:45	**Michael Beer and Matthias Stein**, Handling imprecision in probabilistic models and Bayesian updating
14:45–15:15	Tea/coffee break
15:15-15:50	**Paul E. Barbone and Assad A. Oberai**, Efficiently computing uncertainty in solutions of inverse diffusion problems
15:50-16:25	**Ellen Simoen,** Resolution and uncertainty analysis in Bayesian vibration-based model updating
16:25-17:00	**E.Z. Moore, Kevin D. Murphy and J.M. Nichols,** Practical implementations of a Bayesian, model-based SHM methodology
17:30	Coach to Chilworth Manor
19:30	Dinner Reception

Thursday, 22nd March

09:30 -10:50	Plenary session - **James L. Beck**, Bayesian system identification and robust response predictions with applications in structural dynamics
10:50-11:20	Tea/coffee break
11:20-11:55	**Bill Lionheart,** Applied inverse problems
11:55-12:30	**Keith Worden,** Bayesian approaches to nonlinear system identification
12:30-13:50	Lunch
13:50-14:25	**Costas Papadimitriou**, Bayesian uncertainty quantification in structural dynamics simulations using high performance computing techniques
14:25-15:00	**Christophe Lecomte, Jon J. Forster, Brian R. Mace and Neil S. Ferguson,** Bayesian parameter estimation at mid-frequency from transfer function measurements
15:00-15:30	Tea/coffee break
15:30-16:05	**Richard Dwight**, Reducing uncertainties in aeroelastic flutter boundaries using experimental data
16:05-16:40	**Hamed H. Khodaparast, John E. Mottershead, Yves Govers, Michael Link and Richard Dwight,** Interval model updating of the AIRMOD structure
16:40-17:15	**Siu-Kui Au**, Bayesian operational modal analysis: From theory to practice
17:30	Finish

REGISTRATION REQUESTED WITH JON LAWN: j.lawn@soton.ac.uk

Bayesian operational modal analysis: From theory to practice.

Siu-Kui AU

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Abstract

Ambient vibration tests have attracted increasing attention over the last few decades because they can be performed economically with the structure under working condition without artificial loading. Ambient modal identification or ‘operational modal analysis’ techniques do not require knowledge of the loading but they assume that it is statistically random. A Bayesian approach provides a fundamental means for extracting the information in the data to yield information about the modal parameters consistent with modeling assumptions. Issues do exist in the implementation and interpretation of results. This talk presents an overview of a Bayesian frequency-domain approach for operational modal analysis. Issues of theoretical, computational and practical nature are discussed, drawing experience from a number of field applications.

Efficiently computing uncertainty in solutions of inverse diffusion problems

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March 1, 2012

Abstract

In many problems, one observes the response of a system and desires to infer the system properties from the response. Such a problem can be called an inverse problem, and is often formulated in a Bayesian context. A formal solution for the a posteriori probability distribution is readily available, but is difficult to work with practically. A typical application of the solution is to compute the MAP (Maximum A posteriori Probability) estimate of the solution. This solution represents the most probable estimate of system properties that is consistent with the observed system response.

In practice, finding the MAP estimate may require minimizing a very high dimensional function ($O(10^5)$ parameters) with complex (e.g. discretized nonlinear PDE) constraints. Because of the large number of parameters involved, the algorithm to reach the minimum must scale as efficiently as possible. Hessian evaluation is typically out of the question, since the most efficient direct eval-uation of the Hessian requires O(N) solutions of the constraint equations, and $O(N^2)$ storage of the result. Hence, gradient techniques are typically employed; an adjoint method gives gradient information in O(1) constraint solutions, and so iteration times scale tractably.

In addition to knowing the MAP estimate, it is often desirable in practice to know the range of uncertainty in the estimate of system properties. Estimating the uncertainty in the solution, however, requires the Hessian to be computed at the MAP point. Thus, it seems, computing the uncertainty in such a large scale problem is intractable. We exploit results from inverse problems and computational linear algebra to show that, in general, for an ill-posed inverse problem, the Hessian can be accurately approximated with relatively little computation over and above that used in finding the MAP estimate. This method can thus be used to compute the uncertainty in an inverse problem solution relatively efficiently. We describe the approach formally in a relatively general setting, and demonstrate it with examples in inverse diffusion.

Bayesian approaches to nonlinear system identification

Keith Worden

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Abstract

We present ways of identifying nonlinear differential and difference equations from measured time data. An approach based on Markov Chain Monte Carlo is covered in the talk and model selection as well as parameter estimation are discussed.

Bayes in high-dimensions: efficient particle filters

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Abstract

Bayes theorem formulates the data-assimilation problem as a multiplication problem and not an inverse problem. In this talk we exploit that using an extremely efficient particle filter on a highly nonlinear geophysical fluid flow problem of dimension 65,000. The proposal density of the particle filter is chosen such that all particles end up close to the observations, so in the high probability regions, and at the same time ensemble collapse is avoided by construction.

The example is borrowed from geophysical fluid dynamics and consists of a quasi-two-dimensional fluid in the chaotic regime, solving the so-called barotropic vorticity equation. In controlled experiments we is show that only 24 particles are needed to find a mean close to the truth. We explore rank histograms to show that each of the particles is indistinguishable from the truth, showing that the particle filter is performing correctly. Finally plans for applications to numerical weather prediction and climate models are discussed.

Bayesian system identification and robust response predictions with applications in structural dynamics

James L. Beck

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Abstract

The goal of system identification for dynamic systems is to use experimental data to improve mathematical models of the input-output behavior of a system so that they make more accurate predictions of the system response to a prescribed excitation. A common approach is to take a parameterized model of the system and then use system data to estimate the value of the uncertain model parameters. It is unrealistic, however, to expect any model to be an exact representation of the system behavior and so one cannot expect true parameter values, and often the estimation gives non-unique results. Furthermore, in order to make more robust predictions, the uncertainties in modelling the system, as well as in modelling the future system input, should be explicitly treated; in particular, because of the approximate nature of any system model, one should explicitly treat the uncertain prediction error (the difference between the outputs or state of the real system and those of the system model). An overview of a complete probabilistic framework for system identification based on Bayesian updating is presented that addresses these points. First, a brief overview of probability logic is given because this provides a rigorous foundation for the Bayesian approach; it provides a multi-valued propositional logic for quantitative plausible reasoning based on probability models where the "probability of a model" is a measure of its relative plausibility within a proposed set of models. Instead of using system data to estimate the model parameters, Bayes' Theorem is used to update the relative plausibility of each model in a stochastic model class, which is a parameterized set of input-output probability models for the system behavior together with a prior probability distribution over this set that expresses the initial plausibility of each model. Then to perform posterior robust predictive analyses, the entire model class is used with the probabilistic predictions of each model being weighted by its posterior probability, in accordance with the Total Probability Theorem. An additional level of robustness can be performed by combining the robust predictions of each model class in a set of candidate stochastic model classes for the system, where each contribution is weighted by the posterior probability of the corresponding model class. These robust analyses involve integrals over high-dimensional parameter spaces that usually cannot be evaluated analytically. Useful computational tools for these evaluations are Laplace's method of asymptotic approximation and Markov Chain Monte Carlo methods. Examples in structural dynamics are given to illustrate the Bayesian system identification framework.

Handling imprecision in probabilistic models and Bayesian updating

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Abstract

The nature of engineering information is frequently diverse and data are limited so that a quantitative modeling is difficult. For example, statistical data may be rare, measurements may be imprecise, sample elements may have been generated under inconsistent environmental conditions, the conditions on the construction site may deviate from the conditions under which laboratory tests have been performed, subjective expert knowledge and experience may be available whereby different opinions exist. This complexity and inconsistency of information requires a comprehensive modeling. A classification of information into uncertainty, associated with probabilistic characteristics, and imprecision, associated with non-probabilistic characteristics, is helpful for this purpose. So it becomes clear that the sole selection of a traditional probabilistic model would introduce unwarranted information by condensing imprecision into single values, e.g. for distribution parameters, or by ascribing random characteristics to imprecise variables without statistical justification. On the other hand, a fuzzy model or an interval model would waste statistical information. To remedy this discrepancy, advancements have been made in two directions. First, Bayesian approaches have been utilized to include expert assessments via subjective probabilities. Second, the framework of imprecise probabilities has been developed to link uncertainty and imprecision in one model. A natural advancement is now a combination of imprecise probabilities with Bayesian approaches. Efforts have been made to consolidate a mathematical basis for this symbiosis.

In this paper, combinations of imprecise probabilities with Bayesian approaches are examined from an engineering point of view. Fuzzy probability theory is utilized to take into account stochastic uncertainty and imprecision simultaneously. The uncertainty of input information is translated into probabilities in the results whilst input imprecision is reflected as imprecision of the probabilistic results. In the proposed fuzzy Bayesian approach two typical cases for input imprecision are investigated; imprecision in the subjective probability statements for the prior distribution and imprecision in the available data. These basic cases, as well as a combination thereof, are examined by way of an example for the quantification of the compressive strength of concrete based on a small sample. Features of the proposed quantification approach are discussed in the context of Bayesian theory and illustrated with the aid of variants of the example.

Bayesian inference for adversarial parameters in diagnostic and threat detection tests

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Abstract

Diagnostic tests are ubiquitous in medicine, signal processing, natural disaster warnings, and even finance. Due to statistical variability’s, such tests are not perfect and are prone to misdiagnoses. The efficacy of such tests is assessed by two parameters, test sensitivity and test specificity. These parameters are adversarial. Consequently, a Bayesian analyses of these parameters calls for a joint prior distribution with negative dependence on the unit square. We discuss two such families of prior distributions and invoke them on synthetic data to demonstrate their usefulness.

Resolution and uncertainty analysis in Bayesian vibration-based model updating

Ellen Simoen and Geert Lombaert

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Abstract

Vibration-based model updating makes use of experimental modal characteristics such as natural frequencies and mode shapes to calibrate a numerical model of structures. In order to quantify uncertainties due to measurement and modeling errors in vibration-based model updating, a Bayesian framework can be adopted. In this contribution, a comprehensive resolution and uncertainty analysis is presented which allows for a detailed investigation of the results of Bayesian model updating schemes. The proposed resolution analysis encompasses several techniques. Besides the computation of classical posterior statistics such as standard deviations and covariance matrices, an eigenvalue analysis is proposed that yields additional insight into which directions in the parameter space – or linear combinations of parameters – are best resolved from the available information. By linking the eigenvalue problem to information entropy measures, it can be deduced to what degree the different directions are resolved, in absolute terms and relative to each other.

Reducing uncertainties in aeroelastic flutter boundaries using experimental data

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Abstract

Flutter prediction as currently practiced is usually deterministic, with a single structural model used to represent an aircraft. By using uncertainty quantification we compute the impact of structural variability on the altitude at which flutter occurs, and observe that under certain conditions moderate structural uncertainties can lead to large uncertainties in the location of the flutter boundary. Therefore we aim to reduce this epistemic uncertainty by identifying structural parameters using experimental data - in particular we use modal eigenvalues measured at conditions prior to the onset of flutter. The identification procedure uses a high-fidelity coupled fluid-structure solver with compressible Euler for the fluid part, a Bayesian framework, and polynomial-based uncertainty quantification techniques to perform the model updating accurately. The method is applied to the Goland wing, and we investigate how far the uncertainty in the flutter boundary can be reduced using measurements at different distances from the boundary.

Interval model updating of the AIRMOD structure

Hamed H. Khodaparast1), John E Mottershead 2), Yves Govers3),
Michael Link4) and Richard Dwight5)

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Abstract

The accuracy of finite element models of aircraft structures can significantly affect their predicted aeroelastic behaviour. Deterministic finite element model updating has become a reliable tool for improving the accuracy of finite element model predictions. Measured dynamic responses, such as natural frequencies and mode shapes, are generally used to identify or update the values of unknown structural parameters.

Although deterministic finite element model updating is frequently used in industry, it is not capable of accounting for and correcting the inherent irreducible system uncertainty brought about by variability in unknown structural parameters. Stochastic model updating techniques have emerged as a valuable alternative to conventional methods in order to overcome this difficulty. In stochastic model updating, a number of modal responses are measured and the variability, in the form of pdfs or intervals, of unknown structural parameters is identified based on the statistics of the measurements. Interval model updating [1] is one recent technique developed for this purpose and is used in this paper.

The problem of interval model updating can be greatly simplified by using parameter vertex solutions under conditions that are very often applicable in large-scale model updating. In such cases the choice of physically meaningful updating parameters can be obscured by the scale and complexity of the problem at hand, so that an often workable approach is to use subsystem material parameters such as the elastic modulus and mass density. Then, the following conditions apply to the global mass and stiffness matrices: (i) they become linear functions of the updating parameters; and (ii) they can be decomposed into nonnegative-definite substructure mass and stiffness matrices. If these conditions hold and the output data are the eigenvalues of the dynamic system then interval model updating by parameter vertex solutions is applicable. These conditions are valid for the test case studied in this paper.

The test case is the DLR AIRMOD structure, a replica of the GARTEUR SM-AG19 benchmark structure. Fourteen eigenvalues (mode 1 to 8, 10 to 12, 14 and 19 to 20) are considered as output data for updating. The other modes are used for validation purposes. Eighteen updating parameters, mostly parameters related to joints, are selected. The interval model updating technique using parameter vertex solutions is used to update the modal responses of the model, based on the measured natural frequencies. The variability in the measured data arises from disassembly and reassembly of the physical structure.

[1] H.H. Khodaparast, J.E. Mottershead and K.J. Badcock, Interval model updating with irreducible uncertainty using the Kriging predictor, *Mechanical Systems and Signal Processing*, 25(4), 2011, 1204-1226.

Bayesian uncertainty quantification in structural dynamics simulations using high performance computing techniques

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Abstract

Bayesian inference is used for quantifying and calibrating uncertainty models in structural dynamics based on vibration measurements, as well as propagating these modeling uncertainties in simulations to achieve updated robust predictions of system performance, reliability and safety. The Bayesian tools for identifying system and uncertainty models as well as performing robust prediction analyses are Laplace asymptotic approximation techniques and more accurate stochastic simulation algorithms (e.g. MCMC, Transitional MCMC, MCMC with delayed rejection). These tools require a moderate to very large number of repeated system analyses to be performed over the space of uncertain parameters. Consequently, the computational demands depend highly on the number of system analyses and the time required for performing a system analysis.

High performance computing techniques are integrated with Bayesian techniques to efficiently handle large-order models of hundreds of thousands or millions degrees of freedom, localized nonlinear actions activated during system operation, and stochastic loads. Fast and accurate component mode synthesis (CMS) techniques are proposed, consistent with the finite element model parameterization, to achieve drastic reductions in computational effort. Further computational savings are achieved by adopting automated multi-level sub-structuring methods to substantially speed-up computations, while parallel computing algorithms can be used to efficiently distribute the computations in available GPUs and multi-core CPUs. Application of the framework to structural health monitoring, damage identification and updating remaining structural reliability is emphasized. The proposed approach is demonstrated using applications in civil and vehicle engineering.

Practical implementations of a Bayesian, model-based SHM methodology

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Abstract

In this work a Bayesian, model-based approach is taken to identify damage in a rectangular plate that has been fully clamped in a test fixture. Specifically, the damage is a single, straight crack that has been milled into the plate. The methodology involves using the free vibration response of the plate, arising from a single impact. Data are gathered from three resistive strain gages, placed at arbitrary orientations and locations far from the crack. The experimental time responses are then used to estimate the crack parameters (size, location, and orientation) that characterize the damage in an efficient finite-element model of the plate. The approach is demonstrated effective in identifying the crack parameters, as well as their associated credible intervals. The results show that even with limited, noisy vibration data valuable information regarding the damage state can be successfully estimated.

Data assimilation for fluid mechanics: A comparison of Bayesian and filtering methodologies

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Abstract

Utilizing state-of-the-art MCMC sampling techniques, we determine the posterior distribution of state for Navier-Stokes equation on the torus informed by noisy observations. We then use this as a gold standard against which we evaluate the accuracy of various ad-hoc algorithms in reproducing its moments. The inexpensive ad-hoc algorithms, based on Gaussian approximations, are used in atmospheric data assimilation. We have shown that the ad-hoc algorithms that are used in practice recover only reasonable estimates of the mean of the posterior distribution and fail to recover the statistical variation.

Furthermore, we show that the methods become unstable and heuristic methods which are utilized in practice to improve filter performance all modify the covariance matrix and can be categorized as stabilization mechanisms. Additionally, we have rigorously identified the mechanism of stabilization for the simplest such algorithm and proven convergence of the estimator from this algorithm to a neighborhood of the true signal, which is bounded by the size of the observational noise. Hence, the simplest such algorithm can be tuned to recover a good estimator of the mean and this estimator converges to the true signal in the zero noise limit.

Bayesian parameter estimation at mid-frequency from transfer function measurements

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Abstract

In this talk, we first discuss how uncertainty in the parameters of a linear dynamic system affects the properties of its vibratory response. Although the transfer functions of such linear systems can be characterised by simpler and simpler statistics for increasing frequency ranges, the characterisation of the responses in the mid-frequency range is challenging. One reason for that is the theoretical and practical difficulty of identifying and estimating the statistical information that is relevant for specific random transfer functions. The randomness introduced by error in measurements further complicates the task. Here, we demonstrate that, based on transfer function measurements, the parameters of a particular one-dimensional system can be practically identified by using a Bayesian approach when the measurement error is a Gaussian process. The model and practical aspects of the (Markov Chain Monte Carlo) Bayesian approach are detailed and statistical features of the uncertain system such as multimodality are highlighted.

Applied inverse problems

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Abstract

In this talk I will survey some inverse problems in imaging including examples from medical, industrial and security applications. When working with applied inverse problems the "golden rule" is to ask the problem owner: What do they measure? What do they already know? What do they want to know? We can then formulate a model for the measurement, investigate the issues of sufficiency of data, stability of solution and incorporation of a priori information. Specific examples will include electrical impedance tomography in medical imaging, polarized light tomography for stress measurement and x-ray tomography for airport security. Underlying this standard approach to applied inverse problems is the Bayesian approach which in practice is rarely made explicit.