Near-capacity joint source and channel coding of symbols from an infinite set

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Presented by

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Part 1 - Background

Robert G. Maunder, Wenbo Zhang, Lajos Hanzo

Outline

- Symbol values from an infinite set
- Elias Gamma (EG) code
- EG-CC SSCC benchmarker
- Capacity loss analysis
- Conclusions

* Separate Source and Channel Coding (SSCC)
* Convolutional Code (CC)

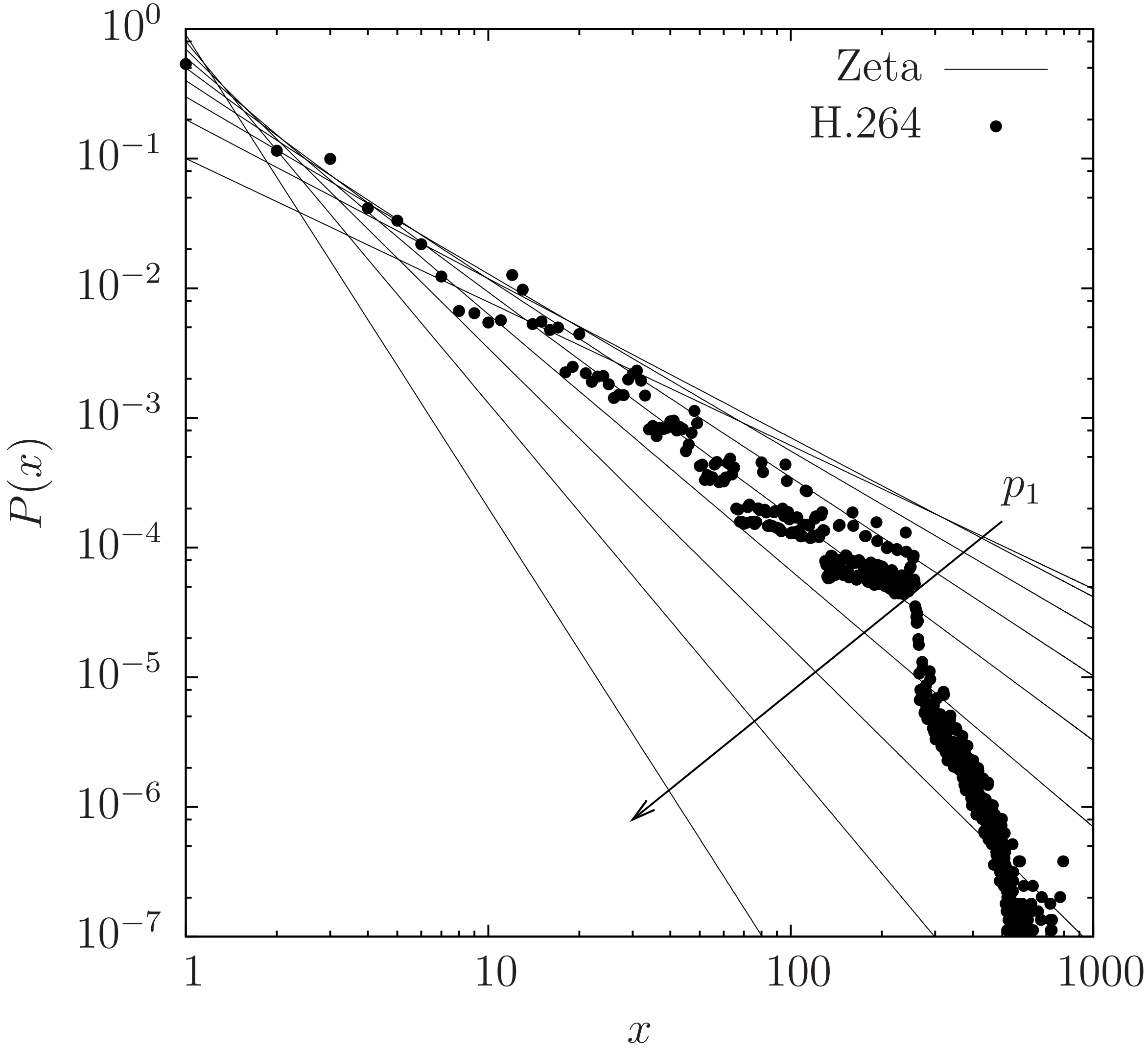
Background

	Finite symbol set e.g. $\{a, b, c, \ldots, z\}$	Infinite symbol set e.g. $\mathbb{N}_1 = \{1, 2, 3, \ldots, \infty\}$
Separate Source and Channel Coding (SSCC)	• Huffman code • Shannon-Fano code	• Unary code • Elias Gamma code
Joint Source and Channel Coding (JSCC)	• Variable Length Error Correction (VLEC) code	**?**

When decoding symbol values selected from an infinite set:

- existing SSCC schemes have significant capacity loss;
- existing JSCC schemes have infinite complexity.

Symbol values from an infinite set



Zeta distribution

$P(x) = \frac{x^{-s}}{\zeta(s)}$,

$\zeta(s) = \sum_{x\in\mathbb{N}_1} x^{-s}$,

$s > 1$,

$p_1 = 1/\zeta(s)$.

Symbol entropy

$H_X = \sum_{x\in\mathbb{N}_1} P(x) \cdot \log_2(1/P(x))$.

Here, $p_1 \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$

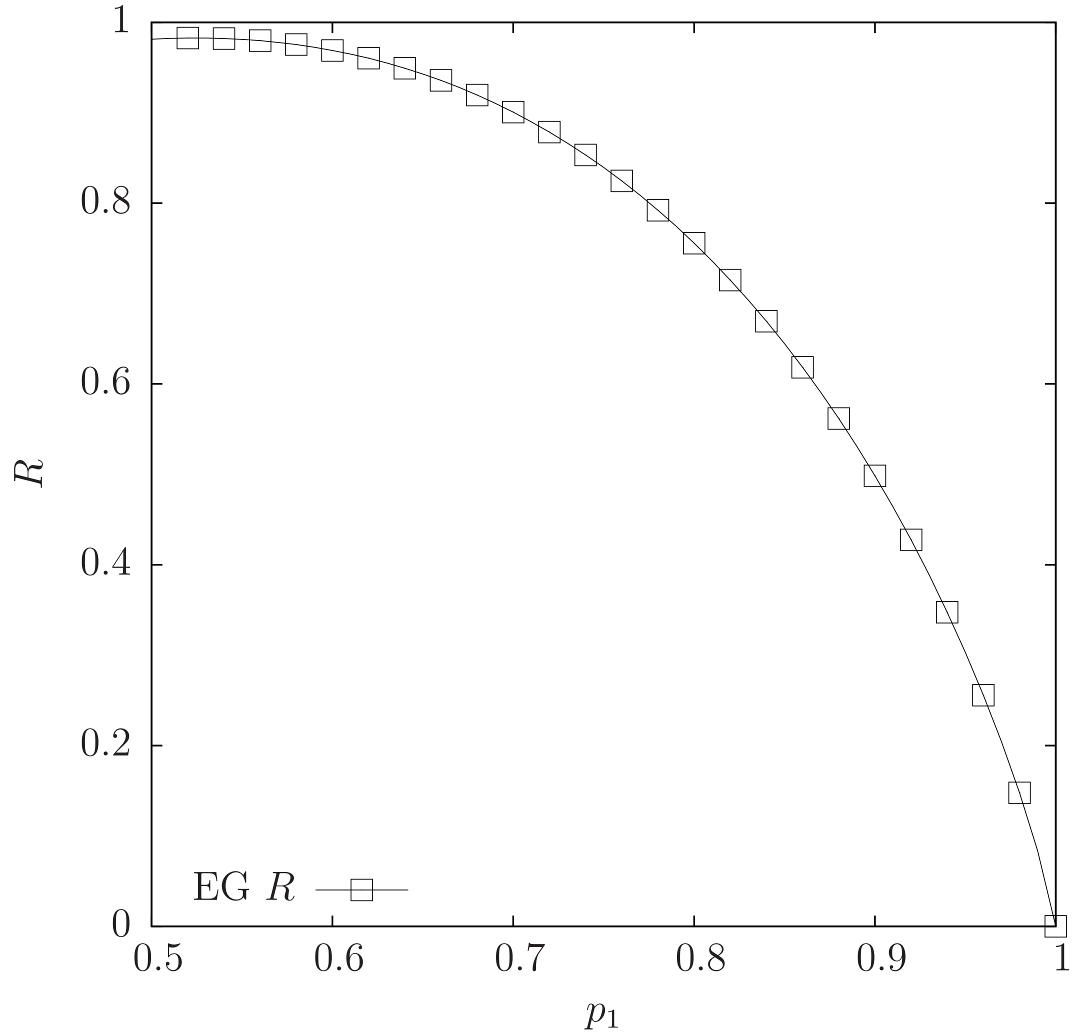
x_i	$P(x_i)$			EG(x_i)
	$p_1 = 0.7$	$p_1 = 0.8$	$p_1 = 0.9$	
1	0.7000	0.8000	0.9000	1
2	0.1414	0.1158	0.0717	010
3	0.0555	0.0374	0.0163	011
4	0.0286	0.0168	0.0057	00100
5	0.0171	0.0090	0.0025	00101
6	0.0112	0.0054	0.0013	00110
7	0.0079	0.0035	0.0007	00111
8	0.0058	0.0024	0.0004	0001000
9	0.0044	0.0017	0.0003	0001001
10	0.0034	0.0013	0.0002	0001010

Table 1: The first ten codewords of Elias Gamma (EG) code.

Elias Gamma code

Average codeword length
$l = \sum_{x \in \mathbb{N}_1} P(x)(2\lfloor \log_2(x) \rfloor + 1)$.

Elias Gamma coding rate



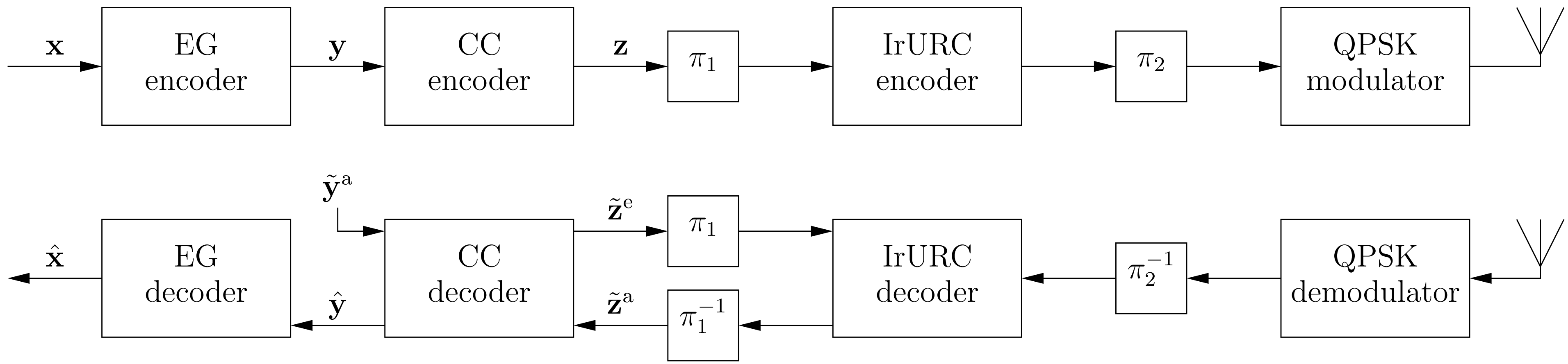
Coding rate of EG code for zeta distribution.

Coding rate

$$R = \frac{H_X}{l}.$$

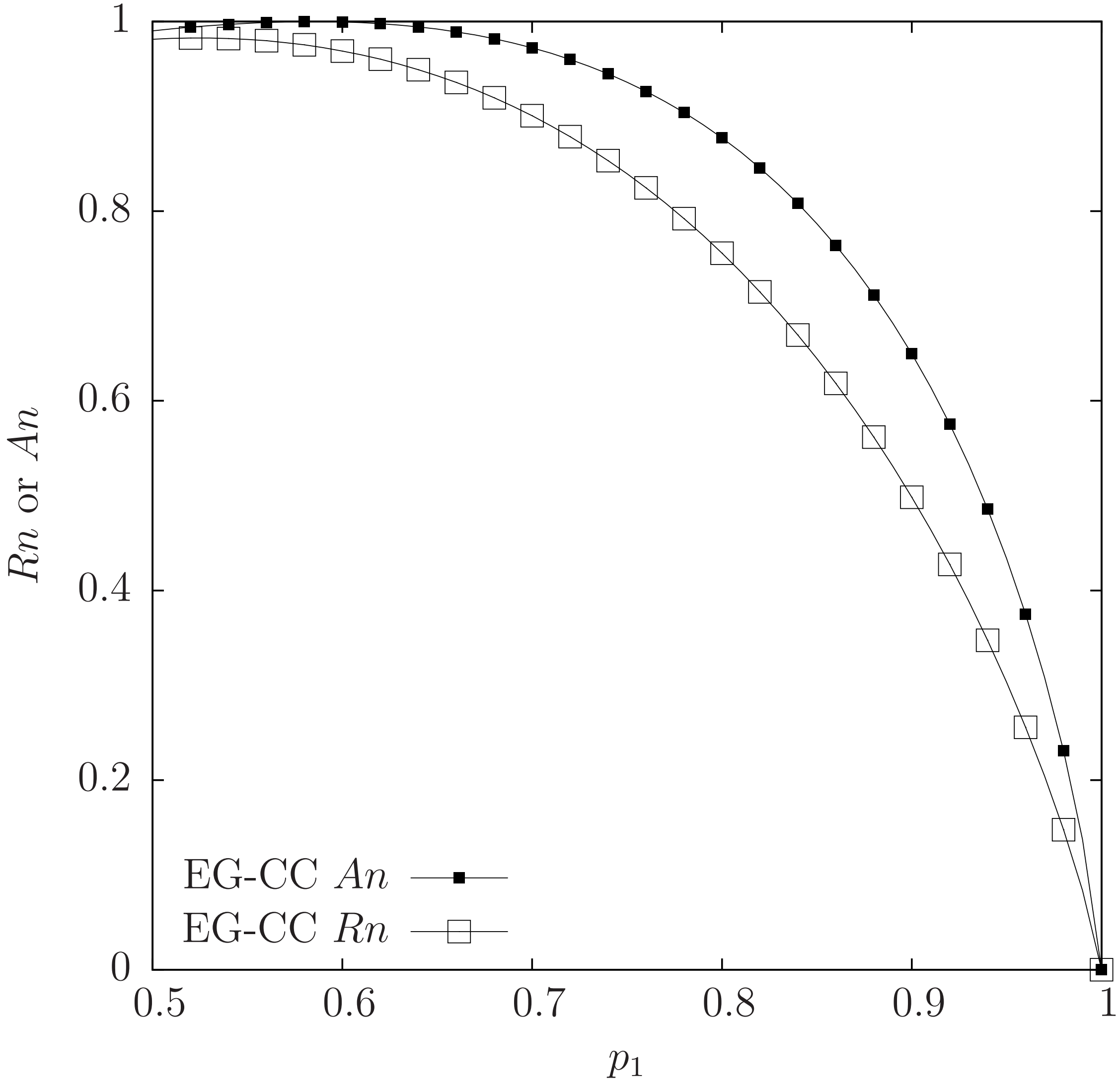
Region above curve represents residual redundancy

EG-CC SSCC benchmarker



- ⇨ EG code is identical to $k = 0$ Exponential-Golomb code.
- ⇨ Convolutional Code (CC) with $n = 2$ encoded bits, 4 states and recursive generator polynomial.
- ⇨ Irregular Unity Rate Code (IrURC), whose components have $n = 1$ encoded bit, 2, 4 or 8 states and recursive generator polynomial.
- ⇨ Quaternary Phase Shift Keying (QPSK) with Gray mapping and puncturing.

Significant EG-CC capacity loss



Rn and An of EG-CC scheme, for zeta distribution.

A is the area beneath the EXtrinsic Information Transfer (EXIT) curve of the EG-CC decoder.

n is the number of encoded bits produced by the CC encoder.

The region above the An curve represents residual redundancy exploited for error correction.

The region between the curves represents un-exploited residual redundancy, giving capacity loss.

Conclusions

- All previous JSCC schemes have infinite complexity when decoding symbols selected from infinite sets.
- SSCC benchmarker suffers from significant capacity loss.

Part 2 - Unary Error Correction Codes

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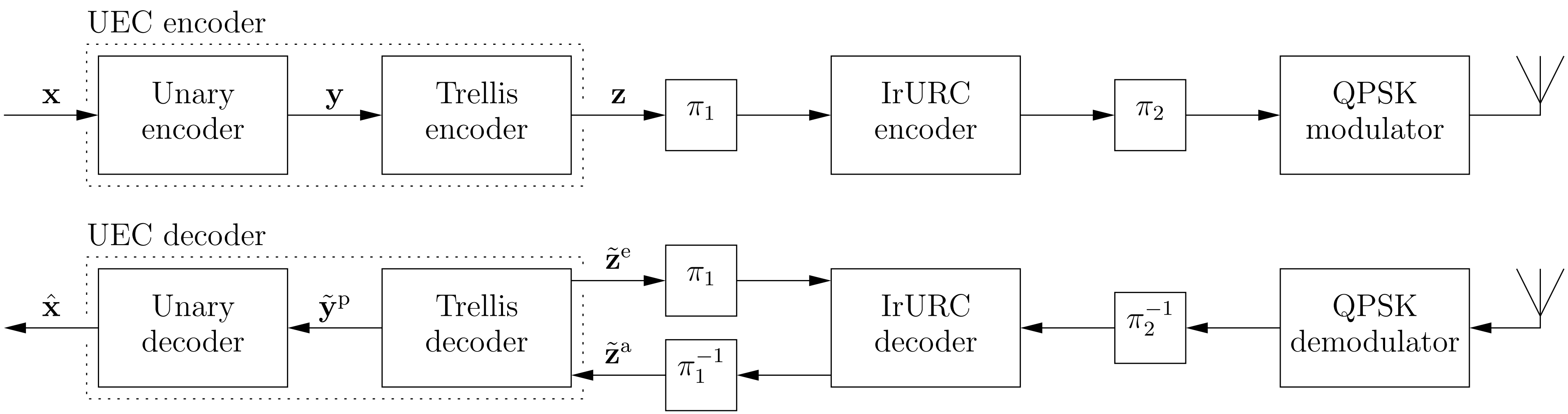
Outline

- ❑ Proposed JSCC scheme using UEC code
- ❑ Near-capacity analysis
- ❑ Error ratio performance
- ❑ Conclusions

* Joint Source and Channel Coding (JSCC)

* Unary Error Correction (UEC) Code

Proposed JSCC UEC scheme



⇨ Replace EG code with a unary code.

⇨ Replace CC code with a novel trellis code, having $n = 2$ encoded bits and r states.

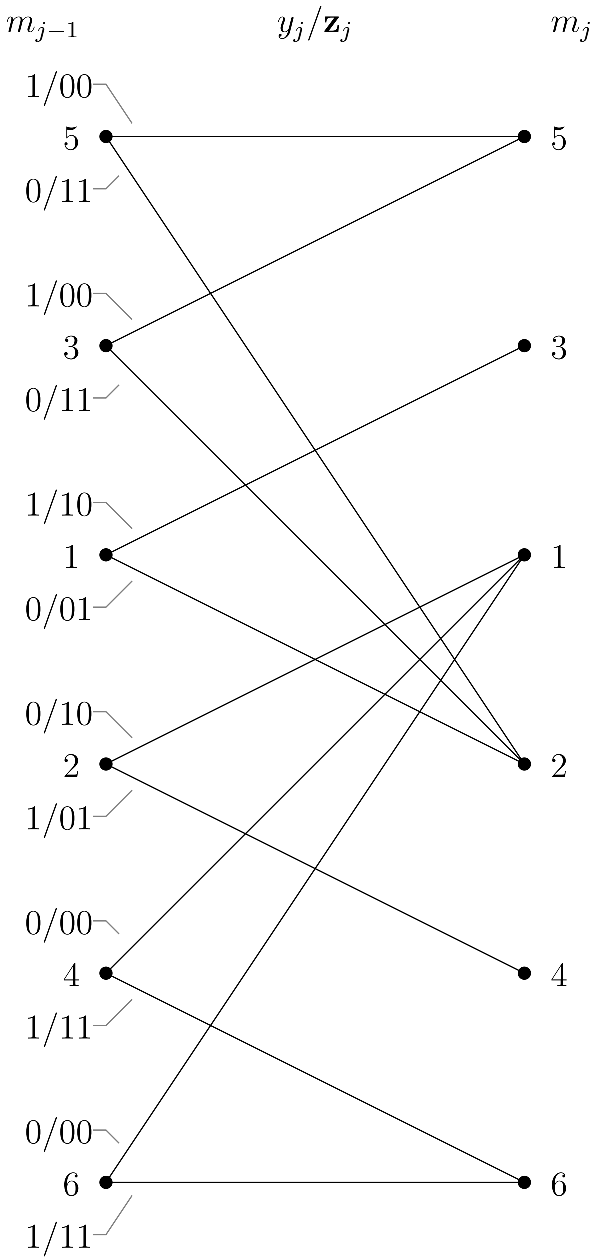
x_i	$P(x_i)$			Unary(x_i)	EG(x_i)
	$p_1 = 0.7$	$p_1 = 0.8$	$p_1 = 0.9$		
1	0.7000	0.8000	0.9000	0	1
2	0.1414	0.1158	0.0717	10	010
3	0.0555	0.0374	0.0163	110	011
4	0.0286	0.0168	0.0057	1110	00100
5	0.0171	0.0090	0.0025	11110	00101
6	0.0112	0.0054	0.0013	111110	00110
7	0.0079	0.0035	0.0007	1111110	00111
8	0.0058	0.0024	0.0004	11111110	0001000
9	0.0044	0.0017	0.0003	111111110	0001001
10	0.0034	0.0013	0.0002	1111111110	0001010

Table 2: The first ten codewords of unary and Elias Gamma (EG) codes.

Unary code

Average code-word length $l = \sum_{x \in \mathbb{N}_1} P(x)x$

l becomes infinite for $p_1 < 0.608$



Trellis code

Here, the trellis has $r = 6$ states.

Encoding begins in state $m_0 = 1$.

e.g. for symbols $\mathbf{x} = [4, 1, 2, 1, 3, 1, 1, 1, 2, 2]$,

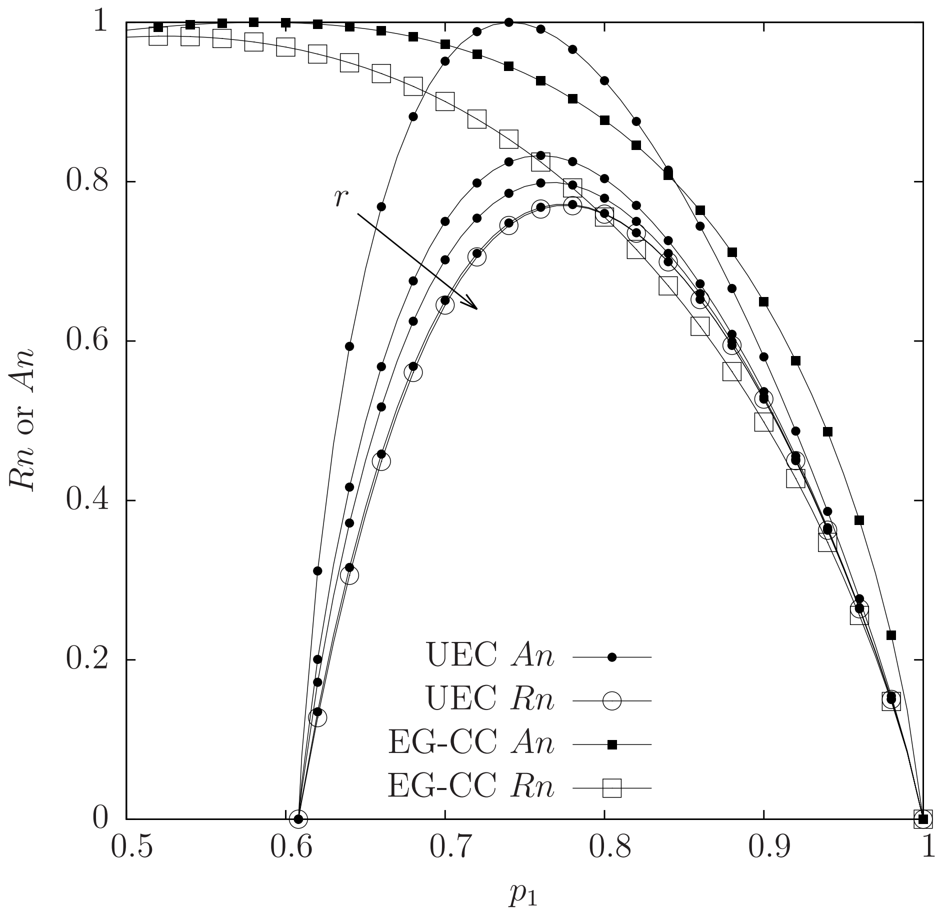
⇨ $\mathbf{y} = [111001001100001010]$.

⇨ $\mathbf{m} = [1, 3, 5, 5, 2, 1, 3, 2, 1, 3, 5, 2, 1, 2, 1, 3, 2, 4, 1]$.

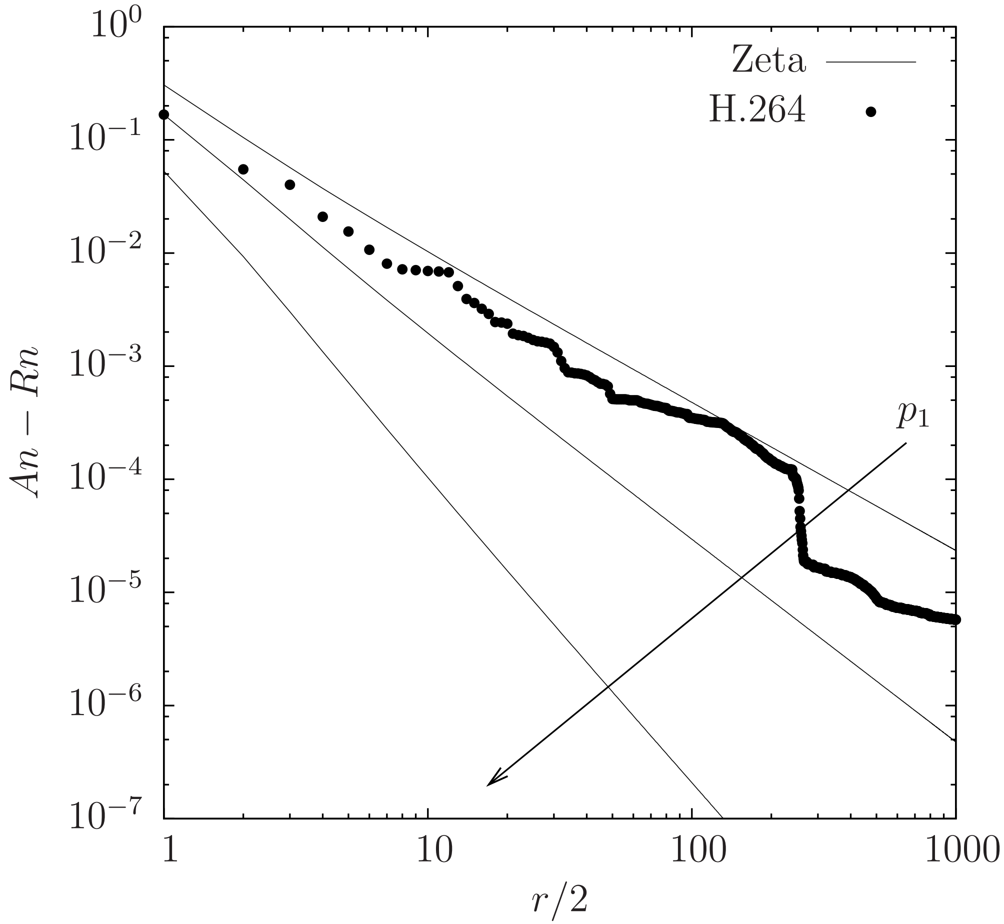
⇨ $\mathbf{z} = [100000111010111010001110011001110100]$.

Each transition occurs with a different probability, which is exploited during soft-in soft-out decoding.

Vanishing UEC capacity loss

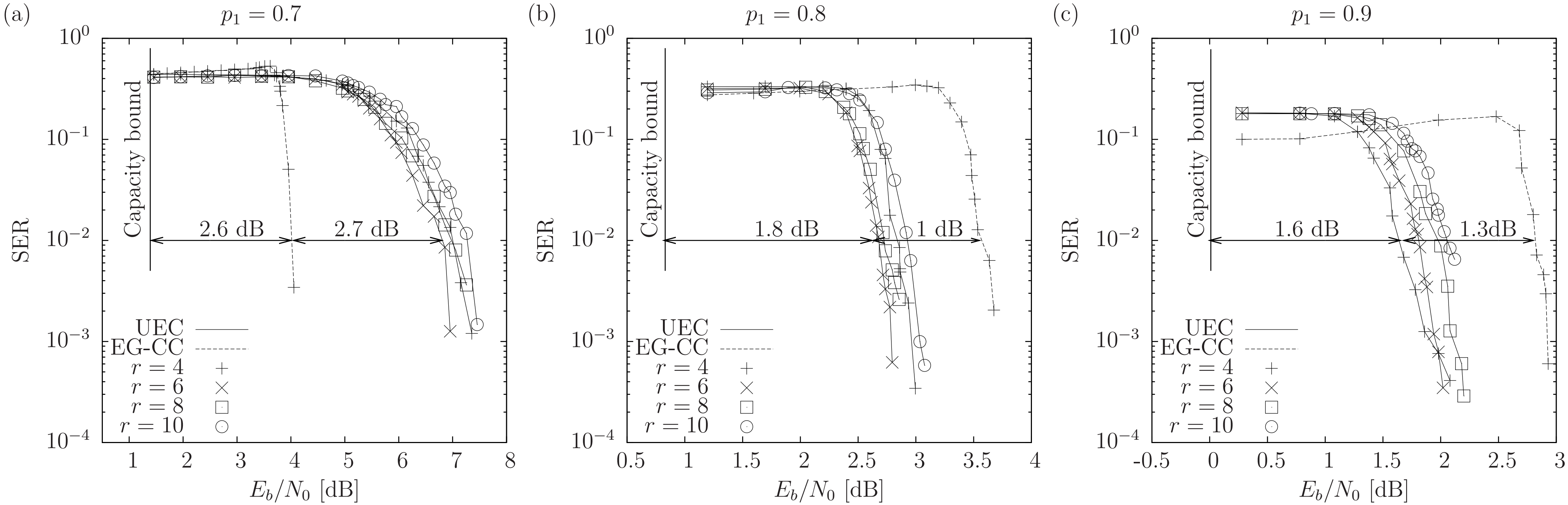


Rn and An of EG-CC scheme and UEC scheme having $r \in \{2, 4, 6, 30\}$ states, for zeta distribution.



Capacity loss in UEC scheme, for zeta distribution having $p_1 \in \{0.7, 0.8, 0.9\}$.

Symbol Error Ratio (SER) Performance



SER performance of EG-CC and schemes, for zeta distribution having $p_1 \in \{0.7, 0.8, 0.9\}$. Uncorrelated narrowband Rayleigh fading channel with QPSK modulation. 10^4 symbols per frame and up to 10^4 Add-Compare-Select (ACS) operations per symbol.

Conclusions

- SSCC benchmarker suffers from significant capacity loss.
- Proposed JSCC UEC scheme has only moderate complexity and its capacity loss asymptotically approaches zero as the number states r increases.
- As much as 1.3 dB gain within 1.6 dB of capacity bound, without any increase in transmission energy, duration, bandwidth or decoding complexity.
- However, the proposed UEC has an infinite average codeword length for zeta distributed source symbols having $p_1 < 0.608$, as well as poor SER performance for $p_1 = 0.7$.

Part 3 - Elias Gamma Error Correction Codes

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Outline

- ❑ Proposed JSCC scheme using EGEC code
- ❑ Near-capacity analysis
- ❑ Error ratio performance
- ❑ Conclusions

* Joint Source and Channel Coding (JSCC)

* Elias Gamma Error Correction (EGEC) Code

d_i	Unary(d_i)	EG(d_i)	x_i	Unary(x_i)	FLC(t_i)	t_i
1	1	1	1	1		0
2	01	010	2	01	0	0
3	001	011	2	01	1	1
4	0001	00100	3	001	00	0
5	00001	00101	3	001	01	1
6	000001	00110	3	001	10	2
7	0000001	00111	3	001	11	3
8	00000001	0001000	4	0001	000	0
9	000000001	0001001	4	0001	001	1
10	0000000001	0001010	4	0001	010	2

Table 3: The first ten codewords of various source codes.

Elias Gamma code revisited

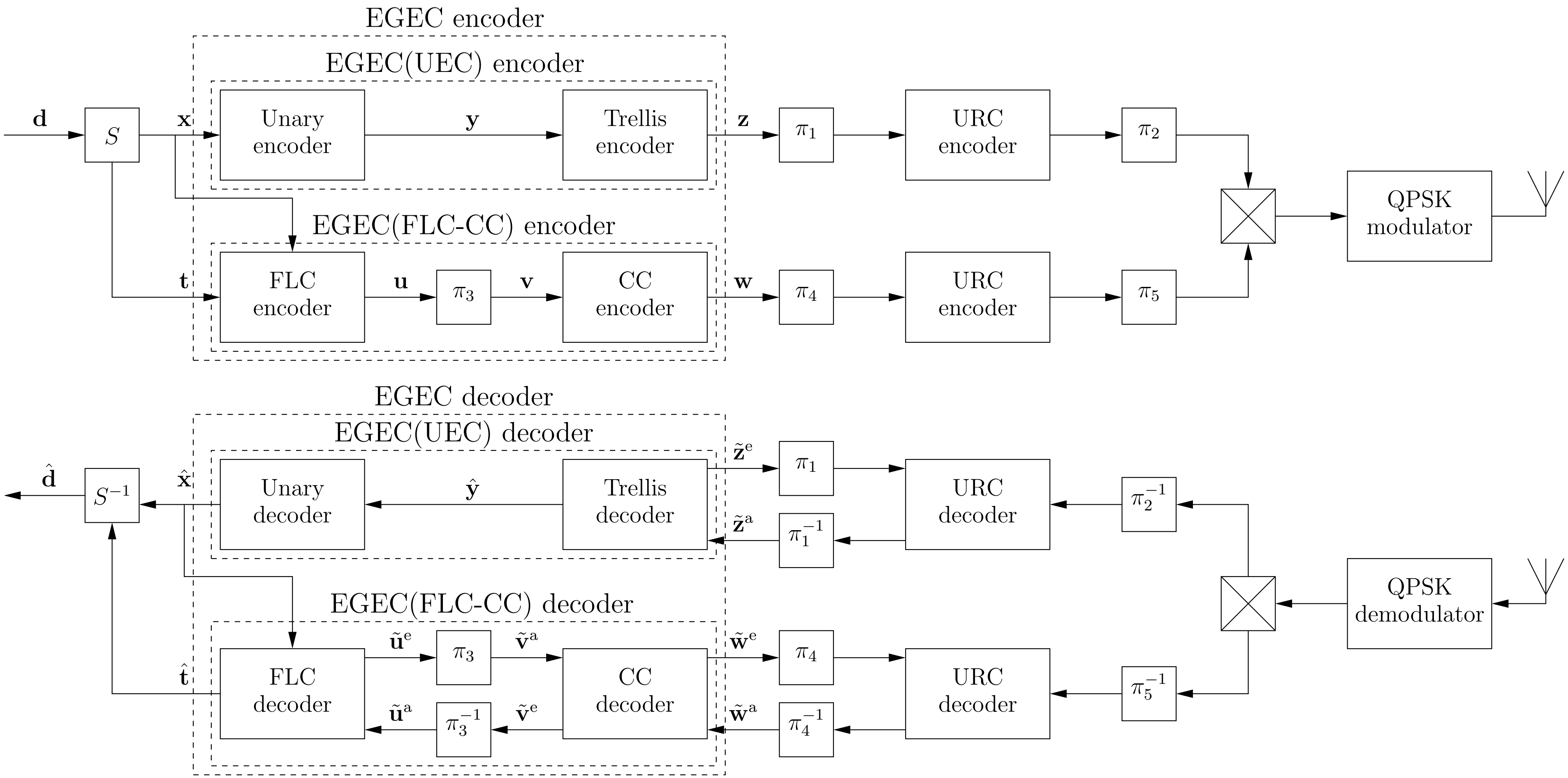
An Elias Gamma (EG) codeword EG(d_i) can be thought of as a concatenation of codewords from a unary code and a Fixed Length Code (FLC).

$x_i = \lfloor \log_2(d_i) \rfloor + 1$

$t_i = d_i - 2^{\lfloor \log_2(d_i) \rfloor}$

$d_i = 2^{x_i - 1} + t_i$

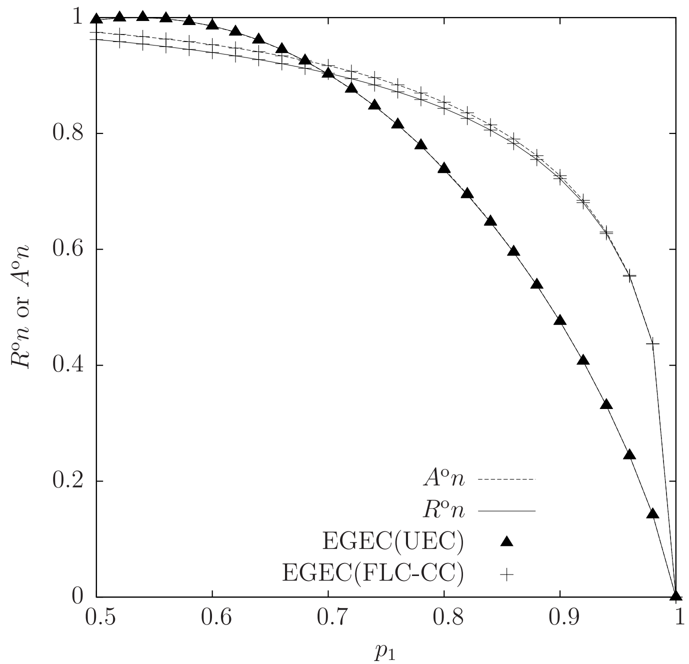
Proposed JSCC EGEC scheme



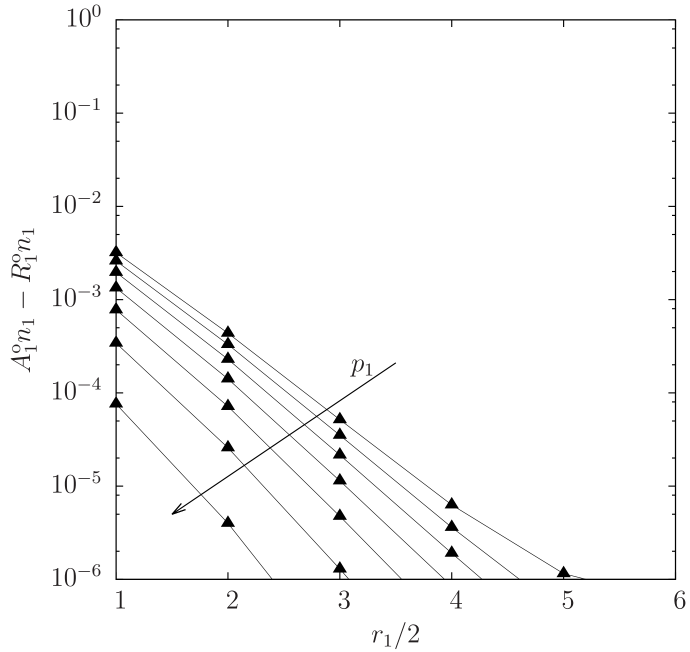
⇨ π_2 and π_5 can use different puncturing rates, to achieve Unequal Error Protection (UEP).

⇨ The FLC decoder only engages in iterative decoding for symbols satisfying $\hat{x}_i \leq x_{\max}$.

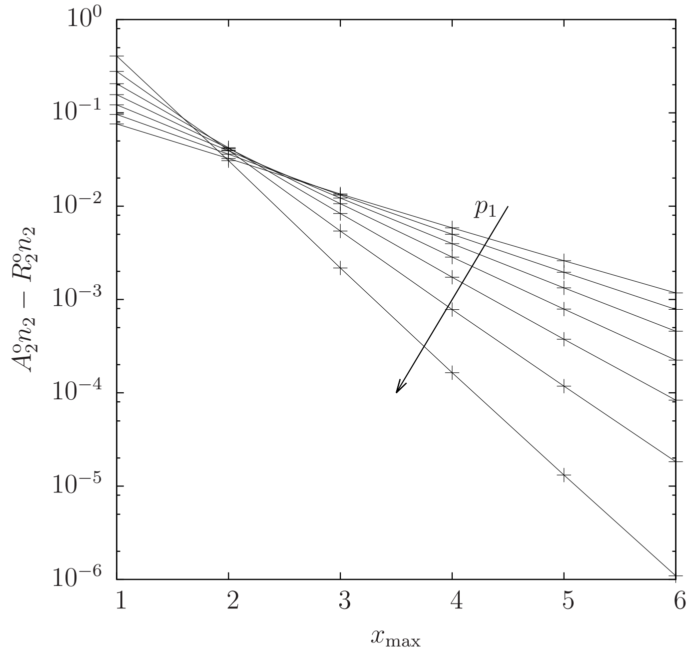
Vanishing EGEC capacity loss



$R^{\circ}n$ and $A^{\circ}n$ of EGEC(UEC) scheme having $r_1 = 4$ states and EGEC(UEC) scheme having $x_{\max} = 3$, for zeta distribution.

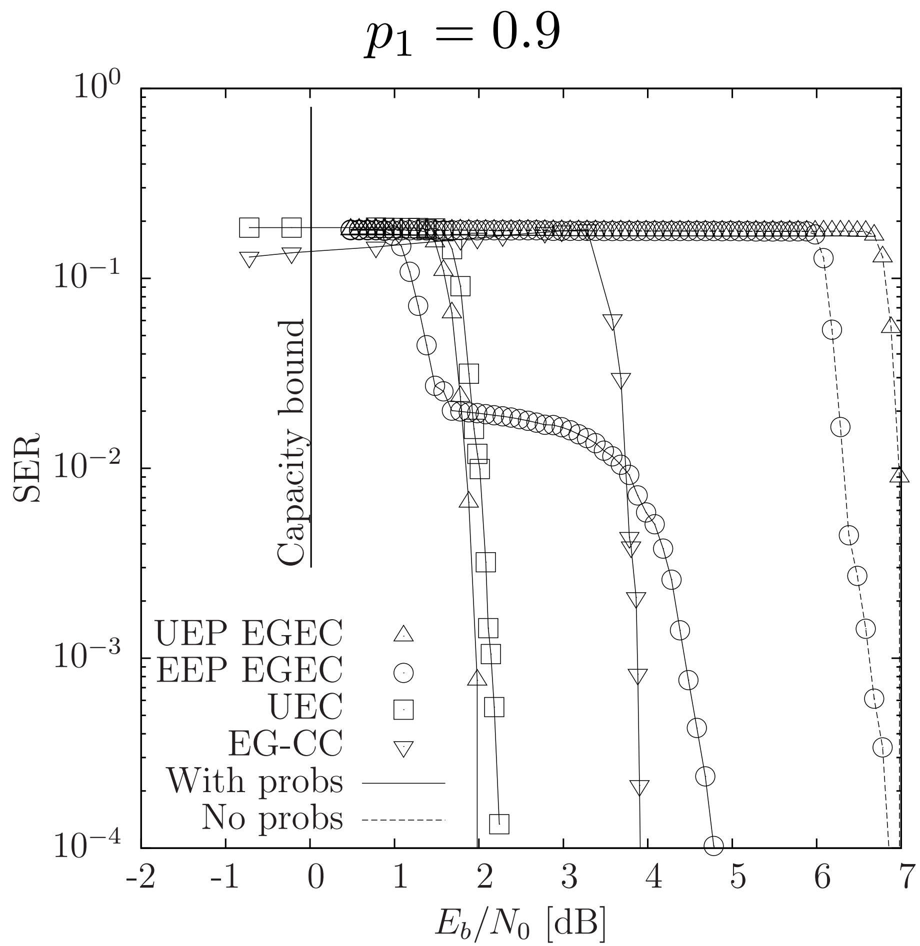


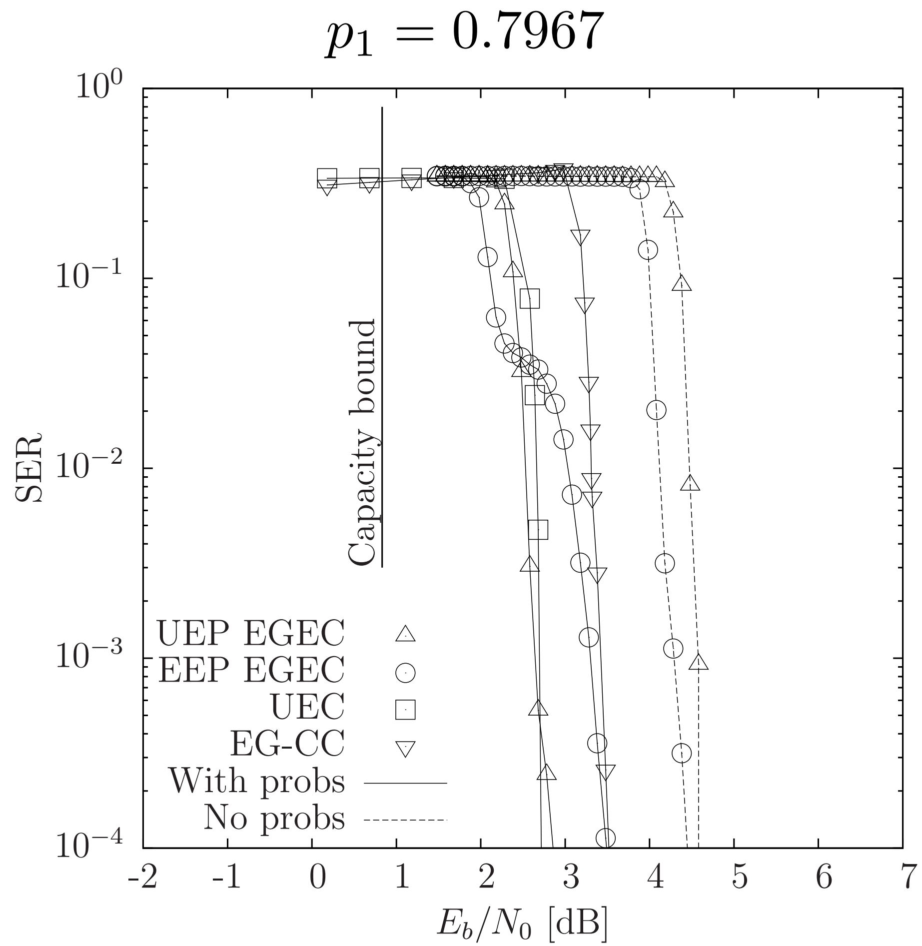
Capacity loss in EGEC(UEC) scheme, for zeta distribution having $p_1 \in \{0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95\}$.



Capacity loss in EGEC(FLC-CC) scheme, for zeta distribution having $p_1 \in \{0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95\}$.

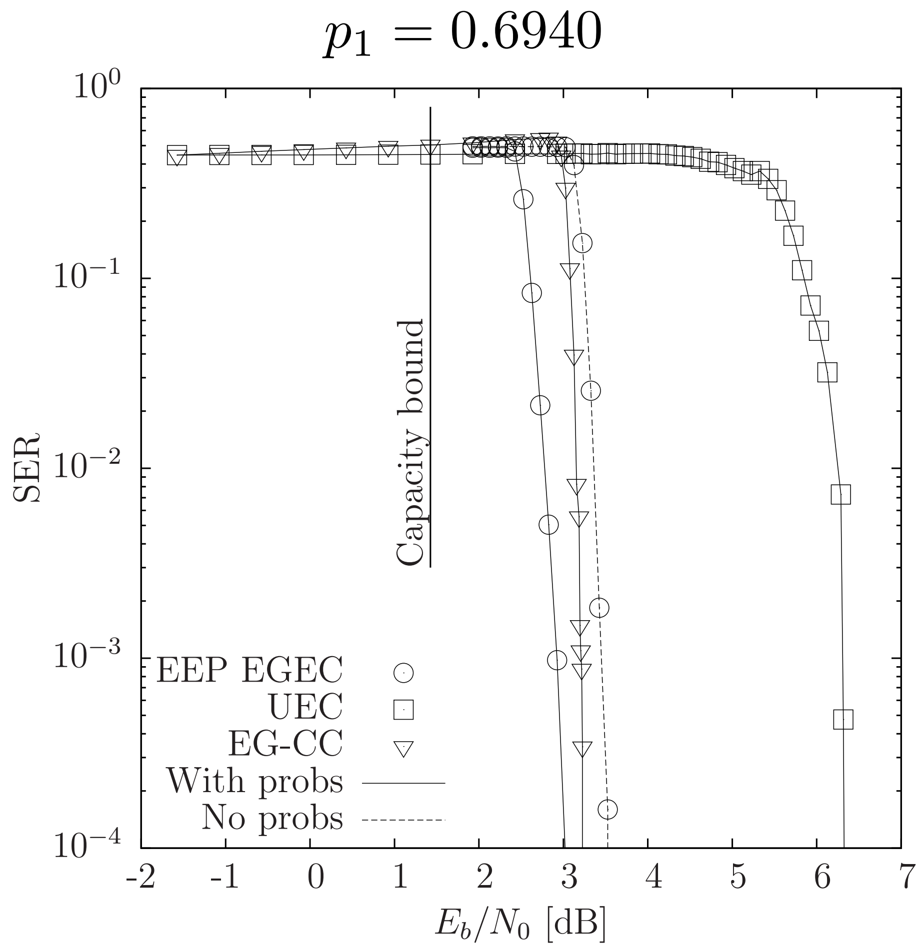
Symbol Error Ratio (SER) Performance

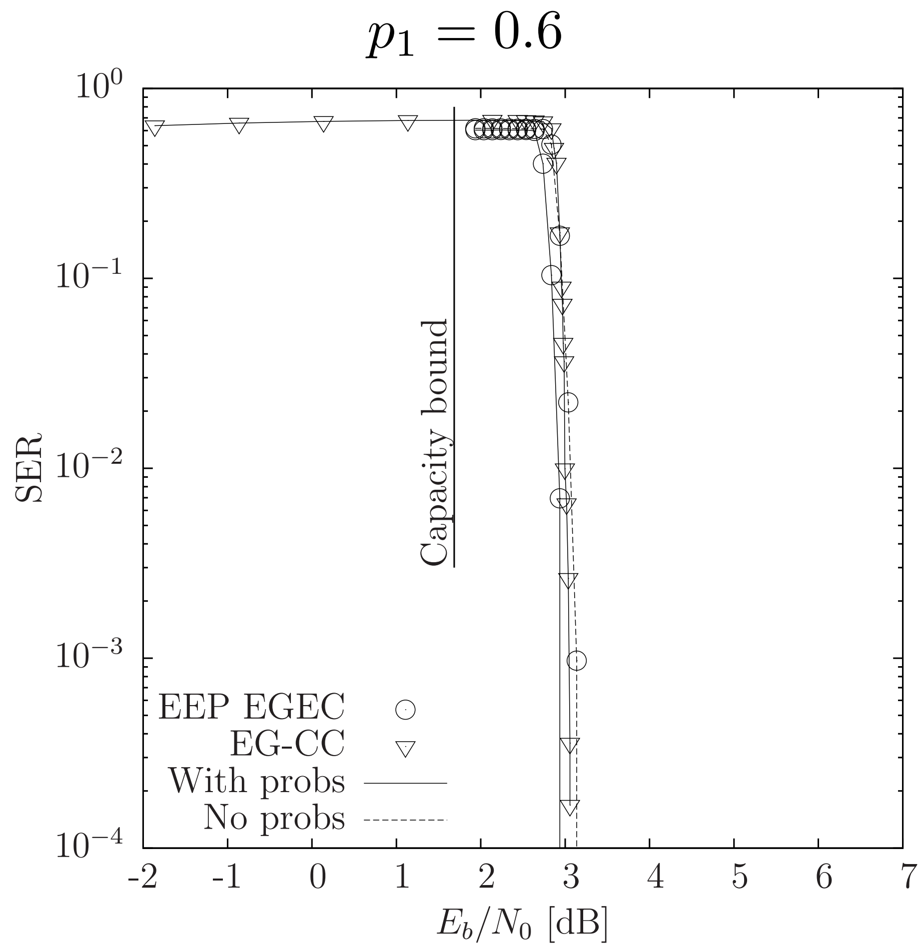




SER performance of EGEC scheme and various benchmarkers, for zeta distribution. Uncorrelated narrowband Rayleigh fading channel with QPSK modulation. 2×10^4 symbols per frame. Results marked 'No probs' were obtained without knowledge of the source symbol distribution at the EGEC decoder.

Symbol Error Ratio (SER) Performance





SER performance of EGEC scheme and various benchmarkers, for zeta distribution. Uncorrelated narrowband Rayleigh fading channel with QPSK modulation. 2×10^4 symbols per frame. Results marked 'No probs' were obtained without knowledge of the source symbol distribution at the EGEC decoder.

Conclusions

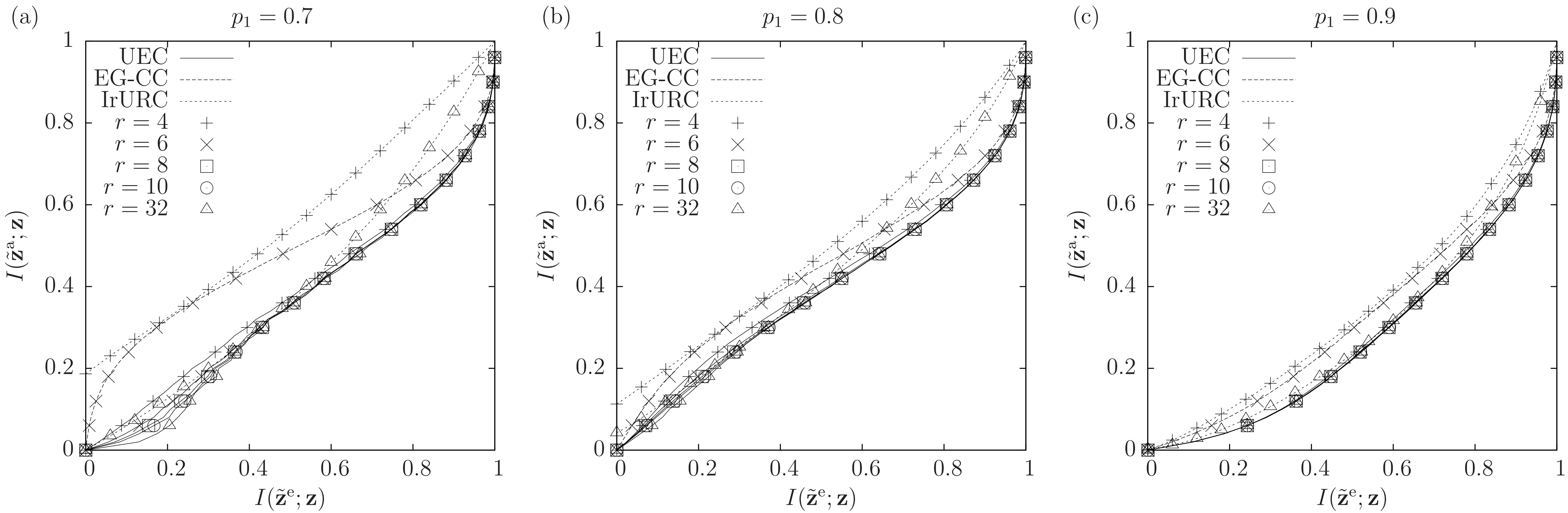
- SSCC benchmarker suffers from significant capacity loss for zeta distributed source symbols, having $p_1 \in \{0.9, 0.7967\}$.
- UEC benchmarker has an infinite average codeword length for zeta distributed source symbols having $p_1 < 0.608$, as well as poor SER performance for $p_1 = 0.7$.
- Proposed JSCC EGEC scheme supports $p_1 < 0.608$, has only moderate complexity and its capacity loss asymptotically approaches zero as the number states r_1 used in the EGEC(UEC) scheme increases and as the value of $x_{\max}$ used in the EGEC(FLC-CC) scheme increases.
- For each value of p_1 considered, the proposed EGEC scheme offers the best SER performance.

Thank you!

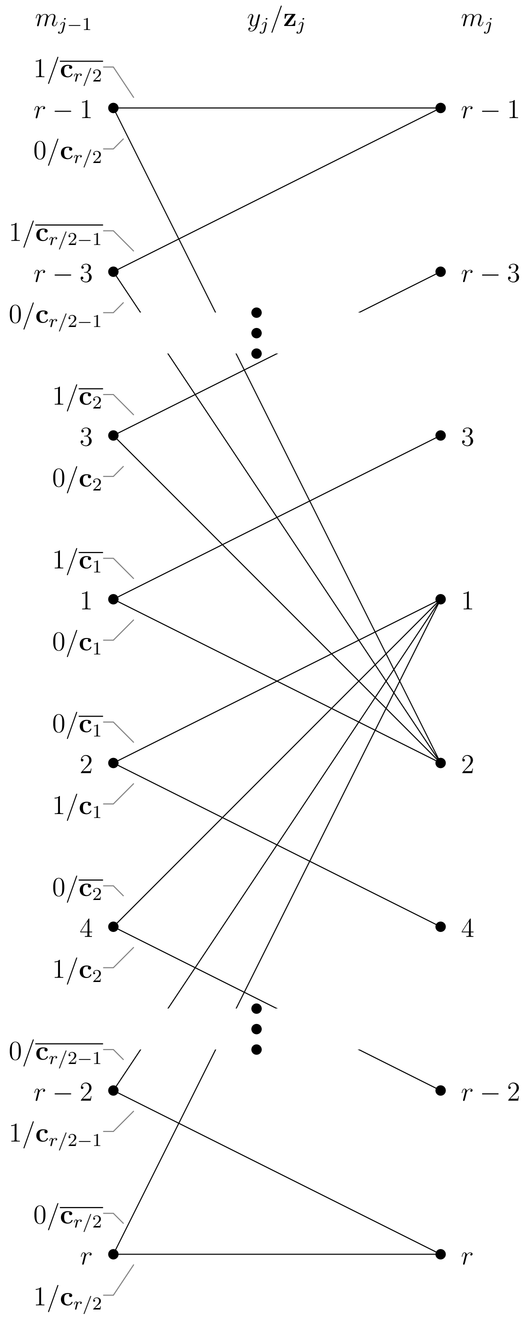
Maunder, R.G., Zhang, W., Wang, T. and Hanzo, L. (2013) A unary error correction code for the near-capacity joint source and channel coding of symbol values from an infinite set. IEEE Transactions on Communications, 61, (5), 1977-1987. http://eprints.soton.ac.uk/341736/

Zhang, W., Maunder, R.G. and Hanzo, L. (2013) On the complexity of unary error correction codes for the near-capacity transmission of symbol values from an infinite set. In, 2013 IEEE Wireless Communications and Networking Conference (WCNC), Shanghai, CN, 2795-2800. http://eprints.soton.ac.uk/344059/

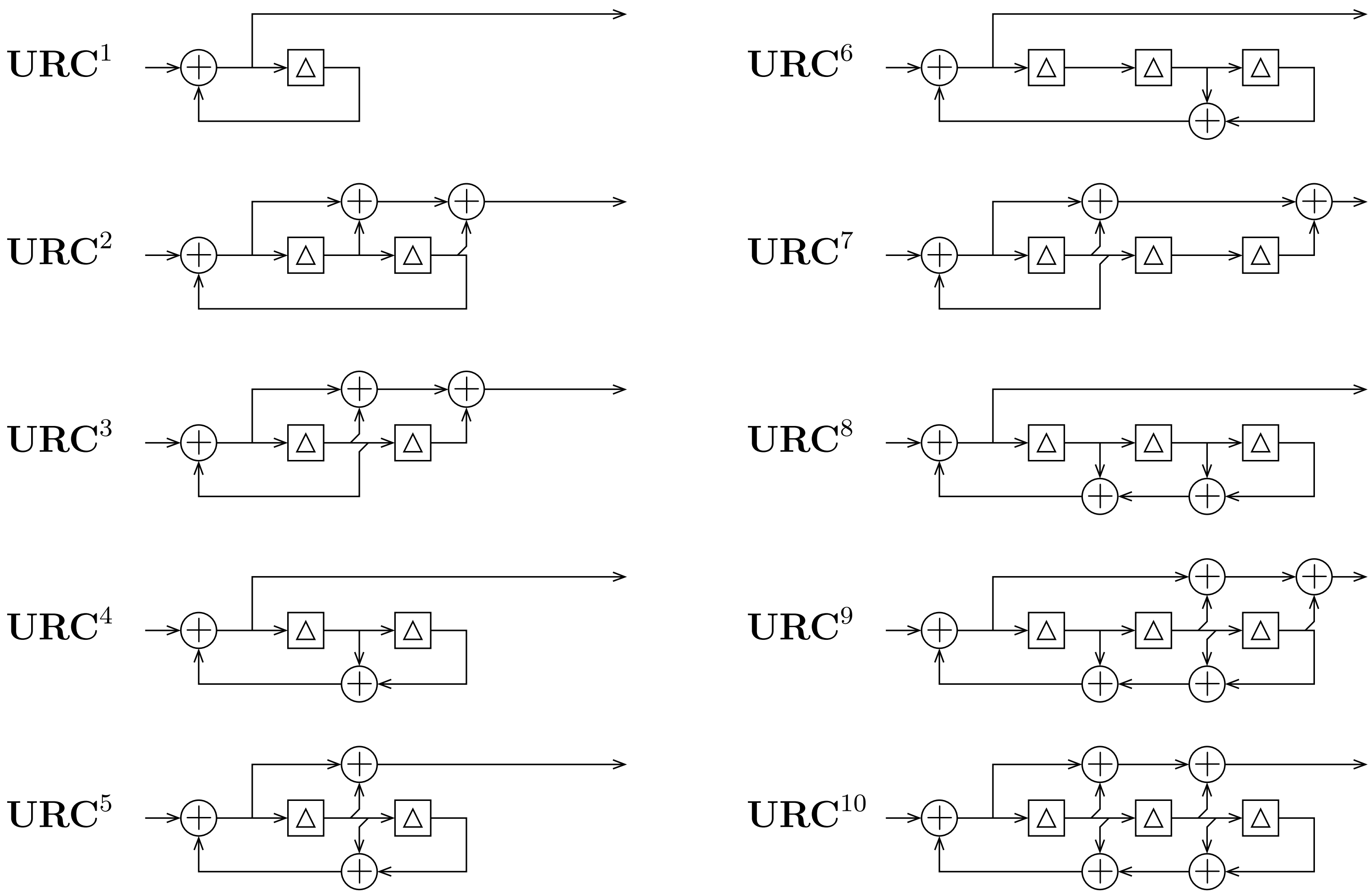
Wang, T., Zhang, W., Maunder, R.G. and Hanzo, L. (2014) Near-capacity joint source and channel coding of symbol values from an infinite source set using Elias Gamma Error correction codes. IEEE Transactions on Communications, 62, (1), 280-292. http://eprints.soton.ac.uk/346658/



Inverted EXIT curves for the UEC decoder having $r \in \{4, 6, 8, 10, 32\}$ states and EG-CC decoder having $r = 4$ states, where $p_1 \in \{0.7, 0.8, 0.9\}$. Corresponding EXIT curves are provided for the IrURC schemes at the lowest E_{b}/N_0 values that facilitates the creation of an open tunnel with the EXIT curves of the $r = 32$-state UEC and the $r = 4$-state EG-CC. Uncorrelated narrowband Rayleigh fading channel with QPSK modulation.


m_{j-1}
$y_j/\mathbf{z}_j$
m_j
$1/\overline{\mathbf{c}_{r/2}}$
$r-1$
$0/\mathbf{c}_{r/2}$
$1/\overline{\mathbf{c}_{r/2-1}}$
$r-3$
$0/\mathbf{c}_{r/2-1}$
$1/\overline{\mathbf{c}_2}$
3
$0/\mathbf{c}_2$
$1/\overline{\mathbf{c}_1}$
1
$0/\mathbf{c}_1$
$0/\overline{\mathbf{c}_1}$
2
$1/\mathbf{c}_1$
$0/\overline{\mathbf{c}_2}$
4
$1/\mathbf{c}_2$
$0/\overline{\mathbf{c}_{r/2-1}}$
$r-2$
$1/\mathbf{c}_{r/2-1}$
$0/\overline{\mathbf{c}_{r/2}}$
r
$1/\mathbf{c}_{r/2}$
$r-1$
$r-3$
3
1
2
4
$r-2$
r

$$P(m, m') = \begin{cases} \frac{1}{2l}\left[1 - \sum_{x=1}^{\left\lceil \frac{m'}{2} \right\rceil} P(x)\right] & \text{if } m' \in \{1, 2, \ldots, r-2\}, m = m' + 2 \\ \frac{1}{2l} P(x) \Big|_{x = \left\lceil \frac{m'}{2} \right\rceil} & \text{if } m' \in \{1, 2, \ldots, r-2\}, m = 1 + \mathrm{odd}(m') \\ \frac{1}{2l}\left[1 - \sum_{x=1}^{\frac{r}{2}-1} P(x)\right] & \text{if } m' \in \{r-1, r\}, m = 1 + \mathrm{odd}(m') \\ \frac{1}{2l}\left[l - \frac{r}{2} - \sum_{x=1}^{\frac{r}{2}-1} P(x)\left(x - \frac{r}{2}\right)\right] & \text{if } m' \in \{r-1, r\}, m = m' \\ 0 & \text{otherwise} \end{cases} \tag{1}$$


URC^1
URC^2
URC^3
URC^4
URC^5
URC^6
URC^7
URC^8
URC^9
URC^{10}

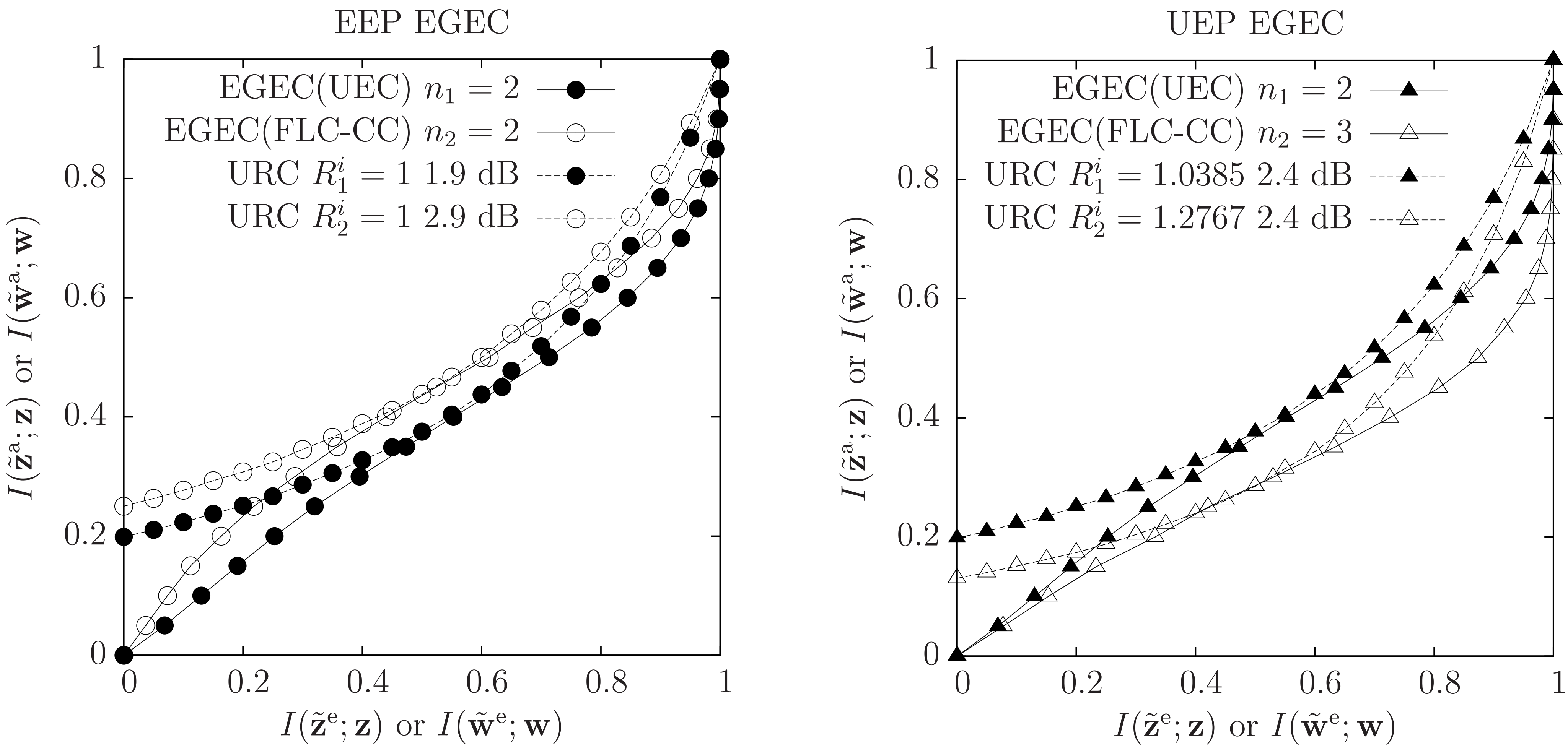
p_1	URC component code fractions α									
	$r = 2$	$r = 4$				$r = 8$				
	(2,3)	(7,5)	(7,6)	(4,7)	(6,7)	(8,B)	(D,C)	(8,F)	(B,F)	(E,F)
0.7	0	0	0.44	0	0.44	0	0.10	0	0.02	0
	0.35	0	0	0.18	0.17	0.05	0	0.25	0	0
0.8	0.18	0	0.71	0.10	0.01	0	0	0	0	0
	0.30	0	0.33	0.27	0.10	0	0	0	0	0
0.9	0	0	0.33	0	0	0.09	0.58	0	0	0
	0	0	0.85	0	0	0.02	0.13	0	0	0

Table 4: The fraction of the IrURC input bit sequence that is encoded by each component code.

Decoder	r	$\max^*$	add	ACS
$n = 2$-bit CC Viterbi decoder $\hat{\mathbf{y}}$	4	2	8	18
$n = 2$-bit CC BCJR decoder $\tilde{\mathbf{z}}^{\mathrm{e}}$	4	10	22	72
$n = 2$-bit Trellis BCJR decoder $\tilde{\mathbf{y}}^{\mathrm{p}}$	4	7	20	55
	6	11	30.5	85
	8	15	40.5	115.5
$n = 2$-bit Trellis BCJR decoder $\tilde{\mathbf{z}}^{\mathrm{e}}$	4	10	22	72
	6	16	32	112
	8	22	42	152
URC BCJR decoder	2	6	19	49
	4	14	37	107
	8	30	73	223

p_1	Scheme	r	R_o	A_o	R_i	η	E_b/N_0 $[dB]$ for $C=\eta$	E_b/N_0 $[dB]$ for $A_i=A_o$	E_b/N_0 $[dB]$ for open tunnel
0.7	EG-CC	4	0.4503	0.4861	1	0.9006	1.39	2.03	3.5
	UEC	4	0.3226	0.3751	1.3958			2.70	3.8
		6		0.3510				2.09	3.7
		8		0.3412				1.85	3.7
		10		0.3361				1.72	3.6
		32		0.3253				1.46	3.4
0.8	EG-CC	4	0.3779	0.4387	1.0048	0.7594	0.83	1.96	3.1
	UEC	4	0.3797	0.4019	1			1.24	2.4
		6		0.3896				1.01	2.0
		8		0.3853				0.92	1.8
		10		0.3833				0.90	1.8
		32		0.3801				0.84	1.8
0.9	EG-CC	4	0.2492	0.3247	1.0578	0.5272	0.01	1.72	2.2
	UEC	4	0.2636	0.2682	1			0.11	0.9
		6		0.2651				0.04	0.9
		8		0.2642				0.02	0.8
		10		0.2639				0.01	0.8
		32		0.2636				0.01	0.7

Outer coding rate R_o, inner coding rate R_i and total throughput η for two schemes with different values of p_1 and r. Three categories of E_b/N_0 where $C=\eta$ and $A_i=A_o$ in theory, and where tunnel is open in simulation, respectively.



Inverted EXIT curves for the EGEC(UEC) decoder having $r_1 = 4$ and EGEC(FLC-CC) decoder having $x_{\mathrm{max}} = 3$, where $p_1 = 0.7967$. Corresponding EXIT curves are provided for the URC schemes at the lowest E_{b}/N_0 values that facilitates the creation of an open tunnel with the EXIT curves of the EGEC scheme. Uncorrelated narrowband Rayleigh fading channel with QPSK modulation.

p_1	Scheme			n	r	R^{o}	A^{o}	R^{i}	η	E_{b}/N_0 [dB] for $C = \eta$	E_{b}/N_0 [dB] for $A^{\mathrm{i}} = A^{\mathrm{o}}$	E_{b}/N_0 [dB] for open tunnel	Complexity
0.9	EGEC	EEP	UEC	2	4	0.2378	0.2378	1.0578	0.5272	0.01	2.4	3.9	267
			FLC-CC	2	4	0.3609	0.3636						
		UEP	UEC	2	4	0.2378	0.2378	1.1251			0.1	1.0	286
			FLC-CC	3	4	0.2406	0.2424	1					
	UEC			2	4	0.2636	0.2682	1			0.1	1.5	250
	EG-CC			2	4	0.2492	0.3247	1.0578			1.6	2.4	257
0.7967	EGEC	EEP	UEC	2	4	0.3721	0.3721	1	0.7620	0.84	1.6	2.9	338
			FLC-CC	2	4	0.4229	0.4283						
		UEP	UEC	2	4	0.3721	0.3721	1.0385			0.9	2.4	379
			FLC-CC	3	4	0.2820	0.2855	1.2767					
	UEC			2	4	0.3810	0.4041	1			1.3	2.5	331
	EG-CC			2	4	0.3810	0.4410	1			2.0	3.0	322
0.6940	EGEC	EEP	UEC	2	4	0.4533	0.4535	1	0.9066	1.43	1.5	2.5	431
			FLC-CC	2	4	0.4533	0.4599						
	UEC			2	4	0.3112	0.3654	1.4565			2.7	4.5	614
	EG-CC			2	4	0.4533	0.4877	1			2.0	3.0	410
0.6	EGEC	EEP	UEC	2	4	0.4906	0.4910	1	0.9690	1.69	1.8	2.8	547
			FLC-CC	2	4	0.4699	0.4766						
	EG-CC			2	4	0.4845	0.4998	1			2.0	3.0	522

Outer coding rate R^{o}, inner coding rate R^{i} and total throughput η for various schemes with different values of p_1, n and r. Three categories of E_{b}/N_0 where $C = \eta$ and $A^{\mathrm{i}} = A^{\mathrm{o}}$ in theory, and where tunnel is open in simulation, respectively.