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FACULTY OF LAW, ARTS & SOCIAL SCIENCES

School of Social Sciences

Portfolio based VaR model: A Combination of Extreme Value Theory (EVT) and Dynamic Conditional Correlation (DCC) model

by

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To My Family

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

FACULTY OF LAW, ARTS & SOCIAL SCIENCE
SCHOOL OF MANAGEMENT

Doctor of Philosophy

Portfolio based VaR model: A Combination of Extreme Value Theory (EVT) and Dynamic Conditional Correlation (DCC) model

JO-YU WANG

This thesis fills a gap in the risk management literature and expands the understanding of the portfolio value at risk (VaR) by providing a theoretical market risk measurement of a portfolio (called "GEV-DCC model"), which combines the tail dynamic conditional correlation (tail-DCC) and extreme value theory. According to the spirit of VaR, the tail distribution is more important than the entire distribution, as well as the correlation in the tail area between various assets. The main advantage of this approach is the increase of accuracy in the parameter estimation of the tail distribution and more consistent correlation measurement for VaR. The results from this method are compared with four other conventional VaR approaches; GARCH model, RiskMetrics, stochastic volatility, and historical simulation. Furthermore, three quality measures are applied to evaluate the suitability, conservativeness, and magnitude of loss of the forecasted VaR, which offer more information from the forecasted VaR pattern.

Applying 16 major equity index returns from developed and emerging markets, this study finds that the GEV-DCC model offers a more accurate coverage across the blocks in the three hypothetical portfolios (the developed equity markets, Asian and Latin American equity markets) compared with the four competing models. The uncovered rates of the GEV-DCC model with the 5-day block approach are generally close to the given probability (α) set in the VaR calculation. These consistent results can also be found in the robustness test with the shorter forecasting period. In the quality checks, the GEV-DCC presents a relatively stable pattern in the daily and 10-day VaR results. In addition, the GEV-DCC model also provides satisfactory results in the conservativeness and potential loss tests although no direct evidence indicates that it delivers the best result in these two checks. We also find significant differences between the original DCC and the tail-DCC. This evidence shows that the correlations between equity markets in the left tail are significantly higher than the ones in the right tail, and there are significant changes (generally rising) in the tail-DCC patterns around the period of financial crisis in the third quarter of 2008. The results from this study could potentially provide a critical reference for investors in measuring or managing the market risk.

Key Words: Value at Risk, VaR, Extreme Value Theory, EVT, Market risk, Risk management

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DECLARATION OF AUTHORSHIP

I, ……………………..JO-YU WANG……………………, declare that the thesis entitled

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……………………..………………………………………………………………………

and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

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Signed: ………………………………………………………………………..

Date:……………………………………………………………………………

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Terminology

α	A small probability in the tail of the return distribution
Q	Average value exceeding corresponding VaRs of the violated observation
D	Average distance between forecasted VaRs and the actual returns of non-violated observations
ETV	Extreme value theory
HS-250	Historical simulation with 250 previous returns
GEV	Generalized extreme value distribution
MSE	Mean squared error
POT	Peak over threshold
VaR	Value at Risk

Chapter 1 Introduction

1.1. Motivation and Aim

Over the last three decades, both the importance of risk measurement and the efficiency of risk management tools have increased dramatically. Following the havoc to international equity markets in 1987, the concept of risk management or risk control has become widely spread across the markets. In recent years, there has been a rapid increase of uncertainty around the world economy; international financial markets have been seriously afflicted with various crises, and investors and financial institutions have frequently suffered huge losses. In 1988 the Basel Committee on Banking Supervision raised the profile of risk management in the banking sector by finalizing the Basel Capital Accord, and then in 1996 they included a capital requirement for market risk in an amended Basel Accord. A well-known investment bank, J.P. Morgan, publicised their techniques and strategies for risk management in 1995. Following these early beginnings there was an explosion of various risk models proposed based on a wide variety of different assumptions and aspects. The Value at Risk (VaR) model was derived under this context, and primarily used for measuring market risk, defined as a decrease in the value of a position due to the changes in the financial market prices. However, in the last decade or so, several financial crises have still had serious impacts on the financial markets despite these risk measures being in place, implying that there might still be something lacking in the current system of risk management. In addition, due to increasing globalization of the financial markets and significant advances in trading system technology, the impact of a certain financial crisis anywhere can spread very rapidly around the world and have significant consequences that would not have been conceivable ten or twenty years ago. Thus, the inadequacy of existing risk management system offers plenty of scope for academic discussion, research and development.

The difficulties of risk measurement and management in the financial markets have spawned voluminous research. Most of the research in the risk-modelling field focuses on the distribution of financial returns. Some emphasize a parametric method by assuming that the financial returns follow a specific distribution, for example the normal distribution, although this popular assumption does not actually hold true in real financial markets (J.P.Morgan (1996), Christoffersen and Diebold (2000), Pafka and Kondor (2001), and Bauwens and Laurent (2005)). Alternatively, some apply non-parametric techniques to avoid the issue of the distribution assumption in parametric approach (Beder (1995), Hendricks (1996), Barone-Adesi et al. (1998), Barone-Adesi et al. (2002), and Boudoukh, Richardson, and Whitelaw (1998)). Yet the two methods mentioned above still have significant weaknesses when measuring market risk. A group of researchers suggest that for risk management the estimation of return distributions should only focus on the estimation of tail distribution by applying extreme value theory (EVT)[1], which increases the accuracy of the parameters estimation (McNeil (1999), Lauridsen (2000), and McNeil and Frey (2000)) — This is the major inspiration for this research. McNeil (1999) is one of the pioneers in the studies using EVT for measuring market risk. He compared VaR model based on EVT to other popular measures and suggested that EVT models provides the best results in measuring market risk of DAX index. In the wake of the uncertainty over the global stock market we face today, the correlation forecast of financial assets for risk management is a new issue, and yet paramount to portfolio management. In this thesis, the practical risk measurement of a portfolio is

[1] In extreme value theory, the tail-distribution is called generalized extreme value distribution (GEV). The details of this distribution and its derivatives will be discussed in Chapter 2.

reconsidered, based on the major characteristics of financial returns such as the information from extreme returns and their dynamic correlation[2]. Before providing such a portfolio VaR measure, several critical issues need to be discussed and accounted for. Firstly, although the concept and the model of VaR have been researched and developed over the last twenty years, most previous research focused on modelling univariate VaR rather than the VaR of a portfolio because of the complexity of multivariate analysis. However, financial institutions normally hold portfolios including a large number of assets; there is thus a strong need for a comprehensive portfolio VaR model for measuring (or managing) market risks and managing the portfolio.

Secondly, it is widely known that the correlation between financial assets plays a critical role in portfolio management. However, the question of how to appropriately measure the correlation of the individual assets in the portfolio for risk modelling is still inconclusive. Although a group of scholars have proposed different type of dynamic conditional correlation (DCC) models since Engle's (2002) seminal work, it is still unclear if the DCC model and its derivatives are suitable for risk modelling. Thus, a new approach for measuring dynamic correlations for portfolio VaR needs to be discussed. In this new approach of correlation, two critical concepts (called "seriality" and "correlation with seriality") emphasizing the importance of order in the time series are introduced and applied in the estimation of dynamic correlations. Thirdly, the methods of VaR evaluation in previous research mainly concentrated on the number of violated VaRs, and non-violated VaRs were entirely ignored. In this thesis, we suggest that VaR models should be tested from two aspects: the characteristics of violated and non-violated VaRs. Thus, we can have a comprehensive understanding of the VaR model.

To illustrate the application of the portfolio VaR model suggested in this study, an empirical study with the daily returns of sixteen equity indices over a sample time period spanning from

[2] In this study, the method for accounting extreme return and dynamic conditional correlation is proposed and symbolized by GEV-DCC. The DCC here indicates the concept of dynamic conditional correlation.

January 1990 to April 2010 is also provided. The dataset comprises six developed equity indices, six Asian equity indices, and four Latin American equity indices.

The results show that compared with other approaches the GEV-DCC model is generally the best portfolio VaR measurement in the coverage tests, and also provides satisfactory patterns in the quality checks. In the univariate analysis, the coverage test results of the GEV models (with different blocks), as well as historical simulation and the GARCH model, provide accurate daily-VaR forecasts although the GEV model is not satisfactory in the developed markets. In the correlation analysis, the results of the tail-DCC show that there are some imbalance phenomena and changeable patterns within the conditional correlations between the left and the right tails, which is consistent with previous research in this area. These results imply that the relationships between the equity markets tend to co-move in the downturn period rather than in the upturn period.

In the portfolio VaR analysis, we find that the GEV-DCC model not only offers the most accurate coverage but also has the most reasonable quality VaR patterns. For robustness, using a shorter estimation period GEV-DCC model provides consistent results and VaR patterns, indicating that the GEV-DCC model is a stable VaR measure. In the VaR quality checks, from all observations in each VaR series we find the GEV-DCC offers the smallest variation VaR patterns, and historical simulation performs the worst. From the non-violated (D) observation check, the GEV-DCC model tends to be lightly conservative and the GARCH model (historical simulation) offers the best (worst) results in this test. On the other hand, in the check of violated (Q) VaR observations the GEV-DCC, as well as the GARCH model, has equivalently small potential losses exceeding VaR.

1.2. Research questions and thesis structure

The following perspectives are the critical research questions we try to answer in this thesis.

1. How is extreme value theory (EVT) applied in measuring univariate VaR and what is the advantage in using this approach?
2. How do we measure the time-varying correlation, suitable for application in risk management of the extreme returns selected from various financial return sequences? And what is the difference between this correlation and the conventional one?
3. How do we apply the bottom-up procedure to measure the VaR of a portfolio by using the time-varying correlation model to aggregate the individual VaRs as a whole?
4. How do we use the VaR patterns and their corresponding return sequences to comprehensively evaluate the overall performance of VaR models? (i.e. the coverage, stability, conservativeness, and potential losses of VaR models.)
5. How do we apply this portfolio VaR model to real datasets?

To set out the details regarding the objectives and research questions aforementioned, the rest of this thesis is organized as follows. In chapter 2, the fundamental theory of market risk measurement is introduced. The first section shows the context and the introduction of risk management policies in international financial markets. In addition, the distinct concept and the basic calculation of VaR, and the debate in related literature are also explicated in this chapter. Moreover, three types of application for value at risk in practice are explained. In the remaining two sections, the use of extreme value theory for measuring market risk and the advantages of this are set out in detail. In the final section, two crucial concepts (seriality and correlation with seriality) are defined and the importance of them in the correlation analysis is introduced in detail. A simple and explicit mathematical process for the portfolio VaR formula is also shown in this section.

Chapter 3 reviews critical literature in this field. In general, the methods of VaR modelling are separated into three categories: parametric model, non-parametric approach, and semi-parametric model, according to their assumption of the distributions of financial returns. The results in current literature are quite contrary due to the diverseness of the model assumptions and the datasets used within their research. In addition, we also provide an example to explain the importance of order in correlation calculations. In the last section, current evaluation methods of VaR model are reviewed.

Chapter 4 mainly covers the methodology and the datasets used in this thesis. The first section introduces some basic econometric functions, which are used to test the properties of time series data. The following sections describe the estimation of the model suggested by this thesis, and present four competing models which are very common in the literature. In section 4.5, two quantitative approaches widely used for backtesting the credibility of the VaR models are introduced. Furthermore, three benchmarks are suggested here to examine the quality of the VaR model as regards to stability, conservativeness, and the magnitude of violation.

The results and findings are shown in Chapter 5. The first part answers the first research question and presents the results of the univariate VaR measured by GEV and four competing models. Overall, the GARCH model, historical simulation and the model based on extreme value theory perform equally well in the two types of backtesting. The correlation analysis provides direct and significant evidence of fatness and asymmetry in the distributions of the index returns based on the method of tail-DCC. In the portfolio VaR analysis, we provide the comprehensive evidence that the GEV-DCC model suggested in this thesis is the best portfolio VaR measurement compared with the competing models. In addition, for robustness, a portfolio VaR analysis based on 10-day return sequences and a different period of sample sets are shown in the end of this chapter. Thus, all the evidence in this thesis shows that extreme value theory could be applied for measuring market risk more accurately for both univariate and portfolio conditions. Chapter 6 marks the conclusion of the thesis, including a comprehensive summary, the potential contribution of the thesis, research limitations, and suggestions for future research in this line.

Chapter 2 Theory of Market Risk Measurement

2.1 Introduction

This chapter aims to set out the theoretical foundations of the well-known market risk measure, Value at Risk (VaR), and to explain its application in various financial institutions. It commences with a review on the background of VaR, the fundamental theory behind VaR, and the methods of VaR evaluation, in Section 2.2. We also present the important debate started by Artzner et al. (1999) of VaR modelling and regulatory measurement of market risk. In Section 2.3, a basic VaR formula will be exhibited, and the critical part of this formula, conditional volatility, will be discussed for the univariate and multivariate cases. This section shows the roles of Engle's (1982) volatility model and its derivatives in the VaR model. In section 2.4, we introduce three main applications of VaR. Two of these applications have been used in financial institutions for years, but the third is new to practice. Compared with the ex post nature of the previous two applications, the third application emphasizes the ex-ante application of VaR tools, which uses the VaR to evaluate the marginal risk-based performance of the new investment or diversification before they are made. In the last two decades, many institutional investors have suffered huge losses in a series of serious financial market collapses. Thus, we look at whether it is helpful for financial institutions to apply extreme value theory in measuring VaR during international financial market events. Therefore, in Section 2.5, the nature and philosophy of extreme value theory will be discussed in detail. We will also show the mathematical process of VaR modelling with this EVT. The final section of this chapter is the conclusion.

2.2 The Value at Risk (VaR) Framework

2.2.1 Background

When it comes to the history of risk management and value-at-risk, it is evident that some events have evolved indirectly as a forerunner of the system of risk management nowadays, for example the Herstatt event and the Basel Committee accord. In June 1974, the German regulators withdrew the licence of the Bank of Herstatt, a midsize private bank, due to its lack of liquidity and over-indebtedness in its foreign exchange position. Because of different time-zones between Herstatt and its counterparts, a number of banks in New York had sent their payment in Deutsch Marks to exchange for U.S. Dollars in the previous trading day. However, with Herstatt's suspension, they did not then receive their payment in U.S. Dollars and lost the full amount that they had sent. This event let each central bank to realize the need for an international organization to coordinate global transactions. Responding to the Herstatt crisis, the Basel Committee on Banking Supervision (BCBS), under the Bank for International Settlement (BIS), was formed by the G-10 countries in 1975 and designed to oversee the regulation of cross-jurisdictional situations and capital requirements.

In July of 1988, the Basel Committee announced a minimum requirement of capital standard (hereafter, the 1988 Basel Accord), which mainly required commercial banks to maintain enough capital, say 8% of its risky asset, against credit risk. According to the 1988 Basel Accord, bank capital is divided into two areas: core capital and supplementary capital. Under the 1988 Basel Accord, all the bank's assets are assigned a risk weight according to the credit quality of the corresponding counterparty. Roughly speaking, this weighting is from zero for cash or claims on OECD[3] central governments up to 100% for the private sector. After the deregulation of financial markets in the 1990s, commercial banks now actually not only take credit risk but also market risk. Therefore, the Basel Committee published an amended version of the Basel Accord in 1996 (the 1996 Basel Accord) to incorporate market risk into the requirement of minimum capital. The main feature of the 1996 Basel Accord is to allow banks to use internal models for measuring market risk, with the exception of the standardised measurement method. Since then, the term, value-at-risk, formally became a substantial concept in these fields[4]. The announcement of the 1996 Basel Accord fanned the wave of research in VaR and its derivatives, and since then the use of VaR in the banking sector has remained strong for many years. For instance, the investment bank, J.P. Morgan[5], was one of the earliest financial institutions to start using the VaR model in measuring market risk of their daily trading positions. Uptake was encouraged even further following the announcement by the U.S. Securities and Exchange Commission (Commission) of a new rule (Securities Act Release No. 7368) in January of 1997 that required publicly traded firms to have to do quantitative and qualitative disclosures of their market risk in their annual report.

Under Basel II, published in 2004 and effected in 2007, the internal model is permitted to measure credit risk via external rating agents or internal rating models. Nowadays, the concept and technique of value at risk have been widely applied to other categories of risk measurement (Crouhy, Galai and Mark (2000) and Gordy (2000)). Jorion (2002)

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[3] Organisation for Economic Co-operation and Development (OECD) is an international organisation whose main mission is to stabilize the economies of its members.

[4] In fact, J.P. Morgan had done some research similar to this concept during the late eighties, which can be referred to in Guldimann (2000).

[5] Actually, J.P. Morgan can be viewed as the creator of the concept of value at risk. In the earlier stages, they had serious discussions about the importance of fluctuation in the value of a trading position and its earnings. As an investment bank, they were more concerned with the variation of price than earnings. Thus, “value” at risk set the tone of their risk management.

investigated the use of VaR and information quality in the top-twelve largest commercial banks in the U.S., according to their trading positions from 1995 to 1999, and found that the VaR disclosures of these commercial banks were insubstantial and relatively uninformative. However, VaR still plays a significant role in risk management within the banking sector and lately, VaR applications have extensively spread to all the financial industries. Another comprehensive survey of the disclosures of the top 50 banks in the world can be referred to in Pérignon and Smith (2010).

2.2.2 The General Concept of Value at Risk

The concept and model of value-at-risk was first proposed by J.P Morgan in 1994 and became a standard method to measure the risk of a risk position (Alexander, 2005). It describes the worst loss of a risk factor over a particular horizon with a given level of confidence (1-α) if the market is hit by a certain shock (Jorion, 2007). In other words, value at risk offers a simple single number to summarise the maximum potential loss with a given likelihood if the market in the next trading day is bad. Therefore, the value at risk for a long position at the α percentile can be simply defined as

$$\Pr(r_t \leq \mathrm{VaR}_\alpha) = \alpha \qquad (2.1)$$

where r_t and 1-α denote the asset return and confidence levels respectively, for example 95% or 99%. Eq. (2.1) could be rewritten as

$$F_r(\mathrm{VaR}_{\alpha,t}) = \int_{-\infty}^{\mathrm{VaR}_{\alpha,t}} f(r_t)dr \qquad (2.2)$$

Where F_r and $f(r_t)$ are the cumulative distribution function (CDF) and probability distribution function (PDF) of a return, r_t. Under the original normal distribution assumption to asset return, value at risk may be calculated by the following formula.

$$\mathrm{VaR}_\alpha = \mu_r + z_\alpha \sigma_r \qquad (2.3)$$

where μ_r and σ_r indicate the mean and standard deviation of the asset return, respectively. Eq. (2.3) statistically describes the value at risk for a long position as equal to the mean of the financial return plus the critical value (Z_α) with a given probability (α) times the volatility of the financial return. For simplicity, one might assume that the density of financial returns follows a normal distribution. Consequently, if the given probability is α=0.01, then $Z_\alpha < 0$ and the value at risk happens in the left area of the distribution of financial returns, indicating the downside risk of the long position. In contrast, the value at risk of a short position presents in the right area of the corresponding distribution. Alternatively, the concept of value-at-risk for a long position can be shown in Figure 2.1. Geometrically, the value at risk of an asset return is the point corresponding to the critical value based on a given probability (α) in the left tail and the probability density function, f(r), as shown in Eq. (2.3) and Figure 2.1. It is obvious that the value at risk of a single asset is affected by at least three key factors: the probability (α); the volatility of the financial return (σ_r); and the probability density function, f(r). According to the Basel regulation, commercial banks should be tested for performance at the 1% level of their internal risk model for local supervision. Engle (1982) and Bollerslev (1986) lift the curtain on the research of conditional volatility, demonstrating that the volatility of financial returns varies with previous information. In recent research, the assumption of the distribution to financial returns has been modified according to several major properties in practice, for example, fatness and skewness. In addition, the correlation between individual assets also plays a crucial role in calculating value at risk, especially for portfolio value at risk.

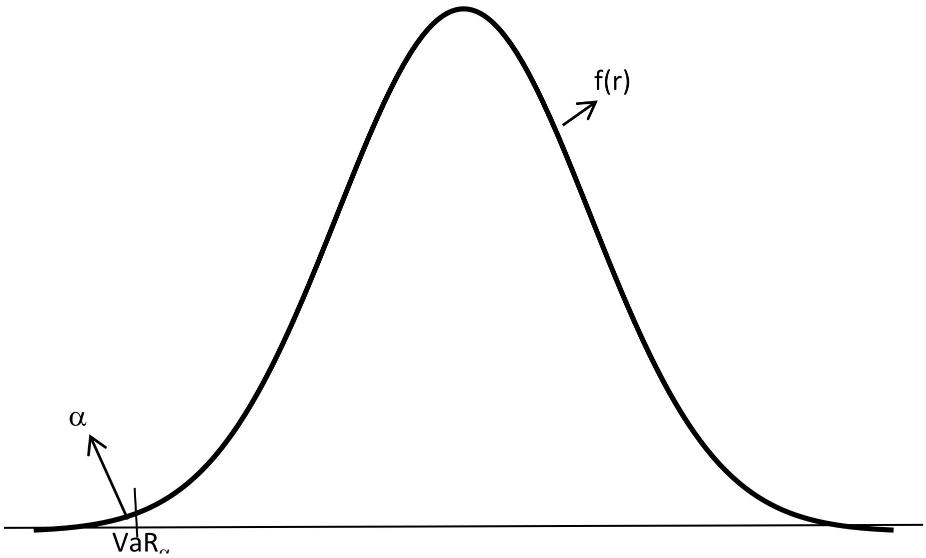

Figure 2.1 The concept of Value at Risk (VaR)

Nowadays, value at risk has become a standard and widespread risk measure. This can be attributed to several reasons. First of all**,** from the perspective of authority, the amendment of the Basel Accord in 1997 allowed banks to use internal market risk management models in order to fulfil the requirements for capital adequacy. This amendment sparked a surge of research around value-at-risk as a so-called internal model, which could then be used by the financial institutions. As a result, banks can now use their own VaR models as the basis for determining the required capital to hold against market risk. Since internal models are seen as requiring less risk capital than the Basel's standard method, most financial intermediaries and their managers who use VaR benefit from both a lower capital requirement and overall better performance. It is therefore not surprising that VaR has been promoted officially as a good risk measure in practice. Financial institutions have now taken one step further in this direction by applying the same concept to credit risk and operational risk. Saunders and Allen (2002) offers a comprehensive discussion and application for applying the VaR model to credit risk measurement and operational risk.

Secondly**,** the simple and clear content of VaR and the convenient calculation procedures make it more attractive for financial institutions to adopt as their management practice compared to many other traditional ones. A well-designed VaR system could offer management level an explicit profile that shows how each individual asset in the portfolio affects the performance. In addition, Stulz (1996) suggested that companies could stand to benefit from better risk management in general, for example in improving capital structure and the reduction of taxes or avoiding the costs of bankruptcy.

In practical implementation, computation of value at risk involves choosing: a proper α; a time horizon; the frequency of the data; the cumulative distribution function of the asset return of a particular financial position; and the amount of the financial position. Assume α, the time horizon, frequency of asset returns, and financial position are given, one might find that the calculation of value at risk would be strongly affected by the assumption of the distribution of the asset return. In previous research, most studies assumed that asset returns distributed normally, however, this is not the case and would mislead the distribution of financial assets. The earliest paper to my knowledge, Fama (1965), suggested that the Gaussian or normal distribution is not an adequate representation of financial assets. Besides, the existing literature on interest rates, exchange rates, and stock returns provides strong evidence that the distribution of price change in these financial assets has a fat tail and is significantly non-Gaussian distributed. Lau et al. (1990) also showed evidence of asymmetry to stock price changes. For the long position (short), since VaR presents a particular value to the left- (right-) tail of the distribution given a confident level, the actual distribution of the tail should be at the core of the VaR calculation. Several tail-related studies indicated that most financial time series are fat-tailed (Danielsson and De Vries, 1997, Loretan and

Phillips, 1994, Hols and De Vries, 1991)[6]. These studies provide strong evidence that the original normality assumption of the financial asset return should be modified to a distribution which is closer to the reality.

2.2.3 The VaR Debate

Various VaR methods have been applied in many different financial institutions (Pérignon and Smith, 2010), however, there is still some debate and criticism at a theoretical level about its actual value and use. Overall, these comments cannot obscure the virtues of VaR, especially for practitioners. Artzner et al. (1999) suggested that a coherent risk measure, Π, should have to satisfy four properties[7]: translation invariance; subadditivity; positive homogeneity; and monotonicity.

Artzner et al. (1999) showed that VaR is not a coherent risk measure due to its lack of the subadditivity property, with the exception of the linear combination portfolio. Furthermore, they proposed an extension of VaR, called tail conditional expectation (Tail-VaR or TCE), which is the expected value of the value exceeding VaR. For a financial return, r, the TCE of a long position with probability, α, can be defined as

$$\mathrm{TCE}_{\alpha}(\mathrm{r}) = \mathrm{E}\left[\mathrm{r}\middle|\mathrm{r} \leq \mathrm{r}_{(\alpha)}\right] \tag{2.4}$$

where $r_{(\alpha)}$ is the lower quantile of return distribution. If we set $r_{(\alpha)}$ equal to VaR_{α}, then TCE can be presented as an expected value of returns smaller than VaR_{α}.

$$\mathrm{TCE}_{\alpha} = \mathrm{E}\left[\mathrm{r}\middle|\mathrm{r} \leq \mathrm{VaR}_{\mathrm{r},\alpha}\right] \tag{2.5}$$

[6] This area of research starts with Mandelbrot (1963).

[7] The four characters can be displayed in the following equations.
(1)Translation invariance: $\Pi(X + a) = \Pi(X) - a$
(2)Monotonous: for all assets, X and Y, with $X \leq Y$, then $\Pi(x) \leq \Pi(y)$
(3)Positively homogeneous: for $h > 0$, $\Pi(hX) = h\Pi(X)$
(4)Subadditivity: for all X and Y, $\Pi(X + Y) \leq \Pi(X) + \Pi(Y)$
More details of these four properties can be found in Frittelli and Gianin (2002).

The TCE_α in Eq. (2.5) reflects the mean loss exceeding the quantile of VaR_α. Specifically, TCE_α of a long position can be rearranged as a direct formula[8].

$$TCE_\alpha = VaR_\alpha + E[r - VaR_\alpha | r \leq VaR_\alpha] \quad (2.6)$$

Similar concepts of risk measure are discussed in the literature, for instance, conditional value at risk (CVaR), expected shortfall (ES), and worst conditional expectation (WCE). A detailed survey of these is provided by Acerbi and Tasche (2002). The major concept of subadditivity is that the diversification effect could reduce the total risk of a portfolio. In other words, the risk of a merged portfolio is never larger than the sum of the risk of the stand-alone portfolio. In response to this criticism of VaR, Danelsson et al. (2005) offered an explicit demonstration indicating that VaR does satisfy subadditivity in the tail area with a fat-tail distribution, although VaR does lack this feature across the entire distribution. Since the key point of the fatness property of financial returns and the calculation of VaR are both focused on the tail area, the problem of a lack of overall subadditivity might not really be that serious. Moreover, Dhaene, Goovaerts and Kaas (2003) suggested that risk measures should reflect the economic elements used to measure risk. They also provided some illustrations to explain that the four features of coherence might lead the risk measure to be too restrictive to be actually usable in any given economic situation. Dhaene et al. (2008) investigated the use of risk measures for the aspect of setting solvency capital requirements. They concluded that coherent risk measures in solvency capital requirements could be too subadditive and might lead to an increase of risk in the case of a merger. In addition, they suggested that VaR satisfies the regulators' conditions,

[8] The TCE_α in Eq. (2.6) can be re-written as a risk measure of a short position, $TCE_{(1-\alpha)}$, where α is a probability in the right tail, and (1-α) is a cumulative probability. Thus, TCE of a short position is

$$\begin{aligned} TCE_{(1-\alpha)} &= E[r | r \geq VaR_{r,(1-\alpha)}] \\ &= VaR_{(1-\alpha)} + E[r - VaR_{(1-\alpha)} | r \geq VaR_{(1-\alpha)}] \end{aligned}$$

and is the most efficient capital requirement in its cost bracket. In summary, the debate around the lack of subadditivity is still an open issue. To date, VaR is the most popular risk measure in the field due to its intuition and efficiency in calculating.

2.2.4 Regulatory market risk measurement and evaluation

The earliest rule concerning market risk was introduced by the Basel Committee on Banking Supervision (BCBS) in 1995, requiring that commercial banks should, at least, hold a certain minimal amount of capital[9] against their potential losses from any transactions in the trading book[10]. Obviously, the Basel regulation emphasizes that capital charge should be associated with the bank's risky assets, and the first step of this is to verify the quantity of market risk in the trading book. According to the Basel Committee on Banking Supervision (1996b, 2005), two methods (the Standardized Method and the Internal-Models Approach) are allowed to be applied when measuring market risks.

The Standardized Measurement Method

The Standardized Method is a bottom-up approach, composed of calculating the market risks from four risk factors (interest rate, equity position, foreign exchange and commodities risk) based on specific guidelines. Then the whole market risk is accessed via the summation of all of these individual risks. Thus, it can be expressed as the following formula.

$$\mathrm{TMR} = \sum_{i=1}^{4} \mathrm{MR_i} \qquad (2.7)$$

where TMR means total market risk, and MR is individual market risk, including interest rate risk, equity position risk, foreign exchange risk, and commodities risk.

[9] According to the 1988 Basel Accord, the principal form of eligible capital to cover market risks consists of shareholders' equity and retained earnings (tier 1 capital) and supplementary capital (tier 2 capital).

[10] The definition of the trading book is the account of security positions mainly for trading purposes.

The risk weight and capital change of each instrument are associated with their individual characteristics, as referred to in the Basel Committee on Banking Supervision (1996b).

The Internal-Models Methodology

BCBS (1995) allows the banks to measure their capital charge of market risk based on a daily VaR at 99% confidence interval (i.e. α=0.01), with at least one year of historical observations. In the calculation, a 10-day VaR of price movement must be used to display the bank's market risk. In other words, the minimum "holding period" will be ten trading days. Banks could use VaR numbers calculated according to shorter holding periods (for example, daily frequency) scaled up to ten days by the square root of time. The capital charge of market risk to this method can be displayed as

$$\mathrm{MRC}_{\mathrm{internal,t}} = \max\left(\mathrm{k}\frac{1}{60}\sum_{\mathrm{i}=1}^{60}\mathrm{VaR}_{\mathrm{t-i}}, \mathrm{VaR}_{\mathrm{t-1}}\right) + \mathrm{SRC}_{\mathrm{t}} \qquad (2.8)$$

In Eq. (2.8), k[11] means a multiplication factor set by individual supervisory authorities on the evaluation of the quality of the VaR model, subject to an absolute minimum of 3. The last term of Eq. (2.8), SRC, is the specific risk charge associated with interest rate related instruments and equity securities as defined in the standardised approach that are not measured in the internal model.

The Basel rules in backtesting the internal-model

Although the internal model is allowed to be used for market risk measurement, BCBS also require a method of backtesting at the same time. According to BCBS (1996a), the backtesting programme generally consists of a periodic comparison of the daily

[11] The minimum of k is 3. Its increases are displayed in Table 2.1.

forecasted VaR with the daily actual return of the portfolio. Ideally, in a long (short) position, the forecasted VaRs tend to be smaller (larger) than 99% of the corresponding actual returns of the portfolio if the confidence level of the VaR model is equal to 0.01. The main point of backtesting is to examine if the bank's 99th quantile VaR covers 99% of the actual returns. Those actual returns uncovered by the forecasted VaRs are regarded as exceptions or violations. Based on using the most recent 12 months of data (about 250 historical observations) in the backtesting procedure, the result is then divided into three zones by counting the number of exceptions. As shown in Table 2.1, the number of exceptions in the green zone goes up to 4, and the increase of k remains at zero. The result of backtesting in this zone indicates that the VaR model is accurate. In the yellow zone, the number of exceptions go from 5 to 9 and the k spans 0.40 to 0.75, suggesting that the VaR model is more likely to be inaccurate rather than accurate. The VaR model is extremely unlikely to be an accurate model if the backtesting result falls in the red zone.

Table 2.1 Three zones of exception in regulatory backtesting

Zone	Number of exceptions	Increase in scaling factor (k)	Cumulative probability
Green zone	0	0.00	8.11%
	1	0.00	28.58%
	2	0.00	54.32%
	3	0.00	75.81%
	4	0.00	89.22%
Yellow zone	5	0.40	95.88%
	6	0.50	98.63%
	7	0.65	99.60%
	8	0.75	99.89%
	9	0.85	99.97%
Red zone	10 or more	1.00	99.99%

Source: BCBS (1996a), table 2.

2.2.5 VaR in other Risk measures

This sub-section reviews applications of VaR in the different types of risk measurements such as credit risk and operational risk. Liquidity risk will not be discussed here since the liquidity risk will be reflected within the market risk and credit risk according to the types of financial assets (Muranaga and Ohsawa, 1997, Bessis, 2010). There are four main credit risk models widely discussed in the literature: J.P. Morgan's CreditMetrics, Moody's KMV model[12], the CreditRisk+ proposed by Credit Suisse Financial Products (CSFP), and McKinsey's CreditPortfolioView model. Credit risk is usually measured once a year because rating information is only available once a year. Compared to the VaR in market risk, the VaR model of credit risk tries to answers the following question: "If next year is a bad year, how much will I lose on my loans and loan portfolio?" CreditMetrics is a VaR framework introduced in 1997 by J.P. Morgan, measuring the risk of non-tradable assets and private bonds, and it is calculated based on four factors: (1) borrower's credit rating, (2) borrower's rating transition matrix, (3) recovery rates on a defaulted loan, and (4) credit spreads and yields in the bond or loan market. Credit VaR is the difference between the expectation of discounted value of a loan based on the yields in the loan market and the expected recovery value of a loan according to J.P. Morgan's credit migration matrix. The other three credit risk models are default only models, i.e. credit migration is not considered. KMV derives the actual probability of default, the Expected Default Frequency (EDF), for each borrower based on a Merton's (1974) type model of the firm and the firm's capital structure, the return volatility and asset value. CreditRisk+ assumes that a borrower's default process is a binominal process, and the probability of default in a given period is the same for any other month. Moreover, it assumes the default events are independent with each other. Both KMV and CreditRisk+ are firm-specific credit risk models, McKinsey's CreditPortfolioView model is a

[12] KMV is the trademark of the KMV corporation found by Stephen Kealhofer, John McQuown and Oldrich Vasicek in 1989. KMV was acquired by Moody's Analytics in 2002.

multi-factor model which considers the default probability according to the market conditions and credit cycles, i.e., the macroeconomic factors. Specifically, the default probability is modelled as a logistic function based on current or lagged macroeconomic variables, as well as the firm or industrial level variables.

Since the loans are not publicly traded, the market value of the loans and their volatility cannot be observed. Most credit risk models have similar weaknesses such as parameterisation by judgment and data blanks (Jackson and Perraudin, 2000). In addition, the results of credit risk models cannot be compared with each other due to the lack of accessibility of data.

Lately, the concept of VaR has been applied in the modelling of operational risk, defined as the risk of direct or indirect loss resulting from inadequate or failed internal processes, human error and errors from external events. Normally, modelling operational risk requires the setting classification of the operational risk events in the first step, and gives each of these events an occurrence probability according to modelling, historical records, and expert experience in the second step. The third step is to assess the expected financial impact of each operational risk event. However, the events of operational risk and their occurrence probabilities would shift according to changes in the system, staff and/or procedures. Similar to credit risk modelling, it is difficult to make comparisons of different operational risk models due to the different assumptions in the parameterisation.

2.3 Foundation of VaR calculations

According to Eq. (2.3), it is clear that the calculation of VaR should be stressed on the standard deviation of the asset return, σ_r, usually called volatility. As mentioned in the previous section, volatility is an indicator of risk. Thus, a large number of volatility models have been employed as part of VaR measurement. The main line behind VaR calculations assumes that the financial returns follow a particular process or distribution. Following this hypothesis, the parameters of the distribution would be estimated and then VaR would be derived. In this section, this simplistic method of VaR calculation will be reviewed.

2.3.1 VaR calculation with univariate Conditional volatility

In the classic portfolio theory proposed by Markowitz (1952), standard deviation of returns (or volatility) is used to represent the risk of a particular asset, and investors should look to choose the most efficient frontier to be part of their portfolio. In other words, market participants prefer to settle their portfolio either in the area of minimum risk given expected return or maximum expected return within a sticking level of risk. After this, standard deviation was regarded as a major indicator of risk and was used as such across various fields.

Fama et al. (1969) initiated the event study method focusing on investors' behaviour in response to particular events. In their research, they provided explicit evidence suggesting that investors' behaviour would significantly respond to past events, which implies that market participants would refer to past information before their actual action. In the aspect of volatility, with the same concept as Fama et al. (1969), a time-varying variance model which took into account previous information was proposed by Engle (1982), named autoregressive conditional heteroscedasticity (ARCH). In his seminal paper, he modified Granger and Andersen's (1978) work so that

$$r_t = \varepsilon_t h_t^{\frac{1}{2}} \tag{2.9}$$

$$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 \qquad (2.10)$$

where ε_t is white noise with zero mean and unit variance, and h_t represents conditional variance depending on past information. Under the assumption of normality of returns, $r_t|\psi_{t-1} \sim N(0, h_t)$, the model of conditional variance can be straightforwardly displayed as

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i h_{t-i} \qquad (2.11)$$

where if p=1, then $h_t = \alpha_0 + \alpha_1 h_{t-1}$ is called ARCH(1).

From Markowitz (1952) through Fama et al. (1969) to Engle (1982), this line of research has developed gradually into a vital area which incorporates conditional volatility into the calculation of VaR. However, several features found in the reality of financial markets are not entirely captured by Engle's (1982) model, and this has attracted numerous attention and a lot of critical models[13]. Engle et al. (1987) extended the simple ARCH technique of measuring conditional variance to the ARCH in mean (ARCH-M) model, suggesting that the conditional variance is a determinant of the current risk premium.

In general, rational investors would correct their behaviour from any previous forecast error before carrying out their next move. Accordingly, a generalized ARCH (GARCH) model was proposed by Bollerslev (1986), which incorporates an error-correction mechanism to the ARCH model. He endowed the ARCH model with a more flexible structure, which enabled it to not only include the lagged conditional variance but also to consider previous error terms. Assuming a random variable, r_t, is formed by

$$r_t = E(y_t|\Psi_{t-1}) + \upsilon_t \qquad (2.12)$$

[13] A comprehensive survey and discussion can be found in Bollerslev, Engle and Nelson (1994), and Laurent, Bauwens and Rombouts (2006).

Where the error term, $\upsilon_t \backsim \cdot\ (0, h_t|\Psi_{t-1})$, the conditional variance, GARCH (p,q) can be expressed as

$$h_t = c + \sum_{j=1}^{p} b_j h_{t-j} + \sum_{i=1}^{q} a_i \upsilon_{t-i}^2 \qquad (2.13)$$

where the lagged parameters, p and q are non-negative integers (i.e. $p, q \in \mathbb{N}$). When q=0, then the GARCH model reduces to its original ARCH model. In the simplest case of Eq. (2.13) we assume that the conditional variance at time t, h_t, is only affected by the information one period ahead, denoted as GARCH(1,1)

$$h_t = c + b\, h_{t-1} + a\, \upsilon_{t-1}^2 \qquad (2.14)$$

Bollerslev's (1986) model includes a critical economic meaning by implying that investors might correct their investment based on their forecasting error in the last period. This arrangement is significantly more pertinent to market reality than the ARCH model, and theoretically offers a better performance in conditional variance forecasting.

Engle (2001) and Tsay (2005) provided a distinct explanation for applying the generalized ARCH model to the VaR calculation. The most well-known application of the GARCH model to the VaR model is the RiskMetrics model proposed by JP Morgan (1996). Assuming that the random variable $r_t = \sigma_t \varepsilon_t$ and $\varepsilon_t \rightarrow \text{i.i.d}\ N(0,1)$, RiskMetrics can be shown as a normal Integrated GARCH(1,1) model where the autoregressive parameter is set from their large-scale survey. Thus, the RiskMetrics model can be clearly expressed as

$$h_t = \lambda h_{t-1} + (1 - \lambda) r_{t-1}^2 \qquad (2.15)$$

where λ is the decay factor and equal to 0.94 and 0.97 for the daily and monthly data sets, respectively. Thus, a simple daily volatility process can be easily formed as

$$h_t = 0.94h_{t-1} + 0.06r_{t-1}^2 \quad (2.16)$$

It goes without saying that the GARCH model captures some critical characteristics of financial returns, for example, serial correlation, volatility clustering, and fatter tails. However, other substantial properties in the reality of financial markets can end up omitted. In the past twenty years, a large number of models have been proposed by econometricians looking at alternative aspects of the financial markets. Nelson (1991) (hereafter EGARCH) suggested that conditional volatility responds asymmetrically to positive and negative error terms and thus might be more suitable in the real world. With the same view point, Glosten et al. (1993) (hereafter the GJR model) proposed a similar model which emphasized the impact of previous forecast errors. In other words, most of these asymmetric GARCH models stressed that conditional volatility would respond more strongly to negative news than to positive news.

Since the major feature of conditional variance was not fully encompassed by GARCH models with the assumption of normality, some researchers started looking to relax this restriction. For instance, Bollerslev (1986) mentioned that the GARCH model could be fitted under other conditional densities. As was already known, previous models based on the assumption of normality might not adequately account for leptokurtosis or the fatness of the tail. In this case, the standardized residuals from the estimated models often present as leptokurtic, and thus might cause an underestimation in the VaR estimation and forecast with the assumption of normality. Bollerslev (1987) provided a direct example by applying the GARCH model with the standard student t distribution to fit the returns of the stock index and exchange rate. He also concluded that the relatively simple GARCH(1,1) model with student t distribution fits the data series well, despite not knowing the real distribution of residuals. Nelson (1991) also used alternative densities[14] to fit conditional variance. Mittnik and Paolella (2000) demonstrated that related GARCH models with asymmetric generalized t distribution (t_3) yield a significantly better in-sample fit. Furthermore, they suggested that the asymmetric power GARCH model with t_3 considerably outperforms its simpler counterparts.

To bring the models closer to the market reality, various conditional densities to innovations have been adopted to fit different time series models. Some previous research focused on the feature of the fat-tail, whilst others emphasized the feature of skewness or asymmetry. Using the related GARCH models one can capture some particular properties of financial data, for example, thick tail, serial correlation, and asymmetric effect. According to the VaR calculation shown in Eq. (2.3), conditional volatility indeed plays a crucial role in VaR forecasting. Being able to use time-varying volatility gives financial institutions a more dynamic risk management mechanism.

2.3.2 Multivariate Conditional volatility and Correlation

The discussion in the previous subsection focused on the univariate VaR calculation based on GARCH related models. However, there is still a need for a multivariate version since portfolios include a range of financial assets. Consequently, it is necessary to clarify the relationship among the financial assets in a portfolio for the purpose of diversification, asset allocation, and risk management. The process of a random return vector with $N \times 1$ dimension, $\mathbf{r}_t$, can be expressed as

[14] Nelson (1991) assumed the variable is an i.i.d sequence drawn from generalized error distribution (GED).

$$\mathbf{r}_t = \mathbf{H}_t^{\frac{1}{2}} \boldsymbol{\varepsilon}_t \tag{2.17}$$

where $\boldsymbol{\varepsilon}_t$ is an $N \times 1$ white noise vector and $\mathbf{H}_t$ is an $N \times N$ covariance matrix. Then the VaR formula of a multivariate case can be analogized from Eq. (2.3) as

$$\mathrm{VaR}_{\alpha,t} = \boldsymbol{\omega}'\boldsymbol{\mu}_{r,t} + z_\alpha \sqrt{\boldsymbol{\omega}'\mathbf{H}_t\boldsymbol{\omega}} \tag{2.18}$$

where $\boldsymbol{\omega}$ is an N×1 weight vector of assets in the portfolio, and $\mathbf{H}_t$ means the covariance matrix.

Bollerslev, Engle and Wooldridge (1988)[15] proposed the first multivariate conditional volatility model, applying a stacking operator and the feature of symmetry in the covariance matrix to transfer it as a vector with $N(N+1)/2$ elements. Since then, the conditional covariance matrix has been formed as

$$\mathrm{vech}(\mathbf{H}_t) = \mathbf{C} + \sum_{i=1}^{q} \mathbf{A}_i \,\mathrm{vech}\left(\boldsymbol{\epsilon}_{t-i}\boldsymbol{\epsilon}_{t-i}'\right) + \sum_{j=1}^{p} \mathbf{B}_j \,\mathrm{vech}\left(\mathbf{H}_{t-j}\right) \tag{2.19}$$

where $E(\boldsymbol{\epsilon}_t) = \mathbf{0}$, $\mathrm{Var}(\boldsymbol{\epsilon}_t) = \mathbf{H}_t$, and $\mathrm{vech}(\cdot)$ indicates the column stacking operator of the lower portion of a symmetric matrix. The main contribution of the VECH model is to describe the process of variance not only based on the previous term, but also the previous covariance. However, the major difficulty to this model is the number of parameters. Although the number of parameters in the VECH model is reduced by the symmetry of the covariance matrix, it still has $\frac{1}{2}N(N \times 1)[1 + N(N+1)\,(p+q)/2]$ parameters in Eq.(2.19) which need to be estimated. Take for example a GARCH(1,1), let $\mathbf{r}_t$ be a tri-variable matrix (i.e. N=3), then the total number of parameters that need to be estimated is 78, rising to 210 as N=4. However, even when the question of parameters was addressed through Engle and Kroner (1995) suggesting a diagonal representation

15 There are two different abbreviations to this model. Tsay (2005) used VEC to denote this model, and Brooks (2008) applied VECH as a symbol for this model. In this thesis, I follow Brooks' (2008) notation.

(DVECH) model to reduce the number of parameters to $(N+1)^2$, there remained another significant instability with the model. Specifically, the main weakness of the VECH model is that it could not guarantee a positive definite covariance matrix[16]. Thus, statistical tests for each parameter might not be carried out. In practice, it is difficult to check if the parameterization of $\mathbf{H}_t$ is positive definite for all values of $\boldsymbol{\epsilon}_t$. To overcome this drawback, Engle and Kroner (1995) provided a BEKK[17] model which can be expressed as

$$\mathbf{H}_t = \mathbf{C}_0'\mathbf{C}_0 + \sum_{i=1}^{q} \mathbf{A}_i'\boldsymbol{\epsilon}_{t-i}\boldsymbol{\epsilon}_{t-i}'\mathbf{A}_i + \sum_{j=1}^{p} \mathbf{B}_j'\mathbf{H}_{t-j}\mathbf{B}_j \qquad (2.20)$$

Compared with Eq. (2.19), one might find that the BEKK model skilfully decomposes each of the parameter matrices as a product of two of the same matrices in order to obtain a positive definite covariance matrix. The BEKK model is a critical milestone in the multivariate GARCH modelling because of its positive definiteness. The multivariate GARCH model has been widely applied and extended in literature. Bollerslev, Engle and Nelson (1994), Laurent, Bauwens and Rombouts (2006), and Silvennoinen and Terasvirta (2008) provide more detail on the related models. Higgs and Worthington (2004) applied the BEKK model in volatility transmission. Ledoit, Santa-Clara and Wolf (2003) offered alternative estimations in various covariance matrix models by estimating the diagonal and off-diagonal coefficients separately. Lopez and Walter (2001) evaluated the performance of several covariance matrix forecasts and concluded that implied covariance from option price outperforms the BEKK and the diagonal VECH models. There might be a possible bias to this, attributed to the normality assumption. Bauwens

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[16] In linear algebra, an n×n matrix, **A**, is regarded as positive definite if $\mathbf{z}'\mathbf{A}\mathbf{z} > 0$ for all non-zero vector **z.** More details of the definition can be referred to in section 14.2 of Harville (2008). The importance of positive definiteness to a covariance matrix is to enable the square root of the covariance matrix to exist, which is a critical part in the inference procedure.

[17] This model is referred to by the acronym of the original authors in earlier versions as proposed in Baba et al. (1991).

and Laurent (2005) adopted a multivariate skew-student distribution to measure VaR, however, they suggested the performance was not significantly different to other models. One possible reason for this is that some extreme values in the sample could have influenced the accuracy of the estimation. Accordingly, one potential answer to this issue might be to refer to extreme value theory, which will be discussed in section 2.5.

2.3.3 Conditional correlation

Provoked by the development of multivariate conditional volatility models, some researchers went further into its derivative, looking at conditional correlation. Bollerslev (1990) argued a constant conditional correlation model (CCC) as follows, assuming the conditional correlation matrix, ρ, is constant through time

$$\mathbf{H}_t = \mathbf{D}_t \boldsymbol{\rho} \mathbf{D}_t = \left[\, \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}}\,\right] \quad (2.21)$$

where $\mathbf{D}_t = \mathrm{diag}\left(h_{11,t}^{1/2}, \cdots, h_{NN,t}^{1/2}\right)$ and $\boldsymbol{\rho}$ is the constant conditional correlation matrix. The two main advantages of this model are to reduce the unknown parameters and to link correlation with the conditional covariance matrix. Even though the assumption of constant conditional correlation is frequently criticized for being far from reality, it is still reasonable to use this to describe the relationship of financial assets in the short-term. Tse (2000) and Bera and Kim (2002) applied several major equity market returns to examine the constancy of correlation, and some significant evidence was found against the assumption of constant correlation. Engle (2002), Tse and Tsui (2002)[18], and Christodoulakis and Satchell (2002) offered similar models describing time-varying conditional correlation. The process of the covariance matrix in the first two models can

[18] In Tse and Tsui (2002), they assume the conditional correlation matrix is generated from a recursive pattern, $\boldsymbol{\rho}_t = (1-\theta_1-\theta_2)\bar{\boldsymbol{\rho}} + \theta_1\boldsymbol{\Gamma}_{t-1} + \theta_2\boldsymbol{\psi}_{t-1}$, where θ_1 and θ_2 are scalar parameters, and $\boldsymbol{\psi}_{t-1}$ indicates a functional form depending on the standardized residuals.

be modified as $\mathbf{H}_t = \mathbf{D}_t \boldsymbol{\rho}_t \mathbf{D}_t$, and the process of dynamic conditional covariance can be expressed as

$$\mathbf{H}_t = (1 - \alpha - \beta)\overline{\boldsymbol{\rho}} + \alpha(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}) + \beta\mathbf{H}_{t-1} \tag{2.22}$$

where α and β are scalar parameters, and $\overline{\boldsymbol{\rho}}$ is the unconditional correlation matrix. Then, the dynamic conditional correlation (DCC) would be obtained via

$$\rho_{ij,t} = \frac{h_{ij,t}}{\sqrt{h_{ii,t}\, h_{jj,t}}} \tag{2.23}$$

Or in the matrix case

$$\boldsymbol{\rho}_t = \mathbf{J}_t\, \mathbf{H}_t\, \mathbf{J}_t \tag{2.24}$$

where $\mathbf{J_t} = \mathrm{diag}\left(\mathbf{h}_{11,t}^{-1/2}, \cdots, \mathbf{h}_{NN,t}^{-1/2}\right)$ is an inverse matrix of $\mathbf{D}_t$.

Based on Engle and Kroner's (1995) BEKK model and Engle's (2002) DCC model, numerous extensions and modifications have been proposed. Hafner and Franses (2003) offered a generalized DCC model, which guaranteed the positive definiteness of the covariance matrix by squaring the values of all correlation parameters. Cappiello, Engle and Sheppard (2006) suggested an asymmetric generalized DCC (AGDCC) according to investors' expectation

$$\mathbf{H}_t = (\overline{\mathbf{P}} - \mathbf{A}'\overline{\mathbf{P}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{P}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}) + \mathbf{A}'(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1})\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} + \mathbf{G}'\mathbf{n}_{t-1}\mathbf{n}'_{t-1}\mathbf{G} \tag{2.25}$$

where **A**, **B**, **G** are N×N parameter matrices, and $\mathbf{n}_t = I(\varepsilon_t < 0) \circ \varepsilon_t$, $I(\cdot)$ is a k×1 indicator function which is equal to 1 if the investors' expectation is true and 0 otherwise (the operator " ∘ " means the Hadamard product). Similarly, Cajigas and Urga (2005) also suggested an AGDCC model with an asymmetric multivariate Laplace distribution (AML). Billio et al. (2006) proposed a flexible DCC (FDCC) model to release the constant dynamics of the covariance process from Engle's (2002) scalar version.

Apart from GARCH-related conditional correlation, Bodurtha and Shen (1999) offered an alternative approach to calculate time-varying correlation, called implied correlation. However, implied correlation is not appropriate to be applied in the calculation of VaR since implied correlation describes the relationship between a financial instrument and its derivative.

2.3.4 Correlation and dependence

In the previous sections, the issues focused on were around risk management in a univariate situation (i.e. in a single asset case). Risk measurement of a portfolio involves more complicated issues and the modelling of the correlation, or dependence structure, of a multivariate case. Since Markowitz (1952) showed that optimal portfolio selection was impacted by the concept of correlation, correlation has played a central role in financial theory and the measure of correlation has garnered a lot of attention. However, the application of, and limitations to, correlation and dependence are still unclear and debatable. There are three main approaches to measure the relationship between different financial assets: conventional Pearson's linear correlation; the tail dependence parameter; and rank correlation (particularly, Spearman's rank correlation). However, the rank correlation coefficient is only used for measuring the relationship between two ordinal variables, and thus is not appropriate for risk management. Accordingly, the first two will be discussed in this section due to their popularity in the literature and their potential suitability for risk management.

According to the calculation in Eq. (2.18), linear correlation[19] measures the product of the distance to their means of two random variables scaled by their standard deviations.

[19] Specifically, the linear correlation means Pearson's linear correlation.

It can be expressed by using a discrete form

$$\rho_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{\sum_{i=1}^{n_x} \sum_{j=1}^{n_y} (x_i - \mu_x)(y_j - \mu_y)}{\sigma_x \sigma_y} \qquad (2.26)$$

where ρ_{xy} takes the values in [-1,+1]. The Pearson's linear correlation is criticized from two main perspectives. According to its definition, linear correlation measures the linear relationship between two random variables. However, there could be a non-linear relationship in financial markets. Specifically, it does not guarantee that two random variables are entirely independent when their linear correlation is zero. Moreover, on the aspect of risk management, it is well known that the estimation of tail distribution is definitely more important than looking at the entire distribution as a whole. The estimation of linear correlation with the whole sample set would not be appropriate for measuring correlation (Embrechts et al. (2002)). In the context, Lhabitant (2002) and Kat (2003) mentioned that a method for conditional correlation could be applied to measure the extremal correlation in that particular tail area. In terms of dynamic conditional correlation, the so-called DCC-related model, measuring the dynamic correlation conditioned on past information, is another method to overcome the non-linear correlation problem.

Another widespread method to measure correlation is the parameter of tail dependence deriving from copula[20]. Over the past decade, copula has been extensively applied to various fields, especially in biostatistics and risk management. Copula offers a reasonable avenue to overcome the difficulties in multivariate distribution. Let $F(x_1, x_2)$ be a bivariate joint distribution function with margins F_1 and F_2. Then there exists a copula C, such that for all real numbers x_1 and x_2, it has the equality

[20] The concept of copula was initially proposed by Sklar (1973).

$$\Pr(X_1 \le x_1, X_2 \le x_2) = F(x_1, x_2) = C\big(F_1(x_1), F_2(x_2)\big) \qquad (2.27)$$

$$f(x_1, x_2) = f_1(x_1) \cdot f_2(x_2) \cdot c\big(F_1(x_1), F_2(x_2)\big) \qquad (2.28)$$

where $F_i(x_i)$ is a cumulative marginal distribution, $0 \le F_i(x_i) \le 1$, and $f(x_i)$ is a marginal distribution function, $0 \le f_i(x_i) \le 1$ i=1,2. Accordingly, a copula (c) is used to unite two different uniform variables into one dimension, and describes the degree of dependence of the two variables. As shown in Eq. (2.28), a two dimensional joint distribution function can be decomposed into its margins and a copula, which completely describes the dependence between the two variables. In addition, the components in Eq. (2.28), $F_1(x_1)$ and $F_2(x_2)$, can be seen as two variables with uniform distribution. In most cases, the multivariate distribution is too complicated to obtain or too difficult to calculate, however a description of the margins, F_1 and F_2, is relatively easy to acquire.

The most popular copula function is the Gaussian copula, which uses a mild assumption of normality and can be expressed as

$$C(u_1, u_2; \rho) = \Phi_\rho[\Phi^{-1}(u_1), \Phi^{-1}(u_2)] \qquad (2.29)$$

Its probability density function (pdf), by definition of normal distribution, is displayed as

$$c(u_1, u_2; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2}\boldsymbol{\Psi}'(\mathbf{R}^{-1} - I_2)\boldsymbol{\Psi}\right] \qquad (2.30)$$

where $\Psi = \big(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\big)'$, and the correlation matrix in this case is a 2×2 dimension, which is a dependence structure describing the dependency between X_1 and X_2. In a similar fashion, the student t distribution and other distributions can be applied in a copula function. The details of the general copula family can be referred to in Joe (1997) and Nelsen (2006). Recently, the copula function has been applied to measure the market linkage via alternative aspects. Some application of the copula family to the dependence structure in exchange rates can be found in Patton (2006a), Patton (2006b), and Bartram et al. (2007); for the equity market refer to Longin and Solnik (2001), Hartmann et al. (2004), Poon et al. (2004), and Jondeau and Rockinger (2006). Most empirical research suggests that the dependence of financial markets tends to be much stronger in a downturn market than in an upturn period. Moreover, an asymmetric dependence structure can be found in both the left and right tail (Patton, 2006b).

From the viewpoint of risk management based on the tail area, one might consider the relationship of the extremes to the assets in the portfolio. Thus, Coles et al. (1999) suggested a coefficient of tail dependence conditioned on a certain threshold

$$\chi = \lim_{z \to z^*} \Pr(Y > z | X > z) \tag{2.31}$$

where z^* is the upper limit of the support of the common marginal distribution. The inspiration in Eq. (2.31) is consistent with the multivariate case of extreme value theory based on the method of peak over threshold. In other words, χ measures the probability of one variable falling in the extremal area given that another one is extreme as well.

Although the copula structure offers a reasonable approach to attain the dependence structure of different variables, it still has some drawbacks. Firstly, the class of copula functions is now very vast. Accordingly, it is often difficult to select the best one to fit the data (see Panchenko (2005) and Jondeau et al. (2007)). Although, various approaches to identify appropriate goodness-of-fit tests have been carried out, there is still no conclusive answer to this issue. More details and various goodness-of fit tests are explored by Genest et al. (2009). Secondly, the result of the dependence structure might statistically explain the relationship between two financial risk factors, however, in economics it is often difficult to interpret the connection between different markets. The main reason for this is that the foundation of the copula is constructed on probability theory without the feature of sequence or time-order as mentioned in the preceding section. For example, Eq. (2.31) measures the extremal dependence based on the probability of $Y > z$ given $X > z$. Under these circumstances, it could also cause the same dependence structure with a different time horizon. Take for instance the extremal dependence between the bond and equity markets, how could one explain if the coefficient of tail dependence came out at 0.4, which according to this model roughly means the probability of the two markets crashing at the same time is about 0.4. However, we know that the two markets tend towards a trade-off relationship rather than a consistency one. By relying only on the risk management models, a hedge strategy might be overly biased towards this condition and thus be ineffective.

2.4 Application of VaR

In this section, some applications of VaR are reviewed. I follow Jorion's (2007) suggestion that there are three stages of application to the VaR measurement and concept. In the early stages, financial institutions were relatively passive in their use of VaR, with it merely being used to reveal their risk in the annual report. As it became more familiar and more widespread, some of them started using VaR as a tool to defend market risk for portfolio optimization. In recent years, VaR has been widely applied in company-wide risk management and performance measurements.

2.4.1 Passive application: Risk reporting

Following the Basel Accord announcement in 1996[21], commercial banks have developed their own internal models for measuring market risk and generally VaR is the most popular one, even though the methodologies are diverse. Since VaR summaries the risk of the banks' portfolio into a simple value accessible to management level and shareholders, it can be used as a foundation to evaluate the risk of financial institutions. Regulators take risk disclosure very seriously after the 1988 Basel Accord, especially within the banking sector. To echo the Basel Accord, the International Accounting Standards Committee (IASC) published the International Accounting Standard NO.32 (IAS.32) in 1995, suggesting rules on qualitative and quantitative disclosures of risk associated with financial instruments. Various sources of risk are included in IAS32, such as credit, market, interest, and liquidity risk. The spirit of IAS32, then, led the International Accounting Standards Board (IASB)[22] to announce the rules of financial instrument disclosure in the draft of the International Financial Reporting Standard NO.7 (IFRS.7) on 22nd July 2004, which came into effect on 1st January 2007. In November 2009, IFRS 9 was issued as a replacement for IAS39, focusing on the classification and measurement of financial instruments, it will come into effect on 1st January 2013. Moreover, the Securities and Exchange Commission of the U.S. (Commission) aligned their national standards, GAAP, toward IFRS in 2007. Thus, risk disclosure in financial statements has become a requirement for nearly all financial institutions. To date, more than one hundred countries have adopted IFRS as the foundation of their regulation. In other words, risk evaluation and disclosure is a standard requirement in the financial statement and regulated by local and international authorities.

[21] See BCBS (1996a).

[22] The International Accounting Standards Board (IASB) succeeded the International Accounting Standards Committee (IASC) on 1st April 2001.

Another requirement for risk disclosure often comes from the shareholders. After the economic crisis in the 1980s, shareholders have increasingly asked for transparency in financial statements, particularly with regards to the complicated financial instrument transactions made in many financial institutions. Besides controlling risk taking, management can set the boundary of risk taking in their line of business for the treasury department. Thus, traders are restricted to only making transactions within this boundary. A well-known case of risk reporting to management is found at J.P. Morgan, who require the managers of every business line to report the risk of their position at 4:15 p.m. every day. Then the management level can assess how risky their portfolio is under this mechanism. In other words, the board can straightforwardly understand how much money they might lose if the market was bad the following day.

As mentioned above, risk disclosure is required by regulators and shareholders. The VaR technique is the most popular one used by financial institutions. Table 2.2 shows an example of a typical risk disclosure at J.P. Morgan Chase. They used various risk measures, both statistical and non-statistical, to estimate the risk-taking for their portfolio. The VaR results displayed in Table 2.2 are estimated by historical simulation. It not only shows the firm-wide VaR, but also the individual VaR of each financial instrument. Using the VaR of market risk, the diversification effect can be found, showing the difference between the firm-wide market risk at the end of 2009 ($129), the sum of the market risk to the four segments ($228) is $99. The main advantage to risk disclosure is that it offers comprehensive information to management and shareholders about how risky their position is and how much money they might lose in the next trading period. A similar VaR-based risk disclosure can also be found in the 2009 annual reports of other financial institutions, for instance the Bank of America (p.94), the Royal Bank of Scotland (p.163), the Deutsche Bank (p.84), and the Accounts and Report of Lloyds TSB bank. Nowadays, VaR measure is the mainstream for risk disclosure, although each financial institution modifies the technique slightly for their own particular needs, the details of which are not generally available.

Table 2.2 99% confidence-level VaR
Investment Bank trading VaR by risk type and credit portfolio VaR
source: JP Morgan Chase & Co. 2009 annual report, p.127.

As of or for the year end, Dec. 31,[a] in millions	2009			2008			At Dec. 31,	
	average	min	max	average	min	max	2009	2008
By risk type:								
Fixed income	$ 221	$ 112	$ 289	$ 181	$ 99	$ 409	$ 123	$ 253
Foreign exchange	30	10	67	34	13	90	18	70
Equity	75	13	248	57	19	187	64	69
Commodities	32	16	58	32	24	53	23	26
Diversification	(131)[b]	NM[c]	NM[c]	(108)[b]	NM[c]	NM[c]	(99)[b]	(152)[b]
Trading VaR	$ 227	$ 103	$ 357	$ 196	$ 96	$ 420	$ 129	266
Credit portfolio	101	30	221	69	20	218	37	171
Diversification	(80)[b]	NM[c]	NM[c]	(63)[b]	NM[c]	NM[c]	(20)[b]	(120)[b]
Total trading and credit portfolio VaR	$ 248	$ 132	$ 397	$ 202	$ 96	$ 449	$ 146	317

a. The results for the year end, December 31, 2008, include five months of heritage JPMorgan Chase & Co. only results and seven months of combined JPMorgan Chase & Co. and Bear Stearns results.

b. Average and period-end VaRs were less than the sum of the VaRs of its market risk components, which is due to risk offsets resulting from portfolio diversification. The diversification effect reflects the fact that the risks were not perfectly correlated. The risk of a portfolio of positions is therefore usually less than the sum of the risks of the positions themselves

c. Designated as not meaningful ("NM") because the minimum and maximum may occur on different days for different risk components, and hence it is not meaningful to compute a portfolio diversification effect.

2.4.2 Active application: Performance measurement and management

Performance management plays a crucial role in business management, particularly in selecting the method of measurement. In the early stages, practitioners and researchers put their focus for performance measurement on the return-side, such as the return on equity/capital (ROE, ROC) or the return on investment (ROI). However, performance assessment only based on the return-side is insufficient to meet the needs of contemporary business and market conditions. Take the banking sector as an example, banks exchange various risk takings for returns. Over the past twenty years, the banks have received huge losses in market collapses, yet even in everyday situations they take some risks and losses as well. Thus, for performance measurement in related industries, not only the return-side has to be considered but also the aspect of risk. From the firm-wide level, capital is used as a buffer to risk (Berger, Herring and Szego (1995), Copeland and Weston (1992), and Lucas (2001)). If the losses from various risks are vast, banks have to recover these losses via their capital if they do not have enough retained earnings. Theoretically, higher risk businesses or activities need more capital to absorb potential losses. Consequently, when using the term “capital” in performance measures, such as “return of capital”, it might be more appropriate to employ risk-adjusted-capital (Mittnik, Paolella and Rachev, 2000) rather than the book value of capital. In addition, return on capital can be extended to be a “return on risk-adjusted capital” (RORAC). Other similar concepts based on risk-taking are the risk adjusted return on capital (RAROC) and the economic value added (EVA)[23] or economic profit (EP), or the shareholder value added (SVA). Generally speaking, the formula of RAROC and EVA can be expressed as

$$\mathrm{RAROC} = \frac{\text{operating profits}}{\text{risk adjusted capital}} \tag{2.32}$$

$$\mathrm{EVA} = \text{operating profit} - \text{capital} \times \text{cost of capital} \tag{2.33}$$

The focus of this section is on the capital aspect, since the other items in Eq. (2.32) and (2.33) are not really discussed in this thesis. For the firm-wide level, I have looked at capital allocation as the critical issue. However, performance management needs to be broken down to business level or segments at least. Thus, the formulas should be rearranged as

$$\mathrm{RAROC_i} = \frac{\text{operating profit}_\mathrm{i}}{\text{risk adjusted capital}_\mathrm{i}} \tag{2.34}$$

$$\mathrm{EVA_i} = \text{operating profit}_\mathrm{i} - \text{cost of capital} \times \text{allocated capital}_\mathrm{i} \tag{2.35}$$

[23] EVA is a trademark of the consulting firm, Stern Stewart & Co.

As mentioned above, riskier businesses and operations need more capital than the ones with lower risk. Thus, a high-risk segment would be allocated more capital. There are several methods for looking at capital allocation, however there is not the scope to cover all this here. More details can be found in Albrecht (2004), Urban et al. (2004) and Stoughton and Zechner (2007). In Urban et al.'s (2004) research, they concluded that capital allocation is more affected by risk measure than the method of allocation. Briefly, there are two steps in the procedure of calculating capital allocation. The first step is to select the risk measure. Stoughton and Zechner (2007) suggested the minimal level of allocated capital to each segment should be the amount of VaR for each segment. However, the sum of VaR for each segment might exceed the actual firm-wide total due to the effect of diversification. Then, the second step is to set the method of allocation based on the VaR obtained from step one. To date, the best-practice method of capital allocation is still inconclusive, but the general concept behind all the various approaches is consistent with RORAC or EVA. In summary, according to the foundation of financial theory, the segment with more risk tends to be allocated more capital.

The role of VaR in risk-based performance management is critical, especially when linked with the compensation system. For example, if the compensation package is positively associated with the performance measure, EVA, the managers will try to either improve operating profit or to reduce allocated capital. Since the business or segment with higher risk will be allocated more capital, the manager's bonus will be indirectly affected by this mechanism, as shown in Eq. (2.35). With rational behaviour, the managers of each segment will thus consider carefully the trade-off between risk and return for every project or investment, even those which are routine. Moreover, risk-based performance management can prevent situations where managers might pursue the personal short-run performance and ignore the long-run profit of the firm. As described by James (1996), the RAROC system developed by the Bank of America has now been implemented to each segment level and has become a critical performance measurement. Moreover, risk-based capital allocation is broken down to various dimensions, for example, the level of individual products, transactions, and customer relationships. Accordingly, it is obvious that VaR's active role in performance evaluation systems is getting more important.

2.4.3 Diversification tool

In the previous two sections, the VaR function in risk reporting, controlling, and its role in performance management are merely a lagging indicator, visible ex post. However, management might not be satisfied with this, since it does not help some losses still being unavoidable in the future. Although active application in risk based performance management could fill this gap in ex ante warning of risk, using the compensation system would still take several years. Several mechanisms using derivatives of VaR could provide management with an ex ante vision in decision-making. For example, application of VaR implemented before investment could help management to reduce the total risk via diversification. In other words, it could make it clear that how the new investment could affect the increment of the total VaR. For example, the incremental VaR (also called marginal VaR, denoted as MVaR) of a new project could be obtained from the following equation. Suppose a new investment, Δx, is added into the current portfolio, the question "what is the difference between the VaRs before and after this investment?" will be asked.

$$\text{marginal VaR}_{\Delta x} = \frac{\Delta \text{VaR}_{\text{all}}}{\Delta x} \quad (2.36)$$

It is of particular interest to investors what the effect of a new trade, Δx, would be to the current portfolio. The spirit of Eq. (2.36) is the risk increment of every dollar investment. Assume two options of the new trade, x and y, and MVaR_x and MVaR_y are their marginal VaR, respectively. If x and y are equivalent on the return-side, the decision for the new investment is obviously made on the project with the smaller MVaR. However, if there is a discrepancy between x and y in the risk-side and return side, then the investment of x and y is quite vague. Theoretically, under the perfect conditions, each unit of risk should have the same return.

$$\frac{R_1}{\text{VaR}_1} = \frac{R_2}{\text{VaR}_2} = \cdots = \frac{R_n}{\text{VaR}_n} \quad (2.37)$$

This concept as shown in the equation above is a well-known performance indicator, proposed by Sharpe (1966) or Treynor (1965). The spirit of Eq. (2.37) indicates that each unit risk in the portfolio obtained an equivalent return. Thus, an optimal portfolio can be derived in this condition. In addition, any new investment or project can then be tested to see if they increase or decrease the total risk of the portfolio.

Secondly, from the aspect of total risk the management has to decide which one of the following two alternatives is their operating objective.

Alternative 1:

$$\begin{cases} \max: R_{\text{all}} = \sum_{i=1}^{n} r_i \times \omega_i \\ \text{st.} \ : \ \text{VaR}_{\text{all}} \le \mathcal{U} \end{cases} \quad (2.38)$$

Alternative 2:

$$\begin{cases} \min: \text{VaR}_{\text{all}} \\ \text{st.}: R_{\text{all}} = \sum_{i=1}^{n} r_i \times \omega_i \ge \mathcal{L} \end{cases} \quad (2.39)$$

where R_{all} and r_i indicate returns of the entire portfolio and the individual asset, respectively, and ω_i is the proportion of the individual asset in the portfolio. $\mathcal{L}$ and $\mathcal{U}$ are the lower and upper boundaries to the portfolio return and VaR_{all}. There is no standard rule for choosing between alternative 1 or 2, since each business line might encounter varied market conditions and business strategies. For example, the bond market is generally more stable than the equity market. Thus, an equity fund might set alternative 2 as its strategy, and a bond fund might find alternative 1 more suitable. Then in a volatile or a tranquil period, you can modulate the lower and upper boundaries.

Finally, the efficient frontier can be built with the VaR. For example, the investors in the UK equity market can use the forecasted VaR of the constituent stocks of the FTSE100 index and their expected returns to present the efficient frontier based on VaR. Based on a monthly efficient frontier, one could find which stocks fall around the efficient frontier. Consequently, investors could obtain an optimal investment of these constituent stocks. Moreover, they can adjust their portfolio according to the change of the monthly efficient frontier.

2.5 Extreme Value Theory

As mentioned in the beginning of this chapter, one of the critical factors in the calculation of value at risk is the density estimation of financial returns, particularly in the tail area. For the purpose of increasing the accuracy of the forecasted VaR, extreme value theory straightforwardly selects certain extreme values from the available sample to fit the tail distribution instead of estimating the whole distribution with the entire range of samples[24]. In this section, two different approaches (Block maxima and peak

[24] Two very thorough textbook examples of extreme value theory can be found in Leadbetter, Lindgren and Rootzen (1983) and Embrechts, Kluppelberg and Mikosch (1997).

over threshold) of sampling procedure for extreme value theory and their application in the VaR calculation are introduced. Both of these two methods construct a series of extremes, however the procedure and assumptions used are substantially different.

In financial markets, both positive and negative extremes are seen as a risk to investors. Therefore, VaRs to the long and short positions are demonstrated in this section. Fortunately, we can simply inverse a long position of a financial return to a short position with a minus sign.

2.5.1 Extreme Values based on block maxima

The first method of sampling procedure for extreme value theory is called block maxima, which means that each extreme[25] (the maximum and minimum observations) is sampled in a fixed block period. The procedure of VaR calculation for this method can be divided into several steps. The first step is to opt for a block period, for example one week, ten days, or one month. The second step is to select the extreme value in each block, and then the maximum and minimum extremes are collected to fit the tail distribution of financial returns. Let $r_1, r_2, \cdots, r_N$ be a series of asset returns without autoregression. One can extract the maximum observation from each block that includes n observations. The first maximum extreme value in the first block is denoted as $r_{max,1n} = \max(r_1, r_2, \cdots, r_n)$. In this manner, a sequence of extremes, $r_{max,1n}$, $r_{max,2n}, \cdots, r_{max,gn}$, can be extracted from each block, where $g \times n = N$. For these extremes, Fisher and Tippett (1928) suggested that if there exists a constant $c_n>0$ and a constant $d_n \in R$, then the variable (x_j) obtained from the normalized extremes will converge to a specific distribution, $H(\cdot)$. This can be

[25]Generally, most natural sciences focus on the maximum observations, for example, the wind speed and flood level. Thus, the maximum observations of those variables are collected and analysed. In finance, stock prices going up and dropping down are both common risks for investors in short and long positions.

mathematically expressed as

$$x_j = \frac{r_{max,jn} - d_n}{c_n} \xrightarrow{d} H_{max}(\cdot) \qquad (2.40)$$

where $1 \leq j \leq g$, and g is the number of the block. If $H_{max}(\cdot)$[26] is a non-degenerate distribution[27] then it must belong to one of the following three standard extreme value distributions.

Type 1: Fréchet distribution[28],

$$\Phi(x; k_n) = \begin{cases} 0\ , & x \leq 0 \\ \exp\left[(-x)^{\frac{1}{k_n}}\right]\ , & x > 0 \end{cases} \qquad (2.41)$$

Type 2: Weibull distribution[29],

$$\Psi(x; k_n) = \begin{cases} \exp\left[-(-x)^{\frac{1}{k_n}}\right]\ , & x < 0 \\ 1, & x \geq 0 \end{cases} \qquad (2.42)$$

Type 3: Gumbel distribution[30],

$$\Lambda(x) = \exp[-\exp(-x)], \quad x \in R \qquad (2.43)$$

The figures for these three extreme distributions are shown in Figure 2.2. One can see

26 The subscript, max, in Eq. (2.40) means that each extreme return is the largest in its block.

27 A clear definition of non-degenerate distribution can be found in Chung (2001).

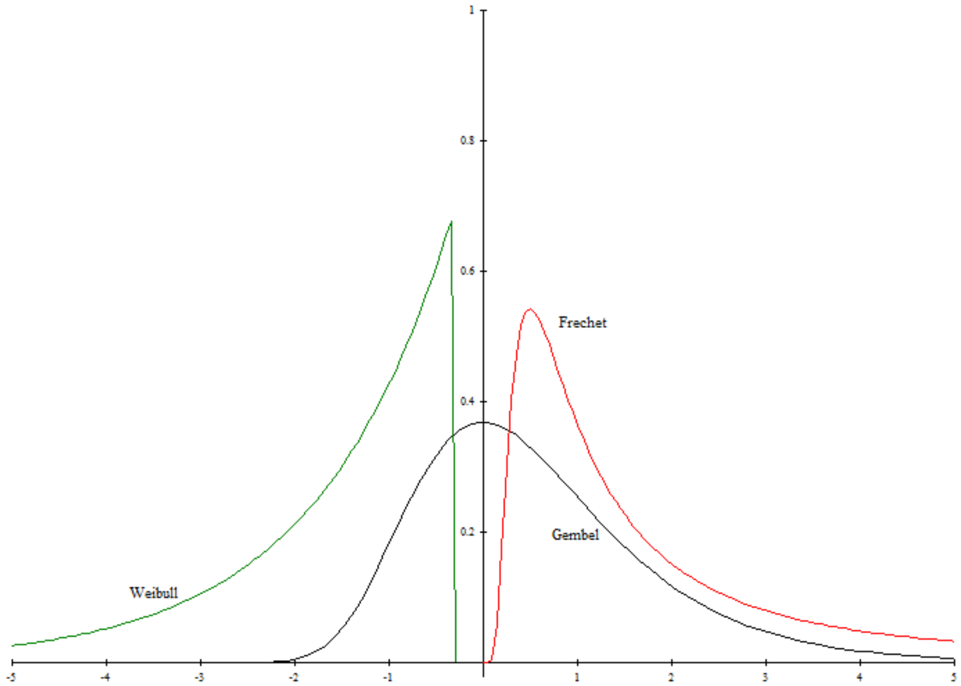
28 This distribution is proposed by Maurice Fréchet in 1927 and the density function can be formulated as $f(x) = \frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1} \exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right]$, where α is the shape parameter and β is the scale parameter.

29 Weibull distribution was proposed by Waloddi Weibull in 1939, and is widely used in the material sciences.

30 Gumbel distribution was proposed by the German mathematician, Emil Gumbel, in 1960 and has been applied in particular for the modelling of meteorological phenomena such as annual flood flows. Its probability density function can be expressed as $f(x) = \frac{1}{\sigma}\exp(-z - \exp(-z))$, where $z = {(x-\mu)}/{\sigma}$, μ is the location parameter and σ is the distribution scale.

that the Gumbel distribution has the thinnest tail, and the Fréchet distribution is the thickest.

Figure 2.2 The three types of distribution for the extreme value



Jenkinson (1955) provided a representation with three parameters to generalize these three extreme value distributions above as follows.

$$H(r_{max,jn}; k_n, c_n, d_n) = \begin{cases} \exp\left\{-\left[1 + k_n\left(\frac{r_{max,jn}-d_n}{c_n}\right)\right]^{\frac{1}{k_n}}\right\} & , \text{ if } k_n \neq 0 \\ \exp\left\{-\exp\left[-\left(\frac{r_{max,jn}-d_n}{c_n}\right)\right]\right\} & , \text{ if } k_n = 0 \end{cases} \quad (2.44)$$

where j=1,2,…,g. The parameters (d_n and c_n) in Eq. (2.44) indicate the location parameter and scale parameter respectively, and, k_n is a shape parameter. When $k_n < 0$ and $k_n > 0$ correspond to the Fréchet distribution with a heavy tail and the Weibull distribution with a limited tail, respectively. The case of $k_n = 0$ is associated with the Gumbel distribution. This generalization is known as generalized extreme value (GEV) distribution for maxima extremes. Eq. (2.44) can be rewritten for minimum extremes (i.e. long position) in the analogy.

$$H\left(r_{min,jn}; k'_n, c'_n, d'_n\right) = \begin{cases} 1 - \exp\left\{-\left[1 + k'_n\left(\frac{r_{min,jn} - d'_n}{c'_n}\right)\right]^{\frac{1}{k'_n}}\right\}, & \text{if } k'_n \neq 0 \\ 1 - \exp\left\{-\exp\left[-\left(\frac{r_{min,jn} - d'_n}{c'_n}\right)\right]\right\}, & \text{if } k'_n = 0 \end{cases} \tag{2.45}$$

All the parameters in Eq. (2.39) and (2.45) can be derived by using the maximum likelihood estimation. From the perspective of risk management for financial time series, the Fréchet distribution with a thick tail is the most suitable to fit the distribution of asset returns. However, other features of the returns of financial assets cannot be completely excluded. Consequently, GEV distribution with a shape parameter will be applied to fit the tail distribution of equity returns. Then the probability density function (pdf) of GEV with maximum extremes, $h(\cdot)$, can be directly obtained by the first order of differentiation of Eq. (2.39).

$$h\left(r_{max,jn}; k_n, c_n, d_n\right) = \frac{1}{c_n}\left(1 + k_n\frac{(r_{max,jn} - d_n)}{c_n}\right)^{\frac{1}{k_n}-1} \exp\left[-\left(1 + k_n\frac{(r_{max,jn} - d_n)}{c_n}\right)^{\frac{1}{k_n}}\right] \tag{2.46}$$

Finally, the VaR for the long and short position can be obtained with a given probability, α. Take for example a long position, the VaR formula for the original return based on Eq. (2.45) is

$$\mathrm{VaR} = d_n' - \frac{c_n'}{k_n'}\left\{1 - [n\ln(1-\alpha)]^{k_n'}\right\} \quad (2.47)$$

The distribution above is based on an i.i.d assumption. However, for most financial returns data might contain a cluster phenomenon on the basis of information signalling theory. In this case, the cumulative density function would be arranged as

$$H(r_{max,jn}; k_n, c_n, d_n, \theta) = \exp\left\{-\left[1 + k_n\left(\frac{r_{max,jn} - d_n}{c_n}\right)\right]^{\frac{\theta}{k_n}}\right\} \quad , \text{ if } k_n \neq 0 \quad (2.48)$$

where θ is called the extremal index describing the clustering effect or series dependence in a stationary series. When θ is close to 1, it indicates that the return series is weakly dependent. In contrast, the dependence effect is stronger with a smaller extremal index, θ. Unfortunately, most financial returns might have a weaker dependence but remain stationary in the return level. Analogically, the probability density function can be derived in the same way as shown in Eq. (2.49).

$$h(r_{max,jn}; k_n, c_n, d_n, \theta)$$
$$= \frac{1}{c_n}\left[\left(1 + k_n\frac{(r_{max,jn} - d_n)}{c_n}\right)\right]^{\frac{\theta}{k_n} - 1} \exp\left[-\left(1 + k_n\frac{(r_{max,jn} - d_n)}{c_n}\right)^{\frac{\theta}{k_n}}\right] \quad (2.49)$$

The parameters in Eq. (2.49): the location parameter (d_n), scale parameter (c_n), extremal index (θ) and tail index (k_n), can be estimated by the maximum likelihood estimator (MLE). The log-likelihood function would be:

$$\ln L(r_{max,1n}, r_{max,2n}, \cdots, r_{max,gn}) = \ln \prod_{j=1}^{g} h(r_{max,jn})$$

$$= g\ln(\theta c_n) - \left(\frac{\theta}{k_n} + 1\right) \sum_{j=1}^{g} \left(1 + k_n \frac{r_{max,jn} - d_n}{c_n}\right) - \sum_{j=1}^{g} \left(1 + k_n \frac{r_{max,jn} - d_n}{c_n}\right)^{\frac{\theta}{k_n}} \quad (2.50)$$

2.5.2 Extreme Values based on the peak over threshold method (POT)

The aforementioned extreme value sampling method encounters some intractable defects. First, as mentioned above, the length of the block is still difficult to define. Unfortunately, the parameters of the GEV distribution are strongly affected by the selection of the block. Secondly, the procedure of sample selection to block maxima needs a long range of data to collect enough extremes to accurately fit to the tail distribution. However, this is difficult in practice. For instance, it would not be possible in cases such as annual environment data or some newly issued financial instruments. Consequently, to overcome these drawbacks, a modern sample selection procedure, called the peak over threshold[31], has been proposed, which pays attention to the values which exceed a certain threshold or a particular hurdle. In other words, POT considers the distribution of exceedances over a certain threshold rather than the original data. For illustration, let $\{r_t\}$ be a random sequence of a certain financial return with a distribution function, F. The POT method concentrates on the conditional distribution F_u constructed by the values above a given threshold, u. In this manner, F_u can be defined as

$$F_u(y) = \Pr(r_t - u \leq y | r_t > u)$$

[31] Actually, the most complete and earliest reference to the peak over threshold (POT) method is Todorovic and Zelenhasic (1970). The original purpose of this method was developed for the natural sciences, for example, flood estimation and air pollution research. A more comprehensive discussion can be found in Smith (2002). Although this sampling concept in extreme value theory is not new, its application in financial areas is still quite modern.

$$= \frac{\Pr(u \le r_t \le y + u)}{\Pr(r_t > u)} = \frac{\Pr(r_t \le y + u) - \Pr(r_t \le u)}{1 - \Pr(r_t \le u)}$$

$$= \frac{F(y+u) - F(u)}{1 - F(u)} = \frac{F(r_t^*) - F(u)}{1 - F(u)} \tag{2.51}$$

where r_t is a random return, u is a given threshold, and y= r_t - u are the values over u. For the return sequence, r_t, Eq. (2.51) focuses on a distribution given a positive threshold, u, and a particular distribution can be derived using a succession of algebra such as

$$\Pr(r_t - u \le y | r_t > u) \approx 1 - \left[1 - \frac{ky}{c - k(u-d)}\right]^{\frac{1}{k}} \tag{2.52}$$

where k, c, and d are the parameters of shape, scale, and location, respectively. Eq. (2.52) is the upper tail distribution based on extreme values exceeding the threshold u. In Eq. (2.52), the case $k > 0$ corresponds to heavy-tail distribution where tail decay is like power functions, such as the Pareto, Student t, and Fréchet distributions. The case $k = 0$ suits normal, exponential, and lognormal situations where the tail decays exponentially. Then the case $k < 0$ adapts to the distribution with a short tail, for instance, uniform and beta distributions. Pickands III (1975) suggested that the conditional excess distribution function, $F_u(y)$, can be usefully approximated by a limiting distribution $G_{k,\sigma}(y)$ when $u \to \infty$.

$$F_u(y) \approx G_{k,\sigma}(y), \ u \to \infty \tag{2.53}$$

where k and σ are the shape and dispersion parameters obtained from statistical estimation.

$$G_{k,\sigma,u}(y) = \begin{cases} 1 - \left(1 - \frac{k}{\sigma} y\right)^{\frac{1}{k}}, \text{if } k \neq 0 \\ 1 - \exp\left(\frac{-y}{\sigma}\right) \ , \text{if } k = 0 \end{cases}, \quad y = r. - u \tag{2.54}$$

Since y represents the part of the return exceeding the threshold, $y \in [0, (r_{max} - u)]$. Eq. (2.54) is called generalized Pareto distribution (GPD). In its earlier stages, GPD was generally applied to diversiform environmental science. Nowadays, it is employed to measure market risk in rare market conditions. From this standing point, Eq. (2.54) can be inverted to be a VaR measure after a sequence of algebraic procedures.

$$VaR = u + \frac{\sigma}{k}\left[\left(\frac{N}{n_u} p\right)^k - 1\right] \tag{2.55}$$

where N and n_u indicate the total number of observations and the number of observations above the threshold u.

However, in financial markets, investors would be more interested in the lower tail with a lower threshold v, which can be obtained from similar distributions for the long position.

$$Pr(r_t - v \leq y | r_t < v) \approx \left[1 - \frac{ky}{c - k(v-d)}\right]^{\frac{1}{k}} \tag{2.56}$$

By analogy, GDP and its VaR formula for the long position can be expressed as follows.

$$G_{k,\sigma,v}(y) = \begin{cases} \left(1 - \frac{k}{\sigma} y\right)^{\frac{1}{k}}, \text{if } k \neq 0 \\ \exp\left(\frac{-y}{\sigma}\right) \ , \text{if } k = 0 \end{cases} \quad y = r. - v \tag{2.57}$$

$$\mathrm{VaR} = \mathrm{v} + \frac{\sigma}{\mathrm{k}}\left[1 - \left(1 - \mathrm{p}\frac{\mathrm{N}}{\mathrm{N}-\mathrm{n_v}}\right)^{\mathrm{k}}\right] \qquad (2.58)$$

where v indicates the lower threshold of the return level.

2.6 Portfolio VaR measure

Based on the theories presented in Sections 2.3 and 2.5, a comprehensive theory and method for measuring portfolio VaR is provided in this section, a combination of extreme value theory and the dynamic relationship between individual assets. This section starts by linking extreme value theory with the method of variance/covariance, and then discusses the nature of correlation. In addition, two critical concepts, called seriality and correlation with seriality[32], will be defined. Moreover, three frequently used measures of correlation and some of their shortcomings will be discussed. In the final subsection, the measure of portfolio VaR will be carried out, and some of its advantages will be explained.

According to Sections 2.3 and 2.5, the calculation of portfolio VaR can be divided into two steps. A portfolio VaR could not be calculated using the multivariate version of Eq. (2.47) since that would only provide a possible set of spaces for portfolio VaR, which is not meaningful in practice. Accordingly, the secret for measuring a portfolio VaR is to find a way to aggregate individual VaRs. In the first step, individual VaRs are measured based on the method of extreme value theory. Avoiding the fallacy described in Section 2.3.3, the extreme value sampling procedure, block maxima, is adopted in this thesis. Thus, Eq. (2.47) can be used to measure the original risk of each asset in the portfolio. Based on the method of variance-covariance, Eq. (2.18) can be rearranged as the

[32] The concept of seriality was proposed by Kammerer (1919). He was a biologist and the formulator of “the law of seriality”, offering a systematization of serial data, for example, homologous and analogous, pure and hybrid, and other types of sequences. However, his research was arrested on 23th September 1926 due to his suicide.

following equation by assuming the mean return of the portfolio approaches zero.

$$\mathrm{VaR}_\alpha = z_\alpha\sqrt{\boldsymbol{\omega\Sigma\omega'}} \tag{2.59}$$

In the univariate case, Eq. (2.59) will be reduced back to Eq. (2.3) without the mean return. Theoretically, the VaR (for a long position) obtained from extreme value theory is also a particular point in the horizontal axis of the distribution diagram as shown in Figure 2.2. Thus, VaR calculated by extreme value theory can be represented by the new volatility of a new distribution.

$$\mathrm{VaR}_{ex} = z_\alpha\sigma_r = \widetilde{\sigma_r} \tag{2.60}$$

For the bivariate condition, Eq. (2.59) can be rearranged as a matrix version.

$$\mathrm{VaR}_\alpha = z_\alpha\left\{[\omega_1 \quad \omega_2]\begin{bmatrix}\sigma_1^2 & \sigma_{12}\\ \sigma_{12} & \sigma_2^2\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}\right\}^{1/2}$$

$$= z_\alpha(\omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_{12})^{1/2} \tag{2.61}$$

According to the concept of Eq. (2.60) and the basic theory of statistics, Eq. (2.61) can be represented as a summation of all the individual VaRs and the relationship between individual assets. This can be formulated as

$$\mathrm{VaR}_\alpha = (\omega_1^2 z_\alpha^2\sigma_1^2 + \omega_2^2 z_\alpha^2\sigma_2^2 + 2\omega_1\omega_2 z_\alpha^2\rho_{12}\sigma_1\sigma_2)^{1/2}$$

$$= (\omega_1^2\tilde{\sigma}_1^2 + \omega_2^2\tilde{\sigma}_2^2 + 2\omega_1\omega_2\rho_{12}\tilde{\sigma}_1\tilde{\sigma}_2)^{1/2}$$

$$= \left(\omega_1^2\mathrm{VaR}_{1,ex}^2 + \omega_2^2\mathrm{VaR}_{2,ex}^2 + 2\omega_1\omega_2\rho_{12}\mathrm{VaR}_{1,ex}\mathrm{VaR}_{2,ex}\right)^{1/2} \tag{2.62}$$

Using the approach for a portfolio VaR, Eq. (2.62) can be extended to a multivariate case.

$$\mathrm{VaR}_{\alpha,p} = \left(\sum_{i=1}^{d}\omega_i^2\mathrm{VaR}_{i,ex}^2 + 2\sum_{\substack{i=1\\ i\neq j}}^{d}\sum_{\substack{j=1\\ j>i}}^{d}\omega_i\omega_j\rho_{ij}\mathrm{VaR}_i\mathrm{VaR}_j\right)^{1/2} \tag{2.63}$$

A similar approach to Eq. (2.63) was applied in Longin (2000), however, they found a paramount issue that needed to be addressed, which will be elaborated on in the second step of measuring portfolio VaR. The main advantage of the method of Eq. (2.63) in measuring portfolio VaR is that it is conceptually easy to understand and directly echoes Markowitz's (1952) thoughts regarding the perspective of portfolio theory. In addition, it can be easily applied to monitor the marginal risk contribution for the entire portfolio risk. In other words, using Eq. (2.63) when measuring portfolio risk could achieve the purposes of controlling the total risk of a portfolio and monitoring the marginal risk contribution of each asset in the portfolio, as mentioned in Section 2.4. However, this approach still has a palpable weakness in aggregating individual risk as a portfolio VaR, particularly with the relationship[33] between the individual VaRs. Although Eq. (2.63) has been applied in some research looking at measuring portfolio risk, the role of the correlation is mainly restricted within the property of linearity (Embrechts, McNeil and Straumann, 2002). Specifically, if the relationship between two random variables is non-linear, the relationship cannot be captured by the (Pearson) correlation. Accordingly, the principal objective of the second step in calculating portfolio VaR is to modify the method of aggregation in Eq. (2.63).

[33] The relationship between individual assets in Eq. (2.62) and (2.63) is characterized by using Pearson's linear correlation (also called the Pearson correlation). According to Pearson (1920), the original concept of correlation was initiated by August Bravais, a geologist who also wrote on astronomy, physics, meteorology and the theory of probabilities.

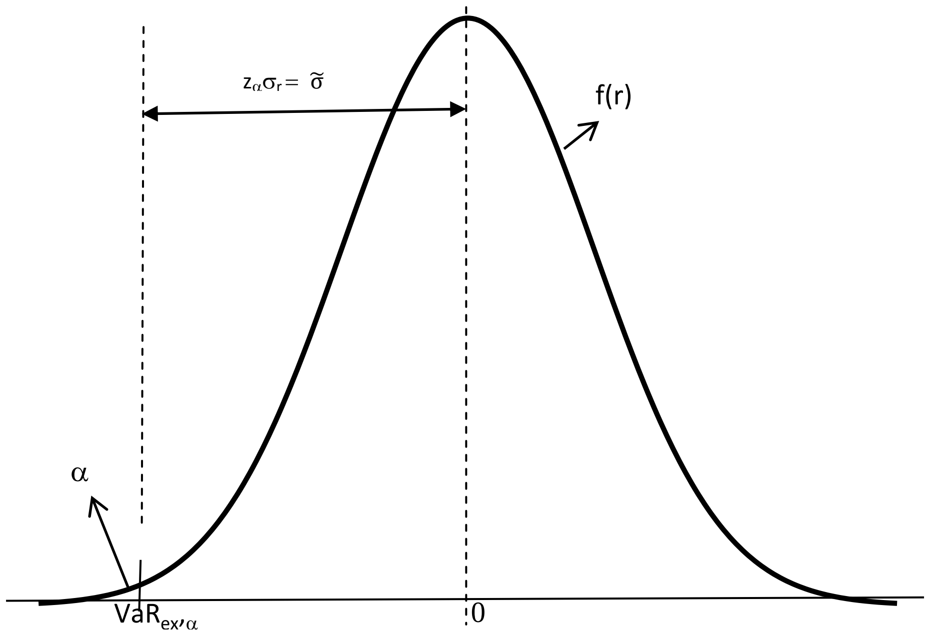


Figure 2.3 The analogy of VaR based on EVT with standard deviation of financial return

Before discussion of the second step, two important concepts of time series data mentioned in the beginning of this section should be defined. First, in this thesis a special term, called seriality, is defined as the order of occurrence of a set of time series data.

Definition 1:Seriality is defined as the order of occurrence by time of a random time serial variable.

The concept of seriality suggests that the order of time series data has natural properties. For example, the price of an asset at time t is derived from its price at time t-1, which synthesizes investors' views and behaviour happening between t and t-1. Thus, the seriality of a sequence should not be changed if the sequence happened. The concept of seriality is not new in the literature. An Austrian biologist, Kammerer (1919) is generally credited as the first one to describe the term of seriality. Kammerer describes the feature of seriality by stressing the recurrence or clustering of the same or similar events in a time horizon or a certain space, but here the definition of seriality emphasizes the order of occurrence of a same or similar event. In Kammerer's theory, a random event occurs in a short time period more than its expectation of general intuition, indicating the existence of an unexplained physical force or statistical rule provoking this behaviour. For financial time series data, for example, the stock price reflects investors' views about underlying assets, and investors' behaviour changes are reflected on stock returns. In this manner, financial time series data sets have both features of seriality as defined by Kammerer and this thesis because investors' behaviour is naturally assumed to be continuous based on its occurrence. In other words, investors' behaviour at time t is driven by how they acted at time t-1. The importance of seriality is to supplement the lack of four moments in describing the characteristics of financial time series data. In the classical theory of statistics, it is known that the features of a distribution of a set of random data could be entirely described by its four moments, i.e. mean, variance, skewness, and kurtosis. However, one might obtain the same four moments from two different financial time series data sets with totally different patterns or ordering of the data. In this case, from the aspect of the four moments, the two sets of data are statistically alike with each other because of the same mean, variance, skewness, and kurtosis, and maybe even their distributions. Yet the two sets of time serial data probably actually have different patterns, i.e. different seriality, in the time horizon. This is a particular feature of time serial data, which does not show in cross-sectional data or in that collected from questionnaires. It is not easy to find the importance of seriality of single financial time series, and it is impossible to disturb the order when we are measuring the serial correlation since the order exists within the time series data. Thus, this thought can be used in the next concept.

The second step of portfolio VaR calculation involves a new concept we need to define, a derivative constructed on the definition of seriality, called "correlation with seriality".

Definition 2: correlation with seriality. If the relationship between two sets of time series *data is calculated based on the order of their original occurrences, then any correlation is defined to be a correlation with seriality.*

According to the definition above, correlation with seriality is the relationship between two sets of time serial data derived from their patterns. In other words, correlation with seriality describes the degree of the co-movement of two sequences within their own patterns, and it emphasizes that the relationship between two sets of time serial data needs to be reasonably measured in accordance with the order of occurrence. Failure to take into account this property can render any conclusions meaningless. For example, if one would like to measure the dependence between the return of the FTSE 100 and the S&P 500 over thirty years, it is meaningless to calculate the conditional probability of co-occurrence of the decline of S&P 500 in 2008 and the one of the FTSE 100 in 1987. The former, in the United States, was caused by the financial crisis of subordinated debts, and the latter was triggered by the well-known black Monday[34]. Statistically, it is possible that the two events would occur again in the future at the same time even though the probability is extremely low. However, from the viewpoint of risk management, we are interested in the relationship in their future pattern, and the degree of their co-movement. Specifically, the co-movement or the correlation of two sets of time series data is more meaningful than the pure probability of a co-occurrence of two particular events.

For illustration, we provide an example with two original five-year financial index returns (the Hang Seng Index and the Nikkei 225 Index) as shown in Figure 2.4 panel (a) and (b). In panel (c), a changed Nikkei 225 (i.e. the returns from 1st March 2007 to 31th March 2008 is moved backward to 2nd January 2006.) is made. Under the peak over threshold (POT) sampling procedure as discussed in Section 2.5.2, conditional correlation of the

[34] Black Monday is the market crash on Monday (October 19, 1987), starting from Hong Kong and spreading to European stock markets, and finally the stock market of the United States. The stock markets around the world crashed, causing a huge value of losses in a very short time.

Hang Seng Index and the Nikkei 225 (ρ_{ab}) would be equal to the conditional correlation of the Hang Seng Index and the changed Nikkei 225 (ρ_{ac}). This phenomenon can be attributed to the lack of seriality[35] in pairs of extreme series. Under these circumstances, several issues can arise. Firstly, from the viewpoint of conditional correlation (conditional on extremes collected by POT approach), it can cause serious confusion that $\rho_{ab}=\rho_{ac}$ even if they do not have the same patterns. In addition, in the conditional correlation ρ_{ac}, it is difficult to offer an explanation as to why the extremes of the Hang Seng Index in 2008 might be associated with the ones of the Nikkei 225 in the middle of 2007. In other words, if the data in panel (c) is real, how could we explain that the market shock happening in the Hong Kong stock market in 2008 could be related to another shock in the Japanese stock market one and a half years earlier? Finally, it is a troublesome puzzle to decide the threshold to make the number of extreme values of the two return series equivalent. Obviously, none of the three issues above is easy to solve, and, to my best knowledge, none of them has yet been successfully resolved. A similar question also arises with regard to the method of tail dependence in Eq. (2.66) and (2.68). The best method to solve this issue is to calculate the conditional correlation with data sets that are arranged according to their time of occurrence. In other words, the calculation of conditional correlation has to be time-matched with each other, consistent with the concept of seriality.

[35] Generally speaking, the feature of seriality refers to synchronous trading behaviour. Synchronicity to extremes indicates that an extreme value in each fixed period would be accompanied by another extreme value in the same time interval.

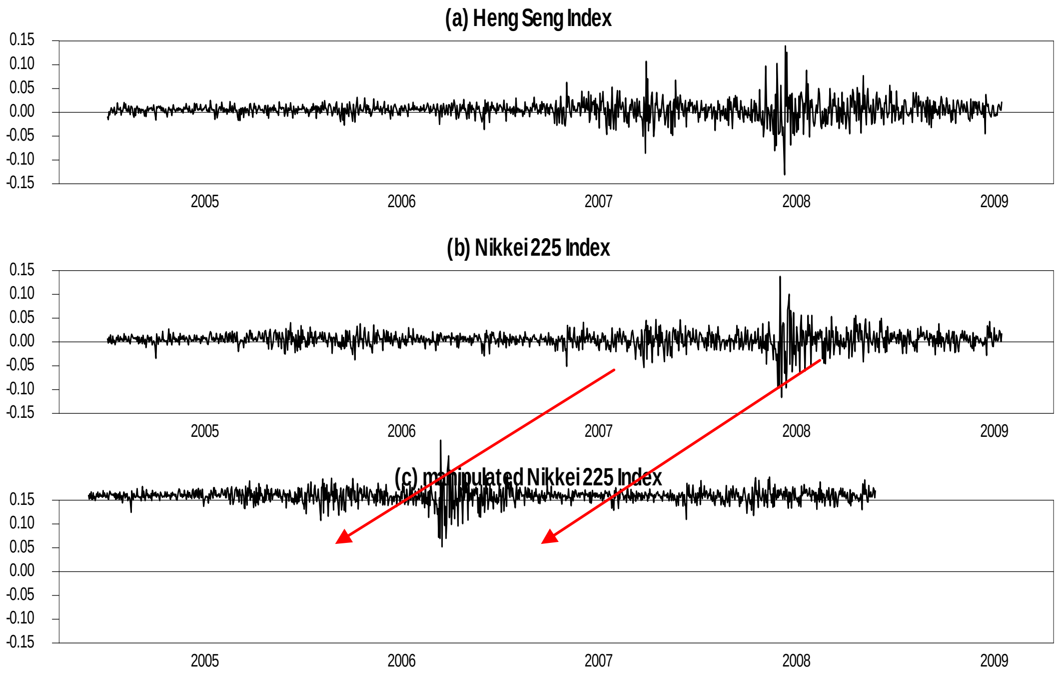


Figure 2.4 Returns of HSI and Nikkei225 index

According to Eq. (2.63), the portfolio VaR can be expressed as the square root of the sum of each squared individual VaR, and their interaction in the portfolio. One critical component, the coefficient of Pearson correlation, is applied to describe the relationship between individual assets in the portfolio. In recent years, there has been a wide-ranging debate on how to measure the relationship between two random variables. McNeil, Frey and Embrechts (2005) suggest that three main approaches are generally applied in measuring the relationship between two random variables. They are Pearson's linear correlation, rank correlation, and tail dependence (TD) based on the method of copula. Pearson's correlation has been shown in Eq. (2.26).

Two common measures of rank correlation, Spearman's rho (ρ^S_{XY}) and Kendall's tau (τ), have been discussed and applied in the literature. However, they might not be helpful when measuring the relationship between two sets of time serial data due to their main weaknesses, especially in risk measurement. The first one, Spearman's rank correlation, can be expressed as Eq. (2.64).

$$\rho_{XY}^{S} = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)} \tag{2.64}$$

Where $d_i = R_{X_i} - R_{Y_i}$, means the difference between the ranks of the two random variables, and is used to transfer data to a rank series for the purpose of avoiding the numerical scale issue. The second rank correlation can be regarded as an estimate of the concordance between two random sequences.

$$\tau = \Pr\big((X_1 - X_2)(Y_1 - Y_2) > 0\big) - \Pr\big((X_1 - X_2)(Y_1 - Y_2) < 0\big) \tag{2.65}$$

As with Eq. (2.65), Kendall's τ can be explained as the probability of the concordance of X and Y, minus the probability of discordance. In other words, Kendall's τ describes the degree of co-monotonic increase or decrease of the two sequences.

Thirdly, we look at tail dependence, which emphasizes the relationship between the variables in the tail area. Eq. (2.67) and (2.69) describe the tail dependence in the upper and lower tail, respectively.

$$\lambda_{XY,\ell}^{TD} = \lim_{q\to 0} \Pr[X \le F_X^{-1}(q) | Y \le F_Y^{-1}(q)] \tag{2.66}$$

$$= \lim_{q\to 0} \frac{\Pr\left(X \le F_X^{-1}(q), Y \le F_Y^{-1}(q)\right)}{\Pr\left(Y \le F_Y^{-1}(q)\right)}$$

$$= \lim_{q\to 0} \frac{C(q,q)}{q} \tag{2.67}$$

$$\lambda_{XY,u}^{TD} = \lim_{q\to 1} \Pr[X > F_X^{-1}(q) | Y > F_Y^{-1}(q)] \tag{2.68}$$

$$= \lim_{q\to 1} \frac{\Pr\left(X > F_X^{-1}(q), Y > F_Y^{-1}(q)\right)}{\Pr\left(Y > F_Y^{-1}(q)\right)}$$

$$= \lim_{q\to 1} \frac{1-\Pr\left(X\le F_X^{-1}(q)\right)-\Pr\left(Y\le F_Y^{-1}(q)\right)+\Pr\left(X\le F_X^{-1}(q),Y\le F_Y^{-1}(q)\right)}{1-\Pr\left(Y\le F_Y^{-1}(q)\right)}$$

$$= \lim_{q\to 1} \frac{1-2q+C(q,q)}{1-q}$$

$$= 2-\lim_{q\to 1} \frac{1-C(q,q)}{1-q} \tag{2.69}$$

where F_x and F_y are the cumulative density functions of X and Y, and q is the tail probability in a certain criteria. From the risk management perspective, especially as relates to extreme value theory, the concept of tail correlation is doubtless commendable, since we care more about the prices of financial instruments that concurrently run to the extreme rather than those that stay around the average price at the same time. As a result, the key point of correlation of the time serial data sets should be punctuated at the tail areas. However, there might be a considerable drawback in applying this measure to time serial data. The tail dependences shown in Eq. (2.66) and (2.68), based on the method of copula, are drawn as a conditional probability. Take, for example, the lower tail dependence in Eq. (2.66), if q approaches to zero, then the cumulative density functions, $F_X(q)$ and $F_Y(q)$, approach to zero as well. From the probability aspect of the X and Y sequences, $F_X^{-1}(q\to 0)$ and $F_Y^{-1}(q\to 0)$ are two extremely left points in the tail area of f(x) and f(y). If X and Y are regarded as two general return sequences, the conditional probability in Eq. (2.66) implies the likelihood that X's return drops in the extremely left area given that Y's return falls in the same area. An obvious fallacy might be caused, as mentioned before, in cases like the S&P 500 and FTSE 100, because this measure of correlation lacks seriality. Mari and Kotz (2001) suggested that a special term, called "statistical dependence", might be more suitable, especially as these events may not coincide at the same moment.

One might still be concerned with the applicability of the correlation in Eq. (2.63) in measuring portfolio VaR, particularly in non-linear cases. To address this uncertainty, Kat (2003) suggests using conditional correlation. Accordingly, Engle's (2002) seminal DCC model describing the change of non-linear correlation over time can be utilized for aggregating individual risk into portfolio VaR. Furthermore, Lhabitant (2002) also applied a similar approach to calculate conditional correlation between two financial instruments.

According to the discussion above, the modified Pearson's correlation based on the DCC model is the only one in compliance with the concept of correlation with seriality. Rank correlation, like Pearson's, measures the relationship between two variables based on the ordering of observations. However, the rank correlation in Eq. (2.64), in fact describes the relationship between the ranks of X and Y, which is obviously different from the correlation of X and Y. From another angle, although the method of tail dependence is not entirely consistent with the spirit of correlation with seriality, its approach of separating the upper and lower tail is still worthy to be apply in the modified Pearson's correlation, especially in the extremes sampling procedure of block maxima. Roughly speaking, the conditional correlation of the upper and lower tail can be calculated with respect to the maxima and minima sampled from the method of block maxima (minima). More details of this forecasting procedure will be given in Chapter 4.

Now that we have defined the concept of correlation with seriality, step two in calculating the portfolio VaR can be achieved by modifying Eq. (2.63) with the dynamic conditional correlation based on extreme values.

$$\mathrm{VaR}_{\alpha,p} = \left(\sum_{i=1}^{d} \omega_i^2 \mathrm{VaR}_{i,ex}^2 + 2 \sum_{\substack{i=1 \\ i \neq j}}^{d} \sum_{\substack{j=1 \\ j > i}}^{d} \omega_i \omega_j \widehat{\rho_{ij}} \mathrm{VaR}_i \mathrm{VaR}_j \right)^{1/2} \qquad (2.70)$$

where $\widehat{\rho_{ij}}$ is the dynamic conditional correlation between asset i and j. The main advantage of Eq. (2.70) is that this method not only accounts for the risk of extreme market conditions based on EVT, but also considers the variability within the conditional correlation. This means fund managers could monitor not only the risk of individual assets, but also any changes in the relationship between assets. This indeed would help them in portfolio management, as mentioned in Section 2.4. When considering whether a new asset should be added into the portfolio or not, the manager might apply this model to find the contribution of the new asset in risk reduction. Everything we have discussed above is designed to try and make the risk measurement more accurate and suitable for practical use.

2.7 Conclusion

This chapter detailed some important theories of VaR and its applications in financial institutions. Several key points regarding the VaR calculation are introduced in Section 2.2, including the background and the general concept of VaR. We also showed the criticism and debate surrounding VaR modelling. In Section 2.3, we explained the calculation of VaR in more detail, and the roles of correlation between different assets in the portfolio VaR calculation. Moreover, three measurements of relationship commonly used in the literature were introduced and discussed.

Section 2.4 looked at how VaR could be applied in practice, starting from simple risk reporting, through risk control or monitoring, and finally to the aspect of actively risk-based performance management. Some of these have already been applied in financial institutions, other potential applications, for example active risk management, are linked with the performance and compensation systems. By applying VaR in these ways, financial institutions could be more efficient and effective in risk management, as well as in performance management. The active use of VaR is also of value for the management system, giving them greater awareness of which product, business line, or profit-centre takes more risk in its position and offering financial institutions a judgement criterion for a new investment or new project, based on their marginal contribution towards the firm-wide total risk. Although it is clear that many financial institutions derive great benefits from their use of VaR, the VaR disclosed by these different financial institutions could not be compared with each other, due to the divergence of the method and the assumption of VaR.

In the Section 2.5, the theory of extreme value, its two sampling approaches, and VaR modelling with a single asset based on those extreme returns were shown. However, the multivariate VaR model is less prominent in the related research, and the method used in my thesis fills this gap in the portfolio VaR modelling area. In Section 2.6, we took into account the extreme market conditions and time-varying relationship between individual financial assets to form a VaR method for a multivariate case. In addition, two unique and critical characteristics of time series data, less discussed in the literature, were also defined in Section 2.6. They are "seriality" for the individual asset and "correlation with seriality" for the relationship between individual assets, which do not generally present in cross-sectional data or data from questionnaires. A lack of either (seriality or correlation with seriality) could cause serious errors in the calculations, and even if results could be obtained their correlation would be difficult to explain. Based on these two concepts, we suggested a suitable measurement of correlation, considering the characteristics of financial data, to use in the portfolio VaR modelling.

Finally, a modified dynamic conditional correlation (DCC) synthesized with individual VaR based on extreme value theory model was obtained. This method also considered the property of the fatness of financial returns in forecasting VaR. In practice, it provides the financial institutions with a clear overview of their market risk, looking at both the individual asset level and the portfolio as a whole.

Chapter 3 Literature Review

3.1 Introduction

As the general financial uncertainty increased during the 1990's, there was an intensive push to do research by financial institutions, regulators and academics to try and develop more sophisticated models for measuring market risk. The major objectives of this chapter are to review the related literature in VaR modelling and the main methods of model evaluation. As mentioned in Section 2.2.1, the VaR concept proposed by J.P. Morgan has long been the standard for measuring market risk, and some of the academics followed J.P. Morgan's idea by focusing on the parametric model. In this line of research, various (G)ARCH related models were developed and applied in measuring market risk, for example, the RiskMetrics model and the variance-covariance method. Other researchers found an alternative route to overcoming the shortcomings of parametric models by concentrating on the non-parametric model[36].

In more recent years, many studies have tried to fill the middle ground between the parametric and non-parametric approach by developing the alternative method, or so called semi-parametric method, and adding in risk management. The core concept, extreme value theory, used to be applied in hydrology but is now adapted to measure market risk in rare conditions (e.g., Embrechts et al. (1997), Embrechts et al. (1999), Longin (2000), McNeil and Frey (2000), Gencay et al. (2003), McNeil et al. (2005), and Gilli and Kellezi (2006)).

[36] The non-parameter models do not assume any distribution to financial returns, but use the α-quantile (or α-percentile) to calculate the VaR.

This chapter looks at the approach of these different VaR measurement models, dividing them into three main categories. The key features of each approach and its application will be reviewed and discussed. Section 2 looks at the parametric models, starting with the RiskMetrics approach introduced by JP.Morgan (1996). All approaches in this category assume a hypothetical distribution of financial returns before measuring VaR. Next, non-parametric models and their variants are presented in Section 3. This line of research overcomes some of the difficulties found in the parametric approach, but still encounters its own obstacles. In Section 4 of this chapter, a semi-parametric approach is presented which does not focus on the whole density of financial returns but instead just pays attention to the extreme returns. In Section 5, various performance evaluation approaches, or so called backtesting, will be reviewed, and some of their difficulties will be discussed as well. Section 6 presents the conclusion.

3.2 Parametric model

According to Eq. (2.3), the volatility of a financial asset plays a critical role in the calculation of VaR. Thus, when we come to look at VaR, the estimation and forecast of volatility needs to be discussed. The development of volatility research directly led to the progression of risk management, especially to the development and refinement of VaR models.

3.2.1 Univariate VaR

Since the seminal paper of Markowitz (1952), the volatility of financial assets has become an important indicator of market risk. In addition, stimulated by the concept of asset risk, the capital asset pricing model (CAPM) was proposed and developed by Treynor (1961), Sharpe (1964), and Lintner (1965). In the CAPM, a market risk measure is obtained by using an indicator, beta, relative to the risk of market portfolio. These two measures (volatility and beta) of market risk, however, do not answer the essential and fundamental question: “how much money are investors going to lose if tomorrow is a bad day in the market?” Volatility and beta merely describe the degree of uncertainty in the market, and cannot quantify any real potential loss of the investors’ holding positions. Market participants are generally looking for a simple measure to uncover how the value of their current portfolio is going to be affected in the next trading period. As a result, a third measure is proposed to fill the gap and answer the question posed above. In this section, this third market risk measure will be discussed in detail.

As is well known in basic statistics and financial theory, the volatility of a financial asset describes the tendency of the price to fluctuate away from the mean. Reflecting the rapidly changing market conditions and getting a more accurate simulation of the reality of markets, Engle (1982) proposed a time-varying concept of the volatility process, called heteroscedasticity, based on the past square of the error term. After Bollerslev’s (1986) generalization of the ARCH model (GARCH) and Bollerslev’s (1990) multivariate version (MGARCH) which followed in this line of research, a series of more complicated versions of the volatility process were created and volatility estimation and forecasting made a remarkable splash in the area of risk management. Copious extensions and variants of the GARCH models have been introduced to encompass various different emphases. Some academics pay attention to the asymmetric response of investors (Nelson (1991), Engle and Ng (1993) and Glosten et al. (1993)), whilst others might focus on more fundamental issues, for example the density of financial returns (Bauer (2000), Giot and Laurent (2004)and Bauwens and Laurent (2005)). Falling somewhere between the two, Engle and Ng (1993) offered a comprehensive discussion in the modelling of asymmetric GARCH and suggested the GJR model fitted better than other asymmetric models.

For the purpose of offering an explicit risk measure, JP.Morgan (1996) proposed a concept of risk measure, called value at risk (VaR), and taking inspiration from Engle’s (1982) and

Bollerslev's (1986) works created a simple measure of market risk model, called RiskMetrics. The basic concept of VaR is defined as an amount of loss on a position with a given probability over a fixed horizon. In other words, it reflects how the value of an investors' portfolio behaves in the extreme market conditions which could occur in the future. The common RiskMetrics model assumes that the returns of financial assets follow a conditional normal distribution with zero mean and conditional variance, which can be expressed as an exponentially weighted moving average (EWMA) process of their historical squared returns. The formula of VaR with a given significant level $1-c$ can be set as $VaR_{\alpha} = Z_{\alpha}\hat{\sigma}_t$[37], where Z_{α} indicates a specific quantile of potential loss distribution of a portfolio and $\hat{\sigma}_t$ means the forecasting conditional volatility (or time-varying volatility) estimated by a certain parametric model. Intuitionally, calculation of VaR here should put the stress on the conditional volatility (Christoffersen and Diebold (2000)). As a result of several market crunches, VaR became the third most popular type of model for measuring market risk in the late 1990's (based on the parametric method) and it only gets more popular with time. Most uses of VaR concentrate on modelling a conditional volatility process of financial returns, although Jorion (1995) implied that ARCH models provide poor volatility forecasts. The past decade has witnessed a rapid development of different techniques for calculating VaR, and it has become a well-known measure in this field due to its simplicity and user-friendliness. In JP.Morgan's (1996) technical document, as the precursor in this line of research, they suggested a GARCH(1,1) process to exchange rate movement with a decay parameter, λ=0.94. Nowadays, RiskMetrics is widely used by practitioners as a substantial tool in modelling volatility because of its easy implementation. However, it might display under- or overestimated risk because of the lack of any great deliberation in its distribution assumption. The VaRs obtained from RiskMetrics are calculated under the unrealistic assumption of normality to assets' returns, and the critical

[37] In this setting, the mean of return to a financial asset is assumed to be zero in the long term due to the efficient market hypothesis.

character of a fat-tail to the distribution is completely neglected. Even though the effect of thickness is minor when calculating VaR at the 95% significant level, it would be significantly different in a higher confidence level (for example, at the 99% of the Basel’s standard requirement). Thus, this assumption of financial returns behaviour might cause market risk to be underestimated when calculating VaRs based on the RiskMetrics approach (Pafka and Kondor, 2001).

Focusing on various volatility approaches, Christoffersen et al. (2001) tested the performance calculated by the original RiskMetrics, GARCH(1,1), an option-based implied volatility, and a stochastic volatility with S&P 500 returns, suggesting that the VaR measure based on the GARCH(1,1) approach generally excels over the others in the 5% and 10% level of significance. These findings are consistent with previous research, stating that significant improvements might be found by releasing the restrictions of RiskMetrics (Danielsson and De Vries (2000) and Engle and Manganelli (2004)). In addition, the failure identification of all these four competing approaches at the 1% level could simply be attributed to extreme returns.

Risk management also corresponds to the holding capital for banking sectors. Berkowitz and O’Brien (2002) conducted an alternative value-at-risk model, ARMA(1,1)-GARCH(1,1), to calculate the risk in the top six large commercial banks in the U.S. They were the first to display direct evidence on the performance of the GARCH model with real data. Their results showed that the banks’ risk measures were too conservative, which would cause the banks to hold too much unnecessary capital and decrease their operating performance. By contrast, the GARCH model of the banks’ profit and loss (P&L) generally provided for lower VaRs and was better at predicting changes in volatility. Because of the latter, the GARCH model permits comparable risk coverage with less regulatory capital.

For the purpose of capturing the effect of the fat tail on the density of financial returns, Mittnik and Paolella (2000), Angelidis et al. (2004), Bams et al. (2005) and Hartz, Mittnik and Paolella (2006) presented some basic and practical methods of risk measurement. They applied various GARCH related models with non-normal distribution to capture the effect of fatness on major exchange rate and primary international equity indices. Their work indicates that VaR measures based on the GARCH approach with student-t distribution are more effective than other measures based on the normal and generalized error distribution (GED). In Mittnik and Paolella (2000), they even took the non-balance response of investors into account to form an asymmetric power ARCH (APARCH) model, and it outperformed other simple GARCH models with t density. Giot and Laurent (2004)[38] adopted the APARCH model with t distribution to calculate 1-day-ahead VaRs of the CAC40 index and S&P 500 futures contracts, and two major exchange rates. In this work, however, they found that the APARCH provides only an equivalent performance to the method based on the realized volatility approach. Echoing the related studies above, and keeping an eye on the skewness or asymmetry (third moment), some studies looked at the excess kurtosis (fourth moment), that is not captured by GARCH with normal density. Harvey and Siddique (1999, 2000) indicated that there is also a dependence in the conditional skewness and possibly in the kurtosis of stock return as well. Wilhelmsson (2009) extended Jensen and Lunde (2001) and Forsberg and Bollerslev's (2002) work and proposed an NIG-autoregressive conditional density (ACD) model applying in market risk measurement. In this manner, it seems all the characters of return density, from the first to the fourth moment, can be included. As a result, the VaR measured by the NIG-ACD model displayed a very competitive performance against other parametric approaches. Similarly, So and Yu (2006) adopted seven GARCH related models to measure the VaR of a range of equity indices and exchange rates. Unfortunately, the results were inconsistent. Generally speaking, the models based on t distribution gave better 1% VaR

[38] The details of this model can be seen in Lambert and Laurent (2001).

estimates than normal distribution in the long position, however this was not the case for the short positions.

Alternatively, some academics apply GARCH models taking into account the relationship between derivatives and their underlying assets. Spurred on by the delta-GARCH model proposed by Hsieh (1993) and the gamma-normal introduced by Wilson (1994), Fallon (1996) then generalized these two thoughts and proposed a volatility process, gamma-GARCH[39]. They used daily returns of four individual stocks listed in the NYSE and S&P 500 index, for a period over twenty-six years, to evaluate the performance of these risk measures. Surprisingly, the results showed that even the gamma-type measure outperformed the ones based on delta, however, both the two models performed poorly in certain situations.

After the first and second energy crisis occurred in the 1970s and 80s, crude oil and the energy market as a whole were spotlighted in hedge investors' energy portfolios. In the past decade, the price of crude oil peaking to a historical high has also caught international investors' and governments' attention and made them cautious. Thus, there has been a big focus in the academic community towards applying diverse parametric VaR approaches to measuring the market risk of crude oil contracts. For example[40], Giot and Laurent (2003a), Fan et al. (2008), Hung et al. (2008), and Agnolucci (2009) suggested that volatility forecasts based on skewed and fat-tail distributions offer more accurate predictions than other competing approaches.

To date, a number of studies looking at a variety of GARCH related models have been applied to measure the VaR of various markets. However, there has not been a conclusive result to give us clear direction and guidance towards which model is the best overall and which might

[39] The gamma of a derivative security is the second derivative with respect to the underlying asset. In a portfolio context, the gamma of a portfolio is the matrix of the second derivative of the portfolio with respect to the vector of underlying assets.

[40] Here, Giot and Laurent (2003a) applied the ARARCH model with t distribution, Fan et al. (2008) and Agnolucci (2009) suggested the GARCH model with generalized error distribution (GED), and Hung et al. (2008)borrowed a GARCH with a heavy-tailed (HT) distribution proposed by Politis (2004).

be best suited for measuring the VaR of various different data.

3.2.2 Multivariate models

The research mentioned in the last section concentrated mostly on the VaR measure for univariate models. With the progress in methodology of time-series analysis, some researchers turned to looking at estimating the total risk of a portfolio. The earliest version of a multivariate GARCH (MGARCH) model can be found in Bollerslev, Engle and Wooldridge (1988), namely the VECH model. The researchers first applied the conditional volatility process as a univariate case, and then applications to VaR with the multivariate model were carried out. Engle and Kroner (1995) (hereafter called BEKK) partially surmounted the over-parameters phenomenon found in the VECH model by suggesting a diagonal representation in the covariance matrix. This significantly improves the efficiency in parameter estimation, especially in large portfolios. Moreover, BEKK also guarantees that a positive definite to the covariance matrix could be obtained, avoiding the uncertainty of parameter testing and inference in the VECH model.

Coming at it from another angle, some researchers built diversified multivariate GARCH, similar to the BEKK model. Engle, Ng and Rothschild (1990) argued that asset excess returns are driven by specific factors, and the conditional covariance of return might be affected by these factors. Several extensions of the factor model were proposed in the literature. For example, Ng, Engle and Rothschild (1992) provided a dynamic multi-factor GARCH in stock returns and Vrontos, Dellaportas and Politis (2003) suggested a full-factor multivariate GARCH.

When it comes to risk management of multi-asset portfolios, attention should also be directed towards the relationship amongst assets. For this reason, some multivariate GARCH models stress the correlations of various assets. Indeed, from both the viewpoint of risk management and portfolio diversification, it is important to clarify the relationship between various assets. Consequently, Bollerslev (1990) presented a multivariate GARCH(1,1) with a constant conditional correlation (CCC), investigating the short-run relationship of five weekly European exchange rates. In addition, Tse (2000) and Bera and Kim (2002) provided some robust evidence against the hypothesis of constant correlation by examining several major equity indices. Echoing this, Engle (2002), Tse and Tsui (2002)[41], and Christodoulakis and Satchell (2002) offered similar models describing inconstant conditional correlation, also called time-varying correlation[42]. Multivariate GARCH models with time-varying correlation theoretically offer more accurate estimations and are intuitionally closer to reality. However, one major drawback to the DCC model reduces its accuracy: it assumes that the pattern of the dynamics of the conditional correlation in the market is not changeable since the parameters, α and β (or θ_1 and θ_2 in Tse and Tsui (2002)), are time-invariant. Therefore it erroneously concludes that the conditional correlation of all assets has the same dynamic pattern. From the aspect of VaR based on a MGARCH model, Engle (2002) displayed some rough results showing that MGARCH-DCC related models, especially mean reverting (MR) ones, offer better outcomes than other methods. A flexible DCC (FDCC) model and then its generalization version are proposed by Billio et al. (2006) and Billio and Caporin (2009) respectively, and both of these two models are applied in asset allocation based on conditional correlation. Taking into account the effect of asymmetry in conditional correlation, Cappiello, Engle and Sheppard (2006) generalized the asymmetric effect in conditional correlation, including the allowance of time-varying patterns of conditional correlation with the original

[41] Tse and Tsui (2002) assume the conditional correlation matrix is generated from a recursive pattern, $\boldsymbol{\rho}_t = (1-\theta_1-\theta_2)\overline{\boldsymbol{\rho}} + \theta_1\boldsymbol{\Gamma}_{t-1} + \theta_2\boldsymbol{\psi}_{t-1}$, where $\boldsymbol{\psi}_{t-1}$ indicates a functional form dependant on the standardized residuals.

[42] For the purpose of identification between Engle's (2002) dynamic conditional correlation (DCC) model and Tse and Tsui's (2002) time-varying correlation (TVC) model, they are marked as DCC and TVC, respectively.

DCC model, and called it asymmetric generalized DCC (AGDCC). The evidence in their research showed that asymmetry widely exists in the conditional volatility of international equity returns, but less so in international government bonds. However, asymmetric conditional correlation is present in both bond and equity returns. Although the advantages to this model are manifest, the cumbersome procedure in parameter estimation is the major shortcoming, even in the diagonal version.

Alternatively, as suggested by Giot and Laurent (2004), the concept of VaR in the univariate case can simply be extended to the multivariate version. In this manner, the density of the multivariate case should be accurately calculated before measuring the portfolio risk. Looking at this, Giot and Laurent (2003b) and Bauwens and Laurent (2005) proposed a practical and flexible method to introduce skewness into the multivariate symmetric density and to improve the model creditability. In Giot and Laurent's (2003b), they suggested an AR-APARCH[43] model with a multivariate skewed student distribution and time-varying correlation structure, as proposed by Tse and Tsui (2002). They offered a comprehensive empirical result, both in long and short positions, on three individual stock returns listed on the NYSE. They also offered direct evidence that the multivariate GARCH model with skewed distribution improves the performance of VaR estimations. Staying in the spirit of the DCC model, Bauwens and Laurent (2005) applied a multivariate GARCH model with skewed student-t distribution for measuring VaR of two stock indices and three major exchange rates. The results of their work are consistent with the previous study of the univariate model. Moreover, the model they proposed improves the quality of out-of-sample VaR forecasts when compared with a symmetric one.

In general, the practicability of the multivariate GARCH model is limited by the fact that too

[43] AR in this contraction means that the return in the mean equation follows a first order autoregression. The Asymmetric Power ARCH (APARCH) model is proposed by Ding, Granger and Engle (1993), which includes Bolleslev's (1986) GARCH and five other models.

many parameters need to be estimated, and this is quite time consuming. To address this difficulty, Ledoit et al. (2003) suggested a flexible multivariate GARCH (Flex MGARCH) model, which skilfully separates the covariance as diagonal and off-diagonal parts and estimates each respectively. In addition, he also provided comprehensive evidence of the improvements of this model, comparing it with other multivariate models in VaR measurement of equity indices. Ledoit et al.'s (2003) study contributed not only by extending the academic field, but also by developing methodology which can easily be applied to real markets, for example the risk management of banks, mutual funds, and other financial institutions. As a mature technique of (multivariate) conditional volatility for estimating and forecasting, the covariance-related method [44] has become the most well-used and representative VaR measure.

3.2.3 The application of Copulas method

As mentioned in the previous section, the relationship between the various assets is an essential issue for practitioners and academics to consider for several reasons. For example, portfolio selection, hedging strategies, and measuring the risk of the portfolio would all be strongly associated with this issue. In the early years, under the normality assumption, linear correlation was applied as a dependence measure for integrating the risk of the individual components which made up a portfolio. In more recent years, some oppositional perspectives have suggested that linear correlation might only be applicable under the particular conditions found with the normality assumption and thus not really suitable for real life situations (Embrechts, McNeil and Straumann (2002)). Nowadays, some academics strive to measure

[44] Sometimes, the variance-covariance method is called the "delta-normal" method. In the univariate case, the variance-covariance method is reduced to the variance model, which is called exponentially weighted moving average (EWMA) in JP Morgan's method.

the relationship between the various assets in a portfolio. For example, the copulas[45] method has been broadly applied in the statistical and biostatistics literature. This approach deems that any joint distribution can be represented in terms of a copula and marginal distribution functions. This representation demonstrates that it is possible to individually specify each variable's marginal distribution and the dependence relation that links these marginals into a joint density.

To my best knowledge, most studies apply copulas as a tool of dependence measure rather than as a risk scale. For example, Longin and Solnik (2001), Poon, Rockinger and Tawn (2004), and Hartmann, Straetmans and Vries (2004) all applied alternative copulas, investigating linkage or dependence of international markets. In general, focusing on the contemporaneous correlations in the tails, all these papers suggested that dependence is high when the market conditions are bad, and low in good market conditions. There is also an asymmetric phenomenon between international markets, suggesting that international markets tend to go down together but go up separately. This result is also found in Jondeau and Rockinger (2006). This is an important discovery for hedging strategy and implies that it is difficult to hedge effectively if the market collapses. Inspired by these results, and a similar concept found in Engle (2002) and Tse and Tsui (2002), Patton (2006b) extended the unconditional copula theory to the conditional case, and then proposed a time-varying conditional dependence model with asymmetry for capturing the rapidly changing market conditions.

Some scholars use copula to obtain joint density for risk measurement. Cherubini and Luciano (2001) allocated capital to each business unit based on their risk, measured by a copula approach. Although there was a lack of backtesting in their research, they presented a new application of VaR in a trade-off relationship between risk and allocated capital. Poon,

[45] The original concept of copula was developed by Sklar(1973). More detail about the copula family and their application in finance can be found in Joe (1997) and Nelsen (2006).

Rockinger and Tawn (2004) offered a static VaR calculation for a particular date, comparing it to another market risk measure and expected shortfall, as proposed by Artzner et al. (1999). However, they did not display the results of the backtesting between the VaR and the expected shortfall. Along the line of Patton's (2006b) job, Palaro and Hotta (2006) proposed a symmetrised Joe-Clayton (SJC) copula for measuring VaR. Compared with other conventional VaR approaches, they suggested that the SJC approach provided good performance for VaR one day ahead measuring, especially in extremal fields. Applying the copulas method, Rosenberg and Schuermann (2006) presented a comprehensive research study into measuring the total risk of particular bank holding companies with nine-year quarterly data; calculating market, credit, and operational risk individually and then aggregating the three types of risk. However, they only provided static VaR estimates without backtesting the results and thus it could not be compared with the other measures.

Although the merit of the copulas method in measuring VaR is remarkable, it still poses an arduous problem for practitioners because there is not a standard rule to identify which copula function to use in practice. In addition, it is often cumbersome for multivariate cases when calculating VaR, especially in high dimensions.

3.3 Non-parametric model

As mentioned in the previous section, the major drawback of the parametric VaR model is the use of the assumption of the hypothetical normal distribution of financial returns when estimating VaR – this bears little or no resemblance to the real life situation. As a result, an alternative concept without any distribution assumptions, so called non-parametric models, was derived. Historical simulation (HS) is a common approach for measuring market risk. Its convenience in calculation has led it becoming one of the most popular methods for the banking sector. The spirit of HS is to obtain an asset's return distribution based on its historical data. Stimulated by the drawbacks of the parametric models, Beder (1995) was a pioneer in this field, using HS to calculate VaR by using conventional HS with a range of artificial data. He argued that the concepts of the VaR models were seductive, but they still had some difficulties. Even for the simplest one, like HS, the outcome could be affected by the horizon selection. In addition, employing long horizons of real exchange rates, Hendricks (1996) compared the HS approach (non-parametric) with the variance-covariance method (parametric), suggesting that HS with long period data, say over 1,250 days, offers very accurate coverage for both the 95^{th} and 99^{th} percentile risk measures. Unfortunately, similar comparisons in Vlaar (2000) suggested that the performance of the HS approach is merely satisfactory, even if a long range of historical data is included. In general, Hendricks (1996) could not specify which approach offered superior performance. Thus, he made an important suggestion for further research aimed at combining the best characteristics of the two approaches in measuring VaR.

Although the traditional HS approach has some obvious merits in its convenience and avoidance of making any assumptions on the density of financial returns, it is still criticized for ignoring the impacts of serial correlation and the heteroscedasticity of volatility. The equal weight given to past returns is also an issue of the HS approach which has received criticism. Spurred on by Hendricks' (1996) suggestion, several variants of HS were proposed. For the purpose of purging the noisy information in raw data, a new approach, named filtered historical simulation (FHS) [46] was introduced by Barone-Adesi et al. (1998) and Barone-Adesi et al. (1999). In the first step of the FSH approach, an ARMA or GARCH model is adopted to wash off the serial correlation and heteroscedasticity of volatility in the financial time series. Then, an independent and identical density (i.e. i.i.d) applied to the conventional HS approach. They also offered backtesting residual sequence is obtained and of

[46] The FHS approach has also been extended to allow for more complicated volatility models, see Audrino and Barone-Adesi (2005).

empirical studies with the FHS approach for a derivative portfolio in Barone-Adesi et al. (2002). Overall, the evidence showed that FHS is accurate at shorter time horizons, but it tends to become more conservative with long period forecasting. Using a similar procedure, David Cabedo and Moya (2003) and Costello et al. (2008) utilized ARMA models to remove the effect of autoregression, measuring the market risk to oil markets. Their work, in general, suggested that the HS approach with ARMA provides a better VaR estimation than those provided by the standard HS approach or the variance-covariance method.

Boudoukh, Richardson, and Whitelaw (1998) (hereafter BRW model) proposed an alternative approach, directly addressing the difficulty of using equal weighting for all past returns. The spirit of their paper is that over the longer simulation period the older data might be less relevant to the current situation. Accordingly, in this work the BRW approach combined the RiskMetrics' concepts into the HS approach by giving each past return an exponentially calculated weighting before implementing standard HS. Applying a range of financial returns, including exchange rate, spots of Brent crude oil, S&P 500 index, and bond index, the results in Boudoukh, Richardson, and Whitelaw (1998) show significant improvements in statistical performance over the two competing methods, standard HS and RiskMetrics. However, BRW is criticized on the grounds that it is an indirect and somewhat inefficient way of allowing for stochastic volatility (Hull and White (1998)). Besides, Hull and White (1998) suggested that HS can be improved by taking into account the volatility changes experienced during the period covered by the historical data. They presented a variant of the HS model incorporating volatility updating schemes. They used about 9 years of daily data on 12 different exchange rates and 5 different equity indices to offer evidence that their approach is better than the BRW approach in 1-percentile estimates. Since the results of these various VaR models based on HS are inconsistent with each other, Pritsker (2006) suggested that practitioners needed to have awareness of the different properties of the different VaR models based on historical simulation approaches before they adopted one as a tool for market risk measurement. Pritsker (2006) found that both the BRW and the FHS approach for measuring market risk responded sluggishly to changes in conditional volatility, and responded asymmetrically to large price moves, i.e. risk estimates increased after large losses, but not after gains. In summary, the accuracy of the HS based approach to VaR models is significantly different depending on the length of the horizon.

3.4 Semi-parametric model

The semi-parametric model stands in the midpoint between the parametric and the non-parametric approach. Most methods in this field do not adopt entire samples in parameter estimation, but rather they take the extreme ones through various sampling procedures. The concern regarding this method is whether it can appropriately deal with the tail-area density of financial returns. In the last twenty years, international investors have suffered severely from different financial disasters, for example the American stock market crash in 1987, the Asian financial crisis in 1997, and the international credit crunch in 2008.[47] However, traditional risk measures failed to capture the probability of these rare events and this might mislead investors about the potential risk of investors' positions. Extreme value theory (EVT) fills this gap by focusing on the behaviour of the tail of the distribution of the asset return. In other words, extreme value theory models just the conditional tail distribution, rather than the whole distribution, to calculate VaR, which provides a more accurate method to measure the risk within an investors' portfolio (Embrechts, Resnick and Samorodnitsky, 1999). This theory has been developing rapidly and there have been a large number of applications in the related field of finance (Longin (1996), Longin (2000), McNeil and Frey (2000), Bali (2003), Gilli and Kellezi (2006), and the references therein). In this section, the concept of extreme value theory with two different sampling procedures, maxima of block and peak of threshold, will be introduced in detail. In addition, some empirical applications to diversified markets or instruments will be reviewed. The weaknesses of these methods will be discussed in this section as well.

[47] Several regional financial crises with various impacts happened in other markets during the 90's: the western European exchange rate mechanism crisis in 1992, the Mexican crisis in 1994-1995, the Russian crisis and the LTCM Hedge fund crisis in 1998, and the Brazilian crisis in early 1999.

3.4.1 Block Maxima

Using the method of block maxima, the extremes are sampled (the biggest or smallest) within a fixed time horizon, for example, one week or one month. The sampled extremes are then applied to fit the density of the tail-area and to measure market risk. The core thinking behind this method puts more emphasis on the rare events in each time horizon, believing that the source of the tail distribution could provide more precise parameter estimations, especially for the maxima likelihood estimation. Fisher and Tippett (1928) suggested that the distribution of extremes should belong to one of three densities: Gumbel[48], Fréchet[49], and Weibull[50]. Jenkinson (1955) provides a representation with three parameters to generalize the three extreme value distributions, named generalized extreme value distribution (GEV). Academics and practitioners have tried to disentangle the distribution of asset returns for several decades, especially those of extreme returns. Gettinby et al. (2004) applied various distributions to fit the extremes of UK daily stock price changes from 1975 to 2000, and they empirically argued that GEV distribution might fail to capture a fatter tail compared to generalised logistic (GL) distribution. However, from the theoretical viewpoint, GEV statistically offers more creditability than the others. Longin (1996) investigated the asymptotic behaviour of the distribution of extreme returns (from this approach) to the most traded stocks in the New York Stock Exchange. He suggested that the asymptotic distribution of extreme returns is a stable Fréchet distribution. This finding is consistent with Fisher and Tippett (1928) and Jenkinson (1955).

In recent years, this line of research has been broadly applied to measuring VaR. Cutler et al.

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48 Gumbel distribution was proposed by the German mathematician, Emil Gumbel, in 1960 and has been applied in particular for modelling meteorological phenomena such as annual flood flows. Its probability density function can be expressed as $f(x) = \frac{1}{\sigma}\exp(-z - \exp(-z))$, where $z = {}^{(x-\mu)}/_{\sigma}$, μ is the location parameter and σ is the distribution scale.

49 This distribution was proposed by Maurice Fréchet in 1927 and the density function can be formulated as $f(x) = \frac{\alpha}{\beta}(\frac{\beta}{x})^{\alpha+1}\exp(-(\frac{\beta}{x})^{\alpha})$, where α is the shape parameter and β is the scale parameter.

50 Weibull distribution was proposed by Waloddi Weibull in 1939, and is widely used in material sciences.

(1989) triggered this area of study focusing on extreme returns. They applied seven measures to analyse long-range monthly stock price changes, from 1926 to 1985, and annual returns from 1871 to 1986, and they implied that most extreme returns were actually associated with major news, in particular with bad news. In addition, Jondeau and Rockinger (2003) offered a comparison on left and right tail extreme returns with five mature market indices and fifteen emerging market indices. In general, the characteristic parameters of the left and right tails were equal, which means that the asymmetry of extreme returns in the left and right tail is not significantly different. After several market collapses, Këllezi and Gilli (2000)[51] offered a comprehensive illustration covering twenty years of data, applying the extreme value method for VaR measuring. However, they did not offer any backtesting of their results.

To date, extreme value theory with the BM sampling method has been widely applied for measuring market risk across various fields. Lauridsen (2000) applied extreme value theory based on the BM procedure to measure the market risk of two Danish banks with 14 years of daily returns, offering distinct evidence that this method was superior to the method with normal distribution. This finding was consistent with Ho et al. (2000), who offered a simple sensitivity analysis with major Asian stock indices and demonstrated the that the performance of the GEV method was much stronger than the other techniques. They also pointed out the fact that the estimated parameters might be affected by the size of window selected. Moreover, they indicated that extreme returns, both minima and maxima ones, could be described well within an extreme value theory framework. Along this line, Longin (2000) applied the extreme value model with BM to calculate the VaRs of S&P 500 returns over three frequencies, daily, five-day, and ten-day returns. Compared with the GARCH method and RiskMetrics with normal and historical distribution, the evidence showed that the extreme value method is conservative in the high confidence level. To calculate the risk of a portfolio

[51] The new version of this paper can be referred to in Gilli and Këllezi (2006).

with two assets, Longin (2000) provided a simple ad hoc method to aggregate the risk. Although it is criticized as feasible only in normal distribution or linear combination portfolios, until recently it was difficult to straightforwardly calculate portfolio VaR any other way. Byström (2004) tried to evaluate the extreme value model from the alternative aspect, he found that the unconditional extreme value model tends to be too conservative during more tranquil periods.

Bali (2003) used extreme value theory to assess the VaR of various interest-rate related securities with long period data sets from the US markets. They indicated that generalized extreme value distributions worked surprisingly well for capturing the extremes in the interest rate market. In addition, they also suggested that the performance of extreme value theory in VaR measurement is significantly more precise than the standard approach. Krehbiel and Adkins (2005) used the extreme value model for measuring the price risk in the energy market.

The primary advantage of the extreme value method with block maxima sampling approach is that it is able to reduce dependencies in the raw data. Unfortunately, due to the nature of the procedure, only the most extreme values are recorded and the others are discarded, thus some important information within other observations in the same block might be neglected (Diebold, Schuermann and Stroughair, 2000). In addition, the selection of block size is still a contentious question. Furthermore, Lauridsen (2000) and Ho et al. (2000) indicated that the estimated parameters are very sensitive to the choice of block size and, unfortunately, as of now there has been no standard selection rule to overcome this weakness. Some researchers offer alternative solutions to this, for example, Coles (2001) argued for using yearly maxima in order to avoid seasonality and to reflect the rare events. However, for the financial assets with short historical horizons, large blocks might cause less extreme observations and reduce the accuracy of parameters estimation. To this problem of parameters and block size, a more reasonable suggestion is made by Christoffersen et al. (1998), who proposed that ten to fifteen trading days as a block is required to capture independent and identical features of the extreme observation.

3.4.2 Peak over threshold

Instead of the block maxima procedure mentioned above, an alternative efficient sampling approach can be employed to fit the tail distribution, called peak over threshold (POT). In this procedure, extreme values are defined as those over a given threshold value. In other words, this method is more concerned with particular large losses than with whole loss observations. For modelling the behaviour of these exceedances[52], Pickands (1975) showed that the generalized Pareto distribution (GPD) is the only non-degenerate distribution which approximates the distribution of return exceedances. In the past ten years, extreme value theory with POT (hereafter, the GPD model) has been widely applied in measuring the market risk of various financial instruments, suggesting that using extreme density formed by the observations in the tail area to calculate VaR is more precise than other standard approaches (e.g, McNeil and Saladin (1997), Neftci (2000), and Gencay et al. (2003)).

As a pioneer in this field, McNeil (2005)[53] offered a general good-practice guide for fitting tail area distribution. Looking at Danish fire losses from 1980 to 1990, the evidence suggested that GPD is a useful method for estimating the tails of loss severity distributions. Consigli (2002) used a GPD model for liquid bonds, equity markets and emerging bond markets. The GPD method consistently offered an accurate tail approximation in the liquid equity market. However, for the emerging bond market the accuracy of this approach was restricted by the

[52] The word “exceedance” is not a regular word but an idiom in this field. In this section, I follow the wording in Davison and Smith (1990) and Reiss and Thomas (2007).

[53] The earliest version of this paper refers to a working paper in 1996. Barone-Adesi et al. (2002) called this method “*filtered*” VaR.

length of data set available. For emerging equity markets, Gencay et al. (2003), Gençay and Selçuk (2004), Maghyereh and Al-Zoubi (2006) and Bao et al. (2006) cover a range of comparisons of VaR forecasts between the extreme value models and other conventional approaches, focusing on the period of the Asian financial crash in 1997. In their results the GPD model was less satisfactory in the coverage rate, although it was still better than some other approaches (Bao et al. (2006)). Generally speaking, in Gençay and Selçuk's (2004) investigation, the GPD model fitted the tail distribution of financial returns in emerging markets well and offered an accurate VaR estimation in both long and short positions, compared with other standard methods. However, it failed to present the best performance in all emerging markets, especially Korea and Turkey. Similar results for the GPD model are found in Bali and Gokcan (2004) and Maghyereh and Al-Zoubi (2006) who looked at monthly hedge fund returns and daily equity returns in the Middle East and North Africa respectively. Looking at the derivatives market, Brooks et al. (2005) applied non-parametric and semi-parametric methods to measure the VaR of three derivatives traded on the London International Financial Futures and Options Exchange (LIEFT). They suggested that the GPD model offers a reasonable performance compared to other conventional approaches, particularly in the period of the Asian financial crisis.

For energy markets, the GPD model is applied to measure the risk of the electricity and oil markets by Byström (2005) and Marimoutou et al. (2009), respectively. In summary, most previous research deems that GPD is an indispensable tool in market risk management for a range of markets.

Although the GPD model is more popular in this line of research, and has the advantage of efficiency of data use, it still has two main drawbacks. Firstly, the parameters estimated can be affected by the choice of the threshold, especially in small sample cases (Jondeau and Rockinger, 2003). Huisman et al. (2001) provided a robust small sample bias-corrected estimator based on the linear regression of Hill's (1975)[54] tail estimates. However, this issue is still contentious. This impact of the different thresholds can also be found in McNeil (2005) and Brooks et al. (2005), particularly with regards to the shape parameter. Secondly, the tail-index estimators could be significantly biased by a non-iid series. Most evidence in previous research supports the idea that there are clustering and autoregressive effects in financial series against the iid assumption. Moreover, Danielsson and De Vries (2000) argued that extreme values, by their very definition, only happen infrequently and might thus be associated with different events. Consequently, they would not exhibit particularly strong time dependence in financial time series data.

An alternative conditional GPD method is proposed by McNeil and Frey (2000), who suggest a two stage method, evading the iid problem, to calculate VaR. In the first stage, iid residuals of the financial return are obtained after GARCH filtering[55], which washes off the autoregressive effect in financial time series data. In the second step, the iid residuals would be applied to fit ETV with a generalized Pareto distribution and to calculate VaR. McNeil and Frey's (2000) method has been applied fruitfully in a variety of studies (see Lauridsen (2000), Byström (2004), Byström (2005), Fernandez (2005), and Maghyereh and Al-Zoub (2006)). However, the GARCH filter is not a monotonic transformation from the original return level to residuals; thus, one might miss some information and characteristics when conducting this procedure. Moreover, the number of extreme values after the GARCH-filter is significantly reduced (Lauridsen's (2000)). From this aspect of the GPD model, although there is no direct evidence on this issue, theoretically the results might still be affected by the non-iid feature of financial data.

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[54] The estimation techniques for tail indexes for particular distributions proposed by Hill (1975) can be applied in risk management which focuses on the probability of tail area.

[55] Some similar procedures are adopted to wash-off the effect of autoregression, for example, MA(1) and ARMA(p,q) were employed in Stephan and Whaley (1990) and Stoll and Whaley (1990) to exclude the serial correlation effect.

3.4.3 Comparison

The nature of extreme value theory in VaR measurement is to estimate the tail distribution of the financial returns, rather than the entire distribution. The main advantage of only focusing on the extremes is that it improves the performance and efficiency of parameter estimation. This concept involves the two different approaches to sampling mentioned in the previous sections, block maximum (BM) and peak over threshold (POT). Two types of tail distribution generated from GEV and GPD methods can be used for estimating VaR. In some circumstances, they can be easily transferred to each other. In other words, when the threshold is approaching to the right-end or left-end point, then GEV is also approaching to GPD. However, there is still some dispute about these two approaches.

Although both the two methods have been applied as a market risk measurement, it is still unclear which one is more appropriate to projecting VaR. Both of the two approaches have been criticized for specific demerits, although some of these criticisms are not relevant when using them for the financial markets. Most comments regarding the BM sampling procedure focus on the wastage of data, since only one observation is taken in each block (Coles (2001) and Gilli and Kellezi (2006)). For a newly issued financial instrument, it is difficult to implement the VaR model, regardless of the model choice. Another problem with this method is how to decide the length of the block. Christoffersen et al. (1998) offered a simple rule to solve this problem, which suggests ten to fifteen trading days is the optimum as a block for financial data. In addition, some evidence in the previous research has demonstrated that financial markets exhibit a weekend or Monday effect; thus, a weekly block might be a good alternative approach to prevent omissions of extremes happening on a Friday or Monday. Therefore, the two main weaknesses to the BM method can be addressed. The main advantage of this approach is that it does not assume financial returns have to be entirely independent

(Jondeau and Rockinger, 2003). Moreover, the extremes extracted by this sampling procedure, by its very nature, can be seen as non-dependent series (especially in the case of large blocks). Looked at in this light, VaR measures based on block maxima demonstrate fewer impacts from extreme selection.

Looking at the alternative approach, the main difficulty with POT is that the threshold can be difficult to decide. If the threshold is set too high, then the number of extreme values will be reduced and the accuracy of tail estimation might be diminished as well. In contrast, if the threshold is too low, the estimation would be inefficient and the accuracy of the tail estimation would be similarly affected. Thus, the threshold choice is a trade-off decision between accuracy and efficiency. Although Hull and White (1998) and Gonzalo and Olmo (2004) offered some guidance on how to choose a proper threshold, it is still not definitively clear cut. A more serious problem with this method is found in the multivariate case. The conditional correlation between various assets cannot be calculated with direct interpretation since the number of extreme values in each series might be different, and a mis-matching phenomenon can be expressed even if the number of extremes are equal. More specifically, one might intend to calculate the (conditional) correlation between two financial returns, r_x and r_y, based on the extremes above their certain thresholds, u_x and u_y. The conditional correlation could be presented as $\rho_{xy} = \mathrm{corr}(r_x > u_x, r_y > u_y)$, where u_x and u_y are the thresholds to r_x and r_y, respectively. Consequently, the extreme values of r_x and r_y might not come from the same period or might be mismatched in the time horizon. Thus, it is difficult to explain the implication of any conditional correlation, because it has integrated two different events from two different time horizons as mentioned in Section 2.6. Even if the numbers of extreme values from various series are equivalent, the implications of the conditional correlation might still be inexplicable.

To compare POT with the BM approach, most characteristics of a return series could be conserved in the series of the extreme values generated from the block maxima procedure, although it does still have some shortcomings which cannot be overcome. POT, on the other hand, has a fatal drawback because it completely disregards the sequential characteristic of financial time series data.

3.5 Performance evaluation

In the previous sections, a large number of VaR measures based on different assumptions were reviewed. Although the pros and cons and the applications of each approach has been displayed, it is still unclear as to how we should best evaluate these methods. Consequently, in this section the verification schemes of VaR measures will be reviewed in detail.

There are a number of diverse ways to check model validation, such as backtesting, stress testing, sensitivity analysis, and scenario analysis. For simplicity, in general the VaR model evaluation or performance evaluation focuses on the procedure of backtesting, which is a comparison between the VaR numbers and the actual returns after VaR modelling and forecasting. Briefly, there are two main reasons why financial institutions need to assess their VaR measures. The first comes from the regulatory authority requirement for market discipline. The amended Capital requirement was proposed by the Accord Basel Bank Supervisors Committee in 1996, suggesting that market risk can been measured by internal models proposed by financial institutions instead of the standard model (see Basel Committee on Banking Supervision, 1996a; Basel Committee on Banking Supervision, 1996b). The current regulatory market risk framework requires that the internal model is reviewed by local authorities according to its VaR result based on at least a one-year period with ten days forecasting. The second driver to evaluate the VaR measure comes from the management, who like to understand their operating performance and the quantity of risk in their portfolio. Thus, selecting an appropriate evaluation procedure or approach for the VaR model is as critical as selecting the best form of VaR modelling. Under the circumstances mentioned above, evaluating the accuracy of the VaR models is thus a necessary exercise.

3.5.1 Regulatory evaluation method

Generally speaking, the object of regulatory evaluation is to reflect the market risk in the regulatory capital of commercial banks, as described in Section 2.2.4, requiring that the banks hold enough capital to absorb any potential losses. As shown in Eq. (2.8), the market risk capital (MRC)[56] is set with an indicator parameter, reflecting the backtesting performance of the internal model. The backtesting method proposed by the Basel Committee has several drawbacks. It assumes that the violation sequence follows an iid process, but some research has suggested instead the presence of a clustering phenomenon within the violation series (Berkowitz and O'Brien, 2002). Moreover, the minimum number of backtesting observations required is 250 (i.e. about one year of daily returns) which seems insufficient to examine the creditability of the VaR measure. Another weakness of the regulatory approach is that it suggests a method to convert the daily VaR number to k-day VaR by using the square root of k, i.e. $\sqrt{k}$. Christoffersen, Diebold and Schuermann (1998) provided mathematical demonstrations and practical examples to show that this method is not a proper way to scale the time horizon. According to the discussion above, the regulatory backtesting method is not a good method to evaluate the VaR model.

3.5.2 Coverage test

Following the basic requirement of backtesting regulated by the Basel Committee as mentioned in the previous section and Section 2.2.4, two widespread tests are proposed to examine the accuracy of the VaR measure. Kupiec (1995) assumed that estimated losses (VaR numbers) follow a binomial process, either smaller or larger than actual losses, and thus could

[56] Currently, $\mathrm{MRC}_t = \max\left(\mathrm{VaR}_t(0.01), S_t \frac{\sum_{i=0}^{59} \mathrm{VaR}_{t-i}(0.01)}{60}\right) + c$, in which S_t is a multiplication factor, which is divided into three categories corresponding to the back-testing results.

be evaluated for accuracy via a likelihood test (denoted as LR_K), marked as a proportion of failures. Under perfect conditions, the expected number of exceptions[57] should be equivalent to the sample size times the tail probability (α) as shown in Figure 2.1. In this situation, likelihood test statistics, LR_K, would approach to zero. An under- or over-estimated potential loss to the next trading period would drive LR_K toward the positive or negative, respectively. Kupiec's (1995) method has been widely applied in this line of research, particularly with unconditional coverage tests (UCT). Pérignon and Smith (2008) stepped further ahead to extend this model to multivariate cases. However, Kupiec's test only looked at whether the proportion of the reported VaR sequence violated by the corresponding actual return was equal to expectation or not. In this manner, the power of this test might be obstructed by, at least, two ignored features of these violations. The first defect is that the UCT related approach might fail to detect VaR measures that are systematically under- or over-estimating risk. The second is that UCT related methods do not take into account whether the property of dependence exists in the violation sequence. If violation at time t could be presaged by the one at time t-1 (or the so-called clustering effect) accordingly the probability to the violation at time t will be unity. To overcome this second deficiency, Christoffersen (1998) modified Kupiec's statistics by adding an independence test to form a combination test which takes into account coverage ratio and the independence of violations. Basically, Christoffersen's (1998) independence check was constructed on a Markov test by forming a two by two contingency table with the numbers of violations to time t and t-1. Specifically, the Markov independence test examines whether the proportion of the violations following the previous violation is equal to the proportion of violations following the previous non-violations or not. Several studies have progressed the independence test along further, following Christoffersen (1998). Christoffersen and Diebold (2000) suggested a convenient and powerful model-free runs test

[57] In this field, an exception or violation is defined as an actual loss larger than the estimated (reported) VaR.

for testing the independence of the violation sequence. Alternatively, an independence test focused on the time elapsed between violations, called the duration-based approach, was proposed by Christoffersen and Pelletier (2004), who argued that if VaR violations are purely independent of each other, then the durations between the violations should be independent as well. In the case of independence, the time between violations in the series of VaR violations should not present any duration of dependence. Following the moment estimation technique, Bontemps (2008) and Bertrand et al. (2009) modified Christoffersen and Pelletier's (2004) model as a GMM duration-based test, which provides a better parameter estimation.

All the evaluation methods mentioned above are built on testing the original violation sequence or its derivatives. Lopez (1999)[58] provided a different approach for violation testing. In Lopez's method, he takes the evaluation further by adding the square of the difference between the VaR estimates and actual returns. A loss function generated by this device not only keeps the advantages of the original method of Kupiec (1995), but also takes into account the magnitude of the violations. Thus, the verification work is based on testing average sample losses. However, this mean test towards losses might be easily and strongly affected by extreme values. Alternatively, Christoffersen et al. (2001) suggested a complicated back-testing procedure within a GMM framework, and Kerkhof and Melenberg (2004) focused on density risk in measuring risk.

[58] An early and concise version can be referred to in Lopez (1998).

3.6 Conclusion

This Chapter reviewed the critical literature of three different types of VaR models and the major methods of model evaluation. Lately, more attention has been given to VaR models that consider the distribution modelling of financial returns. Some academics have conventionally tried to measure market risk via parametric models, as shown in Section 3.2. The GARCH model is still widely used in this line to model time-varying volatility and VaR modelling. Alternatively, some methods do not assume any kind of distribution of the financial data. For example, historical simulation has been used for a wide variety of markets over the last two decades to fit the empirical distribution and to calculate VaR.

The two types of VaR measures mentioned above, which focus on fitting the whole distribution of financial returns, were developed and widely used over the past ten years; however, international financial markets still suffered a series of market collapses. Consequently, extra effort has been made to investigate the risk associated with these crashes. In this manner, it is suggested that the extreme value in return series delivers more useful information about market risk. Thus, extreme value theory (EVT) is borrowed to evaluate the risk in financial markets. This approach is divided into semi-parametric models that look at extremes and sampling procedure. The definition of the extreme values is a critical issue in this field, since the results are significantly affected by the selection of the extremes. In Section 3.4, two approaches for selecting the extreme values (block maxima and peak over threshold) are reviewed and discussed. Both of them have some drawbacks, although some researchers have offered various remedies to counteract these. We further show the troublesome problems of these two approaches which might affect the VaR modelling, although not significantly. Generally, most VaR-related literature concentrates on the univariate VaR modelling and the model evaluation of the violations: this is a major gap in the literature.

Identical portfolios from different or even the same VaR models can provide different risk forecasts (e.g., Marshall and Siegel (1997) and Christoffersen et al. (2001)). This phenomenon implies that market participants have some difficulties in the model selection and usage of VaR numbers. In Section 3.5, we discussed the regulatory method of VaR evaluation and the two conventional VaR backtesting approaches widely used in related literature. However, these methods focus on the proportion of violated returns rather than the non-violated or even the whole pattern of observations. This is a further gap in the literature.

This thesis attempts to fill these two gaps. To achieve this goal, a portfolio VaR model is proposed that considers the financial market collapse conditions, the estimation accuracy of return distribution, and the correlations between the financial assets. In other words, this portfolio VaR model bridges the univariate VaR model using a special tail-DCC suggested in this thesis. Furthermore, we fill the gap of the VaR model evaluation by offering the quality measures from the whole pattern and those from non-violated observations, delivering a comprehensive understanding of the VaR model to academics, practitioners, and financial institutions.

Chapter 4 Methodology and Data

4.1 Introduction

In this chapter, we present some basic econometric tests, the methodology of portfolio VaR modelling, and the methods of VaR model evaluation used in this thesis. In the final part, a description of the data sample and the estimation approach applied are exhibited. In Section 4.2, several fundamental econometric statistics of time series data will be employed to test the characteristics of the data sets. The method of estimation of the parameters and its procedure are discussed in Section 4.3. The description of dynamic conditional correlation based on the extreme returns is also explained in this section.

In the thesis we apply four competing models against the GEV-DC model. Section 4.4 outlines the fundamentals of the four competing models, which are widely used approaches in practice and related literature. They are the GARCH (1,1) model, the RiskMetrics model, Multivariate stochastic volatility, and historical simulation.

In Section 4.5, we present the methods of performance evaluation for all VaR models applied in this thesis. In addition, the elemental concept, the procedure of backtesting, and the theory of the evaluation of model performance are explained in detail. Furthermore, two critical indicators used to evaluate the suitability of the forecasted VaR sequences are proposed in this section. These two methods offer different viewpoints for the model evaluation.

Section 4.6 covers the range of the data sample sets used in this thesis, from developed equity markets to emerging markets. In order to be consistent with previous research and avoid the data contaminated by government policies, more equity indices are used to demonstrate VaR modelling. In addition, some important histories of these equity markets are briefly laid out, which might help us to understand the market risk or help us to interpret the origins of market risk. We offer the profile of the data sets from the general descriptive summary. Some analysis based on the quantile-quantile plot (QQ plot) and autocorrelation function (ACF) are also provided. Obviously, most of the data sets are stationary and tend to be independent. In general, the developed equity markets tend to be independent and the Latin-American emerging equity markets have some autoregressive effects in the lag one level. In addition, the Q-Q plots show that some returns of each index fall into the extreme area, indicating that it is appropriate to apply extreme value theory to fit the tail-distribution.

4.2 Basic econometric tests

In this section, some econometric approaches used in this thesis for testing the characteristics of time series data are introduced and presented. Obviously, financial data such as trading price is significantly different to those in other forms, as, for example, collected from questionnaires, since the former has the property of continuity by time. Thus, it is worthwhile making a clear explanation of the financial time series data used, as well as its properties.

4.2.1 Time series data

Empirical research around related financial issues should be constructed using financial data. Brooks (2008) suggests that generally three types of data could be employed in quantitative analysis of financial issues, they are: time series data, cross-sectional data, and panel data. Time series data are collected over a specified period and consist of one or several variables, based on a regular frequency. In contrast, cross-sectional data are one or several variables collected at a single point in time. The panel data are the sets of data collected by time and by object. In this thesis, a range of time series data are used for measuring the market risk of equity markets. Thus, it is essential to realize the properties of financial time series data before we use it. Financial time series data often differs from macroeconomic data due to their frequency, accuracy, seasonality, and other characteristics. Moreover, financial time series data are also often regarded as noisy, with a lack of normality, and having different patterns with different frequencies.

4.2.2 Basic econometric tests

Normality test

In conventional theory and empirical studies, it is generally assumed that financial returns follow the normal distribution. However, to date it is evident that some characteristics of financial returns do not fit the assumption of normality. One of the most widely applied tests for normality is the approach proposed by Bera and Jarque (1981) (the BJ test). The statistics primarily examine the normality assumption by testing whether the coefficient of skewness and excess kurtosis are appropriate for normal distribution. The statistics of skewness (sk) and kurtosis (k) are given by

$$\mathrm{sk} = \frac{\mathrm{E}(\varepsilon_r^3)}{(\sigma_r^2)^{3/2}}$$

$$\mathrm{k} = \frac{\mathrm{E}(\varepsilon_r^4)}{(\sigma_r^2)^2}$$

where the residual of the forecasted return is set as $\varepsilon_{i,r} = r_i - \hat{\mu}_r$, and $\hat{\mu}_r$ is the estimated mean of return. Then the statistic of the BJ test is

$$\mathrm{BJ} = \mathrm{N}\left[\frac{\mathrm{sk}^2}{6} + \frac{(\mathrm{k}-3)^2}{24}\right] \qquad (4.1)$$

where N is the number of samples. Since the kurtosis of normal distribution is 3, thus the excess kurtosis has to be minus 3. The BJ test statistic asymptotically follows a chi-square distribution with a degree of 2.

Stationarity test

As mentioned in related textbooks (Tsay, 2005), stationary sequences can be sorted into strictly and weakly stationary according to the stability of the four moments. Time series data is said to be strictly stationary if the distribution of $\{y_t, \cdots, y_{t+m}\}$ equals to the one of $\{y_{t+k}, \cdots, y_{t+m+k}\}$ to all the values of k. In other words, all the four moments keep constant with any k. By contrast, the weakly stationary one focuses on the first and the second moment. Time series data is said to be weakly stationary if the mean and variance of $\{y_t, \cdots, y_{t+m}\}$ is equal to the ones of $\{y_{t+k}, \cdots, y_{t+m+k}\}$ for any k. Generally, it is rare to see only strictly stationary sequences in the financial markets. Thus, most discussions in finance about stationarity mainly concentrate on the weakly stationary rather than the strictly one, because the first and second moment often represent the mean return and risk of financial assets.

The most important function of stationarity is to avoid the spurious inference. Granger and Newbold (1974) suggested that the null hypothesis ($H_0: \beta_1 = \beta_2 = \cdots = \beta_k = 0$) might be rejected by the conventional F test under non-stationary situations, because the statistic of F would not follow Fisher's F distribution in this circumstance. In that manner, obviously the statistic inference might be easily misled by the characteristics of the financial time series. Therefore, the test of stationarity is essential and should be done before the application of other econometric models. Several approaches have been employed to test this feature. In this thesis, three main approaches are applied to test the stationarity of the sample sets, they are the Dickey-Fuller (DF) test proposed by Dickey and Fuller (1979), the Augmented Dickey Fuller test (ADF) proposed by Dickey and Fuller (1981), and Phillips and Perron's (1988) PP test[59]. Assume the AR(1) process of a certain return sequence is formulated as

$$r_t = c + \beta r_{t-1} + a_t \tag{4.2}$$

where c could be a constant or a constant with a time trend, i.e. $c = c_0 + \alpha t$, and a_t is a white noise disturbance term. Generally speaking, if the condition $|\beta| > 1$ holds then the return process follows an explosive pattern. In financial markets, this theoretical hypothesis is often ignored since none of the financial returns or prices retain this pattern in the long term. For $\beta=1$, r_t follows the non-stationary process of a_t. In the case of $\beta<1$, r_t will converge to a certainly stable level as $T \to \infty$ and any effect of shock in the market will be smoothed out in the long run. In this manner, r_t is called a stationary sequence. The original DF focuses on testing the parameter, β, and sets the null hypothesis $H_0: \beta = 1$ versus the alternative hypothesis $H_a: \beta < 1$. Generally, its statistics could be presented as

$$DF = \frac{\hat{\beta}-1}{SD(\hat{\beta})} \tag{4.3}$$

where $\hat{\beta}$ is the least squares estimate of β. Moreover, Dickey and Fuller (1981) extend the original DF to verify if a non-stationary characteristic exists in the higher order process, for example $AR(p)$, and $p>1$ and $p \in N$. Accordingly, Eq. (4.2) can be reworked as

$$r_t = c + \beta r_{t-1} + \sum_{i=1}^{p} \theta_i \Delta r_{t-i} + a_t \tag{4.4}$$

and Eq. (4.3) can still be applied to test the stationarity of the equation above. This is called the augmented DF test. Since the sequence of returns is the order differencing of the price level, Eq. (4.4) can be rearranged as follows by subtracting r_{t-1}.

59 The PP test is an extension of Phillips' (1987) work, by setting the mean equation with a drift, or a drift and a linear trend. The original approach of Phillips (1987) is $Z_\alpha = \frac{N(\hat{\alpha}-1)-(1/2)(s_{N\ell}^2 - s_u^2)}{(T^{-2}\sum_{\ell}^{N} r_{t-1}^2)}$ or $Z_t = \left(\sum_{\ell}^{N} r_{t-1}^2\right)^{\ell-2} \frac{(\hat{\alpha}-1)}{s_{N\ell}} - \left(\frac{(s_{N\ell}^2 - s_u^2)}{2}\right)\left[s_{N\ell}\left(T^{-2}\sum_{\ell}^{N} r_{t-1}^2\right)^{-1/2}\right]$, where Z_α is a transformation of the standardized estimator $N(\hat{\alpha} - 1)$ and Z_t is a transformation of the regression t statistics.

$$\Delta r_t = c + \beta' r_{t-1} + \sum_{i=1}^{p} \theta_i \Delta r_{t-i} + a_t \tag{4.5}$$

Alternatively, following the same concept of the ADF test, the Phillips and Perron (1988) test stresses the issue that the return sequence from a certain generating process might have a higher order of autocorrelation. Whilst the augmented Dickey–Fuller test addresses this issue by introducing lags of Δr_t as the regressors in the test equation as shown in Eq. (4.5), the PP test makes a non-parametric correction to the t-test statistic. The two hypothetical regression models are

$$r_t = \hat{\mu} + \hat{\alpha} r_{t-1} + u_t \tag{4.6}$$

$$r_t = \tilde{\mu} + \tilde{\beta}\left(t - \frac{1}{2}N\right) + \tilde{\alpha} r_{t-1} + \tilde{u}_t \tag{4.7}$$

The t-test statistics of parameters in the model above are

$$t_{\hat{\alpha}} = {(\hat{\alpha} - \alpha)\left[\sum (r_{t-1} - N^{-1} \sum r_{t-1})^2\right]^{1/2}} \Big/ {\hat{s}} \tag{4.8}$$

$$t_{\hat{\mu}} = {(\hat{\mu} - \mu)\left\{\sum (r_{t-1} - N^{-1} \sum r_{t-1})^2 / \sum r_{t-1}^2\right\}^{\frac{1}{2}}} \Big/ {\hat{s}} \tag{4.9}$$

$$t_{\tilde{\mu}} = {(\tilde{\mu} - \mu)} \Big/ {\sqrt{(\tilde{s}^2 c_1)}} \tag{4.10}$$

$$t_{\tilde{\beta}} = {(\tilde{\beta} - \beta)} \Big/ {\sqrt{(\tilde{s}^2 c_2)}} \tag{4.11}$$

$$t_{\tilde{\alpha}} = {(\tilde{\alpha} - \alpha)} \Big/ {\sqrt{(\tilde{s}^2 c_3)}} \tag{4.12}$$

where $\hat{s}$ and $\tilde{s}$ denote the standard errors of the two regression models, respectively, and c_j means the jth element of the matrix of dependent variables, $(\mathbf{X}'\mathbf{X})^{-1}$. A transformation of the standardized estimator $N(\hat{\alpha} - 1)$ and Z is a transformation of the regression t statistic.

4.3 Estimation and Forecast

This section describes the method of parameter estimation and the steps of VaR forecasting used in this thesis. In the first part, the estimation of parameters in measuring individual VaRs and dynamic conditional correlations will be explained. The second part elaborates on the method of VaR forecasting based on one- and 10-day ahead.

4.3.1 Estimation of parameters

Parameters in individual VaR

Based on the extreme value theory model, the distribution of selected extremes would converge to a generalized extreme value distribution (GEV) as shown in Eq. (2.44). Ignoring the case of k_n=0, the probability density function can be derived from Eq. (2.44) by a simple differentiation.

$$h\left(r_{max,jn}; k_n, c_n, d_n\right) =$$

$$\frac{1}{c_n}\left(1 + k_n \frac{\left(r_{max,jn} - d_n\right)}{c_n}\right)^{\frac{1}{k_n}-1} \exp\left[-\left(1 + k_n \frac{\left(r_{max,jn} - d_n\right)}{c_n}\right)^{\frac{1}{k_n}}\right] \qquad (4.13)$$

where n is the number of observations in the block and j= $1,2,\cdots,g$. d_n, c_n, and k_n indicate the location parameter, scale parameter and shape parameter, respectively. Under the assumption of independence to return consequences, the parameters of the GEV distribution can be obtained via the method of maxima likelihood. The log-likelihood function can be shown as

$$\ln L\left(r_{max,1n} r_{max,2n} \cdots r_{max,jn}; c_n, d_n, k_n\right)$$

$$= -g\ln c_n + \left(\frac{1}{k_n} - 1\right)\sum_{j=1}^{g} \ln\left[1 + \frac{k_n\left(r_{max,jn} - d_n\right)}{c_n}\right]$$

$$-\sum_{j=1}^{g}\left[1+\frac{k_n(r_{max,jn}-d_n)}{c_n}\right]^{1/k_n} \quad (4.14)$$

There is no standard guidance as to the selection of the length of the block. The main advantage of extreme value theory is that it improves accuracy in the estimation of parameters. If n is too big then obviously fewer extreme observations can be obtained. In contrast, if n is too small then the result might lose the spirit of extreme value theory. Moreover, the accuracy of the estimation might be affected in this manner. In this thesis, for the most robust check, n is set to be 5, 10, and 22 corresponding to one week, two weeks and one month.

After obtaining the estimated parameters, the critical point of the extreme sequence can be gained by inverting the accumulative density function (CDF) given a confidence level (α).

$$H\left(r_{max,jn}; k_n, c_n, d_n\right) = \alpha^*$$

$$= \exp\left\{-\left[1+k_n\left(\frac{r_{max,jn}-d_n}{c_n}\right)\right]^{\frac{1}{k_n}}\right\}$$

The critical value of the extreme level is

$$r_{max}^* = d_n - \frac{c_n}{k_n}\left[1-\left(-\ln(\alpha^*)\right)^{k_n}\right] \quad (4.15)$$

According to the order statistics, the relationship between extreme returns and original returns in probability is

$$H_n(x) = \alpha^* = \Pr(r_{max} \le x)$$

$$= \Pr(r_1 \le x, r_2 \le x, \cdots, r_{max} \le x)$$

$$= \prod_{i=1}^{n} \Pr(r_i \le x)$$

$$= \prod_{i=1}^{n} [F_r(x)] = [F_r(x)]^n$$

Since $F_r(x)$ is the probability, α, in the original return level in the right tail, the final relationship between the extreme level and original return is

$$\alpha^* = [1 - \alpha]^n \tag{4.16}$$

Taking Eq. (4.16) into Eq. (4.14), the individual VaR would be carried out in the original return level.

$$\mathrm{VaR} = d_n - \frac{c_n}{k_n}\left[1 - \left(-n\ln(1-\alpha)\right)^{k_n}\right] \tag{4.17}$$

Parameters in the model of dynamic conditional correlation

The second part of this subsection is to account for the parameters in the DCC model, which has been generally regarded as having a critical role in the related literature. The original application of the DCC model focuses on the dynamics of the conditional correlation based on the whole sequence. Specifically, in the procedure of estimation, whole samples are applied to estimate the conditional correlation and its dynamics. However, from the perspective of risk management it is reasonable to focus only on the tail-correlation rather than including the whole distribution. In this thesis, the main purpose for the calculation of DCC is to aggregate the individual VaRs into a portfolio VaR, and thus the main spirit of DCC applied in this thesis is to discover the dynamic pattern of any two extreme return sequences obtained from the block maxima sampling procedure. As shown in Eq. (2.22) and (2.23), the dynamic conditional correlation is obtained via the process of conditional covariance, modelled by the generalized autoregressive conditional heteroskedasticity (GARCH) model. The parameters, α and β, in Eq. (2.22) mainly describe the time-varying pattern of conditional volatility. However, it also implies that each individual return series has the same dynamics in the original model, which is not appropriate in real life. Consequently, a generalized DCC model, similar to Hafner and Franses (2009), is applied to estimate the time-varying relationship between individual assets in this thesis, conditioned on previous information.

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$$

$$\mathbf{R}_t = \mathrm{diag}(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t\mathrm{diag}(\mathbf{Q}_t)^{-1/2} \tag{4.18}$$

$$\mathbf{Q}_t = (\overline{\mathbf{P}} - \mathbf{A}'\overline{\mathbf{P}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{P}}\mathbf{B}) + \mathbf{A}'\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}\mathbf{A} + \mathbf{B}'\mathbf{Q}_{t-1}\mathbf{B} \tag{4.19}$$

where $\mathbf{A}$ and $\mathbf{B}$ are k-dimension matrices, and $\overline{\mathbf{P}}$ is the unconditional correlation which is obtained from the sample correlation. For the purpose of reducing the number of estimated parameters, $\mathbf{A}$ and $\mathbf{B}$ are assumed to be a diagonal matrix. Then, the generalized dynamic conditional correlation can be calculated via Eq. (2.23). Since investors might have long or short positions, intuitionally the correlation of financial assets to long and short positions would be different due to the asymmetric effect. In other words, the correlations of two return sequences in the left and right tail tend to be different. Accordingly, the generalized (diagonal) DCC to any pair return sequence will be calculated twice based on those maximum (right tail) and minimum (left tail) returns sampled through the BM approach, called tail-DCC. There are two main advantages to calculating conditional correlation as above. Firstly, it is needless to model asymmetry in the covariance pattern, since conditional covariance patterns of the left and right tails will be estimated separately. Secondly, separate calculation of conditional correlation avoids a contamination from the maximum returns when we calculate the correlation of the left tail. The major benefit of this approach is that to any two return sequences investors would more likely care about the concurrent likelihood in the left or right tail based on their extreme returns rather than the entire returns.

Estimating the generalized DCC in Eq. (4.18) and (4.19), a quasi-maximum likelihood is adopted, maximizing the log-likelihood function.

$$\mathrm{L}(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln|\mathbf{H}_t(\theta)| + \mathbf{r}'_t\mathbf{H}_t^{-1}(\theta)\mathbf{r}_t\right] \tag{4.20}$$

Following Engle's (2002) two stage method, the likelihood can be divided into two parts, the volatility term and the correlation term.

$$L(\boldsymbol{\theta}) = L_V(\boldsymbol{\theta}_V) + L_C(\boldsymbol{\theta}_C) \qquad (4.21)$$

$$L_V(\boldsymbol{\theta}_V) = -\frac{1}{2}\sum_{t=1}^{T}[n\ln(2\pi) + \ln|\mathbf{D}_t|^2 + \mathbf{r}_t'\mathbf{D}_t^{-2}\mathbf{r}_t] \qquad (4.22)$$

$$L_C(\boldsymbol{\theta}_C) = -\frac{1}{2}\sum_{t=1}^{T}[\ln|\mathbf{R}_t| + \boldsymbol{\varepsilon}_t'\mathbf{R}_t^{-1}\boldsymbol{\varepsilon}_t - \boldsymbol{\varepsilon}_t'\boldsymbol{\varepsilon}_t] \qquad (4.23)$$

The parameters obtained in maximizing Eq. (4.21) are then applied in maximizing Eq. (4.23). Then all the parameters in the generalized DCC model are finally obtained. An alternative distribution with fatness property can also be applied in Eq. (4.20) to (4.23). In this thesis, to capture the fat-tail characteristics consistently, a multivariate student t distribution is applied to estimate the pattern of the covariance matrix. Several papers in this area proposed similar GDCC models with the asymmetry effect based on a skewed multivariate student t distribution, emphasizing that the correlation between financial assets would be changed under different market conditions. Yet this is not an appropriate case in this thesis, since the dynamic conditional correlation in this thesis is estimated based on the extreme values, separating minima and maxima.

In this thesis, after the data sets are collected from DataStream, Excel 2007 is used to arrange the data and the professional software, RATS v.6.35 and v.7.0, used to estimate the parameters such as Eq. (4.14) and Eq. (4.21) to Eq. (4.23). As mentioned above, the VaR and DCC can be calculated after we obtain all the parameters.

4.3.2 VaR forecast

Basically, the Basel regulation requires the banks to offer information relating to a ten-day VaR estimate. Yet this seems not to meet the demand when using VaR for risk management. Thus, financial institutions mainly predict the VaR as one-day ahead in their portfolios.

Logically, the estimated critical return given a probability at time t is used to forecast the VaR at time t+1.

$$E_t(r_t^*|\alpha) = VaR_{t+1} \tag{4.24}$$

Taking a similar concept, the estimated dynamic conditional correlation at time t is regarded as the relationship between individual assets in the next trading period. After receiving all the estimated parameters, the portfolio VaR can be calculated according to Eq. (2.63). For the purpose of backtesting the performance of this model of VaR, a method of fixed-window rolling samples is adopted to forecast the VaR sequence. In this thesis, sixteen-year daily returns are applied to estimate dynamic conditional correlation since most of the observations would be filtered out by the procedure of extreme value selection.

4.4 Competing models

In order to provide some significant evidence that the performance of the portfolio VaR measure suggested by my thesis is theoretically and empirically better than related ones, some comparisons among the various measures needs to be made. In this section, four conventional VaR measures in this area are applied as competing models. One of the competing approaches is a non-parametric model, the others are parametric. All of these methods are widely used in financial institutions, especially in the banking sector. As the basic concept of VaR discussed in Section 2.2, the main factor of a VaR calculation is the process of estimated return volatility. Various VaR measures have assumed different dynamic patterns of financial returns. All of these dynamics or assumptions can be divided into the three categories, discussed in Section 3.2 to 3.4. Generally, the competing approaches assume the measure of portfolio VaR can be formulated as

$$VaR = \boldsymbol{\omega}'\mathbf{r}_t + z_\alpha\sqrt{\boldsymbol{\omega}'\boldsymbol{\Sigma}_t\boldsymbol{\omega}} \tag{4.25}$$

Alternatively, for convenience, the mean of daily returns for each financial asset is assumed to be zero. Thus, Eq. (4.25) can be reduced to a pure volatility version

$$\mathrm{VaR} = z_{\alpha}\sqrt{\boldsymbol{\omega}'\boldsymbol{\Sigma}_t\boldsymbol{\omega}} \tag{4.26}$$

where $\mathbf{r}_t$ is the matrix including the mean return of all individual assets, $\boldsymbol{\omega}$ is the matrix of weight for each individual asset in the portfolio, and $\boldsymbol{\Sigma}_t$ indicates the covariance matrix of the portfolio. This section describes the four approaches, all theoretically modelling diversified viewpoints in the hypothetical process of return volatility and widely used in risk management. Consequently, they are appropriate to be the alternative methods against the GEV approach with DCC model proposed by this thesis. As mentioned in Section 4.3.1, Excel 2007 is used to arrange the data and RATS is used to estimate the parameters shown in this section. VaR can be calculated after we obtain all the parameters.

4.4.1 GARCH model

As discussed in Section 2.2, volatility is one of the critical factors in measuring VaR. Since the dynamic volatility process, ARCH and GARCH models, were proposed by Engle (1982) and Bollerslev (1986), numerous (G)ARCH models have been proposed focusing on different aspects in this area. Since then, conditional volatility of financial returns has played a critical role in measuring market risk. In particular, these volatility models are at the centre of the measurements for VaR. Thus, it is both necessary and worthwhile to compare the performance in VaR measuring of the GARCH-related model with the one from the GEV-DCC model.

In the portfolio case, the measure of VaR based on multivariate GARCH (denoted as VaR_{MG}) can be displayed as Eq. (4.25) or (4.26). The critical point of the two measures is the process of estimated return volatility, $\boldsymbol{\Sigma}$. For the issue of positive-definite, and reducing the number of estimated parameters, the dynamic process of covariance in Eq. (4.26) is simply assumed as a BEKK GARCH model without exogenous variables, as shown below. For avoiding cumbersome calculations, the order of lag in the process of return volatility is set to unity

$$\mathbf{\Sigma}_t = \mathbf{C}_0'\mathbf{C}_0 + \boldsymbol{\alpha}_1'\mathbf{a}_{t-1}\mathbf{a}_{t-1}'\boldsymbol{\alpha}_1 + \boldsymbol{\beta}_1'\mathbf{\Sigma}_{t-1}\boldsymbol{\beta}_1 \quad (4.27)$$

where $\mathbf{a}_{t-1}$ is the residual vector at time t-1. In the two variables case, Eq. (4.27) can be shown as a matrix version

$$\mathbf{\Sigma}_t = \mathbf{C}_0'\mathbf{C}_0 + \begin{bmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}' \begin{bmatrix} a_{1,t-1}^2 & a_{1,t-1}a_{2,t-1} \\ a_{2,t-1}a_{1,t-1} & a_{2,t-1}^2 \end{bmatrix} \begin{bmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}$$

$$+ \begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}' \mathbf{\Sigma}_{t-1} \begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix} \quad (4.28)$$

Although the mean of equity returns tends to be zero, for the purpose of accuracy VaRs are still measured with this term as shown in Eq. (4.25). Thus, the last step of this approach is simply to substitute Eq. (4.28) into the Eq. (4.25) VaR formula.

4.4.2 RiskMetrics model

As mentioned in Section 3.2.1, RiskMetrics (denoted VaR_{RM}) is the earliest VaR measure (proposed by J.P. Morgan in 1996). For a long position, VaR_{RM} can be expressed as shown below.

$$VaR_{RM} = z_\alpha \sigma \quad (4.29)$$

where z_α is a critical point of normal distribution given a probability α. The multivariate case can be presented as Eq. (4.25) or (4.26). Obviously, based on the concept of volatility of financial returns, a variation of returns is deemed the main indicator of market risk. However, the conventional variance is calculated based on the spirit of equally weighted previous information, and estimated volatility might thus be easily affected by particular shocks (positive or negative) that happened a long time ago. Moreover, it seems unreasonable to say that the current return volatility has the same degree of influence as another volatility which happened a long time ago. Consequently, RiskMetrics adopted volatility based on the exponentially weighted moving average (EWMA) method which could eliminate the influence from large shocks in the economy by giving a decay factor. The original exponentially weighted moving average variance is formulated as in the following equation.

$$\sigma^2_{t+1|t} = (1-\lambda)\sum_{t=1}^{T}\lambda^{t-1}(r_t - \bar{r})^2 \tag{4.30}$$

where λ is the decay factor, which indicates the amount of volatility at time t affected by previous volatility. In practice, the average financial return is set as zero, and this equation can be rearranged as a tractable version[60].

$$\sigma^2_{t+1|t} = \lambda\sigma^2_t + (1-\lambda)r^2_t \tag{4.31}$$

Furthermore, J.P. Morgan suggested that the decay factor, λ, equals to 0.94 and 0.97 for daily and monthly data, respectively. Analogically, the covariance can be derived with the similar form

$$\sigma^2_{XY,t+1|t} = \lambda\sigma^2_{XY,t} + (1-\lambda)r_{X,t}r_{Y,t} \tag{4.32}$$

From the aspect of portfolio risk management, Eq. (4.31) and (4.32) can be extended to the multivariate version. The dynamic process of stacked covariance of financial returns can be shown as the equation below.

$$\boldsymbol{\sigma}^2_{t+1|t} = \begin{bmatrix} \sigma^2_{X,t+1|t} \\ \sigma^2_{XY,t+1|t} \\ \sigma^2_{Y,t+1|t} \end{bmatrix}$$

[60] By assuming the mean of financial returns is zero, then the exponentially weighted moving average volatility (or variance) of the financial return can be obtained as

$$\begin{aligned}\sigma^2_{t+1|t} &= (1-\lambda)\sum_{i=0}^{T}\lambda^i \mathbf{r}^2_{t-i} = (1-\lambda)\left(\lambda^0 r^2_t + \lambda^1 r^2_{t-1} + \lambda^2 r^2_{t-2} + \cdots\right) \\ &= (1-\lambda)r^2_t + \lambda(1-\lambda)\left(\lambda^0 r^2_{t-1} + \lambda^1 r^2_{t-2} + \lambda^2 r^2_{t-3} + \cdots\right) \\ &= \lambda\sigma^2_t + (1-\lambda)r^2_t\end{aligned}$$

$$= \begin{bmatrix} \lambda_X & 0 & 0 \\ 0 & \lambda_{XY} & 0 \\ 0 & 0 & \lambda_Y \end{bmatrix} \begin{bmatrix} \sigma^2_{X,t|t-1} \\ \sigma^2_{XY,t|t-1} \\ \sigma^2_{Y,t|t-1} \end{bmatrix} + \begin{bmatrix} 1-\lambda_X & 0 & 0 \\ 0 & 1-\lambda_{XY} & 0 \\ 0 & 0 & 1-\lambda_Y \end{bmatrix} \begin{bmatrix} r^2_{X,t} \\ r_{X,t}r_{Y,t} \\ r^2_{Y,t} \end{bmatrix} \quad (4.33)$$

where λ_X, λ_Y, λ_{XY} are the decay factors of σ_X, σ_Y, and σ_{XY}, respectively. In the original RiskMetrics model, all the decay factors of financial return volatility were identical: 0.94 for daily data. Thus, the VaR based on RiskMetrics can be presented as

$$\mathrm{VaR}_{t+1,RM} = \boldsymbol{\omega}'\mathbf{r}_t + z_\alpha\sqrt{\boldsymbol{\omega}'\boldsymbol{\sigma}^2_t\boldsymbol{\omega}} \quad (4.34)$$

4.4.3 Multivariate stochastic volatility

Both the previous two models focus on using the time series model to describe the co-movement of covariance between various financial returns. However, these time-varying covariance matrices can be estimated by other approaches. Harvey, Ruiz and Shephard's (1994) stochastic volatility has already been demonstrated as successful in presenting the jump-diffusion process volatility. Accordingly, the third VaR competing model is based on the concept of stochastic volatility (denoted VaR_{SV}), which is an alternative method to capture the dynamics of return volatility. In this section, a multivariate stochastic volatility proposed by Harvey, Ruiz and Shephard (1994) is briefly introduced and applied for measuring the portfolio VaR (denoted VaR_{SV}), by estimating the covariance in advance. The univariate model can be simply shown as an AR(1) process, as below.

$$\ln r^2_t = \mu + \ln\sigma^2_t + \ln a^2_t,\ t = 1, \cdots, T \quad (4.35)$$

$$\ln\sigma^2_t = \alpha_0 + \alpha_1 \ln\sigma^2_{t-1} + \nu_t \quad (4.36)$$

where the original mean equation is set as $r_t = \mu + a_t,\ a_t = \sigma_t\epsilon_t$, a_t follows a normal distribution with zero mean and unit variance, and ν_t are iid $N(0, \sigma^2_\nu)$. For convenience, the stationary assumption is set, $|\alpha_1| < 1$. As with the exponential GARCH (EGARCH), the stochastic volatility model working in logarithms guarantees the variance to be positive over time. Harvey, Ruiz and Shephard (1994) suggested that the parameters in the volatility equation could be estimated by the quasi-maximum likelihood method, computed using the Kalman filter. They also provided a multivariate stochastic volatility model as follows. Let r_t be an $N \times 1$ vector, with elements

$$r_{i,t} = \mu_i + a_{i,t}\sigma_{i,t} \tag{4.37}$$

$$\ln\sigma_{i,t} = \alpha_{0,i} + \alpha_{1,i}\ln\sigma_{i,t-1} + \nu_{i,t} \tag{4.38}$$

where $i = 1, \cdots, N,\ t = 1, \cdots, T$. In this thesis, a quasi-maximum likelihood method is used to estimate the parameters in Eq. (4.37) and (4.38) by using the RATS v.7.0 package procedure. The non-diagonal element of the covariance matrix can be derived by $(\pi^2/2)\rho_{ij}^*$, where $\rho_{ii}^* = 1$.

$$\rho_{ij}^* = \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(n-1)!}{\left(1/2\right)_n n}\rho_{ij}^{2n},\quad i \neq j, i, j = 1, \cdots, N \tag{4.39}$$

where $(x)_n = x(x+1)\cdots(x+n-1)$. With the covariance matrix obtained from Eq. (4.38) and (4.39), the portfolio VaR can be measured by applying Eq. (4.26).

4.4.4 Historical simulation (HS)

Historical simulation is the most popular non-parametric approach to forecast VaR in practice. The advantages and application of this method have been discussed in Section 3.3. For the purpose of eliminating the noisy information embedded in the return series, a filtered historical simulation proposed by Barone-Adesi et al. (1999) is adopted. The spirit of this approach is to form a modified filtered historical simulation and to compare with the VaR measure suggested by this thesis. This competing model follows Barone-Adesi et al's suggestion that a GARCH model can be used as a filter to generate i.i.d. residuals from the return series. The choice of filter for each return sequence is made up of two criterions: Akaike information criterion (AIC) and Bayesian information criterion (BIC).

$$\mathrm{AIC} = \frac{2}{n}\ln(\text{value of likelihood function}) + \frac{2}{n}(\text{no. of parameter}) \qquad (4.40)$$

$$\mathrm{BIC}(\ell) = \ln\left(\bar{\sigma}_{\ell}^{2}\right) + \frac{\ell \ln(n)}{n} \qquad (4.41)$$

where n is the sample size, and ℓ is the lag of AR model.

After obtaining the i.i.d. filtered return series, the procedure of historical simulation in measuring portfolio VaR can be separated into four steps. Firstly, the number of historical data used to perform the empirical distribution needs to be appropriately accounted for. Three sample sizes (250, 750, and 1,250 observations corresponding to one, three, and five years) are taken into account for measuring the VaR. Secondly, the selected samples are utilized to estimate standard deviation and empirical distribution. The third step is setting the probability of confidence level. Then the VaR of each financial asset can be obtained based on Eq. (2.3). Finally, all individual VaRs are aggregated into a portfolio VaR.

4.5 Backtesting

In the related forecasting research, it is essential to detect the quality of forecasting obtained from various risk measures, especially for those that have been applied in financial institutions. The basic concept behind the method of backtesting suggested by the Basel Committee on Banking Supervision (1996) has been further developed and widely applied across the banking sector for several years. In general, the substance of these diverse backtesting approaches is to compare the forecasted risk derived from internal models with the actual returns. Thus, the internal models can be refined and improved based on the results of the backtesting.

The first sub-section reviews the backtesting approach suggested by the Basel Committee on Banking Supervision in 1996, and discusses its main defects. In the second part of this section, some statistical tests that can be used to compare the quality of the alternative risk measures are discussed.

4.5.1 Backtesting procedure

Using internal risk models to measure market risk is now the approved alternative method for the practitioners. However, as set out in the Basel's regulations (see BCBS (1995)), and designed to ensure the accuracy of these risk forecasts, financial institutions adopting internal risk models are required to backtest the performance of their approaches and report the results to the local authorities. The committee believes that the procedure of backtesting properly provides the opportunity to the designers of risk measures to understand how to incorporate a variety of market circumstances into their methodologies.

As mentioned above, most backtesting procedures typically consist of a series of comparisons between the daily return of a hypothetical portfolio and the forecasted VaR. Conceptually, the measure of value at risk is an estimate of the amount that might happen in the next trading period with a given probability. Since the holding positions usually involve a large amount of capital, the Basel Committee requires that the backtesting should be implemented with a 99% level of confidence of risk measure, which means that, on average, 99% of daily returns to a long position are bigger than the corresponding VaR. In other words, a well-designed VaR model will cover 99% of market variation in its estimate.

The complete procedure of backtesting adopted in this thesis can be separated into several main steps. After selecting the methodology of risk measurement, the first step is to do the one-day-ahead out of sample forecast of VaR, which represents the VaR of the next trading period. The second step is to create the VaR series by using the method of fixed-window rolling sample forecasting, and then repeating step one. For example, according to step one, the historical observations within the period from 2nd January 1990 to 29th December 2006 are used to estimate the model parameters and forecast the VaR on 2nd January 2007. Similarly, the samples within 3rd January 1990 to 2nd January 2007 are applied for the VaR on 3rd January 2007, and so on. The last VaR (30th April 2010) is forecasted based on the period from 6th May 1993 to 29th April 2010.

The third step is to compare the actual return series with the VaR sequence obtained from the risk measurements. Based on the theory, the VaR_t is applied to forecast the market risk or potential loss in the next trading period, therefore the comparison will be made between the forecasted value-at-risk at time t (VaR_t) and the actual portfolio return at time t+1 (r_{t+1}). In the fourth step, the violation series is obtained based on the actual return and its corresponding VaR. Simply put, an actual return (of a long position) smaller than the corresponding forecasted VaR is regarded as a violation, e_t. Then, the violation function can be defined as

$$\begin{aligned} e_t &= I(VaR_{t-1}|r_t) \\ &= \begin{cases} 0, & \text{if } r_t \geq VaR_{t-1} \\ 1, & \text{if } r_t < VaR_{t-1} \end{cases} \end{aligned} \tag{4.42}$$

where e_t is a series with T-1 elements.

Finally, the performance of risk measure can be tested based on the unconditional approach proposed by the Kupiec (1995) method. The violation rate (VR) is calculated as

$$VR = \frac{\sum_{t=1}^{N} e_t}{N} \tag{4.43}$$

where N is the total number of observations within the backtesting period, which is from 2nd January 2007 to 30th April 2010. Theoretically, VR of a well-designed VaR measure will approximate the value of α, which is the probability in Eq. (2.1). Statistically, the alternative hypothesis is set as $H_a: VR \neq \alpha$, against the null hypothesis, $H_0: VR = \alpha$. The statistics of the unconditional test are

$$LR_{un} = 2\ln[(1-VR)^{N-x}VR^{x}] - 2\ln[(1-\alpha)^{N-x}\alpha^{x}] \qquad (4.44)$$

where x is the total number of violations.

Alternatively, based on Kupiec's (1995) work, Christoffersen (1998) concentrates on the consequences of the independence of the violations. He emphasizes that a good risk measure should not only meet the criteria of VR=α, but also have to avoid the clustering (autoregressive) effect in the violation series. Since if e_t could be applied to forecast e_{t+1}, then the probability of occurrence of e_{t+1} might not be equal to α. Assuming the transition probability matrix and the approximate likelihood function of the binary exceedance series can be shown as

$$\Pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix}$$

$$L(\Pi_1; e_1, \cdots, e_N) = (1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}} \qquad (4.45)$$

where $\pi_{ij} = \Pr(e_t = j | e_{t-1} = i)$. The estimate of Π_1 can be obtained by the observed outcomes of the exceedance series.

$$\widehat{\Pi}_1 = \begin{bmatrix} \frac{n_{00}}{n_{00}+n_{01}} & \frac{n_{01}}{n_{00}+n_{01}} \\ \frac{n_{10}}{n_{10}+n_{11}} & \frac{n_{11}}{n_{10}+n_{11}} \end{bmatrix} \qquad (4.46)$$

To test the independence of the violation series, a corresponding independent transition matrix is assumed as

$$\Pi_2 = \begin{bmatrix} 1-\pi_2 & \pi_2 \\ 1-\pi_2 & \pi_2 \end{bmatrix}$$

where $\widehat{\pi}_2 = (n_{01} + n_{11})/(n_{00} + n_{10} + n_{01} + n_{11})$, indicating the probability of a value as 0 followed by the value as 1. The likelihood under the null is

$$L(\Pi_2; e_1, \cdots, e_N) = (1-\pi_2)^{(n_{00}+n_{10})}\pi_2^{(n_{01}+n_{11})} \qquad (4.47)$$

Then, the independence test and LR test to the same hypothesis proposed by Christoffersen (1998) can be expressed as

$$\mathrm{LR_{ind}} = -2\ln\left[\frac{\mathrm{L}(\widehat{\Pi}_2; e_1, \cdots, e_\mathrm{N})}{\mathrm{L}(\widehat{\Pi}_1; e_1, \cdots, e_\mathrm{N})}\right] \tag{4.48}$$

$$\mathrm{LR_{cc}} = \mathrm{LR_{uc}} + \mathrm{LR_{ind}} \tag{4.49}$$

In the backtesting procedure, the conditional and unconditional assessments shown in Eq. (4.44) and (4.49) are mainly applied to test the quality of the risk measures.

4.5.2 Other performance tests

The two backtesting measures mentioned in the previous section are coverage tests, which focus on testing whether the ratio of coverage and type I error of the VaR measure are consistent or not. There are alternative methods to evaluate the performance of VaR measurements. In this section, the performance evaluations are conducted based on stability, conservativeness, and magnitude of potential losses over VaR. Hendricks (1996) developed an application with similar measures to test the performance of VaR models, however the details were unfortunately not numerically evident in his research.

In general, risk management departments would continuously monitor the performance of the portfolios held by financial institutions in the aspect of potential gains and possible losses. To maximize the profit of their positions, the portfolio is adjusted daily according to the monitoring report. In general, the daily market condition would not be significantly changed, except in very rare cases. Consequently, if the risk measure reflected the market condition truly and appropriately, the daily pattern of risk forecasting of a good risk measure would not be volatile. Bearing in mind the transaction fee, investors does not want to have to be continually tuning the portfolio following highly fluctuating forecasted risk. Therefore, it is critical to assess the stability of the risk measure, because a volatile risk measure could not usefully be used in practice. Thus, a stability test for the VaR forecast is essential,

$$\mathrm{MSE} = n^{-1} \sum_{t=1}^{n} [\mathrm{VaR}_t - E(\mathrm{VaR})]^2 \qquad (4.50)$$

where n is the number of observations in the backtesting period, and E(VaR) means the expected value of VaR in the backtesting period[61]. As mentioned above, the measure of VaR is regarded as the minimum required capital for financial institutions. In this manner, a VaR measure with a larger MSE means that the financial institutions need to adjust the amount of required capital frequently, which is unlikely to be implemented in practice. In other words, it would be preferable for a VaR model to have appropriate stability, particularly when it is used for daily forecasts.

From another perspective, as mentioned in Section 2.4.2, the VaR can also be a tool in capital allocation and capital charge. Therefore, if the risk measure is too conservative (also called overestimating) then the financial institutions might find themselves required to hold a larger number of capital than they really should. This has a critical impact to financial institutions that how they use their capital. In contrast, if the risk measure tends to under-estimate the risk of the portfolio then the financial institutions are likely to go bankrupt more easily due to the lack of enough capital. Thus, an ideal and efficient risk measure needs to offer accurate daily risk forecasts without too much or too little protection. With a long position, two simple tests are proposed[62]

$$D = \frac{\sum_{i=1}^{m} (\mathrm{VaR}_i - R_i)_{\mathrm{VaR}_i \le R_i}}{m} \qquad (4.51)$$

$$Q = \frac{\sum_{i=1}^{N-m} (R_i - \mathrm{VaR}_i)_{R_i < \mathrm{VaR}_i}}{N-m} \qquad (4.52)$$

[61] Eq. (4.50) is a natural variance formula. However, it is obvious that the forecasted VaR sequence of a well-designed VaR measure will stay around the expectation of VaR if the return series has less fluctuation. According to Montgomery, Jennings and Kulahci (2008), the MSE is formulated as Eq. (4.50).

[62] Eq. (4.51) and (4.52) are for the long positions, they can be rewritten for the short positions: $D = \frac{\sum_{i=1}^{m} (R_i - \mathrm{VaR}_i)_{\mathrm{VaR}_i \ge R_i}}{m}$ and $Q = \frac{\sum_{i=1}^{N-m} (\mathrm{VaR}_i - R_i)_{R_i > \mathrm{VaR}_i}}{N-m}$. The main statistical properties of these two measures (D and Q) are shown in appendix A.

where m is the number of non-violated observations and $N - m$ is the number of violations (N is the total number of observations in the backtesting period). The D statistics in Eq. (4.51) describes the average distance between the VaR measures and the actual returns in the case that the actual returns do not exceed the forecasted VaRs, i.e. the non-violations. The main aim of D is to evaluate the conservativeness of the VaR measurement. In general, a VaR measure with a larger D means that the approach is likely to be too conservative in measuring market risk and thus this would cause financial institutions to lose their efficiency in capital usage.

In Section 4.5.1, the test of coverage mainly focuses on the assessment of the violation ratio, however the Q statistics in Eq. (4.52) looks at the magnitude of the violation. Following the concept of "loss function" proposed by Lopez (1999), a good VaR measure must not cause too much loss. Thus, the potential loss of those violated observations needs to be reviewed. In other words, it presents the average potential loss as more than the VaR numbers and evaluates the performance of the VaR measures from the perspective of the quantity of the loss.

Using these three performance measures, one can have a clearer overview and understanding when assessing various VaR approaches.

4.6 Data

This section presents the descriptions of the data sets and some of their statistical characteristics. This thesis applies daily equity index data, however the VaR method suggested in this thesis can also be applied to measure the VaR of other financial data, for example exchange rates or interest rates.[63] For comparing the risk of equity indices with previous research, the daily closing price of six developed equity markets, six East Asian emerging equity markets, and four Latin American emerging equity markets are included in this thesis[64]. The six developed equity market indices include the Standard & Poor 500 (SP 500) of the United States, the FTSE 100 of the United Kingdom, the Nikkei 225 of the Japanese equity market, the Toronto Stock Exchange (TSX) index of the Canadian equity market, the Deutscher Aktien index (DAX) of the German equity market, and the Continuous Assisted Quotation 40 (CAC 40) of the French stock market. The indices of the East Asian emerging equity markets are the Hang Seng Index (HSI) of the Hong Kong equity market, the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) of the Taiwanese equity market, the Korea Composite Stock Price Index (KOSPI) of the South Korean equity market, the FTSE Bursa Malaysia Kuala Lumpur Composite Index (KLCI) index of the Malaysian equity market, the Jakarta Composite Index (JCI) of the Indonesian equity market, and the Stock Exchange of Thailand index (SET) of the Thai equity market. The third data set is mainly taken from the emerging equity markets in Latin America. They include the Merval

[63] According to the economic theory, the exchange rate and interest rate are generally led by the policies of the central bank such as open market operations and re-discounting rate adjustments (Taylor, 2001). Taking into consideration of the influence of governments' policies, the main equity indices employed here are effected less by these policies and reflect more truthful market information. Thus, the main equity indices are employed to model daily VaR.

[64] In this field, equity indices are commonly used for measuring VaR. For example, Longin (2000) measured the VaR of the S&P500 index, Ho et al. (2000) analysed the parameter sensitivity of major Asian equity indices, Jondeau and Rockinger (2003) and Longin (2005) applied the method of extreme value theory to estimate left- and right tail-distribution of equity indices. Gencay et al. (2003) and Gençay and Selçuk (2004) investigated the VaR of long and short positions for a series of emerging equity indices. In addition, Christoffersen et al. (2001), Consigli (2002), Angelidis et al. (2004), Giot and Laurent (2004), Bao et al. (2006), Hartz, Mittnik and Paolella (2006) measured the VaR of various equity indices.

index of the Argentinean equity market, the Brazilian Bovespa index, the Chilean IGPA index, and the Mexican Bolsa index. All of the three sets of daily closing price are collected from DataStream. The equity indices used in this thesis are the most important indices in their markets. Thus, the question of survivorship does not come into consideration. All of these indices have the corresponding index derivatives such as index futures and options contracts for market traders. In addition, the survivorship of the equity index does not make any influence to the VaR modelling. Thus, we do not have to be concerned about this issue. To the issue of investability, it is obvious that investors cannot make the long or short positions on an equity index. However, it does not mean that measuring the VaR of equity index returns is meaningless. As is already known, these indices have their corresponding derivatives such as index futures and index options contracts, and measuring VaR of the long and short positions of equity indices offers the investors a comprehensive understanding of the market risk profile of these derivatives' underlying assets. There is no doubt that the VaR models in this thesis can be applied to various financial data. As mentioned in Chapter 1, the main objective of this thesis, as well as previous research, is to provide a better approach for measuring market risk, rather than to establish which asset is the best for investment. To the issue of changes in the constituent stocks of the equity index, it is shown and explained in Appendix B that the effect of changes in constituent stocks to VaR modelling can be ignored.

The sample period for the developed market indices and the Asian emerging market indices is from the 2nd January 1990 to 30th April 2010. In order to avoid the situation that market risk may not be reflected in the price because of the thin trading phenomenon in the early stages of the Latin American equity markets, those price sequences start from the 2nd January 1995 and end on the 30th April 2010. In addition, the sample sets used in this thesis span many equity markets and the business days of these equity markets are not the same. Some special and traditional holidays may only exist in one country or one regional market and not in all global equity markets, for example, bank holidays in the UK, the Chinese New Year holiday in Taiwan. This phenomenon might cause indices to suffer data mismatching, moreover, the inference and analysis might then be made incorrectly. To overcome this inherent issue within the data sets, two general approaches are adopted[65]. Firstly, for obtaining the entire time serial data a method of interpolation is utilized to reform the data sequence by inserting a hypothetical observation. This remedy is constructed on the grounds that the index price changes linearly, and thus the price at time t could be seen as the midpoint between the price at t-1 and t+1. The second method simply uses the previous closing price in place of the missing ones. However, the index closing price is not always linear over time, and the method of replacement seems unsuitable for situations where there is a long range of data missing. To avoid contamination from the over-repair of data, the index price of those particular dates will be excluded if more than half of the equity markets in the data set are closed. The method of interpolation is adopted if only one day of each equity market is closed. After the series of data collation was completed there were 5,255 observations in the developed equity market, 5,287 observations in the Asian emerging equity market, and 3,945 observations in the Latin American emerging equity market.

The continuously compounded returns of the individual indexes were calculated as the first order difference of the natural logarithm of each series[66] (Cont, 2001).

$$r_t = \ln(p_t) - \ln(p_{t-1}) \tag{4.53}$$

The portfolio return is set as an equally weighted return of each individual return (p.

[65] It is not suitable to exclude all days with a missing observation, since most of the missing data happened in a single equity market at a time. The exclusion of the missing data would thus drop more observations. For example, the bank holidays are only in the UK equity market, then the observations for the same date of all the other equity markets will be excluded for the purpose of consistency. This treatment would thus cause more influence to the data.

[66] With the issue of dividends, stock dividend (ex-right) will not affect the index price and its return. However, the opening index price on the ex-dividend date will be affected by the amount of ex-dividend. The index prices used in this thesis are closing prices, which are not influenced by any one single event of the ex-dividend. Besides, the effect of the ex-dividend will be diversified by the weight of the individual stocks. Thus, the influence of the dividend issue can be ignored. To the issue of changes in constituent stocks of equity indexes can be seen in Appendix B, where the effect is shown to be trivial.

390,Bessis, 2010).

$$r_{p,t} = \sum_{d=1}^{N} \omega_i r_{i,t}$$

where d is the number of assets in the portfolio ($1 \leq d \leq N$) , $r_{p,t}$ is the portfolio return at time t, ω_i is the weight of each asset in the portfolio, and $r_{i,t}$ is the return of each individual asset at time t. In the following subsections, each index sequence will be introduced fully, with its main history, regulations, and critical properties.

4.6.1 Six developed equity markets[67]

Standard & Poor 500

This index has been published by the Standard & Poor group since 1957 and has been broadly regarded as the best gauge of the equity market in the U.S., capturing about 75% coverage of market capitalization. It covers the prices of 500 large-cap common stocks frequently and publicly traded in both the two main exchanges: the Nasdaq and the New York Stock Exchange. The 500 U.S.-based companies span various industries and are chosen by the Standard & Poor Index Committee based on critical criteria, for example, market capitalization, liquidity, financial viability, and sector representation. The Index Committee meets monthly and reviews the pending corporate actions which may affect the constituency of the S&P 500.

FTSE 100

[67] The information of the six developed market indices please refer their official website: Standard & Poor 500 (http://www.standardandpoors.com/indices/main/en/eu), FTSE 100 (http://www.londonstockexchange.com/exchange/prices-and-markets/stocks/indices/summary/summary-indices.html?index=UKX), Nikkei 225 (http://www.tse.or.jp/english/market/topix/comparison.html), TSX (http://www.tmxmoney.com/HttpController?GetPage=EquityIndices&SelectedIndex=0000&IndexID=0000&Exchange=T&SelectedTab=QuoteResults&Language=en), DAX (http://www.boerse-frankfurt.de/en/equities/indices), CAC 40 (http://www.euronext.com/trader/summarizedmarket/stocks-2634-EN-FR0003500008.html?selectedMep=1).

The FTSE 100 index is composed of the 100 most highly capitalised companies listed on the London Stock Exchange (LSE), presenting approximately 81% of the UK market capitalization. It is calculated in real-time by the FTSE Group, owned by The Financial Times and the London Stock exchange. The components of the FTSE 100 are reviewed quarterly by the FTSE UK regional committee, based on at least 20 trading records of individual stocks. A security will be included in the FTSE 100 index if its market value ranking rises to 90 per cent quantile or above of all current FTSE 100 shares. In the same manner, a current security in the FTSE 100 will be dropped if its market value ranking falls to the 111 quantile or below.

Nikkei 225

The Nikkei 225 index is calculated daily by the Nihon Keizai Shimbun (Nikkei) newspaper and is an equity index comprised of 225 stocks listed on the first section of the Tokyo Stock Exchange (TSE). Now it is one of the most watched indexes of the Asian equity market. The components of the Nikkei 225 index are reviewed every October, but an extraordinary review will take place if necessary. In 2000, faced with the evolution of the industry structure, both of these two reviews were redefined. The main purpose of the periodic review is to annually reconsider individual company issues from the aspect of changes in the industrial and market structures, and the extraordinary review is designed for deleting and adding constituents in response to special developments, for example, mergers or bankruptcies.

Toronto Stock Exchange (TSX) Composite index

The Toronto Stock Exchange (TSX) was established in 1852 and formally incorporated in 1878. The TSX index, managed by Standard and Poor's, includes 245 large-cap companies and covers about 70% market capitalization of all companies listed in the TSX. All the securities under consideration for addition to or deletion from the index will be assessed by the Index Committee on the basis of 12-month data ending the month prior to the quarterly review. The assessment is based on several points, such as the weight of an individual component to the TSX composite index and their liquidity based on trading volume. Generally, the quarterly review months are March, June, September and December.

Deutscher Aktien index (DAX)

The DAX index reflects the segment of blue chips listed in the Prime Standard Segment and comprises the 30 largest and most actively traded companies listed at the Frankfurt Stock Exchange (FSE). The selection of companies for the DAX index are generally based on two quantitative criteria, turnover and market capitalization. All the constituents included in the DAX are generally reviewed by the Management Board every three months. The review is divided into ordinary adjustments and extraordinary adjustments. The former is decided based on the rules of fast exit, fast entry, regular exit, and regular entry. The latter is designed for the occurrence of specific events such as insolvency.

CAC 40

The CAC 40 index takes its name from the Paris Bourse's early automation system Cotation Assistée en Continu (CAC, called "Continuous Assisted Quotation"). It comprises 40 companies selected from among the 100 largest and most traded stocks listed on the NYSE Euronext Paris[68]. Responding to the changes, the periodic review is made quarterly by an independent Index Steering Committee. In principle, the quarterly adjustment of the weighting of constituents is carried out after the markets close, on the third Friday of March, June, September and December. In addition, the free floats and capping factors of the companies included in the CAC 40 are reviewed annually. Corporate events, for example mergers and acquisitions, might affect the composition of the CAC 40 index.

[68] NYSE Euronext has a series of merger histories over the last few years. In March of 2000, it was announced that the exchanges of Amsterdam, Brussels, and Paris planned to merge into the Euronext exchange and Euronext Paris was set as the headquarters of the new exchange. In 2002, the Lisbon exchange was merged into Euronext as well. Then, in 2007, the New York Stock Exchange (NYSE) and Euronext merged, to become NYSE Euronext.

Figure 4.1 Daily index price and return of S&P 500

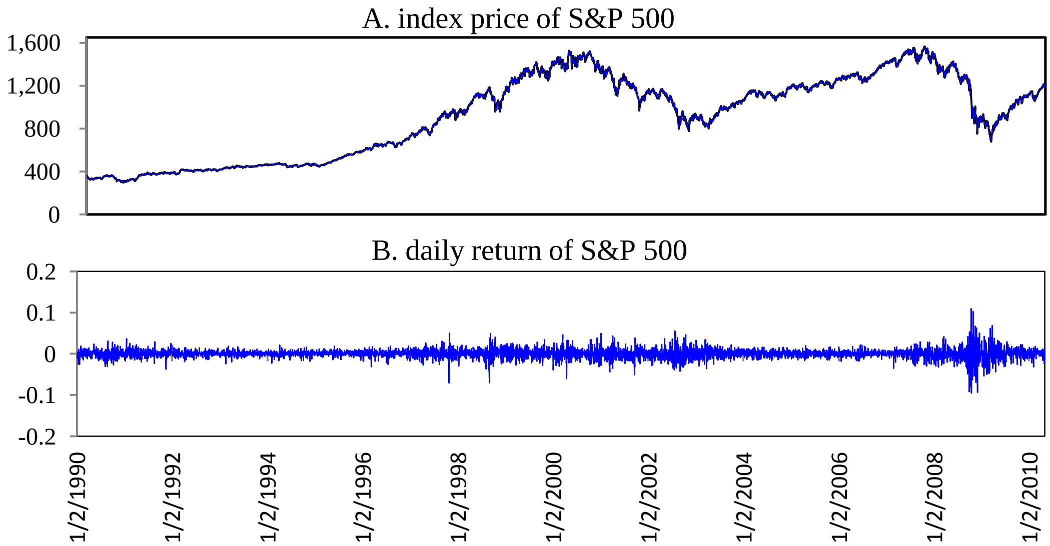


Figure 4.2 Daily index price and return of FTSE 100

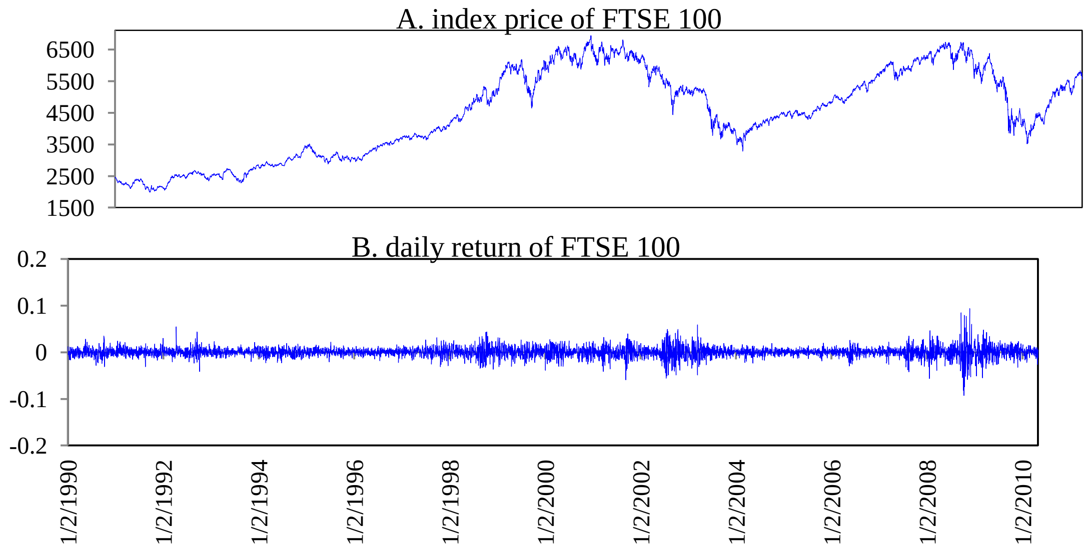


Figure 4.3 Daily index price and return of NIKKEI 225

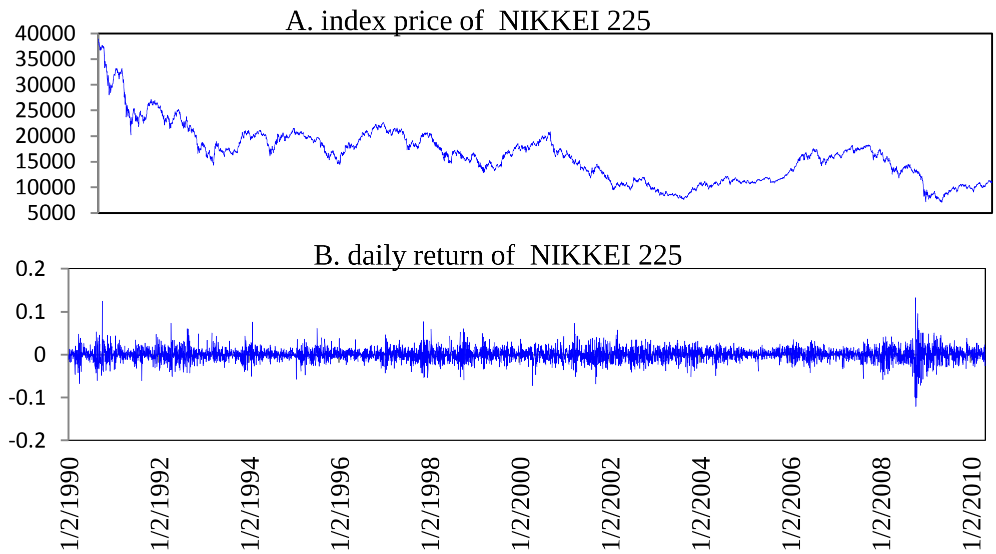


Figure 4.4 Daily index price and return of TSX

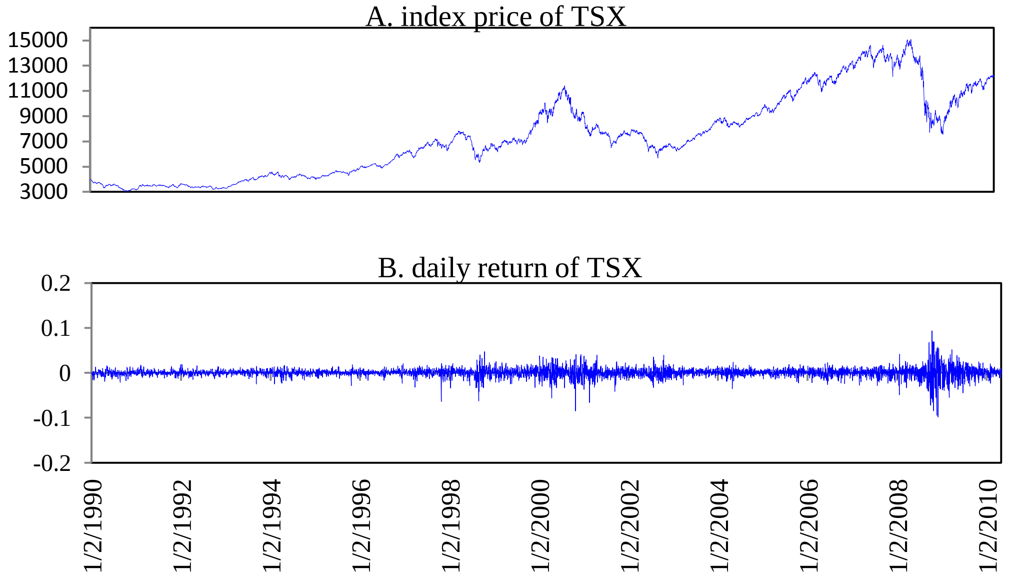


Figure 4.5 Daily index price and return of DAX

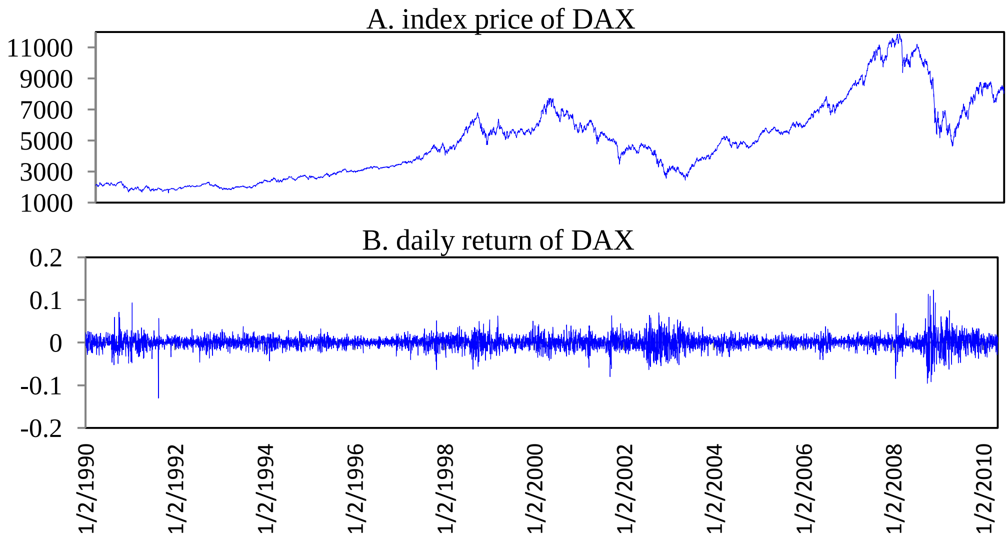
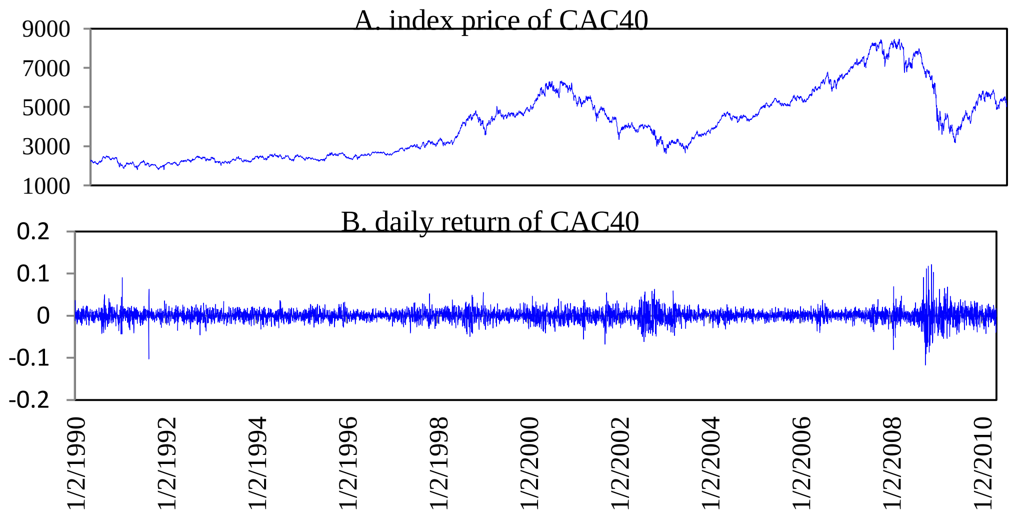


Figure 4.6 Daily index price and return of CAC 40



Summary

The patterns of the developed equity markets are presented in Figures 4.1 to 4.6. Generally speaking, the indices of the developed equity markets have consistent patterns, with the exception of the Nikkei 225. Excluding the Nikkei 225, the other five indices were moving significantly upwards from 1990 to 2000, and then sharply fell to their record low around the third quarter of 2002 because of the burst of the internet bubble. After climbing to a new high in 2007, the indices suddenly dropped to another new record low due to the financial crisis of the subordinated bonds happening around 2008 to 2009. The index of the Nikkei 225 has had a clear downturn trend since 1990 and it reached its global low towards the end of 2002 when the investors were widely disillusioned with the dot-com bubble. After that, it went up slowly until dramatically dropping again in the first quarter of 2008. From the viewpoint of the volatility pattern of daily returns, the Nikkei 225 and the CAC 40 have more volatile dynamics than other indices, and the TSX has relatively small volatility, except for the period of 2008 to 2009.

4.6.2 Six emerging equity markets in East Asia[69]

Hang Seng Index

The Hang Seng Index (HSI) is one of the earliest stock market indexes in Hong Kong, and was launched on 24 November 1969. Now the HSI comprises 45 individual stocks listed on the main board of the Stock Exchange of Hong Kong (SEHK). To reflect the market conditions properly, the components are selected from four sub-sectors: finance, utilities, properties, and commerce and industry, and are reviewed quarterly. A new company is eligible for selection into HSI if its market capitalisation and total turnover is among the top 90% of

[69] For information on the six emerging market indices please refer to their official website:
Hang Seng Index (http://www.hsi.com.hk/HSI-Net/), TAIEX (http://www.twse.com.tw/en/),
KOSPI (http://eng.krx.co.kr/m1/m1_4/m1_4_2/m1_4_2_1/UHPENG01004_02_01_01.html),
KLCI (http://www.bursamalaysia.com/website/bm/market_information/fbm_klci.html),
JCI (http://www.idx.co.id/Home/MarketInformation/MarketIndex/tabid/110/language/en-US/Default.aspx),
SET (http://www.set.or.th/en/products/index/setindex_p1.html).

all the primary shares listed on the SEHK. Then all of the eligible candidates will be finally reviewed based on their company performance, capitalization and turnover ranking, and the representation of the sub-sectors.

Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX)

The TAIEX is the most widely quoted of all the Taiwan Stock Exchange Corporation (TWSE) indices, which includes all of the common stocks listed on TWSE, excluding the full-delivery stocks and newly listed stocks less than one calendar month old. One of the special characteristics of the TAIEX (or of the individual stock) is the limitation of ±7% price fluctuation to the previous closing price during a trading day, thus only a maximum of 7% of price change is allowed at the market close. Since 2005, the first-five trading days of any new listing stock are not confined by this restriction. Before the 2nd January 1998, the stock market opened from 9:00 A.M. to 12:00 noon, Monday to Friday, and 9:00 to 11:00 A.M on Saturday. In order to have consistency with other sequences, the observations on Saturday are excluded.

Korea Composite Stock Price Index (KOSPI)

The Korea Composite Stock Price Index (KCSPI) was introduced in 1972 with a base index of 100 set to January 4 of the same year, which covers 35 constituent stocks selected from the stocks traded on the Primary Board of the Market. After a serious reform of this index, a new method of market capitalization based index (symbol: KOSPI) comprising all stocks listed on the Korean Exchange was introduced in 1983. The new KOSPI is currently the main representative for the performance of the Korean equity market. It was assigned a base index of 100 on the 4th January 1980.

FTSE Bursa Malaysia Kuala Lumpur Composite Index (KLCI)

The original stock market index was launched in 1970 and covered 30 industrial stocks. To better reflect the market condition and industrial structure, the KLSE index with 83 companies was introduced in 1986. Similarly, in 2009, Bursa Malaysia reformed the index as the FTSE Bursa Malaysia Kuala Lumpur Composite Index (FTSE BM KLCI or, for brevity, “KLCI”) which comprises the top 30 largest-cap shares listed on the Main Board of the Bursa Malaysia Exchange. The eligibility requirements are based on the free float share price and its liquidity. The index is biannually reviewed by the FTSE Bursa Malaysia Index Advisory board in June and December.

Jakarta Composite Index (JCI)

The first stock exchange of Indonesia was opened in 1912 by the Dutch government in Batavia. After that, the stock exchange was closed and reopened several times. Finally, a new stock exchange, named the Jakarta Stock Exchange (JSX), was started in 1977, and then was privatized in 1992. In September 2007, the Jakarta Stock Exchange merged with another exchange, the Surabaya Stock Exchange, to form the Indonesian Stock Exchange (IDX). The daily trading is divided into two sessions, session one is from 9:30 A.M to 12:00 noon and session two is from 1:30 P.M. to 4:00 P.M.

Stock Exchange of Thailand (SET) index

The stock market and Bangkok Stock Exchange (BSE) were initiated in the early 1960s. After a serious amendment in regulations and formation, the Securities Exchange of Thailand (SET) was founded and officially started trading on the 30th April 1975. The SET Composite Index is the main indicator representing the price movement for all common stocks traded on the main board of SET, except for those stocks suspended for more than one year.

Figure 4.7 Daily index price and return of HSI

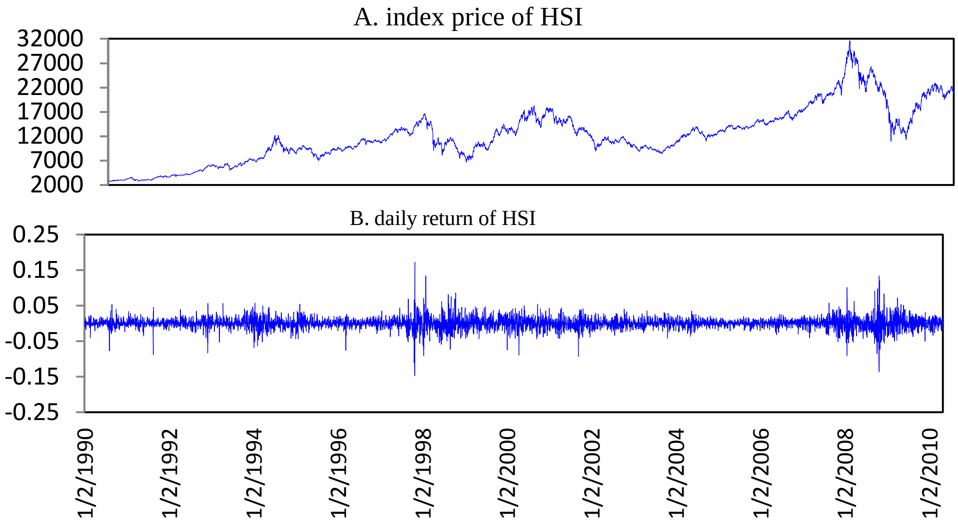


Figure 4.8 Daily index price and return of TAIEX

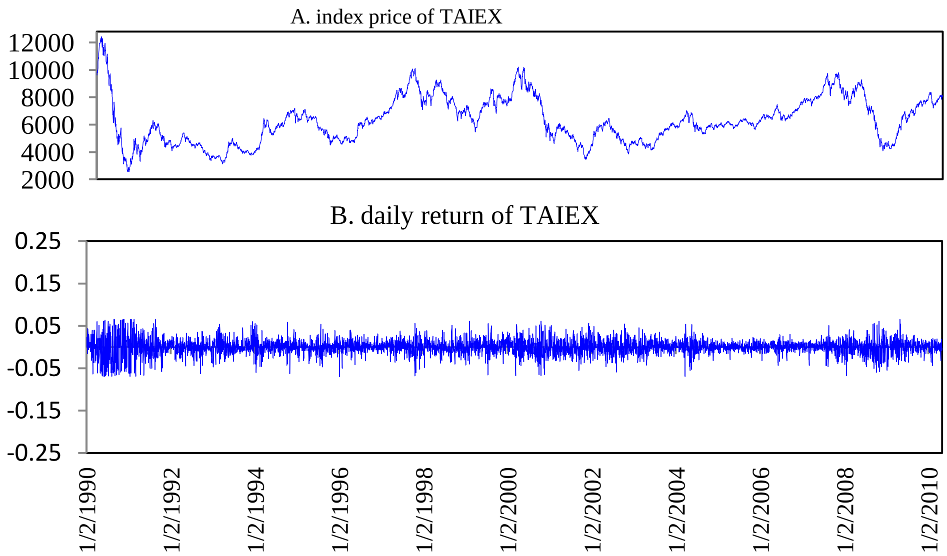


Figure 4.9 Daily index price and return of KOSPI

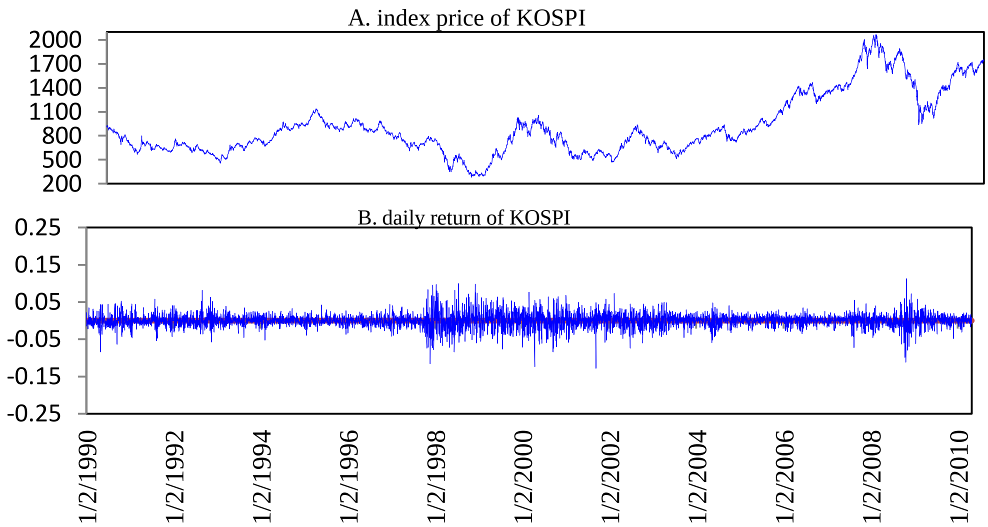


Figure 4.10 Daily index price and return of KLCI

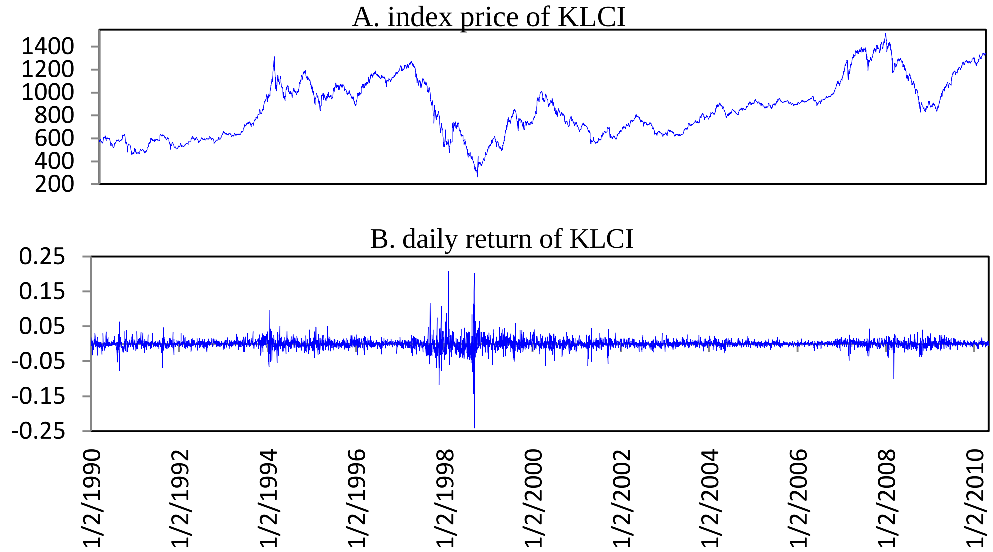


Figure 4.11 Daily index price and return of JCI

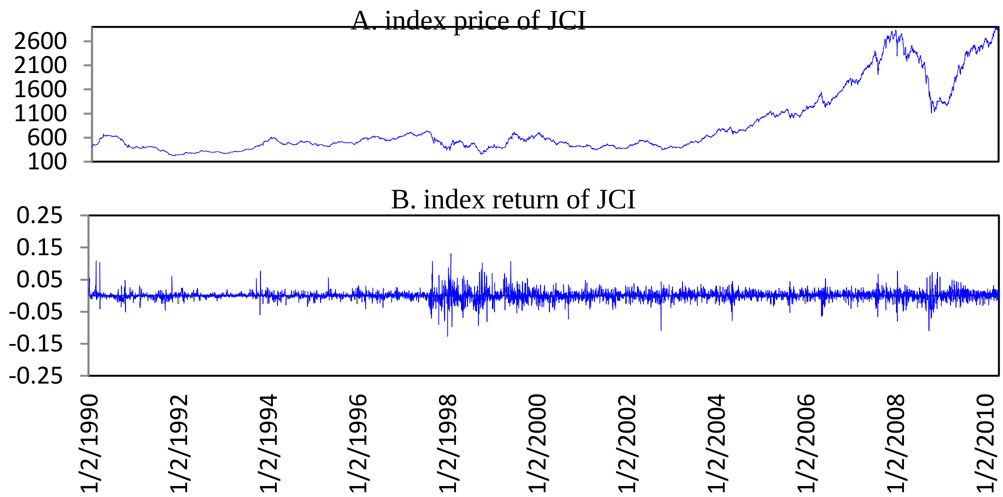
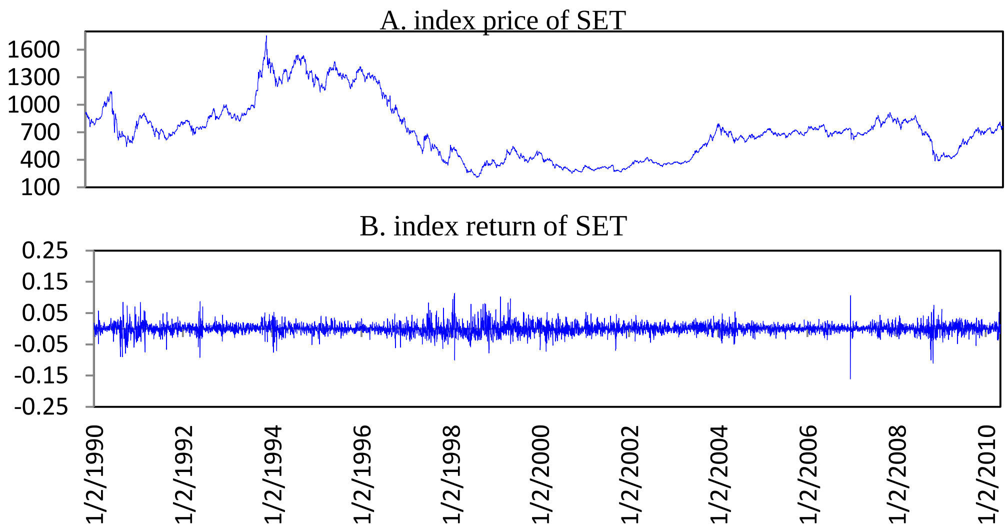


Figure 4.12 Daily index price and return of SET



Summary

The patterns of indices and volatility of the six emerging equity markets in East Asia are shown above in Figures 4.7 to 4.12. Most of them demonstrate the significant impact of the Asian financial crisis in 1997, with slightly less effect evident in the TAIEX, and then a sudden fall in 2008. From the aspect of the dynamics of volatility, the six indices have a similar pattern: high volatility around 1997 to 1998 and 2008 to 2009, covering the period of the financial crises. The volatilities of the HIS, TAIEX, and KOSPI are significantly larger than the ones of KLCI, JCI, and SET, even in the two periods of financial crisis. In principle, HIS and KOSPI have a similar pattern, depressed by the Asian financial crisis, they went on an upward trend to a historical high in 2007, and then sharply dropped in the first quarter of 2008 due to the storm of the credit crunch. A similar pattern happens in the SET and KLCI, however, the SET kept dragging during the downturn whereas the KLCI slightly moved up to its new record high at the end of 2007. The pattern of TAIEX is quite fluctuant, falling from a historical high in February 1990 to its historical low in October 1990. After that, it reached several regional peaks and then went down in the third quarter of 2008. In the Indonesian equity market, the JCI stayed at a low level from 1990 to 2004, although it dropped during the period of the Asian financial crisis. After 2004, JCI went on an escalating trend, peaking in the fourth quarter of 2007, and suddenly dropping down from 2008 to 2009.

4.6.3 Four emerging equity markets in Latin America [70]

Merval index

The Merval index includes the most important stocks traded on the Buenos Aires Stock

[70] Information about the four emerging market indices can be found on their official websites:
Merval Index (http://www.merval.sba.com.ar/Default.aspx),
Bovespa Index (http://www.bmfbovespa.com.br/en-us/markets/equities.aspx?idioma=en-us),
IGPA Index (http://www.bolsadesantiago.com/index.aspx),
Bolsa (http://www.bmv.com.mx/).

Exchange in the Argentina Republic. To date, 14 individual securities have been covered by the Merval index based on their market share, number of transactions and quotation price. The base of the Merval was set on the 30th June 1986. The corporations and weighted prices that compose the Merval are reviewed quarterly according to their market share during the previous period.

Bovespa index

The Brazilian stock market has a long history, which can be traced back to the St. Paulo stock exchange founded in 1890. It includes 50 stocks traded on the St. Paulo Stock, Mercantile & Futures Exchange (BM&FBOVESPA) and represents about 70% of the market capitalisation of the market value trade in the Bovespa. The index and its components are regularly reviewed every four months. A security will be added to the index if the trading value participation is higher than 0.1% of the total and the presence of the trading session covers more than 80%. On the other hand, a security might be excluded from the index if it no longer fits the inclusion criteria, or if the company files for bankruptcy.

IGPA index

The Indice General de Precios de Acciones (IGPA) is the main representative of the market performance of the Chilean stock market, and is a market capitalization weighted index measuring price variations of all the stocks listed on the Santiago Stock Exchange. This index is annually reviewed by the Committee of the Board, based on the frequency of trade and the sector representativeness. The index was developed with a base level of 100 on the 30th December 1980. The Chilean economy experienced a sharp crisis at the end of the 1990s, but it did not have any significant impact on the equity market. Since then, the stock market in Chile has experienced a long period of stability.

Bolsa index

The Bolsa index (sometimes called IPC, in Spanish, Índice de Precios y Cotizaciones) composes 35 individual stocks traded on the Mexican Stock Exchange (MSE), which is the second largest stock exchange in Latin America after the Brazilian BM&FBOVESPA. Now the Bolsa index is the main representative of the overall Mexican market performance. The selection of the component stock of the Bolsa index is based on two indicators. Firstly, whether the individual stocks come within the top 45 daily turnover ratio. Secondly, if the market cap value adjusted by floating shares is bigger than or equal to 0.1% of the total market cap value. These constituents are normally reviewed once a year, and currently there is no provision for any special review.

Figure 4.13 Daily index price and return of Merval

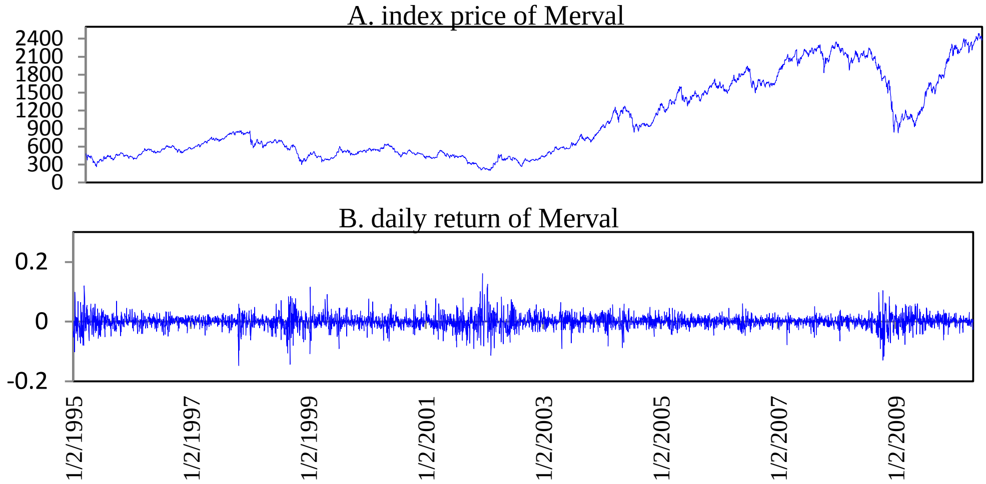


Figure 4.14 Daily index price and return of Bovespa

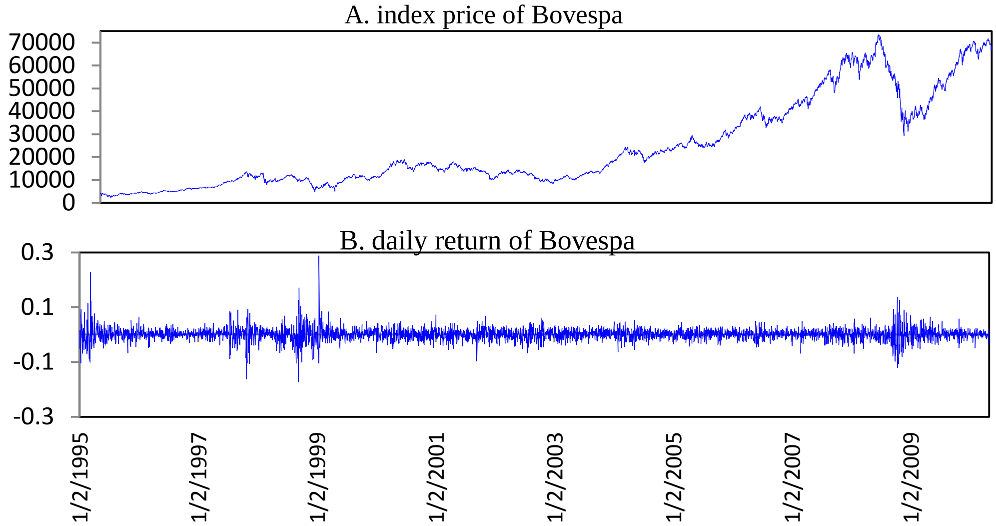


Figure 4.15 Daily index price and return of IGPA

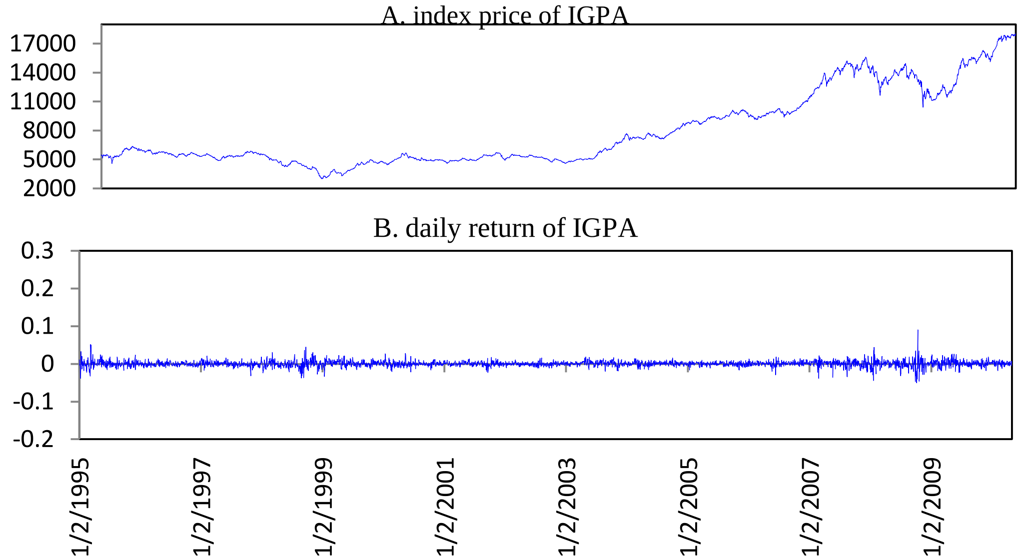
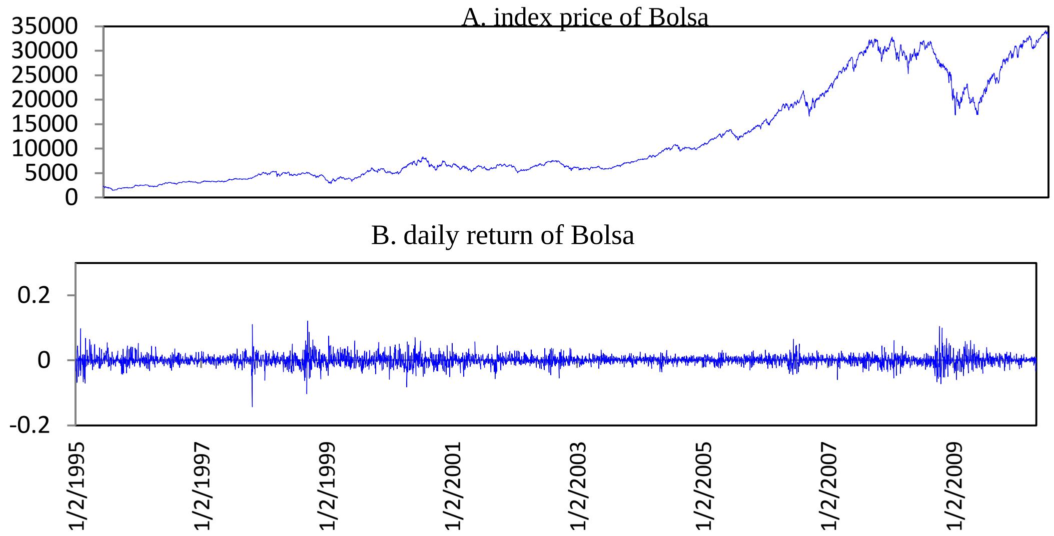


Figure 4.16 Daily index price and return of Bolsa



Summary

The patterns of index level and daily return of the four equity markets in Latin America are displayed above, Figures 4.13 to 4.16. The four indices had very similar patterns in the price level, which went up slowly before 2005, kept climbing to a peak in 2008, and then sharply fell in the fourth quarter of 2008. After that, the indices rocketed up to another historical high in the second quarter of 2010. From the fluctuation pattern of the indices, the Merval has the most volatile pattern due to the economic and currency crisis between 1999 and early 2003, and the credit crisis in 2008. In the volatility pattern of IGPA, it has the smallest movement of all the Latin-American emerging equity markets. The Bovespa has the second-largest volatility, likely as the result of several economic crises in early 1995, 1999, and late 2008. These series of crises had some critical impacts on the stock market, especially in the period of the crisis of subordinated debt.

4.6.4 Data preliminaries

The aim of this sub-section to view the sample sets used in this thesis as a whole, to understand the basic characteristics of the original return sequences. All samples mentioned in the previous section will be described in detail. Table 4.1 provides the basic statistics of each individual sequence from the index price and index return. The statistics in the fourth column are the four moments of each series of the sample, and the JB statistics are in the last column. The statistics in panel A show the indices' price and the corresponding moments present their basic features. In panel B, more specific information about the index returns is provided as well. Generally speaking, the mean return of most equity markets tends to be zero, excepting the IGPA (Chile) and the Bolsa (Mexico), and three (Nikkei225, TAIEX, and SET) out of the fourteen statistically zero-mean indices display a tendency with negative average return, although they are not significant. The equity market of Japan (Nikkei 225) has the smallest mean return, and this might be associated with the downturn of the Japanese economy over the past twenty years. For volatility, most indices demonstrate similar degrees of fluctuation, and overall the TSX and IGPA have the lowest volatility. In contrast, the Merval and the Bovespa have equally high variations of index return, which could be attributed to the several financial and economic crises happening in Argentina and Brazil. The developed equity markets tend to be left-skewed whereas the Latin-American emerging markets generally skew to the right. In the Asian emerging equity markets, the asymmetry of the distribution of these indices is quite obscure. The value in the column of kurtosis indicates that all the individual series are leptokurtic, implying the property of a fat-tail in the index return sequence. In the last column, the JB statistics consistently suggest that the distribution of each index does not follow the normal distribution; even the multivariate Q test for each market portfolio rejects the assumption of normality. In the left column of Figure 4.17, 4.18, and 4.19, the Q-Q plots also provide robust evidence supporting this suggestion.

The information in Table 4.2 shows the stationarity statistics based on the Dickey-Fuller test and the Phillips-Perron test. These two approaches provide similar results, suggesting that each return series is significantly stationary. Combined with the ACF of index returns in the right column of Figure 4.17, 4.18, and 4.19, it is obvious that most of the time series applied tend not to have autoregressive correlation. Although some of them are slightly autoregressive, for example JCI, KLCI, SET, and Bolsa, this is only in the lag one level. The autoregression of the IGPA index is a special case and lasts for the lag two level. Roughly speaking, the emerging equity markets have a stronger autoregressive effect than the developed equity markets, due to the issue of transparency in the market (Gelos and WEI, 2005, Lang and Maffett, 2011). Similar evidence can be found in comparing the HSI (TAIEX or KOSPI) with the other three indices of equity markets in Southeast Asia. This phenomenon might imply that more developed equity markets tend to reflect the information to the market more efficiently. According to the evidence above, the sequences of index return tend to be serially independent over time, and this hypothesis can support us in measuring the individual VaRs based on Eq. (2.47). In other words, the evidence above stands for the assumption of stationarity and independence, and offers a convenient tool for the estimation of VaR as well.

Table 4.1 Descriptive statistics

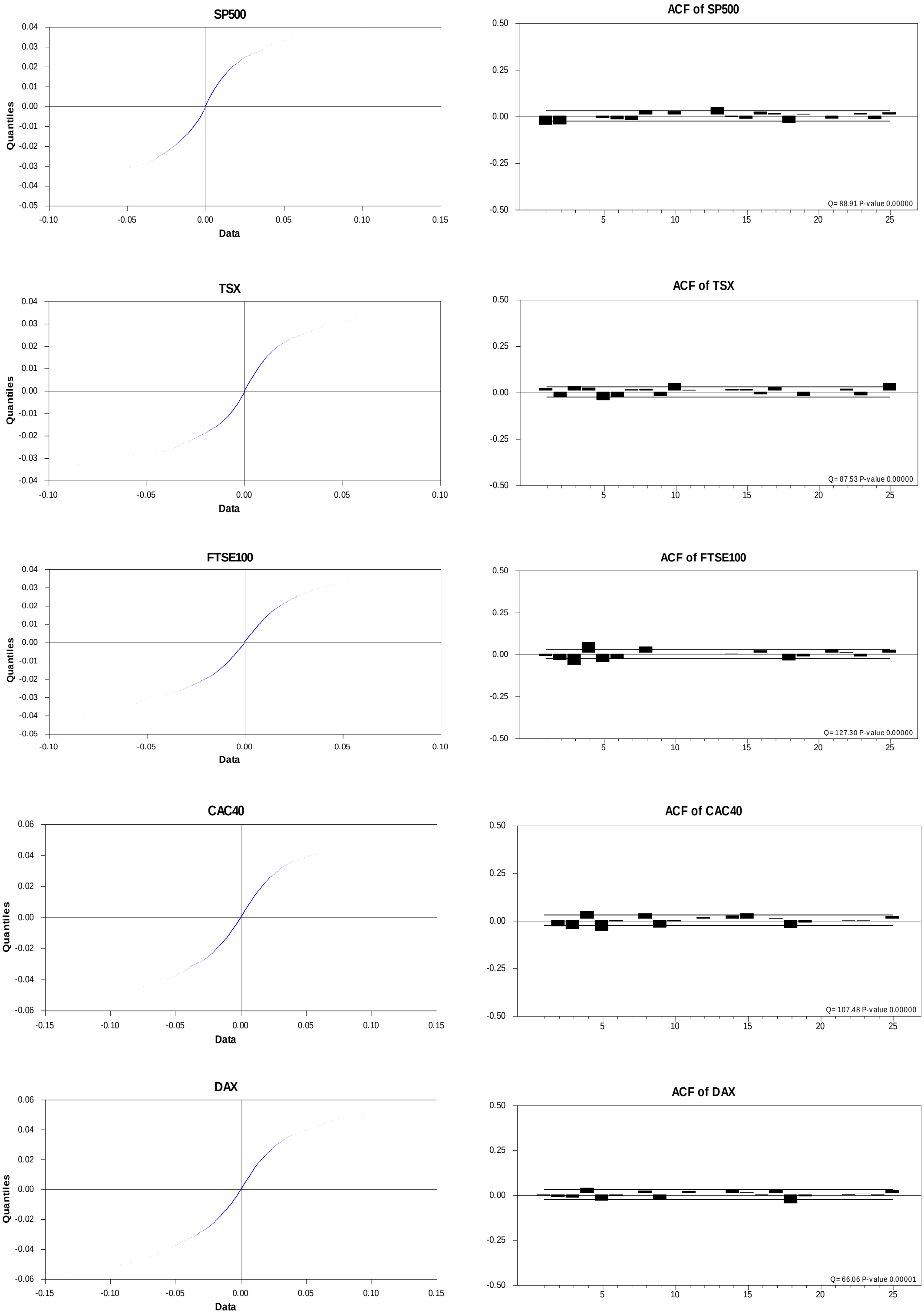
Panel A: Price level	Max.	Min.	Mean [a]	Std.	Skewness	Kurtosis	JB test
Developed equity market							
S&P500	1565.15	295.46	923.89	375.39	-0.2080	-1.3490	4369
FTSE 100	6930.20	1990.20	4499.49	1358.77	-0.1399	-1.2304	348
Nikkei 225	38915.87	7054.98	16253.64	5305.49	0.8893	1.5943	1249
TSX	15073.13	3009.90	7420.06	3196.37	0.4810	-0.7868	338
DAX	11862.87	7054.98	4862.91	2453.41	0.7631	-0.0689	511
CAC 40	8461.44	1798.59	4078.30	1689.33	0.6567	-0.4792	428
Asian emerging market							
HSI	31638.22	2736.55	12193.88	5517.50	0.5191	0.1933	245
TAIEX	12424.53	2560.47	6035.03	1655.42	0.4957	0.0476	217
KOSPI	2064.85	280.00	912.54	371.75	1.0528	0.4066	1013
KLCI	1516.22	262.70	857.54	248.33	0.3201	-0.7512	214
JCI	2971.25	223.25	836.71	663.74	1.5883	1.3259	2610
SET	1753.73	207.31	711.31	318.67	0.7416	-0.0471	485
Latin American emerging market							
Merval	2487.76	200.86	1007.98	643.91	0.7452	-0.8886	495
Bovespa	73516.00	2138.20	24038.62	19053.43	1.0245	-0.1996	495
IGPA	18039.09	2973.92	7917.99	3817.08	0.9472	-0.3882	614
Bolsa	34134.23	1447.52	11984.16	9672.04	0.9342	-0.6148	696
Panel B: Return level	Max.	Min.	Mean [b]	Std.	Skewness	Kurtosis	JB test [c]
Developed equity market							
S&P500	0.1096	-0.0947	0.0227	0.0115	-0.2062	9.4817	19710
FTSE 100	0.0938	-0.0926	0.0159	0.0113	-0.1191	6.7795	10070
Nikkei 225	0.1323	-0.1211	-0.0237	0.0154	-0.0240	5.6957	7099
TSX	0.0937	-0.0979	0.0211	0.0104	-0.7751	11.6599	30276
DAX	0.1237	-0.1306	0.0262	0.0154	-0.1310	6.0992	8155
CAC 40	0.1214	-0.1174	0.0158	0.0146	-0.0432	6.9774	10655
Asian emerging market							
HSI	0.1725	-0.1473	0.0380*	0.0168	0.0123	9.6887	20679
TAIEX	0.0655	-0.0698	-0.0035	0.0182	-0.1005	3.6862	3002
KOSPI	0.1128	-0.1280	0.0123	0.0185	-0.1236	4.4899	4454
KLCI	0.2082	-0.2415	0.0162	0.0142	0.3787	44.1551	429622
JCI	0.1313	-0.1273	0.0380**	0.0153	0.0099	9.6229	20399
SET	0.1135	-0.1606	-0.0031	0.0171	-0.0132	6.5792	9353
Latin American emerging market							
Merval	0.1611	-0.1476	0.0406	0.0225	-0.1934	5.1691	4416
Bovespa	0.2882	-0.1723	0.0697**	0.0234	0.4762	12.8014	27086
IGPA	0.0906	-0.0502	0.0305**	0.0079	0.1349	9.3372	14342
Bolsa	0.1215	-0.1431	0.0667**	0.0165	0.0741	6.0888	6097

a. The significance tests of mean in price level are not made in this table, since the purpose of the information in panel A is to provide an overview and a brief understanding of each equity index. *(**) means the variable is significant at 5% (1%) level.

b. The mean of daily return in panel B is multiplied by 100.

c. The JB statistics of mean are significant at the 1% level. The multivariate Q tests (MV-Q) proposed by Hosking (1980) to these three portfolios are also done but not presented in this table. The assumption of normality is significantly rejected across the portfolios by the statistics of MV-Q.

Figure 4.17 Q-Q plot and ACF of the developed equity markets



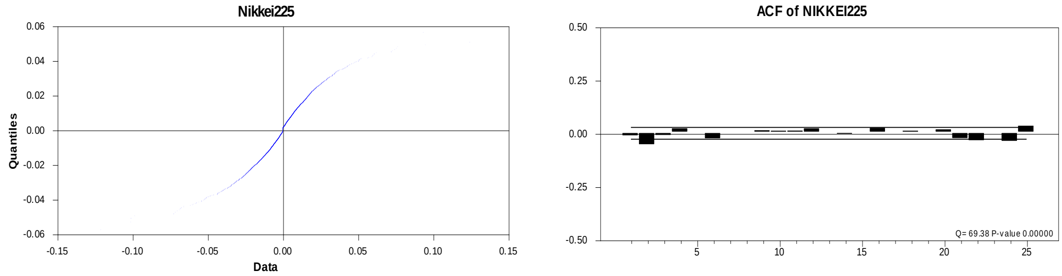
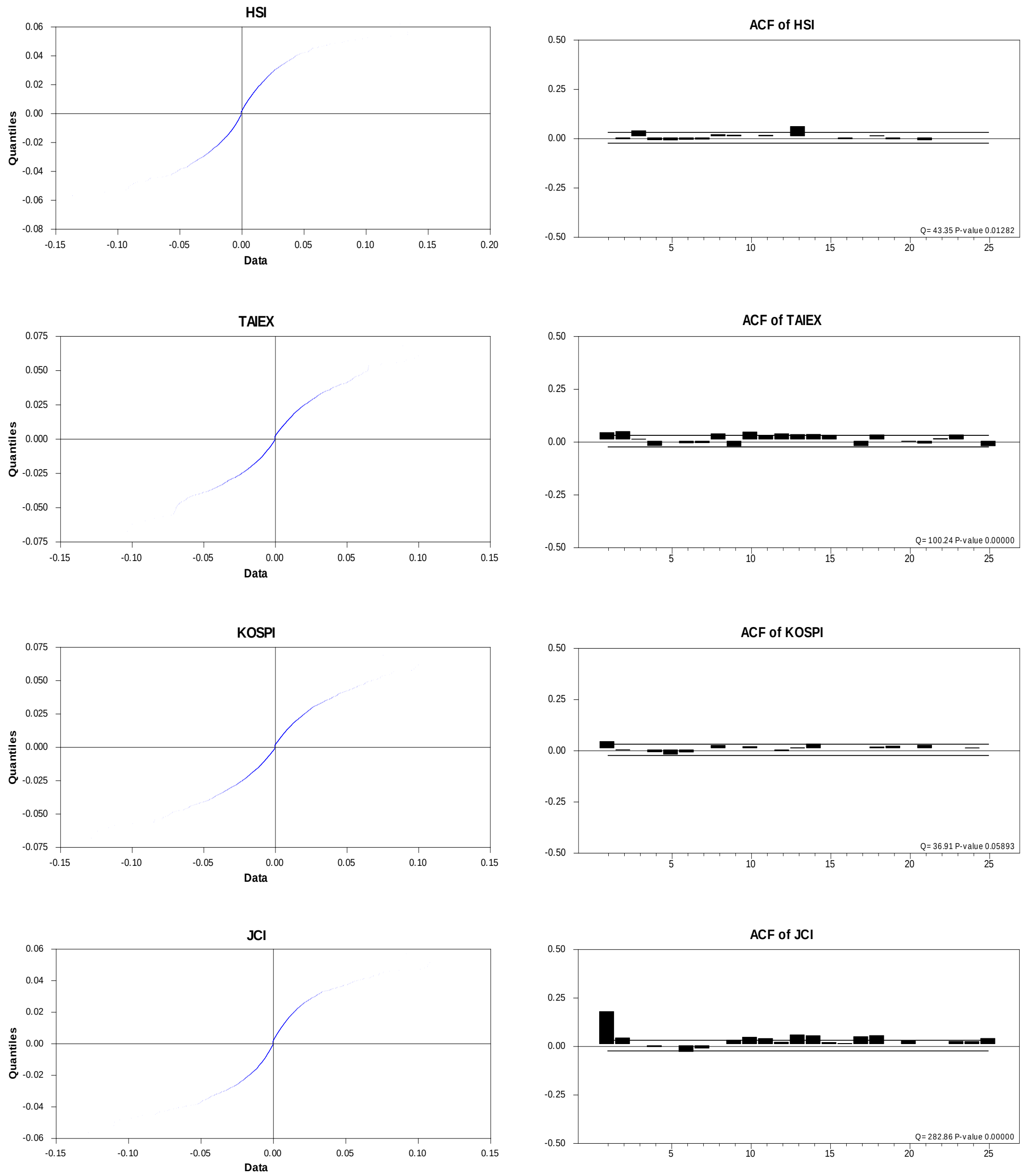
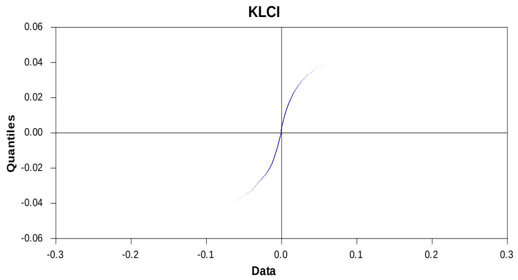
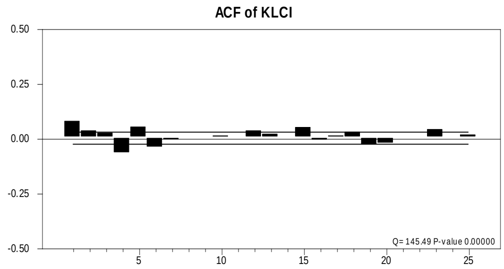
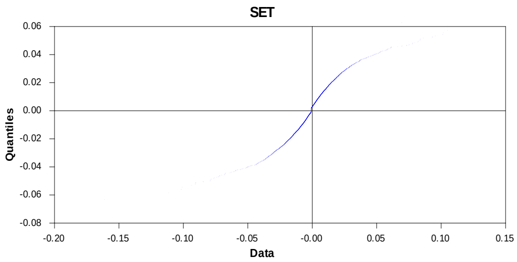


Figure 4.18 Q-Q plot and ACF of the Asian emerging equity markets









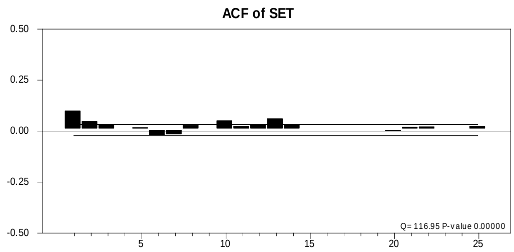
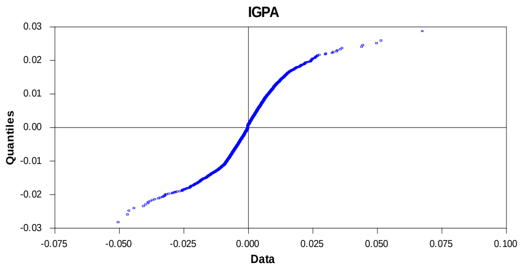
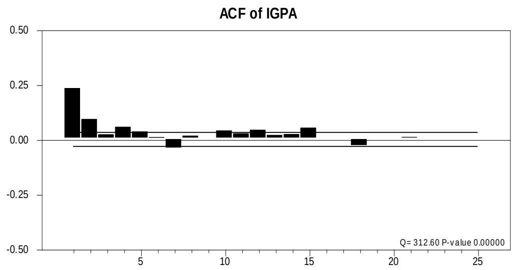
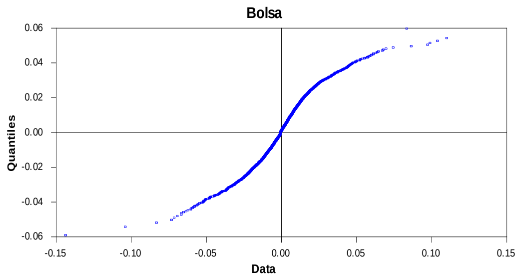
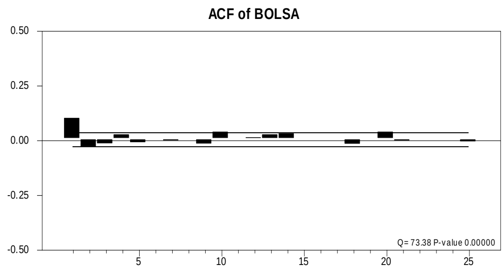


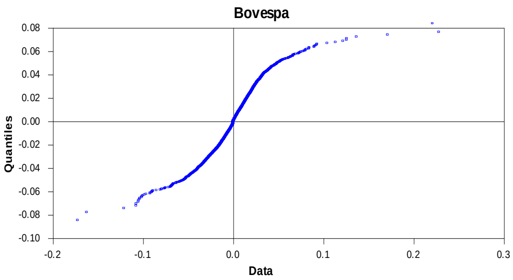
Figure 4.19 Q-Q plot and ACF of the Latin-American emerging equity markets

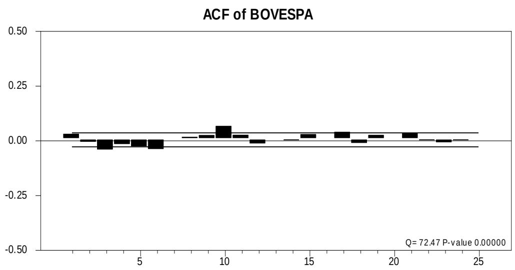


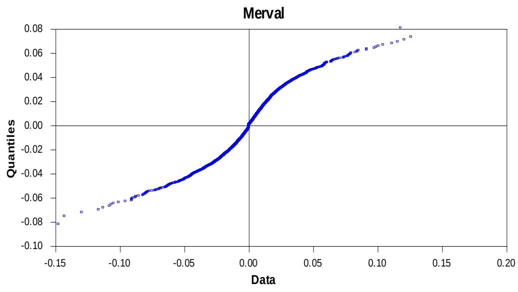












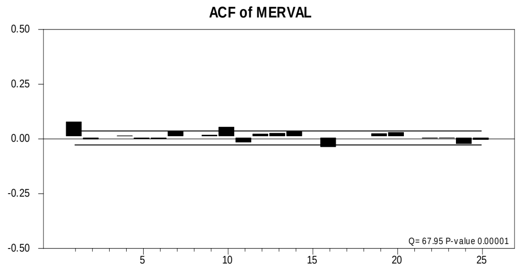


Table 4.2 Stationarity test of index returns

	Dickey-Fuller test	Phillips-Perron test
Developed equity market		
S&P500	-76.5488	-76.7677
FTSE 100	-73.9064	-74.0593
Nikkei 225	-73.7259	-73.8508
TSX	-71.0308	-71.0379
DAX	-73.3267	-73.3625
CAC 40	-72.6398	-72.7094
Asian emerging market		
HSI	-72.4791	-72.4933
TAIEX	-69.7824	-69.8772
KOSPI	-69.7862	-69.7687
KLCI	-67.2462	-67.3097
JCI	-60.8525	-60.8682
SET	-66.0716	-66.2565
Latin American emerging market		
Merval	-58.3342	-58.3143
Bovespa	-61.1877	-61.1884
IGPA	-49.5367	-49.8062
Bolsa	-56.9016	-56.7781

Note: The critical values of the Dickey-Fuller test and the Phillips-Perron test in 1% and 5% are -3.435 and -2.863, respectively.

4.7 Conclusion

This chapter included the methods of estimation and forecasting, the backtesting procedure, and the data samples used in this thesis. Four well-known and widely used VaR models chosen as the competing models were also introduced. They covered both the parametric and non-parametric approaches. In Section 4.2, some fundamental econometric tests applied to realize the characteristics of the equity indices were explained.

Section 4.3 explained the method of parameter estimation and the one-day forward VaR forecast which includes the method of maximum likelihood function utilized in estimating the parameters in the generalized extreme value distribution. Another task of this section was the description of dynamic conditional correlation. To reflect the market condition and risk in the tail-area appropriately, a generalized dynamic conditional correlation based on the selected extremes is used to estimate the relationship between various asset returns. This approach stresses the conditional correlation in the extreme event rather than the entire sequence. The concept of the VaR forecast is accounted for in this section as well.

In Section 4.4, we briefly outlined the alternative VaR methods which are used to compare with the one proposed in this thesis. They are the GARCH(1,1) model, RiskMetrics, multivariate stochastic volatility, and historical simulation. The first three models are based on the parametric approach and the method of historical simulation is non-parametric. All the VaRs forecasted by these approaches are compared with the semi-parametric approach proposed by this thesis.

In Section 4.5, the procedure of backtesting is elaborated upon. Based on Kupiec (1995) and Christoffersen's (1998) method, the exceedance function is firstly carried out and then the violation ratio will be achieved. In addition, Christoffersen's (1998) independence test will also be applied in the performance comparison.

Section 4.6 described the data sets used in this thesis, and the general descriptive summary was presented and discussed. Some graphic (QQ plot and ACF) analysis was also provided. According to the evidence of the Q-Q plots, some of the observations of the return sequences fall into the extreme area, indicating that it is appropriate to apply extreme value theory to fit the tail-distribution. In addition, Several ACF of the indices indicate that they are weakly dependent but most of the data sets are stationary (according to the results of the two stationarity tests) and tend to be independent. Roughly speaking, the developed equity markets tend to be independent; in contrast the Latin-American emerging equity markets have some autoregressive effects in the lag one level.

Chapter 5 Results and Findings

5.1 Introduction

In this chapter, the results and findings of the empirical thesis will be presented in three subsections; starting with the univariate analysis, then through the discussion of correlations based on extreme values, and ending in the multivariate VaR analysis. In Section 5.2, the market risk of each equity index is measured according to its basic approach without any assumption about the correlation amongst various equity indices. In Section 5.3, the extremal correlations, or so-called tail correlations, are presented and analysed. Substantial evidence will show that the distribution of financial returns is asymmetric and thick.

In Section 5.4, the results of the multivariate VaR forecasts and the outcomes of quality checks are presented and discussed. In this chapter, Kupiec's (1995) unconditional coverage test and Christoffersen's (1998) conditional coverage test are applied to backtest the VaR forecasts of the GEV-DCC model and the four competing models. The quality of VaR models are also provided with three quality measures including adaptability, conservativeness and magnitude of violation. The backtesting results of the portfolio VaR show that the GEV-DCC model offers an accurate coverage in general. In the quality of portfolio VaR patterns, the GEV-DCC model demonstrates the stable VaR sequences and produces satisfactory results in the conservativeness and magnitude of violation. Section 5.5 marks the implications of the results by offering a simple illustration. Section 5.6 presents the conclusion.

5.2 Univariate Analysis

This section explains and presents the procedure of estimation, the results of the univariate VaR, and analyses the backtesting performance for each return series. Two main coverage tests will be applied in evaluating the performance of each VaR model, but the final decision will be based on Christoffersen's (1998) approach due to the extra independence test. In addition, three other benchmarks will also be used to check the quality and suitability of these VaR models.

5.2.1 Individual VaR based on Extreme Value Theory

5.2.1.1 Estimation of the tail distribution

As mentioned in Section 2.5.1 and 3.4.1, Jenkinson's (1955) generalized distribution based on the block maxima with three parameters is used to fit the tail-distribution. As noted in section 4.5, a rolling procedure is adopted to forecast the VaR of the next period. Thus, each rolling period would produce three parameters for GEV distribution. The mean of the estimated parameters (and their standard deviations in parentheses) are displayed in Table 5.1, including three different time block spans: one week (5 days), two weeks (10 days), and one month (22 days). The parameters of both the long and short positions are investigated as well. The results from Table 5.1 indicate that more than 99% of the scale and location parameters of the long and short positions of all equity indices are significant at the 1% level. In addition, most of the tail parameters are significant at either the 1% or the 5% level. These evidences imply that the estimated parameters fit the GEV distribution appropriately[71]. According to the backtesting

[71] Although ACF in Figures 4.17 to 4.19 show that autocorrelation in most of equity indices is indistinct and weak, Eq. (2.49) and (2.50) are estimated for examining the extremal index. An alternative approach proposed by Embrechts, Kluppelberg and Mikosch (1997) is also employed to estimate the extremal index, θ. The asymptotical estimate of the extremal index can be formulated as

$$\theta = \frac{1}{n}\frac{\ln\left(1 - {}^{K_u}/_{g}\right)}{\ln\left(1 - {}^{N_u}/_{gn}\right)}$$

where Nu is the number of returns exceeding a certain high threshold, Ku is the number of blocks in which this period, from 2nd January 2007 to 30th April 2010, there are in total about 850 observations to each parameter. It is reasonable that location parameters of minimum extremes are negative, and the ones of maximum extremes are positive. The evidence of the standard deviations show that the estimated parameters are stable over time, which is consistent with the evidence from the patterns of the daily parameters shown in Figure C-1 to Figure C-16 in Appendix C. In these figures, most of the parameters vary within a small range, but several parameters are unexpectedly large or small due to the issue of convergence. In addition, the distributional stabilities of the parameters are also investigated based on two types of stationarity tests in Table C-1 in Appendix C. The results indicate that most of the parameter distributions are stable.

Overall, the three parameters from larger blocks tend to be larger. Take the long position for example, the location parameter spans from -0.0036 (IGPA) to -0.0133 (Bovespa) for n=5 and from -0.0082 (IGPA) to -0.0297 (Merval) for n=22 in Table 5.1. Similar results can be found in the panel of the short position. The scale parameter still goes slightly up by the length of the block even though it does not rise significantly. Overall, the scale and location parameters, both of the long and short position, of the Bovespa (Brazil) and Merval (Argentina) across all blocks are larger (in absolute value) than the other equity indices, which suggests that the market risks in these two equity markets are larger. On the other hand, the results also suggest that the tail parameters are more likely to be negative, which further implies that the extreme value distribution of these return series is potentially suitable for the Fréchet distribution. This finding is generally consistent with previous research in this area, such as Danielsson and De Vries (1997), Longin (1996), and McNeil (1998). The Fréchet distribution corresponds to a thick process of financial returns, thus, the characteristic of the fat-tail in the data sets can be

threshold is violated, and g and n are the number of blocks in the sequence and the length of the blocks. The results of these two approaches are mainly consistent with the ACF, indicating most of the return sequences are very slight auto-correlative. Overall, the estimated θ is between 0.93 and 0.97, thus, the weak auto-correlation effect in return sequences is reasonably neglected in this thesis.

captured appropriately. As shown in Figure 5.1, the magnitude of fatness is positively related to the absolute value of the tail parameter, based on the zero mean and unit scale parameter. The red dashed curve with K_n= -0.6 has the thickest tail, and the one with -0.2 has a thinner tail. As shown in panel A of the long position, the tail parameters are between -0.0477 (DAX) and -0.1985 (KLCI) for the 5-day block and between -0.0709 (Nikkei 225) to -0.4541 (JCI) for the 22-day block in panel C, indicating that the asymptotic distribution of extremes tends to have thicker tails across the blocks. The results in the pattern of estimated parameters and their indication of extreme distribution of daily returns as shown above are generally consistent with Longin’s (2000) results. Furthermore, the results in Table 5.1 are more robust compared with Longin (2000), because they include the mean values from the backtesting period rather a single result at a particular date.

Previous research in related areas has provided some robust evidence to suggest that the distributions of financial asset returns are skewed, mainly caused by information asymmetry and investors’ preference (Post, Van Vliet and Levy, 2008). However, less attention has been paid to the distribution of extreme values. Thus, it is essential and critical to test if a difference exists between the left and right tails. The results of the equality test of estimated parameters for the left and right tails are shown in Table 5.2. The first and second columns describe the equality test[72] of the tendency of dispersion and centralization between the left- and right-tail, respectively, and the same tests of the tail index are displayed in the last column. It is natural that the location parameters in the left (minimum returns) and right tails (maximum returns) are significantly different. Most results show that the scale and tail parameters of the long positions are significantly different from the parameters in the short positions, indicating the shapes between the left-tail and right-tail of GEV distributions are generally different. Although some of the equality tests are not rejected, such as the scale

[72] The main objective of Table 5.2 is to test the equality of estimated parameters in the long and short positions. The hypothesis are: H_0: $c'_n = c_n$ for the scale parameter, and H_0: $k'_n = k_n$ for the tail parameter.

parameters of the Nikkei225 and Merval in panel A, the KOSPI in panel B, and the Bolsa and TAIEX in panel C, none of these exceptions goes across all blocks. Thus, the majority of results in Table 5.2 clearly imply that market participants have different behaviours in the left and right tail. One possible explanation for this is that investors respond differently to bad and good news (see Braun, Nelson and Sunier (1995)).

Based on the Basel rules, the banks are required to expose the potential firm-wide risk for the next 10-day by 1% probability. Thus, 10-day returns of each index are created to perform extreme value distribution and to measure 10-day VaRs. The results of the estimated parameters and the standard deviations with 10-day returns are shown in Table 5.3. The patterns of the parameters are shown in Figure C-17 to C-32 in Appendix C. The results of the standard deviation and the figures show that the parameters of the 10-day return are stable, although they are larger in the larger blocks. Similar to the results of daily returns, the distributional stabilities of the parameters are tested by stationarity tests as shown in Table C-2 in Appendix C. Overall, the estimated parameters of 10-day returns are reasonably stable on average.

In general, more than 99% of observations of the scale parameters in the left and right tails of all equity markets are significant in 1%. Consistent with the daily results in Table 5.1, the scale parameters of the Bovespa and Merval are larger on average. However, the scale and tail parameters in panel A of Table 5.3 are systemically larger than the parameters in Table 5.1. For example, the average locations of long positions in Table 5.3 are 0.0417, 0.0373, and 0.0361 for the 5-, 10-, and 22-day block respectively, against 0.0086, 0.0087, and 0.0095 in Table 5.1. The results of lower frequency returns provide some interesting new information. Firstly, the scale parameters in panel A (n=5) are the highest. Secondly, under the 10-day frequency, the scale parameters tend to be higher than the ones under the daily pattern. These two outcomes suggest that the VaR based on 10-day returns might be higher than the daily ones due to its larger scale parameters. In the tail parameters, the estimated tail in the panel of n=5 and n=10 (for the short position only) are more significant than the case of n=22; and the significance in the short positions is clearer than in the long positions. One interesting result is that some tail parameters (k_n) tend to be positive, such as the long position in panel C, although they are not statistically significant. As mentioned in section 2.5.1, the null hypothesis (i.e. K_n=0) of the tail parameter cannot be rejected, implying that in some cases the 10-day based extreme value distribution could fit the Gumbel distribution. Overall, an extreme value distribution with negative tail parameter is still the most appropriate one for 10-day returns. With the location parameter, most results in n=10 and n=22 are significant at 1%, but unexpectedly not significant in the case of n=5 (although the majority of the developed equity indices in the short position, and a few indices of Latin American markets are significant at 1%). This phenomenon indicates that the average extreme returns of the 10-day return series in panel A (n=5) tend to be zero, particularly for the Asian and Latin American equity markets. The results of the location parameters in Table 5.3 tend to be negative in long positions and positive in short positions, but most results in the case of n=5 of both the long and short panels are not substantially significant to reject the null hypothesis.

Comparing the results of extreme value distribution with daily and 10-day returns as shown in Table 5.1 and Table 5.3, it is natural that the low frequency extreme returns (10-day returns) are more volatile than high frequency ones (daily returns). For example, the scale parameters are between 0.0044 (IGPA) and 0.0124 (Bovespa) for the long position with n=5 in panel A of Table 5.1, corresponding to the one from 0.0247 (TSX) to 0.0622 (Merval) in Table 5.3. Besides, the tail parameters in Table 5.3 are systematically smaller than the ones in Table 5.1. As we know that the function of the tail parameter describes the thickness of extreme distribution (as shown in Table 5.1), this implies that the extreme distribution based on 10-day returns has a thicker tail than the distribution obtained from daily returns.

Figure 5.1 Generalized extreme value distribution with different tail parameters

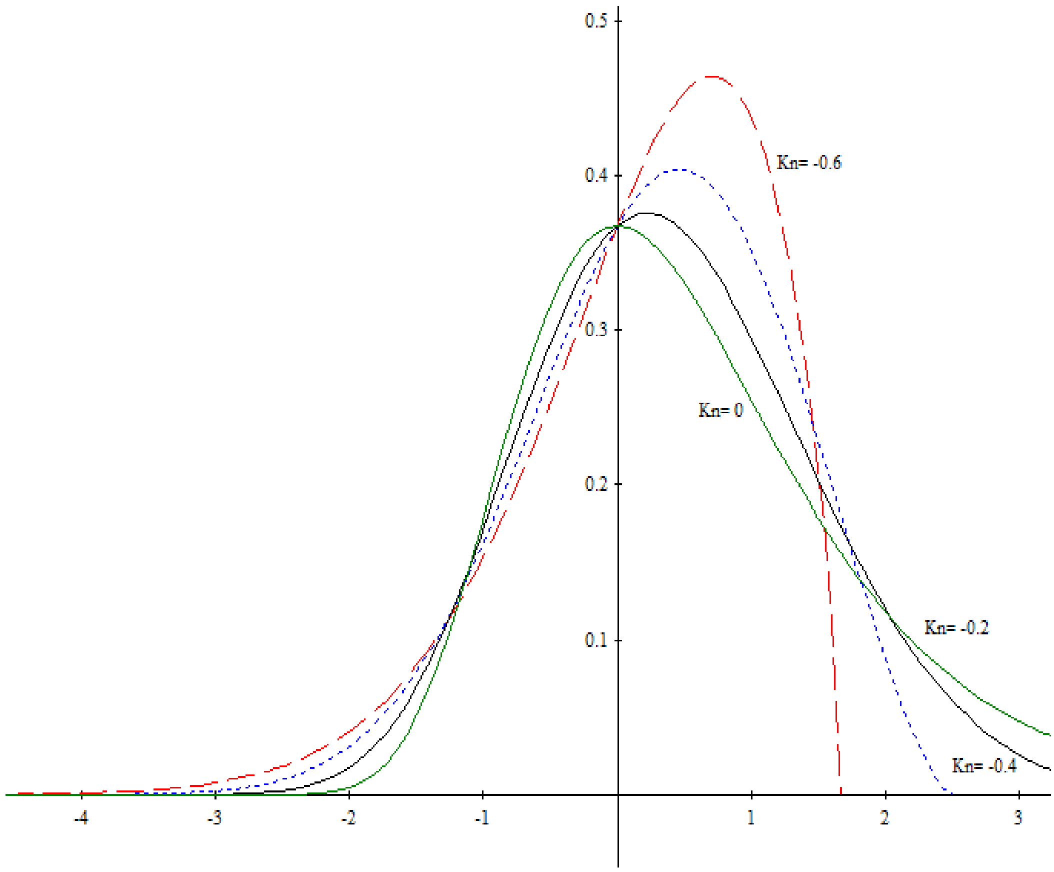


Table 5.1 The mean of estimated parameters of GEV distribution with the 5-day block and daily returns

	Long position			Short position		
Panel A:n=5	Scale parameter (c'_n)	Location parameter (d'_n)	Tail parameter (k'_n)	Scale parameter (c_n)	Location parameter (d_n)	Tail parameter (k_n)
S&P500	0.0058**	-0.0064**	-0.1746**	0.0058**	0.0076**	-0.1088**
	(0.0004)	(0.0036)	(0.0757)	(0.0005)	(0.0034)	(0.0626)
FTSE100	0.0055**	-0.0052**	-0.1777**	0.0052**	0.0065**	-0.0494
	(0.0005)	(0.0042)	(0.0698)	(0.0003)	(0.0029)	(0.0584)
CAC40	0.0062**	-0.0073**	-0.0965	0.0060**	0.0078**	-0.0616
	(0.0006)	(0.0054)	(0.0933)	(0.0003)	(0.0031)	(0.0648)
DAX	0.0081**	-0.0095**	-0.0447**	0.0079**	0.0107**	-0.0085
	(0.0004)	(0.0054)	(0.0603)	(0.0005)	(0.0065)	(0.0734)
TSX	0.0084**	-0.0097**	-0.0899**	0.0081**	0.0108**	-0.0347
	(0.0004)	(0.0069)	(0.0702)	(0.0006)	(0.0031)	(0.0528)
Nikkei225	0.0086**	-0.0104**	-0.0800	0.0086**	0.0111**	-0.0416
	(0.0003)	(0.0057)	(0.0632)	(0.0006)	(0.0063)	(0.0640)
IGPA	0.0044**	-0.0036**	-0.0853	0.0048**	0.0046**	0.0002
	(0.0003)	(0.0012)	(0.0511)	(0.0005)	(0.0002)	(0.0276)
Bolsa	0.0093**	-0.0096**	-0.0877	0.0098**	0.0117**	-0.0547
	(0.0006)	(0.0067)	(0.0608)	(0.0007)	(0.0057)	(0.0589)
Bovespa	0.0124**	-0.0133**	-0.1243**	0.0130**	0.0167**	-0.0391
	(0.0005)	(0.0006)	(0.0194)	(0.0015)	(0.0074)	(0.0419)
Merval	0.0130**	-0.0126**	-0.1567**	0.0130**	0.0147**	-0.0787
	(0.0008)	(0.0005)	(0.0443)	(0.0009)	(0.0050)	(0.0465)
HSI	0.0089**	-0.0093**	-0.1828**	0.0094**	0.0112**	-0.0882**
	(0.0008)	(0.0076)	(0.0642)	(0.0007)	(0.0063)	(0.0468)
TAIEX	0.0099**	-0.0103**	-0.0992	0.0102**	0.0117**	-0.0294
	(0.0006)	(0.0038)	(0.0698)	(0.0009)	(0.0044)	(0.0574)
KOSPI	0.0113**	-0.0118**	-0.1049**	0.0110**	0.0128**	-0.0810**
	(0.0004)	(0.0087)	(0.0755)	(0.0003)	(0.0066)	(0.0754)
KLCI	0.0085**	-0.0068**	-0.1985**	0.0089**	0.0084**	-0.1220**
	(0.0004)	(0.0031)	(0.0307)	(0.0007)	(0.0037)	(0.0551)
JCI	0.0074**	-0.0066**	-0.1551**	0.0079**	0.0072**	-0.1257
	(0.0011)	(0.0091)	(0.0848)	(0.0005)	(0.0058)	(0.0413)
SET	0.0095**	-0.0104**	-0.1029**	0.0103**	0.0110**	-0.0626*
	(0.0007)	(0.0079)	(0.0555)	(0.0003)	(0.0087)	(0.0880)
Panel B:n=10	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	0.0062**	-0.0096**	-0.1957**	0.0055**	0.0109**	-0.2097**
	(0.0004)	(0.0031)	(0.0660)	(0.0003)	(0.0016)	(0.0663)
FTSE100	0.0057**	-0.0081**	-0.2557**	0.0048**	0.0091**	-0.1547**
	(0.0005)	(0.0045)	(0.0745)	(0.0003)	(0.0026)	(0.0615)
CAC40	0.0059**	-0.0100**	-0.2035*	0.0054**	0.0108**	-0.1968*
	(0.0004)	(0.0035)	(0.0577)	(0.0003)	(0.0025)	(0.0526)
DAX	0.0076**	-0.0138**	-0.1133**	0.0066**	0.0146**	-0.1599**
	(0.0003)	(0.0031)	(0.0465)	(0.0003)	(0.0035)	(0.0768)
TSX	0.0082**	-0.0140**	-0.1551	0.0074**	0.0147**	-0.1742**
	(0.0003)	(0.0035)	(0.0526)	(0.0004)	(0.0020)	(0.0462)
Nikkei225	0.0091**	-0.0154**	-0.0609	0.0081**	0.0157**	-0.1113*
	(0.0002)	(0.0032)	(0.0440)	(0.0002)	(0.0002)	(0.0224)
IGPA	0.0042**	-0.0058**	-0.1743*	0.0043**	0.0071**	-0.0780
	(0.0003)	(0.0002)	(0.0476)	(0.0002)	(0.0002)	(0.0364)
Bolsa	0.0089**	-0.0143**	-0.1894*	0.0089**	0.0166**	-0.1964**
	(0.0005)	(0.0006)	(0.0211)	(0.0004)	(0.0032)	(0.0435)
Bovespa	0.0125**	-0.0206**	-0.1624**	0.0109**	0.0233**	-0.1650**
	(0.0004)	(0.0044)	(0.0337)	(0.0004)	(0.0005)	(0.0185)
Merval	0.0139**	-0.0198**	-0.2103**	0.0121**	0.0215**	-0.2282**
	(0.0005)	(0.0006)	(0.0183)	(0.0006)	(0.0040)	(0.0418)
HSI	0.0094**	-0.0146**	-0.2049**	0.0091**	0.0163**	-0.1795**
	(0.0007)	(0.0077)	(0.0690)	(0.0004)	(0.0005)	(0.0234)
TAIEX	0.0105**	-0.0160**	-0.1178	0.0098**	0.0179**	-0.0758
	(0.0004)	(0.0041)	(0.0633)	(0.0005)	(0.0035)	(0.0546)
KOSPI	0.0116**	-0.0174**	-0.1638**	0.0107**	0.0190**	-0.1617**

	(0.0004)	(0.0052)	(0.0526)	(0.0003)	(0.0058)	(0.0719)
KLCI	0.0092**	-0.0113**	-0.2767**	0.0092**	0.0133**	-0.2174**
	(0.0006)	(0.0050)	(0.0598)	(0.0003)	(0.0050)	(0.0541)
JCI	0.0068**	-0.0094**	-0.3354**	0.0073**	0.0107**	-0.3138**
	(0.0001)	(0.0001)	(0.0107)	(0.0002)	(0.0001)	(0.0236)
SET	0.0096**	-0.0151**	-0.1720**	0.0101**	0.0164**	-0.1561**
	(0.0004)	(0.0040)	(0.0339)	(0.0003)	(0.0044)	(0.0490)

Panel B:n=10	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	0.0070**	-0.0132**	-0.1875*	0.0058**	0.0138**	-0.2798*
	(0.0005)	(0.0025)	(0.0549)	(0.0004)	(0.0004)	(0.0546)
FTSE100	0.0063**	-0.0112**	-0.3028*	0.0052**	0.0116**	-0.2055**
	(0.0005)	(0.0007)	(0.0324)	(0.0003)	(0.0011)	(0.0682)
CAC40	0.0064**	-0.0133**	-0.2415*	0.0054**	0.0135**	-0.3077*
	(0.0005)	(0.0019)	(0.0633)	(0.0004)	(0.0013)	(0.0662)
DAX	0.0076**	-0.0185**	-0.1553*	0.0069**	0.0184**	-0.2172*
	(0.0006)	(0.0037)	(0.0592)	(0.0005)	(0.0002)	(0.0584)
TSX	0.0086**	-0.0185**	-0.1844**	0.0080**	0.0192**	-0.2187*
	(0.0004)	(0.0003)	(0.0316)	(0.0004)	(0.0045)	(0.0672)
Nikkei225	0.0095**	-0.0207**	-0.0709	0.0087**	0.0206**	-0.1640
	(0.0003)	(0.0002)	(0.0476)	(0.0003)	(0.0003)	(0.0261)
IGPA	0.0043**	-0.0082**	-0.3227*	0.0041**	0.0095**	-0.2228
	(0.0004)	(0.0012)	(0.0744)	(0.0003)	(0.0003)	(0.0477)
Bolsa	0.0095**	-0.0199**	-0.2722	0.0096**	0.0221**	-0.2737*
	(0.0006)	(0.0038)	(0.0565)	(0.0006)	(0.0024)	(0.0421)
Bovespa	0.0129**	-0.0288**	-0.2112*	0.0108**	0.0294**	-0.2853**
	(0.0005)	(0.0043)	(0.0456)	(0.0009)	(0.0006)	(0.0719)
Merval	0.0151**	-0.0297**	-0.2520	0.0139**	0.0298**	-0.2074
	(0.0006)	(0.0050)	(0.0660)	(0.0008)	(0.0041)	(0.0510)
HSI	0.0101**	-0.0199**	-0.2707**	0.0094**	0.0213**	-0.2695**
	(0.0006)	(0.0009)	(0.0228)	(0.0006)	(0.0006)	(0.0213)
TAIEX	0.0120**	-0.0234**	-0.0729	0.0103**	0.0242**	-0.0727
	(0.0005)	(0.0005)	(0.0338)	(0.0005)	(0.0005)	(0.0468)
KOSPI	0.0126**	-0.0236**	-0.2153*	0.0109**	0.0247**	-0.2617*
	(0.0004)	(0.0025)	(0.0325)	(0.0003)	(0.0019)	(0.0427)
KLCI	0.0114**	-0.0178**	-0.2423**	0.0107**	0.0185**	-0.2335**
	(0.0006)	(0.0043)	(0.0501)	(0.0003)	(0.0009)	(0.0157)
JCI	0.0077**	-0.0136**	-0.4541**	0.0085**	0.0151**	-0.3730**
	(0.0005)	(0.0065)	(0.0422)	(0.0002)	(0.0003)	(0.0275)
SET	0.0105**	-0.0207**	-0.2480**	0.0108**	0.0230**	-0.2022**
	(0.0003)	(0.0004)	(0.0259)	(0.0004)	(0.0024)	(0.0443)

Note: ** (*) means more than 99% (95%) of the observations are significant at 1% (5%) level. The numbers in parentheses are the standard deviation of the estimated parameters.

Table 5.2 The test of equality of the estimated parameters of GEV distribution in the left and right tails

Panel A: n=5	P-value of scale parameter equality test	P-value of location parameter equality test	P-value of tail parameter equality test
S&P500	0.0619	0.0000	0.0000
FTSE100	0.0000	0.0000	0.0000
CAC40	0.0000	0.0000	0.0000
DAX	0.0000	0.0000	0.0000
TSX	0.0000	0.0000	0.0000
Nikkei225	0.8376	0.0000	0.0000
IGPA	0.0000	0.0000	0.0000
Bolsa	0.0000	0.0000	0.0000
Bovespa	0.0000	0.0000	0.0000
Merval	0.3075	0.0000	0.0000
HSI	0.0000	0.0000	0.0000
TAIEX	0.0000	0.0000	0.0000
KOSPI	0.0000	0.0000	0.0000
KLCI	0.0000	0.0000	0.0000
JCI	0.0000	0.0000	0.0000
SET	0.0000	0.0000	0.0000
Panel B: n=10	**P-value of scale parameter equality test**	**P-value of location parameter equality test**	**P-value of tail parameter equality test**
S&P500	0.0000	0.0000	0.0000
FTSE100	0.0000	0.0000	0.0000
CAC40	0.0000	0.0000	0.0130
DAX	0.0000	0.0000	0.0000
TSX	0.0000	0.0000	0.0000
Nikkei225	0.0000	0.0000	0.0000
IGPA	0.0000	0.0000	0.0000
Bolsa	0.1939	0.0000	0.0000
Bovespa	0.0000	0.0000	0.0525
Merval	0.0000	0.0000	0.0000
HSI	0.0000	0.0000	0.0000
TAIEX	0.0000	0.0000	0.0000
KOSPI	0.0000	0.0000	0.4914
KLCI	0.0026	0.0000	0.0000
JCI	0.0000	0.0000	0.0000
SET	0.0000	0.0000	0.0000
Panel C:n=22	**P-value of scale parameter equality test**	**P-value of location parameter equality test**	**P-value of tail parameter equality test**
S&P500	0.0000	0.0000	0.0000
FTSE100	0.0000	0.0000	0.0000
CAC40	0.0000	0.0000	0.0000
DAX	0.0000	0.0000	0.0000
TSX	0.0000	0.0000	0.0000
Nikkei225	0.0000	0.0000	0.0000
IGPA	0.0000	0.0000	0.0000
Bolsa	0.0337	0.0000	0.5184
Bovespa	0.0000	0.0000	0.0000
Merval	0.0000	0.0000	0.0000
HSI	0.0000	0.0000	0.2533
TAIEX	0.0000	0.0000	0.9054
KOSPI	0.0000	0.0000	0.0000
KLCI	0.0000	0.0000	0.0000
JCI	0.0000	0.0000	0.0000
SET	0.0000	0.0000	0.0000

Table 5.3The mean of estimated parameters fitting the GEV distribution with 10-day returns

	Long position			Short position		
Panel A: n=5	Scale parameter (c'_n)	Location parameter (d'_n)	Tail parameter (k'_n)	Scale parameter (c_n)	Location parameter (d_n)	Tail parameter (k_n)
S&P500	0.0248**	-0.0002	-0.0712*	0.0268**	0.0066**	-0.1772**
	(0.0018)	(0.0003)	(0.0354)	(0.0027)	(0.0004)	(0.0174)
FTSE100	0.0253**	-0.0012	-0.0712**	0.0269**	0.0062**	-0.1865**
	(0.0020)	(0.0055)	(0.0399)	(0.0017)	(0.0003)	(0.0116)
CAC40	0.0341**	-0.0016	-0.0870*	0.0347**	0.0082**	-0.1891**
	(0.0066)	(0.0063)	(0.0661)	(0.0025)	(0.0008)	(0.0164)
DAX	0.0340**	0.0003	-0.0423	0.0395**	0.0081**	-0.1744**
	(0.0018)	(0.0004)	(0.0198)	(0.0025)	(0.0006)	(0.0101)
TSX	0.0247**	0.0020	-0.0446	0.0280**	0.0062**	-0.2337**
	(0.0015)	(0.0002)	(0.0278)	(0.0032)	(0.0004)	(0.0143)
Nikkei225	0.0370**	-0.0059**	-0.1390**	0.0365**	0.0041	-0.1462**
	(0.0010)	(0.0009)	(0.0590)	(0.0020)	(0.0008)	(0.0258)
IGPA	0.0273**	-0.0041**	-0.1094**	0.0291**	0.0032	-0.2149**
	(0.0013)	(0.0010)	(0.0229)	(0.0017)	(0.0010)	(0.0165)
Bolsa	0.0437**	-0.0040	-0.0952*	0.0468**	0.0095**	-0.1370**
	(0.0072)	(0.0045)	(0.0392)	(0.0053)	(0.0049)	(0.0163)
Bovespa	0.0517**	-0.0031	-0.0349	0.0609**	0.0121**	-0.1244**
	(0.0015)	(0.0014)	(0.0124)	(0.0053)	(0.0010)	(0.0044)
Merval	0.0622**	-0.0006	-0.1127**	0.0657**	0.0086	-0.1771**
	(0.0023)	(0.0009)	(0.0272)	(0.0027)	(0.0013)	(0.0105)
HSI	0.0458**	0.0032	-0.0656**	0.0509**	0.0061*	-0.1850**
	(0.0022)	(0.0010)	(0.0141)	(0.0065)	(0.0040)	(0.0223)
TAIEX	0.0500**	-0.0011	-0.1646**	0.0501**	0.0042	-0.1783**
	(0.0025)	(0.0012)	(0.0365)	(0.0079)	(0.0041)	(0.0232)
KOSPI	0.0554**	-0.0013	-0.1582**	0.0522**	0.0062	-0.1388**
	()0.0011	(0.0007)	(0.0169)	(0.0011)	(0.0008)	(0.0100)
KLCI	0.0458**	0.0031	-0.1151**	0.0476**	0.0003	-0.1036**
	(0.0013)	(0.0005)	(0.0120)	(0.0005)	(0.0005)	(0.0024)
JCI	0.0493**	0.0052	-0.1072**	0.0522**	0.0044	-0.1566**
	(0.0012)	(0.0008)	(0.0229)	(0.0012)	(0.0014)	(0.0058)
SET	0.0562**	-0.0013	-0.1704**	0.0548**	0.0002	-0.1388**
	(0.0012)	(0.0005)	(0.0215)	(0.0019)	(0.0005)	(0.0096)
Panel B:n=10	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	0.0210**	-0.0092**	-0.0347	0.0220**	0.0167**	-0.1059*
	(0.0015)	(0.0043)	(0.0579)	(0.0023)	(0.0005)	(0.0322)
FTSE100	0.0221**	-0.0113**	-0.0107	0.0225**	0.0167**	-0.1304**
	(0.0010)	(0.0090)	(0.0541)	(0.0017)	(0.0006)	(0.0255)
CAC40	0.0285**	-0.0148**	0.0027	0.0295**	0.0228**	-0.1485**
	(0.0011)	(0.0005)	(0.0377)	(0.0020)	(0.0024)	(0.0301)
DAX	0.0307**	-0.0131**	-0.0298	0.0333**	0.0234**	-0.1329**
	(0.0012)	(0.0050)	(0.0349)	(0.0024)	(0.0008)	(0.0165)
TSX	0.0220**	-0.0068**	-0.0529	0.0224**	0.0161**	-0.1625**
	(0.0012)	(0.0071)	(0.0537)	(0.0025)	(0.0009)	(0.0334)
Nikkei225	0.0328**	-0.0205**	0.0948	0.0316**	0.0190**	-0.0987*
	(0.0011)	(0.0008)	(0.0640)	(0.0017)	(0.0009)	(0.0283)
IGPA	0.0244**	0.0047	-0.0413	0.0262**	0.0126**	-0.1913**
	(0.0011)	(0.0006)	(0.0334)	(0.0009)	(0.0015)	(0.0120)
Bolsa	0.0366**	0.0126**	0.0207	0.0407**	0.0282**	-0.1037**
	(0.0021)	(0.0008)	(0.0560)	(0.0012)	(0.0013)	(0.0087)
Bovespa	0.0481**	0.0167**	0.0250	0.0540**	0.0352**	-0.0940**
	(0.0023)	(0.0019)	(0.0320)	(0.0012)	(0.0008)	(0.0098)
Merval	0.0560**	0.0210**	-0.0476	0.0582**	0.0328**	-0.1402**
	(0.0020)	(0.0013)	(0.0336)	(0.0031)	(0.0018)	(0.0167)
HSI	0.0423**	-0.0128**	-0.0221	0.0431**	0.0242**	-0.1405**
	(0.0019)	(0.0015)	(0.0142)	(0.0023)	(0.0008)	(0.0218)
TAIEX	0.0466**	-0.0177**	-0.1255	0.0427**	0.0223**	-0.1293*
	(0.0019)	(0.0012)	(0.0521)	(0.0035)	0.0011)	(0.0300)
KOSPI	0.0496**	-0.0196**	-0.1038*	0.0466**	0.0252**	-0.0893*
	(0.0011)	(0.0011)	(0.0234)	(0.0018)	(0.0013)	(0.0190)
KLCI	0.0389**	-0.0102**	-0.0240	0.0398**	0.0149**	-0.0518
	(0.0017)	(0.0005)	(0.0215)	(0.0021)	(0.0007)	(0.0164)

	c'_n	d'_n	k'_n	c_n	d_n	k_n
JCI	0.0443**	-0.0103**	-0.0269	0.0467**	0.0210**	-0.1206**
	(0.0023)	(0.0056)	(0.0419)	(0.0032)	(0.0035)	(0.0283)
SET	0.0525**	-0.0198**	-0.1423**	0.0486**	0.0185**	-0.0895**
	(0.0014)	(0.0008)	(0.0238)	(0.0020)	(0.0009)	(0.0160)
Panel C:n=22	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	0.0191**	-0.0192	0.1626**	0.0176**	0.0277**	0.0408
	(0.0012)	(0.0008)	(0.0594)	(0.0013)	(0.0007)	(0.0478)
FTSE100	0.0196**	-0.0226	0.1343*	0.0179**	0.0290**	-0.0123
	(0.0012)	(0.0009)	(0.0677)	(0.0008)	(0.0010)	(0.0222)
CAC40	0.0261**	-0.0300	0.0702*	0.0231**	0.0388**	-0.0381
	(0.0011)	(0.007)	(0.0451)	(0.0014)	(0.0008)	(0.0317)
DAX	0.0289**	0.0290	0.1086	0.0254**	0.0403**	-0.0095
	(0.0021)	(0.0010)	(0.0583)	(0.0016)	(0.0008)	(0.0335)
TSX	0.0213**	-0.0179	0.1280	0.0176**	0.0277**	-0.0527
	(0.0022)	(0.0009)	(0.0645)	(0.0011)	(0.0012)	(0.0418)
Nikkei225	0.0288**	-0.0387**	-0.0354**	0.0275**	0.0374**	-0.0325
	(0.0006)	(0.0014)	(0.0698)	(0.0014)	(0.0011)	(0.0311)
IGPA	0.0224**	0.0163**	0.0410	0.0226**	0.0244**	-0.1265
	(0.0019)	(0.0008)	(0.0552)	(0.0031)	(0.0035)	(0.0446)
Bolsa	0.0355**	0.0324**	0.0822	0.0341**	0.0497**	-0.0265
	(0.0016)	(0.0019)	(0.0382)	(0.0034)	(0.0014)	(0.0428)
Bovespa	0.0436**	0.0398**	0.1589	0.0440**	0.0616**	-0.0126
	(0.0023)	(0.0026)	(0.0387)	(0.0034)	(0.0016)	(0.0425)
Merval	0.0508**	0.0506**	0.0301	0.0466**	0.0613**	-0.0340
	(0.0033)	(0.0026)	(0.0487)	(0.0046)	(0.0026)	(0.0558)
HSI	0.0372**	-0.0336**	-0.0782	0.0361**	0.0473**	-0.0683
	(0.0029)	(0.0020)	(0.0275)	(0.0018)	(0.0009)	(0.0270)
TAIEX	0.0447**	-0.0412**	0.1115	0.0347**	0.0443**	-0.0397
	(0.0026)	(0.0014)	(0.0830)	(0.0033)	(0.0045)	(0.0580)
KOSPI	0.0444**	-0.0428**	0.0216	0.0381**	0.0492**	0.0386
	(0.0014)	(0.0011)	(0.0244)	(0.0013)	(0.0021)	(0.0275)
KLCI	0.0353**	-0.0252**	-0.0892	0.0319**	0.0327**	0.0691
	(0.0022)	(0.0009)	(0.0414)	(0.0010)	(0.0013)	(0.0168)
JCI	0.0390**	-0.0298**	-0.1162	0.0398**	0.0424**	-0.0491
	(0.0014)	(0.0012)	(0.0339)	(0.0017)	(0.0028)	(0.0244)
SET	0.0451**	-0.0435**	0.0372	0.0430**	0.0435**	-0.0379
	(0.0024)	(0.0012)	(0.0503)	(0.0013)	(0.0015)	(0.0136)

Note: ** (*) means more than 99% (95%) of the observations are significant at 1% (5%) level. The numbers in parentheses are the standard deviation of the estimated parameters.

5.2.1.2 Univariate VaR results of the GEV model and VaR backtesting

The results of the one period ahead forecasted VaR with the GEV model are presented in Table 5.4 and 5.5, including Kupiec's (1995) unconditional coverage test and Christoffersen's (1998) conditional test. As mentioned in Section 4.5.1, VR of a well-designed VaR measure will approximate the value of α as shown in Eq. (2.1). Thus, the coverage ratio of a well-calibrated $VaR_{0.99}$ measure would theoretically approach 99%. The VaR model will not be regarded as a good risk measure if the VR is too high (or the coverage ratio is too low). According to Kupiec (1995) and Christoffersen (1998), the null hypothesis of the coverage test is H_0: VR=α, where VR is the violation rate shown in Eq. (4.43). However, one would care more about whether the violation rate exceeds α rather than if it is smaller than α. Thus, the conditional and unconditional coverage tests used in this thesis follow the convention applied by related research to test if the violation rate overtakes the type I error (α). If the statistics exceed the critical point and fall into the area of rejection, the null hypothesis is rejected. Thus, the VaR approach is not a proper market risk measure. On the other hand, the likelihood ratio test of Kupiec (1995) and Christoffersen (1998) might be very small, implying that VR does not equal α. This phenomenon will be discussed and presented with other measures in section 5.2.6. Ideally, VaR forecasted by the GEV model is expected to show theoretically proper potential losses to each equity index.

The main results of $VaR_{0.99}$ and $VaR_{0.95}$ based on the GEV model with daily returns of each equity index are shown in Table 5.4 and 5.5, respectively. The case with 10-day returns is displayed in Table 5.6. Theoretically, the violation rate of $VaR_{0.99}$ ($VaR_{0.95}$) is expected to be 1% (5%). The first and second columns in Table 5.4 show the statistics of the likelihood ratio test of the long and short based on the GEV model with n=5 (i.e. the maxima or minima value is extracted in every five observations with a rolling process). The third and fourth columns are for n=10 and n=22. The mean VaR of each index for various blocks are also in Table 5.4 to Table 5.6. Overall, Latin American indices are riskier than the other two sets of equity indices, and the developed equity indices have less market risks. In the three different blocks, the larger block leads to a smaller market risk. In addition, the forecasted VaR also shows that the long position has a higher market risk than the short one.

Generally speaking, the GEV model with daily returns provides a better VaR forecast in the four Latin American returns and six Asian equity returns than in the six returns from the developed equity markets. Furthermore, the evidence across the three panels shows that the violation rate goes up with the size of the block, particularly in panel A. For example, the violation rate in the long position of the S&P500 rises from 0.0477 for n=5 to 0.0675 for n=22. This consistent pattern can also be found in the long position of the FTSE 100, CAC40, DAX, Nikkei 225, and most of the indices in Latin America. Compared with the results between the long and short position in panel A, there are 10 out of a total of 18 parts in the short positions which are significant at 1% or 5% to both Kupiec (1995) and Christoffersen's (1998) statistics, corresponding to none in just the long position. For both the Latin American results in panel B and the Asian equity results in panel C, the GEV model offers satisfactory VaR forecasts in both the long and short positions, although most of them are only accepted at 95%. However, one finds that most violation rates in the short positions in Table 5.4 are close to α. In other words, the GEV model produces a more accurate VaR forecast in short positions than in long ones. Thus, it is obvious that the GEV model is more suitable to forecast market risk in the short position than the long position.

Sometimes, the forecasted VaR might be calculated under a tolerant probability, for example 95% (i.e., α=0.05). The outcomes of this condition are presented in Table 5.5, and the performance of the VaR forecast here is not as good as in that of the case of α=0.01. Technically, the absolute VaRs with the condition α=0.05 are smaller than the ones with

α=0.01. Therefore, actual index returns would easily surpass forecasted VaRs.

An important phenomenon can be noted in Table 5.4 and 5.5. The results of the violation rate and backtesting are positively associated with the block size, although this tendency is not significant. These findings indicate that the GEV model with a larger block tends to produce smaller VaRs, and it means that VaRs are easily exceeded by actual returns. Under these circumstances the number of exceedance and the violation rate rises. Similarly, as discussed by Lauridsen (2000) and Ho et al. (2000), both parameters in generalized extreme distribution and forecasted VaRs are very sensitive to the size of the block, and until now there has been no standard selection rule to overcome this weakness. Coles (2001) suggested that yearly extreme values would be a better solution to describe the characteristics of the tail. However, this is restricted by the accessibility of the financial time series. Moreover, a yearly block would produce fewer observations, and the results of the tail distribution estimation might be biased. One possible suggestion is that the choice of block size might depend on the frequency of the market down-turn. If one prefers to take the risk that risk potentially happens once in one week, the size of block would be equal to five and so on.

The backtesting of the daily performance in $VaR_{0.99}$ is superior to $VaR_{0.95.}$ Thus, only the $VaR_{0.99}$ based on the 10-day return sequences are created and examined here. The backtesting results are presented in Table 5.6. Due to the higher diversity in 10-day returns, the coverage performance is dissatisfactory compared with that of the daily results in Table 5.4. Similar to the VaR of daily return, the back-testing results in the developed equity markets are worse than the Latin American and Asian indices. For example, the average violation rate of the long (short) positions are 0.0565, 0.0685, and 0.0565 (0.0295, 0.0388, and 0.0491) for n=5, n=10, and n=22. All of these violation rates are too large for the target, 0.01. Apart from the fluctuation feature of the 10-day return sequence, another cause of the lower coverage rate to forecasted VaR is the serial correlation of the violations. In panel A of Table 5.6, the independence tests are considerably higher than the ones in Table 5.4, implying that the violations are highly correlated. In addition, the GEV model with the 10-day return sequence provides acceptably more accurate results in the short positions in the cases of n=5 and n=10 of the Latin American indices, and for some blocks of the Asian equity market indices. In the developed market, the GEV model provides a similarly poor performance in both the long and short positions.

Table 5.4 Backtesting of 99% quantile daily VaR measured by generalized extreme value (GEV)

	GEV(n=5)		GEV(n=10)		GEV(n=22)	
Panel A	Long	Short	Long	Short	Long	Short
S&P500						
Ave. VaR	-0.0295	0.0281	-0.0277	0.0271	-0.0254	0.0247
Violation (%)	0.0477	0.0373	0.0536	0.0407	0.0675	0.0512
Unconditional test	28.0532	16.5028	35.2779	20.1258	54.5691	32.3251
Independent test	1.6441*	0.9284*	1.0029*	0.6355*	0.4842*	0.5405*
Conditional test	29.6974	17.4312	36.2807	20.7613	55.0533	32.8657
FTSE100						
Ave. VaR	-0.0290	0.0278	-0.0274	0.0266	-0.0249	0.0241
Violation (%)	0.0373	0.0291	0.0442	0.0314	0.0512	0.0210
Unconditional test	16.5028	9.0822	23.9818	11.0416	32.3251	3.4380*
Independent test	1.9977*	0.7099*	1.1295*	0.5333*	0.5405*	0.3248*
Conditional test	18.5006	9.7921	25.1113	11.5749	32.8657	3.7628*
CAC40						
Ave. VaR	-0.0356	0.0348	-0.0338	0.0331	-0.0315	0.0307
Violation (%)	0.0466	0.0221	0.0489	0.0303	0.0605	0.0407
Unconditional test	26.6731	4.1145**	29.4556	10.0447	44.6028	20.1258
Independent test	0.9052*	0.3738*	2.5146*	0.7058*	3.7767*	0.0670*
Conditional test	27.5783	4.4882*	31.9702	10.7505	48.3795	20.1928
DAX						
Ave. VaR	-0.0388	0.0365	-0.0366	0.0357	-0.0334	0.0335
Violation (%)	0.0338	0.0175	0.0373	0.0210	0.0442	0.0210
Unconditional test	13.1340	1.7163*	16.5028	3.4380*	23.9818	3.4380*
Independent test	0.3863*	0.2318*	0.2152*	0.3351*	0.4053*	**0.3351***
Conditional test	13.5203	1.9482*	16.7180	3.7731*	24.3870	3.7731*
TSX						
Ave. VaR	-0.0270	0.0235	-0.0259	0.0228	-0.0235	0.0212
Violation (%)	0.0547	0.0419	0.0373	0.0233	0.0629	0.0210
Unconditional test	36.7844	21.3861	16.5028	4.8377**	47.8570	3.4380*
Independent test	2.6450*	1.3846*	0.2152*	0.2121*	3.3253*	0.3351*
Conditional test	39.4293	22.7707	16.7180	5.0498*	51.1823	3.7731*
Nikkei225						
Ave. VaR	-0.0395	0.0384	-0.0378	0.0371	-0.0359	0.0355
Violation (%)	0.0303	0.0210	0.0314	0.0210	0.0349	0.0210
Unconditional test	10.0447	3.4380*	11.0416	3.4380*	14.2273	3.4380*
Independent test	0.6177*	3.4834*	0.5333*	1.6220*	1.1625*	0.3351*
Conditional test	10.6624	6.9214**	11.5749	5.0600*	15.3897	3.7731*
Panel B	Long	Short	Long	Short	Long	Short
Merval						
Ave. VaR	-0.0623	0.0584	-0.0608	0.0581	-0.0565	0.0544
Violation (%)	0.0129	0.0093	0.0129	0.0093	0.0175	0.0117
Unconditional test	0.2799*	0.0164*	0.2799*	0.0164*	1.7361*	0.1008*
Independent test	1.0137*	0.0656*	1.0137*	0.0656*	2.2015*	0.1028*
Conditional test	1.2936*	0.0821*	1.2936*	0.0821*	3.9376*	0.2036*
Bovespa						
Ave. VaR	-0.0584	0.0578	-0.0554	0.0538	-0.0510	0.0497
Violation (%)	0.0152	0.0140	0.0164	0.0175	0.0187	0.0210
Unconditional test	0.8718*	0.5391*	1.2724*	1.7361*	2.2588*	3.4671*
Independent test	0.7562*	0.1484*	0.6484*	0.5524*	0.4668*	0.3228*
Conditional test	1.6280*	0.6875*	1.9208*	2.2885*	2.7256*	3.7900*
IGPA						
Ave. VaR	-0.0186	0.0190	-0.0179	0.0182	-0.0261	0.0169
Violation (%)	0.0479	0.0164	0.0210	0.0187	0.0199	0.0210
Unconditional test	28.1547	1.2724*	3.4671*	2.2588*	2.8369*	3.4671*
Independent test	5.3866**	0.6484*	0.3228*	0.4668*	3.7715*	1.6169*
Conditional test	33.5414	1.9208*	3.7900*	2.7256*	6.6084**	5.0840*
Bolsa						
Ave. VaR	-0.0416	0.0436	-0.0398	0.0425	-0.0371	0.0400
Violation (%)	0.0187	0.0187	0.0222	0.0199	0.0292	0.0222
Unconditional test	2.2588*	2.2588*	4.1467**	2.8369*	9.1331	4.1467**
Independent test	4.0921**	0.4668*	7.8411	0.3905*	7.4837	0.2630*

Conditional test	6.3509**	2.7256*	11.9878	3.2274*	16.6168	4.4097*

Panel C	Long	Short	Long	Short	Long	Short
HSI						
Ave. VaR	-0.0446	0.0435	-0.0423	0.0421	-0.0383	0.0389
Violation (%)	0.0312	0.0173	0.0323	0.0219	0.0266	0.0300
Unconditional test	10.9094	1.6708*	11.9322	4.0401**	7.1628	9.9197
Independent test	1.5962*	0.2299*	1.4471*	0.3707*	2.3022*	0.7000*
Conditional test	12.5055	1.9007*	13.3793	4.4108*	9.4650	10.6197
TAIEX						
Ave. VaR	-0.0448	0.0435	-0.0439	0.0426	-0.0419	0.0407
Violation (%)	0.0139	0.0092	0.0139	0.0115	0.0185	0.0150
Unconditional test	0.5045*	0.0226*	0.5045*	0.0866*	2.1832*	0.8269*
Independent test	0.8859*	0.0649*	0.8859*	0.1016*	0.4741*	0.7646*
Conditional test	1.3904*	0.0875*	1.3904*	0.1882*	2.6573*	1.5915*
KOSPI						
Ave. VaR	-0.0517	0.0502	-0.0497	0.0489	-0.0453	0.0449
Violation (%)	0.0115	0.0092	0.0115	0.0092	0.0139	0.0139
Unconditional test	0.0866*	0.0226*	0.0866*	0.0226*	0.5045*	0.5045*
Independent test	3.6513*	0.0649*	3.6513*	0.0649*	2.9885*	0.1466*
Conditional test	3.7379*	0.0875*	3.7379*	0.0875*	3.4930*	0.6511*
KLCI						
Ave. VaR	-0.0343	0.0358	-0.0330	0.0355	-0.0296	0.0324
Violation (%)	0.0081	0.0023	0.0104	0.0023	0.0127	0.0035
Unconditional test	0.1494*	3.2613*	0.0058*	3.2613*	0.2555*	2.1700*
Independent test	0.0496*	0.0040*	0.0822*	0.0040*	0.1231*	0.0091*
Conditional test	0.1990*	3.2653*	0.0880*	3.2653*	0.3786*	2.1791*
JCI						
Ave. VaR	-0.0414	0.0406	-0.0410	0.0407	-0.0379	0.0378
Violation (%)	0.0242	0.0185	0.0242	0.0185	0.0185	0.0208
Unconditional test	5.5166**	2.1832*	5.5166**	2.1832*	2.1832*	3.3709*
Independent test	4.6805**	2.0074*	4.6805**	2.0074*	2.2196*	0.3294*
Conditional test	10.1970	4.1906*	10.1970	4.1906*	3.8904*	3.7003*
SET						
Ave. VaR	-0.0430	0.0453	-0.0420	0.0443	-0.0393	0.0421
Violation (%)	0.0092	0.0081	0.0104	0.0104	0.0104	0.0115
Unconditional test	0.0226**	0.1494*	0.0058**	0.0058*	0.0058*	0.0866*
Independent test	8.1919	1.7914*	7.4455	1.3532*	1.3532*	1.1771*
Conditional test	8.2146**	1.9408*	7.4513**	1.3590*	1.3590*	1.2638*

1. ** (*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of likelihood ratio test (unconditional test) and independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical value of conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$. All of the average VaR and violation in the table are significant at 1% level, for convenience, the asterisk symbol of significance is not made.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, says 10^{-10}, is assigned to this term for the purpose of convenient to calculate independent test and Christoffersen (1998) unconditional coverage test.

Table 5.5 95% quantile daily VaR based on the method of generalized extreme value (GEV)

	GEV(n=5)		GEV(n=10)		GEV(n=22)	
Panel A	Long	Short	Long	Short	Long	Short
S&P500						
Ave. VaR	-0.0154	0.0161	-0.0141	0.0148	-0.0124	0.0132
Violation (%)	0.1315	0.0908	0.1444	0.1083	0.1595	0.1269
Unconditional test	36.7910	10.6438	47.4125	20.2951	61.2382	33.1897
Independent test	0.0306*	0.0006*	0.1559*	0.0004*	0.3658*	0.0010*
Conditional test	36.8215	10.6444	47.5683	20.2954	61.6040	33.1907
FTSE100						
Ave. VaR	-0.0164	0.0164	-0.0143	0.0147	-0.0125	0.0128
Violation (%)	0.1118	0.0908	0.1234	0.0990	0.1420	0.1339
Unconditional test	22.5198	10.6438	30.5844	14.8309	45.4053	38.6450
Independent test	0.5079*	0.0922*	0.6163*	0.0114*	1.0170*	0.0768*
Conditional test	23.0277	10.7360	31.2007	14.8423	46.4223	38.7218
CAC40						
Ave. VaR	-0.0209	0.0215	-0.0191	0.0193	-0.0175	0.0176
Violation (%)	0.1141	0.0908	0.1246	0.0920	0.1385	0.1281
Unconditional test	24.0541	10.6438	31.4436	11.2063	42.4571	34.0765
Independent test	4.0213**	0.2455*	3.4467*	0.1236*	2.3160*	0.5796*
Conditional test	28.0753	10.8893	34.8904	11.3300	44.7730	34.6562
DAX						
Ave. VaR	-0.0220	0.0221	-0.0197	0.0200	-0.0174	0.0182
Violation (%)	0.1024	0.0803	0.1222	0.0896	0.1304	0.1024
Unconditional test	16.7980	6.1506**	29.7346	10.0936	35.8772	16.7980
Independent test	4.8265**	0.0287*	3.8903**	0.0865*	4.4059**	0.0552*
Conditional test	21.6245	6.1793**	33.6248	10.1801	40.2831	16.8532
TSX						
Ave. VaR	-0.0137	0.0139	-0.0123	0.0126	-0.0105	0.0111
Violation (%)	0.1525	0.1246	0.1630	0.1339	0.1665	0.1548
Unconditional test	54.6924	31.4436	64.6136	38.6450	68.0557	56.8436
Independent test	0.4641*	0.5345*	0.6904*	2.4845*	0.4400*	0.1635*
Conditional test	55.1565	31.9781	65.3040	41.1294	68.4957	57.0071
Nikkei225						
Ave. VaR	-0.0228	0.0231	-0.0216	0.0214	-0.0196	0.0196
Violation (%)	0.0990	0.0757	0.0943	0.0768	0.1187	0.0861
Unconditional test	14.8309	4.5023**	12.3675	4.8931**	27.2419	8.5178
Independent test	0.0209*	0.8452*	0.1213*	0.7498*	0.0346*	0.2002*
Conditional test	14.8519	5.3475*	12.4888	5.6429*	27.2766	8.7179**
Panel B	Long	Short	Long	Short	Long	Short
Merval						
Ave. VaR	-0.0324	0.0333	-0.0297	0.0302	-0.0272	0.0282
Violation (%)	0.0619	0.0397	0.0689	0.0502	0.0783	0.0584
Unconditional test	1.0364*	2.8112*	2.5201*	0.0004*	5.3766**	0.5266*
Independent test	1.6280*	0.7204*	2.3305*	0.1409*	1.8100*	0.1763*
Conditional test	2.6644*	3.5316*	4.8506*	0.1414*	7.1867**	0.7028*
Bovespa						
Ave. VaR	-0.0317	0.0348	-0.0294	0.0310	-0.0267	0.0281
Violation (%)	0.0771	0.0409	0.0911	0.0514	0.1028	0.0631
Unconditional test	4.9682**	3.2398*	10.7587	0.0152*	16.9465	1.2417*
Independent test	2.7597*	0.6303*	2.8793*	0.1057*	3.9357**	0.0481*
Conditional test	7.7279**	3.8701*	13.6380	0.1210*	20.8823	1.2898*
IGPA						
Ave. VaR	-0.0099	0.0111	-0.0088	0.0101	-0.0143	0.0101
Violation (%)	0.1168	0.0829	0.1262	0.1051	0.0689	0.1227
Unconditional test	25.8178	7.1492	32.5292	18.3199	2.5201*	29.9413
Independent test	15.0128	0.7467*	12.4656	3.4886*	5.4623**	1.9752*
Conditional test	40.8306	7.8959**	44.9948	21.8085	7.9824**	31.9165
Bolsa						
Ave. VaR	-0.0231	0.0256	-0.0206	0.0229	-0.0184	0.0209
Violation (%)	0.0736	0.0467	0.0911	0.0572	0.1157	0.0701
Unconditional test	3.8292*	0.0855*	10.7587	0.3931*	25.0222	2.8243*
Independent test	1.6915*	2.8985*	4.6026**	2.2634*	3.0490*	0.7955*
Conditional test	5.5207*	2.9840*	15.3613	2.6564*	28.0713	3.6198*

Panel C	Long	Short	Long	Short	Long	Short
HSI						
Ave. VaR	-0.0230	0.0248	-0.0213	0.0227	-0.0182	0.0202
Violation (%)	0.1212	0.0843	0.1443	0.1016	0.1617	0.1178
Unconditional test	29.2572	7.7922	47.7882	16.4556	63.8486	26.7885
Independent test	1.5587*	0.6079*	0.7565*	0.2385*	0.4948*	0.0430*
Conditional test	30.8159	8.4001**	48.5447	16.6940	64.3433	26.8316
TAIEX						
Ave. VaR	-0.0247	0.0258	-0.0233	0.0246	-0.0214	0.0230
Violation (%)	0.0727	0.0427	0.0785	0.0473	0.0901	0.0577
Unconditional test	3.6148*	0.4398*	5.5300*	0.0568*	10.3794	0.4524*
Independent test	0.0182*	0.0485*	0.2423*	0.0008*	0.0001*	0.0020*
Conditional test	3.6330*	0.4883*	5.7724*	0.0576*	10.3794	0.4544*
KOSPI						
Ave. VaR	-0.0283	0.0286	-0.0255	0.0266	-0.0216	0.0234
Violation (%)	0.0566	0.0358	0.0635	0.0427	0.0785	0.0531
Unconditional test	0.3296*	1.7653*	1.3363*	0.4398*	5.5300**	0.0755*
Independent test	0.7212*	0.2727*	0.2870*	0.0485*	0.0116*	0.0571*
Conditional test	1.0509*	2.0380*	1.6233*	0.4883*	5.5417*	0.1326*
KLCI						
Ave. VaR	-0.0173	0.0189	-0.0145	0.0162	-0.0123	0.0141
Violation (%)	0.0393	0.0312	0.0566	0.0462	0.0843	0.0704
Unconditional test	0.9828*	3.2206*	0.3296*	0.1179*	7.7922	2.9513*
Independent test	0.7385*	3.0190*	1.4226*	1.8203*	7.2597	2.9115*
Conditional test	1.7212*	6.2396**	1.7523*	1.9381*	15.0519	5.8628*
JCI						
Ave. VaR	-0.0200	0.0216	-0.0181	0.0199	-0.0160	0.0173
Violation (%)	0.0889	0.0589	0.1062	0.0727	0.1201	0.0889
Unconditional test	9.8369	0.5937*	19.2004	3.6148*	28.4248	9.8369
Independent test	9.5760	1.9752*	11.1302	1.7429*	10.9155	2.3941*
Conditional test	19.4128	2.5689*	30.3306	5.3576*	39.3403	12.2310
SET						
Ave. VaR	-0.0236	0.0257	-0.0218	0.0235	-0.0190	0.0217
Violation (%)	0.0647	0.0485	0.0727	0.0531	0.0820	0.0670
Unconditional test	1.5649*	6.7134	3.6148*	0.0755*	6.8472	2.0716*
Independent test	4.0022**	0.1921*	2.5329*	0.0571*	1.3135*	0.1470*
Conditional test	5.5671*	6.9055**	6.1477**	0.1326*	8.1606**	2.2185*

1. ** (*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of likelihood ratio test (unconditional test) and independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical value of conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$. All of the average VaR and violation in the table are significant at 1% level, for convenience, the asterisk symbol of significance is not made.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, says 10^{-10}, is assigned to this term for the purpose of convenient to calculate independent test and Christoffersen (1998) unconditional coverage test.

Table 5.6 Backtesting of 99% quantile daily VaR measured by generalized extreme value (GEV) based on 10-day returns

	GEV(n=5)		GEV(n=10)		GEV(n=22)	
Panel A	Long	Short	Long	Short	Long	Short
S&P500						
Ave. VaR	-0.0671	0.0687	-0.0596	0.0614	-0.0671	0.0550
Violation (%)	0.0617	0.0314	0.0768	0.0384	0.0617	0.0454
Unconditional test	46.2211	11.0416	68.7494	17.6834	46.2211	25.3158
Independent test	68.3836	33.5656	82.0366	36.6233	68.3836	35.9105
Conditional test	114.6047	44.6072	150.7860	54.3067	114.6047	61.2263
FTSE100						
Ave. VaR	-0.0693	0.0677	-0.0628	0.0614	-0.0693	0.0557
Violation (%)	0.0594	0.0291	0.0722	0.0407	0.0594	0.0547
Unconditional test	43.0022	9.0822	61.5384	20.1258	43.0022	36.7844
Independent test	85.0357	28.5957	86.2115	30.7447	85.0357	56.0921
Conditional test	128.0379	37.6779	147.7499	50.8704	128.0379	92.8765
CAC40						
Ave. VaR	-0.0907	0.0876	-0.0801	0.0803	-0.0907	0.0728
Violation (%)	0.0640	0.0361	0.0792	0.0442	0.0640	0.0524
Unconditional test	49.5100	15.3505	72.4413	23.9818	49.5100	33.7913
Independent test	89.0570	32.0434	89.5974	44.2096	89.0570	44.8982
Conditional test	138.5670	47.3939	162.0387	68.1914	138.5670	78.6895
DAX						
Ave. VaR	-0.0954	0.1002	-0.0862	0.0894	-0.0954	0.0784
Violation (%)	0.0594	0.0314	0.0640	0.0396	0.0594	0.0512
Unconditional test	43.0022	11.0416	49.5100	18.8913	43.0022	32.3251
Independent test	89.6196	50.7627	98.2322	46.5771	89.6196	53.5122
Conditional test	132.6218	61.8044	147.7422	65.4683	132.6218	85.8373
TSX						
Ave. VaR	-0.0672	0.0665	-0.0608	0.0591	-0.0672	0.0532
Violation (%)	0.0570	0.0314	0.0733	0.0454	0.0570	0.0547
Unconditional test	39.8557	11.0416	63.3190	25.3158	39.8557	36.7844
Independent test	67.9972	29.8041	62.3162	42.7729	67.9972	56.0921
Conditional test	107.8529	40.8458	125.6352	68.0886	107.8529	92.8765
Nikkei225						
Ave. VaR	-0.0970	0.0924	-0.0885	0.0840	-0.0970	0.0778
Violation (%)	0.0373	0.0175	0.0454	0.0244	0.0373	0.0361
Unconditional test	16.5028	1.7163*	25.3158	5.6053**	16.5028	15.3505
Independent test	41.8278	25.9025	46.4164	22.1852	41.8278	39.4385
Conditional test	58.3307	27.6189	71.7322	27.7906	58.3307	54.7890
Panel B	Long	Short	Long	Short	Long	Short
Merval						
Ave. VaR	-0.1574	0.1610	-0.1430	0.1469	-0.1290	0.1296
Violation (%)	0.0292	0.0129	0.0315	0.0164	0.0339	0.0222
Unconditional test	9.1331	0.2799*	11.0988	1.2724*	13.1975	4.1467**
Independent test	72.8750	0.0257*	72.2788	0.0258*	71.9028	0.0374*
Conditional test	82.0081	0.3056*	83.3776	1.2982*	85.1003	4.1841*
Bovespa						
Ave. VaR	-0.1439	0.1642	-0.1304	0.1468	-0.1142	0.1271
Violation (%)	0.0175	0.0012	0.0210	0.0023	0.0245	0.0093
Unconditional test	1.7361*	4.7308*	3.4671*	3.1942*	5.6438**	0.0164*
Independent test	30.7140	0.0010*	34.9505	0.0010*	43.8023	0.0256*
Conditional test	32.4501	4.7318*	38.4176	3.1952*	49.4461	0.0421*
IGPA						
Ave. VaR	-0.0657	0.0672	-0.0582	0.0613	-0.0512	0.0553
Violation (%)	0.0502	0.0187	0.0631	0.0280	0.0864	0.0421
Unconditional test	30.9876	2.2588*	48.0003	8.2033	84.0531	21.4717
Independent test	85.9340	0.1035*	95.9676	0.1767*	117.3510	0.1792*
Conditional test	116.9216	2.3624*	143.9678	8.3800**	201.4041	21.6509
Bolsa						
Ave. VaR	-0.1092	0.1242	-0.0988	0.1114	-0.0894	0.1000
Violation (%)	0.0269	0.0140	0.0304	0.0199	0.0386	0.0292
Unconditional test	7.3107	0.5391*	10.0987	2.8369*	17.7594	9.1331
Independent test	44.5563	0.0258*	43.4207	0.0840*	52.4394	0.1047*
Conditional test	51.8670	0.5649*	53.5195	2.9209*	70.1988	9.2378

Panel C	Long	Short	Long	Short	Long	Short
HSI						
Ave. VaR	-0.1210	0.1228	-0.1075	0.1090	-0.0932	0.0991
Violation (%)	0.0335	0.0323	0.0427	0.0404	0.0600	0.0520
Unconditional test	12.9872	11.9322	22.4662	19.9350	44.2858	33.5266
Independent test	38.6259	0.1494*	53.7896	0.1769*	52.1704	63.6743
Conditional test	51.6131	12.0817	76.2558	20.1118	96.4562	97.2009
TAIEX						
Ave. VaR	-0.1196	0.1262	-0.1110	0.0920	-0.1034	0.0985
Violation (%)	0.0277	0.0069	0.0381	0.0289	0.0485	0.0254
Unconditional test	8.0450	0.4016*	17.5073	8.9645	29.2132	6.3195
Independent test	42.7491	0.0162*	52.6629	0.1034*	57.0716	140.3977
Conditional test	50.7942	0.4718*	70.1702	9.0679**	86.2848	256.2839
KOSPI						
Ave. VaR	-0.1334	0.1873	-0.1211	0.0939	-0.1088	0.1050
Violation (%)	0.0196	0.0012	0.0266	0.0300	0.0370	0.0208
Unconditional test	2.7510*	4.8080**	7.1628	9.9197	16.3341	3.3709*
Independent test	23.1131	0.0010*	31.3555	0.2029*	50.1933	25.9607
Conditional test	25.8640	4.8090*	38.5184	10.1226	66.5274	29.3316
KLCI						
Ave. VaR	-0.1128	0.0798	-0.0971	0.0822	-0.0822	0.0783
Violation (%)	0.0115	0.0035	0.0219	0.0046	0.0358	0.0058
Unconditional test	0.0866*	2.1700*	4.0401**	1.3749*	15.1891	0.8003*
Independent test	10.4911	0.0040*	33.2669	0.0040*	47.7097	11.5189
Conditional test	10.5778	2.1741*	37.3071	1.3790*	62.8988	12.3193
JCI						
Ave. VaR	-0.1210	0.1324	-0.1090	0.1028	-0.0941	0.1049
Violation (%)	0.0462	0.0115	0.0520	0.0300	0.0589	0.0242
Unconditional test	26.4455	0.0866*	33.5266	9.9197	42.6927	5.5166**
Independent test	84.2096	0.0254*	96.2150	0.1253*	94.6597	30.1088
Conditional test	110.6551	0.1120*	129.7416	10.0450	137.3524	35.6254
SET						
Ave. VaR	-0.1332	0.1422	-0.1227	0.1056	-0.1097	0.1104
Violation (%)	0.0196	0.0023	0.0219	0.0208	0.0266	0.0185
Unconditional test	2.7510*	3.2613*	4.0401**	3.3709*	7.1628	2.1832*
Independent test	48.4535	0.0040*	48.7725	0.0502*	55.1194	29.0246
Conditional test	51.2045	3.2653*	52.8127	3.4211*	62.2823	31.2078

1. ** (*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of likelihood ratio test (unconditional test) and independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical value of conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$. All of the average VaR and violation in the table are significant at 1% level, for convenience, the asterisk symbol of significance is not made.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and $\pi_{11}{}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, says 10^{-10}, is assigned to this term for the purpose of convenient to calculate independent test and Christoffersen (1998) unconditional coverage test.

5.2.2 VaR backtesting of competing models

The second part of the univariate analysis is the analysis of four competing models, GARCH(1,1), RiskMetrics, stochastic volatility, and historical simulation. The focus of this section is the backtesting results of the competing VaR models, rather than the estimation of parameters in these models since that is not the core focus of this thesis. Some parts of the estimation will be briefly discussed for the purpose of understanding the procedure of VaR. Before the backtesting analysis, the average forecasted VaR of four competing VaR estimates are shown in Table 5.7. In general, the developed equity markets have the lowest market risk and the market risk in Latin American equity markets are the highest. For example, the VaR of Argentinean (Merval) and Brazilian (Bovespa) equity indices indicate that they have higher market risk in both the long and short positions. The former was affected by several economic and financial crises, for example the economic crisis from 1999 to 2002 and the global financial crisis in 2008. Brazil also experienced an economic crisis in 1999 and the global financial crisis in 2008. Unexpectedly, the forecasted VaR of the Chilean equity index is quite low, but it is still consistent with its daily return pattern shown in Figure 4.15.

Table 5.7 Average VaRs of the four competing models

	$VaR_{0.99}$		$VaR_{0.95}$	
Panel A GARCH(1,1)	Long	Short	Long	Short
S&P500	-0.0340	0.0345	-0.0240	0.0245
FTSE100	-0.0325	0.0277	-0.0229	0.0233
CAC40	-0.0417	0.0421	-0.0294	0.0299
DAX	-0.0410	0.0415	-0.0289	0.0295
TSX	-0.0326	0.0332	-0.0230	0.0235
Nikkei225	-0.0388	0.0325	-0.0275	0.0272
Merval	-0.0460	0.0393	-0.0324	0.0330
Bovespa	-0.0475	0.0411	-0.0335	0.0346
IGPA	-0.0211	0.0183	-0.0148	0.0154
Bolsa	-0.0364	0.0318	-0.0256	0.0268
HSI	-0.0462	0.0396	-0.0326	0.0333
TAIEX	-0.0360	0.0305	-0.0254	0.0256
KOSPI	-0.0377	0.0321	-0.0266	0.0270
KLCI	-0.0232	0.0198	-0.0164	0.0167
JCI	-0.0383	0.0391	-0.0270	0.0278
SET	-0.0362	0.0361	-0.0256	0.0255
Panel B RiskMetrics	Long	Short	Long	Short
S&P500	-0.0260	0.0260	-0.0184	0.0184
FTSE100	-0.0260	0.0260	-0.0184	0.0184
CAC40	-0.0327	0.0327	-0.0231	0.0231

DAX	-0.0346	0.0346	-0.0245	0.0245		
TSX	-0.0235	0.0235	-0.0166	0.0166		
Nikkei225	-0.0344	0.0344	-0.0243	0.0243		
Merval	-0.0515	0.0515	-0.0364	0.0364		
Bovespa	-0.0523	0.0523	-0.0370	0.0370		
IGPA	-0.0175	0.0175	-0.0123	0.0123		
Bolsa	-0.0372	0.0372	-0.0263	0.0263		
HSI	-0.0391	0.0391	-0.0276	0.0276		
TAIEX	-0.0380	0.0380	-0.0269	0.0269		
KOSPI	-0.0433	0.0433	-0.0306	0.0306		
KLCI	-0.0337	0.0337	-0.0238	0.0238		
JCI	-0.0359	0.0359	-0.0254	0.0254		
SET	-0.0388	0.0388	-0.0275	0.0275		
Panel C: SV model	Long	Short	Long	Short		
S&P500	-0.0291	0.0241	-0.0141	0.0141		
FTSE100	-0.0430	0.0430	-0.0303	0.0303		
CAC40	-0.0206	0.0204	-0.0145	0.0145		
DAX	-0.0272	0.0272	-0.0191	0.0191		
TSX	-0.0197	0.0197	-0.0138	0.0138		
Nikkei225	-0.0947	0.0947	-0.0670	0.0670		
Merval	-0.0486	0.0486	-0.0343	0.0343		
Bovespa	-0.0426	0.0426	-0.0300	0.0300		
IGPA	-0.0405	0.0405	-0.0286	0.0286		
Bolsa	-0.0485	0.0485	-0.0341	0.0341		
HSI	-0.0311	0.0318	-0.0219	0.0226		
TAIEX	-0.1478	0.1480	-0.1045	0.1046		
KOSPI	-0.0649	0.0318	-0.0321	0.0226		
KLCI	-0.0119	0.0119	-0.0084	0.0084		
JCI	-0.0272	0.0272	-0.0192	0.0193		
SET	-0.0733	0.0733	-0.0518	0.0518		
Panel D: HS model	$VaR_{0.99}$ (n=1250)		$VaR_{0.99}$ (n=750)		$VaR_{0.99}$ (n=250)	
S&P500	-0.0356	0.0290	-0.0379	0.0330	-0.0455	0.0387
FTSE100	-0.0362	0.0300	-0.0381	0.0328	-0.0418	0.0424
CAC40	-0.0440	0.0351	-0.0461	0.0404	-0.0506	0.0546
DAX	-0.0444	0.0364	-0.0445	0.0387	-0.0496	0.0523
TSX	-0.0326	0.0271	-0.0380	0.0308	-0.0434	0.0359
Nikkei225	-0.0424	0.0359	-0.0477	0.0373	-0.0552	0.0429
Merval	-0.0518	0.0466	-0.0545	0.0449	-0.0614	0.0465
Bovespa	-0.0516	0.0471	-0.0544	0.0508	-0.0577	0.0564
IGPA	-0.0237	0.0189	-0.0272	0.0199	-0.0304	0.0238
Bolsa	-0.0394	0.0375	-0.0427	0.0396	-0.0437	0.0449
HSI	-0.0413	0.0384	-0.0438	0.0449	-0.0524	0.0576
TAIEX	-0.0403	0.0338	-0.0408	0.0329	-0.0433	0.0373
KOSPI	-0.0404	0.0355	-0.0454	0.0367	-0.0491	0.0388
KLCI	-0.0250	0.0205	-0.0281	0.0214	-0.0295	0.0230
JCI	-0.0438	0.0364	-0.0479	0.0407	-0.0506	0.0445
SET	-0.0337	0.0318	-0.0364	0.0337	-0.0441	0.0375

Note: The null hypothesis of average VaR (H_0: mean VaR=0) is significantly rejected at 1% level, for convenience, the asterisk symbol of significance is not made. J.P. Morgan's RiskMetrics model is assumed that the mean return is zero. Thus, forecasted VaR on this approach is symmetrical.

5.2.2.1 GARCH (1,1) model

The first competing model is based on a commonly used econometric volatility model proposed by Bollerslev (1986). The detail of this model has been described in Section 4.4.1.

The best choice for the length of AR and MA should not be considered at this stage[73], and thus the GARCH(1,1) model is adopted in this section. The backtesting results of the GARCH (1,1) model are shown in Table 5.7. In general, the GARCH (1,1) model produces very satisfactory backtesting results since most of them are significant at 95%. In addition, the significances of the unconditional and conditional tests are very balanced. However, in the comparison between the long and short positions, the violation rate in the long position tends to be larger than the one of the short position. Moreover, most results show that the violation rate of the short position is closer to 1% or 5%. For example, the average violation rates of the developed, Latin American, and Asian indices in the long (short) positions are 0.0250, 0.0210, and 0.0210 (0.0169, 0.0257, and 0.0196), respectively. Consequently, this outcome indicates that the GARCH (1,1) model is more suitable for measuring VaR in the short positions rather than the long positions, particularly in the developed and Asian equity markets. On the other hand, the forecasted VaRs of the Latin American equity indices cover more actual returns. For example, the average of all the violation ratios of panel B in Table 5.7, are 0.0210 and 0.0257 (0.0616, and 0.0447) for the long and short position of $VaR_{0.99}$ ($VaR_{0.95}$), respectively. The corresponding violation ratios are 0.0250, 0.0169, 0.0759, and 0.0446 in panel A, and 0.0210, 0.0196, 0.0563, and 0.0444 in panel C respectively.

Table 5.8 Backtesting of VaR measured by GARCH model

	$VaR_{0.99}$		$VaR_{0.95}$	
Panel A	Long	Short	Long	Short
S&P500				
Violation (%)	0.0349	0.0116	0.0768	0.0396
Unconditional test	14.2273	0.0964*	4.8931**	18.8913
Independent test	0.0310*	1.1708*	0.5348*	0.0460*
Conditional test	14.2583	1.2672*	5.4279*	18.9373
FTSE100				

[73] There are two main reasons to support this. Firstly, GARCH(1,1) can be used as an approach for extending the RiskMetrics presented in the next section, by releasing the restriction in the parameters. Due to the consistency between these two methods, the GARCH model is set with first order in the AR and MA term. Secondly, the VaR forecasting procedure is based on a fixed-window rolling sample. For each equity index, the forecasting procedure will be repeated more than 850 times. In this manner, the optimal choice of the length of AR and MA might not be identical. Thus, it seems meaningless to decide the order of AR and MA here.

Violation (%)	0.0256	0.0245	0.0722	0.0547
Unconditional test	6.4154**	5.6053**	3.4172*	0.1696*
Independent test	0.1265*	0.1248*	0.6256*	0.5159*
Conditional test	6.5420*	5.7302*	4.0428*	0.6855*
CAC40				
Violation (%)	0.0210	0.0140	0.0850	0.0501
Unconditional test	3.4380*	0.5286*	8.0180	1.8937*
Independent test	1.6220*	0.1242*	4.9523**	1.8808*
Conditional test	5.0600*	0.6528*	12.9703	3.7745*
DAX				
Violation (%)	0.0221	0.0163	0.0698	0.0466
Unconditional test	4.1145**	1.2557*	2.7689*	0.0947*
Independent test	1.4577*	0.1739*	4.0775**	0.1667*
Conditional test	5.5721*	1.4296*	6.8464**	0.2614*
TSX				
Violation (%)	0.0279	0.0093	0.0827	0.0326
Unconditional test	8.1555	0.0182*	7.0576	2.6985*
Independent test	0.0534*	0.0654*	0.3631*	0.8206*
Conditional test	8.2089*	0.0836*	7.4207*	3.5191*
Nikkei225				
Violation (%)	0.0186	0.0256	0.0687	0.0442
Unconditional test	2.2359*	6.4154	2.4680*	0.2709*
Independent test	0.4690*	0.1265*	0.0004*	0.1524*
Conditional test	2.7049*	6.5420*	2.4684*	0.4233*
Ave. violation	0.0250	0.0169	0.0759	0.0446

Panel B	Long	Short	Long	Short
Merval				
Violation (%)	0.0187	0.0234	0.0561	0.0397
Unconditional test	2.2588*	4.8730**	0.2784*	0.8872*
Independent test	1.9895*	0.4161*	0.2726*	1.1525*
Conditional test	4.2484*	5.2891*	0.5509*	2.0396*
Bovespa				
Violation (%)	0.0175	0.0234	0.0619	0.0409
Unconditional test	1.7361*	4.8730**	1.0364*	0.6911*
Independent test	0.2327*	1.0365*	0.2838*	1.2980*
Conditional test	1.9688*	5.9095*	1.3202*	1.9891*
IGPA				
Violation (%)	0.0280	0.0327	0.0666	0.0549
Unconditional test	8.2033	12.1321	1.9595*	0.1828*
Independent test	3.7387*	3.8457**	4.8191**	0.8882*
Conditional test	11.9420	15.9779	6.7786**	1.0710*
Bolsa				
Violation (%)	0.0199	0.0234	0.0619	0.0432
Unconditional test	2.8369*	4.8730**	1.0364*	0.3758*
Independent test	0.2996*	1.0365*	0.0723*	0.0440*
Conditional test	3.1365*	5.9095*	1.1087*	0.4198*
Ave. violation	0.0210	0.0257	0.0616	0.0447

Panel C	Long	Short	Long	Short
HSI				
Violation (%)	0.0173	0.0265	0.0657	0.0461
Unconditional test	1.6643*	7.1482	1.7955*	0.1214*
Independent test	0.5609*	0.5451*	0.5808*	1.6832*
Conditional test	2.2252*	7.6933**	2.3763*	1.8045*
TAIEX				
Violation (%)	0.0265	0.0173	0.0565	0.0358
Unconditional test	7.1482	1.6643*	0.3236*	1.7782*
Independent test	0.5451*	0.4756*	0.0089*	0.0052*
Conditional test	7.6933**	2.1399*	0.3325*	1.7834*
KOSPI				
Violation (%)	0.0219	0.0196	0.0611	0.0415
Unconditional test	4.0296**	2.7425*	0.9198*	0.6031*
Independent test	0.3702*	0.3307*	0.0816*	0.0874*
Conditional test	4.3998*	3.0732*	1.0014*	0.6905*

KLCI				
Violation (%)	0.0173	0.0334	0.0496	0.0519
Unconditional test	1.6643*	12.9664	0.0013*	0.0284*
Independent test	0.5609*	6.8167**	0.6383*	3.0892*
Conditional test	2.2252*	19.7831	0.6396*	3.1176*
JCI				
Violation (%)	0.0277	0.0138	0.0565	0.0438
Unconditional test	8.0294	0.5011*	0.3236*	0.3144*
Independent test	2.1102*	0.1465*	4.5811**	0.4169*
Conditional test	10.1396	0.6476*	4.9047*	0.7313*
SET				
Violation (%)	0.0150	0.0069	0.0484	0.0473
Unconditional test	0.8225*	0.4043*	0.0194*	0.0593*
Independent test	5.2659**	0.0364*	3.7674*	0.2405*
Conditional test	6.0884**	0.4407*	3.7868*	0.2997*
Ave. violation	0.0210	0.0196	0.0563	0.0444

1. **(*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

5.2.2.2 RiskMetrics models

The second competing VaR model was proposed by the famous investment bank, J. P. Morgan, in 1996, and its formula is presented in Eq. (4.29) and (4.31). Following J.P. Morgan's suggestion, the decay factor (λ) is set as 0.94 for the daily returns. The backtesting results of this method are displayed in Table 5.9.

Overall, VaR_{RM} offers a range of satisfactory results for the three sets of return series, particularly the Asian market (panel C). However, it is surprising that the poor back-testing performance in developed equity markets (panel A), has the highest violation rate compared with the other two markets. The average of all the violation rates in panel A are 0.0520, 0.0324, 0.0990, and 0.0761 for the long and short position of $VaR_{RM,0.99}$ and $VaR_{RM,0.95}$, respectively. The highest violation rate (some violation rates are even more than 10%) of the developed equity markets indicates that the RiskMetrics model might not be appropriate to forecast the market risk in the six developed equity markets. This phenomenon implies that the VaR_{RM} might underestimate the market risk in developed equity markets, which are not as tranquil as we generally think[74]. On the other hand, violation rates in the Latin American panel are 0.0292, 0.0216, 0.0611, and 0.0476, and the ones of the Asian indices are 0.0237, 0.0158, 0.0548, and 0.0396. Another possible reason for the poor performance of this approach is the nature of its volatility assumption. According to the discussion in section 4.4.2, the RiskMetrics model can be regarded as a simplified version of the GARCH (1,1) model with a restriction in parameters. Thus, some information in the market might not have been fully described due to the hypothetical value of the decay factor, λ. It is unreasonable to set aside all the return series in the same decay process across several years.

[74] This intuition is a general impression. Theoretically, developed equity markets have stronger transparency and more efficiency due to the quality of investors and market regulation. They also have a good flow of market information. Thus, the index return would be less volatile, and the index return would have lower risk. This should mean that the market risk would be easy to forecast with VaR models.

Table 5.9 Backtesting of VaR measured by RiskMetrics model

	$VaR_{RM,0.99}$		$VaR_{RM,0.95}$	
Panel A	Long	Short	Long	Short
S&P500				
Violation (%)	0.0629	0.0419	0.1141	0.0827
Unconditional test	47.8570	21.3861	24.0541	7.0576
Independent test	0.0502*	0.5522*	0.0724*	0.8597*
Conditional test	47.9071	21.9383	24.1265	7.9173*
FTSE100				
Violation (%)	0.0477	0.0396	0.0966	0.0768
Unconditional test	28.0532	18.8913	13.5760	4.8931**
Independent test	0.0004*	0.0460*	0.9350*	1.1927*
Conditional test	28.0537	18.9373	14.5110	6.0858**
CAC40				
Violation (%)	0.0512	0.0314	0.0990	0.0687
Unconditional test	32.3251	11.0416	14.8309	2.4680*
Independent test	1.2369*	0.7621*	1.1817*	1.5895*
Conditional test	33.5620	11.8037	16.0126	4.0575*
DAX				
Violation (%)	0.0419	0.0233	0.0827	0.0652
Unconditional test	21.3861	4.8377**	7.0576	1.6612*
Independent test	0.0918*	0.4146*	0.3631*	1.2995*
Conditional test	21.4780	5.2523*	7.4207**	2.9607*
TSX				
Violation (%)	0.0710	0.0338	0.1269	0.0966
Unconditional test	59.7728	13.1340	33.1897	13.5760
Independent test	0.7124*	0.0002*	0.0536*	0.0001*
Conditional test	60.4852	13.1342	33.2434	13.5761
Nikkei225				
Violation (%)	0.0373	0.0244	0.0745	0.0664
Unconditional test	16.5028	5.6053**	4.1258**	1.9139*
Independent test	0.0155*	1.1667*	0.0054*	0.5602*
Conditional test	16.5183	6.7721**	4.1312*	2.4741*
Ave violation(%)	0.0520	0.0324	0.0990	0.0761
Panel B	Long	Short	Long	Short
Merval				
Violation (%)	0.0234	0.0164	0.0514	0.0292
Unconditional test	4.8730**	1.2724*	0.0152*	3.9537**
Independent test	0.2104	0.2024*	0.5347*	0.0412*
Conditional test	5.0834*	1.4748*	0.5499*	3.9948*
Bovespa				
Violation (%)	0.0175	0.0152	0.0479	0.0374
Unconditional test	1.7361*	0.8718*	0.0351*	1.3600*
Independent test	0.5524*	0.1743*	0.1815*	0.0161*
Conditional test	2.2885*	1.0461*	0.2166*	1.3761*
IGPA				
Violation (%)	0.0502	0.0304	0.0888	0.0818
Unconditional test	30.9876	10.0987	9.6639	6.6856
Independent test	2.3168*	0.6138*	8.5238	0.4204*
Conditional test	33.3044	10.7125	18.1877	7.1060
Bolsa				
Violation (%)	0.0257	0.0245	0.0561	0.0421
Unconditional test	6.4570**	5.6438*	0.2784*	0.5209*
Independent test	**1.0335***	0.1646*	3.5311*	0.0670*
Conditional test	7.4904**	5.8084*	3.8095*	0.5879*
Ave. violation(%)	0.0292	0.0216	0.0611	0.0476
Panel C	Long	Short	Long	Short
HSI				
Violation (%)	0.0450	0.0300	0.0865	0.0704
Unconditional test	25.0643	9.9020	8.7551	2.9324*

Independent test	0.3522*	0.6991*	1.3960*	0.3073*
Conditional test	25.4164	10.6011	10.1510	3.2397*
TAIEX				
Violation (%)	0.0277	0.0185	0.0681	0.0404
Unconditional test	8.0294	2.1757*	2.3324*	0.7849*
Independent test	0.0676*	0.2616*	0.1121*	00634*
Conditional test	8.0970**	2.4373*	2.4445*	0.8482*
KOSPI				
Violation (%)	0.0185	0.0115	0.0461	0.0265
Unconditional test	2.1757*	0.0853*	0.1214*	5.2303**
Independent test	2.0091*	0.1015*	0.2934*	0.0961*
Conditional test	4.1849*	0.1868*	0.4148*	5.3265*
KLCI				
Violation (%)	0.0081	0.0035	0.0219	0.0127
Unconditional test	0.1511*	2.1757*	7.8468	15.5405
Independent test	0.0495*	0.0091*	1.4710*	0.1229*
Conditional test	0.2006*	2.1848*	9.3178	15.6634
JCI				
Violation (%)	0.0334	0.0196	0.0588	0.0473
Unconditional test	12.9664	2.7425*	0.5855*	0.0593*
Independent test	3.8842**	1.8145*	6.3919**	1.6712*
Conditional test	16.8505	4.5570*	6.9774**	1.7304*
SET				
Violation (%)	0.0092	0.0115	0.0473	0.0404
Unconditional test	0.0233*	0.0853*	0.0593*	0.7849*
Independent test	1.5566*	0.1015*	2.3641*	0.1022*
Conditional test	1.5799*	0.1868*	2.4233*	0.8871*
Ave. violation(%)	0.0237	0.0158	0.0548	0.0396

1. ** (*)means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

5.2.2.3 Stochastic volatility model (SV)

As mentioned in Chapter 2, the process of dynamic volatility can be described using two basic methodologies: the GARCH model and the method of stochastic volatility. In this subsection nothing new to the stochastic volatility process will be provided, except the results of the VaR model based on Harvey, Ruiz and Shephard (1994).

The results of backtesting are shown in Table 5.10. Roughly speaking, this VaR model demonstrated a poor performance, especially in panel A (i.e. the indices in developed equity markets) and C (Asian equity indices). However, the independence test shows that the violations tend to be independent in general. There are two indices (FTSE100 and Nikkei225) significant at the 5% level in the developed markets, and very few parts in the Asian markets ($VaR_{0.95}$ of TAIEX long position and $VaR_{0.95}$ of KOSPI short position) are significant. Most results of conditional tests significantly reject the null hypothesis. Similar to the GARCH (1,1) model, the violation rate in the long pattern is systematically higher than in the short position. A possible explanation is that the stochastic volatility model is good at capturing the property of dynamics, but it might ignore the fatness of the return distribution. Thus, the value of the forecasted VaRs would be systematically smaller than the actual returns. Another cause of the high violation rate is the convergence issue of the stochastic model. On average, the un-converged observations of the developed and Asian markets are about 7% and 10%. Those non-converged observations would cause the forecasted VaR to be larger or smaller than they should be, and thus eventually lead to a higher violation rate. In summary, the stochastic volatility might not be good enough for measuring the VaR of the equity indices. This non-converged issue would cause a significant impact in the analysis of MSE in the later section. From the results of VaR backtesting and the issue of convergence, the stochastic volatility process does not seem to be a good VaR forecast.

Table 5.10 Backtesting of VaR measured by stochastic volatility model

	$VaR_{0.99}$		$VaR_{0.95}$	
Panel A	Long	Short	Long	Short
S&P500				
Violation (%)	0.0559	0.0501	0.1630	0.1350
Unconditional test	38.3104	30.8797	64.6136	39.5851
Independent test	0.0174	0.0055*	0.2259*	0.5343*
Conditional test	38.3278	30.8852	64.8395	40.1194
FTSE100				
Violation (%)	0.0163	0.0151	0.0466	0.0279
Unconditional test	1.2557*	0.8582*	0.0947*	4.5173**
Independent test	0.2017*	0.1737*	0.9052*	0.6000*
Conditional test	1.4574*	1.0319*	0.9999*	5.1173*
CAC40				
Violation (%)	0.1211	0.1059	0.1932	0.1758
Unconditional test	147.2334	118.5645	96.5489	77.5523

Independent test	1.1850*	0.7045*	1.5652*	2.1675*
Conditional test	148.4184	119.2689	98.1142	79.7198
DAX				
Violation (%)	0.0629	0.0489	0.1246	0.1036
Unconditional test	47.8570	29.4556	31.4436	17.4759
Independent test	0.0502*	0.3158*	4.1510**	1.7921*
Conditional test	47.9071	29.7714	35.5946	19.2680
TSX				
Violation (%)	0.1001	0.0617	0.1537	0.1234
Unconditional test	108.0070	46.2211	55.7641	30.5844
Independent test	0.3300*	0.3941*	0.6275*	0.1977*
Conditional test	108.3370	46.6153	56.3916	30.7822
Nikkei225				
Violation (%)	0.0105	0.0070	0.0221	0.0175
Unconditional test	0.0085*	0.3830*	7.6451	10.9820
Independent test	1.3467*	5.6469**	0.2649*	7.1311
Conditional test	1.3551*	6.0298**	7.9099**	18.1131
Ave. Violation (%)	0.0611	0.0481	0.1172	0.0972

Panel B	Long	Short	Long	Short
Merval				
Violation (%)	0.0257	0.0070	0.0561	0.0315
Unconditional test	6.4570**	0.3751*	0.2784*	3.0519*
Independent test	2.4842*	0.0368*	1.5217*	0.7649*
Conditional test	8.9412**	0.4119*	1.8000*	3.8167*
Bovespa				
Violation (%)	0.0386	0.0210	0.0876	0.0631
Unconditional test	17.7594	3.4671*	9.1352	1.2417*
Independent test	0.0297*	0.3363*	0.8236*	0.0481*
Conditional test	17.7891	3.8034*	9.9589	1.2898*
IGPA				
Violation (%)	0.0105	0.0047	0.0187	0.0082
Unconditional test	0.0098*	1.3281*	9.9833	20.7613
Independent test	0.0832*	0.0163*	4.0921**	0.0502*
Conditional test	0.0929*	1.3444*	14.0753	20.8115
Bolsa				
Violation (%)	0.0175	0.0105	0.0397	0.0245
Unconditional test	1.7361*	0.0098*	0.8872*	6.1994**
Independent test	4.4398**	1.3439*	6.0116**	1.1620*
Conditional test	6.1760**	1.3536*	6.8988**	7.3614**
Ave. Violation (%)	0.0231	0.0108	0.0505	0.0318

Panel C	Long	Short	Long	Short
HSI				
Violation (%)	0.0854	0.0588	0.1315	0.1223
Unconditional test	83.2910	42.6487	37.0855	30.0302
Independent test	3.8191*	0.0000*	1.2690*	0.0249*
Conditional test	87.1102	42.6487	38.3545	30.0551
TAIEX				
Violation (%)	0.0669	0.0715	0.0738	0.0865
Unconditional test	54.1555	61.0902	3.9468**	8.7551
Independent test	0.2286*	0.0109*	0.0793*	0.0943*
Conditional test	54.3841	61.1010	4.0261*	8.8494**
KOSPI				
Violation (%)	0.0346	0.0358	0.1107	0.0738
Unconditional test	14.0513	15.1662	22.0568	3.9468**
Independent test	0.3312*	0.2737*	0.0066*	2.3614*
Conditional test	14.3825	15.4398	22.0633	6.3082**
KLCI				
Violation (%)	0.1234	0.1223	0.1684	0.1915
Unconditional test	153.2342	150.9504	70.6319	95.4209
Independent test	9.7390	3.7684*	11.1498	2.3214*
Conditional test	162.9732	154.7189	81.7817	97.7423
JCI				

Violation (%)	0.0565	0.0484	0.1015	0.1003
Unconditional test	39.5193	29.1788	16.4071	15.7492
Independent test	10.7519	3.7674*	10.3078	0.4157*
Conditional test	50.2712	32.9462	26.7149	16.1649
SET				
Violation (%)	0.1084	0.1315	0.1269	0.1534
Unconditional test	124.2964	169.4758	33.4855	56.0370
Independent test	8.0576	9.1699	9.6499	10.7213
Conditional test	132.3540	178.6457	43.1354	66.7582
Ave. Violation (%)	0.0792	0.0781	0.1188	0.1213

1. ** (*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

5.2.2.4 Historical simulation

The last competing model is easy to implement in practice, and is called historical simulation (denoted HS). Actually, according to Pérignon and Smith (2010), approximately 47% of their surveyed banks use the method of historical simulation to compute their VaRs. Thus, it is essential to include this approach to compete against the GEV model. This method has been refined for many years, and a filtered historical simulation proposed by Barone-Adesi et al. (1999) is adopted in this section to measure VaR. The procedures of measurement are discussed in Section 4.4.4.

For the measurement, an i.i.d sequence of each return series is essential. According to Barone-Adesi et al. (1999), this could be made using a GARCH model. The length of lag is tested[75] and the best choice of the various models are shown in Table 5.11. In addition, the backtesting results are exhibited from Table 5.12 to Table 5.13. The evidence in Table 5.11 shows that the length of lag of all the return series are within the boundary using GARCH(2,2), but only three indices fit well using GARCH(1,1). This is quite different from previous research[76] (Barone-Adesi et al. (1999), Longin (2000), and Jalal and Rockinger (2008)). On the other hand, some interesting outcomes are presented in the backtesting results of $VaR_{0.99}$ and $VaR_{0.95}$ in Table 5.12 and Table 5.13. Firstly, the performance of the forecasted VaR based on the HS model negatively relates to the size of n. Clearly, using a longer period of data to estimate the empirical distribution and VaR will be more likely to produce higher violation rates because the VaR is underestimated. For example, all of the conditional and unconditional tests tend to accept the null hypothesis (H_0: exceedance ratio=α) in the part of n=250 in panel A. By contrast, only one in the panel of VaR(n=1250), the long position of DAX, accepts the null hypothesis. Similar findings can be seen in all panels of Table 5.12 and 5.13. Crnkovic and Drachman (1995) emphasize that 1,000 observations should be set as the

[75] The length of lag is tested from GARCH(0,0) up to GARCH(3,3).
[76] The GARCH model in this research is based on GARCH(1,1).

minimum for Monte Carlo analysis. However, VaR estimated with a larger set of data like this would tend to respond to the market condition quite slowly. This might also be the main reason why VaR(n=250) provides the best performance throughout these two tables.

Secondly, the method of historical simulation provides better performance for measuring the VaR of the Asian index return, for both the 99%- and 95%-quantile. There are 24 significant results in both the unconditional and conditional statistics in the Asian panel of Table 5.12, corresponding to 19 and 17 significant results in the panel of the developed markets and the Latin American market, respectively. Similar results can also be found in Table 5.12; there are 22 significant results in the Asian panel against 15 ones in the other two panels. Thirdly, it is consistent with the previous section; the violation rate of the long position systematically tends to be larger than the one of the short position, although some of them do not accept the null hypothesis.

Overall, the historical simulation provides a reasonable performance for measuring VaR. However, the reliability of the results generated with this approach needs to be improved since most of the significant null hypothesis are at the probability of 95%, implying that a five per cent possibility exists and so the violation rate could be larger than α. Pérignon and Smith (2010) offer a fair comment on the HS based VaR model, suggesting that forecasted VaR based on this approach contains very little information about future volatility, and causes the worst backtesting performance.

Table 5.11 Selected GARCH model of individual equity return

Index	Model	Index	Model	Index	Model
S&P500	GARCH(2,2)	Merval	GARCH(1,1)	HSI	GARCH(2,2)
FTSE100	GARCH(2,1)	Bovespa	GARCH(1,2)	TAIEX	GARCH(2,2)
CAC40	GARCH(1,1)	IGPA	GARCH(2,2)	KOSPI	GARCH(1,1)
DAX	GARCH(1,2)	Bolsa	GARCH(2,1)	KLCI	GARCH(2,2)
TSX	GARCH(2,2)			JCI	GARCH(2,2)
Nikkei225	GARCH(2,1)			SET	GARCH(2,2)

Table 5.12 99% quantile VaR based on the historical simulation

	VaR(n=1250)		VaR(n=750)		VaR(n=250)	
Panel A	Long	Short	Long	Short	Long	Short
S&P500						
Violation (%)	0.0477	0.0373	0.0454	0.0338	0.0268	0.0221
Unconditional test	28.0532	16.5028	25.3158	13.1340	7.2661	4.1145**
Independent test	0.8039*	0.2152*	0.34125*	0.3863*	**0.5504***	0.2649*
Conditional test	28.8571	16.7180	25.6570	13.5203	7.8164**	4.3793*
FTSE100						
Violation (%)	0.0349	0.0338	0.0314	0.0279	0.0186	0.0244
Unconditional test	14.2273	13.1340	11.0416	8.1555	2.2359*	5.6053**
Independent test	0.3230*	0.3863*	1.5800*	0.0647*	0.2641*	0.1662*
Conditional test	14.5502	13.5203	12.6216	8.2203**	2.5000*	5.7715*
CAC40						
Violation (%)	0.0291	0.0326	0.0279	0.0233	0.0186	0.0163
Unconditional test	9.0822	12.0718	8.1555	4.8377**	2.2359*	1.2557*
Independent test	3.4829*	0.0037*	2.0903*	0.4146*	1.9949*	0.2017*
Conditional test	12.5651	12.0755	10.2459	5.2523*	4.2308*	1.4574*
DAX						
Violation (%)	0.0244	0.0291	0.0268	0.0256	0.0140	0.0163
Unconditional test	5.6053*	9.0822	7.2661	6.4154**	0.5286*	1.2557*
Independent test	1.1667*	0.0420*	0.9195*	0.5029*	0.1478*	0.2017*
Conditional test	6.7721**	9.1242	8.1855**	6.9184**	0.6764*	1.4574*
TSX						
Violation (%)	0.0407	0.0338	0.0326	0.0326	0.0210	0.0186
Unconditional test	20.1258	13.1340	12.0718	12.0718	3.4380*	2.2359*
Independent test	2.6899*	2.5614*	2.7721*	2.7721*	0.3351*	0.2641*
Conditional test	22.8157	15.6954	14.8439	14.8439	3.7731*	2.5000*
Nikkei225						
Violation (%)	0.0291	0.0314	0.0279	0.0244	0.0175	0.0163
Unconditional test	9.0822	11.0416	8.1555	5.6053**	1.7163*	1.2557*
Independent test	0.7099*	2.9953*	0.8103*	2.7143*	0.5547*	2.4390*
Conditional test	9.7921	14.0369	8.9658**	8.3197**	2.2710*	3.6946*
Ave. violation(%)	0.0343	0.0330	0.0320	0.0279	0.0194	0.0190
Panel B	Long	Short	Long	Short	Long	Short
Merval						
Violation (%)	0.0269	0.0199	0.0245	0.0199	0.0210	0.0199
Unconditional test	7.3107	2.8369*	5.6438**	2.8369*	3.4671*	2.8993*
Independent test	4.0216**	0.3905*	2.7064*	0.3905*	1.6169*	0.3905*
Conditional test	11.3324	3.2274*	8.3501**	3.2274*	5.0840*	3.2274*
Bovespa						
Violation (%)	0.0222	0.0210	0.0222	0.0199	0.0187	0.0210
Unconditional test	4.1467**	3.4671*	4.1467**	2.8369*	2.2588*	3.4671*
Independent test	1.4527*	0.3228*	0.3751*	0.3905*	0.2650*	0.3228*
Conditional test	5.5993*	3.7900*	4.5218*	3.2274*	2.5239*	3.7900*
IGPA						

Violation (%)	0.0339	0.0350	0.0327	0.0280	0.0152	0.0350
Unconditional test	13.1975	14.2939	12.1321	8.2033	0.8718*	14.2939
Independent test	5.9118**	2.3529*	6.2745**	2.0829*	0.7562*	2.3529*
Conditional test	19.1093	16.6468	18.4066	10.2862	1.6280*	16.6468
Bolsa						
Violation (%)	0.0304	0.0210	0.0245	0.0164	0.0210	0.0210
Unconditional test	10.0987	3.4671*	5.6438**	1.2724*	3.4671*	3.4671*
Independent test	5.0109**	0.3228*	2.7064*	0.6484*	0.3363*	0.3228*
Conditional test	15.1096	3.7900*	8.3501**	1.9208*	3.8034*	3.7900*
Ave. violation(%)	0.0284	0.0242	0.0260	0.0211	0.0190	0.0242
Panel C	Long	Short	Long	Short	Long	Short
HSI						
Violation (%)	0.0461	0.0392	0.0392	0.0323	0.0254	0.0242
Unconditional test	26.4132	18.6818	18.6818	11.9124	6.3058**	5.5040**
Independent test	1.8237*	0.0430*	1.6961*	0.0045*	2.5127*	0.1703*
Conditional test	28.2369	18.7248	20.3779	11.9169	8.8186**	5.6743*
TAIEX						
Violation (%)	0.0242	0.0219	0.0219	0.0300	0.0161	0.0196
Unconditional test	5.5040**	4.0296**	4.0296**	9.9020	1.2119*	2.7425*
Independent test	1.1793*	0.2698*	1.4710*	0.0263*	2.4537*	0.2957*
Conditional test	6.6833**	4.2994*	5.5006*	9.9282	3.6656*	3.0382*
KOSPI						
Violation (%)	0.0231	0.0219	0.0265	0.0219	0.0173	0.0161
Unconditional test	4.7446**	4.0296**	7.1482	4.0296**	1.6643*	1.2119*
Independent test	1.3193*	0.2698*	2.3047*	0.2698*	2.2214*	0.1998*
Conditional test	6.0638**	4.2994*	9.4529	4.2994*	3.8858*	1.4117*
KLCI						
Violation (%)	0.0242	0.0265	0.0196	0.0242	0.0127	0.0173
Unconditional test	5.5040**	7.1482	2.7425*	5.5040**	0.2532*	1.6643*
Independent test	2.7354*	2.3047*	1.8145*	4.6841**	1.0234*	2.2214*
Conditional test	8.2394**	9.4529	4.5570*	10.1881	1.2766*	3.8858*
JCI						
Violation (%)	0.0219	0.0219	0.0185	0.0196	0.0161	0.0150
Unconditional test	4.0296**	4.0296**	2.1757*	2.7425*	1.2119*	0.8225*
Independent test	3.2300*	1.4710*	4.1232**	1.8145*	2.4537*	2.7088*
Conditional test	7.2596**	5.5006*	6.2989**	4.5570*	3.6656*	3.5313*
SET						
Violation (%)	0.0231	0.0185	0.0219	0.0254	0.0208	0.0173
Unconditional test	4.7446**	2.1757*	4.0296**	6.3058**	3.3614*	1.6643*
Independent test	5.0271**	0.4749*	5.3939**	0.1302*	8.4138	0.5609*
Conditional test	9.7716	2.6506*	9.4235	6.4361**	11.7752	2.2252*
Ave. violation(%)	0.0271	0.0250	0.0246	0.0256	0.0181	0.0183

1. ** (*)means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

Table 5.13 95% quantile VaR based on the historical simulation

	VaR(n=1250)		VaR(n=750)		VaR(n=250)	
Panel A	Long	Short	Long	Short	Long	Short
S&P500						
Violation (%)	0.1257	0.1094	0.1153	0.1036	0.0733	0.0710
Unconditional test	32.3121	21.0263	24.8362	17.4759	3.7641*	3.0854*
Independent test	0.0068*	0.0048*	0.1017*	0.0032*	1.0551*	0.0135*
Conditional test	32.3189	21.0311	24.9379	17.4790	4.8191*	3.0989*
FTSE100						
Violation (%)	0.1153	0.1106	0.1001	0.1013	0.0617	0.0617
Unconditional test	24.8362	21.7679	15.4754	16.1312	1.0038*	1.0038*
Independent test	0.9323*	0.1107*	2.1713*	0.2090*	0.0748*	0.9246*
Conditional test	25.7685	21.8786	17.6467	16.3401	1.0786*	1.9284*
CAC40						
Violation (%)	0.1141	0.1013	0.1059	0.0885	0.0722	0.0605
Unconditional test	24.0541	16.1312	18.8641	9.5557	3.4172*	0.8192*
Independent test	4.8209**	0.5345*	4.0842**	0.1492*	2.6731*	0.1600*
Conditional test	28.8750	16.6657	22.9483	9.7049	6.0902**	0.9792*
DAX						
Violation (%)	0.1048	0.1001	0.1013	0.0943	0.0640	0.0559
Unconditional test	18.1646	15.4754	16.1312	12.3675	1.4251*	0.2619*
Independent test	5.1980**	0.0089*	9.3364	0.0942*	1.3676*	0.0326*
Conditional test	23.3626	15.4843	25.4676	12.4617	2.7928*	0.2022*
TSX						
Violation (%)	0.1292	0.1176	0.1036	0.1024	0.0629	0.0605
Unconditional test	34.9724	26.4303	17.4759	16.7980	1.2059*	0.8192*
Independent test	0.2668*	0.0566*	1.7061*	0.2209*	3.3253*	0.4624*
Conditional test	35.2392	26.4869	19.1820	17.0189	4.5312*	1.2816*
Nikkei225						
Violation (%)	0.1036	0.0850	0.0803	0.0768	0.0594	0.0594
Unconditional test	17.4759	8.0180	6.1506**	4.8931**	0.6524*	0.6524*
Independent test	0.1747*	0.5843*	2.2400*	1.3014*	0.5367*	1.9436*
Conditional test	17.6506	8.6023**	8.3905**	6.1946**	1.1891*	2.5960*
Panel B	Long	Short	Long	Short	Long	Short
Merval						
Violation (%)	0.0783	0.0654	0.0771	0.0689	0.0549	0.0666
Unconditional test	5.3766**	1.7035*	4.9682**	2.5201*	0.1828*	1.9595*
Independent test	1.8100*	1.2301*	0.7394*	0.8944*	0.3292*	1.1114*
Conditional test	7.1867**	2.9337*	5.7076*	3.4145*	0.5120*	3.0709*
Bovespa						
Violation (%)	0.0911	0.0666	0.0864	0.0631	0.0643	0.0666
Unconditional test	10.7587	1.9595*	8.6193	1.2417*	1.4642*	1.9595*
Independent test	2.1515*	0.0052*	4.6592**	0.0481*	2.1514*	0.0052*
Conditional test	12.9102	1.9647*	13.2785	1.2898*	3.6156*	1.9647*
IGPA						
Violation (%)	0.1063	0.1028	0.0876	0.0911	0.0619	0.1028
Unconditional test	19.0228	16.9465	9.1352	10.7587	1.0364*	16.9465
Independent test	16.9046	0.8397*	14.6400	1.5185*	8.9157	0.8397*
Conditional test	35.9274	17.7862	23.7752	12.2772	9.9521	17.7862
Bolsa						
Violation (%)	0.0888	0.0724	0.0724	0.0666	0.0549	0.0724
Unconditional test	9.6639	3.4791*	3.4791*	1.9595*	0.1828*	3.4791*
Independent test	2.4958*	1.1573*	3.5887*	1.8310*	2.6299*	1.1573*
Conditional test	12.1597	4.6364*	7.0678**	3.7905*	2.8128*	4.6364*
Panel C	Long	Short	Long	Short	Long	Short
HSI						
Violation (%)	0.1234	0.1234	0.0969	0.1142	0.0623	0.0646
Unconditional test	30.8802	30.8802	13.8426	24.3460	1.1133*	1.5514*
Independent test	1.7689*	0.2321*	3.3277*	0.0490*	0.8487*	0.3365*
Conditional test	32.6491	31.1123	17.1703	24.3950	1.9620*	1.8880*
TAIEX						

Violation (%)	0.0900	0.0773	0.0911	0.0854	0.0565	0.0577
Unconditional test	10.3420	5.0931**	10.8956	8.2513	0.3236*	0.4454*
Independent test	0.0000*	0.2976*	0.0446*	0.5319*	0.2367*	0.1899*
Conditional test	10.3420	5.3907*	10.9403	8.7832**	0.5603*	0.6353*
KOSPI						
Violation (%)	0.0715	0.0727	0.0669	0.0750	0.0519	0.0542
Unconditional test	3.2555*	3.5937*	2.0559*	4.3145**	0.0284*	0.1369*
Independent test	0.0345*	0.2045*	0.5036*	0.0015*	2.0191*	2.6849*
Conditional test	3.2901*	3.7982*	2.5596*	4.3161*	2.0474*	2.8218*
KLCI						
Violation (%)	0.0934	0.0842	0.0807	0.0830	0.0496	0.0600
Unconditional test	12.0391	7.7603	6.3659**	7.2823	0.0013*	0.7438*
Independent test	4.9011**	3.1574*	7.0546	2.5378*	6.3732**	5.0462**
Conditional test	16.9402	10.9177	13.4206	9.8201	6.3745**	5.7900*
JCI						
Violation (%)	0.0796	0.0634	0.0669	0.0623	0.0542	0.0519
Unconditional test	5.9279**	1.3239*	2.0559*	1.1133*	0.1369*	0.0284*
Independent test	10.0700	3.1633*	10.1535	1.5337*	8.1576	1.1400*
Conditional test	15.9979	4.4873*	12.2094	2.6470*	8.2946**	1.1684*
SET						
Violation (%)	0.0773	0.0796	0.0738	0.0727	0.0565	0.0554
Unconditional test	5.0931*	5.9279**	3.9468**	3.5937*	0.3236*	0.2207*
Independent test	1.8723*	0.9893*	1.6047*	0.0187*	3.3723*	0.8178*
Conditional test	6.9654**	6.9172**	5.5515*	3.6124*	3.6959*	1.0385*

1. ** (*)means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

5.2.3 Other performance tests

The coverage tests in the previous section follow the conventional procedure in this field of testing if the exceedance ratio is equal to the value of α or not. However, the quantitative test might ignore some critical information behind the two methods it is comparing. In this section, three measures for quality testing are adopted to identify the suitability and quality of the VaR measures. For convenience, three forecasted VaR patterns, from 2nd January 2007 to 30th April 2010, based on the five models are presented, Figure 5.2 to Figure 5.7.[77] The three return

[77] The data used in this thesis includes 16 equity indices. The VaR patterns of three equity returns are discussed

series of equity indices are the S&P500, the Merval index (Argentina), and the Heng Seng index. Overall, the forecasted VaR is very volatile around the 430th to 600th observation, which is the period of financial crisis from the third quarter of 2008 to the end of 2009. Another noticeable point in these figures is that some patterns of the forecasted VaR are too fluctuating to apply in practice even in the crisis period[78]. For example, the patterns of the VaR measured by the GARCH (1,1), stochastic volatility and HS models are the three with the most fluctuation or sudden jumps. As mentioned in section 2.4, VaR is not only a risk measure but also a management tool in practice; it can be applied to adjust the portfolio according to managers' investment philosophy. No matter whether it is a long or short position, a volatile VaR result could not support the manager when considering the weight of an individual asset in a portfolio. In addition, it seems hard to believe that daily potential loss, on average, would be more than 10%, although it occurs in actual daily returns and the forecasted VaR pattern[79]. Yet, the quality of a VaR model in this aspect could not be accounted for by the two coverage tests. Consequently, there is a need to provide a benchmark for evaluating the quality and accessibility of the VaR model.

On the other hand, if the pattern of VaR responded to the market condition sluggishly, then it could not be called a proper risk measure. An ideal measure of market risk should provide the potential loss associated with the current market condition. Thus, the creeping VaR from historical simulation would not be good enough to measure the immediate market risk. The

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in this sub-section.

[78] As mentioned in Section 2.4, the VaR is a risk management system used by financial institutions. Generally, the portfolio would be adjusted for reducing the total risk according to the concept of Eq. (2.37) and (2.38). If the portfolio size is big and its forecasted daily VaR is strongly volatile, the adjustment will be difficult to make in practice for two main reasons. Firstly, the adjustment will be associated with numerous securities transactions, and some transactions of securities with low liquidity are difficult to perform in a certain period. Secondly, any adjustment generally carries a high transaction cost making frequent portfolio adjustment impractical.

[79] In the sample set used in this thesis, there are few actual returns (less than 4 observations) smaller than -10% in the developed and Asian equity indices. Most of these extreme returns cluster around the Asian financial crisis in 1997 and the period of subordinated debt crisis in 2008. However, there are many actual daily returns less than -10% in the Latin American markets, especially in Brazil and Argentina.

VaR pattern based on the HS approach with 1,250 observations moves much too slowly.

The results of the VaR performance based on the three benchmarks are exhibited in Table 5.14. Leaving aside the GARCH (1,1), RiskMetrics, and stochastic volatility models, only the case of the 5-day block of the GEV model and the HS with the shortest estimation period[80], n=250, are included this section because they offer the best performance compared with n=750 and n=1250. The results in the first and second column describe the mean squared error (MSE) on average of the VaR series, which is applied to measure the fluctuation of the VaR pattern. From this angle, the GEV with the 5-day block and the RiskMetrics model produce the equally smallest MSE. By contrast, the historical simulation based on 250 observations has the largest. The stochastic volatility model shows a great diversity in the MSE result. The variability of some indices is small, which means the SV model provides a good forecasting performance in those indices, such as 0.0967 for TSX, 0.1382 for DAX and 0.1518 for CAC40. However, it offers a very lamentable performance in the TAIEX and SET indexes. In Panel E, the MSEs (i.e. from the stochastic volatility model) of the Nikkei225, TAIEX, KOSPI, and SET are very high. According to Eq. (4.50), MSE is the average value of the squared difference between VaR_t and E(VaR). As mentioned in Section 5.2.2.3, some of the forecasted VaR (about 2% to 10%) do not converge and this causes those non-converged VaRs to be extremely high. In this manner, MSEs of Nikkei225, TAIEX, KOSPI, and SET are high. Take TAIEX as an example, its original volatility of VaR is 0.2657, if we exclude the 57 non-converged observations (about 6.5% of whole sample), and the volatility of the rest is 0.1111, then the MSE of the long and short positions are 0.0123 and 0.0087, respectively.

The second benchmark is the D statistics shown in Eq. (4.51), the results of this benchmark are displayed in the third and fourth columns in Table 5.14. It emphasizes the quality of the

[80] In the method of historical simulation (HS), three forecasting periods (250, 750, and 1250 previous returns) are used to forecast one-day ahead VaR. The results in Table 5.12 and Table 5.13 show that HS with a previous 250 historical returns is more appropriate compared to other forecasting periods. Thus, the other two results are not included in the quality checks .

VaR model by calculating the average distance between the non-violated VaRs and the actual returns. The main goal of the D measure is to evaluate the conservativeness of the VaR. Generally, a VaR model which derives a larger number of D indicates that the approach is most likely too conservative in measuring market risk and thus would cause financial institutions to lose their efficiency in capital usage. By contrast, the VaR model with a small D could be a good risk measure if the coverage of the VaR model is also appropriate. The evidence in Table 5.14 shows that the GEV(n=5) and GARCH(1,1) models exhibit equally strong performances, averaging around -0.0415 and -0.0380 for the long position, and 0.0401 and 0.0345 for the short position. This implies that the GEV model and the GARCH model properly forecast the market risk and are less likely to be over-conservative. In contrast, the worst case is the stochastic volatility model. Its average of D measures are -5.2% and 4.8% for the long and short respectively, implying that the SV model is more conservative than the other VaR models.

The last benchmark in this section, Q[81], looks at the magnitude of violation of those violated observations. In other words, it presents the average potential extra loss to each VaR model and evaluates the performance from the aspect of the quantity of the loss. The results for Q are shown in the fifth and sixth columns in Table 5.14. The GARCH (1,1) model demonstrates the best performance, on average -0.0105 for the long position and 0.0086 for the short position. One possible explanation for this is found in the nature of the GARCH model; the original function of the GARCH model is to forecast the return and fit the process of volatility. The average potential losses of the other models are equivalent to each other, but the RiskMetrics model produces the worse result, -0.0163 for the long and 0.0155 for the short position, suggesting the portfolio might encounter a 1.63% (1.55%) loss in the long (short) position on average once a violation occurs.

[81] The details of Q can be found in Eq. (4.52).

Figure 5.2 1%-VaR of long position v.s. actual return of S&P 500 index

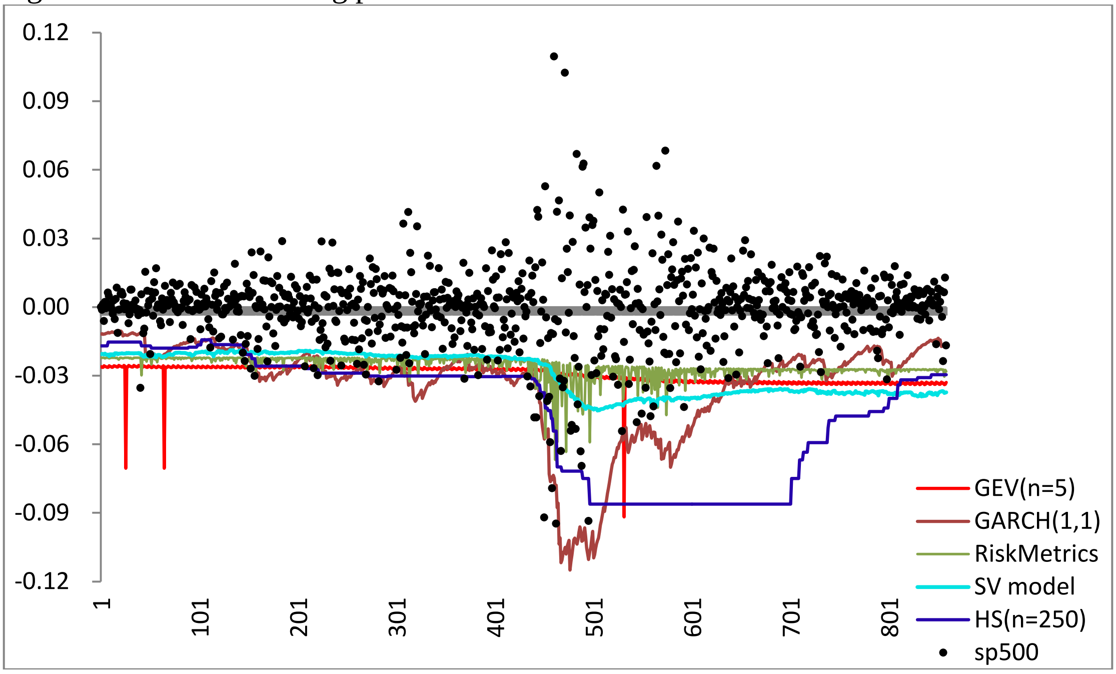


Figure 5.3 1%-VaR of short position v.s. actual return of S&P 500 index

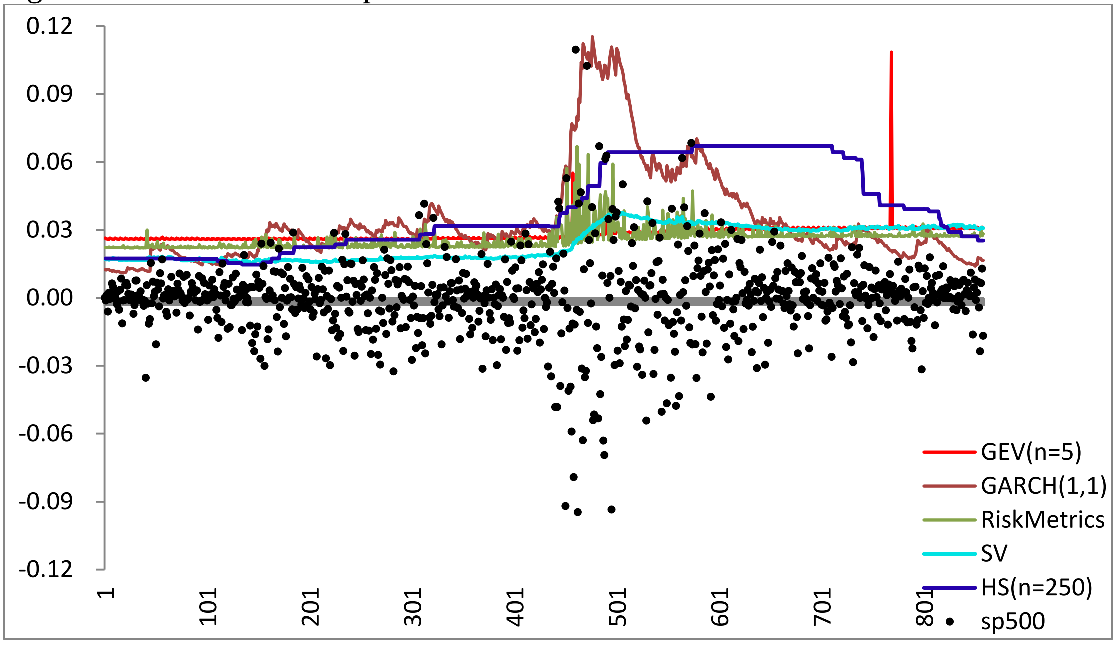


Figure 5.4 1%-VaR of long position v.s. actual return of Merval index

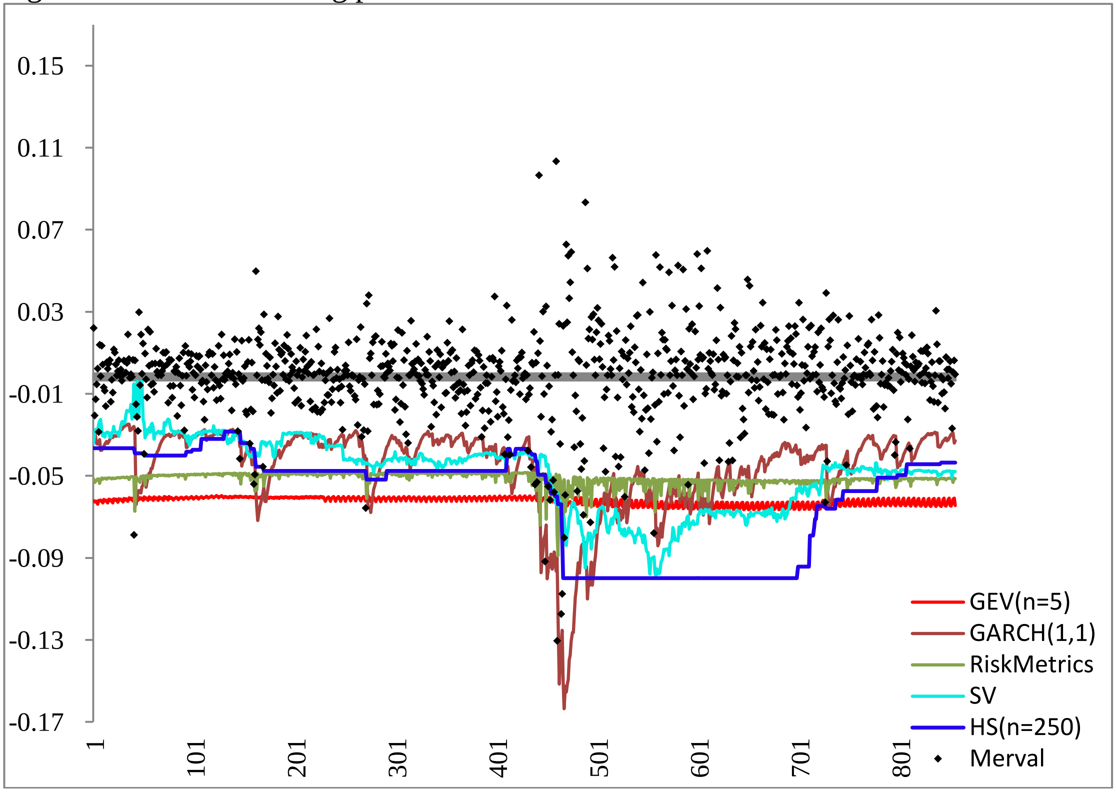


Figure 5.5 1%-VaR of short position v.s. actual return of Merval index

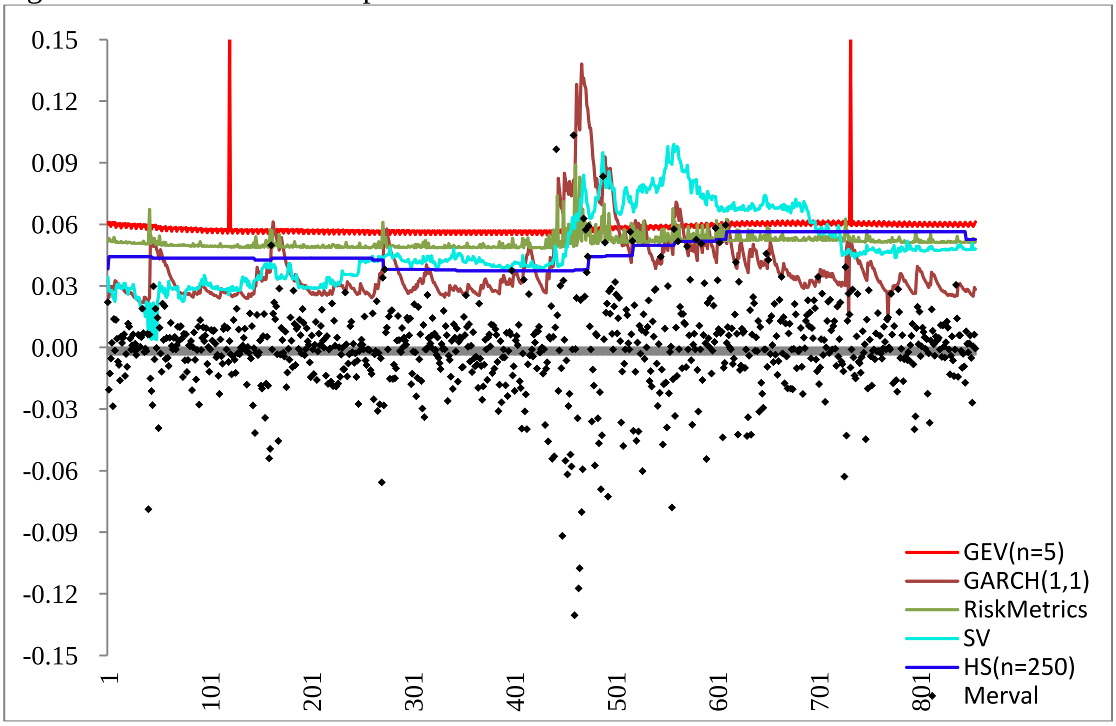


Figure 5.6 1%-VaR of long position v.s. actual return of Hang Seng index

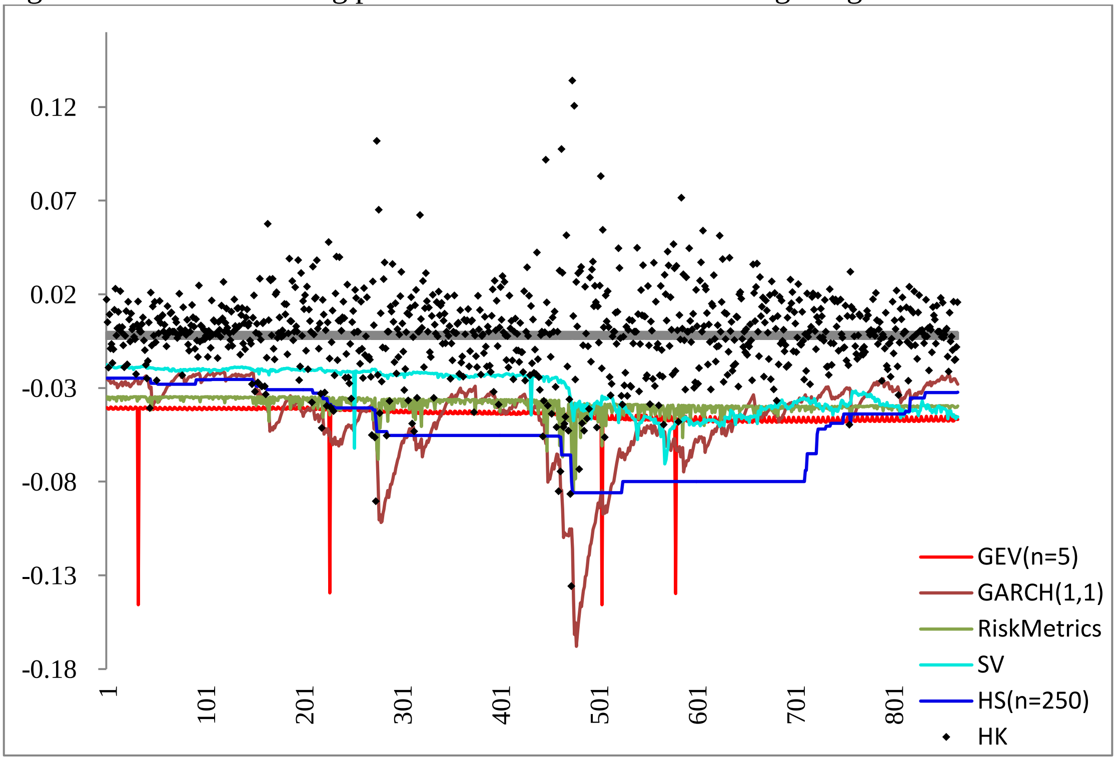


Figure 5.7 1%-VaR of short position v.s. actual return of Hang Seng index

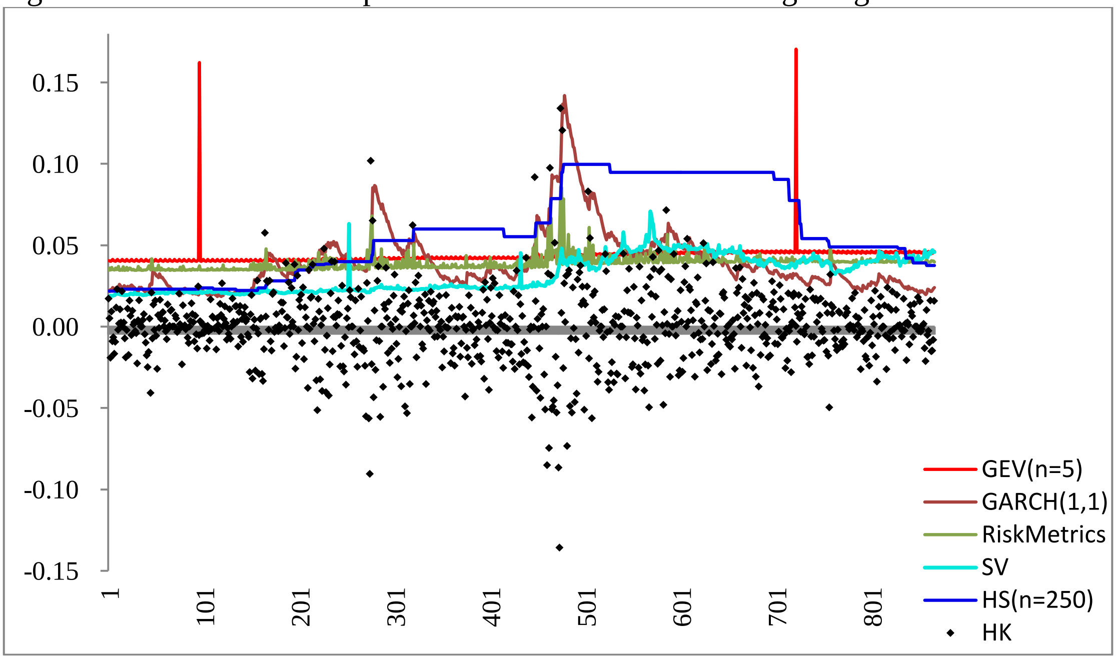


Table 5.14 The comparison of other benchmarks across various VaR models

	MSE (10^{-4})		D		Q	
Panel A: GEV(n=5)	Long	Short	Long	Short	Long	Short
S&P500	0.1797	0.1296	-0.0316	0.0301	-0.0172	0.0178
FTSE100	0.2309	0.0929	-0.0306	0.0293	-0.0157	0.0186
CAC40	0.2487	0.3637	-0.0378	0.0368	-0.0181	0.0320
DAX	0.3505	0.1020	-0.0408	0.0378	-0.0209	0.0354
TSX	0.2290	0.1127	-0.0295	0.0252	-0.0158	0.0155
Nikkei225	0.2651	0.3221	-0.0408	0.0401	-0.0217	0.0190
Merval	0.0365	0.2349	-0.0635	0.0590	-0.0184	0.0159
Bovespa	0.0442	0.6364	-0.0601	0.0585	-0.0212	0.0167
IGPA	0.0358	0.0240	-0.0204	0.0191	-0.0089	0.0140
Bolsa	0.3733	0.2674	-0.0428	0.0445	-0.0106	0.0138
HSI	0.5186	0.3976	-0.0466	0.0448	-0.0158	0.0327
TAIEX	0.1364	0.1774	-0.0455	0.0440	-0.0073	0.0130
KOSPI	0.4574	0.2033	-0.0528	0.0506	-0.0207	0.0148
KLCI	0.0072	0.3287	-0.0349	0.0357	-0.0129	0.0058
JCI	0.1078	0.1078	-0.0434	0.0411	-0.0170	0.0160
SET	0.4257	0.1504	-0.0438	0.0456	-0.0206	0.0129
Ave.	0.2279	0.2282	-0.0415	0.0401	-0.0164	0.0184
Panel B: GARCH	Long	Short	Long	Short	Long	Short
S&P500	4.7684	4.7368	-0.0353	0.0353	-0.0070	0.0076
FTSE100	3.2728	2.3096	-0.0335	0.0288	-0.0091	0.0088
CAC40	5.0468	5.0108	-0.0424	0.0433	-0.0113	0.0119
DAX	4.9998	4.8855	-0.0420	0.0424	-0.0101	0.0096
TSX	4.3115	4.2884	-0.0337	0.0336	-0.0068	0.0074
Nikkei225	4.4266	3.1240	-0.0394	0.0341	-0.0170	0.0093
Merval	4.3465	3.0484	-0.0473	0.0403	-0.0141	0.0087
Bovespa	4.5683	3.2437	-0.0491	0.0419	-0.0099	0.0135
IGPA	1.2924	0.9106	-0.0223	0.0186	-0.0064	0.0042
Bolsa	2.7725	1.9509	-0.0376	0.0326	-0.0098	0.0100
HSI	5.3072	3.7507	-0.0473	0.0409	-0.0120	0.0122
TAIEX	1.4123	0.9991	-0.0372	0.0311	-0.0074	0.0075
KOSPI	3.3097	2.3386	-0.0390	0.0327	-0.0107	0.0060
KLCI	1.0241	0.7258	-0.0240	0.0204	-0.0107	0.0032
JCI	3.3882	3.3866	-0.0404	0.0392	-0.0132	0.0070
SET	1.7969	1.7906	-0.0371	0.0363	-0.0126	0.0101
Ave.	3.5028	2.9063	-0.0380	0.0345	-0.0105	0.0086
Panel C: RiskMetrics	Long	Short	Long	Short	Long	Short
S&P500	0.2256	0.2256	-0.0285	0.0280	-0.0150	0.0143
FTSE100	0.1509	0.1509	-0.0279	0.0277	-0.0143	0.0126
CAC40	0.2579	0.2579	-0.0350	0.0349	-0.0182	0.0232
DAX	0.2049	0.2049	-0.0369	0.0361	-0.0191	0.0272
TSX	0.2215	0.2215	-0.0262	0.0249	-0.0133	0.0140
Nikkei225	0.1770	0.1770	-0.0360	0.0362	-0.0202	0.0168
Merval	0.1346	0.1346	-0.0534	0.0524	-0.0182	0.0128
Bovespa	0.1613	0.1613	-0.0541	0.0530	-0.0232	0.0246
IGPA	0.0620	0.0620	-0.0193	0.0178	-0.0087	0.0077
Bolsa	0.0906	0.0906	-0.0387	0.0382	-0.0115	0.0151
HSI	0.2262	0.2262	-0.0417	0.0408	-0.0144	0.0201
TAIEX	0.0758	0.0758	-0.0394	0.0389	-0.0081	0.0106
KOSPI	0.0833	0.0833	-0.0447	0.0437	-0.0189	0.0158
KLCI	0.0210	0.0210	-0.0343	0.0336	-0.0133	0.0051
JCI	0.1121	0.1121	-0.0382	0.0363	-0.0143	0.0166
SET	0.0635	0.0635	-0.0396	0.0393	-0.0298	0.0117
Ave.	0.1418	0.1418	-0.0371	0.0364	-0.0163	0.0155
Panel D: HS(n=250)	Long	Short	Long	Short	Long	Short
S&P500	6.9402	3.6581	-0.0464	0.0406	-0.0116	0.0134

FTSE100	2.9901	5.1055	-0.0423	0.0442	-0.0133	0.0114
CAC40	4.7332	11.2954	-0.0509	0.0567	-0.0159	0.0231
DAX	4.8315	9.5276	-0.0498	0.0542	-0.0184	0.0214
TSX	5.4064	4.7304	-0.0441	0.0374	-0.0133	0.0133
Nikkei225	8.5312	3.0420	-0.0558	0.0447	-0.0185	0.0184
Merval	6.3989	0.5078	-0.0622	0.0488	-0.0189	0.0168
Bovespa	4.7535	1.0349	-0.0581	0.0497	-0.0172	0.0270
IGPA	1.0020	0.1897	-0.0310	0.0199	-0.0095	0.0078
Bolsa	1.0885	0.8445	-0.0438	0.0396	-0.0084	0.0170
HSI	4.3233	7.2443	-0.0541	0.0594	-0.0121	0.0156
TAIEX	0.5683	1.4113	-0.0438	0.0387	-0.0089	0.0088
KOSPI	3.8319	1.7215	-0.0500	0.0400	-0.0190	0.0150
KLCI	0.5464	0.1710	-0.0298	0.0238	-0.0136	0.0061
JCI	2.2972	2.0854	-0.0518	0.0454	-0.0170	0.0152
SET	3.1637	1.4006	-0.0448	0.0388	-0.0137	0.0108
Ave.	3.8379	3.3731	-0.0474	0.0426	-0.0143	0.0151
Panel E: SV	Long	Short	Long	Short	Long	Short
S&P500	0.3873	0.3873	-0.0237	0.0232	-0.0141	0.0122
FTSE100	3.0977	3.0977	-0.0438	0.0439	-0.0119	0.0083
CAC40	0.1518	1.7574	-0.0250	0.0245	-0.0145	0.0160
DAX	0.1382	0.2309	-0.0301	0.0293	-0.0171	0.0171
TSX	0.0967	0.0967	-0.0232	0.0218	-0.0127	0.0114
Nikkei225	19.1366	19.1366	-0.0955	0.0961	-0.0339	0.0329
Merval	3.1804	3.1794	-0.0506	0.0489	-0.0211	0.0215
Bovespa	1.8747	1.8521	-0.0453	0.0441	-0.0137	0.0212
IGPA	8.2189	8.2201	-0.0415	0.0410	-0.0067	0.0160
Bolsa	5.0162	5.0124	-0.0498	0.0505	-0.0131	0.0150
HSI	1.2355	1.1982	-0.0356	0.0349	-0.0164	0.0179
TAIEX	705.0279	704.9489	-0.1593	0.1603	-0.0129	0.0092
KOSPI	44.5601	1.1982	-0.0721	0.0332	-0.0037	0.0122
KLCI	0.6793	0.6783	-0.0149	0.0141	-0.0079	0.0057
JCI	0.3545	0.3530	-0.0304	0.0287	-0.0170	0.0133
SET	117.8449	117.8256	-0.0839	0.0860	-0.0132	0.0116
Ave.	56.9375	54.3233	-0.0515	0.0488	-0.0144	0.0151

Note: In the Panel E of this Table, MSEs of Nikkei225, TAIEX, KOSPI, and SET (i.e. from stochastic volatility model) are very high. According to Eq. (4.50), MSE is the average value of the squared difference between VaR_t and E(VaR). As mentioned in Section 5.2.2.3, some of forecasted VaR (about 2% to 10%) do not converge and this causes those un-converged VaR to be extremely high. In this manner, MSEs of Nikkei225, TAIEX, KOSPI, and SET are high. Take TAIEX as an example (its original volatility of VaR is 0.2657), if we exclude the 57 un-converged observations (it is about 6.5% of whole sample, and the volatility of the rest is 0.1111), then the MSE of long and short positions are 0.0123 and 0.0087, respectively.

5.2.4 Summary

This section of univariate analysis includes the estimation of the parameters for various GEV models and their results for equity indices. In addition, the analysis of the results of backtesting the GEV model and the competing models were provided as well. Moreover, apart from the two traditional coverage tests, several benchmarks focusing on the variability, conservativeness and potential loss are presented in the last section.

Based on the distribution of generalized extreme values, some specific properties of financial returns are described. For example, the results show that the tail parameters tend to be negative, indicating the characteristic of the fat-tail of financial returns could be captured appropriately. In addition, the results of the equality test of estimated parameters to the left and right tails of the GEV distribution show that the distribution of financial returns tends to be skewed (Peiro (1999) provides a comprehensive discussion for the possible causes of skewness). However, the estimation of the parameters is also highly associated with the size of the block.

From the perspective of coverage tests, the GEV model with the 5-day block and the GARCH(1,1) model provide the best performance for measuring VaR, particularly for the Asian indices. The GEV model with the 10-day block also does a reasonably good job, however, it is not suitable for the developed equity market. Due to the stronger fluctuations in the series of 10-day returns, the VaR sequence of the GEV model produces a poor coverage, i.e. there are too many violations. Looking at the figures, some drawbacks of the models can be found. For example, the pattern of historical simulation responds to the market condition slowly, and the GARCH model is too volatile to be implemented in practice. The results of the RiskMetrics are consistent with Eberlein, Kallsen and Kristen's (2003) suggestion that the VaR pattern is influenced by the fixed decay parameter, and the slow change in volatility produces a slow change pattern in forecasted VaR.

In the final part of this section, three substantial benchmarks (MSE, D, and Q) are used to evaluate the VaR models. Specifically, MSE describes the suitability of the risk measure to be implemented in practice by measuring the variability of whole observations. D focuses on the conservativeness of the VaR measure by calculating the average distance between the forecasted VaRs and their corresponding actual returns of the non-violated observations. In contrast, Q is concerned with the magnitude of the extra loss of the violations. Combined with the coverage tests, one could comprehensively understand the properties of any VaR measure, such as accuracy, variability, conservativeness, and magnitude of loss of the violations. Overall, the GARCH (1,1) model and the GEV (n=5) demonstrate better performances than the other models.

5.3 Correlation analysis

In this section, a different measure of dynamic conditional correlation (DCC), called tail-DCC, is provided. It will then be applied in aggregating the individual VaRs into a portfolio VaR in the next section. The original DCC proposed by Engle (2002), also called linear correlation, mainly focuses on the relationship between the entirety of two sequences, however, the tail-DCC model pays attention only on the tail area of the distribution of financial returns. In this thesis, the observations in the tail area are sampled by the procedure of block maxima (minima) discussed in Section 3.4.1. Some critical features regarding this methodology are stressed and two main properties of tail-DCC are discussed.

5.3.1 The fatness of the distribution

The most significant advantage of tail-DCC for risk management based on extreme returns is that it emphasizes the discrepancies between the left and right tails rather than looking at only one relationship within the whole sequence. This feature highly coincides with the spirit of VaR, because we care more about the big-price changes than the small ones, and the big price changes fall in the tail area of the return distribution. Furthermore, lots of hypothesises in related fields have derived empirical support for suggesting that the distribution of financial returns are skewed (Singleton and Wingender (1986), Lai (1991), and Yan (2005)). The original DCC approach could not fully describe the relationship between the sequences of financial returns, making it inappropriate for risk management. That failing is demonstrated in the patterns of the degrees of freedom (DF) to each index portfolio shown in Figure 5.8 to Figure 5.13. The pattern in black is obtained via the original DCC model looking at whole samples. By contrast, the red and green ones correspond to the DF of left- and right-tail derived from the tail-DCC model respectively. From the aspect of the DCC model, the DF of developed equity markets is the largest one in the data set, winding around 11, compared with

6.5 and 8.5 for Asian and Latin American equity markets respectively. Statistically, the DF of the developed market implies that the distribution of the market is closer to normal distribution than the other two markets. For instance, the critical values of the 1% student distribution with DF=11 and DF=6 are -2.7181 and -3.1427, respectively. The former is much closer to the normal distribution with the same probability (i.e. $Z_{0.01}$= -2.3264). On the other hand, the distribution of the Asian equity market portfolio tends to have a ticker tail on average. One could clearly observe the difference between these markets through the DCC model. However, some critical characteristics of financial returns might be neglected if the analysis was focused on the entire sample. The original DCC model regards the entire sample as a whole, and only provides one correlation representing the whole distribution. However, the tail-DCCs, as shown in red and green, are quite distinct from the original DCC. In the pattern of the developed market, the magnitude of thickness in the left and right tail is quite consistent from 2007 to the third quarter of 2008, and then they significantly disperse in the 450^{th} observation, which is around the beginning of the financial crisis in 2008. Moreover, the imbalance between the left and right tail could naturally be different through the backtesting period. For example, the patterns of the Asian and Latin American markets show that the magnitude of fatness in the left and right tails is obviously different. In the Asian equity market, the left tail is thicker than the right tail, which implies that the risk in the left tail is larger than in the right tail. Clearly, the long position in the Asian equity market is more risky than the short position. However, it is the opposite condition in the Latin America equity market. According to the DF in the left and right tail, the risk in the right tail is higher than in the left tail. In summary, the discussion above stresses that the tail-DCC approach could offer some meaningful characteristics and content from behind the return sequences that are neglected by the original DCC model. In addition, the three findings above can also be found in the low frequency of the extreme observation patterns, for example, the 10-day trading period. Specifically, each extreme return is sampled from ten continuous observations by the rolling window method. The results with the ten-day block are shown from Figure 5.11 to Figure 5.13. In general, the DF of the 10-day block method is likely to be higher than the DF of the 5-day block, indicating that the multivariate limiting distribution has a thicker tail. The results of n=10 are similar to the case of n=5 , in fact, the case of n=22 for the one month block also has consistent results, but they are not displayed in this thesis.

Figure 5.8 The degree of freedom of developed equity markets (n=5)

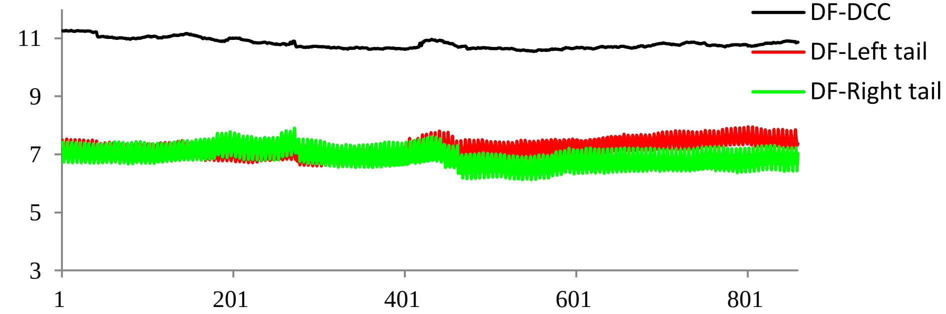


Figure 5.9 The degree of freedom of Asian equity markets (n=5)

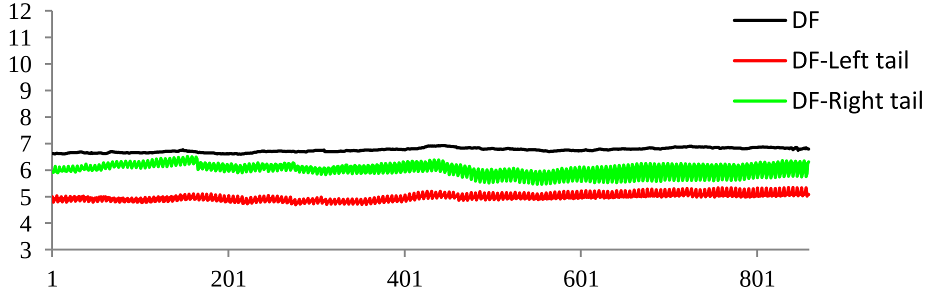


Figure 5.10 The degree of freedom of Latin American equity markets (n=5)

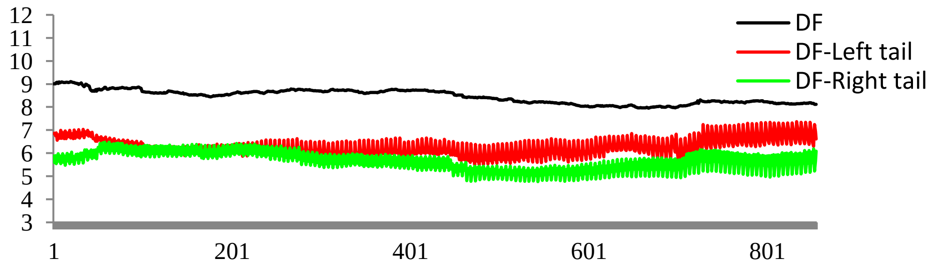


Figure 5.11 The degree of freedom of developed equity markets (n=10)

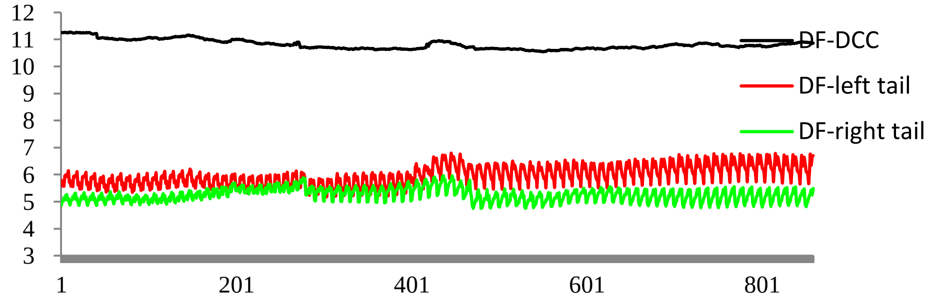


Figure 5.12 The degree of freedom of Asian equity markets (n=10)

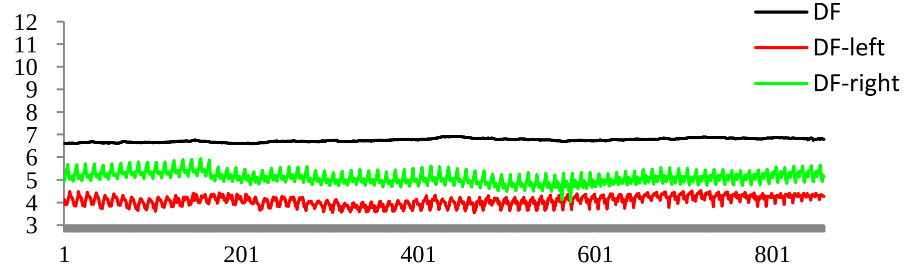
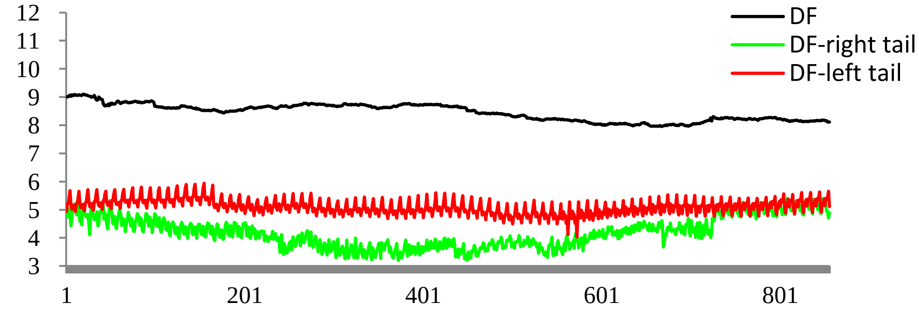


Figure 5.13 The degree of freedom of Latin American equity markets (n=10)



5.3.2 The results of tail-DCC

The second part of this section is graphical analysis of the time-varying correlations. The results of the tail-DCC are exhibited in Figure 5.14 to Figure 5.20, corresponding to the results of the original DCC model shown from Figure 5.21 to Figure 5.23. For convenience, all of the individual equity indexes are labelled as a number from 1 to 6 as displayed in Table 5.15. For example, RO21 with black scatters in panel (a) and (b) of Figure 5.14 indicates the patterns of dynamic conditional correlation between the S&P 500 and TSX indexes in the left and right tails, respectively. The panels (a) and (b) from Figure 5.14 to Figure 5.16 are the tail DCCs between the developed equity markets, corresponding to panel (a), (b), and (c) in Figure 5.21 based on the original DCC model. In the developed equity market, the average dynamic conditional correlation of the left tail is generally between 0.4345 (RO62 for TSX/Nikkei) and 0.8483 (RO54 for CAC40/DAX). On the other hand, the ones for the right tail are from 0.3331 (RO62 for TSX/Nikkei) to 0.8176 (RO54 for CAC40/DAX). In the Latin American equity market, RO41 (for IGPA/Merval) and RO32 (for Bolsa/Bovespa) are the lowest and the highest tail-DCC in both two tails. In the Asian equity market, the pair, HSI/JCI, has the highest tail-DCC in both the left and right tail, 0.5331 and 0.5018, respectively. Yet, RO62 (SET/TAIEX) and RO53 (KLCI/KOSPI) are the lowest tail-DCC in the left and right tail, for 0.2587 and 0.2667, respectively. As in the detailed results shown in Table 5.16, the original DCC model tends to systematically derive higher correlation patterns than the tail DCC model, implying that these non-extreme observations could be highly correlated with each other.

Table 5.15 The numeric labels of the indices

Indices of developed equity market	Corresponding No.	Indices of Asian equity market	Corresponding No.	Indices of Latin American equity market	Corresponding No.
S&P 500	1	HSI	1	IGPA	1
TSX	2	TAIEX	2	Bolsa	2
FTSE 100	3	KOSPI	3	Bovespa	3
CAC 40	4	JCI	4	Merval	4
DAX	5	KLCI	5		
Nikkei 225	6	SET	6		

Table 5.16 Average tail DCC and original DCC

	Left tail		Right tail		Entire sample	
Panel A: developed equity market	Average DCC	SD.	Average DCC	SD.	Average DCC	SD.
RO21	0.7264	0.0675	0.5979	0.1068	0.7012	0.0665
RO31	0.6683	0.0654	0.5450	0.0869	0.5549	0.0674
RO41	0.5879	0.0921	0.5414	0.0933	0.5231	0.0880
RO51	0.5631	0.1017	0.5298	0.1116	0.5251	0.0987
RO61	0.4736	0.1093	0.3686	0.1246	0.0852	0.0365
RO32	0.6332	0.0937	0.5183	0.1292	0.5373	0.0645
RO42	0.5739	0.1155	0.4890	0.1575	0.5305	0.0729
RO52	0.5535	0.1148	0.4730	0.1546	0.5131	0.0813
RO62	0.4345	0.1151	0.3331	0.1432	0.1742	0.0475
RO43	0.7240	0.0690	0.6980	0.1076	0.8568	0.0443
RO53	0.6865	0.0757	0.6037	0.1193	0.8199	0.0454
RO63	0.5158	0.0977	0.3920	0.0947	0.2975	0.0941
RO54	0.8483	0.0445	0.8176	0.0657	0.9482	0.0145
RO64	0.4955	0.0974	0.3828	0.1144	0.3079	0.0924
RO65	0.4811	0.0888	0.3747	0.1522	0.2981	0.1006
Panel B: Latin American equity market						
RO21	0.6134	0.0767	0.3533	0.1396	0.5322	0.0810
RO31	0.5481	0.0712	0.3497	0.1234	0.5250	0.0887
RO41	0.5393	0.0734	0.2940	0.1320	0.5085	0.0975
RO32	0.7483	0.0448	0.5345	0.0823	0.7103	0.0573
RO42	0.6767	0.0581	0.4365	0.0882	0.6105	0.0633
RO43	0.7312	0.0530	0.4693	0.0998	0.6738	0.0491
Panel C: Asian equity market						
RO21	0.4753	0.0846	0.3881	0.0944	0.7012	0.0665
RO31	0.5293	0.0745	0.4454	0.1033	0.5549	0.0674
RO41	0.5331	0.0646	0.5018	0.1200	0.5231	0.0880
RO51	0.5290	0.0701	0.4290	0.0846	0.5251	0.0987
RO61	0.3871	0.1110	0.3986	0.1065	0.0852	0.0365
RO32	0.5022	0.0590	0.4160	0.0824	0.5373	0.0645
RO42	0.3498	0.0922	0.2785	0.1343	0.5305	0.0729
RO52	0.3898	0.0642	0.3072	0.0954	0.5131	0.0813
RO62	0.2587	0.1213	0.3012	0.1067	0.1742	0.0475
RO43	0.4309	0.0861	0.3334	0.1224	0.8568	0.0443
RO53	0.3994	0.0675	0.2667	0.0971	0.8199	0.0454
RO63	0.3460	0.1303	0.3698	0.1113	0.2975	0.0941
RO54	0.4919	0.0767	0.4187	0.1100	0.9482	0.0145
RO64	0.4322	0.0829	0.3623	0.1286	0.3079	0.0924
RO65	0.4078	0.0739	0.3507	0.0837	0.2981	0.1006

According to the graphical evidences of the tail-DCCs as displayed in Figure 5.14 to Figure

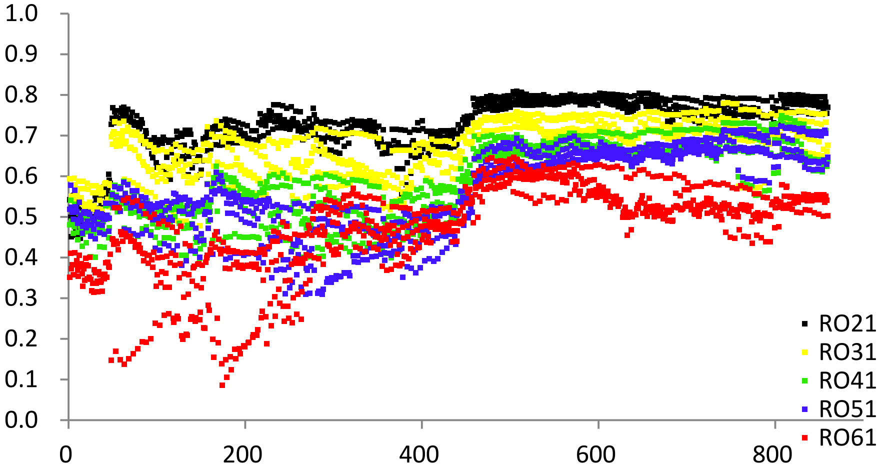
5.23, several interesting findings cannot be ignored. Firstly, most patterns of tail-DCC generally exhibit positive structures, with the exception of several pairs in the Asian market. However, this is reasonable and acceptable as the big price changes of the pair of equity indices tend to rocket and to drop down with each other. Although the correlation theoretically spans between -1 to +1, the equity indices still move together towards the same direction because of the same market information.

Secondly, another consistent characteristic to most patterns of the tail-DCC model is that it is obvious that correlations in the left tail are systematically higher than the ones in the right tail. For example, the correlation of the S&P 500 and TSX indexes in the left tail (panel (a) of Figure 5.14) hovers around 0.7 to 0.8; in contrast, the corresponding correlation in the right tail (panel (b) of Figure 5.14) fluctuates between 0.5 and 0.7. Similarly imbalanced results can also be found in other pairs. The observations of each pair of returns falling in their left tail could be regarded as showing that the markets are in a downslide, and the ones in the right tail imply that the conditions of equity markets are in a trend of escalation. The phenomenon of the correlation of the negative extremes being always greater than the correlation of positive ones is reasonably natural. A possible explanation could be due to investors' expectations or their conservativeness; the investors' behaviour would be affected more strongly by bad news than by good news. Thus, the market indices would move together when the bad news arrived. By contrast, the market indices might not move as consistently with the arrival of good news. Similar results can be also found in previous research (see Jondeau and Rockinger (1999), Longin (2001), Hartmann, Straetmans and Vries (2004), and Poon, Rockinger and Tawn (2004)). However, most of these works focus on the tail dependence by extracting the extreme observations over a particular threshold method, which might cause a confused result (as discussed in Section 2.3.3). Another difficulty of this approach is that the results of this method are highly affected by the choice of the threshold; however, the best way to choose the appropriate threshold is still unclear. Fortunately, the results seem unaffected by the method of dependence.

Thirdly, another phenomenon can be found in the patterns of the tail-correlation - the existence of a structural change or a circulation. The changes in the tail-DCC consistently present in the left and right tail. For instance, the blue scatters, denoted RO51, (tail-DCC between the DAX and S&P 500) in Figure 5.14 exhibit an obvious change around the 450th observation, particularly in the right tail. The tail-DCC spans between 0.4 and 0.55 in the left tail, however, it is around 0.6 to 0.7 in the right tail. The obvious changes happening around the 450th observation in most patterns of tail-DCC could be attributed to the international financial crisis in 2008. Some patterns of correlation present a slight change around the 280th to 300th observation, corresponding to the second quarter of 2008, for example, RO52 (DAX/TSX) in both panel (a) and (b) of Figure 5.15, and RO53 (DAX/FTSE 100) in both panel (a) and (b) of Figure 5.16. Another case of circulation can be found in the Asian equity market, from Figure 5.18 to Figure 5.20. Moreover, the tail-DCC based on the 10-day block also presents similar results from Figure D- 1 to Figure D- 7 in appendix D.

Figure 5.14 The dynamic conditional correlations of developed market indices

(a) Left tail



(b) Right tail

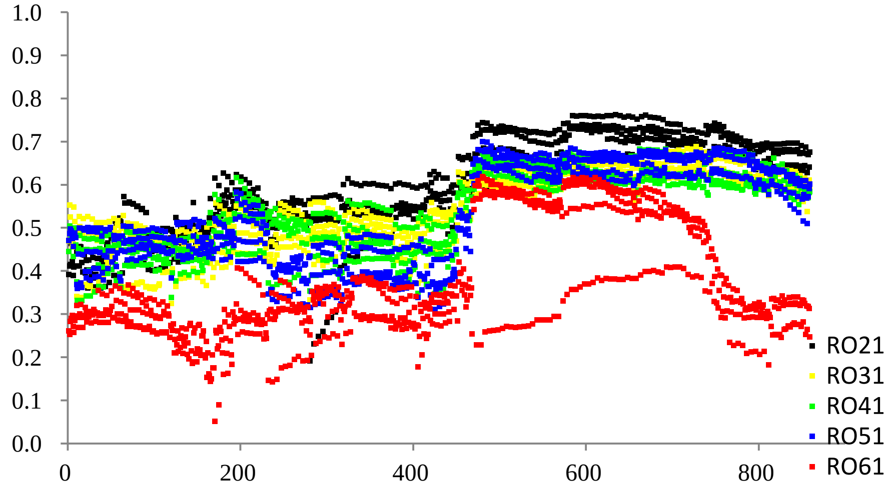
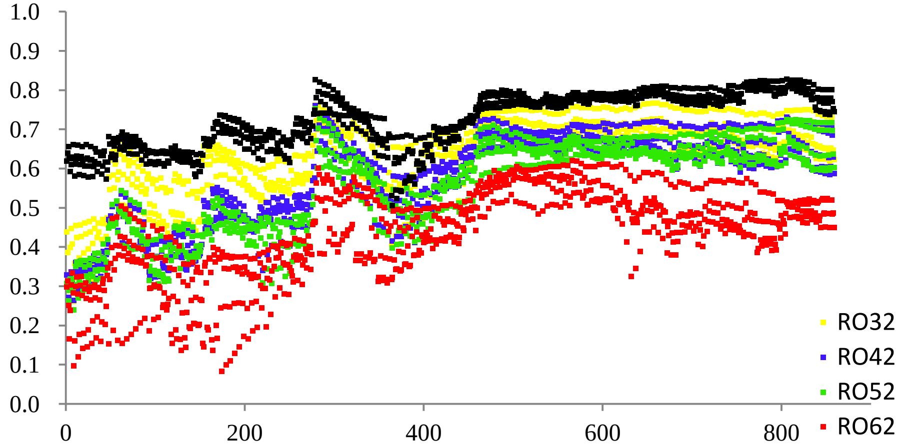


Figure 5.15 The dynamic conditional correlations of developed market indices

(a) Left tails



(b) Right tail

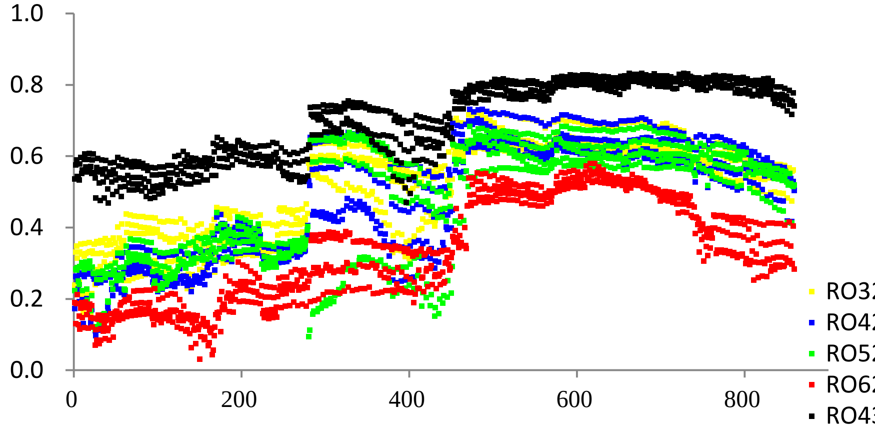
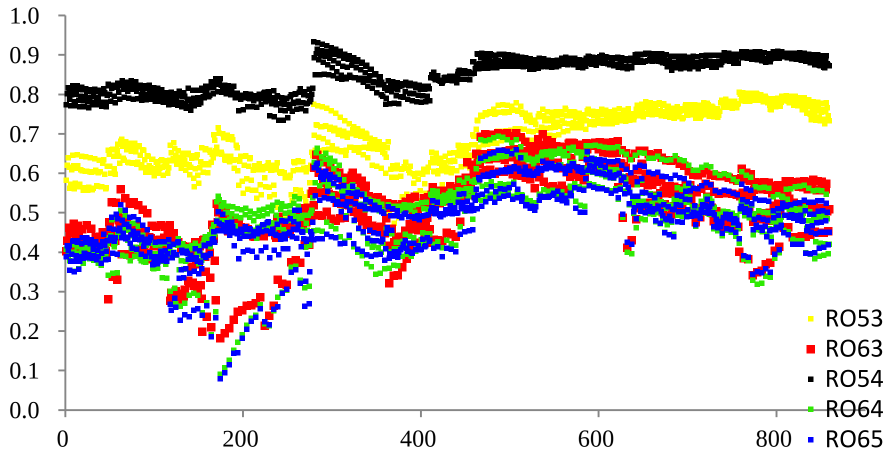


Figure 5.16 The dynamic conditional correlations of developed market indices

(a) Left tail



(b) Right tail

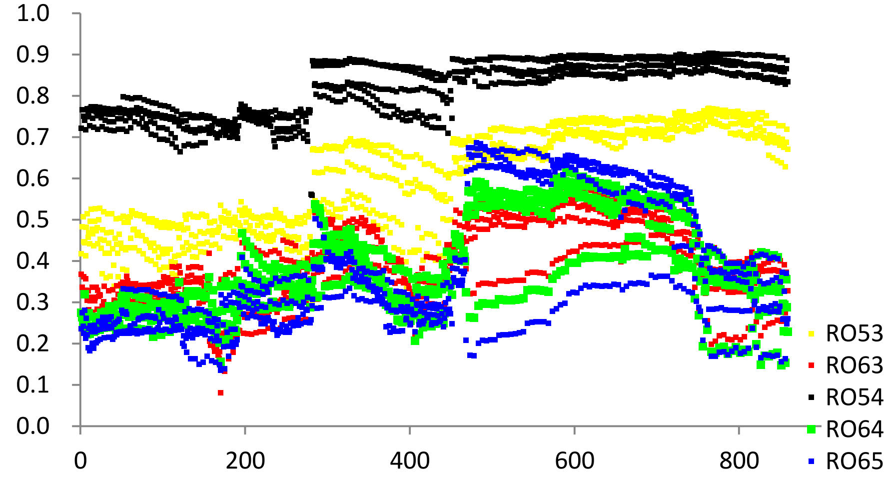
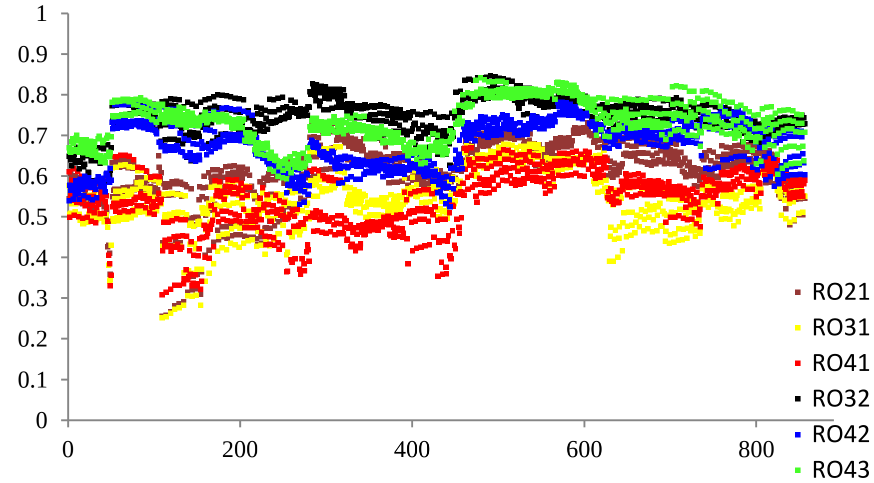


Figure 5.17 The dynamic conditional correlations of Latin American indices
(a) Left tail



(b) Right tail

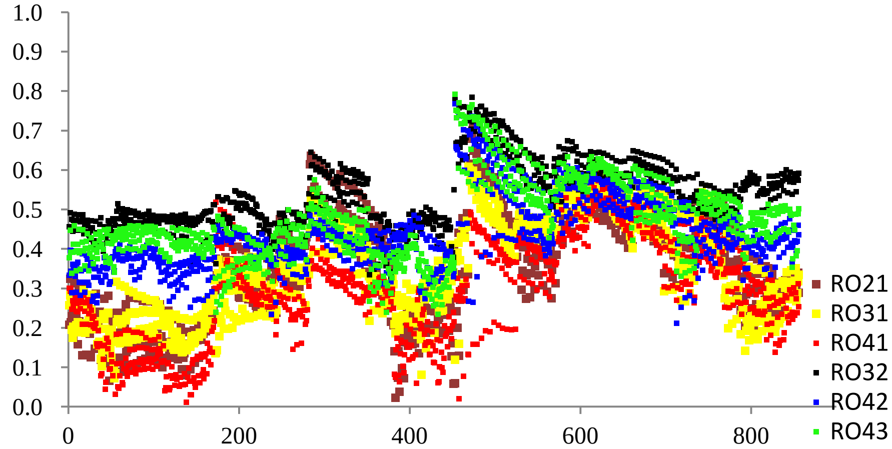
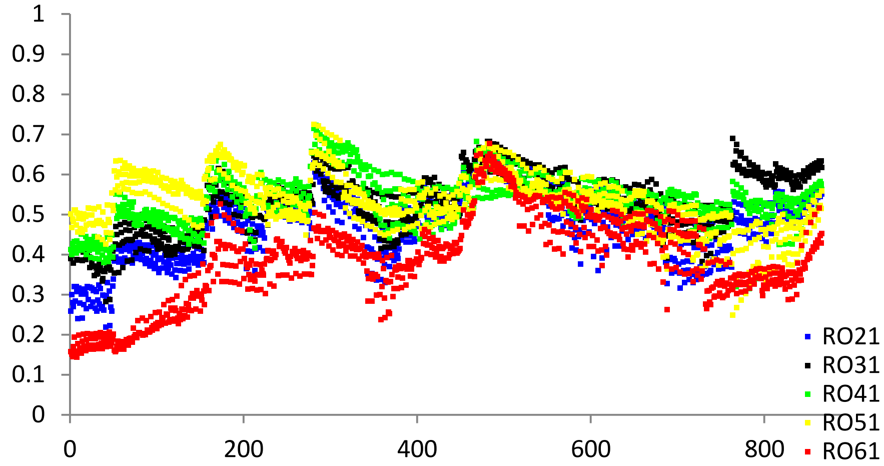


Figure 5.18 The dynamic conditional correlations of Asian indices
(a) Left tail



(b) Right tail

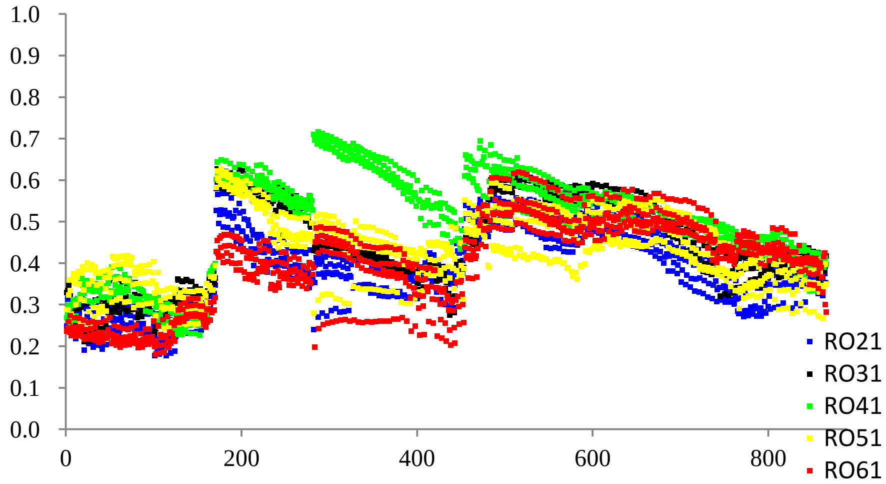
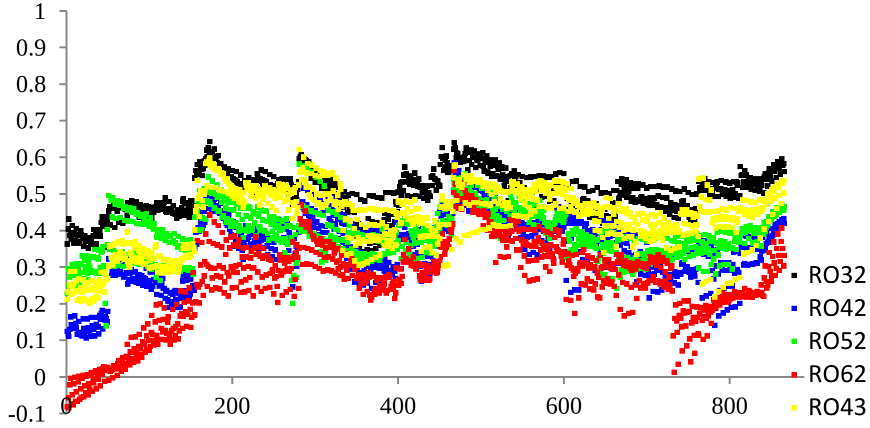


Figure 5.19 The dynamic conditional correlations of Asian indices
(a)Left tail



(b) Right tail

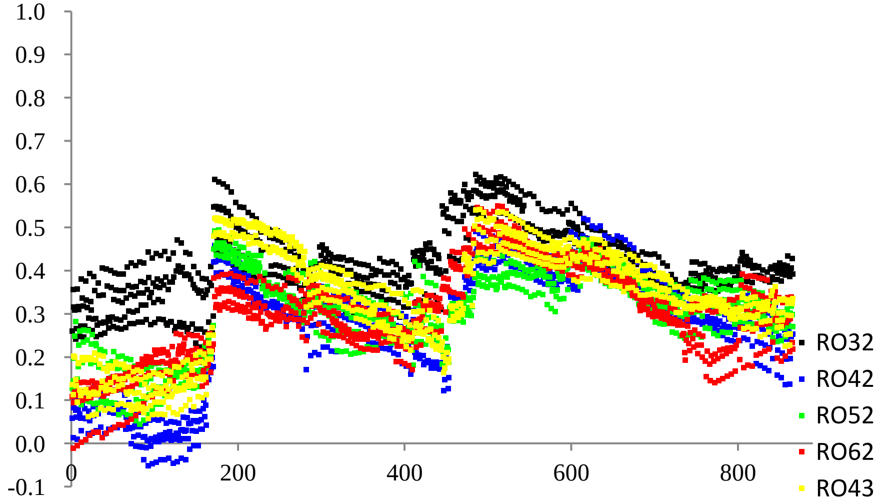
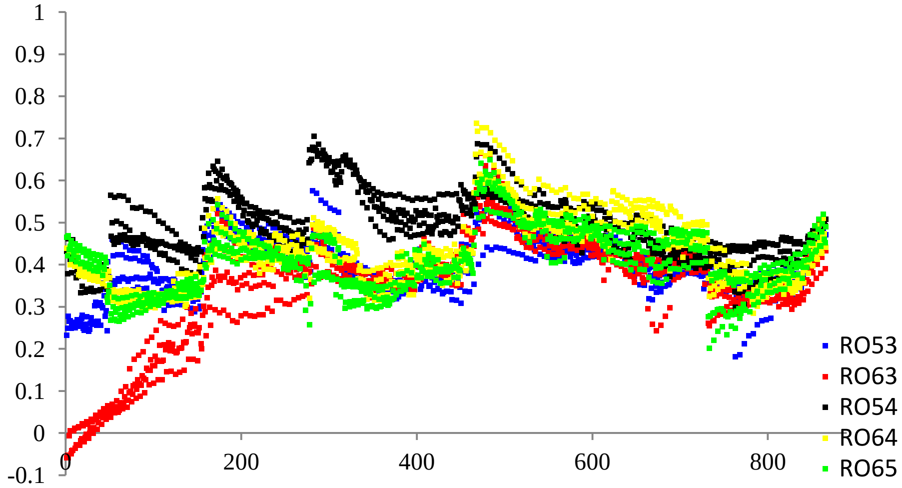


Figure 5.20 The dynamic conditional correlations of Asian indices
(a)Left tail



(b)Right tail

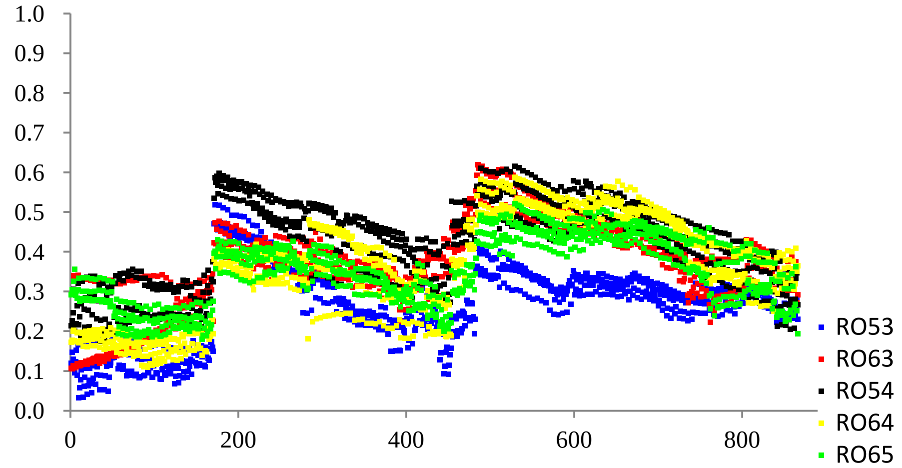
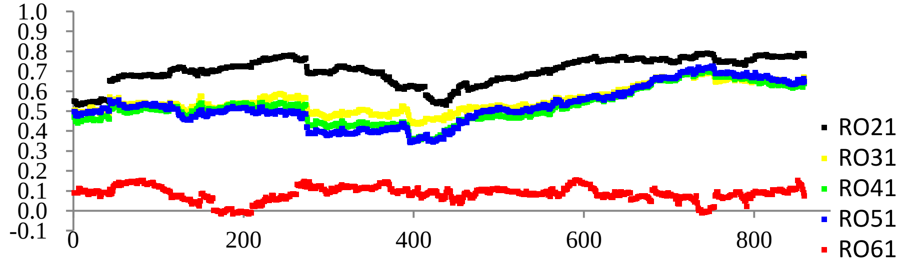
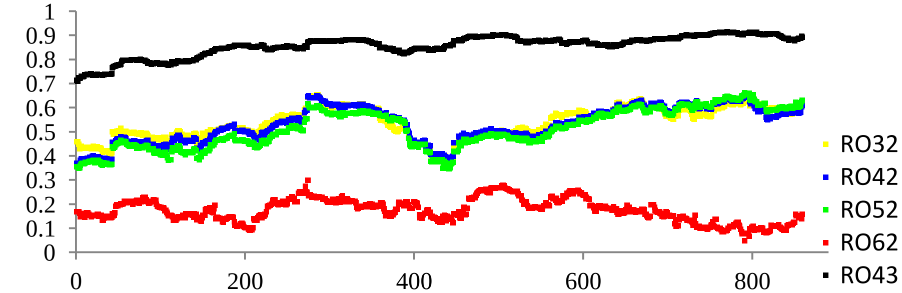


Figure 5.21 The pattern of original DCC model of developed market indices (a)



(b)



(c)

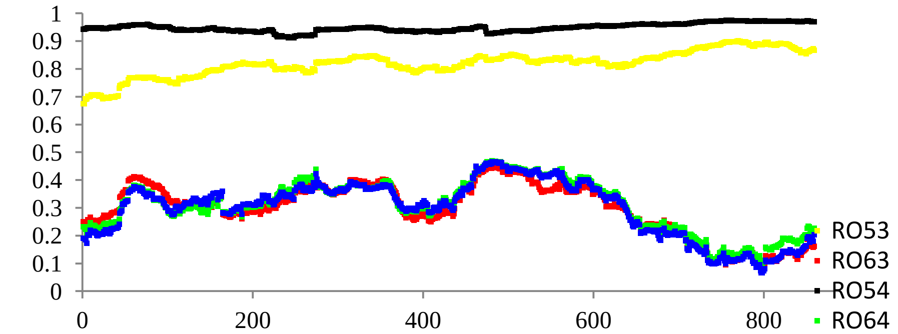


Figure 5.22 The pattern of original DCC model of Asian equity indices (a)

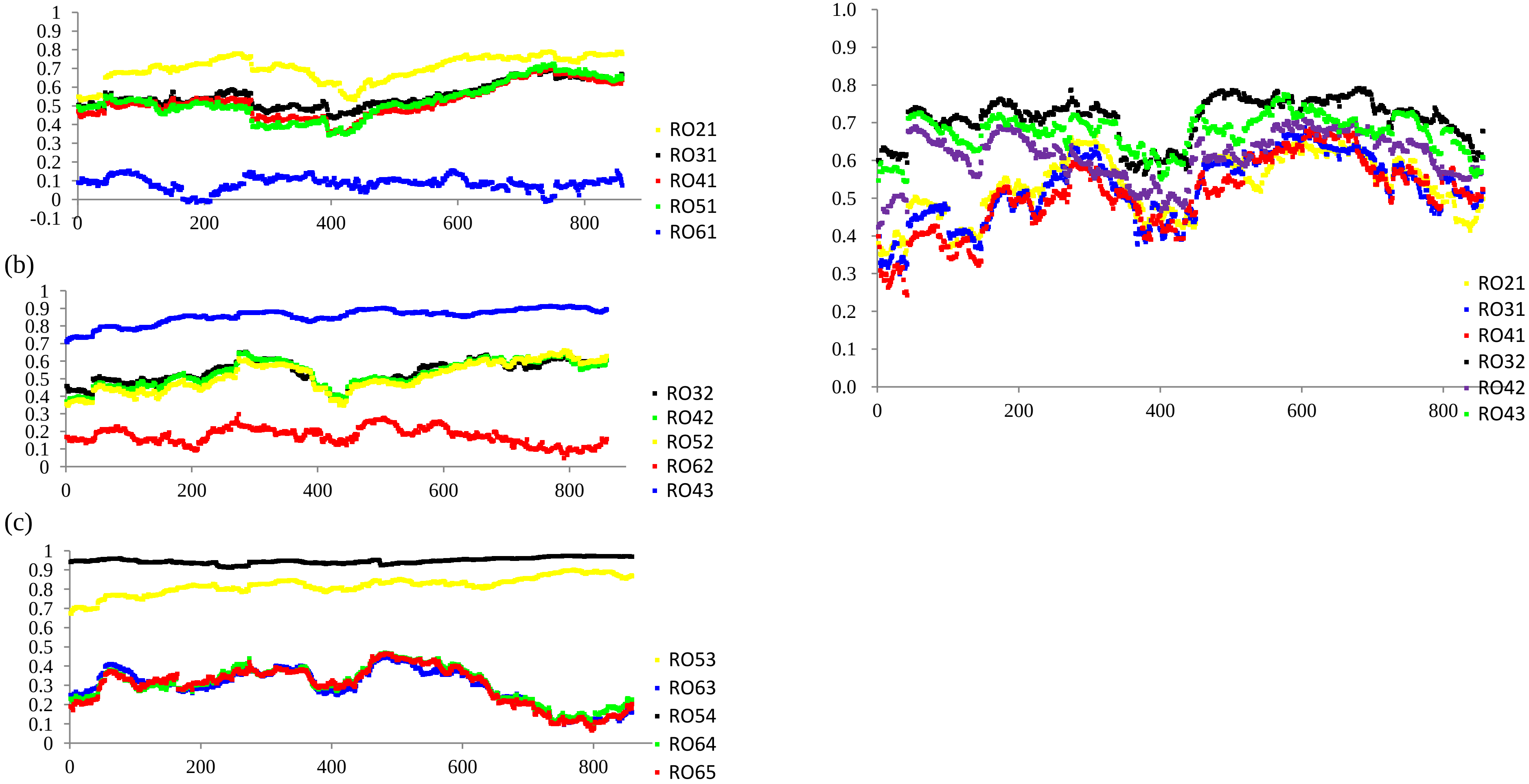


Figure 5.23 The pattern of original DCC model of Latin American indices

5.4 Analysis of Portfolio VaR

Risk management, particularly market risk management, has been under formal development for over twenty years. Most of the previous research focuses on the procedure of risk measurement for a single asset. The key thought behind this is based on the concept of asset mapping. Specifically, a portfolio with multiple assets could be integrated and regarded as a single asset. In this manner, the measurement of VaR becomes a simple task. However, this point of view has some difficulties in practice. For example, the management would lose lots of critical information in the process of asset mapping, particularly with regards to the correlations between various assets. Moreover, the management of a portfolio would stray from the balance of profit maximization and risk minimization. Therefore, there is a need for a new measurement of market risk stepping from the risk measure of a single asset and aggregating the individual risk as a whole portfolio risk. In this subsection, this new measurement is offered, called portfolio VaR. This subsection presents the portfolio VaR of all the models, including the GEV model and the other competing models. In addition, evaluation of its performance is also conducted through the regular backtesting procedure, called coverage test, as discussed in Section 4.5.1. Furthermore, considering the needs of the practitioners, the practicability and adaptability of these VaR models must also to be tested. Therefore, three benchmarks are applied to test how likely it is that the VaR model could be used easily in practice.

5.4.1 Portfolio VaR and backtesting results

The basic statistics of VaR

This subsection graphically presents the patterns of VaR, including the GEV-DCC model and the other four competing models. In addition, the basic statistics of forecasted VaR, shown in Table 5.17 and Table 5.18, and several numerical analyses are provided for backtesting, displayed from Table 5.19 to Table 5.22. According to the nature of VaR methodologies, J.P. Morgan's EWMA model (also called RiskMetrics), the common multivariate GARCH model, and the stochastic volatility model are symmetric[82] between the long and short position. Historical simulation and the GEV-DCC model are better because they measure the VaR from the long or short position. The main evidence shown in Table 5.17 indicates that the methods of historical simulation and the GEV-DCC model tend to derive higher daily risk. For example, the 99%-VaRs of the long (short) positions from HS are 0.0640, -0.0415, and -0.0381 (0.0601, 0.0317, and 0.0306) for the developed equity market, the Latin American, and the Asian equity markets, respectively. Similarly, the ones derived from the GEV-DCC model are -0.0331, -0.0509, and -0.0364 (0.0304, 0.0483, and 0.0390). On the other hand, VaRs from the other models are lower than ±0.0400. It is not surprising that the portfolio of Latin American equity indices is the riskiest one and the portfolio of the developed equity indices has the lowest risk. In addition, the VaRs from HS and GEV-DCC also show that the risks of long positions are generally higher than the short positions, which is consistent with the evidence in the univariate section (Section 5.2). As regards volatility (measured by standard deviation), GEV-DCC is a more stable risk measure, dispersing between 0.0030 and 0.0039 (0.0030 and 0.0045) for the long (short) position. Another point which should not be neglected is the average daily return of the three portfolios: -0.0002, 0.0003, and 0.0002 for the developed, Latin American, and Asian equity markets, respectively.

Graphical analysis of VaR patterns of derived competing models

The patterns of the rolling one-day ahead portfolio VaR are exhibited in Figures 5.24 to 5.26 including the long and short positions and the actual portfolio returns (a clearer figure of the

[82] Since the weighted mean return of the portfolio tends to zero, thus the first term of Eq. (2.18) is neglected.

Latin American one without the SV pattern can be found in Figure F- 1 in the Appendix F[83]). The patterns of 99%- and 95%-VaR are displayed in the (a) and (b) panels, respectively. For instance, Figure 5.24 (a) and (b) present the patterns of 99%- and 95%-VaR of the developed equity markets based on various VaR models, and the actual return. The black scattered points are the equally weighted portfolio returns. The patterns below (above) zero are the VaR for the long (short) position. In Figure 5.24, the noticeable VaR pattern in green, both in the long and short, is derived from the HS method and shows a dramatic rise around the 450th observation. Obviously, it is affected by several extreme observations happening in the third quarter in 2008. This phenomenon persisted until those extremes phased out of the forecasting period (i.e. 250 historical observations). Since all the six developed equity markets experienced a significant collapse in 2008, the empirical distribution was dragged toward the tail area. This could be a good explanation as to why the average VaR and standard deviation of the developed equity market is higher than other two portfolios in Table 5.17. A similar result can also be found in the Asian equity market in Figure 5.26, but not in the Latin American portfolio.

As shown in the original definition and calculation of VaR from Eq. (2.1) to (2.3), volatility plays a critical role in VaR. Thus, three competing models in this thesis are represented focusing on volatility modelling. The first two models, RiskMetrics (in red) and GARCH (in blue) are quite similar, and use the same concept of dynamic volatility. As is already known, the RiskMetrics model is a simplified version of the GARCH model, which has a fixed parameter to daily returns proposed by J.P. Morgan (1996). The main spirit of these two models assumes that volatility is only affected by the one period ahead return and volatility. In other words, they assume that all the market information has been reflected in the previous

83 To avoid the figures of the other four VaR models being obscured by the SV figures, a set of figures without the SV figures are displayed in the Appendix F (Figure F- 1). Then, the patterns of the other four models can be clearly seen and demonstrated.

return and volatility. Moreover, RiskMetrics assumes that 94% of a change of volatility is affected by itself at time t-1. As a result of the nature of RiskMetrics, the influence from those extremes comes in quickly and rapidly phases out as well. Take for example the pattern of the developed market (Asian market), the 99%-VaR of a long position from -0.0357 (-0.0256) dropped to -0.0673 (-0.0575) in 20 trading days and then returned to a more reasonable level in 45 trading days. In contrast to the RiskMetrics model, the pure multivariate GARCH (1,1) model allows for the parameters to change associated with the market condition over time. Thus, the VaR pattern of the GARCH model would not be as deeply influenced by extreme observations. Therefore, the pattern of the GARCH model is smoother than the RiskMetrics one. In fact, the main drawback of RiskMetrics is this restriction of volatility dynamics, which is reflected in its pattern, easily influenced by the extreme return and suddenly soaring to an incredible high. Furthermore, as shown in Eq. (4.33), the identical decay factor in all return sequences seems unreasonable in practice. The third VaR model (in yellow) based on the method of volatility modelling is the stochastic volatility approach (SV). Noticeably, the pattern of SV is more volatile than the first two and the fluctuations seem unrelated with the portfolio returns. One reason for this might be the feature of random walk in the SV model. For example, a local high in the VaR pattern of the developed market obtained from the SV model is around the 400^{th} observation (±0.0565 for the 402^{th}), but the actual portfolio return in that period is relatively stable. On the other hand, the actual portfolio return of the developed market tempestuously varied around the 460^{th} to 490^{th} observation (from -0.0651 to 0.0727), but its VaR pattern in that period kept at a relatively small level. This phenomenon is more obvious in the same period in the Latin American equity market, and the 750^{th} observation at the end of the Asian pattern. All the evidence in the figures apparently shows that the SV model might not be a proper measure of market risk for these portfolios. Compared with the RiskMetrics and GARCH models, it derives a VaR pattern with too many fluctuations due to its randomness. Furthermore, the main difficulty of the SV model is its massive numerical procedure. It involves a maximization process for which it is, in general, difficult to achieve a proper convergence. There are 6.17% and 8.3% (53 and 72 observations) non-converged observations in the developed and Asian equity markets, respectively. Compared with about 2% of non-converged observations of the GEV-DCC model, the SV model seems to need a further refinement in volatility modelling. As a result, the VaR forecast becomes an extremely time-consuming task with the SV model. In fact, all of the three methods encounter a problem, the asymmetry in the long and short VaR patterns. Since some of the volatilities come from a positive return change and others from a negative one, it is unreasonable to just use the positive volatilities in calculating the VaR of long positions.

Table 5.17 Descriptive statistics of 99%-VaR

	HS-L	HS-S	RiskMetrics-L	RiskMetrics-S	GARCH-L	GARCH-S	SV-L	SV-S	GEV-DCC-L	GEV-DCC-S	Return
Panel A: Developed market											
Mean	-0.0640	0.0601	-0.0298	0.0298	-0.0253	0.0253	-0.0168	0.0168	-0.0331	0.0304	-0.0002
SD	0.0325	0.0343	0.0179	0.0179	0.0095	0.0095	0.0101	0.0101	0.0039	0.0036	0.0150
SK	-0.5783	0.5740	-1.8543	1.8906	-0.3052	0.3551	-0.8777	0.9041	-0.6070	1.3046	-0.3696
K	-1.3987	-1.3629	3.2971	3.2981	-1.0473	-1.0473	1.2037	1.2037	6.9675	8.7052	5.6259
Panel B: Latin American market											
Mean	-0.0415	0.0397	-0.0331	0.0331	-0.0269	0.0269	-0.0388	0.0388	-0.0509	0.0483	0.0003
SD	0.0160	0.0143	0.0165	0.0165	0.0071	0.0071	0.0416	0.0416	0.0030	0.0045	0.0159
SK	-0.7284	0.5161	-1.8731	1.8720	-2.2714	2.2714	-1.9172	1.9172	-4.7427	8.3095	-0.3703
K	-1.2337	-1.2516	3.7256	3.7264	11.6628	11.6628	5.3371	5.3371	49.4282	131.0767	4.8003
Panel C: Asian market											
Mean	-0.0381	0.0306	-0.0288	0.0288	-0.0231	0.0231	-0.0159	0.0159	-0.0364	0.0390	0.0002
SD	0.0128	0.0095	0.0129	0.0129	0.0055	0.0055	0.0091	0.0091	0.0034	0.0030	0.0135
SK	-0.4606	0.2404	-1.5922	1.6475	-1.6197	1.7849	-0.8959	0.9264	-3.6208	1.7257	-0.5067
K	-1.1187	-1.2845	3.1277	3.1277	8.3241	8.3241	1.0858	1.0858	57.8726	35.6213	4.3690

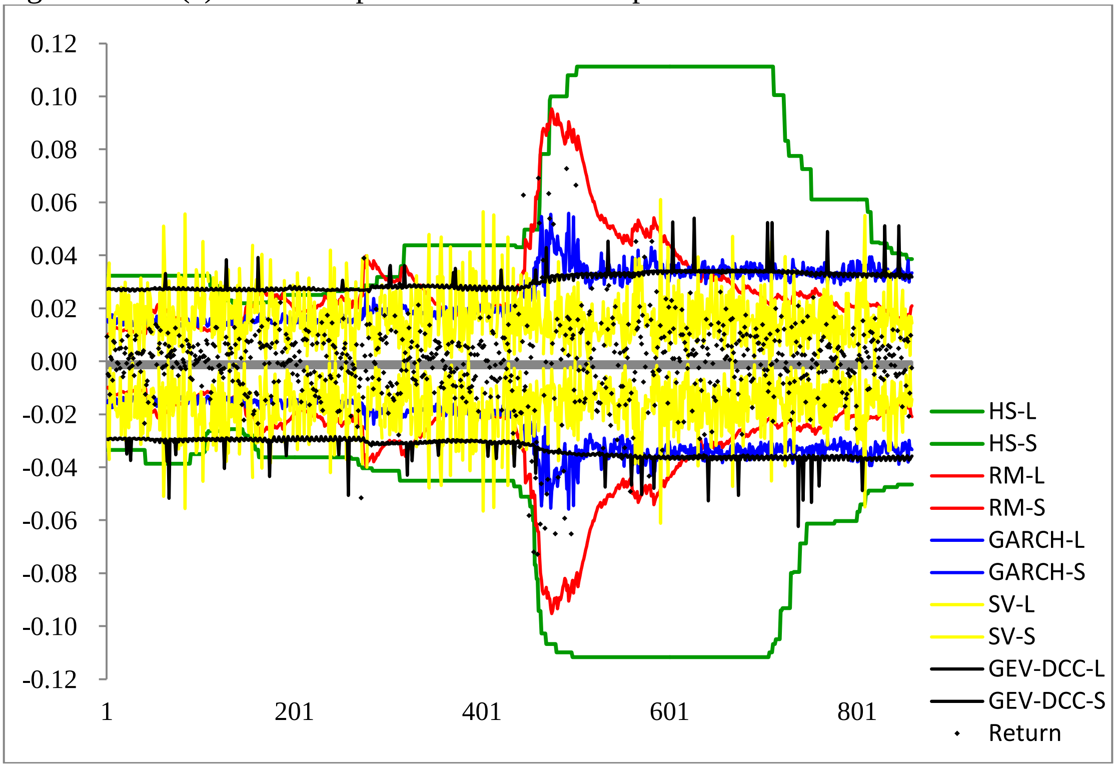
Note: 1. –L (-S) means the long (short) position.

2. The abbreviations of all models: HS is historical simulation based on 250 observations, GARCH means a simple multivariate GARCH(1,1) models, SV is the multivariate stochastic volatility model provided by Harvey, Ruiz and Shephard (1994), and GEV-DCC is the method of extreme value theory which combines the spirit of dynamic conditional volatility.

Table 5.18 Descriptive statistics of 95%-VaR

	HS-L	HS-S	RiskMetrics-L	RiskMetrics-S	GARCH-L	GARCH-S	SV-L	SV-S	GEV-DCC-L	GEV-DCC-S	Return
Panel A: Developed market											
Mean	-0.0417	0.0323	-0.0210	0.0210	-0.0178	0.0178	-0.0119	0.0119	-0.0186	0.0183	-0.0002
SD	0.0215	0.0134	0.0127	0.0127	0.0068	0.0068	0.0071	0.0071	0.0032	0.0029	0.0150
SK	-0.6358	0.5543	-1.8551	1.8906	-0.2967	0.3447	-0.8776	0.9041	-4.6630	4.9471	-0.3696
K	-1.1937	-1.2117	3.2987	3.2981	-1.0753	-1.0753	1.2037	1.2037	36.7699	38.9112	5.6259
Panel B: Latin American market											
Mean	-0.0253	0.0231	-0.0234	0.0234	-0.0188	0.0188	-0.0274	0.0274	-0.0274	0.0283	0.0003
SD	0.0089	0.0083	0.0117	0.0117	0.0051	0.0051	0.0294	0.0294	0.0028	0.0044	0.0159
SK	-0.5312	0.8086	-1.8749	1.8760	-2.2035	2.2035	-1.9172	1.9172	-11.8876	12.7200	-0.3703
K	-1.1981	-0.6926	3.7331	3.7366	11.1493	11.1493	5.3372	5.3372	175.9050	220.1476	4.8003
Panel C: Asian market											
Mean	-0.0220	0.0195	-0.0204	0.0204	-0.0162	0.0162	-0.0113	0.0113	-0.0216	0.0221	0.0002
SD	0.0078	0.0059	0.0091	0.0091	0.0039	0.0039	0.0064	0.0064	0.0043	0.0033	0.0135
SK	0.0618	0.1729	-1.5922	1.6475	-1.5721	1.7299	-0.8939	0.9244	-6.6287	6.0639	-0.5067
K	-1.4299	-0.4820	3.1277	3.1277	7.9473	7.9473	1.0823	1.0823	61.4003	56.0712	4.3690

Figure 5.24 (a).99%-VaR patterns of the developed market and the actual return



(b).95%-VaR patterns of the developed market and the actual return

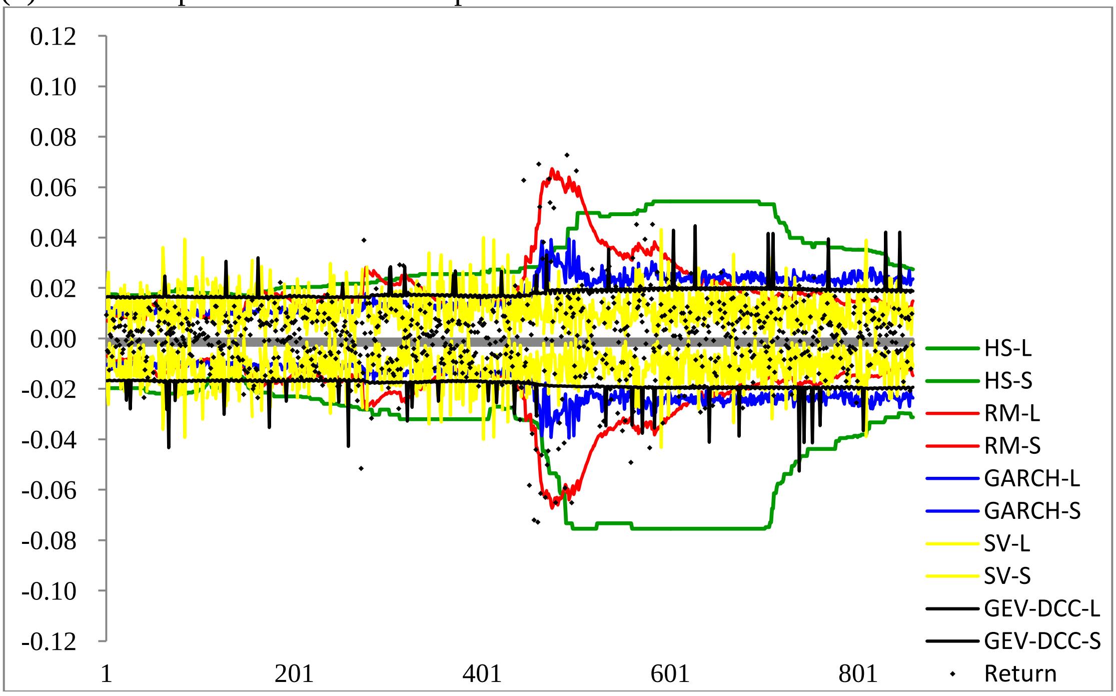
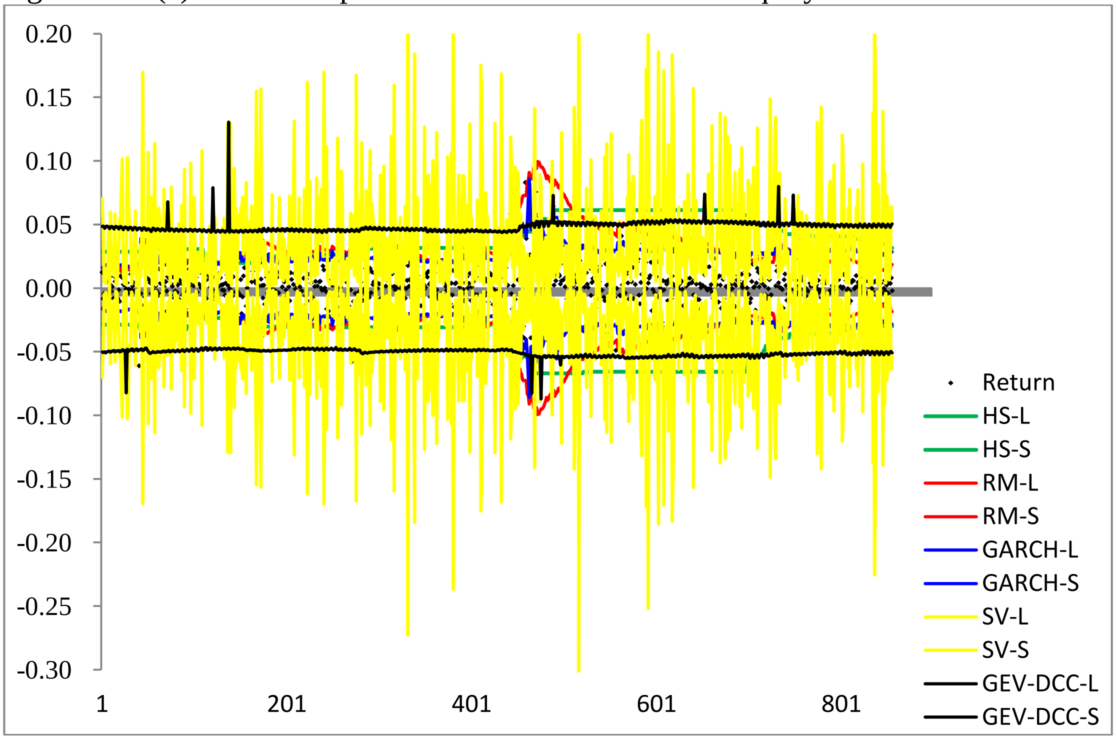
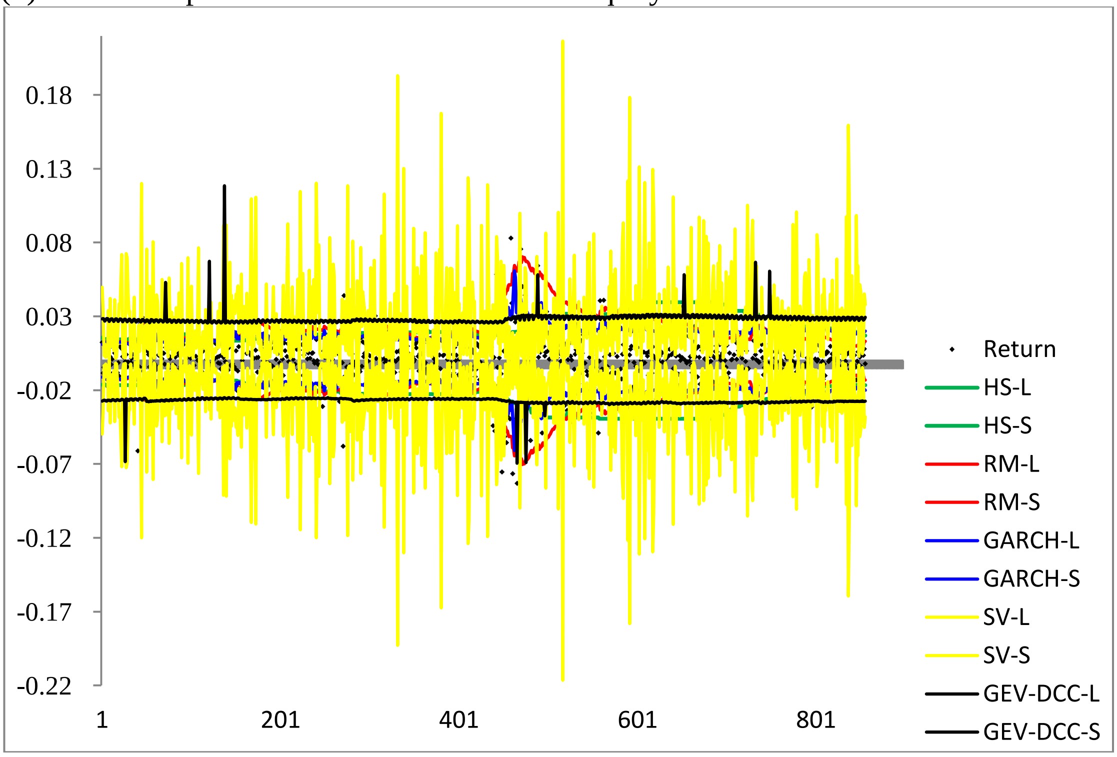


Figure 5.25 (a). 99%-VaR patterns of the Latin American equity market and the actual return[84]

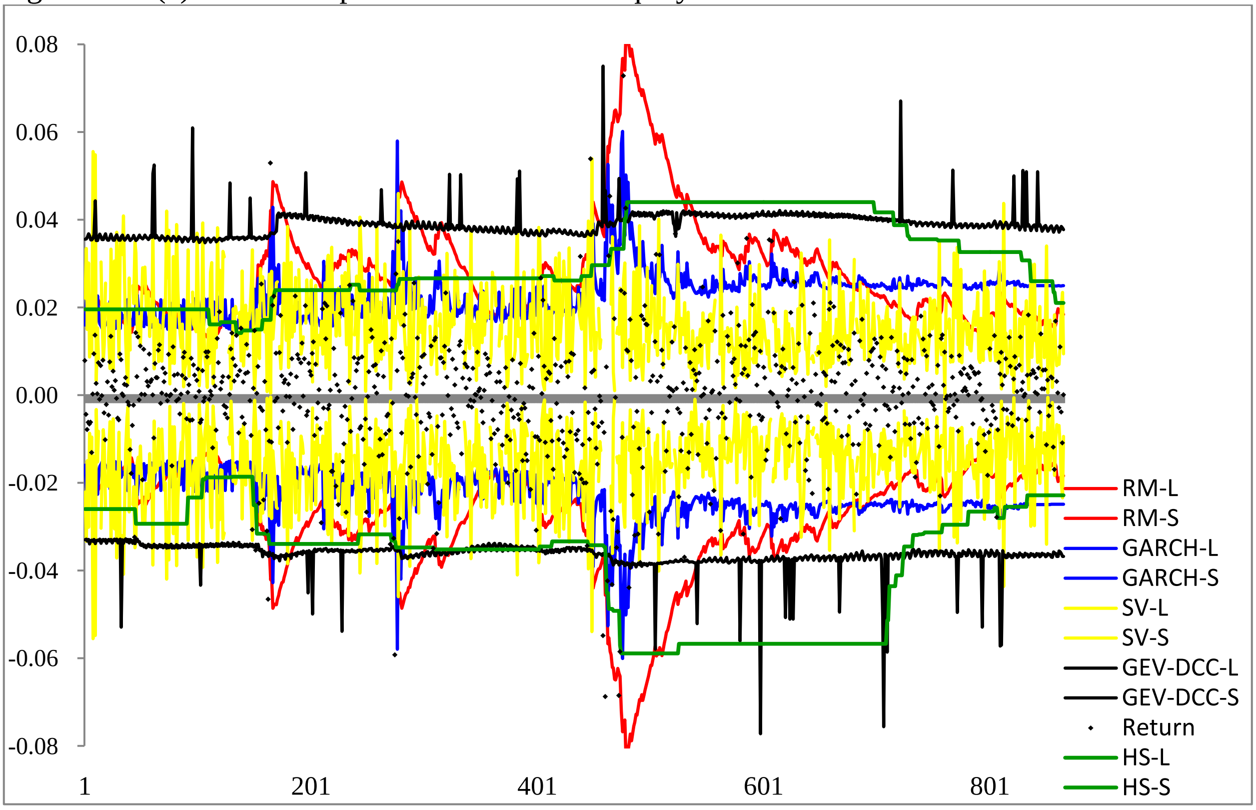


(b). 95%-VaR patterns of the Latin American equity market and the actual returns



[84] Since most patterns are covered by the pattern of the SV model, a clear figure without the SV mode is provided in Appendix F.

Figure 5.26 (a). 99%-VaR patterns of the Asian equity market and the actual returns



(b). 95%-VaR patterns of the Asian equity market and the actual returns

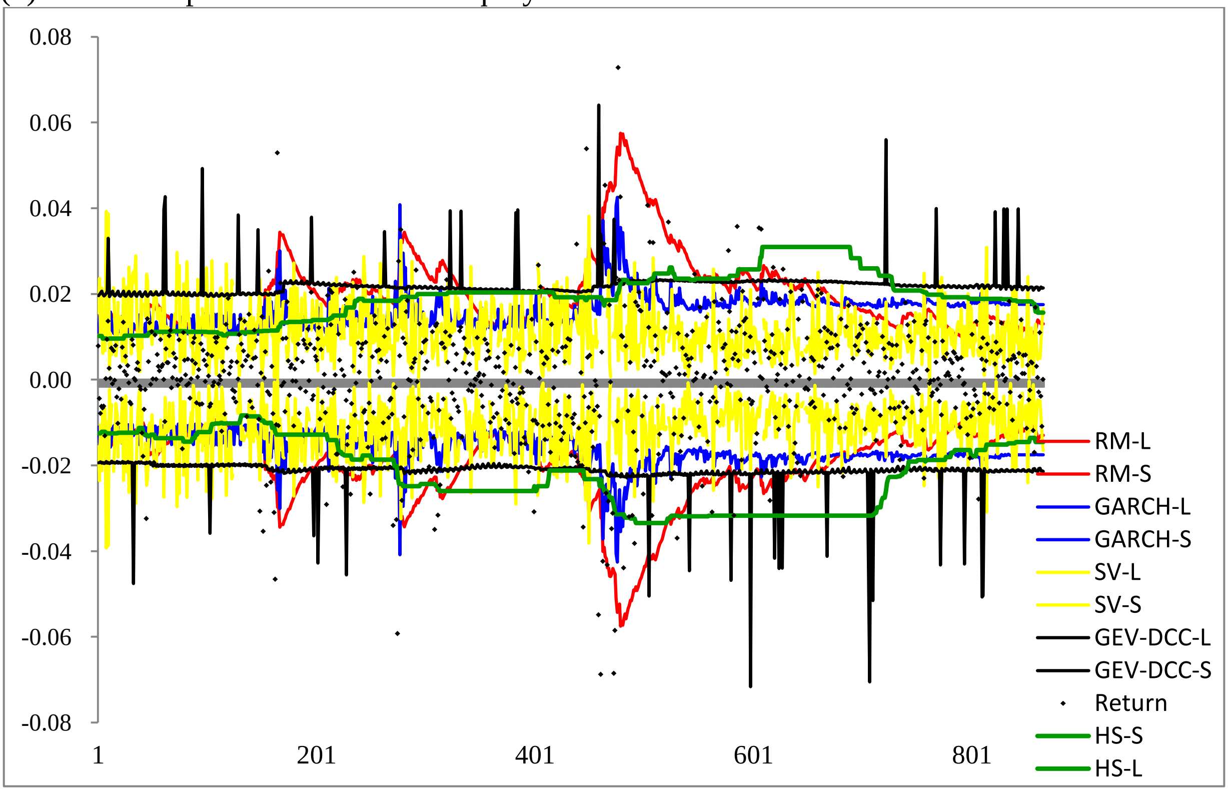
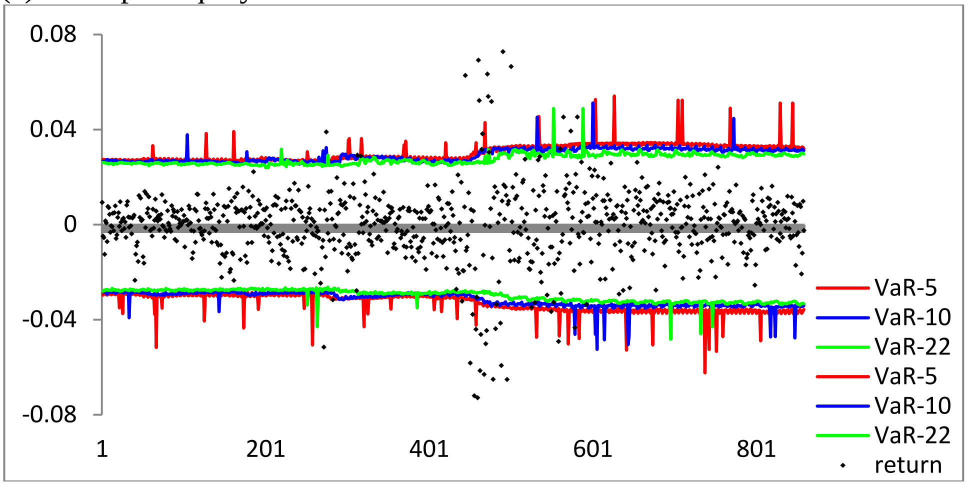
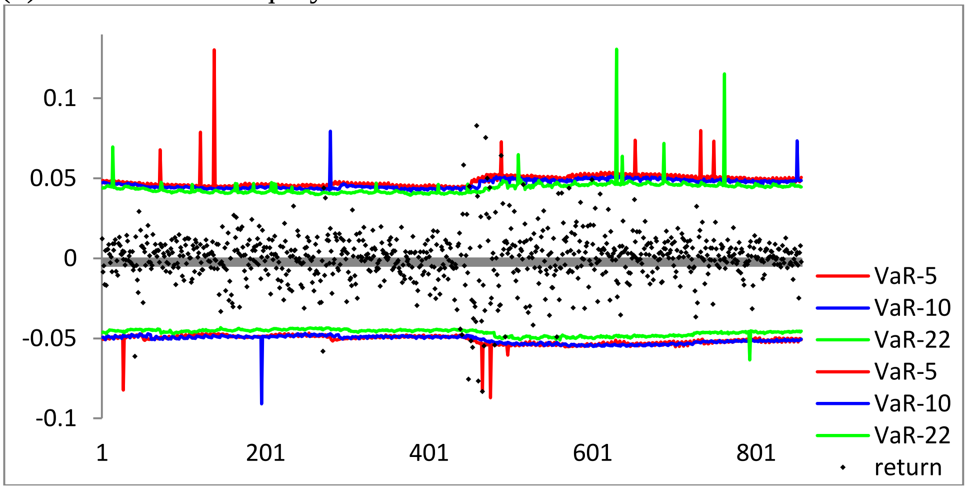


Figure 5.27 The 99%-VaR patterns of the GEV-DCC model with a different size of block

(a)developed equity market



(b)Latin American equity market



(c)Asian equity market

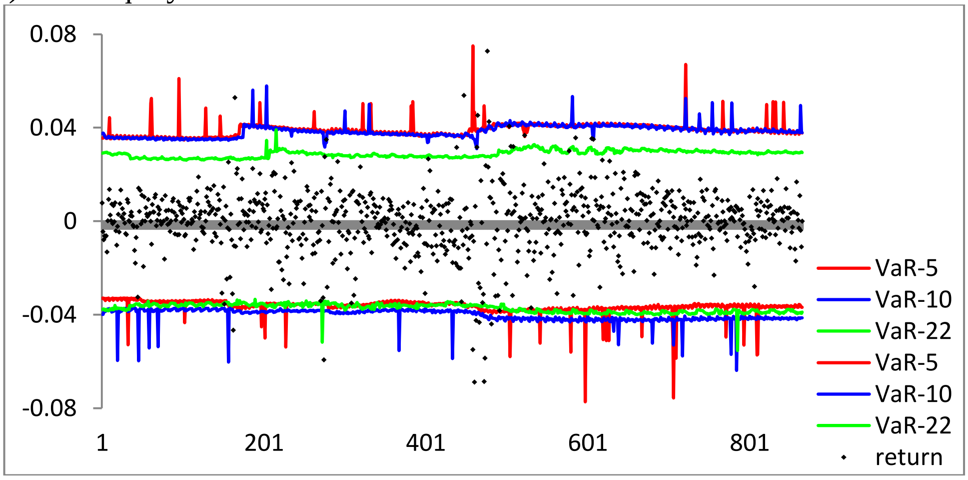
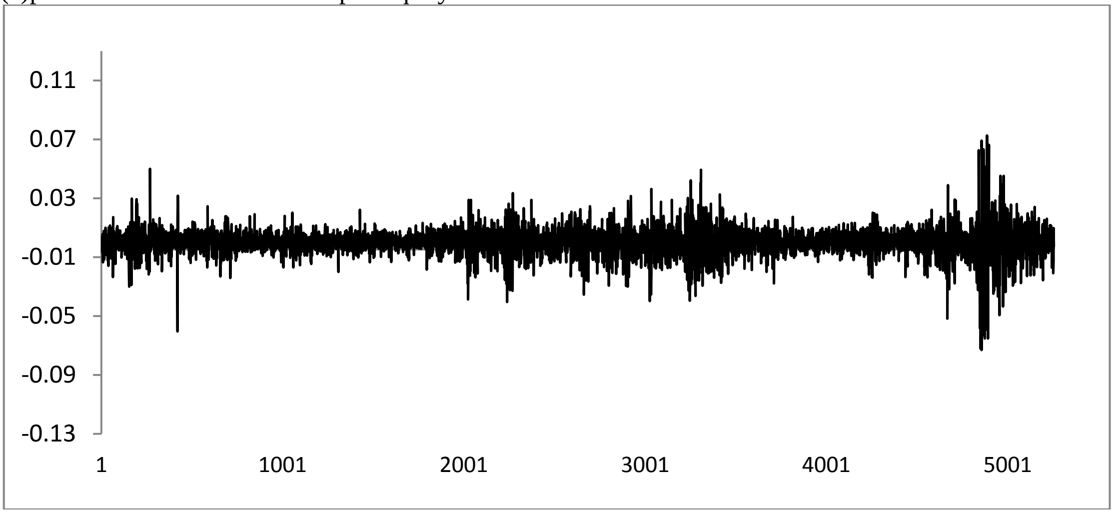
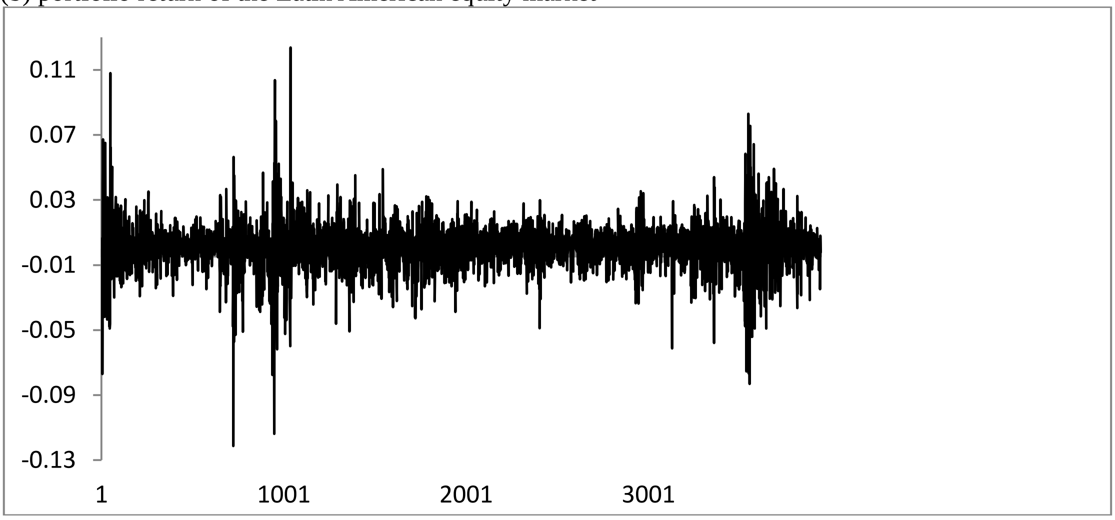


Figure 5.28 The patterns of daily portfolio returns of the equity markets

(a)portfolio return of the developed equity market



(b) portfolio return of the Latin American equity market



(c) portfolio return of the Asian equity market

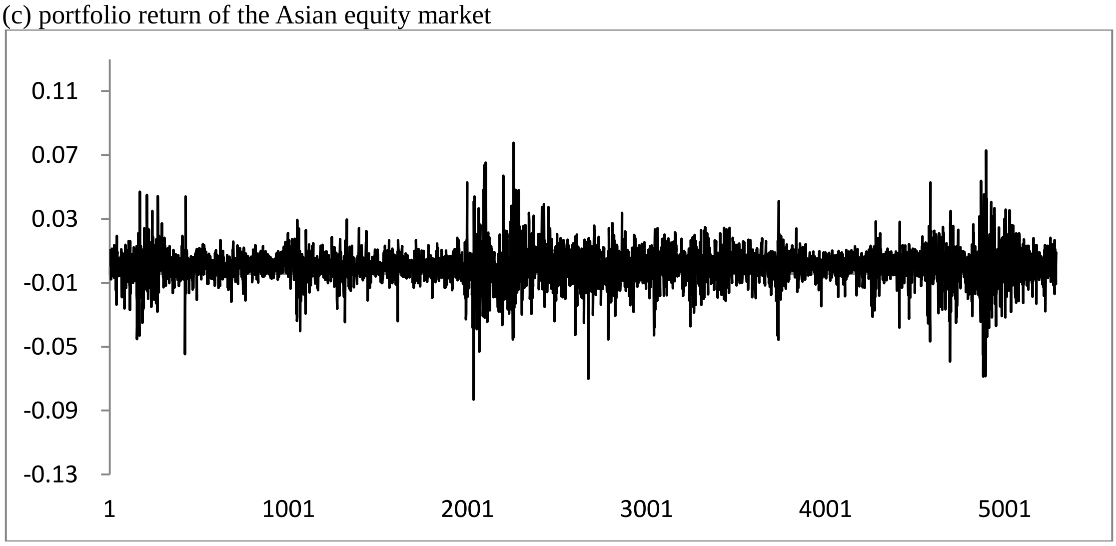
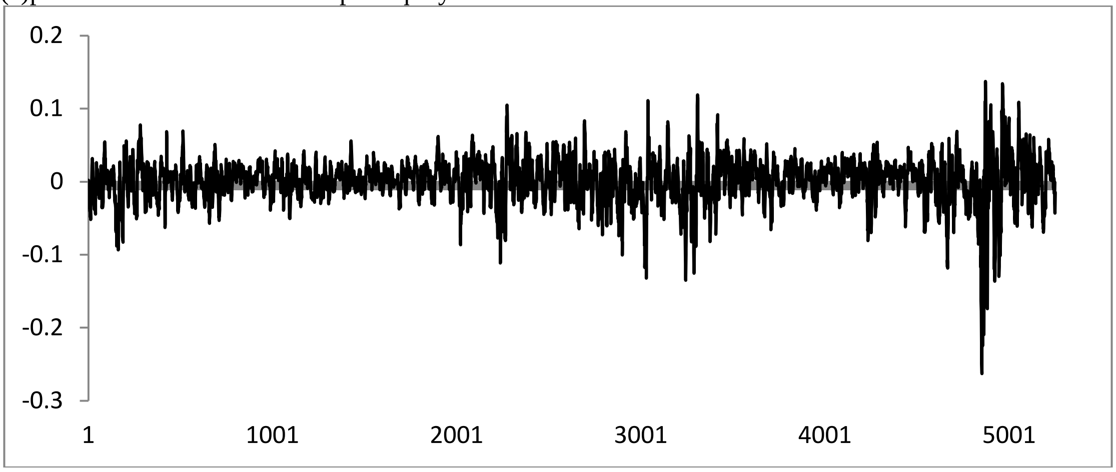
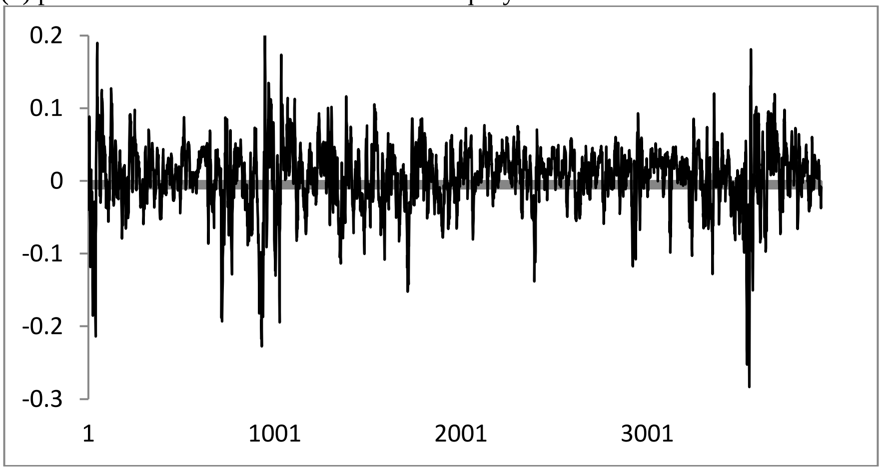


Figure 5.29 The patterns of the 10-day portfolio return of the equity markets

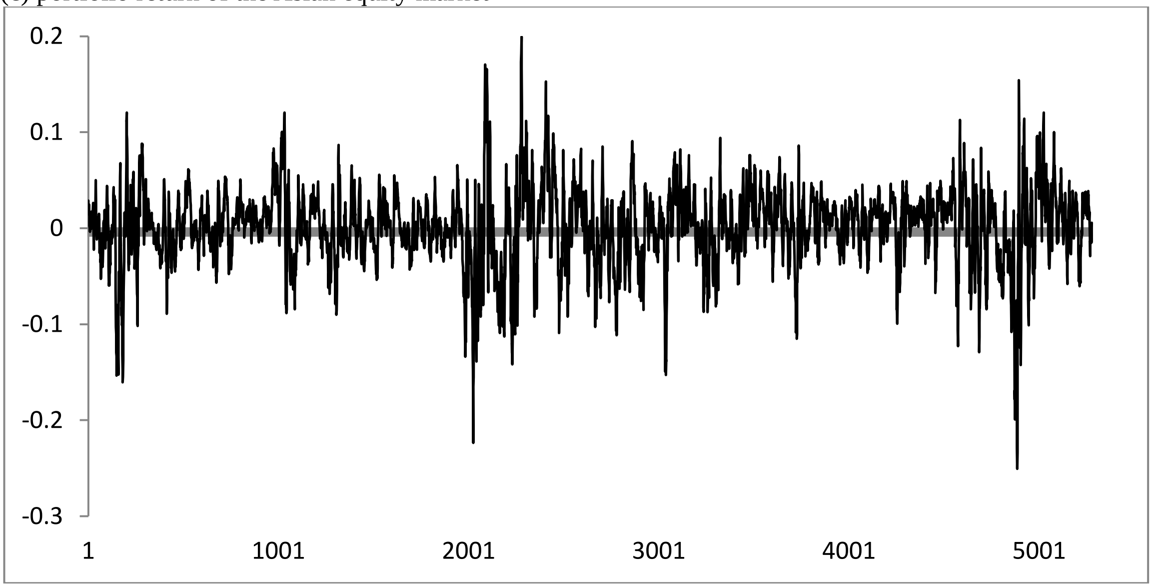
(a)portfolio return of the developed equity market



(b) portfolio return of the Latin American equity market



(c) portfolio return of the Asian equity market



Graphical analysis of VaR patterns derived from the GEV-DCC model

The model suggested by this thesis is the GEV-DCC, which takes into account the univariate case and aggregation of all the individual VaRs. In the first step, the individual VaR of each return series is obtained via the generalized distribution of extreme value proposed by Fisher and Tippett (1928) and Jenkinson (1955), which is discussed in Section 5.2. The second step is to measure an appropriate correlation between the various asset returns; in particular the correlation should have the ability to describe the relationship in both tails. Finally, the third step is to aggregate the individual VaRs as a whole, to be called portfolio VaR. The VaR patterns (in black) of the GEV-DCC present some small variations although they are truly flatter than the other patterns of the competing models. Some of the higher peaks are the un-converged observations, and the small ones are real forecasted VaRs. As expected, the 99%-VaR of the developed equity market portfolio is the smallest level, around -3.3% and 3.0% for the long and short position, respectively. Interestingly, both the long and short VaR patterns of the GEV-DCC model are quite similar to the pattern derived from the GARCH model, particularly after 2008. The portfolio of the Latin American equity market is the riskiest one, and the VaRs of the long and short position are -5.09% and 4.83% on average. Generally speaking, the VaR pattern of the Asian equity market is more volatile, in both the long and short position. Both the VaR of the long and short positions around the 170th observation present a jump. Tracking the reason of the jump, interestingly it stems from the change of correlation between the Asian indices as shown in Figure 5.18 to Figure 5.20, rather than the change in individual VaR.

The VaR patterns with different sizes of block are exhibited in Figure 5.27, including one week (n=5), two weeks (n=10), and one month (n=22). Previous research argued that there is no guidance to decide the size of block, but yet the estimated parameters of generalized extreme distribution are significantly affected by the choice of block size (Lauridsen (2000) and Ho et al. (2000)). As with Lauridsen (2000) and Ho et al.'s (2000) studies, the estimated parameters of the generalized extreme value distribution in Table 5.1 are significantly influenced by the size of the block. However, the evidence in the VaR patterns in Figure 5.27 shows that the size of VaR seems not to be influenced by the block size, except with the short pattern of the Asian equity market in Figure 5.27(c). For example, in Figure 5.27(a), the three patterns in different colours tend to overlap with each other. Even in the period of the third quarter of 2008 (around the 475th observation to the 525th one), there is only a small difference between the VaR pattern in the 22-day block and the VaR pattern in the 5- and 10-day block. A similar phenomenon can also be found in the figures of the Latin American equity market, although the VaR patterns of the long positions are similar to the ones in the developed equity market. In other words, the block size mentioned in Section 5.2.1 would cause an influence in the parameter estimation of the distribution of generalized extreme returns, but it would not significantly affect the forecasted VaR pattern.

Backtesting based on coverage test

The performance evaluation of the VaR model is a critical part of the risk management procedure. This process involves the fundamental question: is this model robust or not? Statistically, a 99%-VaR (or 95%-VaR) sequence should cover 99% (95%) of the actual returns on average. For example, 99% of observations of a 99%-VaR series of a long position, derived by rolling forecasting, should be numerically smaller than the corresponding actual portfolio return. To achieve this task, two main applications, called coverage tests, are applied in this subsection. The results of the coverage test for GEV-DCC and the other four competing models are shown in Table 5.19 to Table 5.22. According to Kupiec (1995), in the coverage test the null hypothesis is that the violation rate equals α[85], against the alternative hypothesis

[85] Eq. (2.1) to eq. (2.3) explain the α.

that the violation rate is larger than α. Christoffersen (1998) refined the coverage test by adding an independence test to the violations. The basic concept of Christoffersen's work assumes that the probability of the appearance of each exceedance should be equal to α. However, the existence of an autocorrelation in the sequence of the exceedance means the exceedance at time t could be an estimator of the exceedance at time t+1. In this case, the occurrence probability of the exceedance at time t+1 would not be equal to α. Christoffersen's (1998) method is applied in this section.

The first part in this subsection is the VaR backtesting of the four competing models, for both the daily and 10-day return sequences, as shown in Table 5.19 and Table 5.20, respectively. Clearly, looking at the 99%-VaR or the 95%-VaR column, RiskMetrics and the Historical simulation method (with 250 historical data observations) provide the best performance in daily VaR backtesting. This result is inconsistent with the outcome in the univariate section, in which the GARCH model offers a better performance in VaR backtesting. Yet, in the portfolio VaR case, the GARCH model only provides a fair performance in the short position of the developed and Latin American equity markets (0.0256 and 0.0268, respectively). Unsurprisingly, the SV model provides the worst performance of the competing models. None of the 99%- and 95%-VaR in the SV panel are significant due to its high violation rate, caused by the randomness of the SV model. Both the GARCH and SV models are time-varying parametric models which involve difficult convergences and tedious calculations. Similarly poor results can be also found in the panel of historical simulation with 1250 historical data. The results of the HS model with n=1250 (hereafter HS-1250) are stable but sluggish. The sluggishness of HS can also be demonstrated in Figure E-1 in Appendix E. It is clear to see the green lines respond to the market condition quite slowly. On the other hand, the case of the HS model with 250 previous observations (hereafter HS-250) provides an excellent backtesting performance, both in the long and short positions. The fact that the HS method with the shorter estimation period produces a better performance than the longer one can be seen as a matter of course since it contains much more current information. Looking at this in more depth, the RiskMetrics model presents only an acceptable performance in the long position of the three portfolios. The violation rates are 0.0279, 0.0280, and 0.0266 for the developed, Latin American, and Asian equity markets, respectively. Unfortunately, the three outcomes are not significant under Kupiec's test. By contrast, the violation rates of the short positions are much better. At least, all of them are significant in both coverage tests at the 95% level. Turning to HS-250, the violation rates of the long positions are significant in both coverage tests, and two out of three of the short positions are significant, at least at the 95% level. In the 95%-VaR column, similar results can be found as well, which demonstrate that the RiskMetrics and HS-250 model provide a better back-testing performance. The magnitude of the violation rate between the long and short positions is obscure, except for the developed equity market. The RiskMetrics model shows that the violation rate in the long position is higher than the short one. However, HS-250 demonstrates the exact opposite. In addition, the one-day ahead VaR forecast and coverage tests are also made based on the 10-day return sequences (the results are exhibited in Table 5.20). The results of the performance of the 10-day VaRs are worse than the daily VaR series. This is perplexing, and one possible explanation to this could be attributed to the stronger fluctuation of the 10-day return series.

The second part of the backtesting looks at the results of the GEV-DCC model and is displayed in Table 5.21 to Table 5.23. As shown in panel A, the results based on the daily sequences are, in general, quite significant. Most of the backtesting results demonstrate that the GEV-DCC model offers a good performance, except for the case of the long position with the 22-day block in the developed equity market. Both the Latin American and Asian equity markets are, at least, significant at the 95% level. It is interesting that the violation rate in the developed market is the highest one and the one in the Latin American market is the lowest.

Intuitionally, as the evidence shows in Table 5.17, the mean VaR of most models would suggest that the Latin American equity market is the riskiest of the three portfolios, but the results of the violation rate put it as the lowest. In fact, the results of the violation rate might properly describe the reality in the market. Referring to the patterns of the three equity market portfolios as shown in Figure 5.28, Latin American equity has a more volatile pattern and those extreme returns will be sampled to form a generalized extreme distribution. As a result, the individual VaR will stay at a significantly high level, and the high level VaR pattern would not be exceeded easily by the returns. In contrast, the returns of the developed equity markets present a smaller fluctuation compared with the other two patterns. Thus, it is natural that the forecasted VaR would remain in the lower level and would be violated easily.

Apart from the backtesting of daily VaR patterns, a backtesting of the 10-day VaR derived by the GEV-DCC model is provided in panel B. The results show that the developed equity market produced a poor performance in the backtesting, and only partial sections of the Asian portfolio are significant at the 95% level. Yet the GEV-DCC model has great success in measuring the 10-day VaR of the Latin American equity market, and most of the violation rates are between 0.0070 and 0.0175, except for the short position of the 10-day block (n=10). Partial results in the Asian market provide an acceptable performance, but the majority of the market is not good enough. Sometimes the VaR might be required in a tolerant condition, for example when α=0.05; the results of $VaR_{0.95}$ are displayed in Table 5.22. Similarly, the GEV-DCC model provides a good performance for the Latin American and Asian equity markets, and the short positions of developed markets in all sizes of blocks are significant at 95%. However, VaR with α=0.05 does not work well in the 10-day return sequences because of the extremely high violation rate.

The results discussed and analysed above are estimated and forecasted with a long-range sample, from 2nd January 1990 to 29th December 2006. For the purpose of robustness testing of the GEV-DCC model, a shorter period of sample set is used to investigate the VaR of the three portfolios using the same procedure, and the results are presented in Table 5.23. The alternative sample set includes 10-year daily returns, from 2nd January 1997 to 29th December 2006[86]. The new period is shorter than the original one, which means that the less extreme returns will be sampled and applied in the estimation of asymptotic distribution of those extremes. Technically, less extreme observations inevitably produce a result with less accuracy. The results generally maintain a consistent pattern, showing that the violation rate in the long positions is higher than the one in the short positions. Most of the results in Table 5.23 are significant at the 95% or 99% confidence level, and the performance is even better than the results in panel A of Table 5.21. One possible interpretation for this is that it is associated with the choice of sample period. Specifically, the original sample with a longer span includes many positive or negative extreme returns happening between 1990 and 1996. Referring to Figure 5.28, for instance, the sample set from 1990 to 1996 of the developed market tends to be stable, but the Latin American market in that period includes many extreme returns. As a result, the VaR pattern of the developed market tends to be high when those relatively smaller extreme returns are excluded. Therefore, a lower level of violation rate to the VaR pattern in the shorter period is found. By contrast, in the Latin American equity market, a smaller VaR pattern is obtained because a series of larger extreme returns in the period of 1995 to 1996 are dropped. Similarly, a higher level of violation rate of the alternative period is found as shown in Table 5.23. In addition, a clearer comparison of the two different periods, with three types of block, is shown in Table 5.24. In general, the two sets of VaR are highly consistent in all the blocks. As in the discussion above, the VaR pattern in the developed market in panel B tends to be higher than the one in panel A. In contrast, the VaR in the Latin American market in panel B is smaller than the one in panel A. The two

[86] The number of observations for the developed market between 1990 and 1996 is 1811, the Latin American market has 513 observations, and the number of observations of the Asian market in this period is 1820.

results of the VaR patterns of the Asian market are more obscure. However, the two results are quite similar. Overall, the evidence in Table 5.24 suggests that the GEV-DCC model is a reliable risk measure to use with different forecast periods.

Comparison

In this part, the three backtesting results of the VaR models are compared with each other; RiskMetrics, Historical simulation, and the GEV-DCC model as shown in panel B and D of Table 5.19 and panel A of Table 5.21. In the backtesting results with the daily return sequence, the GEV-DCC model generally provides a superior backtesting performance than RiskMetrics and HS; although all of them significantly accept the null hypothesis, the violation rate is equivalent to α. For example, the violation rates of the GEV-DCC model with a 5-day block are 0.0244, 0.0105, and 0.0115 in the long position, corresponding to RiskMetrics at 0.0279, 0.0280, and 0.0266, and the HS model at 0.0233, 0.0199, and 0.0150. On the other hand, the GEV-DCC also presents a slightly better violation rate in the short positions. In the 5-day block, the violation rates of the GEV-DCC model are 0.0186, 0.0058, and 0.0081, compared with RiskMetrics (for 0.0035, 0.0117, and 0.0115) and HS (for 0.0198, 0.0269, and 0.0161). Even with a 22-day block, the GEV-DCC model still offers an acceptable violation rate in the long positions (0.0314, 0.0129, and 0.0127) and the short positions (0.0198, 0.0105, and 0.0173).

Apart from the comparison of daily VaR performance, the VaR performance of the GEV-DCC based on the 10-day returns is also compared with the two alternative models. The results are shown in panels B and D in Table 5.20, and panel B in Table 5.21. The results show that the RiskMetrics model is only significant in the short position of $VaR_{0.99}$. Furthermore, the HS-250 model produces the worst VaR performance, and none of the backtesting results is significant. However, the GEV-DCC model is highly significant in the Latin American market and partially significant in the Asian market. The backtesting results in the developed market are quite bad due to the high violation rates and the failure in the independence test of the violations. Unfortunately, the performances of $VaR_{0.95}$ in the three blocks are much worse. From this viewpoint, the GEV-DCC model is likely to be more suitable in the higher probability VaR forecast, for example, the performance of $VaR_{0.99}$ is better than the one of $VaR_{0.95}$. The critical implication from this comparison for investors is that using the GEV-DCC model to measure market risk generally offers the most accurate risk forecast and highest coverage (i.e., the lowest violation rate) in the equity markets, compared with RiskMetrics and historical simulation.

Table 5.19 The backtesting results of competing models based on daily returns

	$VaR_{0.99}$		$VaR_{0.95}$	
Panel A: GARCH model	Long	Short	Long	Short
Developed market				
Violation (%)	0.0524	0.0256	0.1036	0.0652
Unconditional test	33.7913	6.4154**	17.4759	1.6612*
Independent test	1.9864*	1.0381*	8.5902	0.0158*
Conditional test	35.7777	7.4535**	26.0661	1.6770*
Latin American market				
Violation (%)	0.0479	0.0268	0.0946	0.0698
Unconditional test	28.1547	7.2661	12.4924	2.7689*
Independent test	0.2730*	0.5035*	0.1163*	0.5993*
Conditional test	28.4278	7.7696**	12.6087	3.3683*
Asian market				
Violation (%)	0.0461	0.0311	0.0957	0.0715
Unconditional test	26.4132	10.8906	13.2298	3.2555*
Independent test	1.8237*	0.7548*	0.9745*	0.2610*
Conditional test	28.2369	11.6454	14.2043	3.5166*
Panel B: RiskMetrics	Long	Short	Long	Short
Developed market				
Violation (%)	0.0279	0.0035	0.0815	0.0442
Unconditional test	8.1555	2.1301*	6.5973**	0.2709*
Independent test	2.0903*	0.0091*	2.0745*	0.4015*
Conditional test	10.2459	2.1392*	8.6719**	0.6724*
Latin American market				
Violation (%)	0.0280	0.0117	0.0689	0.0444
Unconditional test	8.2033	0.1008*	2.5201*	0.2553*
Independent test	0.0637*	0.1028*	1.5404*	0.0995*
Conditional test	8.2670**	0.2036*	4.0605*	0.3549*
Asian market				
Violation (%)	0.0266	0.0115	0.0843	0.0427
Unconditional test	7.1628	0.0866*	7.7922	0.4398*
Independent test	0.0957*	0.1016*	0.6079*	0.1158*
Conditional test	7.2585**	0.1882*	8.4001**	0.5556*
Panel C: Stochastic volatility	Long	Short	Long	Short
Developed market				
Violation (%)	0.1377	0.1228	0.1861	0.1787
Unconditional test	169.4756	141.3746	83.1288	75.6029
Independent test	2.2229*	1.0713*	5.1132**	0.0502*
Conditional test	171.6985	142.4459	88.2420	75.6530
Latin American market				
Violation (%)	0.1273	0.1379	0.1565	0.1694
Unconditional test	159.0259	180.2627	58.2399	70.7359
Independent test	0.1756*	0.0029*	0.9892*	0.1759*
Conditional test	159.2014	180.2656	59.2291	70.9119
Asian market				
Violation (%)	0.1145	0.1283	0.1660	0.1950
Unconditional test	124.4897	149.4766	62.5851	91.1625
Independent test	2.1264*	3.0055*	1.7760*	2.9875*
Conditional test	126.6161	152.4821	64.3612	94.1500
Panel D: HS (1250)	Long	Short	Long	Short
Developed market				
Violation (%)	0.0314	0.0303	0.1187	0.1129
Unconditional test	11.0416	10.0447	27.2419	23.2819

Independent test	4.7067**	3.2319*	6.3207**	0.7547*
Conditional test	15.7483	13.2766	33.5626	24.0366
Latin American market				
Violation (%)	0.0327	0.0269	0.0970	0.0864
Unconditional test	12.1321	7.3107	13.7077	8.6193
Independent test	1.4245*	0.9150*	1.9990*	0.0135*
Conditional test	13.5567	8.2258**	15.7067	8.6328**
Asian market				
Violation (%)	0.0311	0.0311	0.0911	0.0934
Unconditional test	10.8906	10.8906	10.8956	12.0390
Independent test	1.5985*	0.5432*	0.5194*	2.4154*
Conditional test	12.4891	11.4338	11.4150	14.4545
Panel D: HS (250)	**Long**	**Short**	**Long**	**Short**
Developed market				
Violation (%)	0.0233	0.0198	0.0698	0.0605
Unconditional test	4.8377**	2.8109*	2.7689*	0.8192*
Independent test	0.2121*	0.3926*	9.2289	1.0332*
Conditional test	5.0498*	3.2035*	11.9978	1.8524*
Latin American market				
Violation (%)	0.0199	0.0269	0.0631	0.0864
Unconditional test	2.8369*	7.3107	1.2417*	8.6193
Independent test	0.3905*	0.9150*	4.4253**	0.0135*
Conditional test	3.2274*	8.2258**	5.6670*	8.6328**
Asian market				
Violation (%)	0.0150	0.0161	0.0542	0.0577
Unconditional test	0.8225*	1.2119*	0.1369*	0.4453*
Independent test	0.7654*	0.6573*	6.5778**	0.1899*
Conditional test	1.5879*	1.8692*	6.7146**	0.6353*

1. **(*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

Table 5.20 The backtesting results of competing models based on 10-day returns

	$VaR_{0.99}$		$VaR_{0.95}$	
Panel A: GARCH model	Long	Short	Long	Short
Developed market				
Violation (%)	0.0594	0.0373	0.1327	0.1176
Unconditional test	43.0022	16.5028	37.7136	26.4303
Independent test	43.5569	13.2878	133.1654	78.0734
Conditional test	86.5591	29.7907	170.8790	104.5036
Latin American market				
Violation (%)	0.0536	0.1292	0.1292	0.2619
Unconditional test	35.2779	163.3480	34.9724	184.6410
Independent test	25.8040	52.3276	63.8160	122.1978
Conditional test	61.0819	215.6756	98.7884	306.8388
Asian market				
Violation (%)	0.0559	0.0629	0.1339	0.1804
Unconditional test	38.3104	47.8570	38.6450	82.4693
Independent test	32.0858	7.0693	90.7646	57.3359
Conditional test	70.3963	54.9263	129.4095	139.8053
Panel B: RiskMetrics	Long	Short	Long	Short
Developed market				
Violation (%)	0.0314	0.0012	0.0827	0.0404
Unconditional test	11.0416	4.8080**	7.0576	0.7761*
Independent test	46.1149	0.0010*	87.3528	21.7478
Conditional test	57.1565	4.8090*	94.4104	22.5239
Latin American market				
Violation (%)	0.0199	0.0035	0.0584	0.0589
Unconditional test	2.8369*	2.1700*	0.5266*	0.5937*
Independent test	27.3172	0.0040*	35.6280	47.5612
Conditional test	30.1541	2.1741*	36.1546	48.1548
Asian market				
Violation (%)	0.0164	0.0115	0.0596	0.0520
Unconditional test	1.2724*	0.0866*	0.6785*	0.0301*
Independent test	14.4964	0.0254*	72.0880	55.8347
Conditional test	15.7688	0.1120*	72.7665	55.8649
Panel C: HS (1250)	Long	Short	Long	Short
Developed market				
Violation (%)	0.0407	0.0384	0.1267	0.1035
Unconditional test	20.0984	17.6581	33.1165	17.4255
Independent test	53.0938	0.1272*	126.0825	88.9040
Conditional test	73.1922	17.7853	159.1990	106.3295
Latin American market				
Violation (%)	0.0245	0.0374	0.1051	0.0935
Unconditional test	5.6438**	16.5758	18.3199	11.9025
Independent test	34.2981	41.7700	115.7831	81.1250
Conditional test	39.9419	58.3458	134.1030	93.0275
Asian market				
Violation (%)	0.0323	0.0381	0.1165	0.1027
Unconditional test	11.9124	17.4823	25.9218	17.0760
Independent test	40.1960	44.3690	138.2788	118.8967
Conditional test	52.1084	61.8512	164.2006	135.9727
Panel D: HS (250)	Long	Short	Long	Short
Developed market				
Violation (%)	0.0186	0.0279	0.0593	0.0628
Unconditional test	2.2283*	8.1397	0.6438*	1.1941*
Independent test	20.2107	0.1040*	72.2316	74.3832
Conditional test	22.4391	8.2437**	72.8753	75.5773

Latin American market				
Violation (%)	0.0175	0.0129	0.0678	0.0584
Unconditional test	1.7361*	0.2799*	2.2318*	0.5266*
Independent test	17.3336	22.7622	70.5578	58.4593
Conditional test	19.0698	23.0421	72.7896	58.9858
Asian market				
Violation (%)	0.0254	0.0242	0.0842	0.0750
Unconditional test	6.3058**	5.5040**	7.7603	4.3145**
Independent test	24.8422	26.0659	103.2510	73.1811
Conditional test	31.1480	31.5699	111.0112	77.4956

1. **(*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

Table 5.21 The results of portfolio 99%-VaR based on GEV-DCC model

	GEV(n=5)		GEV(n=10)		GEV(n=22)	
Panel A: daily return	Long	Short	Long	Short	Long	Short
Developed market						
Violation (%)	0.0244	0.0186	0.0244	0.0210	0.0314	0.0198
Unconditional test	5.6053**	2.2359*	5.6053**	3.4380*	11.0416	2.8109*
Independent test	2.7143*	4.1006**	2.7143*	5.7569**	6.6731	3.7799*
Conditional test	8.3197**	6.3365**	8.3197**	9.1949**	17.7147	6.5908**
Latin American market						
Violation (%)	0.0105	0.0058	0.0105	0.0082	0.0129	0.0105
Unconditional test	0.0098*	0.7636*	0.0098*	0.1330*	0.2799*	0.0098*
Independent test	0.0832*	0.0255*	0.0832*	0.0502*	0.1245*	0.0832*
Conditional test	0.0929*	0.7892*	0.0929*	0.1832*	0.4044*	0.0929*
Asian market						
Violation (%)	0.0115	0.0081	0.0104	0.0104	0.0127	0.0173
Unconditional test	0.0866*	0.1494*	0.0058*	0.0058*	0.2555*	1.6708*
Independent test	1.1771*	0.0496*	1.3532*	1.3532*	1.0226*	0.5601*
Conditional test	1.2638*	0.1990*	1.3590*	1.3590*	1.2781*	2.2309*
Panel B: 10-day return	Long	Short	Long	Short	Long	Short
Developed market						
Violation (%)	0.0466	0.0198	0.0547	0.0303	0.0652	0.0384
Unconditional test	26.6731	2.8109*	36.7844	10.0447	51.1798	17.6834
Independent test	74.0098	27.3476	90.3124	43.4727	95.6695	56.9487
Conditional test	100.6829	30.1585	127.0967	53.5173	146.8494	74.6321
Latin American market						
Violation (%)	0.0140	0.0164	0.0152	0.0245	0.0175	0.0070
Unconditional test	0.5391*	1.2724*	0.8718*	5.6438**	1.7361*	0.3751*
Independent test	0.0093*	0.0506*	0.0165*	0.1260*	0.0259*	0.0092*
Conditional test	0.5484*	1.3230*	0.8883*	5.7698*	1.7620*	0.3843*
Asian market						
Violation (%)	0.0186	0.0208	0.0277	0.0231	0.0369	0.0381
Unconditional test	2.2359*	3.3614*	8.0294	4.7446**	16.3101	17.4823
Independent test	28.9539	0.0830*	47.5440	0.0657*	74.6243	0.1762*
Conditional test	31.1898	3.4444*	55.5734	4.8103*	90.9345	17.6585

1. ** (*) means the null hypothesis is not rejected at 1% (5%) level. The critical values of the likelihood ratio test (unconditional test) and the independent test are $\chi^2_{0.01}(1) = 6.6349$ and $\chi^2_{0.05}(1) = 3.8415$. The critical values of the conditional test are $\chi^2_{0.01}(2) = 9.2103$ and $\chi^2_{0.05}(2) = 5.9915$.
2. The numbers in boldface means that a difficulty in the last term of Eq. (4.45) where both π_{11} and n_{11} equal zero and ${\pi_{11}}^{n_{11}}$ cannot be calculated. Therefore, one extreme small number, say 10^{-10}, is assigned to this term for the purpose of convenience to calculate the independent test and Christoffersen's (1998) unconditional coverage test.

Table 5.22 The results of portfolio 95%-VaR based on the GEV-DCC model

	GEV(n=5)		GEV(n=10)		GEV(n=22)	
Panel A: daily return	**Long**	**Short**	**Long**	**Short**	**Long**	**Short**
Developed market						
Violation (%)	0.0768	0.0501	0.0885	0.0629	0.1024	0.0792
Unconditional test	4.8931**	0.0000*	9.5557	1.2059*	16.7980	5.7175**
Independent test	5.8264**	0.6218*	6.2164*	0.0502*	4.8265**	0.5773*
Conditional test	10.7196	0.6218*	15.7722	1.2560*	21.6245	6.2948**
Latin American market						
Violation (%)	0.0549	0.0350	0.0596	0.0397	0.0748	0.0444
Unconditional test	0.1828*	1.9456*	0.6785*	0.8872*	4.1943*	0.2553*
Independent test	3.7644*	0.3199*	5.2433**	1.6642*	4.1351*	2.1144*
Conditional test	3.9472*	2.2655*	5.9218*	2.5514*	8.3294**	2.3697*
Asian market						
Violation (%)	0.0589	0.0346	0.0658	0.0381	0.0785	0.0543
Unconditional test	0.5937*	2.0831*	1.8101*	1.2160*	5.5300**	0.1408*
Independent test	1.9752*	2.3849*	1.8798*	1.8514*	1.0949*	48.2302
Conditional test	2.5689*	4.4680*	3.6898*	3.0674*	6.6250**	48.3710
Panel B: 10-day return	**Long**	**Short**	**Long**	**Short**	**Long**	**Short**
Developed market						
Violation (%)	0.1513	0.0768	0.2037	0.1234	0.2363	0.1723
Unconditional test	53.6285	4.8931**	108.6544	30.5844	149.4658	73.9377
Independent test	119.1629	67.6616	173.6211	100.6029	193.0635	144.1594
Conditional test	172.7914	72.5547	282.2755	131.1874	342.5293	218.0971
Latin American market						
Violation (%)	0.0864	0.0678	0.1308	0.1227	0.1741	0.1121
Unconditional test	8.6193	2.2318*	36.1083	29.9413	75.4992	22.6955
Independent test	92.5322	82.2840	133.8576	95.7197	163.0668	79.8556
Conditional test	101.1515	84.5158	169.9659	125.6609	238.5660	102.5511
Asian market						
Violation (%)	0.0896	0.0715	0.1522	0.1107	0.2099	0.1257
Unconditional test	10.0936	3.2555*	54.9670	22.0568	117.1305	32.6079
Independent test	105.9028	78.6873	150.8256	114.5689	230.3330	130.4648
Conditional test	115.9964	81.9429	205.7926	136.6257	347.4635	163.0728

Note: see the note in table 5.18

Table 5.23 The results of portfolio 99%-VaR based on the GEV-DCC model with a shorter forecasting period

	GEV(n=5)		GEV(n=10)		GEV(n=22)	
	Long	Short	Long	Short	Long	Short
Developed market						
Violation (%)	0.0210	0.0151	0.0233	0.0175	0.0268	0.0163
Unconditional test	3.4380*	0.8582*	4.8377**	1.7163*	7.2661	1.2557*
Independent test	3.4834*	2.6938*	2.9526*	4.4485**	4.0325**	0.1245*
Conditional test	6.9214**	3.5520*	7.7902**	6.1648**	11.2986	1.3802*
Latin American market						
Violation (%)	0.0129	0.0081	0.0152	0.0105	0.0164	0.0117
Unconditional test	0.2799*	0.1378*	0.8718*	0.0098*	1.2724*	0.1008*
Independent test	0.1245*	0.0500*	0.1743*	0.0832*	0.2024*	0.1028*
Conditional test	0.4044*	0.1878*	1.0461*	0.0929*	1.4748*	0.2036*
Asian market						
Violation (%)	0.0104	0.0092	0.0104	0.0081	0.0139	0.0069
Unconditional test	0.0058*	0.0226*	0.0058*	0.1494*	0.5045*	0.4016*
Independent test	1.3532*	0.0649*	1.3532*	0.0496*	0.8859*	0.0364*
Conditional test	1.3590*	0.0875*	1.3590*	0.1990*	1.3904*	0.4380*

1. See the note in table 5.18
2. The results of this table are made with a shorter period. The ten years daily data are from 2nd January 1997 to 29 December 2006.

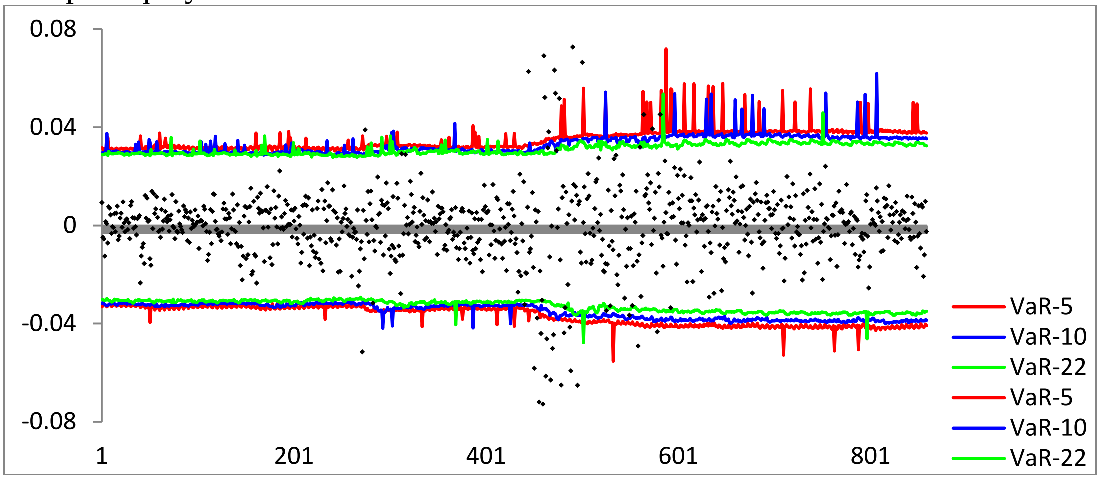
Table 5.24 A comparison of the VaR statistics with two different periods

	GEV-DCC-L			GEV-DCC-S		
Panel A:	n=5	n=10	n=22	n=5	n=10	n=22
Developed market						
Mean	-0.0331	-0.0315	-0.0299	0.0304	0.0292	0.0275
SD	0.0039	0.0030	0.0025	0.0036	0.0027	0.0023
SK	-1.6778	-1.7947	-1.2230	1.8803	1.1349	2.0871
K	6.9675	8.1935	4.9519	8.7052	5.8397	16.3889
Latin American market						
Mean	-0.0509	-0.0509	-0.0464	0.0483	0.0465	0.0439
SD	0.0030	0.0026	0.0020	0.0045	0.0029	0.0048
SK	-4.7427	-4.3082	-1.2966	8.3095	2.5999	11.2145
K	49.4282	61.0197	5.6044	131.0767	24.8214	180.6558
Asian market						
Mean	-0.0364	-0.0402	-0.0372	0.0390	0.0384	0.0288
SD	0.0034	0.0028	0.0018	0.0030	0.0026	0.0015
SK	-6.3828	-3.4599	-1.7419	4.0293	1.5457	0.5150
K	57.6392	20.8678	13.9452	35.3870	8.7618	1.4553
Panel B:	n=5	n=10	n=22	n=5	n=10	n=22
Developed market						
Mean	-0.0369	-0.0352	-0.0329	0.0351	0.0331	0.0311
SD	0.0038	0.0030	0.0023	0.0045	0.0038	0.0023
SK	-0.4063	-0.2016	-0.7770	2.5041	2.0607	1.5208
K	-0.5579	-1.7017	1.5418	11.1360	9.6042	10.2509
Latin American market						
Mean	-0.0491	-0.0474	-0.0444	0.0481	0.0458	0.0430
SD	0.0032	0.0026	0.0019	0.0091	0.0057	0.0044
SK	-7.5405	-7.6041	-3.1171	7.5820	11.6400	16.8581
K	83.8461	110.5461	38.9429	61.0189	155.9128	362.7670
Asian market						
Mean	-0.0410	-0.0390	-0.0367	0.0378	0.0382	0.0366
SD	0.0022	0.0019	0.0018	0.0027	0.0022	0.0019
SK	-3.4808	-3.3781	-4.2767	2.3340	0.0321	-0.0339
K	22.3109	24.1308	46.6033	23.3045	-0.5132	-0.6427

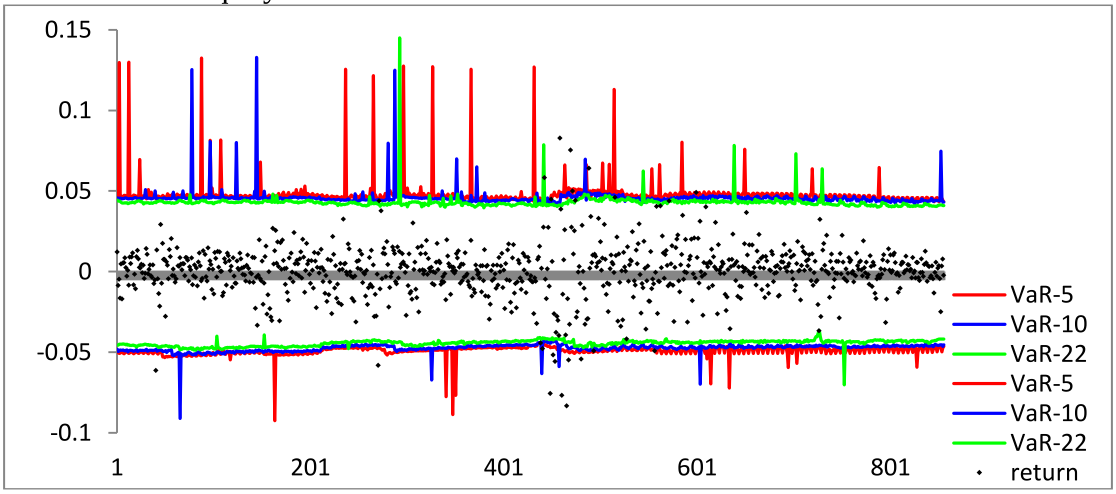
1. The results in panel A are measured by using the data set, which starts from 2nd January 1990 to 29th December 2006, and the results in panel B are obtained from a shorter period sample, from 2nd January 1997 to 29th December 2006.
2. GEV-DCC-L (GEV-DCC-S) means the long (short) position forecasted by GEV-DCC model.

Figure 5.30 The 99%-VaR patterns of the GEV-DCC model with a different size of block with shorter data set

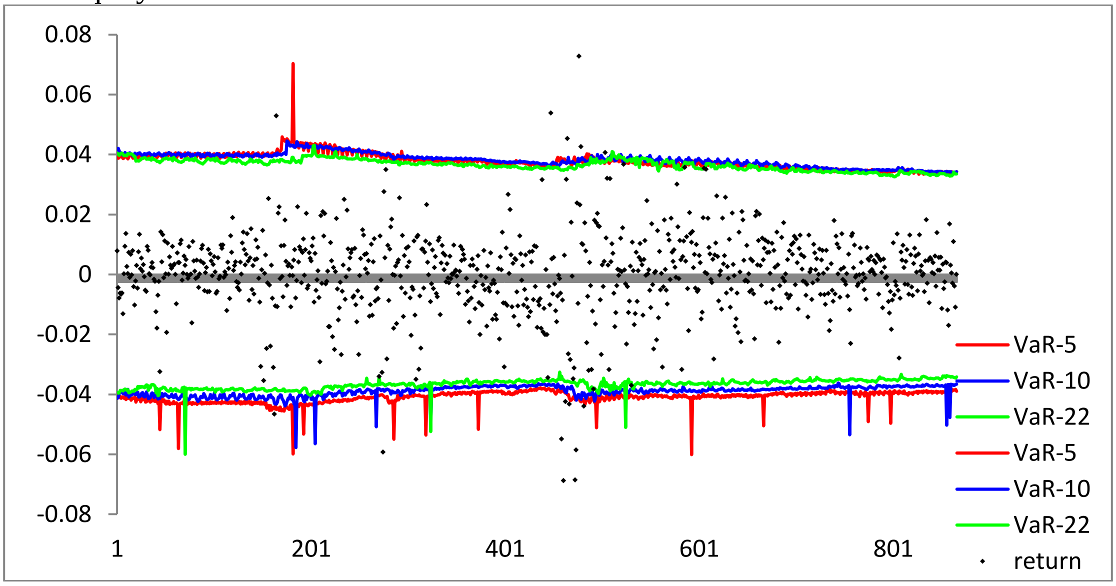
(a)developed equity market



(b)Latin American equity market



(c)Asian equity market



5.4.2 The quality checks of portfolio VaR

In this section, a quality check composed of three measures is set up for examining the practicability and adaptability of the VaR model. In the previous section, the backtesting based on the coverage tests was the main approach for testing if the results of the VaR model were robust. Theoretically, the violation rate is expected to equal the value of α, and thus this is regarded as a good assessment of the ability of the VaR model. However, the coverage tests have their own disadvantages. Owing to the nature of the coverage tests, the violation rate merely reflects the proportion of exceedance in the backtesting sample, making it difficult to describe the quality of those non-exceeded observations and the magnitude of the violations. It is not even possible to show the quality of the VaR pattern using the violation rate. As mentioned in section 2.4, the results of the VaR measurements could be used as a risk report for fund managers when altering the composition of the portfolio, as shown in Eq. (2.38) and (2.39). However, the VaR model would not seem to be a good measuring tool for this if the forecasted VaR pattern is too volatile to be implemented in practice, even if the violation rate is significantly better. Therefore, there is a need here to examine the quality of the VaR model apart from the coverage test. Similar to the section looking at the univariate case, the mean squared error (MSE), D, and Q are calculated to examine the fluctuation, conservativeness, and magnitude of violation. Intuitionally, a VaR model producing a high MSE indicates that those VaR results could encounter problems when it comes to being used in real life. On the other hand, using a VaR model as a market risk measure that has a high D and Q score might cause over- or under-estimation of the quantity of market risk, putting financial institutions at danger of being inefficient or too risky with their capital. Thus, a quality examination of the VaR model is essential, although most previous research in this field just looks at the coverage test. According to Eq. (2.38) and (2.39), and the spirit of the violation rate, the measures Q and D need to be balanced under a proper violation rate and the MSE.

The results of the three measures are exhibited in Table 5.25. In the MSE column, as expected, the MSEs in the GEV-DCC panel are the smallest, indicating that the GEV-DCC model offers a relatively stable VaR pattern and that it would work well as a VaR measure when implemented in practice. In addition, both the results in the daily and 10-day return panel show that the values of the MSE negatively relate to the size of block. For example, MSEs of the 5-day block are 0.1509, 0.0921, and 0.1400 for the three equity portfolios, and 0.0634, 0.0384, and 0.0333 for the block of 22-day. A similar pattern can be found in the GEV-DCC results for the 10-day return sequence in panel B. The values of the MSE in panel C (RiskMetrics) and D (HS method) are much larger than the ones in the GEV-DCC panel, especially those of HS with n=250. Thus, RiskMetrics and HS-250 seem like they could be difficult to use in practice although the coverage tests of the two methods were quite significant (see the previous section). In panel E, the SV model offers quite a volatile VaR pattern in the Latin American market (17.2692). Overall, the GARCH model is the most stable of the four competing VaR models. However, its MSEs (0.9035, 0.5037, and 0.3339 for the three portfolios) are significantly higher than the GEV-DCC model.

The second part looks at the measure D, derived by calculating the average distance between the non-violated VaRs and the portfolio returns. Generally, the results in the D column of Table 5.25 are obscure, but the evidence illustrates that the GARCH and SV models generate an equally small average distance between the VaR and portfolio return in both the long and short position, implying that these two models have less of a tendency towards conservativeness. No matter whether it is 250 or 1250 days, the HS model produces a larger D measure (from -0.0320 to -0.0655 to the long position, and from 0.0266 to 0.0621 to the short position), suggesting that the HS model is quite conservative in forecasting the market risk of the three market portfolios. The GEV-DCC model provides decent results for the VaR pattern of the daily return sequences, but it fares worse in the 10-day return sequences. Intuitionally, this could be attributed to the low violation rate as shown in panel B of Table 5.21; since a conservative VaR model derives higher VaR patterns, and a lower violation rate is obtained. Although the results are not very clear, in summary, the GARCH model, the SV model, and the daily return GEV-DCC demonstrate a good performance and would not be too conservative for measuring market risk in practice.

The third part of the process is to measure the magnitude of the violations for each VaR model. The coverage test could not tell the potential losses when the VaRs were exceeded but the Q measure can fill this gap. As exhibited in the last two columns, all of the models actually provide a good result in the Q measure, with the exception of the GEV-DCC model in the 10-day return pattern. The Q of the GEV-DCC model using the daily returns spans from -0.0086 to 0.0188 (0.0089 to 0.0207) in the long (short) position. In fact, the VaR pattern of the GARCH model offers the smallest Q, from -0.012 to -0.105, in the long position, and from 0.0072 to 0.0115 in the short position. In other words, based on the GARCH model, the portfolio would encounter a loss in the long (short) position, on average, -0.0105 (0.0115) for the developed market, -0.0103 (0.0104) for the Latin American market, and -0.0102 (0.0072) for the Asian market, respectively. Those numbers indicate that once a violation occurs, the value of the portfolio might have an extra loss, in percentage, more than the buffer provided by the VaR. For example, the average value of the violated $VaR_{0.99}$ to the portfolio of the developed market is -0.0337. The mean of the violating returns in this portfolio is -0.0510, and thus the portfolio might have an -0.0173 loss exceeding the average forecasted VaR once a violation happens.

As demonstrated in this chapter, MSE, and the measures D and Q are useful validation tools. Looking at MSE, the GEV-DCC model provides the smallest volatility. Under certain circumstances, VaR could be applied as an indicator for capital reserve to cover potential losses. Measure D helps to evaluate how conservative a model is when estimating risk. A higher D value, such as with the HS model, would cause an inefficiency in funding usage as the VaR overestimates market risk and thus the financial institution holds on to a higher capital reserve than is really necessary. The GARCH, SV and GEV-DCC models provide the smaller D, on average.

A higher Q value means that the VaR is underestimating and the financial institution would be putting itself at risk by not holding on to sufficient capital to cover potential losses, particularly under extreme market conditions. Assessing the Q measure, the GEV-DCC and GARCH models offer a better outcome than the other models, suggesting that the portfolio might encounter smaller extra losses once a violation occurs. To sum up the discussion above, the GEV-DCC model seems to provide the best result taking into account all three requirements.

Table 5.25 The comparison of other benchmark tests between various VaR models in 99%

	MSE (10^{-4})		D		Q	
Panel A: GEV-DCC (daily)	Long	Short	Long	Short	Long	Short
n=5						
Developed market	0.1509	0.1298	-0.0342	0.0315	-0.0173	0.0187
Latin American market	0.0921	0.2066	-0.0519	0.0483	-0.0086	0.0147
Asian market	0.1400	0.1091	-0.0412	0.0392	-0.0127	0.0108
n=10						
Developed market	0.0911	0.0729	-0.0325	0.0304	-0.0188	0.0168
Latin American market	0.0690	0.0861	-0.0519	0.0466	-0.0138	0.0146
Asian market	0.0787	0.0711	-0.0410	0.0387	-0.0143	0.0089
n=22						
Developed market	0.0634	0.0514	-0.0312	0.0287	-0.0176	0.0207
Latin American market	0.0384	0.2323	-0.0475	0.0442	-0.0147	0.0138
Asian market	0.0333	0.0274	-0.0381	0.0293	-0.0151	0.0120
Panel B: GEV-DCC (10-day)	Long	Short	Long	Short	Long	Short
n=5						
Developed market	0.7814	0.9815	-0.0868	0.0882	-0.0529	0.0216
Latin American market	0.8145	0.1640	-0.1378	0.0901	-0.0567	0.0228
Asian market	0.8033	1.5246	-0.1222	0.0930	-0.0474	0.0153
n=10						
Developed market	0.2988	0.4808	-0.0807	0.0778	-0.0503	0.0230
Latin American market	0.5523	0.0701	-0.1275	0.0879	-0.0636	0.0195
Asian market	0.2351	0.5743	-0.1138	0.0920	-0.0389	0.0121
n=22						
Developed market	0.1464	0.2133	-0.0749	0.0695	-0.0497	0.0275
Latin American market	0.2653	0.6147	-0.1159	0.1102	-0.0706	0.0236
Asian market	0.0832	18.5158	-0.1046	0.0825	-0.0419	0.0218
Panel B: GARCH	Long	Short	Long	Short	Long	Short
Developed market	0.9053	0.9053	-0.0270	0.0265	-0.0105	0.0115
Latin American market	0.5037	0.5037	-0.0291	0.0275	-0.0103	0.0104
Asian market	0.3339	0.3335	-0.0249	0.0238	-0.0102	0.0072
Panel C: RiskMetrics	Long	Short	Long	Short	Long	Short
Developed market	3.2009	3.2009	-0.0305	0.0302	-0.0155	0.0123
Latin American market	2.7226	2.7226	-0.0346	0.0332	-0.0187	0.0164
Asian market	1.6414	1.6414	-0.0300	0.0290	-0.0182	0.0150
Panel D: HS	Long	Short	Long	Short	Long	Short
n=250						
Developed market	10.5563	11.7634	-0.0655	0.0621	-0.0154	0.0194
Latin American market	2.5550	0.9229	-0.0429	0.0326	-0.0125	0.0140
Asian market	4.0213	1.8182	-0.0389	0.0310	-0.0116	0.0110
n=1250						
Developed market	3.6496	1.6520	-0.0568	0.0464	-0.0310	0.0313
Latin American market	0.7225	0.9242	-0.0355	0.0326	-0.0143	0.0140
Asian market	0.5393	0.6212	-0.0320	0.0266	-0.0110	0.0097
Panel E: SV	Long	Short	Long	Short	Long	Short
Developed market	1.0213	1.0213	-0.0210	0.0211	-0.0130	0.0102
Latin American market	17.2692	17.2692	-0.0467	0.0463	-0.0125	0.0104
Asian market	0.8225	0.8225	-0.0197	0.0193	-0.0114	0.0083

Note: The results of the 10-day return sequences of the competing models are not shown in this table because of their failure in the coverage tests compared with GEV-DCC model.

5.5 Implication of the results

The results shown in section 5.4 have two vital implications, which are likely to be a valuable reference to the practitioners. For convenience, a long portfolio with a 100-pound value (initial investment) to each equity index is assumed and held by a risk-averse reasonable investor, i.e. this portfolio includes 16 positions, and each investment is 100 pounds. Firstly, from the correlation analysis results presented in panel A of Table 5.16, RO54 (CAC40/DAX) has the highest average correlation in both the left (0.8483) and right tail (0.8176), which means that if the CAC40 index return declines (rises) 10%, then the DAX return will go down (up) about 8.483% (8.176%). By contrast, the correlations of RO62 (Nikkei 225/TSX) are 0.4345 and 0.3331 in the left and right tail, suggesting that the Nikkei225 index return goes down (up) 10% on average but the TSX will only move in the same direction by 4.345% (3.331%). According to portfolio theory, the investor might then consider adjusting the weight of each component accordingly. Alternatively, you could argue that DAX or CAC40 should be sold since they are highly correlated.

Secondly, it is useful and practical to use univariate and portfolio VaR to forecast potential loss in the next trading date. According to the backtesting results, overall the GEV-DCC model offers the best market risk measure. As shown in Table 5.17, GEV-DCC presents average potential losses of -0.0331, -0.0509, and -0.0364 (short for 0.0304, 0.0483, and 0.0390) to developed, Latin American, and Asian equity index portfolios, respectively. Specifically, with 99% probability, the investor's worst loss in the developed equity market will be not more than 19.86 (600*-0.0331) and 18.24 (600*-0.0304) pounds for the long and short positions respectively. The maximum losses to the other two portfolios are 20.36 (400*-0.0509) and 21.84 (600*-0.0364) pounds for the long position (19.32 and 23.4 for the short position). The pound-based loss interpretation provides a clear indication of how to use the VaR numbers. The results of the correlation analysis, portfolio VaR calculation, and backtesting imply that the GEV-DCC model not only theoretically provides accuracy in estimating parameters and fitting extreme distributions, but also offers a good and practical market risk measure.

5.6 Conclusion

This chapter presents the results of various VaR models based on two traditional coverage tests. The content starts with the univariate VaR forecast in Section 5.2, then continues with the correlation analysis based on the tail-DCC model we first suggested in section 5.3, and finally ends in the analysis of a portfolio VaR. In the evaluation of the VaR results, three substantial measures are proposed to test the quality of a VaR model. The MSE focuses on the fluctuation of the VaR pattern, measure D looks at the conservativeness of the risk measure, and Q describes the magnitude of the violations. Combining the three measures with the coverage tests, one could comprehensively understand the properties of a VaR measure, such as variability, conservativeness, magnitude of loss of the violations, and the accuracy of the model. In the correlation analysis, a tail-DCC model is suggested for measuring the relationship between the individual assets in the portfolio. The tail-DCC model emphasizes the correlation in the tail area of the distributions because the most market risk generally happens in the left and right tail rather than across the whole distribution. Therefore, correlation of the extreme observations is more important than assessing the entire sample.

The backtesting results of portfolio VaR are generally consistent with the results of the univariate VaR patterns. In the coverage tests of the univariate case, the GEV model using daily return sequences with a 5- or 10-day block, and the GARCH (1,1) model provide a better performance for measuring VaR. However, according to the results of the MSE measure and graphical analysis, the VaR pattern of all equity markets obtained from the GARCH, HS, and SV models are too volatile to be implemented in practice. Therefore, taking into account the results of the coverage tests and the three measures, the GEV model is the most appropriate model for measuring the risks of the equity indices.

In Section 5.3, substantial evidence of the presence of the thick tail within the distribution of financial returns is provided by using the method of fixed window rolling forecast.

Furthermore, the graphical evidence also indicates that the fatnesses in the left and right tail are significantly different. The analysis of the shape of the tail implies that the distributions of financial returns are time-varying. In this manner, it is natural that the correlations between various asset returns would not be the same in the left and right tail. In section 5.3.2, the correlations in the left and right tails are exhibited, directly suggesting that the correlations are indeed time-varying and different in the left and right tail, although they tend to be positive. Looking at the patterns of tail-DCC, a cycle or a structural change could be found in most of the tail-DCCs examined, particularly in the third quarter of 2008.

In Section 5.4, the evidence suggests that the GEV-DCC model is an appropriate model for measuring portfolio VaR. The backtesting results suggest that the GEV-DCC model offers an accurate coverage in general, and in the three quality measures it demonstrates a stable VaR sequence and decent D and Q measures. Among the four competing models, we found that the VaR derived from the HS model responds to market information slowly, and the longer the period of historical sample used the more sluggishly the VaR pattern reacted to the market condition. A similarly slow response could be found in the results of RiskMetrics as well. Although the HS model and RiskMetrics produce as strong a performance as the GEV-DCC model with the 5-day block in the backtesting of daily returns, the MSE measures of HS and RiskMetrics are larger than the ones from the GEV-DCC model, suggesting that those two models would encounter some difficulties when executed in a real life situation. Moreover, the VaR of the HS model tends to be over-conservative (as shown by it having the highest measure of D), implying the usage of funding might be inefficient. Although the GARCH model is the better one in the univariate case, it could not extend this superiority to the multivariate version because of the high violation rates across all three portfolios. In both the univariate VaR and the portfolio VaR the pattern in the volatility is higher than the other models. In addition, we found that the SV model has difficulty in converging in the multivariate numerical process, and this causes a low coverage (i.e. poor backtesting result) and volatile VaR patterns. We also presented the implications of the results with a simple example in Section 5.5. Generally, compared with other VaR models, using the GEV-DCC model to forecast the risk in equity markets provides the most accurate risk measure and reliable coverage, which is not too volatile, satisfactory VaR patterns and manageable potential loss.

Chapter 6 Conclusions

6.1 Summary of findings and discussion

In the past two decades, risk management, particularly understanding the market risk, has been a critical issue due to the huge losses caused by several financial crises. In more recent years, risk management has also been extended to various other applications within financial institutions, for example for performance evaluation. The main goal of this thesis is investigation of the individual market risk of the major international equity indices using extreme value theory, and the offering of a credible portfolio VaR model taking into account the important characteristics of financial returns such as thickness and asymmetry. In addition, an empirical study with sixteen daily equity indices (six developed equity market indices, six Asian equity market indices, and four Latin American equity market indices) over twenty years, collected from DataStream, is provided. A method of fixed window rolling is applied to forecast one-day ahead tail distribution and VaR. In the accuracy evaluation, the forecasted VaR sequences of the period from 2nd January 2007 to 30th April 2010 (around 850 observations for each index return) are backtested based on two conventional coverage tests. In this thesis, we show that evaluation of the performance of a VaR model should not entirely rely on quantitative coverage tests, but also take into account the quality of the whole VaR pattern in the backtesting period by using three measures describing variability, conservativeness, and the magnitude of the violation. The combination of all these tests gives a more comprehensive understanding of the VaR models' suitability in practice.

The main findings of this thesis are divided into three parts: the univariate VaR, the analysis of tail-DCC, and the outcomes of portfolio VaR. In the univariate VaR analysis, the tail distribution (also called the GEV distribution) is fitted well using extreme value theory across three different lengths of the block maxima approach. For both daily and 10-day returns, the significance, volatilities and stationarity tests (see Appendix C) of the estimated parameters show the high stability of those parameters. The equality tests show significant evidence of asymmetry between the left and right tails of index return distributions. More specifically, the smaller tail parameters in the left side of the GEV distribution indicate a ticker tail, and this implies that the market risks of the long positions are generally larger than the ones of the short positions. However, the GEV distributions of 10-day returns do not fit as well as the daily returns, especially with the longer blocks.

The individual VaRs of the GEV model[87] and the four competing models (GARCH (1,1) model, RiskMetrics, stochastic volatility, and historical simulation) can be obtained by setting a specific quantile, says 1%, in the tail distribution. From the backtesting results of the individual $VaR_{0.01}$, we find that the GEV model with the 5-day block, the HS-250 and the GARCH(1,1) model provide the best coverage ratios with the forecasted daily VaR sequence. Specifically, the violation rates of these three models are statistically equivalent to the confidence level, α[88]. In addition, the GEV model also offers a significantly better performance with the 10- and 22-day blocks. However, the GEV model is not good enough in the developed equity market. Due to stronger fluctuations in the 10-day returns, the GEV model produces the worst coverage in its VaR sequence, i.e. there are too many violations.

Looking at the competing models, some of their drawbacks are evident in their forecasted VaR patterns. For example, the pattern of historical simulation responds to the market condition slowly, and the one from the GARCH model is too volatile to be implemented in practice, although both of their results in the coverage tests were the best amongst all of the

[87] GEV model means a VaR model with generalized extreme value (GEV) distribution obtained from extreme value theory.

[88] The details of α can be referred to in eq. (2.1) in section 2.2.2.

competing VaR models. We also found that the forecasted VaR of the HS model is significantly affected by the number of historical data used in the forecast of empirical distribution. A longer period of historical data produces a slower change in the VaR pattern. This would not be appropriate in practice. The VaR pattern of the GARCH model dramatically peaks at its highest value around the end of 2008 and then drops down quickly in the beginning of 2009, due to its volatility-oriented nature. As mentioned in Section 2.4, a VaR model frequently presenting large changes within its pattern is not appropriate for financial institutions in a real life situation since the transaction costs associated with changing the portfolio constituent will offset the trading profit. The results of RiskMetrics are consistent with Eberlein, Kallsen and Kristen's (2003) suggestion that the VaR pattern is significantly influenced by the fixed decay parameter and highly dominated by previous volatility. As a result, it presents a slow change pattern in forecasted VaR.

In the second part, the measurement of the dynamic conditional correlations between the tail-distributions (call the tail-DCC) is applied, and the results show that this model describes well the time-varying correlations in the tail area between various financial returns. Compared with the original DCC model, tail-DCC is more appropriate for risk management because it independently measures the correlations of the left and right tails rather than only one correlation across the whole distribution. Evidence shows that the tail-DCCs of the left tails are generally higher than the ones of the right tails, implying the equity indices tend to move together in the downturn period rather than in the up-turn period. This phenomenon is consistent with the concept of asymmetry in the financial distribution, but cannot be observed when using just the original DCC model. In the patterns of tail-DCC, we also find important characteristics in the correlation pattern of extreme returns such as structural changes, circulation patterns and asymmetry between left and right tails of the index return distribution. Most tail-correlations are positive which indicates that the big price (extreme price) changes tend to go with each other in the same direction. Finally, the tail-DCC model also captures a structural change or circulation in the pattern of the tail-DCC, providing useful data for analysis.

In the portfolio VaR analysis and backtesting, the results of the coverage tests and quality checks suggest that the GEV-DCC model is the best model for measuring portfolio VaR because of its accuracy with violation rates, its small variation in VaR patterns, and a good performance in conservativeness and in potential loss tests. From the portfolio risk perspective, on average, the Latin American equity market is the riskiest, and the developed equity market has the lowest risk. Generally, VaR in the left tail from the GEV-DCC model is higher than the one in the right tail for both the developed and Latin American equity markets, indicating that the long positions are riskier than the short ones. The results of the portfolio VaR for the four competing models are slightly different from the ones in the univariate cases. Overall, RiskMetrics and HS-250 offer adequate coverage in the backtesting tests. However, the GARCH model is superior to the other three competing models, particularly in the short positions of the developed and Latin American equity markets. From the quality checks, the results of MSE show that the GEV-DCC model presents the most stable VaR patterns, implying that this model is suitable for practical use, whereas the historical simulation and stochastic volatility models produce very fluctuating VaR patterns, unsuitable for real life situations. From the conservativeness of the VaR models (described by the D measure), the results from the GARCH and GEV-DCC demonstrate that both models show only a small tendency towards conservativeness, which means that financial institutions would not be too conservative by reserving too much cash and thus losing their efficiency in capital use. In contrast to this, the evidence also shows that historical simulation is the most conservative VaR model, producing inefficiency of fund use by the financial institutions. On the other hand, the evidence of the magnitude of potential losses (described by the Q measure) shows that the potential losses of the GEV-DCC model are equivalent to the ones of the GARCH and historical simulation models. Although the GEV-DCC model is not always the best in each individual aspect of the quality checks, its results are overall the most reasonable and acceptable.

In the VaR patterns of the four competing models, the historical simulation, like its results in the univariate case, responds to the market information too slowly. This gets worse when using longer periods of historical samples in the estimation of empirical distribution. An equivalently slow response is also found in the RiskMetrics VaR patterns. Although both the HS model and RiskMetrics provide a strong performance in the backtesting of daily returns (equivalent with the GEV-DCC with a 5-day block), there still could be difficulties in implementing them in practice due to their large MSE measures. The GARCH model is the best in the univariate analysis, but its superiority could not be extended to the multivariate version because of the high violation rates in all the three portfolios. We also found equally high violation rates in the VaR patterns of the SV model. From the backtesting results of the 10-day VaR, the GEV-DCC model provides accurate coverage for the Latin American market and for a section of the Asian equity market. However, the 10-day VaR coverage of all the competing models is quite low, indicating that these models are not suitable for 10-day VaR forecasting. To demonstrate the robustness of the GEV-DCC model, a shorter period sample was used for forecasting one-day ahead daily VaR in this thesis, the results show that the GEV-DCC model with a shorter sample period provides a similar significance to the ones using the original period. In summary, the GEV model is indeed superior to the four competing models.

6.2 Contributions and implication

There is still a need for a comprehensive market risk measure due to the losses caused by several financial crises in recent years. Most previous research focused on modelling the VaR measure of a single asset rather than looking at the risk measurement of a portfolio. A well-calibrated VaR model needs to consider the critical characteristics of financial returns. In addition, the measurement of the correlations between individual assets should be accounted for in the portfolio version. As previously mentioned, most research to date does not fulfil all the needs discussed above. Thus, the main contribution of this thesis is to fill this gap in the literature by offering a portfolio VaR model which considers all the important characteristics of financial returns. For increasing the accuracy of return distributions, the suggested portfolio VaR model applies extreme value theory to measure the VaR of a single asset, and a flexible tail-DCC model is then applied to aggregate all the individual VaRs into a whole portfolio VaR. The main advantage of this method is to offer a detailed view of all individual VaRs, and a clear pattern of the relationship (correlations) between the various assets. This study also contributes to the current literature by proposing the critical concept of seriality (and the correlation calculations with seriality) and a special correlation, called tail-correlation. From the results of this correlation model, it obviously describes some special characteristics that the original DCC model could not, such as structural changes, circulation patterns and asymmetry between the left and right tails of the index return distribution. Finally, this thesis also contributes to the existing literature in the VaR model evaluation, considering not only the coverage tests, but also the stability, conservativeness, and magnitude of violations of a VaR model. Whereas the stability test examines the variation of the entire VaR sequence, the tests of conservativeness and the magnitude of the violations pay attention to the quality of the non-violated VaR and the violated observations, respectively. Both of these tests are used to evaluate the quality of the forecasted VaR patterns. From the perspective of practical application, both the over-conservative VaR models or the VaR models incurring a high loss on the occurrence of violations is inappropriate for real life application. In summary, all the three measures suggested by this thesis describe the suitability of the VaR model for real life application.

In this thesis, we suggest a practical portfolio VaR model and a reasonable evaluation method, particularly for financial institutions and individual investors. In addition, this thesis also profiles the market risks of the major international equity indices. Generally, this thesis can be used in two aspects: portfolio management (ex ante) and risk management (ex post). In the aspect of portfolio management, this VaR model can be applied in place of the volatility in the efficient frontier. In addition, the tail correlation supports investors to realize the special relationship between the assets. In the simple application of ex post aspect, it is helpful for financial institutions (or a fund management team) to expand their understanding of the individual and portfolio risks within the portfolio by using the theoretical and practical VaR method.

6.3 Research limitations and future research

This thesis offers a comprehensive portfolio VaR model and new methods of model evaluation. However, it is still limited by three main factors as follows. The first limit of this thesis is with the techniques and development of multivariate extreme returns distribution. Alternative portfolio VaR models may be directly derived from multivariate extreme value distribution without the procedure of dynamic conditional correlation. To our best knowledge, however, this multivariate distribution has not been conducted in the related literature, probably due to its complexity. Although some previous researchers have paid attention to this issue, it is still in the progress of development (see Tawn (1988), Tawn (1990), and Zhang, Wells and Peng (2008)). In this thesis, individual VaR is obtained by inverting the extreme value distribution with a probability. Yet portfolio VaR cannot be carried out with this methodology. Under the best conditions, the portfolio VaR can be shown as a range of sets in the n-dimensional space (i.e. it is not a particular number but a range), but this is meaningless for practitioners. This thesis is also limited by the availability of data. Using real portfolio data from financial institutions is the best approach for VaR model evaluation, however it is very difficult to access a large number of portfolio data from financial institutions. In addition, the details of the VaR models used in the financial institutions are highly confidential. This could be the explanation behind why none of the previous research uses real data in VaR modelling and evaluation. The last main limit of this thesis is the time of calculation, particularly with the portfolio VaR analysis. It is quite time-consuming in the correlation analysis if more equity indices are included into these portfolios. In addition, including more equity index returns increases the difficulty of convergence of the tail-DCC. For example, except for the fitting of maximum likelihood function, $(2 \times d + d \times (d-1)/2)$ parameters and correlations have to be estimated in every step of the fixed-window rolling forecast in the current d-variable vision, and there are about 850 rollings (both in the long and short positions) for each portfolio. Therefore, the scope of the datasets has to be narrowed down to save the time of calculations.

Including the difficulties mentioned above, this thesis could be extended in several new directions for further research if the data and time were available. Firstly, as mentioned above, it is worthwhile to develop and apply the multivariate extreme value distributions in the measurement of portfolio VaR. However, this method needs a large number of historical data to fit the tail distribution of financial returns, which is a laborious problem for financial assets that do not have that much historical data. Moreover, as mentioned in Section 2.4.2, VaR can be applied in the performance system. Exactly how to decompose the firm-wide VaR to the department level VaR is a critical area for further research. Secondly, high frequency data could be applied in a portfolio VaR model for extracting more information from the VaR intra-day pattern, which may be helpful for the day-trading traders. The third is a minor issue in the VaR model evaluation. The majority of the previous research quantitatively examined the performance of VaR models based on Christoffersen's (1998) approach. However, its independence test of violations is strongly sensitive to the new violation, especially in cases where there are only a few violations. Perhaps the best approach would be to extend Christoffersen's (1998) method with the consideration of stability, conservativeness and magnitude of violations.

Appendix A the statistical properties of D and Q and their economic meanings

We show the statistical properties of D and Q in this Appendix, and the statistical and economic meanings of them are provided by an example in the second part.

The first part—the properties of D and Q

According to Eq. (4.52)

$$Q = \frac{\sum_{i=1}^{N-m}(R_i - VaR_i)_{R_i<VaR_i}}{N-m} \quad \text{(A-1)}$$

$$= \frac{\sum_{i=1}^{N-m} R_i - (VaR_1 + VaR_2 + \cdots + VaR_{N-m})}{N-m} \quad \text{(A-2)}$$

According to the definition of VaR, $VaR = \hat{\mu}_r + Z_\alpha \hat{\sigma}_r$, thus

$$Q = \frac{\sum_{i=1}^{N-m} R_i + (\hat{\mu}_{r,1} + Z_\alpha \hat{\sigma}_{r,1}) + (\hat{\mu}_{r,2} + Z_\alpha \hat{\sigma}_{r,2}) + \cdots + (\hat{\mu}_{r,N-m} + Z_\alpha \hat{\sigma}_{r,N-m})}{N-m} \quad \text{(A-3)}$$

$$= \frac{\sum_{i=1}^{N-m} R_i + (\hat{\mu}_{r,1} + \hat{\mu}_{r,2} + \cdots + \hat{\mu}_{r,N-m}) + Z_\alpha(\hat{\sigma}_{r,1} + \hat{\sigma}_{r,2} + \cdots + \hat{\sigma}_{r,N-m})}{N-m} \quad \text{(A-4)}$$

Simply, we assume the average return is zero, e.g., $\hat{\mu}_{r,i} = 0$, i=1,2,…,N-m, then

$$Q \cong \frac{\sum_{i=1}^{N-m} R_i + Z_\alpha(\hat{\sigma}_{r,1} + \hat{\sigma}_{r,2} + \cdots + \hat{\sigma}_{r,N-m})}{N-m} \quad \text{(A-5)}$$

Thus, the distribution of Q is a mixed distribution of R_i and $\hat{\sigma}_i$ (we use this as a symbol of the estimator of σ, however, one might use s or standard deviation). Since R_i and VaR_i are the violated observations in the original return and VaR sequences, for convenience they are assumed to be serially independent.

If $N \to \infty$, then R_i follows a normal distribution, and $\sum_{i=1}^{N-m} R_i$ also follows a normal distribution. According to Kenney and Keeping (1951), the distribution of standard deviation is

$$f_4(s) = 2C_5 e^{-Ns^2/2\sigma^2} s^{N-2} \quad \text{(A-6)}$$

where $C_5 = \left(\frac{N}{2\sigma^2}\right)^{(N-1)/2} \Big/ \Gamma\left(\frac{N-1}{2}\right)$, N is the number of samples, and $\Gamma(\cdot)$ is a gamma function. Thus, the expectation of standard deviation is

$$E(s) = \frac{\left(\frac{2}{N}\right)^{1/2} \Gamma\left(\frac{N}{2}\right)}{\Gamma\left(\frac{N-1}{2}\right)} \sigma \quad \text{(A-7)}$$

Thus,

$$E(s) = b(N)\sigma \quad \text{(A-8)}$$

where $b(N)$ is a function of N, and the asymptotically $b(N) = 1 - \frac{3}{4N} - \frac{7}{32N^2} \cdots$ (Romanovsky, 1925). When $N \to \infty$, $b(N)$ is close to 1. In addition, they also show that the *r*th moment of the distribution of standard deviation as

$$\nu_r = \left(\frac{2}{N}\right)^{r/2} \frac{\Gamma\left(\frac{N-1+r}{2}\right)}{\Gamma\left(\frac{N-1}{2}\right)} \sigma^r \quad \text{(A-9)}$$

Thus, the variance of standard deviation can be given by Eq. (A-9)

$$\mathrm{var}(s) = k(N)\sigma^2 \quad \text{(A-10)}$$

where $k(N) = \frac{1}{2N} - \frac{1}{8N^2} - \frac{1}{16N^3} \cdots$. The details of b(N) and k(N) can be found in Kenney and Keeping (p.171,1951).

According the analysis above, the distribution of Q is a mixed distribution of normal distribution and the distribution of standard deviation as shown in Eq. (A-6). The D measure has the similar definition with Q. Thus, the distribution of D is suggested as a mixed distribution of normal distribution and the distribution of standard deviation as shown in Eq. (A-6).

$$D = \frac{\sum_{i=1}^{m} (VaR_i - R_i)_{VaR_i \le R_i}}{m} \quad \text{(A-11)}$$

$$\cong \frac{Z_\alpha(\hat{\sigma}_{r,1} + \hat{\sigma}_{r,2} + \cdots + \hat{\sigma}_{r,m}) - \sum_{i=1}^{m} R_i}{m} \quad \text{(A-12)}$$

The second part—the statistical and economic meanings of D and Q

In this part, we explain the statistical and economic meanings of Q and D with a simple example from the S&P 500 long position of GEV (n=5) and GARCH (1,1) shown in Table A-1. In the column of D, GEV (n=5) is -0.0316 and GARCH (1,1) is -0.0353. As mentioned in Section 2.4, average VaR is regarded as the amount of fund reserved for covering potential losses. Thus, the statistical meaning of D is that the VaR models with larger D (absolute value), on average, reserve more funds. In Table A-1, the absolute value of GARCH (1,1) (0.0353) is larger than the one of GEV (n=5) (0.0316) showing the differences between the VaRs and their corresponding returns of those non-violated observations of GARCH (1,1), on average, are larger than the ones of GEV (n=5). In other words, financial institutions will reserve more funds for covering potential losses if they use VaR from GARCH (1,1). This implies GARCH (1,1) is more conservative than the GEV (n=5) model. In these circumstances, the fund usage of those financial institutions is inefficient and the average VaR of GARCH (1,1) is reasonably larger than the one of GEV (n=5). The economic meanings of D is -- if the original value of the investment is 1,000,000, and financial institutions secure the funds based on VaR of GEV (n=5) and GARCH (1,1), thus they have to reserve 29,500 (0.0295×1,000,000) and 34,000 (0.0340×1,000,000) against potential losses, respectively. Moreover, financial institutions using GEV (n=5) and GARCH (1,1) models as their risk measures, on average, reserve 31,600 (0.0316×1,000,000) and 35,300 (0.0353×1,000,000) more than the average return on the non-violated trading days. Thus, the VaR model with higher D such as GARCH (1,1) causes the financial institutions to reserve more capital. In this case, financial institutions using GARCH (1,1) model have to reserve more 3,700 (i.e. 35,300-31,600=3,700) than GEV (n=5).

In the part of Q, GEV (n=5) is -0.0172 and GARCH (1,1) is -0.0070. The statistical meaning of Q is that financial institutions will, on average, lose 1.72% and 0.7% of their original investments by using GEV(n=5) and GARCH (1,1) models in the case of violation, after they use their reserved funds (VaR numbers) to cover the losses. Specifically, the economic meaning of Q is that, on average, financial institutions will have the extra losses for 17,200 (1,000,000×0.0172) and 7,000 (1,000,000×0.0070) in the case of violations by using GEV (n=5) and GARCH (1,1) models, respectively.

Table A-1 The example of D and Q

	GEV (n=5)	GARCH(1,1)	GEV (n=5)	GARCH(1,1)
$\overline{\mathrm{VaR}}$	-0.0295	-0.0340	-0.0295	-0.0340
D	-0.0316	-0.0353		
Q			-0.0172	-0.0070

Note: The numbers of average VaR of GEV (n=5) is the long position VaR of S&P 500 in Table 5.4, and the average VaR of GARCH (1,1) is the long position VaR of S&P 500 in the panel A of Table 5.7. The numbers of D and Q are collected from the long positions of S&P500 in the panel A and B in Table 5.14.

Appendix B The changes in constituent stocks of index

Some might concern about the influence from the changes in constituent stocks to the measurement of VaR. In this appendix, it is shown that the impact of the changes in constituent stocks is very small and can be ignored. Assume the observed index price at time t and t+1 are p_t and p_{t+1}, respectively, and the constituent stocks are changed at t+1 (i.e. p_{t+1} is the closing price of the new equity index). Let the unobserved index price at time t+1 is $\hat{p}_{t+1}$ if there is no constituent stock change at t+1. The difference between observed and unobserved index price at time t+1 is

$$\Delta p_{t+1} = p_{t+1} - \hat{p}_{t+1} \qquad \text{(B-1)}$$

Thus, the observed index return (r_{t+1}) at t+1 is

$$\begin{aligned} r_{t+1} &= \ln\left(\frac{p_{t+1}}{p_t}\right) \\ &= \ln(p_{t+1}) - \ln(p_t) \qquad \text{(B-2)} \end{aligned}$$

And, the unobserved index return ($\hat{r}_{t+1}$) at t+1 if there is no change of constituent stock is

$$\hat{r}_{t+1} = \ln\left(\frac{\hat{p}_{t+1}}{p_t}\right) \qquad \text{(B-3)}$$

$$= \ln\left(\frac{p_{t+1} - \Delta p_{t+1}}{p_t}\right) \qquad \text{(B-4)}$$

$$= \ln(p_{t+1} - \Delta p_{t+1}) - \ln(p_t) \qquad \text{(B-5)}$$

Obviously, the effect of change in the constituent stocks of the equity index is Δp_{t+1}. If Δp_{t+1} is small, then the effect of change in the constituent stocks is small. Thus,

$$\hat{r}_{t+1} \approx r_{t+1} \qquad \text{(B-6)}$$

This effect does not happen in TAIEX, KOSPI, SET, JCI, and IGPA index because these indices include all stocks listed in their stock exchanges. That indicates that r_{t+1} and $\hat{r}_{t+1}$ are the same to these indices. The effects of change in constituent stocks on S&P500, FTSE100, Nikkei 225, and TSX (TSX includes 245 large-cap companies) are also extremely small since these indices capture, at least, 70% coverage of market capitalization and more than 100 stocks are included.

Another point is that the effect of change in constituent stocks is diluted with the number of stocks included within these indices. In fact, Δp_{t+1} is extremely small. For example, if stock A is replaced by stock B at time t+1, then, from the perspective of constituent stock, the change of index price, Δp_{t+1}, can be calculated as follows

$$\Delta p_{t+1} = {(p_{B,t+1} \times Q_B - p_{A,t+1} \times Q_A)}\big/_{\text{basis Index price}} \qquad \text{(B-7)}$$

where Q_B and Q_A are the number of outstanding shares of stock B and stock A, and $p_{B,t+1}$ and $p_{A,t+1}$ are the stock prices of stock A and B at time t+1, respectively. Compared with other stocks already included in the index, the number of outstanding shares of the new constituent (i.e., Q_B) is very small. Thus, Δp_{t+1} is also very small. From another aspect, in all the VaR models, r_{t+1} is used to estimate the distribution of equity returns and to forecast the VaR, and r_{t+1} is also used in the backtesting procedure. Thus, the influence from changes in constituent stocks to all models is consistent.

According to the analysis above, it is evident that not all of the equity indices applied in the thesis have the issue of changes in constituent stocks. Even if it is an issue to some of the equity indices, the effect of changes in constituent stocks is very small and can be ignored.

Appendix C The parameter stability of GEV distribution

This Appendix presents the distributional stability of estimated parameters in Section 5.2.1.1. The results in Table C.1 and Table C.2 are the stationarity tests of the estimated parameters based on daily and 10-day returns. The figures in this Appendix provide an overview of the patterns of the parameters. Figure C-1 to C-16 present the patterns of the estimated parameters of daily returns, and Figure C-17 to C-32 are the patterns of the ones of 10-day returns.

Table C. 1 The stationarity test of estimated parameters of GEV distribution with daily returns

	Long position			Short position		
n=5	Scale parameter (c'_n)	Location parameter (d'_n)	Tail parameter (k'_n)	Scale parameter (c_n)	Location parameter (d_n)	Tail parameter (k_n)
S&P500	-11.8949	-10.8167	-10.8501	-14.1893	-28.9946	-27.5118
	(-10.7934)	(-9.3042)	(-9.1458)	(-13.2334)	(-29.0351)	(-28.5112)
FTSE100	-11.9379	-11.3313	-16.5129	-13.8055	-26.7225	-17.3133
	(-10.8575)	(-10.2511)	(-17.0102)	(-13.3914)	(-26.9870)	(-17.9142)
CAC40	-12.7713	-29.4711	-28.4326	-14.5318	-29.3660	-17.6651
	(-11.7871)	(-29.5081)	(-28.4600)	(-13.9446)	(-29.4010)	(-17.8730)
DAX	-17.3402	-26.1481	-19.1443	-15.2020	-29.0979	-22.0152
	(-17.8154)	(-26.4359)	(-20.0978)	(-14.4645)	(-29.1331)	(-22.6345)
TSX	-16.7375	-27.9612	-25.0802	-16.4091	-29.1482	-30.3871
	(-17.2423)	(-28.0710)	(-25.3797)	(-16.4258)	(-29.1852)	(-30.5532)
Nikkei225	-25.7852	-29.1206	-15.1501	-27.2134	-29.4731	-30.9798
	(-26.5291)	(-29.1548)	(-15.1722)	(-27.2177)	(-29.5085)	(-31.1629)
IGPA	-19.6174	-25.1344	-16.5890	-15.4990	-5.6939	-28.9704
	(-20.8542)	(-25.7021)	(-17.0940)	(-14.9165)	**(-2.9881)**	(-29.3840)
Bolsa	-23.7170	-23.9182	-33.0216	-27.3271	-28.8968	-35.7543
	(-24.4150)	(-24.4863)	(-42.2597)	(-27.3065)	(-28.9388)	(-40.0283)
Bovespa	-12.8051	-9.3292	-20.0509	-18.5739	-26.3908	-23.0197
	(-11.1243)	(-6.9815)	(-18.8616)	(-17.7533)	(-26.8070)	(-24.4002)
Merval	-14.1592	-8.0793	-20.4453	-20.3581	-25.6639	-26.3812
	(-12.6070)	(-5.5281)	(-19.2624)	(-19.2685)	(-25.9350)	(30.2035)
HSI	-21.5699	-24.3387	-35.8655	-14.8935	-26.8629	-29.5272
	(-22.8310)	(-25.0470)	(-37.8152)	(-14.1999)	(-27.1433)	(-41.4841)
TAIEX	-26.9718	29.1713	-32.6633	-7.9371	-29.1411	-20.0551
	(-27.4910)	(-29.2048)	(-33.3425)	(-5.4577)	(-29.1776)	(-19.6480)
KOSPI	-22.1514	-29.5824	-23.8747	-21.5744	-29.6178	-27.6075
	(-22.5972)	(-29.6900)	(-24.1454)	(-20.6060)	(-29.6553)	(-28.0070)
KLCI	-19.1398	-16.9810	-21.0919	-24.2489	-12.8133	-30.4220
	(-19.6486)	(-17.1815)	(-21.0013)	(-24.1941)	(-11.8981)	(-33.9341)
JCI	-48.3612	41.5497	-55.8334	-31.4555	-24.2199	-39.1419
	(-88.9760)	(-41.1627)	(-114.4107)	(-32.5828)	(-24.8673)	(-58.7361)
SET	-26.3037	-4.3651	-34.5425	-17.5991	-29.7870	-25.3180
	(-26.3949)	(-2.5594)	(-38.6088)	(-16.0870)	(-29.8189)	(-25.5274)
Panel B: n=10	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	-5.5811	-26.8282	-10.1486	-13.1147	-29.0409	-13.3110
	(-3.5723)	(-27.0953)	(-8.7967)	(-12.4944)	(-29.0797)	(-12.6086)
FTSE100	-8.4741	-4.3942	-13.6417	-11.6445	-3.7800	-6.9939
	(-6.7601)	(-2.6372)	(-13.1852)	(-10.6444)	(-3.2786)	(-5.1567)
CAC40	-7.1377	-20.6753	-25.3739	-9.5596	-28.6942	-18.9518
	(-5.3316)	(-21.5514)	(-25.8120)	(-8.2895)	(-28.7438)	(-19.7112)
DAX	-8.3750	-23.1867	-13.0259	-13.5673	-29.0170	-14.3665
	(-6.8117)	(-23.8605)	(-12.5651)	(-12.7666)	(-29.0537)	(-14.1241)
TSX	-6.3416	-19.4413	-16.8309	-17.3491	-26.6257	-11.1819
	(-5.0838)	(-20.2877)	(-17.2897)	(-18.0173)	(-26.8870)	(-10.2030)
Nikkei225	-8.3278	-26.4907	7.9923	-14.6930	-14.5541	-11.9369
	(-8.2144)	(-26.7298)	(-6.6893)	(-14.4714)	(-13.6212)	(-11.4758)
IGPA	-6.6516	-7.1544	-7.0216	-8.3376	-5.1341	-9.8489

	(-5.0665)	(-8.3357)	(-5.6505)	(-7.6275)	(-4.6986)	(-8.1027)
Bolsa	-9.5228	-3.8973	-20.8081	-8.6709	16.0249	-13.5261
	(-7.6607)	(-4.0166)	(-21.5380)	(-8.2520)	(-16.2973)	(-13.7254)
Bovespa	-16.5023	-15.4625	-22.5076	-12.2709	-9.6142	-19.5915
	(-17.2495)	(-15.7384)	(-23.4033)	(-11.8633)	(-8.3769)	(-21.0030)
Merval	-10.1270	-6.7049	-15.0145	-10.7425	-20.9791	-24.3390
	(-9.2161)	(-4.5220)	(-15.1803)	(-9.3851)	(-21.8661)	(-24.3100)
HSI	-11.6267	-23.7726	-21.0439	-6.4117	-3.5992	-10.5853
	(-11.3248)	(-24.5268)	(-20.9031)	(-4.0510)	(-2.4639)	(-9.2199)
TAIEX	-10.5728	-29.2690	-12.9789	-11.5402	-29.2208	-8.4384
	(-10.3293)	(-29.3034)	(-12.5940)	(-10.8005)	(-29.2557)	(-7.7636)
KOSPI	-13.7293	-29.3448	-21.8819	-15.9341	-29.5972	-25.0966
	(-13.7794)	(-29.3788)	(-21.8151)	(-15.6713)	(-29.6342)	(-25.1960)
KLCI	-16.2997	-24.9002	-22.4330	-14.6288	-19.7253	-20.4918
	(-16.6075)	(-25.5173)	(-22.6427)	(-14.8230)	(-20.6264)	(-20.4909)
JCI	-22.0237	-16.8073	-20.0551	-21.9109	-17.6758	-22.7992
	(-23.2253)	(-17.0589)	(-21.2626)	(-22.6868)	(-18.5195)	(-22.5991)
SET	-20.9328	-29.2688	-13.8830	-18.8238	-28.8492	-19.5524
	(-21.4550)	(-29.3043)	(-13.6574)	(-19.0156)	(28.9030)	(-20.1535)

Panel C: n=22	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	-3.2071	-24.2321	-9.9589	-5.7676	-4.7399	-6.0849
	(-3.2044)	(-24.9255)	(-8.8721)	(-4.7357)	(-5.1718)	(-5.4381)
FTSE100	-2.9043	-2.5934	-8.5821	-3.4161	-24.3818	-11.8715
	(-2.5296)	(-1.3757)	(-8.2279)	(-2.9335)	(-25.0726)	(-11.1697)
CAC40	-4.6162	-28.4827	-23.9945	-6.9925	-28.4839	-15.6603
	(-3.8200)	(-28.5568)	(-24.7379)	(-6.5055)	(-28.5444)	(-15.7605)
DAX	-17.3274	-29.0408	-19.8977	-6.1989	-10.0995	-5.5563
	(-17.9075)	(-29.0774)	(-20.7079)	(-6.0366)	(-9.6141)	(-4.0230)
TSX	-5.6111	-5.4306	-5.9536	-7.6737	26.5767	-13.2280
	(-7.1217)	(-4.2762)	(-7.8095)	(-6.6206)	(-26.8496)	(-12.9286)
Nikkei225	-9.4665	-10.9384	-3.6567	-8.8099	-9.7451	-9.1357
	(-10.4293)	(-10.9421)	(-4.4920)	(-8.9965)	(-9.8797)	(-8.2424)
IGPA	-7.2384	-20.7221	-11.8915	-5.4721	-4.2942	-5.1897
	(-5.4467)	(-21.6105)	(-11.3039)	(-5.9882)	(-4.0686)	(-5.4682)
Bolsa	-10.1652	-11.7714	-9.7763	-11.5263	-25.9850	-20.1920
	(-8.9653)	(-10.9549)	(-9.0465)	(-10.9813)	(-26.3822)	(-20.5559)
Bovespa	-7.7650	-12.5756	-13.4938	-8.9603	-4.7920	-7.8948
	(-8.0684)	(-12.1286)	(-13.6620)	(-10.1612)	(-5.3483)	(-9.3993)
Merval	-11.1388	-18.1064	-13.8067	-5.9493	-16.9299	-13.1816
	(-11.452)	(-18.8590)	(-13.9696)	(-5.7363)	(-17.4208)	(-13.4860)
HSI	-4.4429	-1.7091	-11.0615	-4.0621	-3.4354	-8.0991
	(-3.7492)	(-1.7891)	(-10.8553)	(-3.9824)	(-3.3233)	(-7.3039)
TAIEX	-5.4560	-7.2364	6.4822	-4.8054	-6.3233	-3.8374
	(-4.7292)	(-7.4269)	(-5.6319)	(-5.8223)	(-7.4140)	(-4.3638)
KOSPI	-12.0248	-28.9670	-18.5984	-9.0304	-28.7526	-12.1499
	(-11.7096)	(-29.0079)	(-18.4441)	(-10.4643)	(-28.8089)	(-11.8796)
KLCI	-9.5472	-11.9915	-18.3191	-9.3107	-2.5443	-13.6283
	(-8.4749)	(-11.2660)	(-18.8140)	(-8.8425)	(-1.8914)	(-13.7896)
JCI	-12.8871	-23.4195	-17.5430	-13.5666	-9.7135	-11.5119
	(12.9421)	(-24.0666)	(-17.9648)	(-12.7743)	(-10.1881)	(-11.4033)
SET	-8.0661	-7.5142	-8.1356	-15.3545	-24.3193	-14.1577
	(-7.2939)	(-7.4558)	(-8.1840)	(-15.1896)	(-24.8659)	(-14.1191)

Note: The major numbers and the numbers in parentheses are unit-root test based on augmented Dickey-Fuller test (ADF test) and Phillips-Perron test (PP test). The critical values of the Dickey-Fuller test and the Phillips-Perron test in 1% and 5% are -3.435 and -2.863, respectively.

Table C. 2 The stationarity test of estimated parameters of GEV distribution with 10-day returns

	Long position			Short position		
n=5	Scale parameter (c'_n)	Location parameter (d'_n)	Tail parameter (k'_n)	Scale parameter (c_n)	Location parameter (d_n)	Tail parameter (k_n)
S&P500	-14.2405	-10.5617	-9.6859	-5.0252	-7.8226	-12.0256
	(-13.4792)	(-8.5129)	(-7.3285)	(-2.2448)	(-5.0235)	(-9.9021)
FTSE100	-17.6161	-21.5461	-7.6888	-5.3200	-13.1398	-9.1452
	(-17.8053)	(-22.2200)	(-4.9063)	(-2.4494)	(-11.4373)	(-6.6125)
CAC40	-27.9881	-28.8191	-21.3748	-4.8592	-5.3187	-7.7505
	(-28.0027)	(-28.8514)	(-22.1176)	(-1.9678)	(-2.5765)	(-5.11900)
DAX	-9.1044	-18.7507	-10.5462	-4.9210	-12.0764	-7.3797
	(-6.3912)	(-19.1774)	(-8.2166)	(-1.9944)	(-10.1423)	(-4.5037)
TSX	-7.6522	-15.5438	-10.2995	-5.6174	-11.0124	-15.0759
	(-4.7909)	(-14.4836)	(-8.0369)	(2.9120)	(-9.4687)	(-14.1236)
Nikkei225	-15.9224	-3.3343	-2.4296	-15.3995	-10.4398	-6.9068
	(-14.0735)	(-1.4172)	(-0.7637)	(-15.4020)	(-8.1067)	(-4.6416)
IGPA	-14.1243	-21.6103	-9.0913	-15.7786	-16.9786	-14.0795
	(-13.7772)	(-22.6695)	(-7.0746)	(-15.7035)	(-17.2435)	(-13.4525)
Bolsa	-28.4255	-28.3479	-25.2502	-26.5012	-27.3916	-25.9839
	(-28.5062)	(-28.3913)	(-25.8299)	(-26.6429)	(-27.5936)	(-25.9890)
Bovespa	-10.8031	-4.8806	-13.4908	-26.5012	-5.0495	-19.0667
	(-8.4806)	(-2.0796)	(-11.5901)	(-26.6429)	(-2.6078)	(-19.4515)
Merval	-14.2684	-11.2456	-11.2030	-8.4568	-5.4421	-14.1343
	(-12.4107)	(-8.9477)	(-8.8472)	(-5.7702)	(-2.8946)	(-13.1992)
HSI	-14.1103	-9.3465	-20.4435	-21.3846	-22.3364	-23.7550
	(-12.1231)	(-6.9835)	(-20.0752)	(-21.5424)	(-22.6405)	(-23.4784)
TAIEX	-5.9130	-4.2004	-3.9684	-15.6001	-22.9890	-15.8035
	(-3.1088)	(-2.5711)	(-2.4015)	(-15.4397)	(-23.6604)	(-15.6055)
KOSPI	-22.1363	-14.6320	-5.4086	-10.8889	-10.4570	-11.1397
	(-21.9721)	(-14.1832)	(-2.5695)	(-9.2858)	(-7.9998)	(-10.0920)
KLCI	-21.7220	-12.7107	-21.9150	-25.4476	-15.6567	-51.2518
	(-32.2253)	(-11.5350)	(-74.9154)	(-25.6831)	(-15.8181)	(-60.1691)
JCI	-16.8925	-11.3523	-10.8384	-9.1947	-4.5892	-8.3242
	(-15.5795)	(-10.2995)	(-8.4047)	(-6.5584)	(-2.3588)	(-5.6495)
SET	-15.1888	-14.5052	-4.4140	-9.1369	-11.8261	-9.0275
	(-14.3954)	(-13.7511)	**(-2.4558)**	(-7.5086)	(-10.2154)	(-7.3626)
Panel B: n=10	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	-13.2677	-18.8108	-12.1075	-9.7629	-9.6596	-14.8007
	(-13.3666)	(-18.8919)	(11.7489)	(-9.8142)	(-10.3984)	(-14.4945)
FTSE100	-12.3831	-26.1949	-12.9464	-11.6074	-11.1980	-16.7734
	(-11.8347)	(-26.3873)	(-12.3592)	(-11.1851)	(-10.6698)	(-16.1775)
CAC40	-10.8386	-15.2320	-6.3704	-9.9015	-27.3655	-19.1793
	(-10.3078)	(-14.8059)	(4.5381)	(-9.2388)	(-27.4592)	(-19.6156)
DAX	-8.7841	-29.3146	-12.3569	-10.5779	-11.2728	-18.0050
	(-8.4056)	(-29.3491)	(-11.7780)	(-9.5513)	(-11.7804)	(-17.2332)
TSX	-11.8004	-28.7649	-9.6928	-11.9496	-5.3352	-14.1161
	(-11.3246)	(-28.8011)	(-8.1763)	(-11.7594)	(-4.9703)	(-13.9165)
Nikkei225	-11.4031	-6.4670	-3.6102	-12.9805	-10.9638	-11.3831
	(-12.0707)	(-4.1935)	(-3.3759)	(-13.5449)	(-11.0068)	(-12.4396)
IGPA	-9.7772	-9.7625	-7.5214	-10.3320	-14.6227	-12.3404
	(-9.3048)	(-9.3431)	(-8.1604)	(-9.8034)	(-14.6626)	(-11.8075)
Bolsa	-12.2437	-15.5978	-10.2874	-10.8677	-11.1190	-12.1138
	(-13.0934)	(-15.8535)	(-11.5709)	(-11.5514)	(-11.3418)	(-13.5904)
Bovespa	-11.2223	-6.6605	-13.9496	-10.8677	-14.0436	-15.4884
	(-12.2583)	(-6.2113)	(-14.6029)	(-11.5514)	(-14.0242)	(-15.7542)
Merval	-16.7638	-9.8739	-11.0687	-6.2538	-10.4703	-9.4286
	(-17.4029)	(-9.5818)	(-11.4528)	(-7.3487)	(-11.0514)	(-10.4285)
HSI	-10.7823	-6.0599	-13.3691	-9.3004	-15.8421	-8.5563
	(-10.9009)	(-5.6333)	(-13.3923)	(-10.5373)	(-16.1211)	(-8.3914)
TAIEX	-9.9315	-7.5724	-4.6563	-11.1554	-20.8285	-10.8236
	(-8.7661)	(-7.6841)	(-4.0213)	(-10.5283)	(-21.6622)	(-10.2859)
KOSPI	-21.1348	-9.7907	-10.6572	-19.9862	-11.5704	-22.6768
	(-21.6099)	(-10.7489)	(-10.0239)	(-19.9121)	(-11.1959)	(-24.2057)
KLCI	-35.6420	-11.3689	-42.6632	-20.9402	-20.8223	-29.8253

	(-37.9988)	(-10.8434)	(-71.8306)	(-19.9982)	(-21.0428)	(-33.0710)
JCI	-26.1999	-18.5884	-22.1193	-27.2267	-20.2430	-12.6939
	(-26.3360)	(-19.1703)	(-23.2767)	(-27.2694)	(-21.1725)	(-12.2347)
SET	-20.2537	-20.5526	-10.5869	-15.0721	-11.3097	-19.6300
	(-20.5513)	(-21.3025)	(-9.9632)	(-15.8681)	(-12.261)	(-19.5400)

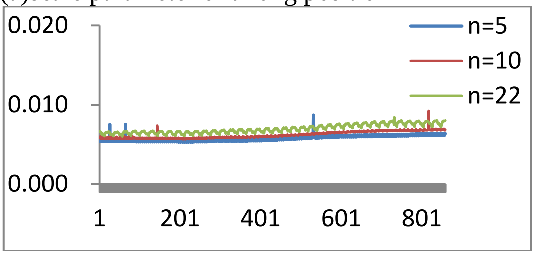
Panel C: n=22	c'_n	d'_n	k'_n	c_n	d_n	k_n
S&P500	-6.8964	-7.4162	-6.5048	-11.1476	-5.7927	-13.4195
	(-7.1390)	(-7.8848)	(-6.2632)	(-10.5683)	(-6.3494)	(-13.2027)
FTSE100	-6.9609	-8.1841	-5.5397	-9.5342	-5.2518	-12.4905
	(-8.2293)	(-8.9767)	(-6.3550)	(-9.3601)	(-6.9651)	(-12.6766)
CAC40	-7.5362	-11.9010	-4.9933	-6.1844	-7.6745	-6.0053
	(-7.5963)	(-11.9104)	(-4.7931)	(-7.2473)	(-8.6405)	(-6.6739)
DAX	-9.1707	-8.6680	-9.7051	-7.6741	-5.1645	-9.0934
	(-10.2934)	(-9.3879)	(-10.2660)	(-8.1659)	(-6.3274)	(-9.7028)
TSX	-16.6130	-14.6268	-8.6242	-6.7170	-3.7517	-6.5477
	(-17.0675)	(-14.7043)	(-8.6079)	(-8.1452)	(-3.3237)	(-6.6330)
Nikkei225	-11.3188	-5.7786	-2.6772	-6.7956	-7.1598	-6.6717
	(-11.0431)	(-7.4722)	(-2.4676)	(-8.9530)	(-8.9324)	(-8.6745)
IGPA	-6.0943	-6.9125	-6.8816	-18.2346	-18.6523	-11.1138
	(-7.6306)	(-8.8172)	(-8.1594)	(-18.8959)	(-19.4721)	(-11.9379)
Bolsa	-6.5086	-6.8676	-7.7681	-6.2669	-7.4696	-7.1369
	(-7.4152)	(-8.6519)	(-7.7601)	(-8.6563)	(-8.4867)	(-9.0923)
Bovespa	-8.1519	-6.4370	-9.7866	-6.2669	-7.9335	-11.2269
	(-8.9229)	(-6.3064)	(-10.6946)	(8.6563)	(-9.2588)	(-12.0849)
Merval	-6.8035	-6.7968	-7.6983	-7.4705	-5.5049	-7.1439
	(-8.2538)	(-8.6898)	(-8.3838)	(-9.1932)	(-7.4132)	(-9.0824)
HSI	-5.7585	-4.4406	-9.6239	-5.2892	-8.0261	-4.7698
	(-7.7155)	(-3.9506)	(-9.4734)	(-4.9465)	(-7.8406)	(-4.2623)
TAIEX	-6.3043	-7.4566	-3.7461	-17.1269	-22.7404	-7.7545
	(-8.6750)	(-7.6181)	(-4.1528)	(-17.6583)	(-23.3606)	(-7.2423)
KOSPI	-7.8030	-7.0861	-6.9443	-14.2796	-5.9407	-15.2089
	(-8.9917)	(-7.5425)	(-5.6801)	(-13.6629)	(-7.1412)	(-15.0092)
KLCI	-12.3064	-10.1394	-12.9818	-15.5857	-11.6314	-13.1565
	(-12.7482)	(-10.8728)	(-13.3805)	(-16.8120)	(-12.6871)	(-14.0563)
JCI	-10.7249	-12.4445	-9.9619	-9.4231	-4.1007	-10.1485
	(-11.0182)	(-13.6464)	(-9.8631)	(-10.2213)	(-4.3598)	(-11.7450)
SET	-10.2118	-18.7656	-9.2328	-12.6907	-10.6917	-17.5757
	(-11.5600)	(-19.3560)	(10.8509)	(-13.2980)	(-11.3488)	(-17.1075)

Note: The major numbers and the numbers in parentheses are unit-root test based on augmented Dickey-Fuller test (ADF test) and Phillips-Perron test (PP test). The critical values of the Dickey-Fuller test and the Phillips-Perron test in 1% and 5% are -3.435 and -2.863, respectively.

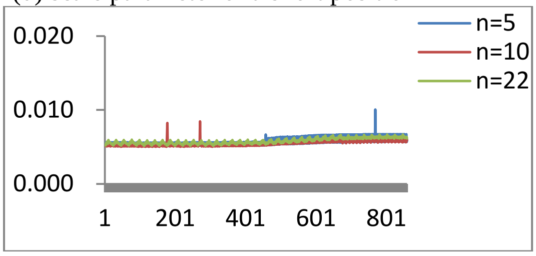
Appendix C

Figure C- 1 The pattern of estimated parameters of S&P 500

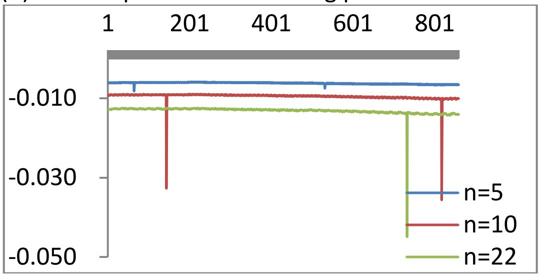
(a)scale parameter of a long position



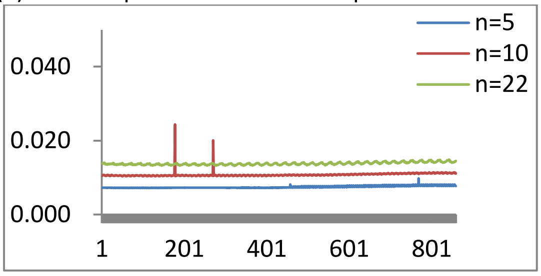
(d) scale parameter of a short position



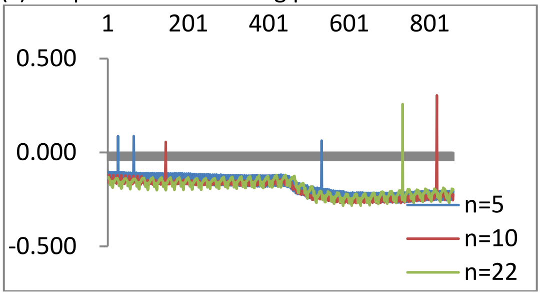
(b)location parameter of a long position



(e) location parameter of a short position



(c) tail parameter of a long position



(f) tail parameter of a short position

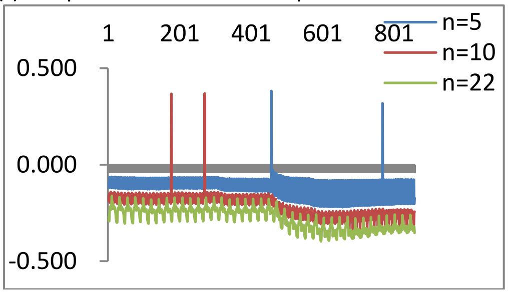
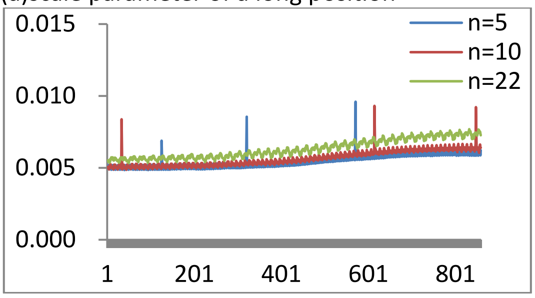
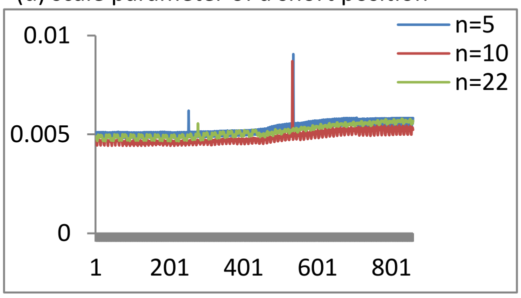


Figure C- 2 The pattern of estimated parameters of FTSE 100

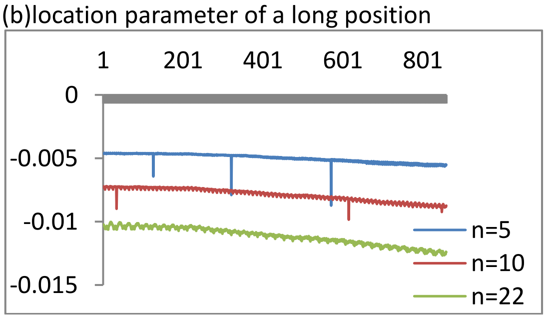
(a)scale parameter of a long position



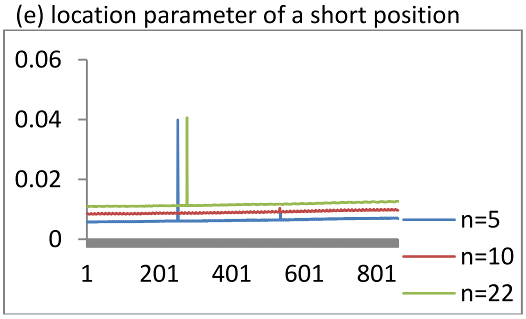
(d) scale parameter of a short position



(b)location parameter of a long position

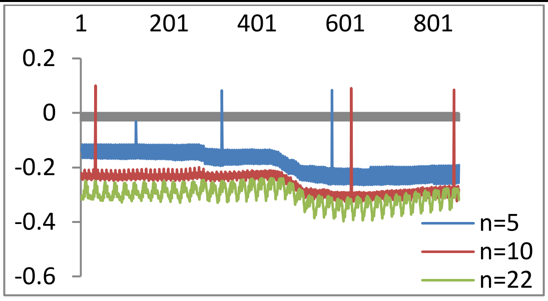


(e) location parameter of a short position



(c) tail parameter of a long position

(f) tail parameter of a short position



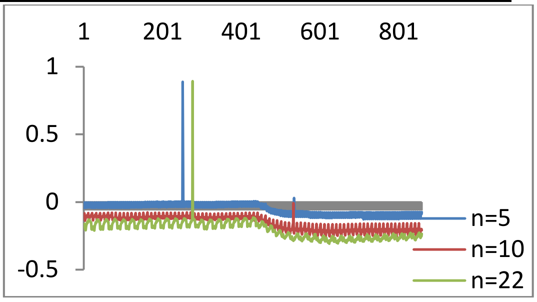
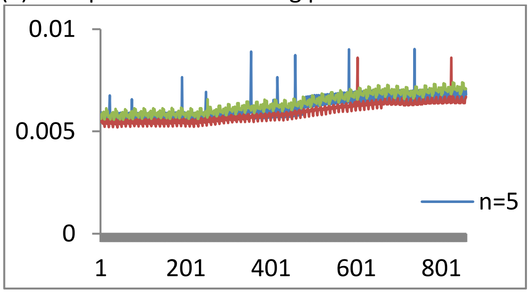
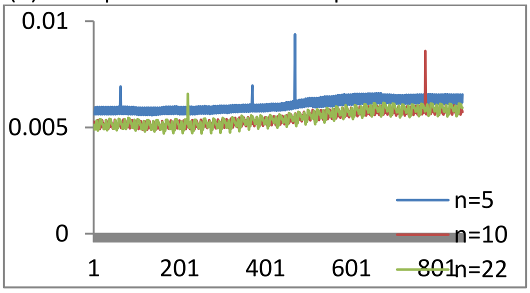


Figure C- 3 The pattern of estimated parameters of CAC40

(a)scale parameter of a long position

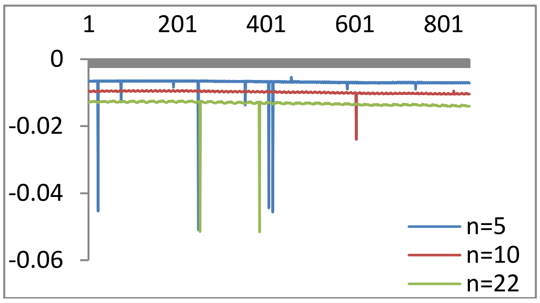
(d) scale parameter of a short position

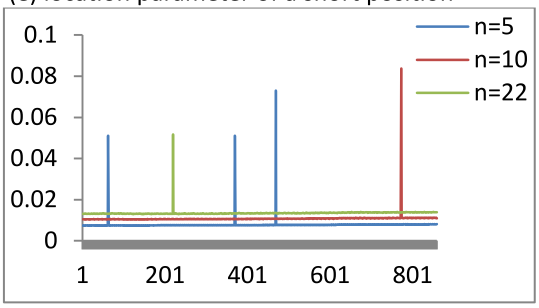




(b)location parameter of a long position

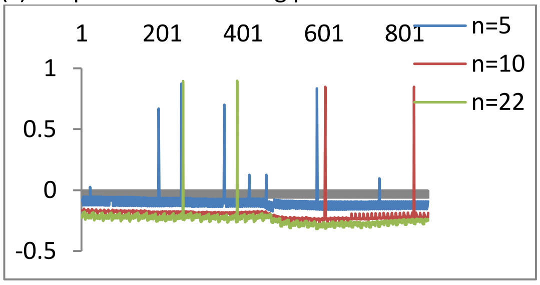
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



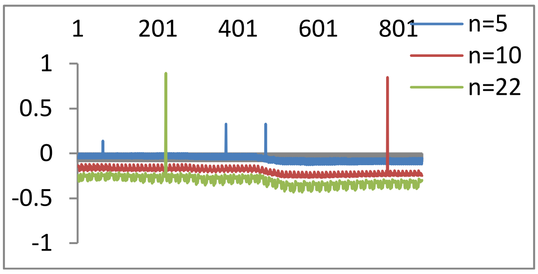
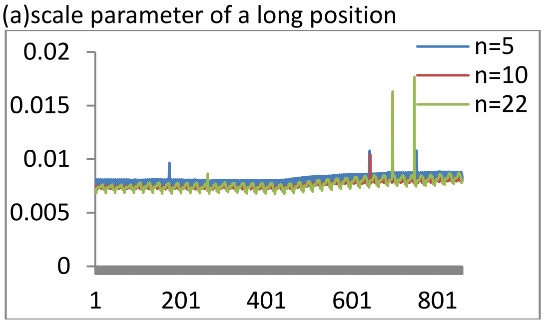
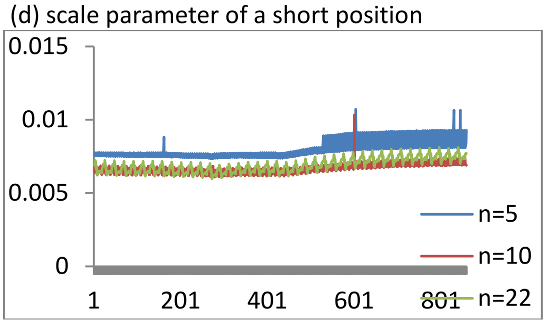


Figure C- 4 The pattern of estimated parameters of DAX

(a)scale parameter of a long position

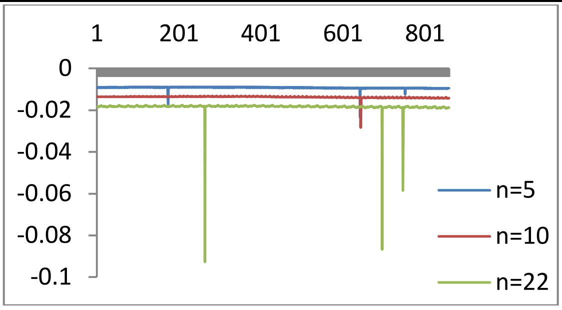
(d) scale parameter of a short position

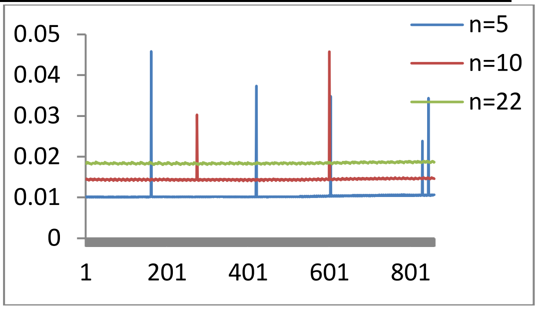




(b)location parameter of a long position

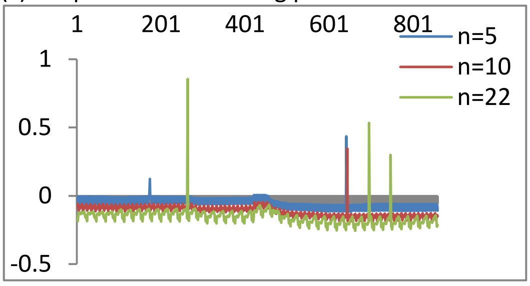
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



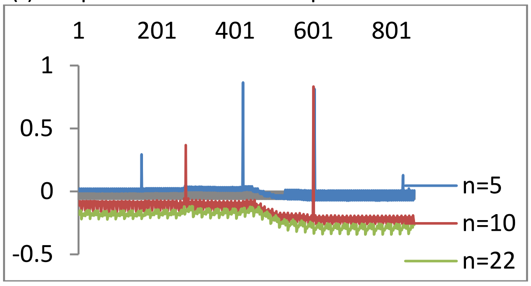
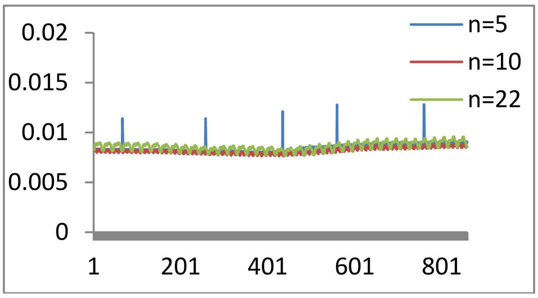
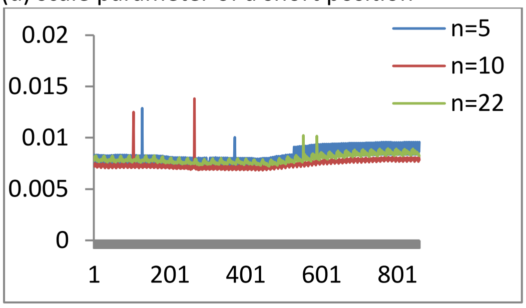


Figure C- 5 The pattern of estimated parameters of TSX

(a)scale parameter of a long position

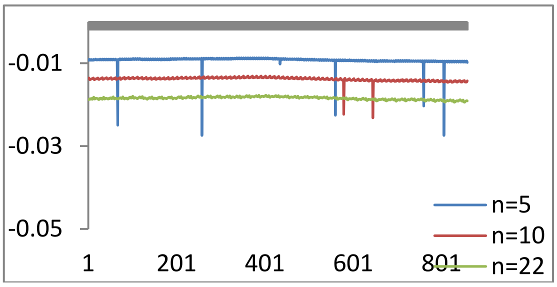
(d) scale parameter of a short position

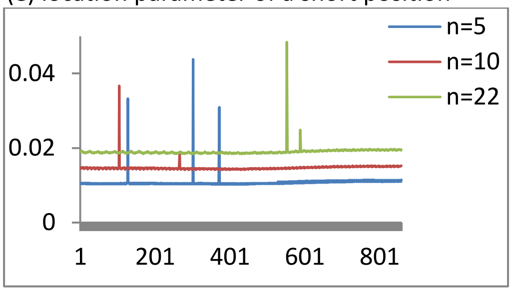




(b)location parameter of a long position

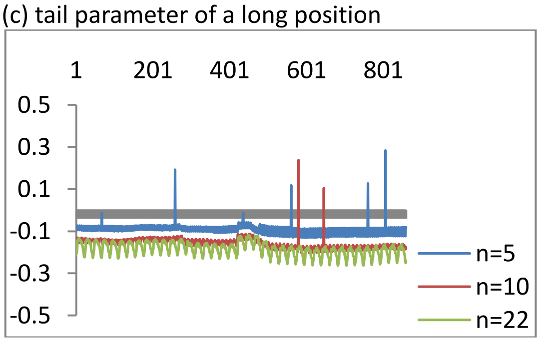
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



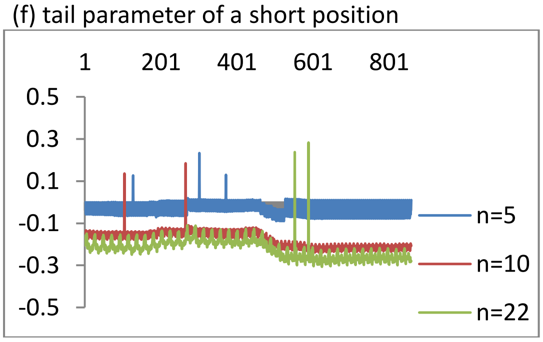
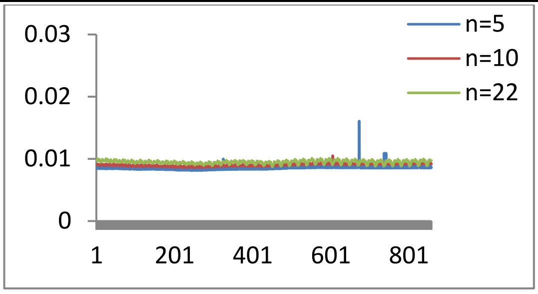
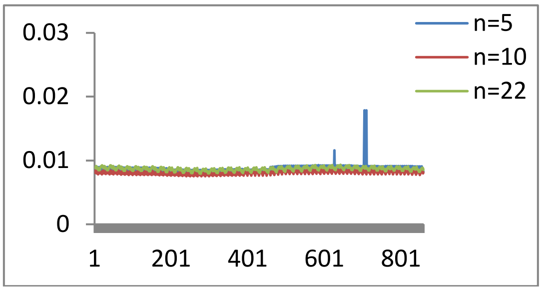


Figure C- 6 The pattern of estimated parameters of Nikkei225

(a)scale parameter of a long position

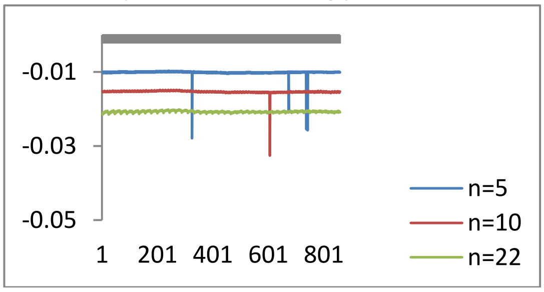
(d) scale parameter of a short position

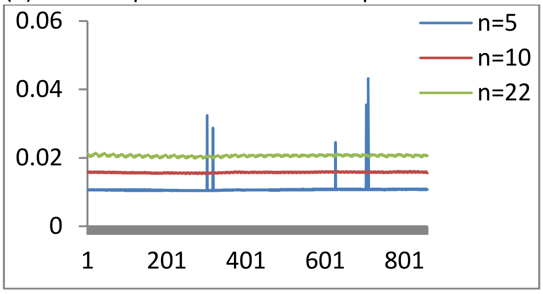




(b)location parameter of a long position

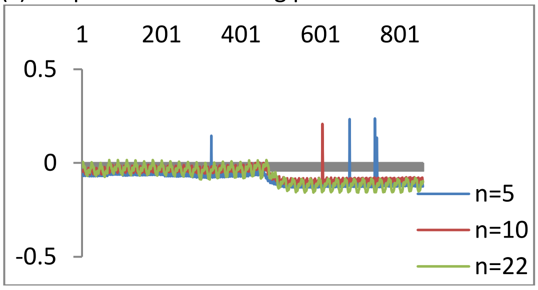
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



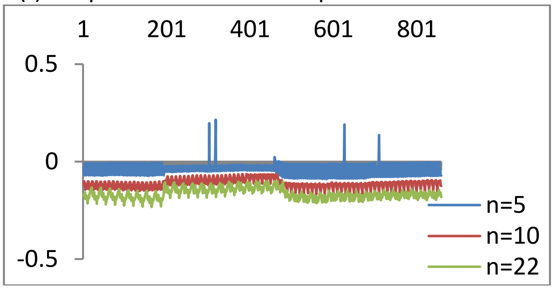
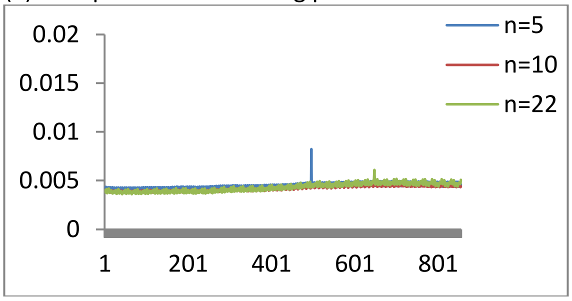
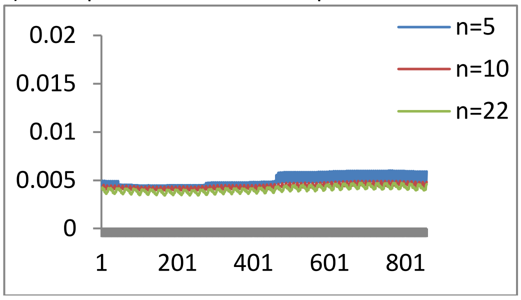


Figure C- 7 The pattern of estimated parameters of IGPA

(a)scale parameter of a long position

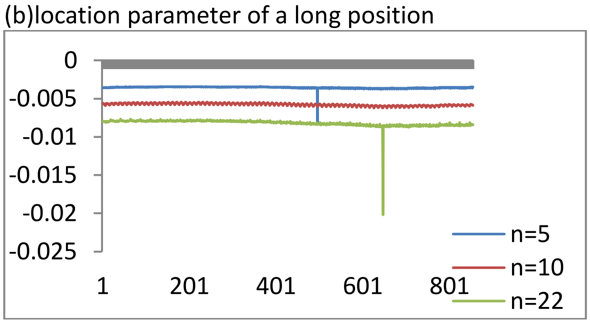
(d) scale parameter of a short position

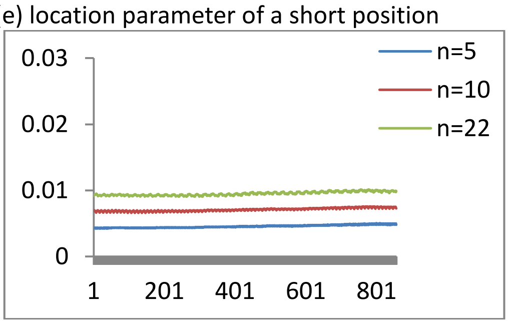




(b)location parameter of a long position

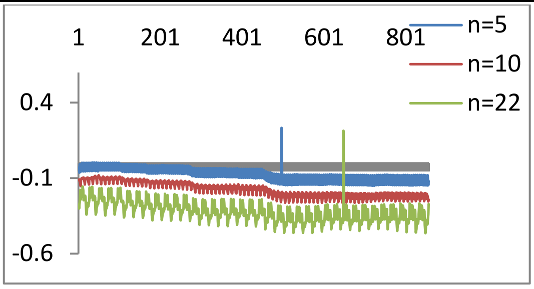
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



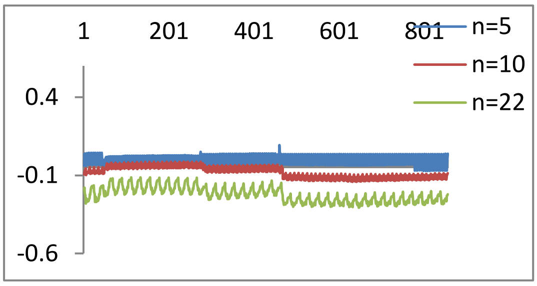
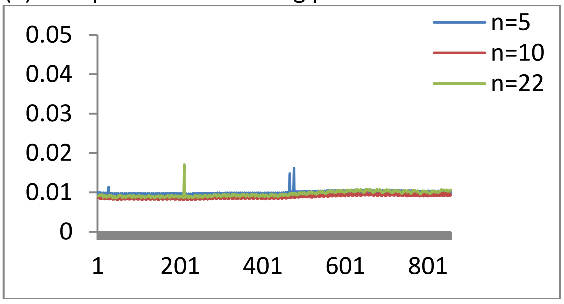
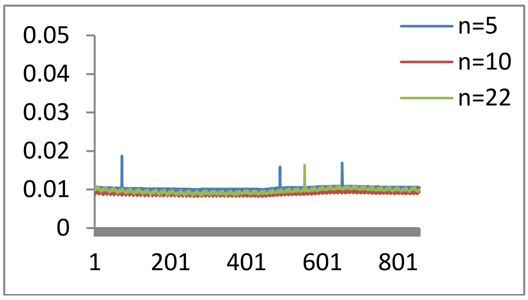


Figure C- 8 The pattern of estimated parameters of Bolsa

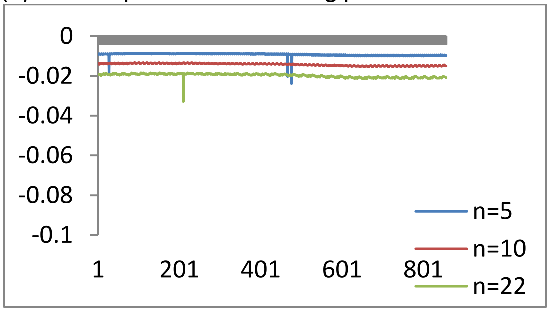
(a)scale parameter of a long position



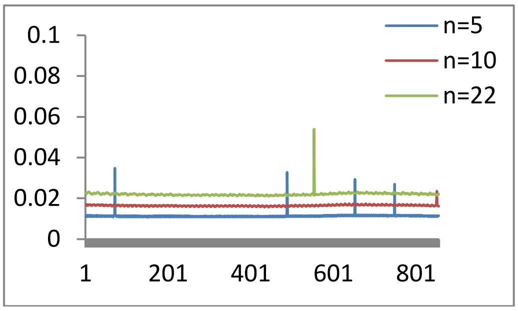
(d) scale parameter of a short position



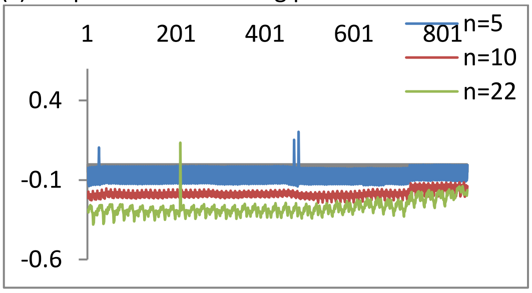
(b)location parameter of a long position



(e) location parameter of a short position



(c) tail parameter of a long position



(f) tail parameter of a short position

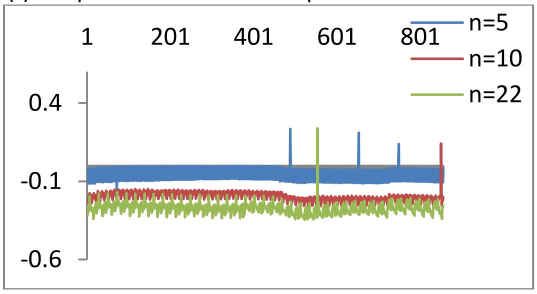
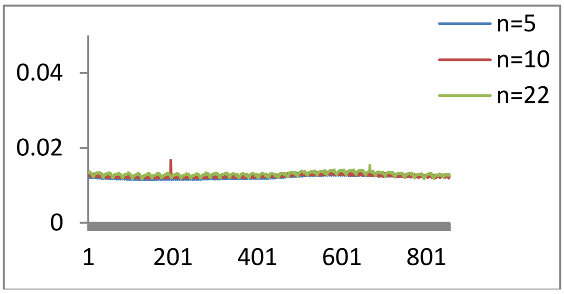
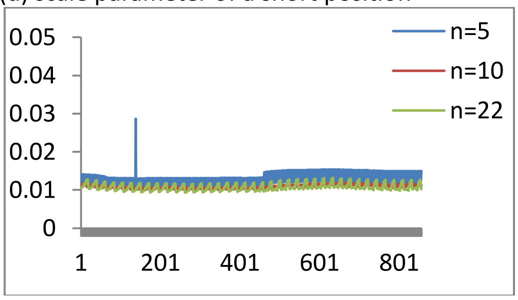


Figure C- 9 The pattern of estimated parameters of Bovespa

(a)scale parameter of a long position

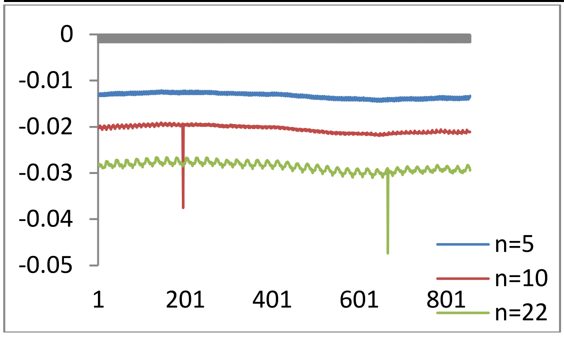


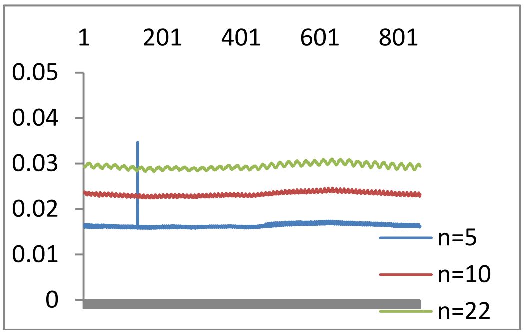
(d) scale parameter of a short position



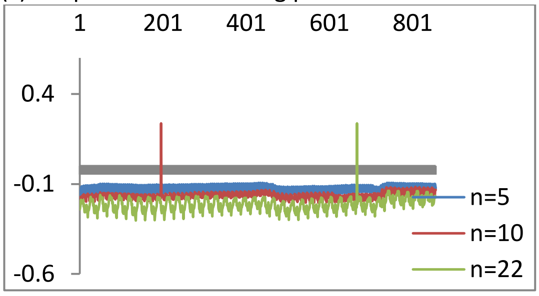
(b)location parameter of a long position

(e) location parameter of a short position





(c) tail parameter of a long position (f) tail parameter of a short position



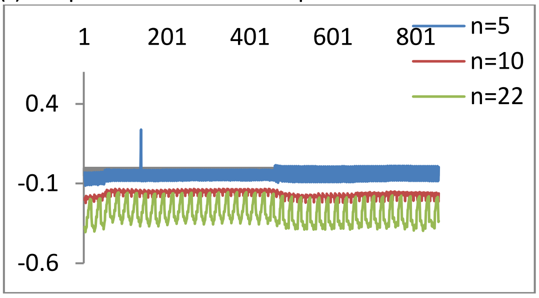
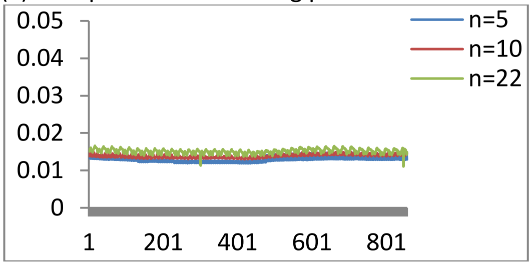
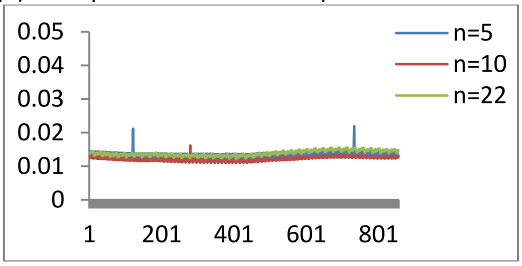


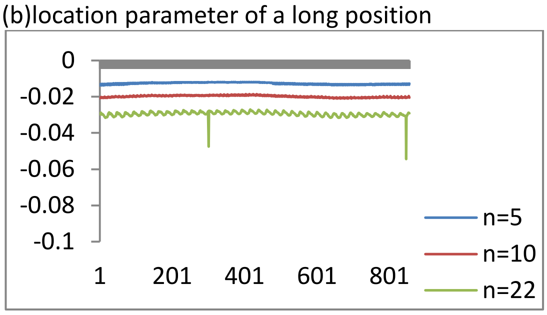
Figure C- 10 The pattern of estimated parameters of Merval

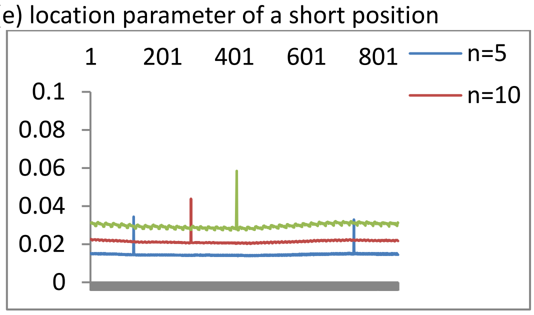
(a)scale parameter of a long position (d) scale parameter of a short position



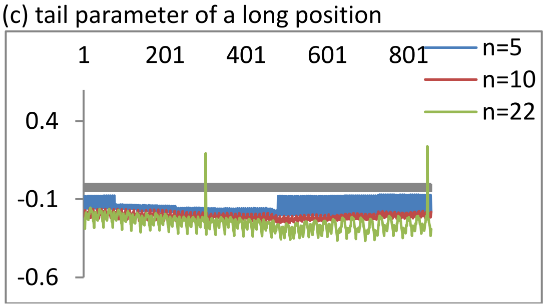


(b)location parameter of a long position (e) location parameter of a short position





(c) tail parameter of a long position (f) tail parameter of a short position



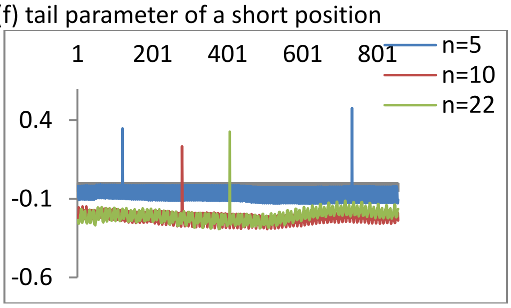
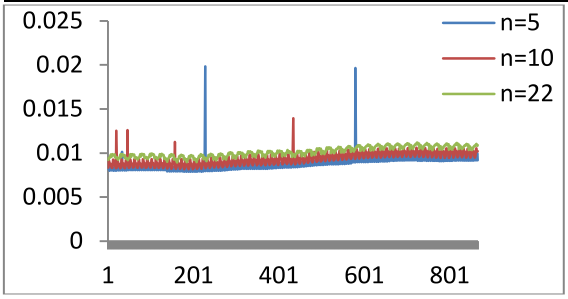
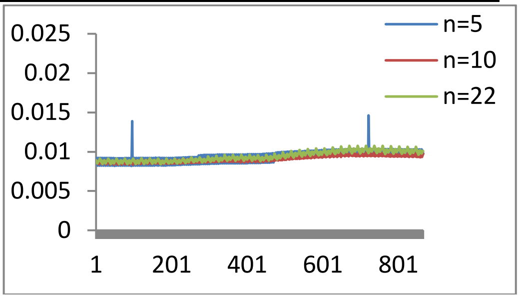


Figure C- 11 The pattern of estimated parameters of HSI

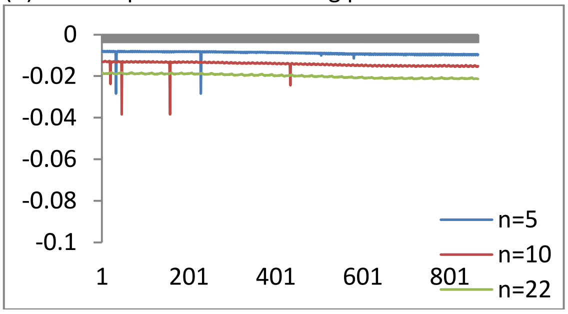
(a)scale parameter of a long position (d) scale parameter of a short position

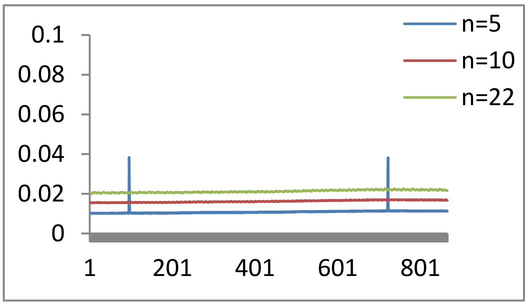




(b)location parameter of a long position

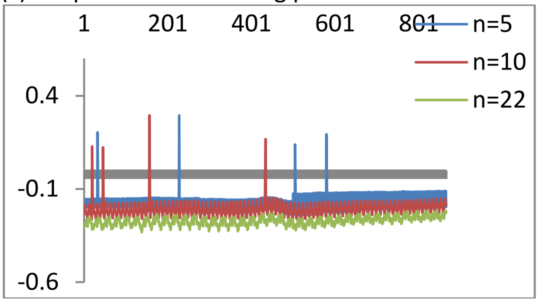
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



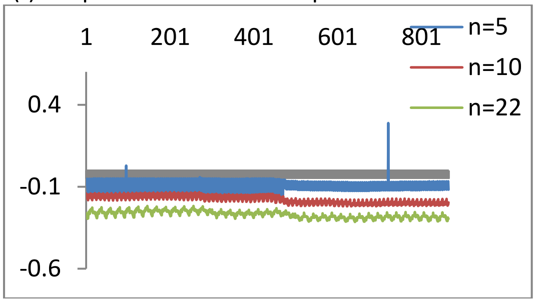
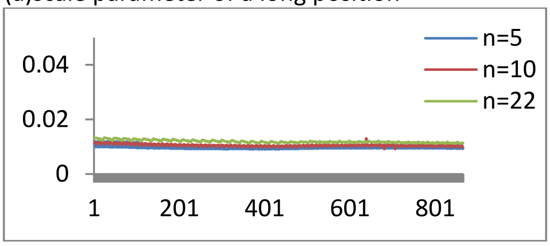
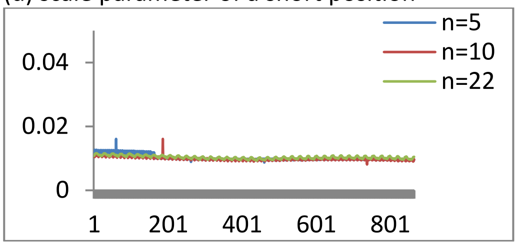


Figure C- 12 The pattern of estimated parameters of TAIEX

(a)scale parameter of a long position

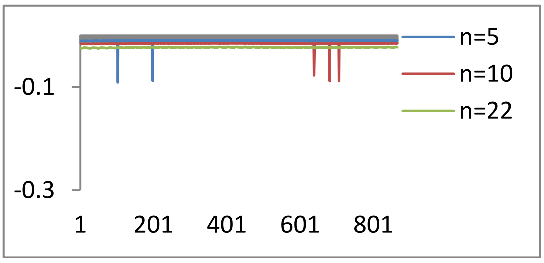
(d) scale parameter of a short position

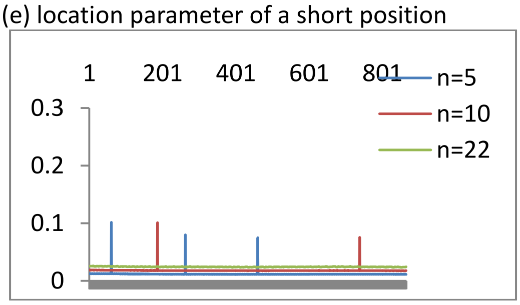




(b)location parameter of a long position

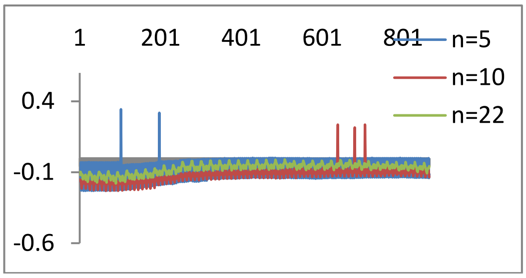
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



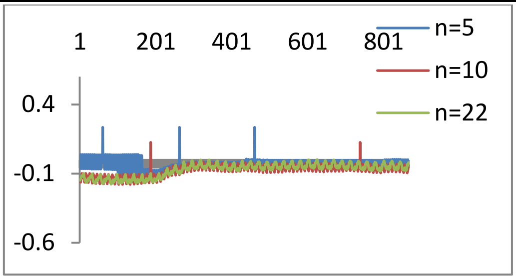
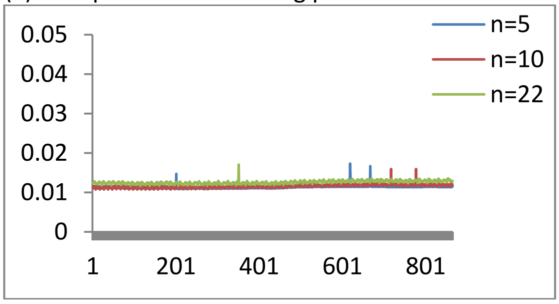
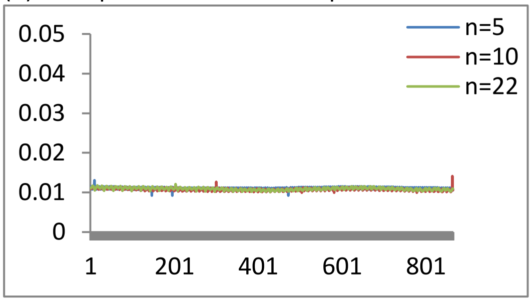


Figure C- 13 The pattern of estimated parameters of KOSPI

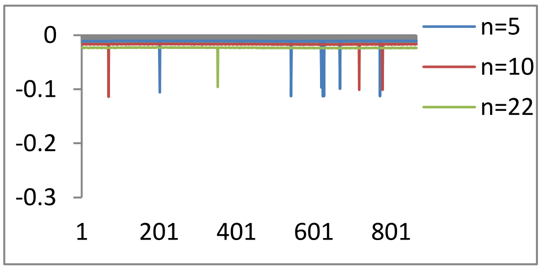
(a)scale parameter of a long position



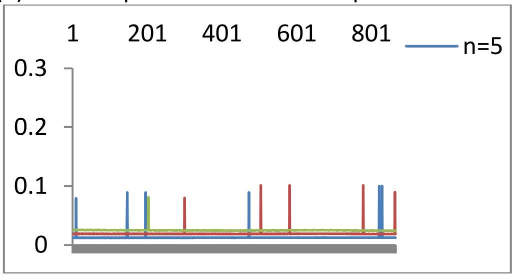
(d) scale parameter of a short position



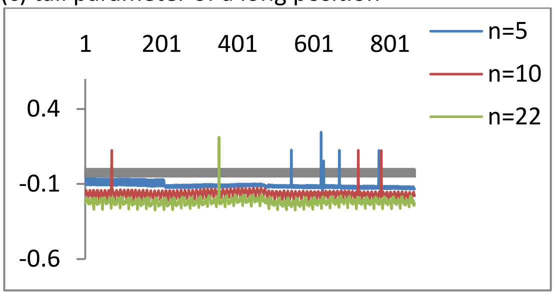
(b)location parameter of a long position



(e) location parameter of a short position



(c) tail parameter of a long position



(f) tail parameter of a short position

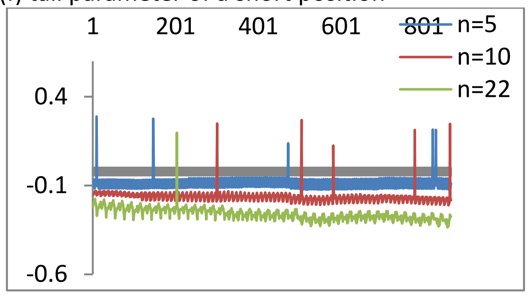
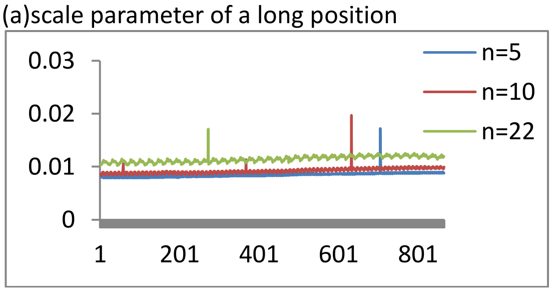
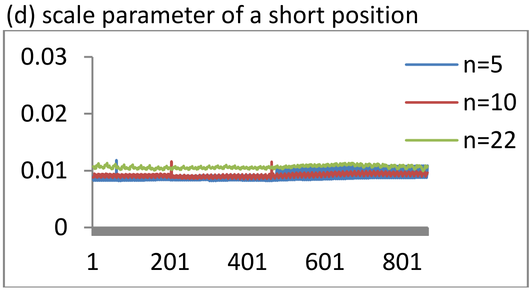


Figure C- 14 The pattern of estimated parameters of KLCI

(a)scale parameter of a long position

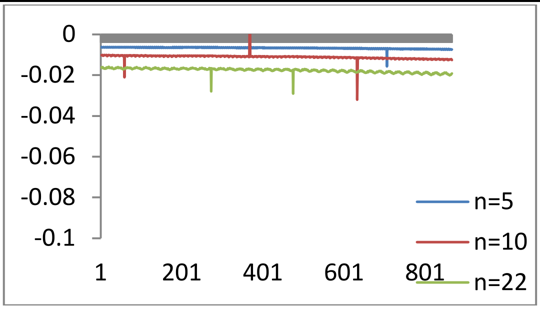


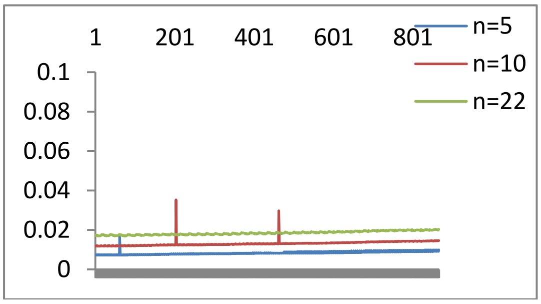
(d) scale parameter of a short position



(b)location parameter of a long position

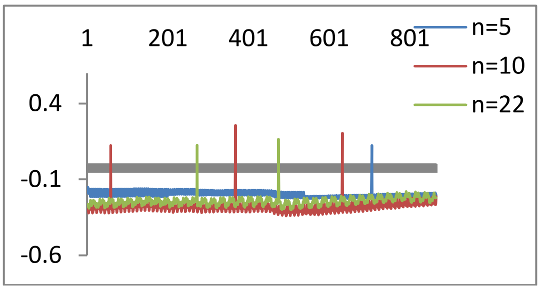
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



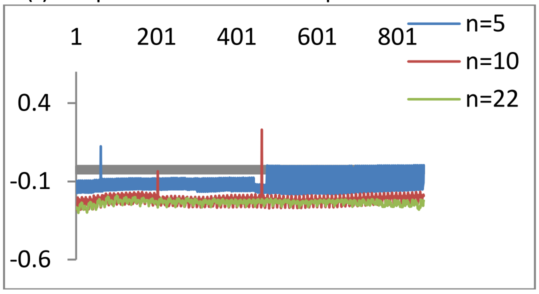
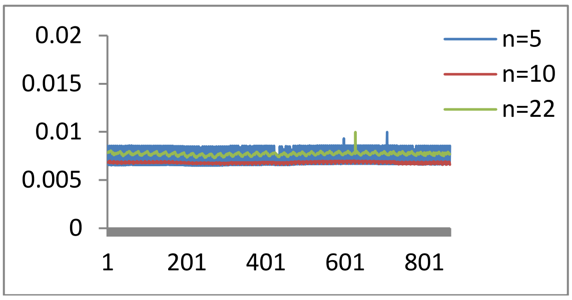
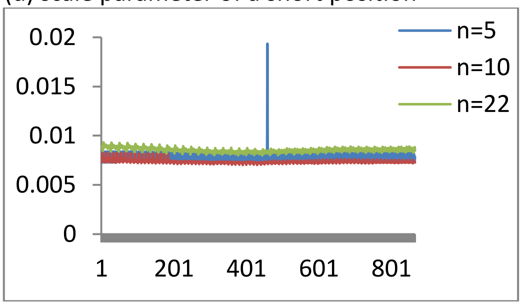


Figure C- 15 The pattern of estimated parameters of JCI

(a)scale parameter of a long position

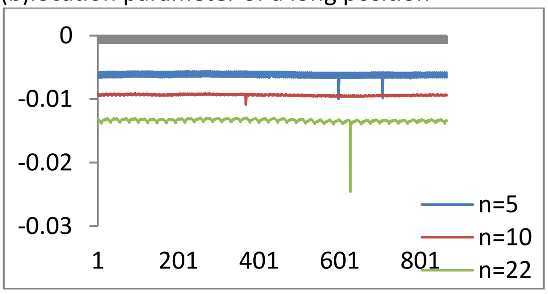
(d) scale parameter of a short position

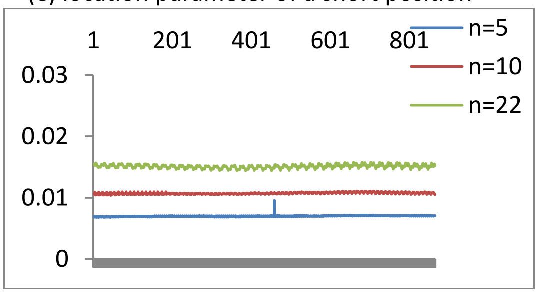




(b)location parameter of a long position

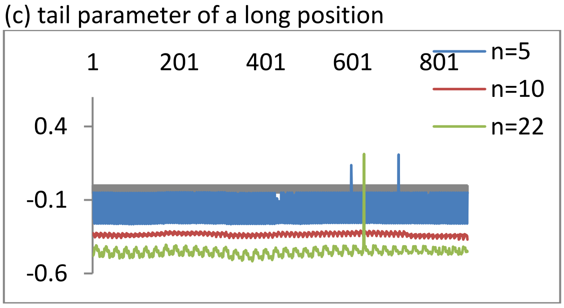
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



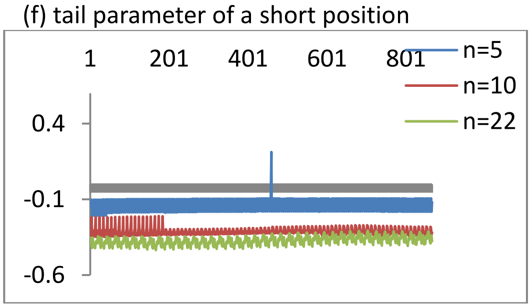
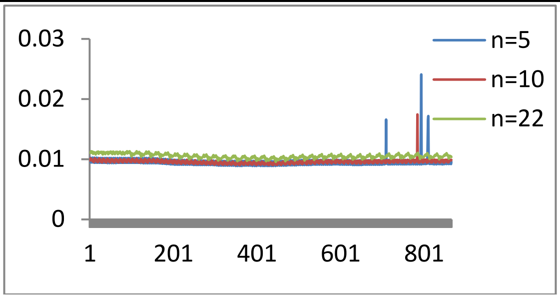
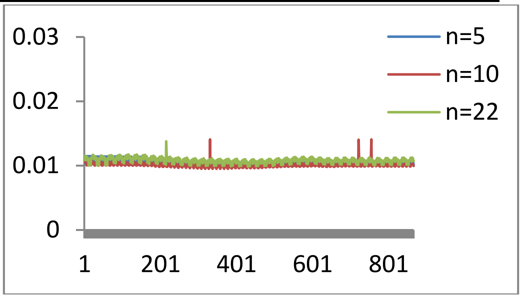


Figure C- 16 The pattern of estimated parameters of SET

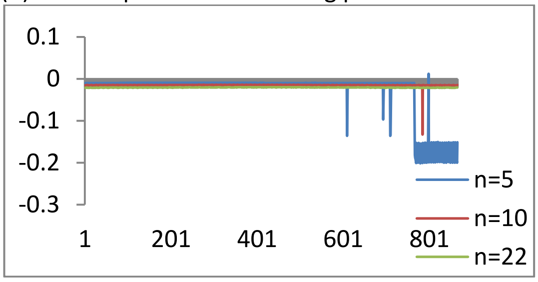
(a)scale parameter of a long position

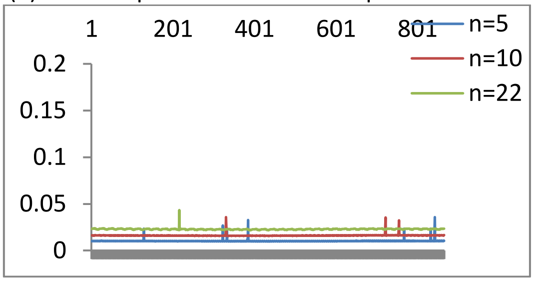
(d) scale parameter of a short position



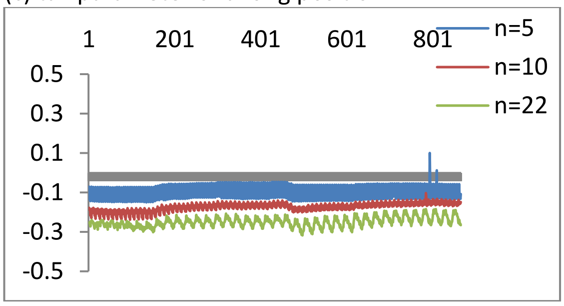


(b)location parameter of a long position (e) location parameter of a short position





(c) tail parameter of a long position (f) tail parameter of a short position



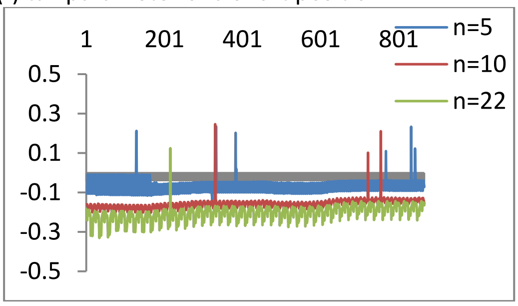
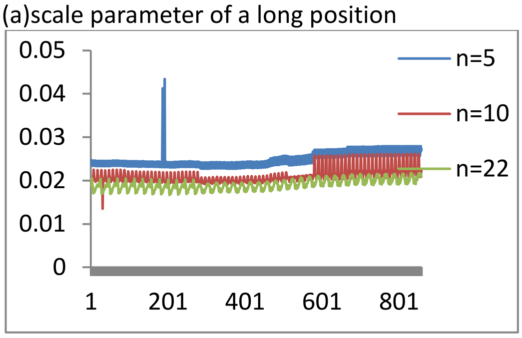
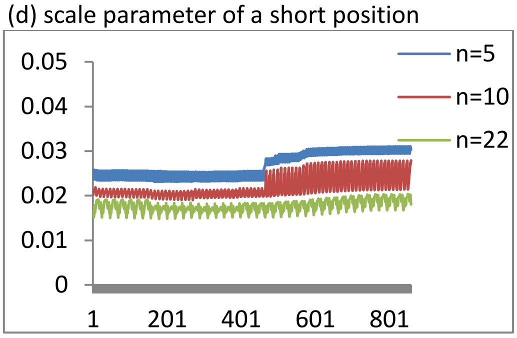


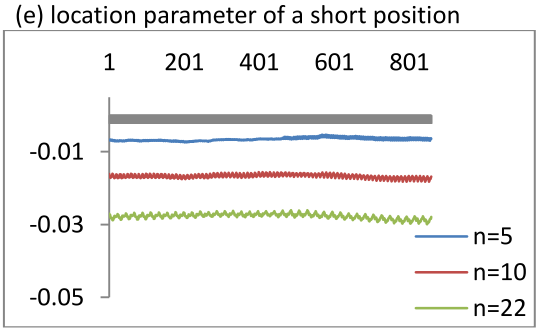
Figure C- 17 The pattern of estimated parameters of S&P 500 (10-day return)

(a)scale parameter of a long position (d) scale parameter of a short position

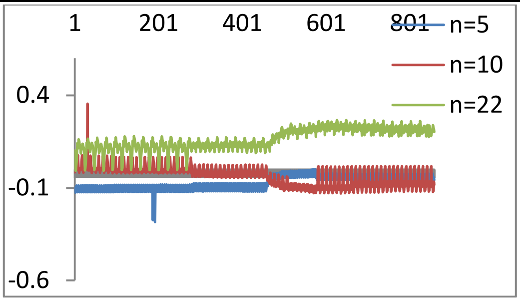




(b)location parameter of a long position (e) location parameter of a short position



(c) tail parameter of a long position (f) tail parameter of a short position



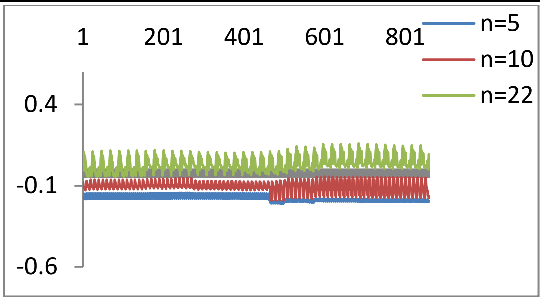
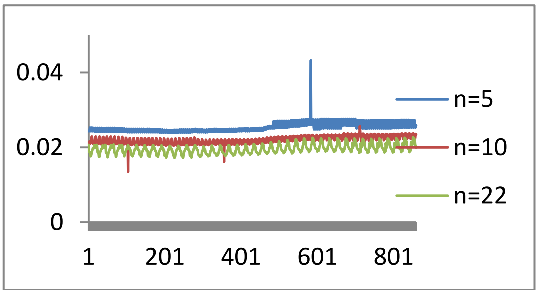
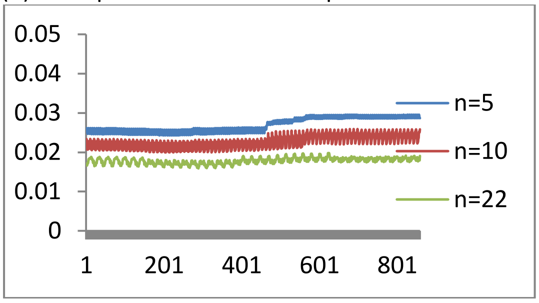


Figure C- 18 The pattern of estimated parameters of FTSE 100

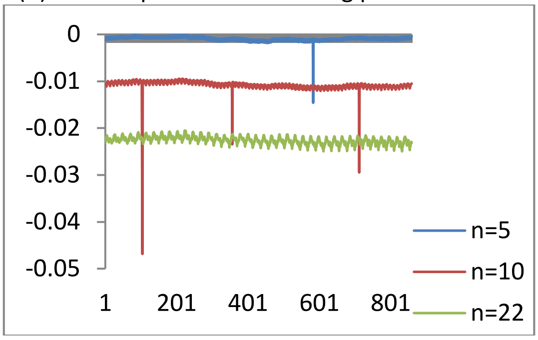
(a)scale parameter of a long position



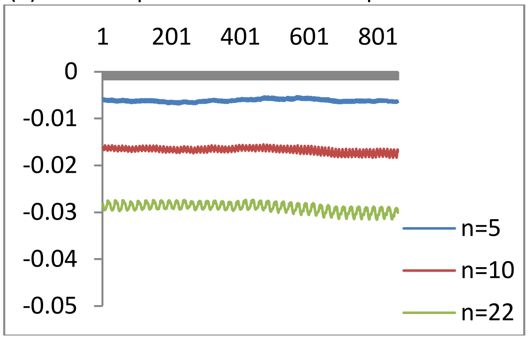
(d) scale parameter of a short position



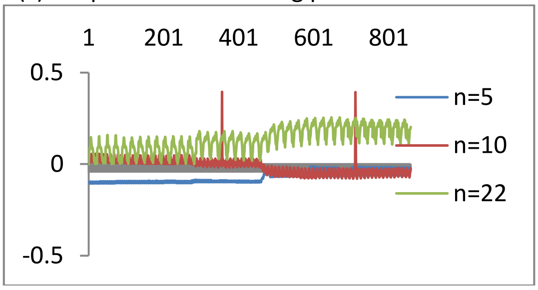
(b)location parameter of a long position



(e) location parameter of a short position



(c) tail parameter of a long position



(f) tail parameter of a short position

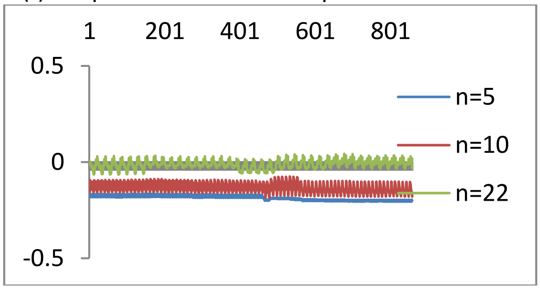
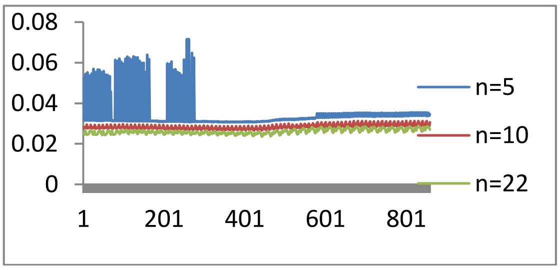
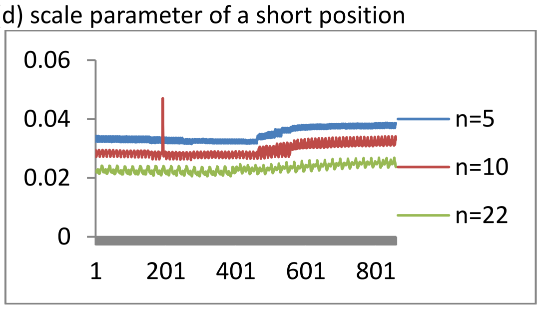


Figure C- 19 The pattern of estimated parameters of CAC40

(a)scale parameter of a long position

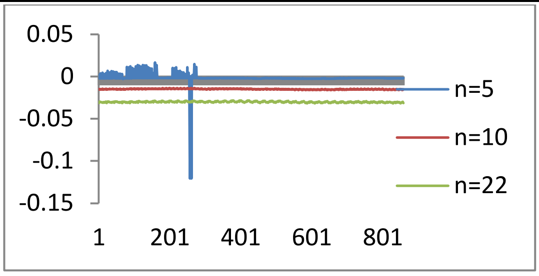


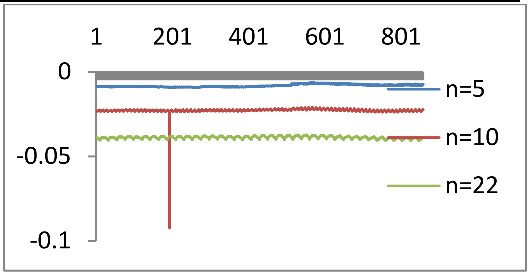
(d) scale parameter of a short position



(b)location parameter of a long position

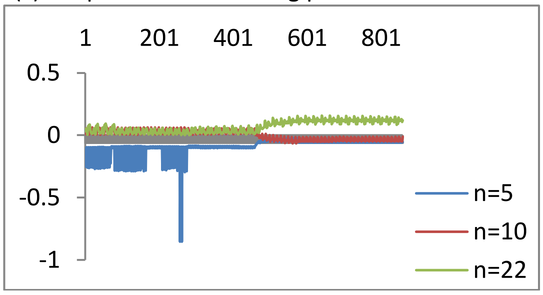
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



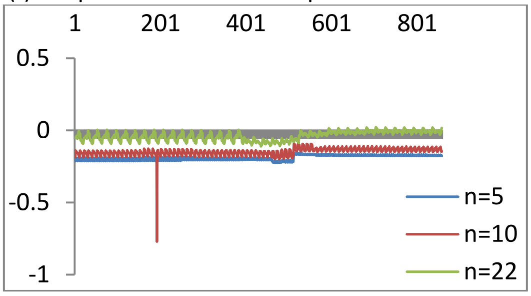
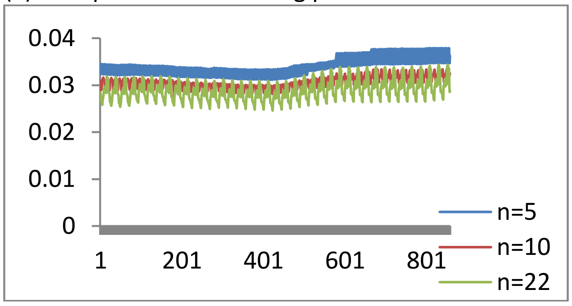
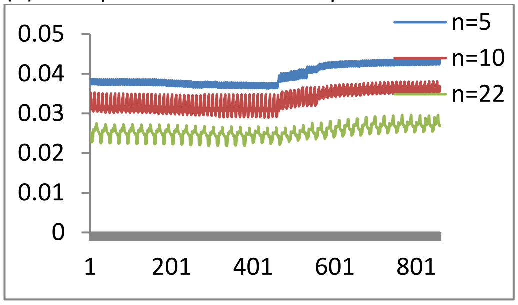


Figure C- 20 The pattern of estimated parameters of DAX

(a)scale parameter of a long position

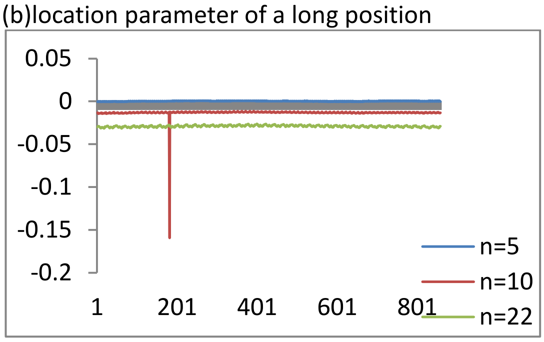
(d) scale parameter of a short position

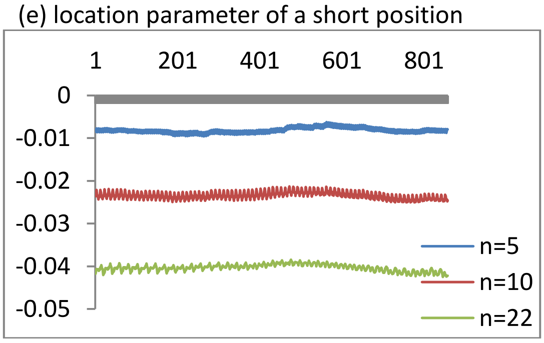




(b)location parameter of a long position

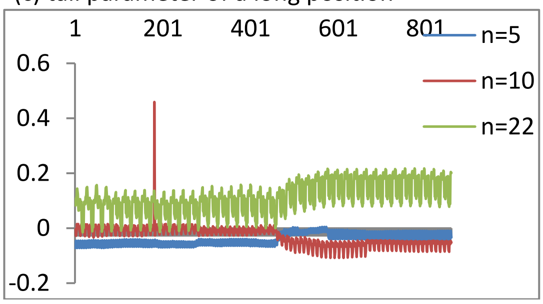
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



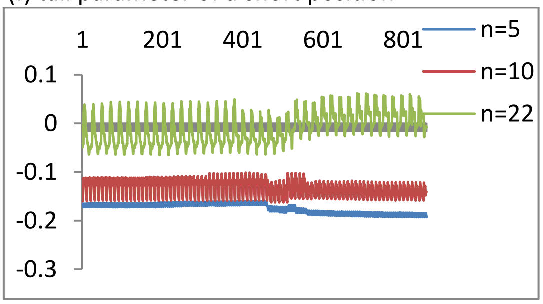
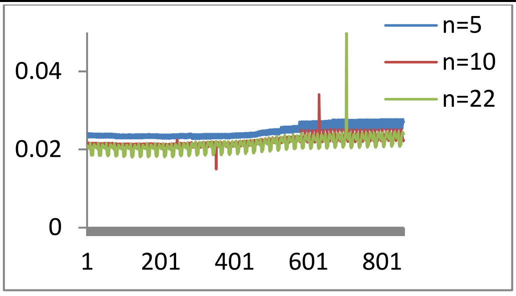


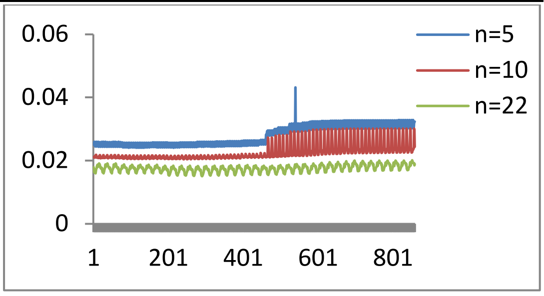
Figure C- 21 The pattern of estimated parameters of TSX

(a)scale parameter of a long position

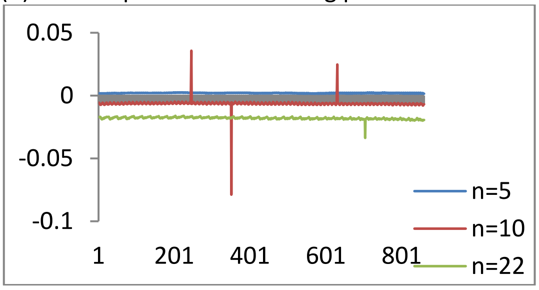
(d) scale parameter of a short position



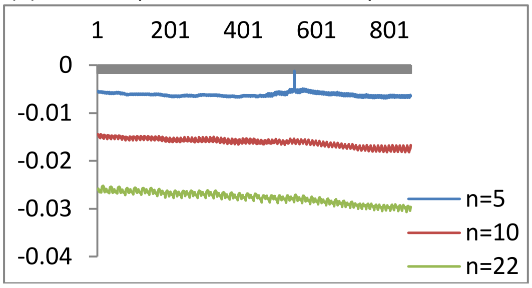
(b)location parameter of a long position



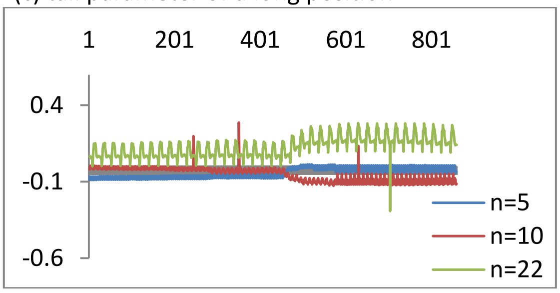
(e) location parameter of a short position



(c) tail parameter of a long position



(f) tail parameter of a short position



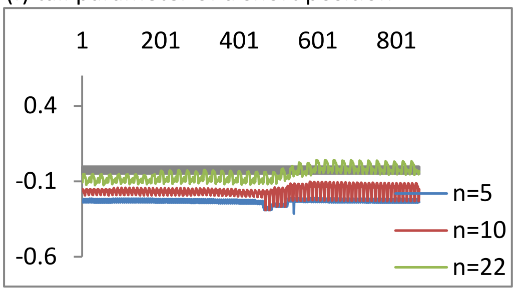
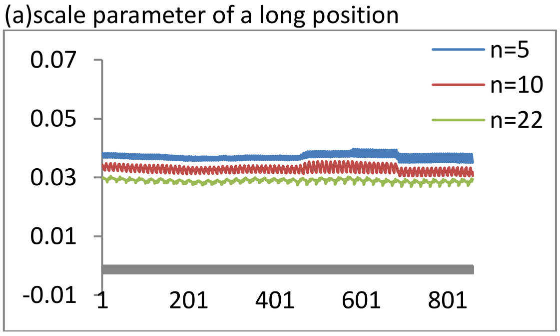
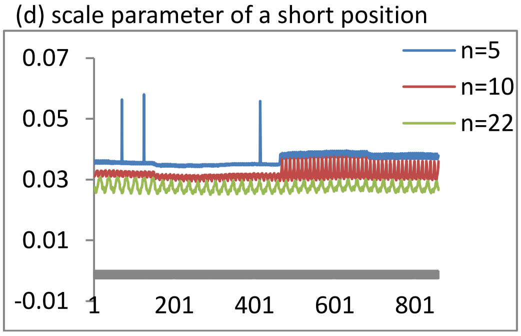


Figure C- 22 The pattern of estimated parameters of Nikkei225

(a)scale parameter of a long position

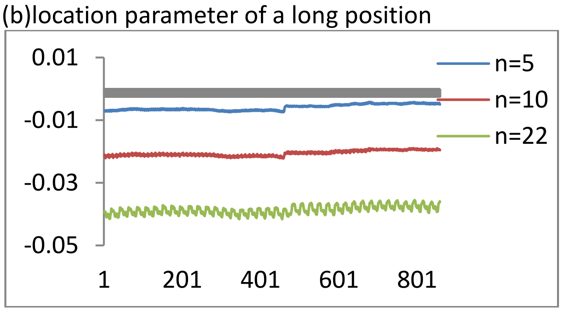
(d) scale parameter of a short position



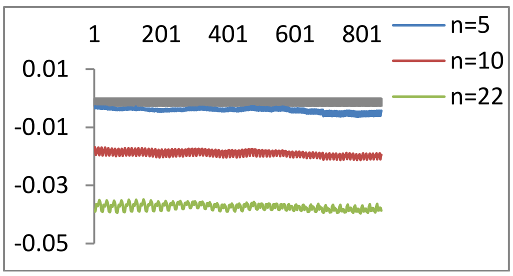


(b)location parameter of a long position

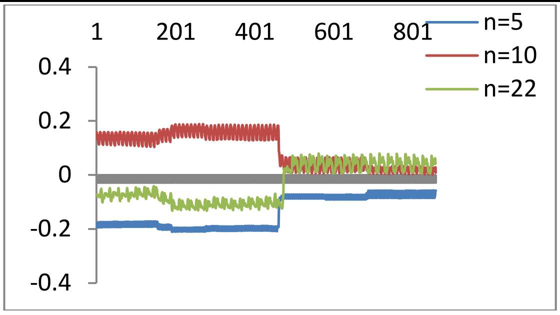
(e) location parameter of a short position



(c) tail parameter of a long position



(f) tail parameter of a short position



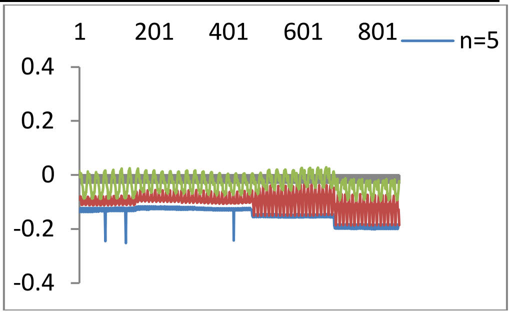
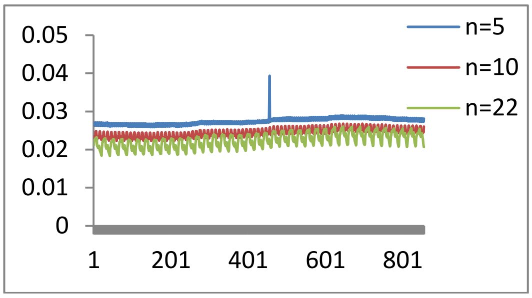
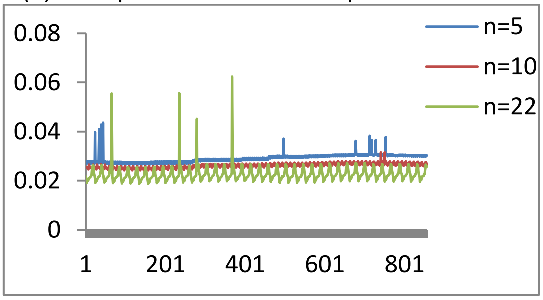


Figure C- 23 The pattern of estimated parameters of IGPA

(a)scale parameter of a long position

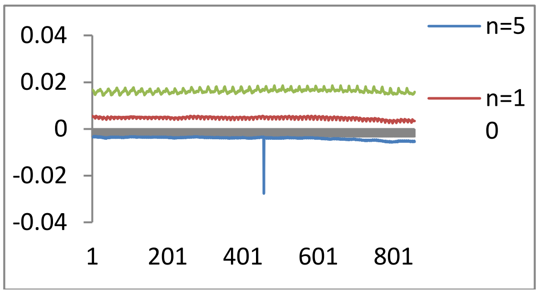
(d) scale parameter of a short position

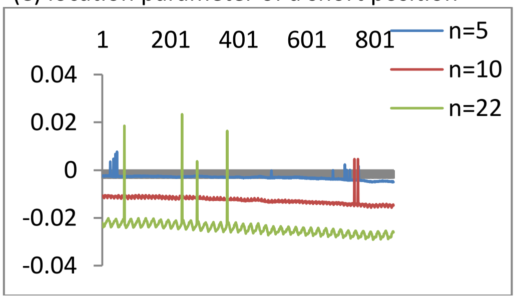




(b)location parameter of a long position

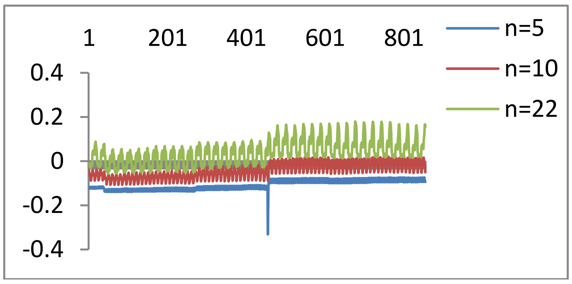
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



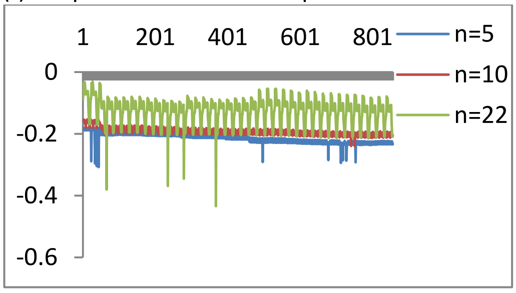
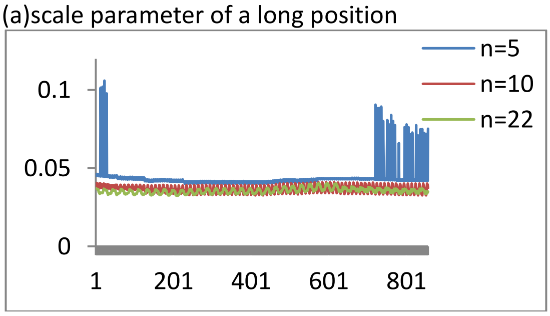
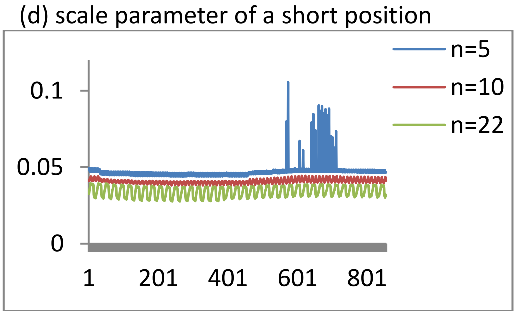


Figure C- 24 The pattern of estimated parameters of Bolsa

(a)scale parameter of a long position

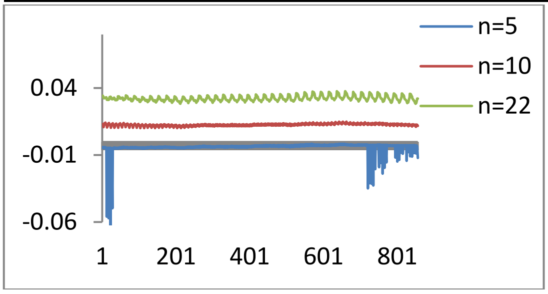
(d) scale parameter of a short position

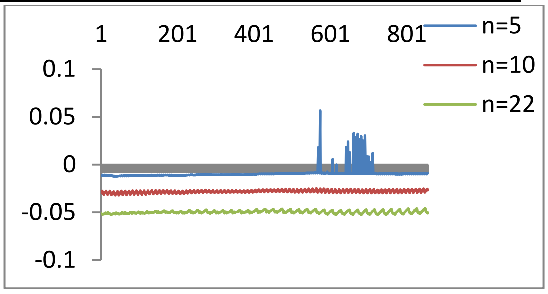




(b)location parameter of a long position

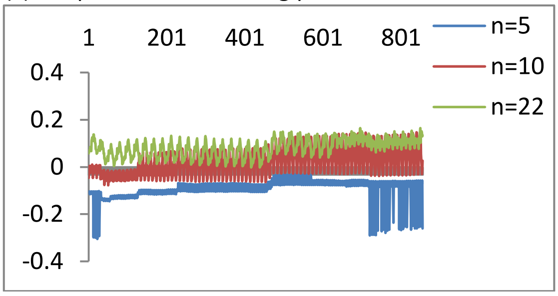
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



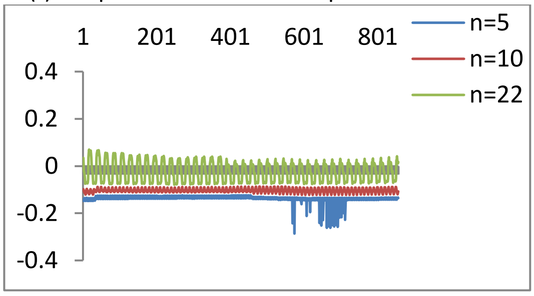
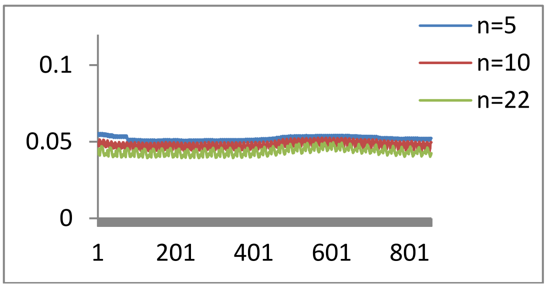
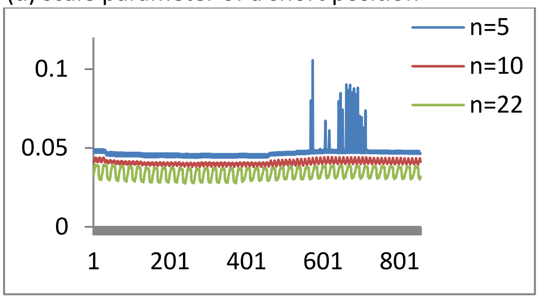


Figure C- 25 The pattern of estimated parameters of Bovespa

(a)scale parameter of a long position

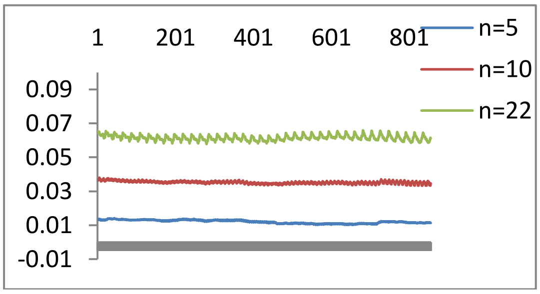
(d) scale parameter of a short position





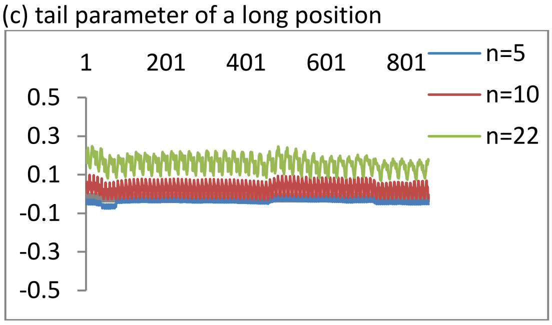
(b)location parameter of a long position

(e) location parameter of a short position



(c) tail parameter of a long position

(f) tail parameter of a short position



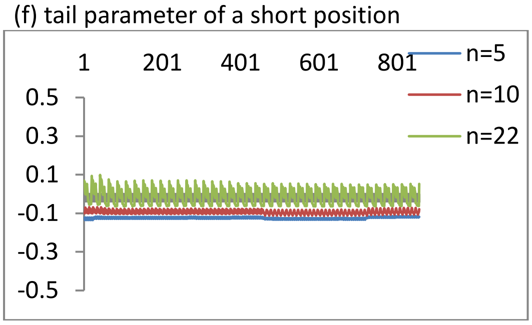
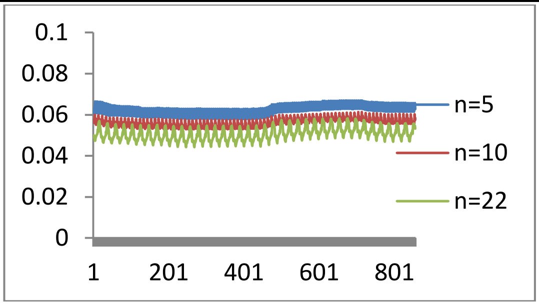
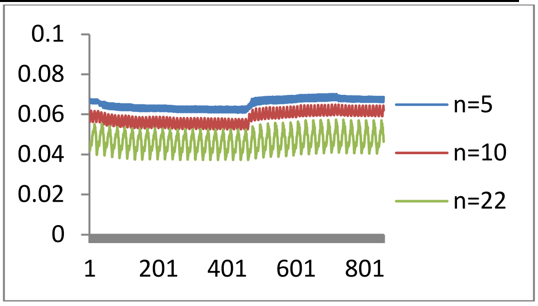


Figure C- 26 The pattern of estimated parameters of Merval

(a)scale parameter of a long position

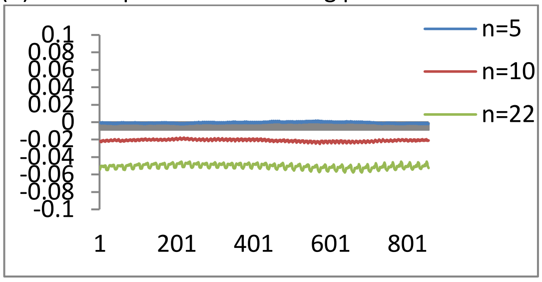
(d) scale parameter of a short position

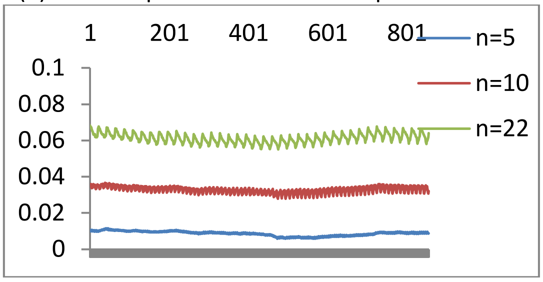




(b)location parameter of a long position

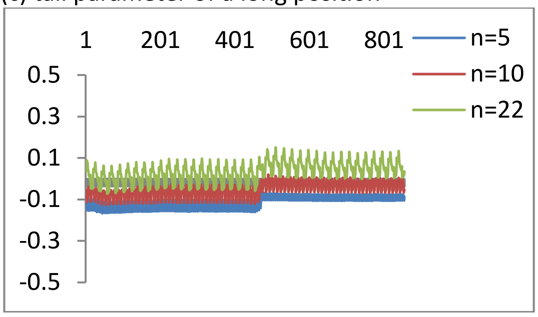
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



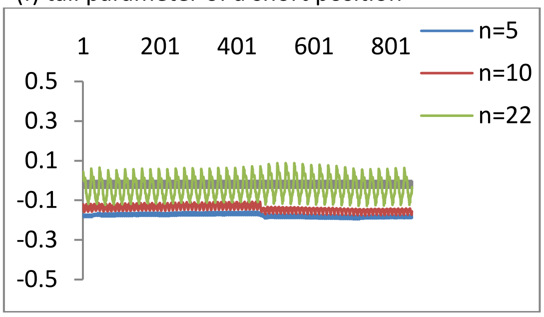
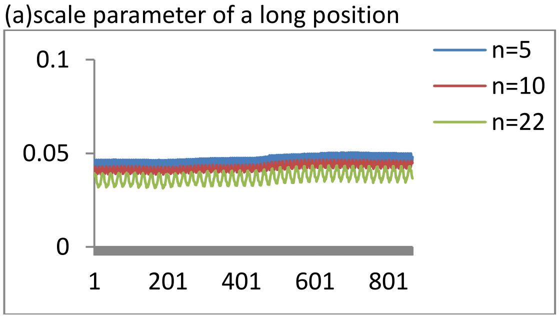
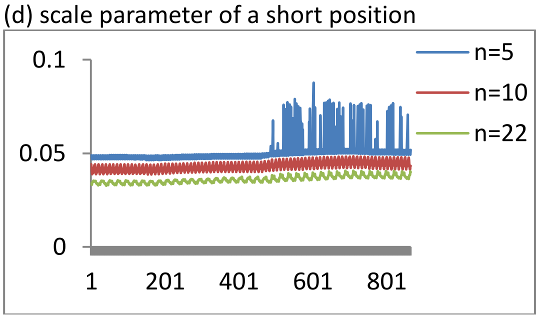


Figure C- 27 The pattern of estimated parameters of HSI

(a)scale parameter of a long position

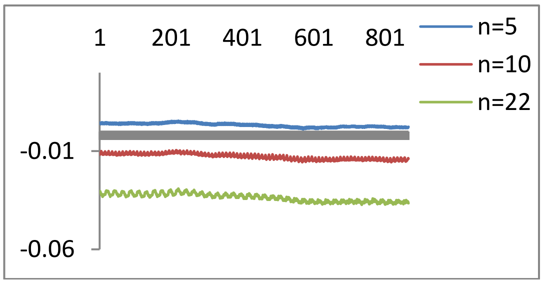
(d) scale parameter of a short position

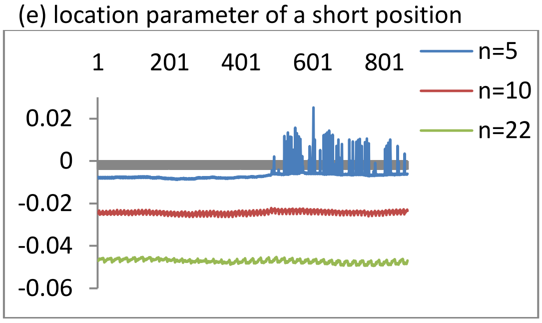




(b)location parameter of a long position

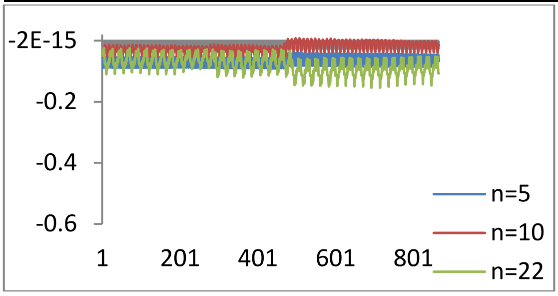
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



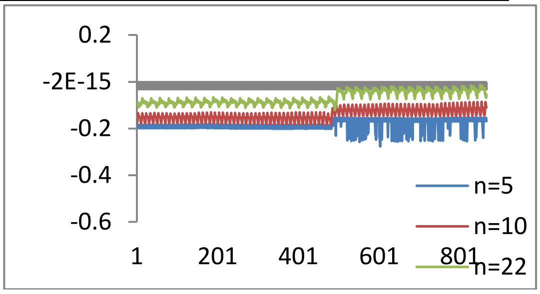
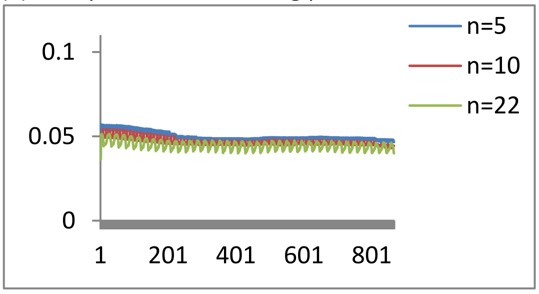
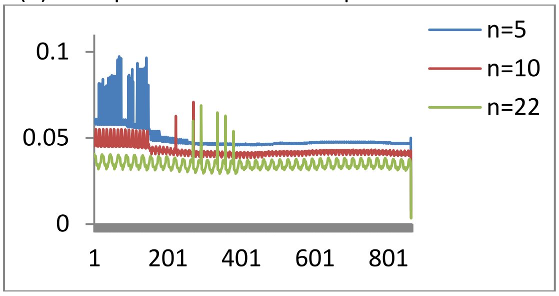


Figure C- 28 The pattern of estimated parameters of TAIEX

(a)scale parameter of a long position

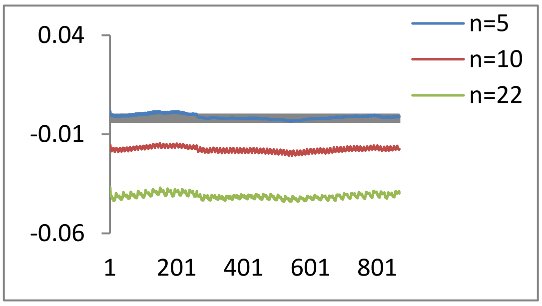
(d) scale parameter of a short position

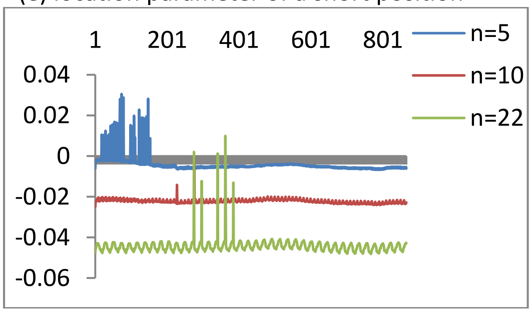




(b)location parameter of a long position

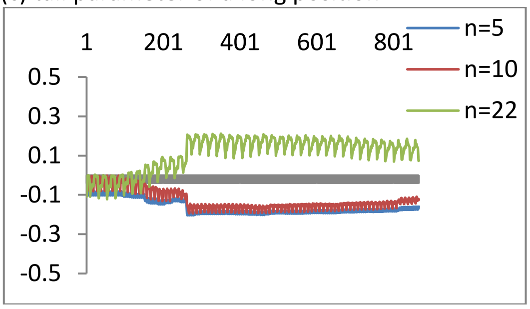
(e) location parameter of a short position





(c) tail parameter of a long position

(f) tail parameter of a short position



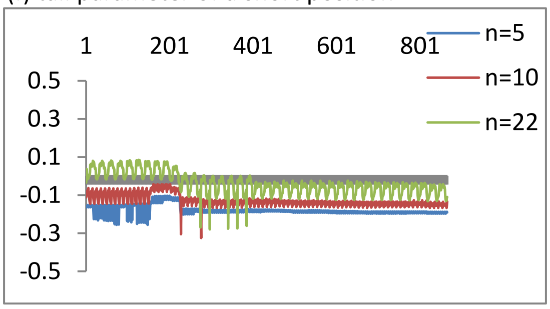
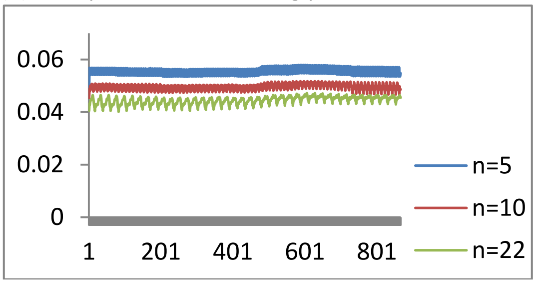
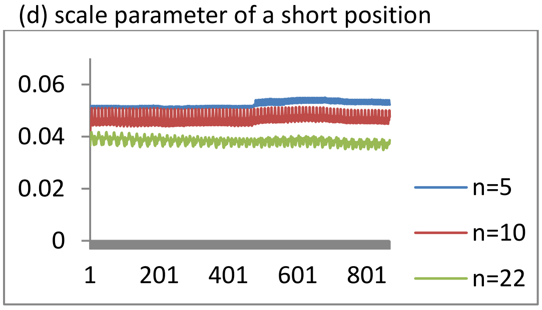


Figure C- 29 The pattern of estimated parameters of KOSPI

(a)scale parameter of a long position

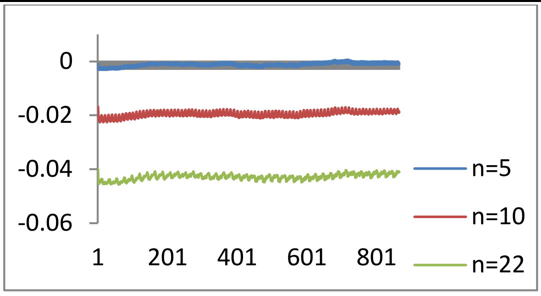
(d) scale parameter of a short position

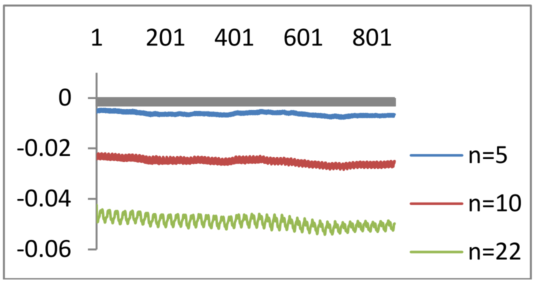




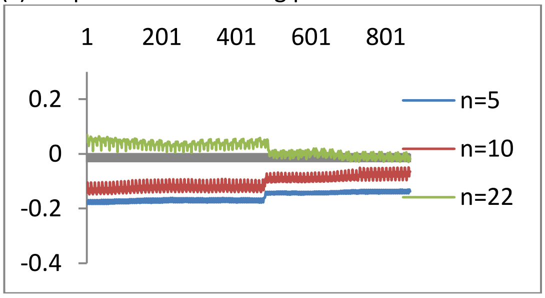
(b)location parameter of a long position

(e) location parameter of a short position





(c) tail parameter of a long position (f) tail parameter of a short position



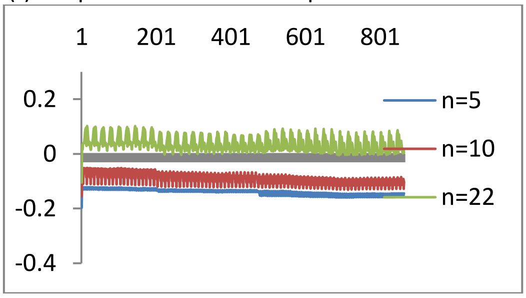
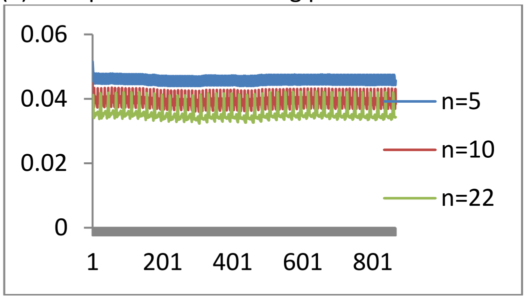
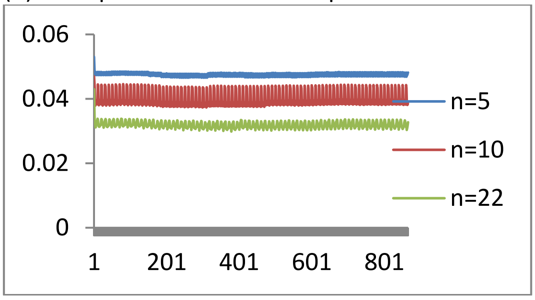


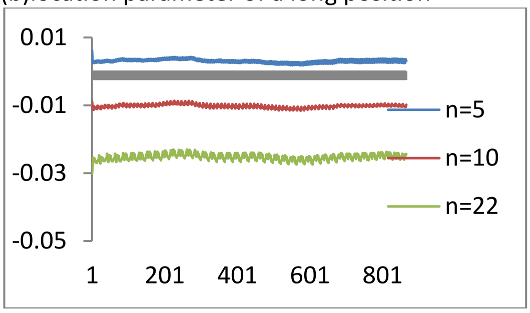
Figure C- 30 The pattern of estimated parameters of KLCI

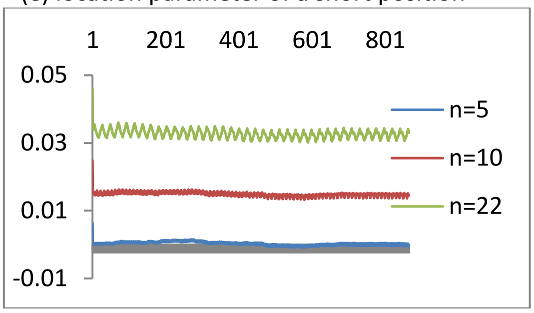
(a)scale parameter of a long position (d) scale parameter of a short position



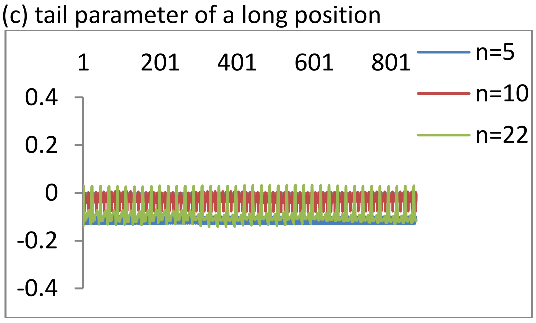


(b)location parameter of a long position (e) location parameter of a short position





(c) tail parameter of a long position (f) tail parameter of a short position



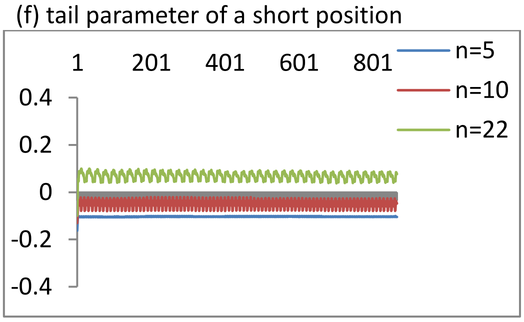
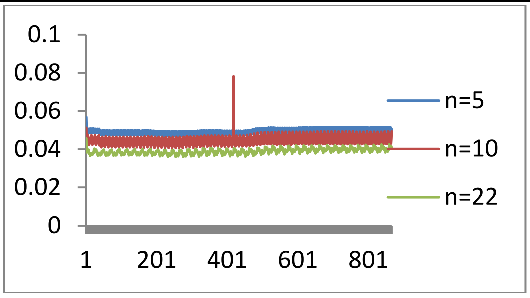
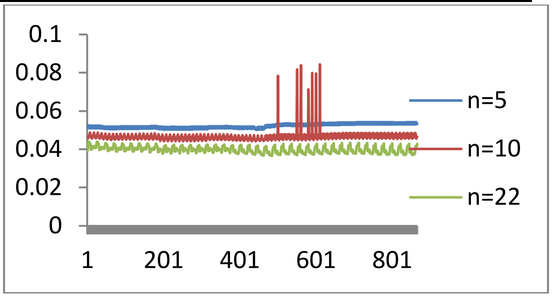


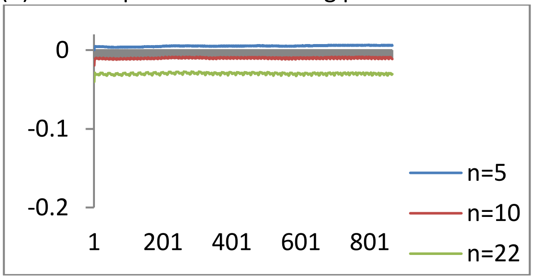
Figure C- 31 The pattern of estimated parameters of JCI

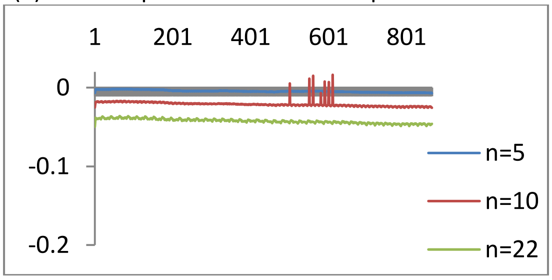
(a)scale parameter of a long position (d) scale parameter of a short position



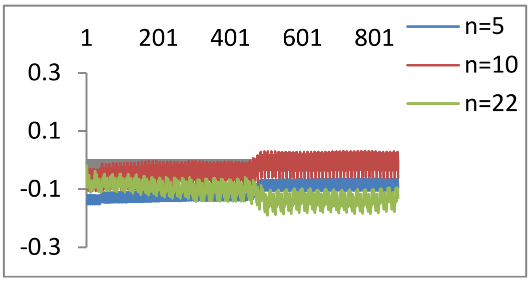


(b)location parameter of a long position (e) location parameter of a short position





(c) tail parameter of a long position (f) tail parameter of a short position



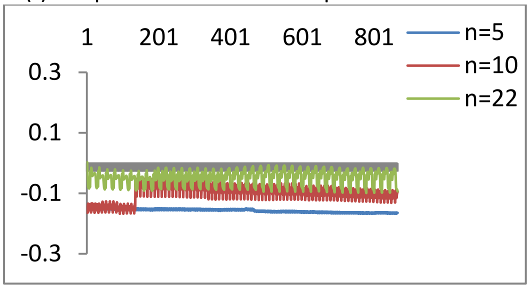
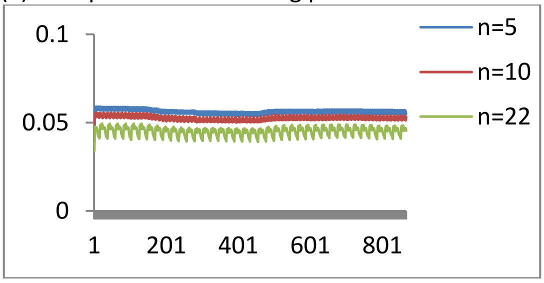
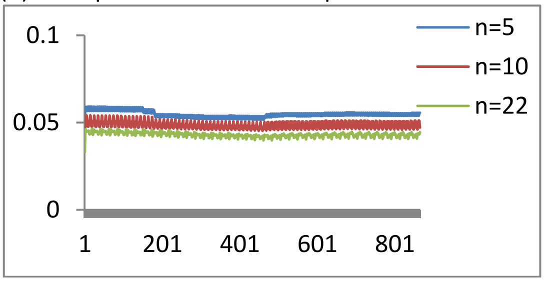


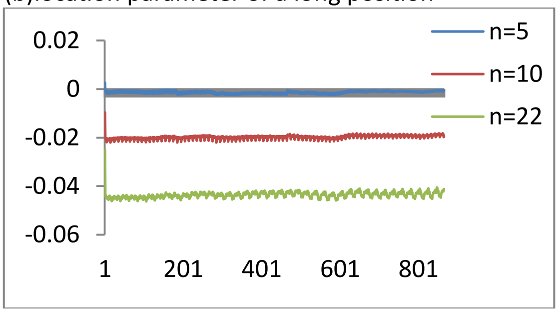
Figure C- 32 The pattern of estimated parameters of SET

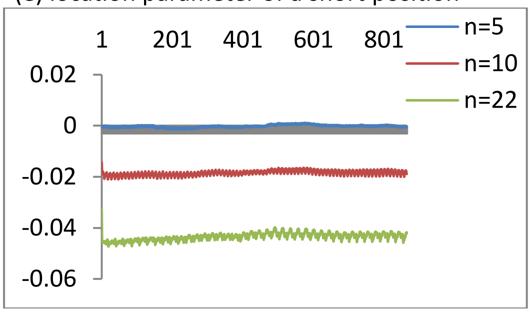
(a)scale parameter of a long position (d) scale parameter of a short position



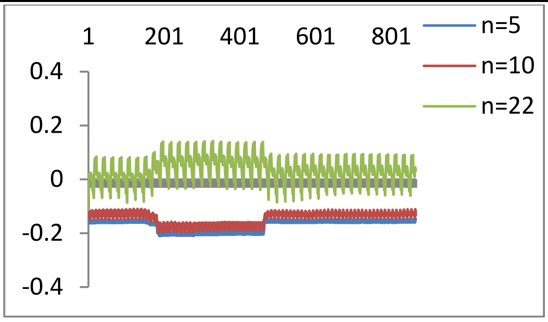


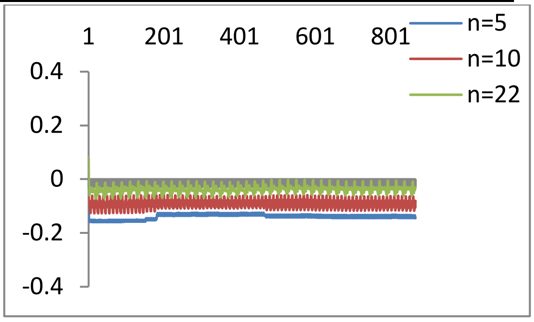
(b)location parameter of a long position (e) location parameter of a short position





(c) tail parameter of a long position (f) tail parameter of a short position

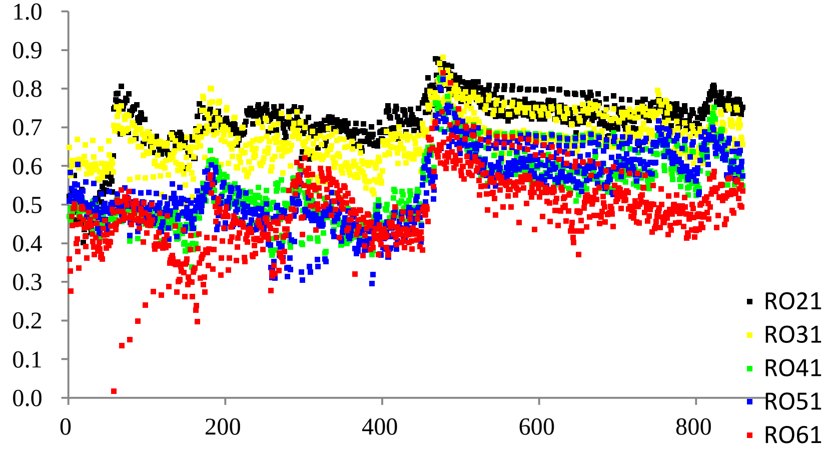

1
201
401
601
801
0.4
0.2
0
-0.2
-0.4
n=5
n=10
n=22


1
201
401
601
801
0.4
0.2
0
-0.2
-0.4
n=5
n=10
n=22

Appendix D The tail-DCC of the market indices

Figure D- 1 The tail-DCC of developed market indices (n=10)

(a) Left tail



(b) Right tail

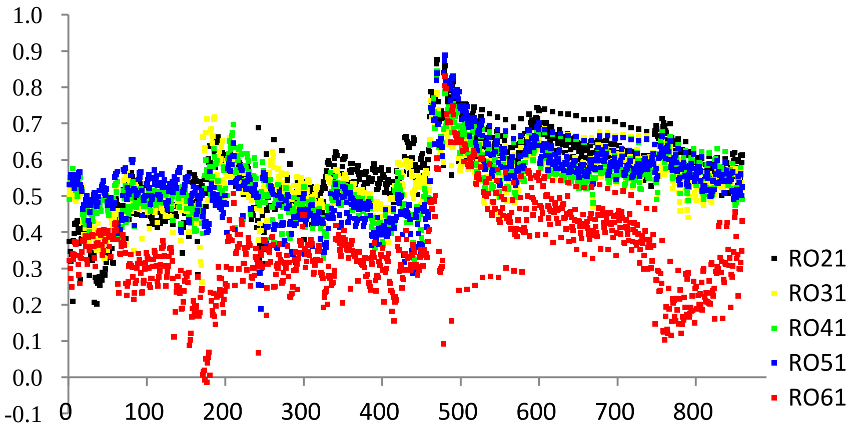
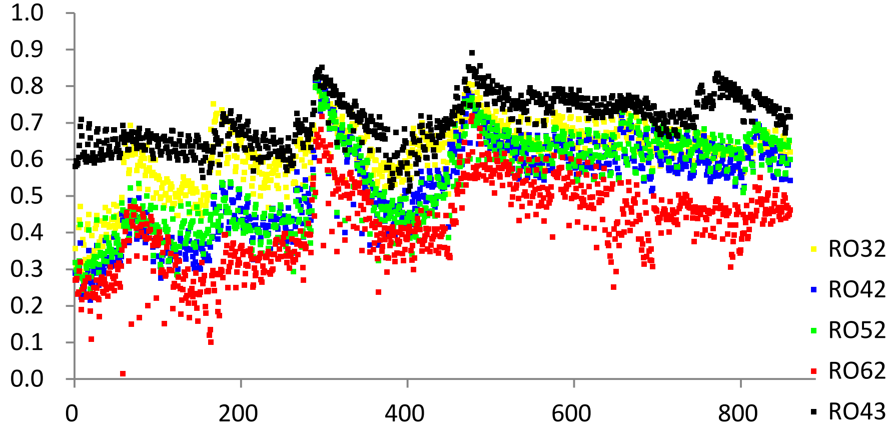


Figure D- 2 The tail-DCC of developed market indices (n=10)

(a) Left tail



(b)Right tail

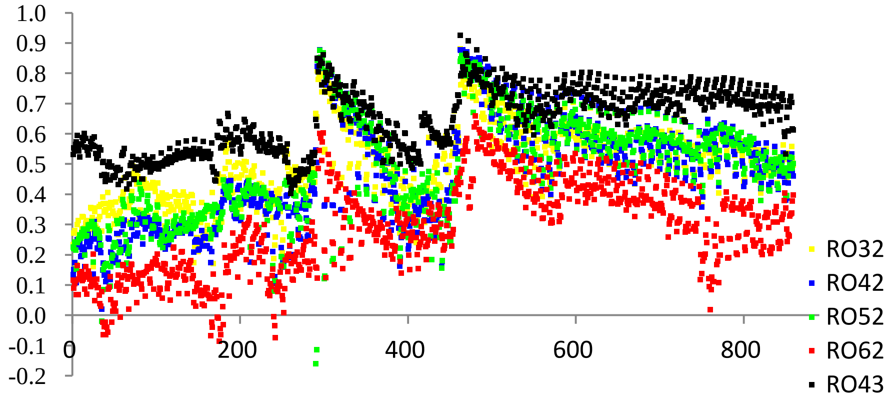
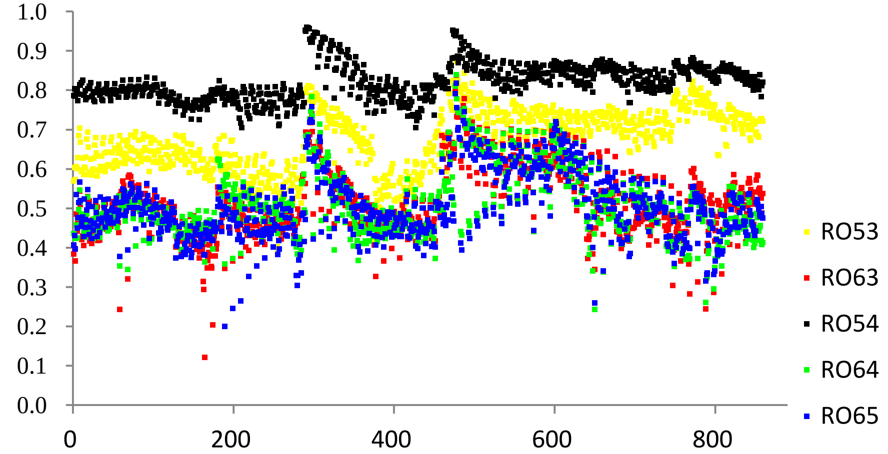


Figure D- 3 The tail-DCC of developed market indices(n=10)

(a) Left tail



(b) Right tail

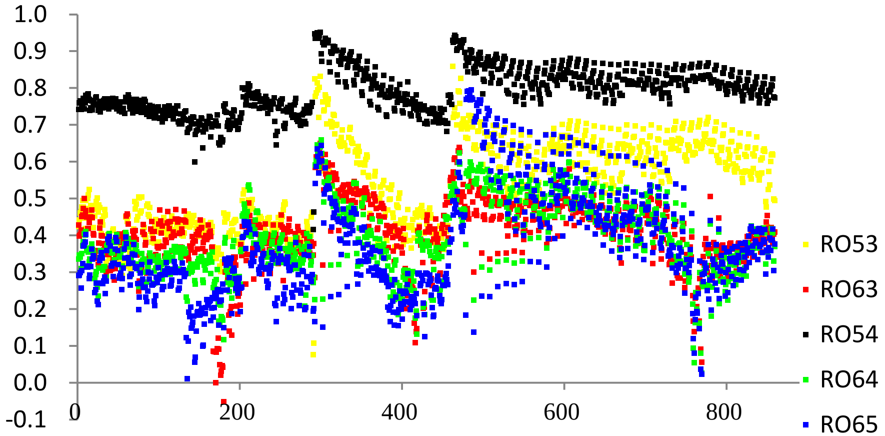
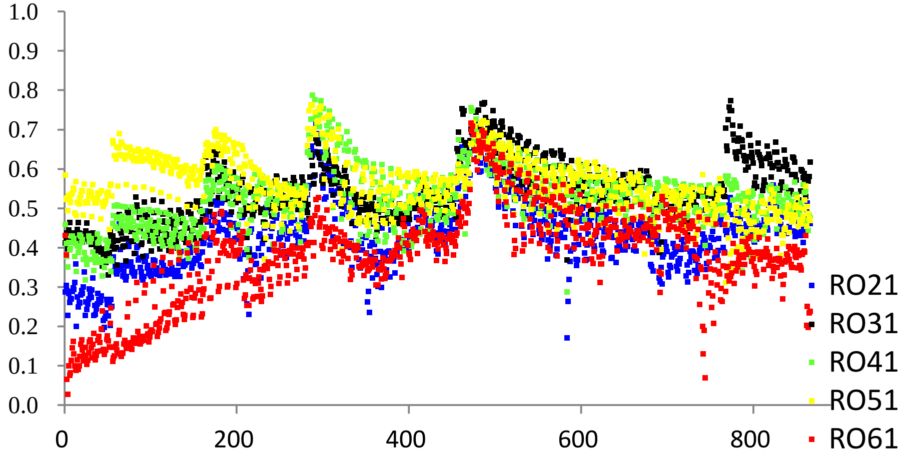


Figure D- 4 The tail-DCC of Asian market indices (n=10)
(a) Left tail



(b) Right tail

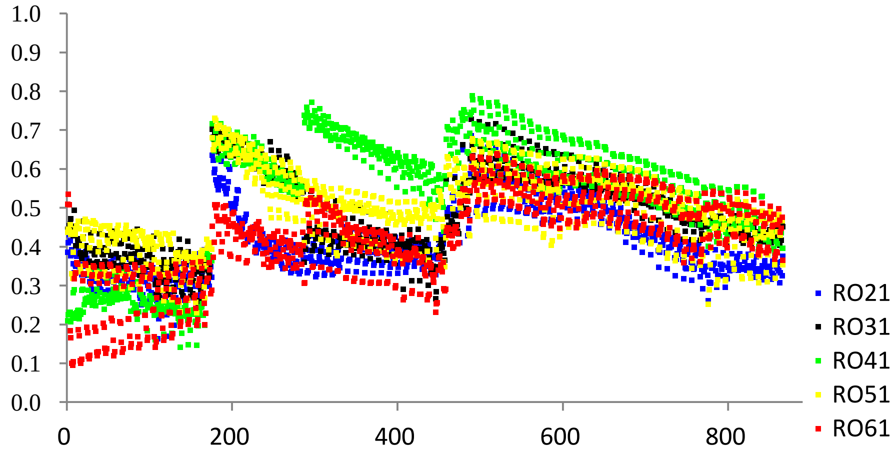
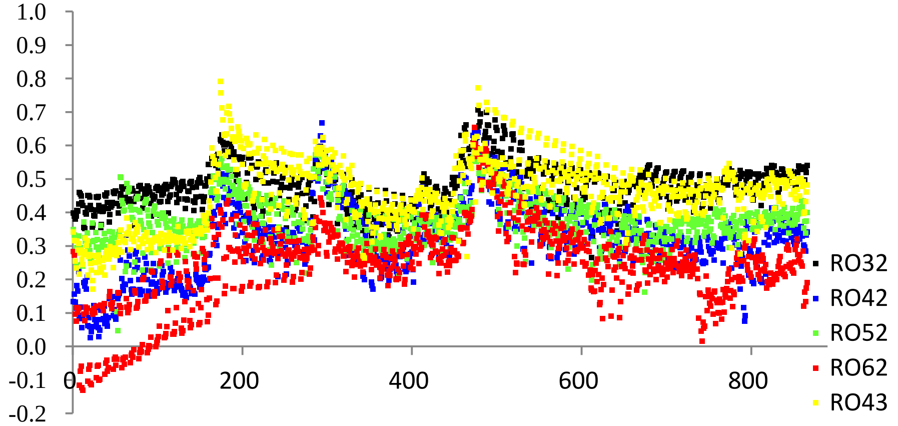


Figure D- 5 The tail-DCC of Asian market indices(n=10)
(a) Left tail



(b) Right tail

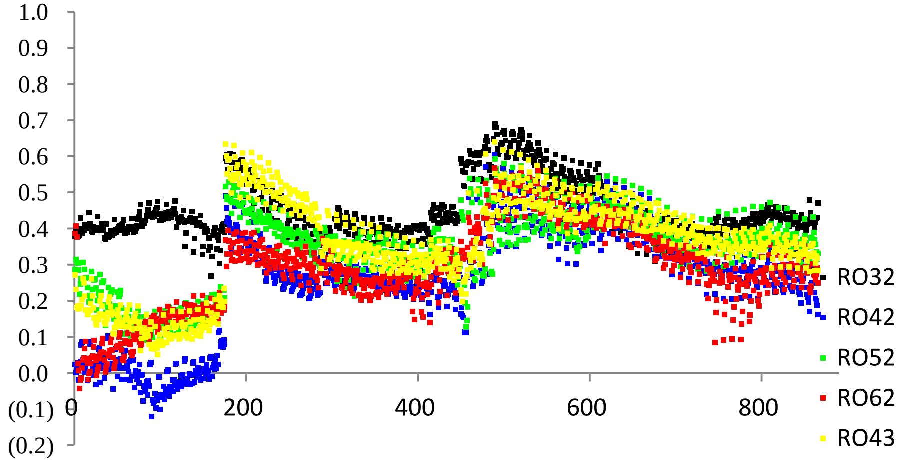
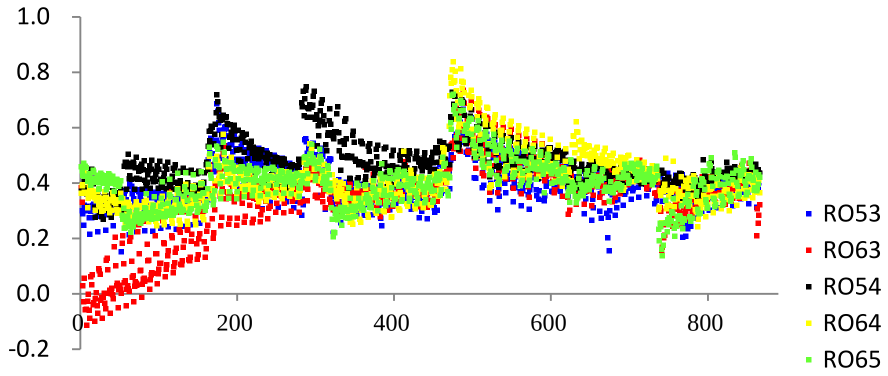


Figure D- 6 The tail-DCC of Asian market indices(n=10)
(a) Left tail



(b) Right tail

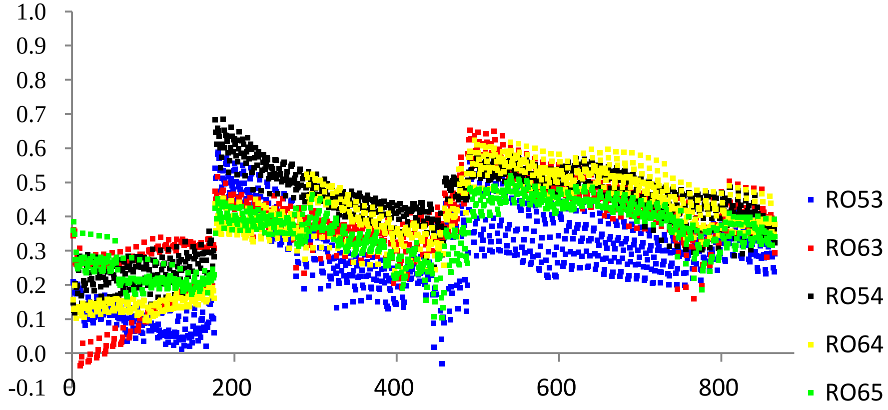
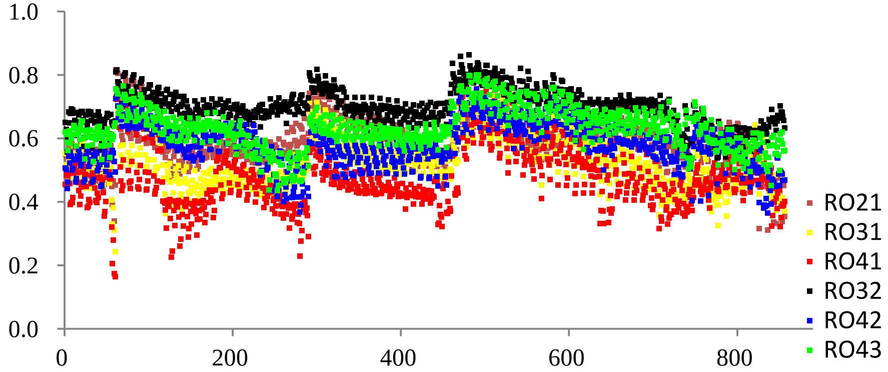
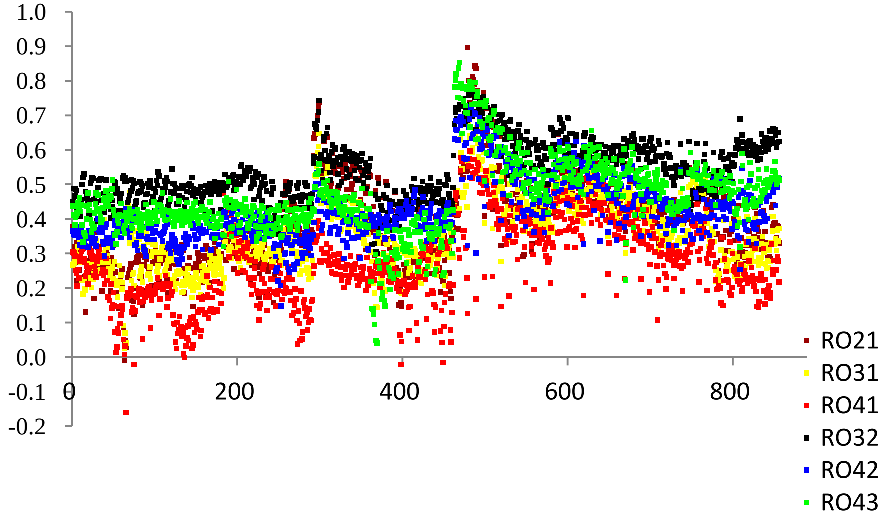


Figure D- 7 The tail-DCC of Latin American market indices(n=10)
(a) Left tail


1.0
0.8
0.6
0.4
0.2
0.0
0
200
400
600
800
RO21
RO31
RO41
RO32
RO42
RO43

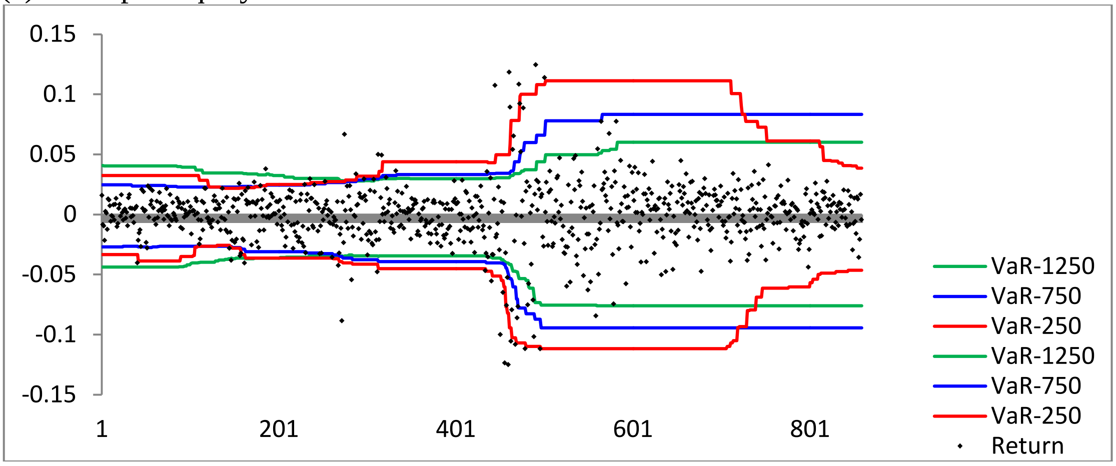
(b) Right tail


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
0
200
400
600
800
RO21
RO31
RO41
RO32
RO42
RO43

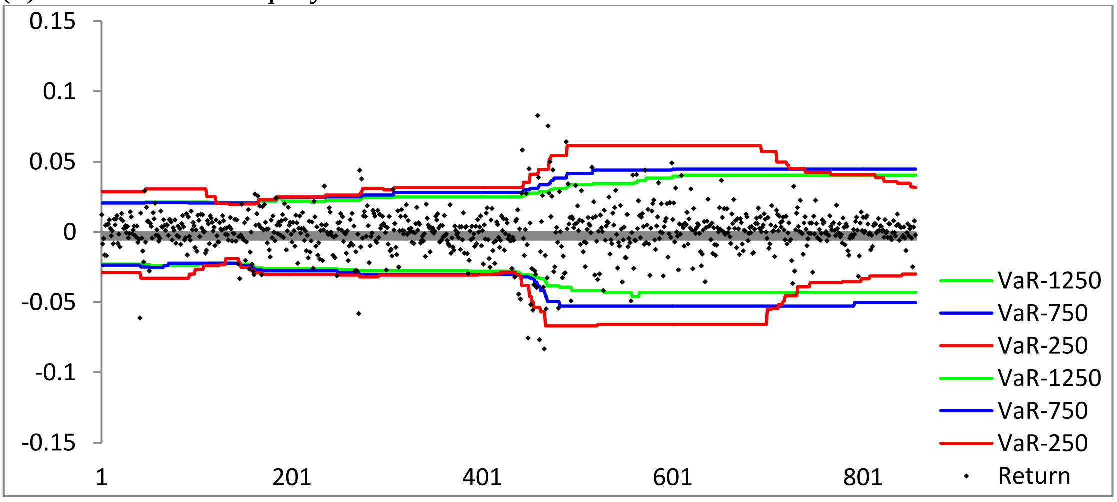
Appendix E VaR pattern of Historical simulation

Figure E- 1 The VaR pattern of Historical Simulation with a different simulation period

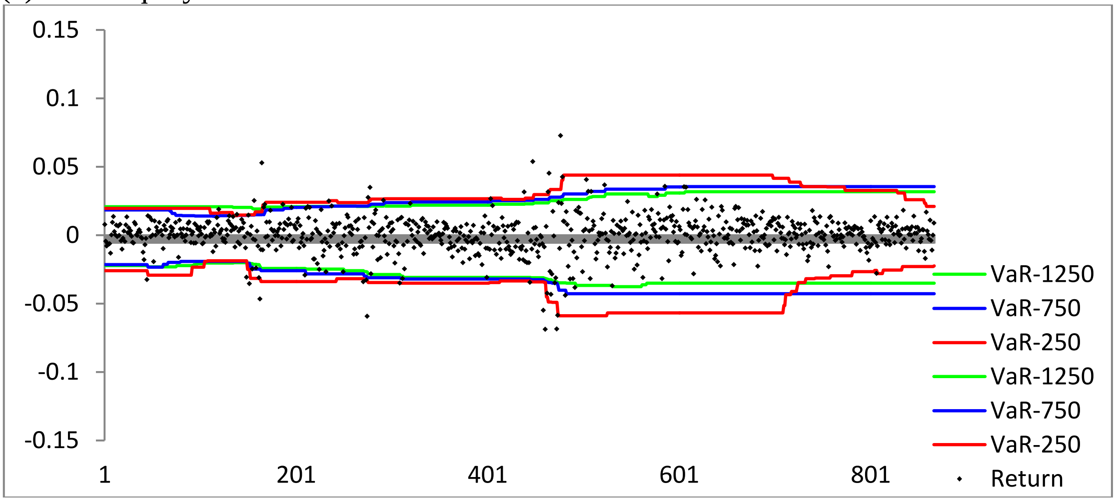
(a)developed equity market



(b)Latin American equity market

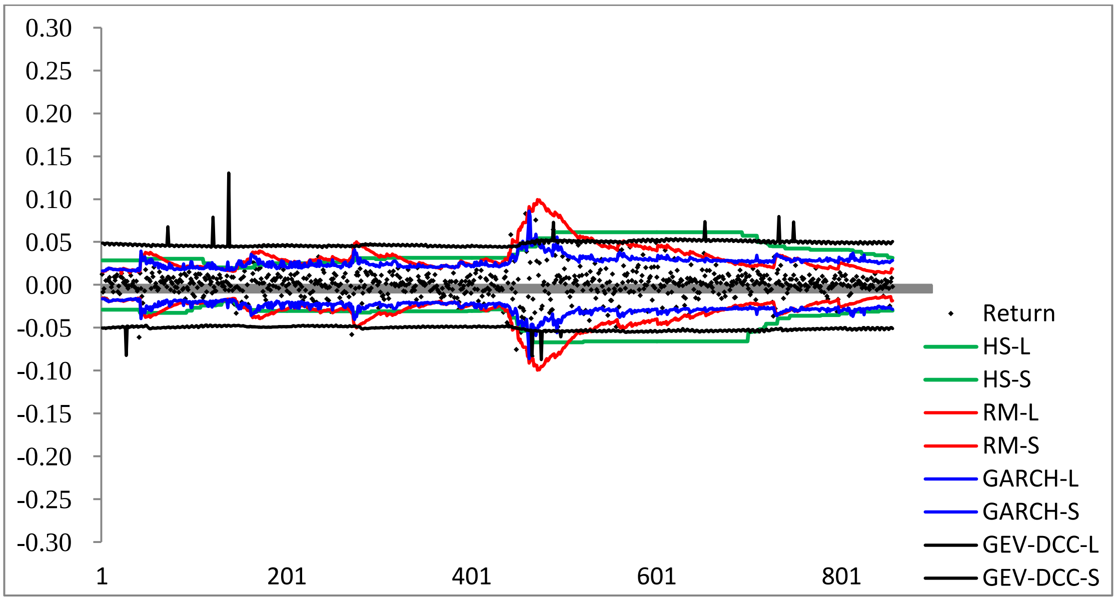


(c)Asian equity market

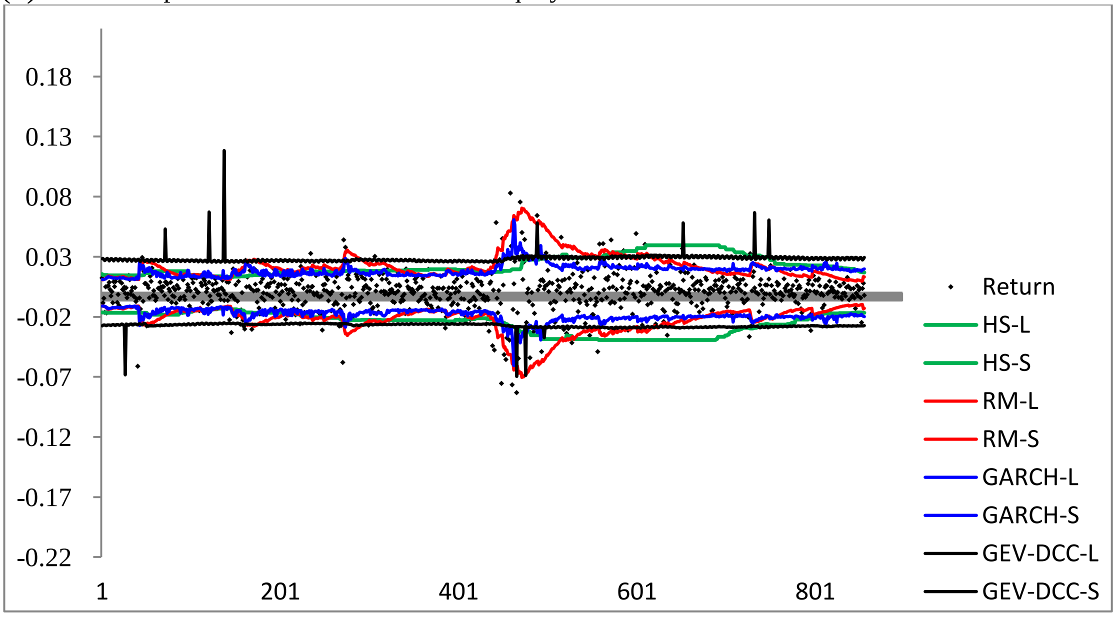


Appendix F The VaR pattern without SV model

Figure F- 1 (a). 99%-VaR patterns of the Latin American equity market and the actual returns without SV model



(b). 95%-VaR patterns of the Latin American equity market and the actual returns without SV model



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