

## Effects of feedback conditions for an online algebra tool

Christian Bokhove & Paul Drijvers

Freudenthal Institute for Science and Mathematics Education, Utrecht University, the Netherlands

Procedural skills and conceptual understanding have been widely debated, especially with regard to algebra. In the mean while the use of ICT in education has increased. In this article we report on one of the design principles for and results of a digital intervention for algebraic expertise. The intervention aimed at improving algebraic expertise and was deployed in 15 grade 12 mathematics classes in 9 secondary schools. In this paper we focus on the implementation of IDEAS feedback and report on the effects of two feedback conditions. Preliminary results show that relevant feedback aids students in learning algebra by decreasing the number of attempts needed for an algebraic task, whilst improving the scores. We conclude there is potential in the use of feedback in an online algebra tool but that further development is needed.

### Introduction

During the last decade the dichotomy between procedural skills and conceptual understanding has been widely debated. It has been a focal point in the so-called 'Math War' (Schoenfeld, 2004) discussion. The debate also influenced the realm of algebraic expertise: should students focus on practising algorithms or on reasoning and problem solving strategies? One approach stresses the fact that computational skills are an essential ingredient for understanding mathematical concepts (US Department of Education, 2007). Another approach starts off with more focus on conceptual understanding (ibid.). Although most experts seem to agree that essentially both are needed, there is no clear agreement on the relationships and priorities among the two. Another development in recent years involves the advent of the use of technology use in mathematics education. The National Council of Teachers of Mathematics (2008) formulated the potential of Information and Communication Technology (ICT) for learning in their position statement. Our research combines the aforementioned elements: we want to use the potential of ICT to address algebraic skills, on both a procedural and a conceptual level, and aim to design and test an intervention doing just that. In this paper we will focus on one of the design principles behind the intervention, the use of item feedback.

### Conceptual framework

For the purpose of this paper we will focus on one of three topics involved in the general framework of our study: theoretical notions of formative assessment and feedback.

#### *Formative assessment*

Black and Wiliam (2004) distinguish three functions for assessment: supporting learning (formative), certifying the achievements or potential of individuals (summative), and evaluating the quality of educational programs or institutions (evaluative). Summative assessment is also characterised as assessment *of* learning and is contrasted with formative assessment, which is assessment *for* learning. Black and Wiliam (1998) define assessment as being 'formative' only when the feedback from learning activities is actually used to modify teaching to meet the learner's needs. From this it is clear that feedback plays a pivotal role in the process of formative assessment.

#### *Feedback*

Both Hattie and Timperley (2007) and Vasilyeva et al. (2007) conducted an extensive meta-review of the effectiveness of different types of feedback. The feedback effects of cues and corrective feedback are deemed best. Seeking feedback is governed by a cost/benefit ratio. In general, feedback is psychologically reassuring, and people like to obtain feedback about their performance, even if it has no impact on their performance. The model provided by Hattie and Timperley (ibid.) distinguishes three questions that effective feedback answers:

Where am I going? (the goals)	FeedUp
How am I going?	FeedBack
Where to next?	Feed Forward

Each feedback question works at four levels (focus of the feedback): the task level: how well tasks are understood/performed (FT), the process level: the main process needed to understand/perform tasks (FP), the self-regulation level: self-monitoring, directing and regulating of actions (FR), and the self-level: personal evaluations and affect (usually positive) about the learner (FS). Hattie and Timperley (2007) also provide some statements on the effectiveness of (combinations of) feedback types, including that FS feedback is least effective, simple FT feedback is more effective than complex FT feedback, FT and FS do not mix well (“Well done, that is correct” is worse than “Correct” only), and that FT is more powerful when it’s about faulty interpretations, not lack of information. Furthermore they state that we should be attentive to the varying importance of the feedback information during study of the task. The three main design principles of our digital intervention involve feedback. In this paper we will not focus on two of these principles, formative scenarios and crises, but only on feedback at the item level. Here, both custom feedback and so-called IDEAS feedback are used to provide more “intelligent” feedback. The accompanying research question, therefore, is: does a variation in feedback type influence scores, attempts and student behaviour and in what way?

## Method

An intervention called “Algebra met Inzicht” [“Algebra with Insight”] is designed in the Digital Mathematical Environment (DME, [www.algebrametinzicht.nl](http://www.algebrametinzicht.nl)). It was field tested in a pilot lesson series by the end of 2010 for 15 groups of grade 12 mathematics students at nine secondary schools throughout the Netherlands (N=334). Schools were randomly allocated to two feedback conditions c1 (N=133) and c2 (N=178). The collected data included results from a pre- and post-test, and the scores and log files of the digital activities. The log files record information on item scores, feedback, answers, and number of attempts. The DME has a provision for feedback by connecting to the IDEAS web service (Heeren & Jeuring, 2010), as well as the feature of providing custom feedback, the latter which is described by Bokhove (2010). Custom feedback consists of feedback that teachers can program themselves, IDEAS feedback consists of a web service that provides feedback automatically. The IDEAS web service is also implemented for other online mathematical environments.

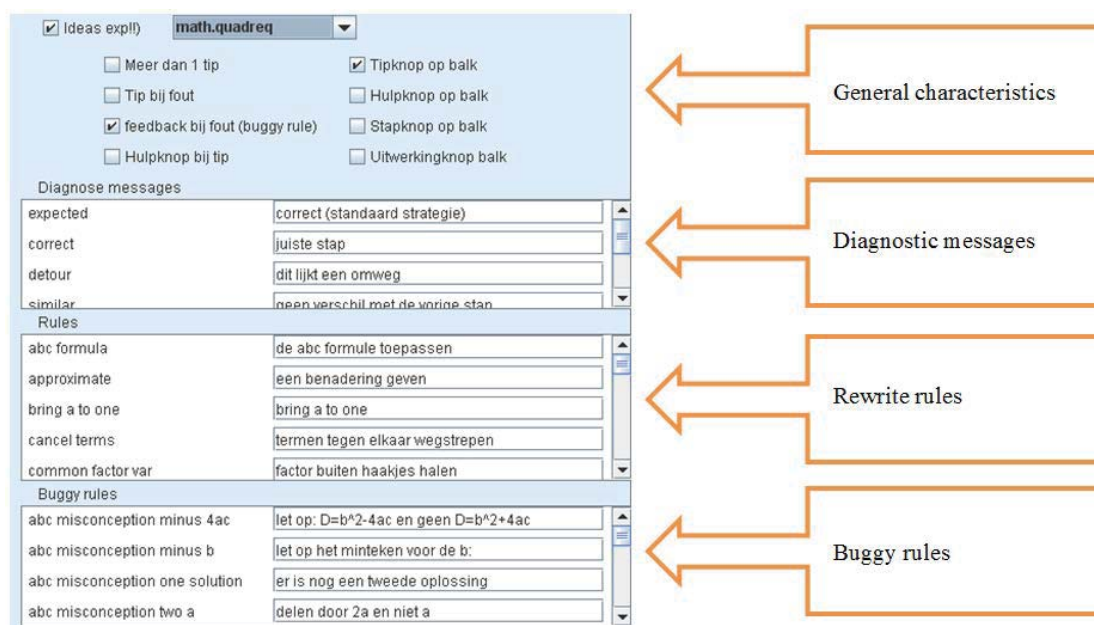


Figure 1: Screenshot of DME's authoring environment for IDEAS feedback

Figure 1 shows the essential characteristics of the IDEAS implementation in the DME. Firstly the *general characteristics* of IDEAS feedback, which includes what feedback is shown when and where. These settings were used to create the two conditions c1 and c2. Secondly IDEAS implements a block of *diagnostic messages*, which concerns feedback on strategy, the ‘correct step’ but also possible

'detours'. The third block of feedback concerns *rewrite rules*, rules that can be applied to an expression. Finally there are *buggy rules*, which describe the feedback that appears when a mistake is made. Using the authoring environment we implemented two series of tasks with both custom and IDEAS feedback we will refer to as d1 and d3. Two versions of these series were made: one for condition c1, and one for condition c2. The first feedback condition c1 consists of IDEAS and custom feedback without buttons in the interface. Feedback is only provided in the stepwise approach.

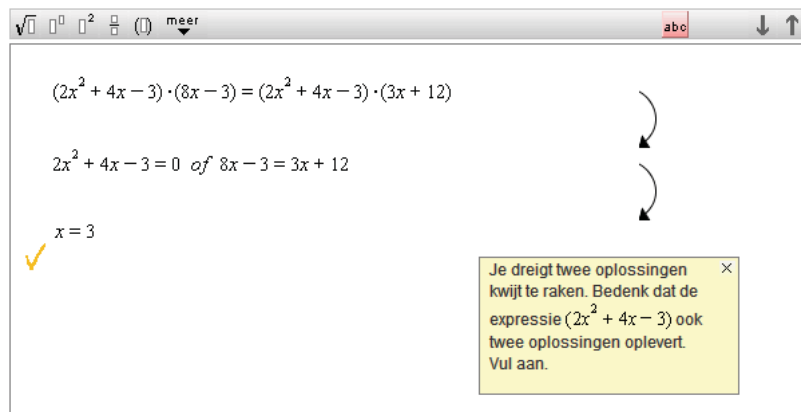


Figure 2: Stepwise custom feedback

To illustrate this, figure 2 shows the solution process for a polynomial equation. The student loses solutions for the equation along the way, and appropriate feedback warns the student that this is happening: "You are about to lose two solutions. Keep in mind that the expression also yields two solutions. Please complete". This feedback is along the lines of 'feedback about the task' (FT). The second feedback condition c2 is essentially the same as c1, but additionally provides several buttons on the screen that could be used for getting hints and solutions of the exercises.

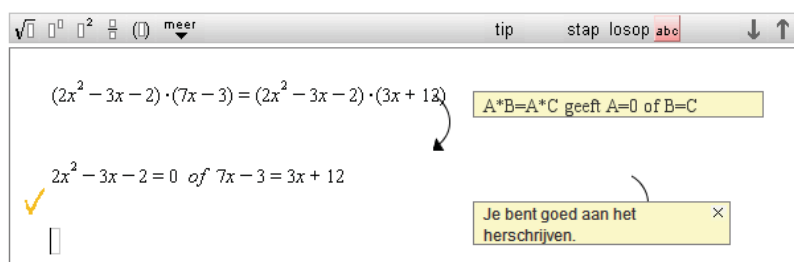


Figure 3: Feedback condition c2, including buttons




Figure 3 shows these buttons for (i) tip, which provides a hint for the next step, (ii) stap, which provides the next step in the solution process, and (iii) los op, which solves the whole equation and thus provides a 'worked example' (Sweller & Cooper, 1985). These buttons can be used by the student at will, providing self-regulatory tools (feedback type FR). In the case of figure the student used the 'stap' button to obtain the next step in the solution process, and feedback "A\*B=A\*C gives A=0 or B=C". After deploying the intervention we will look at scores, attempts and case examples to formulate an answer to our research question.

## Results

First the quantitative findings will be presented and then these will be illustrated by some case examples. As a Kolmogorov-Smirnov test shows that both d1 and d3 scores are not distributed normally ( $Z=5.408$ ,  $p<0.001$ ;  $Z=6.768$ ,  $p<0.001$  respectively), we apply a non-parametric Mann-Whitney test. This test shows that there is a significant difference between the feedback conditions when we look at the score for d1 ( $U=7680.00$ ,  $p<0.001$ ,  $r=-.321$ ) with condition c2 scoring higher than condition c1. According to Cohen (1992) this accounts for a medium effect size. The second series of tasks d3, however, did not show a significant difference ( $U=10560.00$ ,  $p=.531$ )

When considering the number of attempts, this was significantly higher for feedback condition c1 ( $Mdn=126.00$ ) (without extra buttons) than for feedback condition c2 ( $Mdn=105.00$ ),  $U=9904.50$ ,

$p < .001$ ,  $r = .202$ . Although we can classify this as just a small effect, it suggests that the addition of buttons for hints and solutions results in fewer attempts.

Apart from these quantitative results there also is a substantial body of case examples where students have successfully or unsuccessfully used feedback options. We now illustrate the use of IDEAS feedback with three examples from both feedback conditions. The examples show the subsequent steps of a student when confronted with a task (step 0), and the accompanying feedback. Note that feedback was translated from Dutch into English for the purpose of this paper, and that randomization of the tasks means that students received different equations to solve. The symbols ,  and  designate incorrect answers, correct but not final answers and correct answers, respectively.





Step	Student		Feedback
0	Solve:		
1			You are rewriting correctly
2			Hint: rewrite to [expression]=0
3			You are rewriting correctly
4			You have solved the equation correctly

Table 1: Feedback example from condition c1 (stepwise feedback)

In the example in table 1 the student starts of by expanding the left and right hand side of the equation. As there are no feedback-buttons available the system evaluates the expression in step 1 as correct (but not the final solution). The student makes a calculation error in rewriting in the form [expression]=0. Now the system hints that the expression is incorrect and gives feedback. The difficulty of judging what mistake was made instantly becomes apparent: in this case the student already understood he/she should rewrite to [expression]=0 but makes a calculation error. In step 3 this error is corrected. The student then remembers to apply the Quadratic formula and solves the equation correctly. We see that the system does still have difficulties with judging what format of the expression is “good enough” and what isn’t. The evaluation is quite ‘liberal’, giving the notation in step 4 full marks.

The second example in table 2 shows that the addition of buttons helps a student to overcome initial difficulties with the given equation.





Step	Student		Feedback
0	Solve:		Hint: $AB=AC \Rightarrow A=0$ or $B=C$
1			
2			You are rewriting correctly
3			This is not quite the exact format
4			You have solved the equation correctly

Table 2: Feedback example from condition c2 (hints)

After being given a new equation the student uses the ‘tip’ button to get a hint. The student uses the hint to apply the correct strategy, first making a notational error, but correcting this in step 2. After this the student concludes the task in steps 3 and 4. In step 3 the system prompts that the expressions can be simplified.

The third and last example concerns a student that uses the 'los op' button to automatically solve the given tasks, with the system adding the strategies as feedback for every step in the solution process.

Step	Student		Feedback
0	Solve:		$AB=AC \Rightarrow A=0$ or $B=C$
1		✓	Rewrite in form [expression]=0
2		✓	Move constants to the right
3		✓	Free up variable by dividing on both sides
4		✓	Simplify by factoring
5		✓	Use quadratic formula
6		✓	Simplify roots
7		🚩	You have solved the equation correctly

Table 3: Feedback example from condition c2 (solve)

This example was added because it shows the difficulties of evaluating student answers. Most solutions from students combined several steps into one. For example, steps 2, 3 and 4 could easily be combined in one step. The notational issue mentioned earlier also crops up: many teachers would perhaps have given full marks for the solution in step 6, but because the square root *can* be simplified this is not seen as the final solution.

## Conclusion

In this paper we set out to see whether variation in feedback type influences scores, number of attempts and student behaviour. When observing the scores students obtained in the two conditions, we can see that there is a medium effect for the feedback condition including self-regulatory feedback (condition c2). This effect, however, only was apparent in one of the series of tasks. We think we can explain the difference between d1 and d3 in the fact that both series address the category of polynomial equations (Bokhove & Drijvers, 2010). Having solved polynomial equations with feedback in d1 meant that similar types of equations (in d3, which followed after d1) could already be solved, and subsequently no additional form of self-regulation was needed any more in d3. The addition of buttons for feedback also had a, albeit small, effect on the number of attempts. This makes sense as the additional feedback that can be requested discourages more attempts. When looking in more detail at the use of the feedback in the three case examples it is clear that both task-related and self-regulatory (FT and FR) feedback can be used in a formative way *for* the learning of algebra. Students can use the feedback to overcome difficulties and check whether they are on the correct solution path or not.

However, a word of caution is needed. Although research suggests that worked examples are effective (Sweller & Cooper, 1985) students could easily be tempted to 'just push the button' (this is an actual statement from a student) to get full marks. To address this pitfall, a design principle involving fading (Renkl, Atkinson, Maier & Staley, 2002) and formative scenarios (Bokhove, 2008), whereby the amount feedback and worked examples are decreased during the course of the intervention, was applied to the intervention. This approach shows promising results which, however, lie outside of the scope of this paper. The same holds for a third design principle involving crises: non-standard tasks that can't be solved with the standard algorithms. The idea is that these tasks force students to think 'out of the box'. The role of feedback is to provide enough support to students so they won't just give up when confronted with such a crisis.

Although the results of using custom and IDEAS feedback for algebraic expertise are promising, there still are many improvements to be made. These improvements should firstly focus on notational aspects as shown in the case examples. Student motivation declines when they do not get full marks just because the system says so. Secondly feedback should be adapted to the target audience and math curriculum. Clearly, the mathematical language of higher education is different from that of secondary education. It is not a viable option to let all teachers author their own set of feedback comments. One goal in the near future will be to try and provide default values for feedback that applies to the most common student errors and behaviours, resulting in feedback 'out-of-the-box'. It is imperative that the appropriateness and quality of 'intelligent' feedback is improved before we can reap the benefits.

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