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Published in IET Science, Measurement and Technology
Received on 27th May 2011
Revised on 12th October 2011
doi: 10.1049/iet-smt.2011.0111

Special Issue on Computation in Electromagnetics
containing selected, extended papers from CEM 2011

ISSN 1751-8822

Numerical investigation of Fano resonances in metamaterials with electric asymmetry

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Abstract: The excitation of high-quality factor asymmetric Fano-type resonances on a double-layer metafilm structure is investigated through numerical simulation. The study demonstrates that it is possible to design simple structures capable to sustain a very high-quality factor resonance by reducing their radiation losses. An equivalent circuit formed by two linearly coupled resonant RLC circuits is extracted in an attempt to explain the observed Fano resonance through classical circuit theory.

1 Introduction

Electromagnetic metamaterials and their potential applications have attracted significant attention from the scientific community in recent years. The research is driven by the opportunity to achieve electromagnetic properties that lead to new phenomena such as negative refraction, perfect magnetic surface or cloaking [1], which have no equivalents in natural world. Most of such 'exotic' electromagnetic response of metamaterials is because of their resonant behaviour; hence achieving a high-quality factor resonance is essential to obtain an efficient metamaterial-based device. Typically, a metamaterial cell comprises a metallic structure – in which currents can be induced – and a supporting dielectric substrate. Depending on the induced current distribution in the conducting region, different resonance modes can be excited within these cells. Most of these resonant structures do not have a very high resonant quality factor because they suffer significant radiation losses. Moreover, because of their relatively small size with respect to the wavelength of operation, the metamaterial cell structures are not able to provide a large enough volume for the confinement of electromagnetic field necessary to support high-quality factor resonances. However, recent theoretical analysis and experimental results have shown that high-quality factor resonances can exist on small structures. These are mainly because of the existence of asymmetric profile resonances which belong to a larger class labelled as Fano resonances. The asymmetry is caused as a result of the close coexistence of resonant transmission and resonant reflection. Fano resonance is usually related to quantum systems and was first observed in 1961 by Ugo Fano in a quantum mechanical study of the auto-ionising states of atoms. This special type of resonance arises from the constructive and destructive interference of narrow discrete resonance with a broad spectral line. Fano resonances have been observed in different quantum systems such as quantum dots, nanowires and tunnel junctions [2]. Recently, such resonances have also been observed in a number of classical nanoscale oscillators and in several metamaterial films [2–5]. Joe *et al.* [6] have shown that Fano-like resonances exist in any weakly coupled harmonic oscillator, where one of them is driven by a periodic external force. In such coupled systems there are two resonances located close to the eigenmodes of the oscillators. The first resonance has a symmetric profile whereas the second resonance is asymmetric. This odd behaviour is due to the coupling between the oscillators. Initially, the two oscillators have the same phase with the external force. As the frequency of the external force approaches the eigenfrequency of one of the oscillators, it will reach its resonance and its phase will exhibit a π jump. Hence, the oscillator will be out of phase with respect to the driving force. However, the other oscillator will still have the phase of the external force. At the second resonance (eigenmode) there are effectively two forces acting on the first oscillator which are out of phase and cancel each other. The example analysed in [6] demonstrates the resonant destructive interference which is a unique property of Fano resonance.

Previous work has shown that a certain small structural (geometrical) asymmetry has to be introduced to the shape of their conducting elements to excite such resonances [5, 7]. However – as shown in [6] – the necessary conditions to obtain such destructive resonances are close eigenresonances for the two oscillators and a weak coupling existing between the two oscillators. Hence, a geometrical asymmetry of the two oscillators is needed to produce two close-enough eigenfrequencies, whereas the weak coupling is realised by placing the two oscillators physically close enough so that their electromagnetic fields can interact with each other. Based on these observations in our previous work [8] we have shown that similar effects can be achieved without breaking the geometrical symmetry of the resonator but instead introducing an electrical asymmetry through a combination of substrates with different dielectric properties, which is an alternative way of changing the eigenfrequency of the resonators. Making such structures is not easy as different dielectric materials have to be joined and assembled in a film or on a board on which the metal structures of the metamaterial cell are then deposited.

In this paper, the idea of electric asymmetry is expanded and a novel type of metafilm is proposed to simplify the fabrication process. The new approach introduces an extra degree of freedom in the design of the metamaterial unit cell that offers further possibilities in tuning and improving the device.

2 Electrical asymmetric resonator

The metafilm considered in this work consists of identical sub-wavelength metallic inclusions on a dielectric substrate that are arranged in a periodic array. The metallic structures are symmetrically split rings with one-half of the structure being on the top layer of the substrate and the second half on the second layer, as shown in Figs. 1*a* and *b*.

The metallic split rings are assumed to be made of copper with a thickness of 35 μm. The outer radius (r_2) of the ring is 6 mm and the inner radius (r_1) is 5.2 mm. The size of the fundamental cell that is repeated throughout this metafilm is $w \times w = 15 \times 15$ mm. This array is considered in our numerical study as infinitely long in x and y directions. The dielectric substrate is FR4 with the permittivity $\varepsilon_{FR4} = 4.07 + j0.05$. The total thickness of the substrate is $d_1 + d_2 = 3$ mm, with the second layer at a distance of $d_1 = 1.5$ mm from the top surface (Fig. 1*b*). This structure has an inherent effective dielectric permittivity difference between the top and middle layers. Whereas the middle layer will have the effective dielectric permittivity equal to 4.07, which is the FR4's value, the top layer has an effective value lower than that because it is a combination of the FR4's value and $\varepsilon_{air} = 1$, the value for air which is in direct contact with the top layer. Owing to this difference, the two equal metal arcs will have effectively different electrical lengths.

Transmission and reflection of this thin metamaterial was obtained from electromagnetic simulation under normal incidence conditions (Fig. 1*b*).

3 Results and discussions

The metamaterial structure was modelled using commercial software (CST Microwave Studio) with periodic boundary conditions applied to simulate an infinite array. The normal incident waves were forced through the definition of Floquet ports on the top and bottom boundaries of the 3D model (in the z-direction – Fig. 1*b* – not shown). The solution used a frequency sweep between 3 and 12 GHz. The transmission and reflection properties of the structures of Figs. 1*a* and *b* depend strongly on the polarisation of the incident electromagnetic waves. For this configuration a Fano-like resonance response is observed when the electric field of the incident electromagnetic wave is polarised in the y-direction (Fig. 3). The analogy to two weakly coupled oscillators may be helpful here to explain the results obtained. We can consider the two metallic arcs as two oscillators and the incident wave as an external periodic driving force. The eigenfrequencies of the two oscillators are related to their dipole resonance hence the middle arc will resonate first as it has a longer electrical length, being embedded in the FR4 substrate. The top layer arc, being electrically shorter, will resonate at a slightly higher frequency. For the structure shown in Figs. 1*a* and *b* with $\alpha = 30^\circ$ these resonant frequencies are 5.35 and 6.32 GHz, respectively. It is important to note that these values are affected by the thickness of the substrate on which the metal structures are deposited and the values mentioned above were obtained for the thickness given in the previous section. However, when these two arcs are coupled, as they are in reality, these two resonant frequencies are different (Figs. 2 and 3).

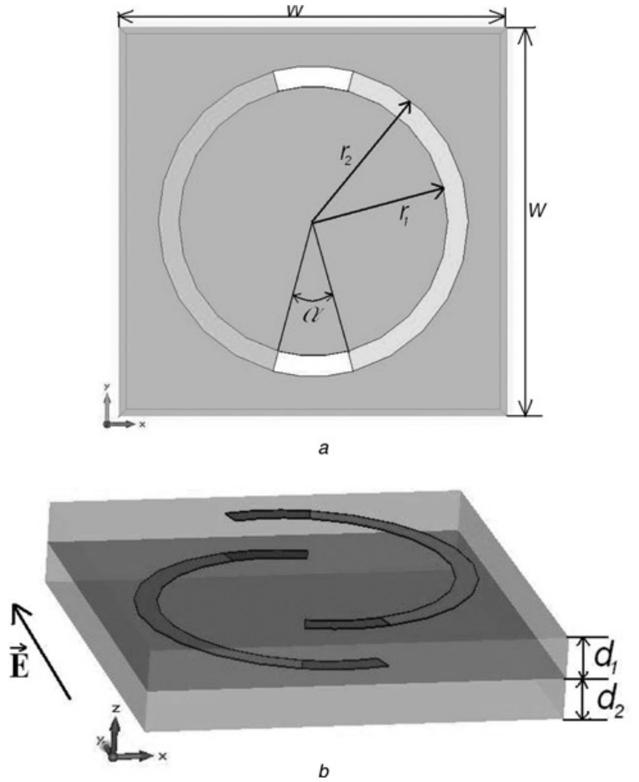

Fig. 1 *Metafilm and the fundamental cell*

a 2D view
b 3D view

As long as the frequency of the incident wave (external driving force) is below the first resonant frequency, the structure does not interact with the incoming wave as induced currents on the metallic arc will be in phase with this wave. Situation changes dramatically when the frequency of the incident wave reaches the dipole resonant frequency of the middle arc. At this point the amplitude of the current on the middle arc is much larger than the amplitude of the current induced on the top arc and it will exhibit a π phase change with respect to the exciting wave; thus, the wave excited by this current will oppose the incoming wave. This will result in a strong reflection from the surface of the metafilm as shown in Figs. 3*a* and *b* (point 1). As the frequency of the incoming wave is further increased and approaches the dipole resonant frequency of the top layer arc, the amplitude of the induced current on this arc increases. However, the current induced in the middle arc is 180° out of phase with the current induced into the top arc; thus, there is a point where the two currents have almost the same amplitude but opposite phases.

At this frequency the fields produced by the two induced currents cancel each other; therefore the incident wave will not be disturbed and it will pass through the metafilm. Point 2 in Figs. 3*a* and *b* illustrates the fact that the incoming wave is not reflected by the metafilm (low value of reflection and high value of transmission). By increasing further the frequency of the external wave the resonant frequency of the top layer arc will be reached; hence the induced current on this arc will reach its maximum amplitude and it will change phase by 180°. The wave excited by this current will now interact with the incoming

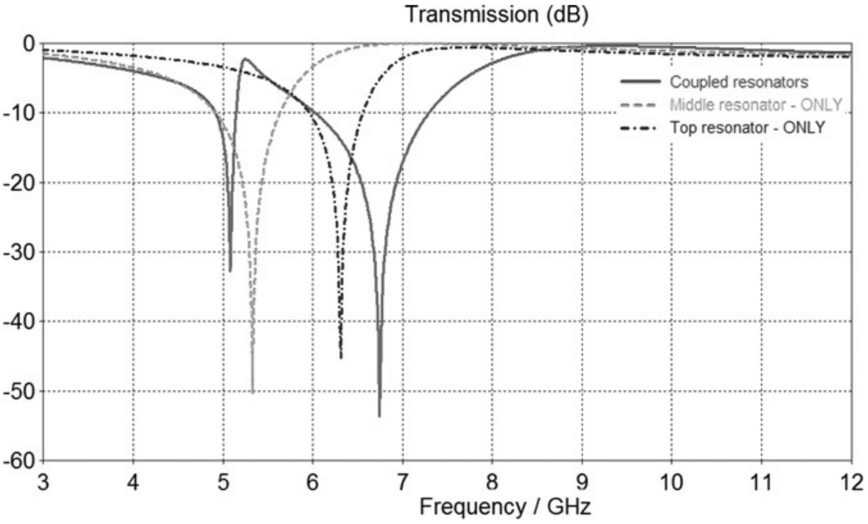

Fig. 2 *Transmission through the metafilm for single-resonator cases and coupled resonator situation*

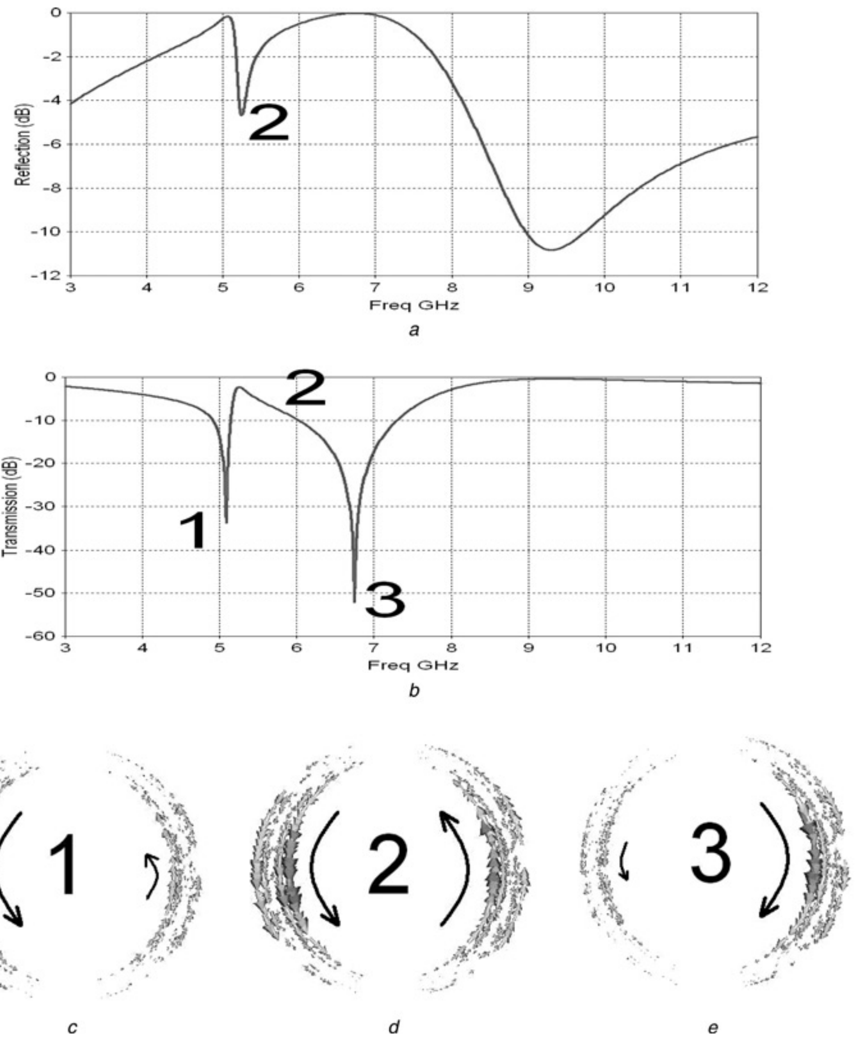

Fig. 3 *Reflection and transmission of the metafilm and the induced current on the two metallic arcs for the three resonant frequencies*

a Reflection
b Transmission
c Current distribution at 5.1 GHz
d Current distribution at 5.31 GHz
e Current distribution at 6.78 GHz

doi: 10.1049/iet-smt.2011.0111

wave and because of its phase it will create again a strong reflection as shown by point 3 in Fig. 3.

A relatively small electrical asymmetry between the two coupled oscillators is needed to produce the Fano-like resonance described above. This asymmetry is necessary to create a narrow frequency domain in which the currents existing on the metal structure of the cell are 180° out of phase and they have large and almost equal amplitudes. This asymmetry can be introduced by changing the physical lengths of the two arcs as shown in [3]; however – for such single-layer structures – the coupling of the two oscillators cannot be altered. The novel two-layered structure presented in this work does not have this limitation as the position of the two arcs can be moved relative to each other and thus the coupling can be changed. The thickness of the substrate can be varied as well, which will also affect the coupling between the two oscillators. To study the above-mentioned effects a series of full-wave models have been set up and solved using similar settings as explained earlier in this section.

In the first set of models, the relative distance between the two arcs has been modified and transmission through the metafilm has been observed. Fig. 4 depicts the fundamental cell for different distances between the two arcs. In Fig. 4*a*, the displacement is taken as zero, this being the initial situation.

The top layer arc is then shifted to the left whereas the middle layer arc is shifted to the right by same distance. The arcs are shifted in 0.5 mm steps from 0 to 5.5 mm. Figs. 4*b* and *c* illustrate a 2.5 and 5 mm shifts, respectively. The frequency response in terms of transmission through the metafilm for different positions of the arcs is presented in Fig. 5.

These numerical results show that as the two arcs are moved, the first and third resonances are shifted in opposite directions. The first resonance drops down to 4.375 GHz from its initial value of 5.1 GHz following a shift of 3 mm from the original position, whereas the third resonance moves upwards to 7.54 GHz for a 2.5 mm shift of the arcs. Subsequently, once the shift is 3.5 mm or more (up to 5.5 mm), the two resonances start to move towards each other. Hence, at a shift of 5.5 mm the first resonance is now at 5.4 GHz whereas the third resonance is at its minimum value of 6.31 GHz. The second resonance – for which maximum transmission through the metafilm is achieved – behaves in a similar fashion, initially shifts towards a minimum value, which is reached at a displacement of 3 mm, and after this point starts moving upwards up to 5.75 GHz, which is its maximum value. At this point the shift is 5.5 mm. The quality factor of this resonance was defined as

$$Q = \frac{f_{rez}}{BW_{3\,dB}} \tag{1}$$

where f_{rez} is the resonant frequency, or the frequency at which maximum transmission is observed (peak), and BW_{3dB} is the

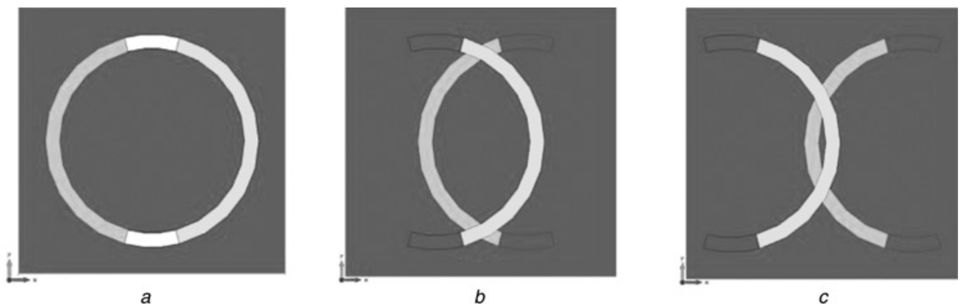

Fig. 4 *Fundamental cell structure of the metafilm for different shifts*

a 0 mm
b 2.5 mm
c 5 mm

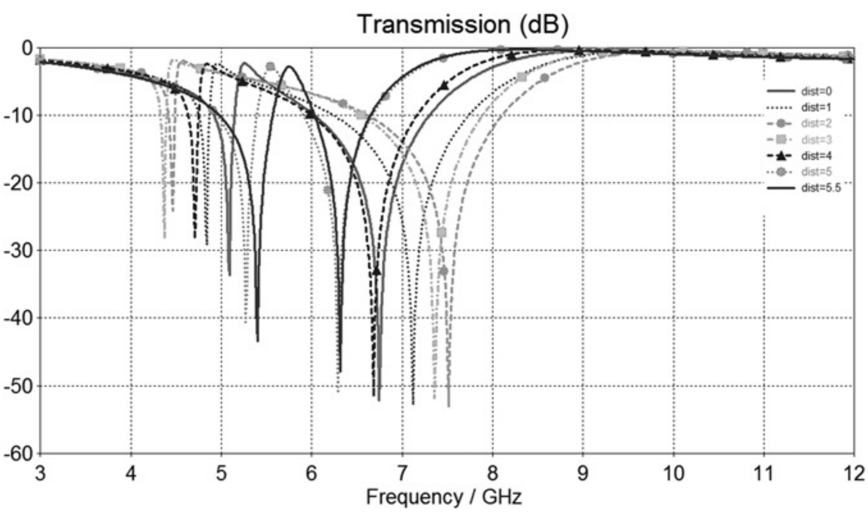

Fig. 5 *Transmission through the metafilm for different shift positions of the arcs*

doi: 10.1049/iet-smt.2011.0111

3 dB bandwidth. The quality factor has been calculated for two situations: a lossless case when the substrate was assumed to have permittivity $\varepsilon_{FR4} = 4.07$, and a set-up in which the loss of the substrate was taken into account by using a complex value of the permittivity $\varepsilon_{FR4} = 4.07 + j0.05$. In the microwave range, considered in this study, the main loss mechanism is because of the dielectric losses within the substrate. It was therefore important to include the dielectric losses in our simulation to understand their effects over the quality factor of the resonance. However, if the structures were scaled to make them work at terahertz frequencies the losses in the metal structures would dominate; therefore structures with less conductive parts would perform better in terms of the quality factor.

It is interesting to observe the variation of the quality factor of this resonance with respect to the shift of the two arcs. For the lossless case, as the arcs are shifted, the quality factor initially drops until it reaches a minimum value of about 4 at the 2.5 mm shift; afterwards it starts increasing and reaches a maximum value of 22 at 5.5 mm (Fig. 6). Similar behaviour is observed if the substrate's loss is considered in calculations. Moreover, all the quality factor values are below the values obtained for the lossless case. The maximum is now 18.5, reached at a shift of 5.5 mm (Fig. 6). This behaviour is not obvious and would be difficult to predict without the help of the full-wave simulation.

The second parameter that was considered in this study is the total thickness of the substrate $(d_1 + d_2)$. The thickness was varied in 0.1 mm steps from 2 mm down to 0.2 mm and the effects on the quality factor and position of the resonances are presented in Figs. 7 and 8, respectively.

It can be noticed that the quality factor has a clear maximum of about 38 for the thickness of 0.5 mm for the lossless case, whereas when the dielectric loss is taken into account this maximum drops to about 14.5 for a total thickness of 0.7 mm. All three resonances shift towards higher values as the total thickness of substrate reduces; however, the shift of the third resonance is slightly bigger than that of the first resonance, which is to be expected as the effective permittivity of the top layer drops faster.

Finally, the effect of the top layer thickness variation was evaluated. For these models the thickness d_2 of the bottom layer was fixed at 1.5 mm and the thickness of the top layer, d_1, was reduced from 1.5 to 0.2 mm in steps of 0.1 mm. The computed quality factor for the lossy substrate showed a monotonic increase with the decrease in the thickness of the top layer down to 0.4 mm. For the 0.3 and 0.2 mm-thick substrates the quality factor drops slightly. However, for the lossless case the quality factor increases as the thickness reduces (Fig. 9). A similar behaviour has been observed in our previous work [8] when the

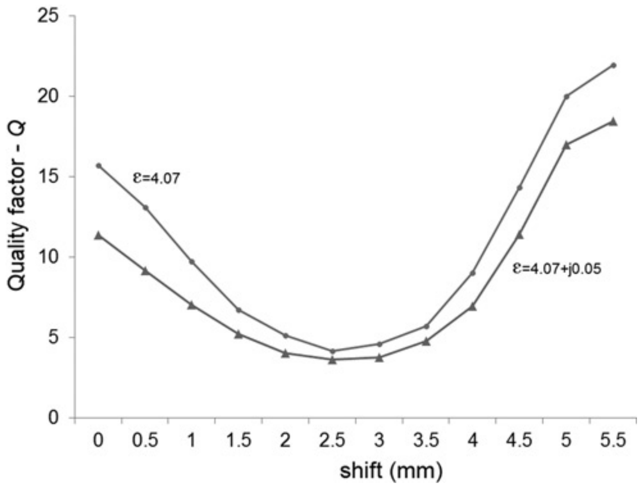

Fig. 6 *Quality factor variation with the shift parameter*

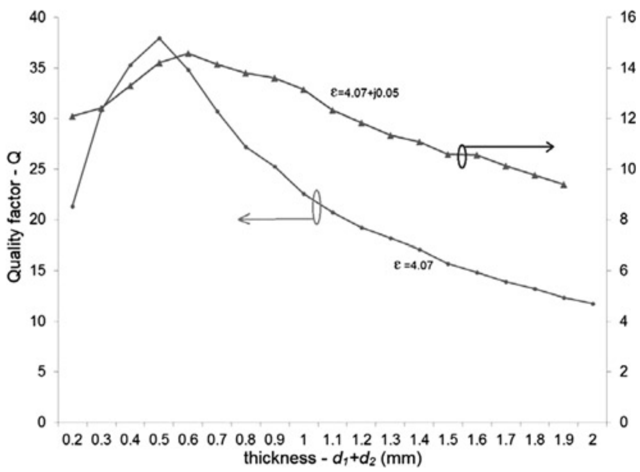

Fig. 7 *Quality factor changes with film thickness*

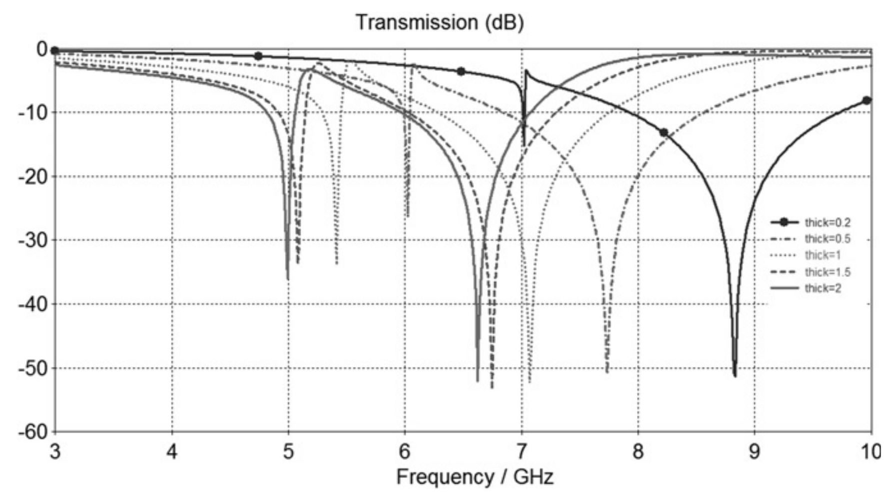

Fig. 8 *Transmission through the metafilm for different thicknesses of the substrate*

IET Sci. Meas. Technol., 2012, Vol. 6, Iss. 5, pp. 386–393
doi: 10.1049/iet-smt.2011.0111

asymmetry between the two oscillators was varied. In the present case, by reducing the thickness of the top layer the electrical asymmetry between the oscillators is reduced; hence the quality factor of the second resonance increases. If the thickness of the top layer were to become zero there would be no electrical asymmetry, hence the two resonances would coincide and the second resonance – that is because of the destructive interference – would not exist.

An equivalent circuit of the unit cell of the metamaterial was extracted in an attempt to analyse and explain further the Fano resonances observed on these structures. As mentioned in the introduction, Joe *et al.* [6] have used two weakly coupled mechanical oscillators to show the existence of resonant destructive interferences. Similarly, two coupled electrical resonant circuits could be used to illustrate the same phenomena. The circuit that mimics the behaviour observed in our full-wave simulation is shown in Fig. 10. It consists of two resonant RLC loops coupled through a capacitor and a mutual inductor. L1 and the C2 represent the self-inductance and self-capacitance of the top layer arc. R1 accounts for the losses within the top layer. Similarly, the middle layer is described by the inductor L2,

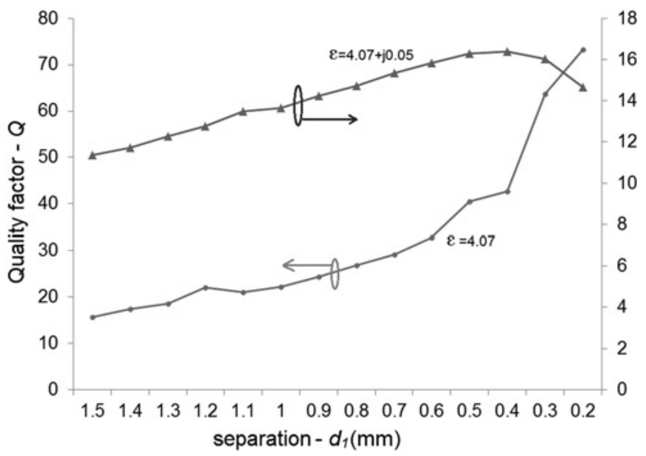

Fig. 9 *Effect of top layer thickness on the quality factor*

capacitance C3 and resistance R2. The capacitor C1 models the coupling through electric field between the two resonators, while the Mutual1 represents the magnetic coupling between the two layers. This circuit was build and simulated in frequency domain using ADS (Advanced Design System circuit simulator). The two transmission lines TL1 and TL2 connected between terminal 1 and terminal 2 are ideal transmission lines and they model the fact that the ports of the exciting wave in the full-wave model are 20 mm away from the metamaterial film. Therefore there is a phase change between the source and surface of the film that has to be accounted for in the equivalent circuit.

The values of the components of the equivalent circuit were extracted using a quasi-static solver. The coupling between different parts of the two-layered structure film needed extra attention as the effects of the neighbour cells has to be taken into account in the equivalent circuit. The values obtained for the geometry described in detail in Section 2 are presented in Table 1. It can be noticed that the coupling capacitance and inductance change with the position of the two arcs, which is to be expected; however, the explanation for the actual values and their variation with the shift is not straightforward. The variation of the capacitance C1 increases with the shift, which intuitively should be expected as the two arcs are getting closer and above each other (Fig. 4). However, the value of C1 has to be computed taking into account the position of the arcs with respect to their neighbours as this will affect the total value of C1. The effect of the position of the neighbouring cells may be seen more clearly in the magnetic coupling variation. For a zero shift the two arcs from the same cell are relatively far apart from each other (the distance between the arcs is 10.2 mm – see Fig. 1), but they are much closer to the arcs in the

Table 1 Values of the components of the equivalent circuit

L1 = L2 = 1.4 nH	0 mm shift	2.5 mm shift	5 mm shift
C2 = 1.223 pF	C1 = 0.755 pF	C1 = 0.955 pF	C1 = 1.165 pF
C3 = 0.741 pF	M1 = 0.535 nH	M1 = 0.0305 nH	M1 = 0.522 nH

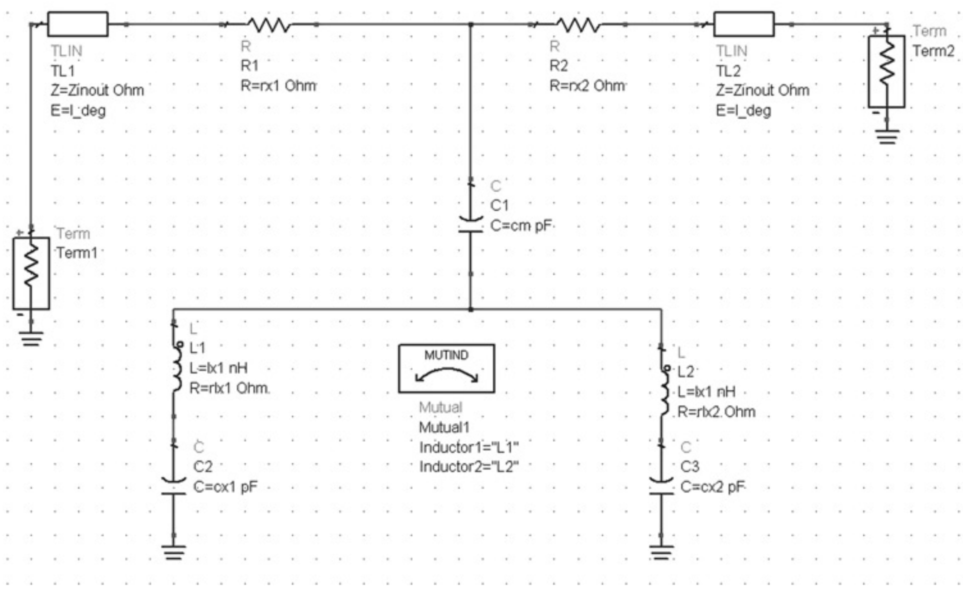

Fig. 10 *Equivalent circuit used in this study*

neighbouring cells (only 3 mm apart); hence the rather large value of mutual coupling.

As the shift is increased the two arcs will get closer but they will also move further from their neighbours and thus the coupling will drop until it reaches a minimum value when the distance between the arcs in the same cell and the distance to the neighbour cell are equal. After this point, as the shift increases further, the mutual coupling is now dominated by the coupling of the arcs in the same cell and increases with the shift (Table 1).

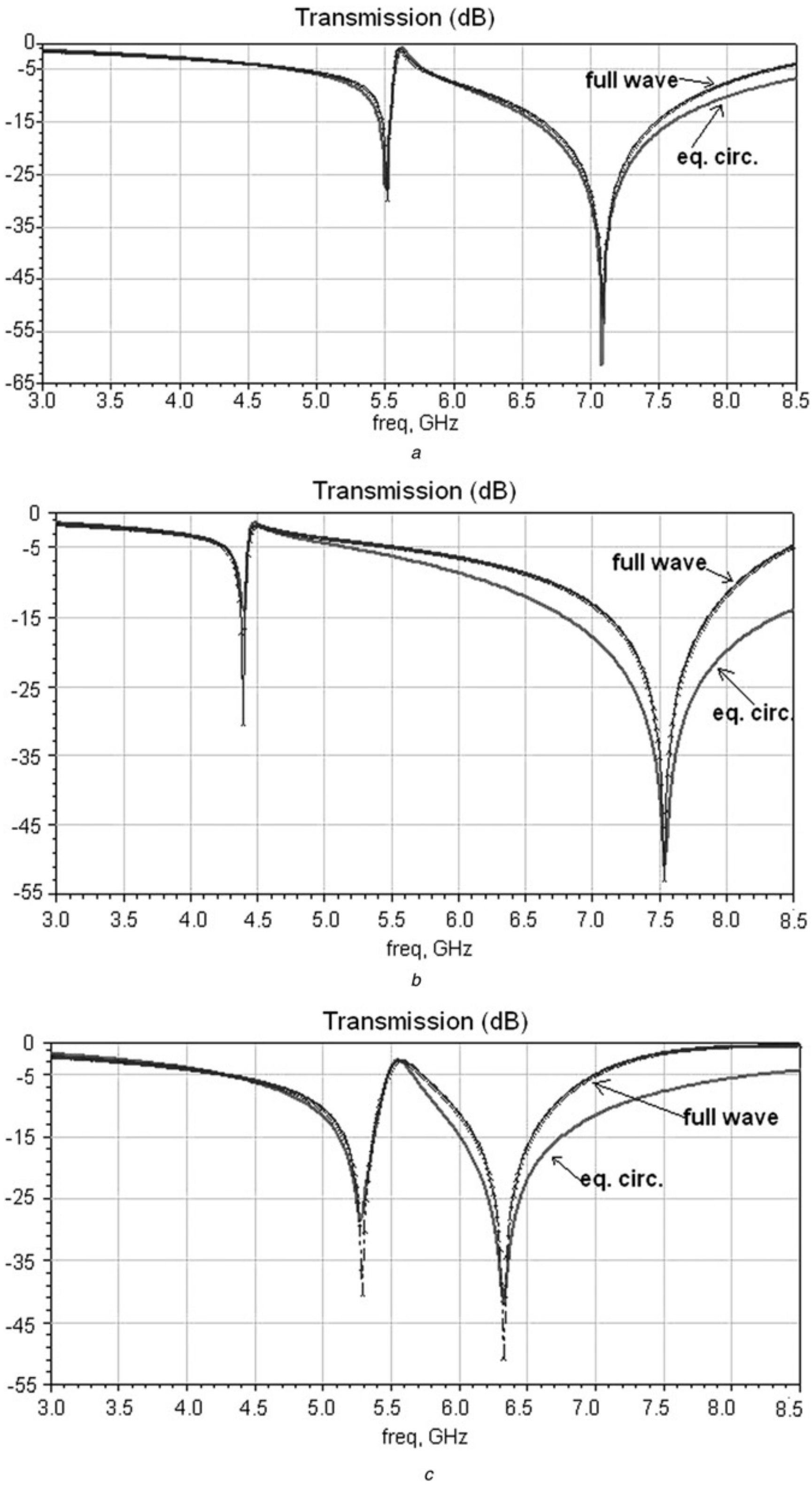

Fig. 11 *Comparison of transmission through the metafilm response*

a 0 mm
b 2.5 mm
c 5 mm

doi: 10.1049/iet-smt.2011.0111

The equivalent circuit results were compared with the full-wave simulation results in terms of the transmission coefficient and good agreement was obtained as shown in Fig. 11.

4 Conclusions

A novel two-layered metamaterial surface has been introduced. The metafilm can sustain very high-quality resonances because of the electrical asymmetry created by the different dielectric loading of the two metallic parts. The existence of the Fano-type resonance is demonstrated through numerical simulation. The main advantage of the approach proposed here is the ease of fabrication of such structures as compared with the previous electrical asymmetric structures that needed two different dielectric materials to be combined in the same substrate. The Fano resonance observed in these structures can be explained and modelled by the coupling of two oscillators. This has been shown by extracting an equivalent circuit model with two coupled RLC resonating circuits.

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