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Implications of failure criteria choices on the rapid concept design of composite grillage structures using multiobjective optimisation

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Abstract Grillage topologies are commonly used in many composite structural applications to produce low mass designs that have a high stiffness. While composite failure criteria are being compared in many different simple structures, for example plates and tubes, literature must also compare more complicated applications, including grillages, as there are distinct differences in behaviour. This paper therefore performs analysis of grillage structures with more up to date failure criteria, taken from the world wide failure exercise, than previously investigated. The grillage theory selected is that of Navier theory with elastic equivalent properties due to its low computational expense for use with a genetic algorithm to optimise a composite structure. The results take an example from leisure boatbuilding showing the grillages produced from the different limit states, comparing the cost and mass. The final results show that the method allows a rapid analysis of grillages and that the selection of the limit state has an important effect on the optimised grillage topology.

Keywords Failure Criterion · Rapid Structural Analysis · Grillage Theory

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NOMENCLATURE

1 Introduction

Composite materials are used within a large number of products in many industries from aerospace through to automotive and boatbuilding among many others. They are generally utilised for their ability to be created with properties specific to the task required. They exhibit high strength to weight ratio, excellent corrosion resistance and a large design freedom. To model the constraints of the material properties on the composite a number of different failure criteria are used. These failure criteria have been applied to different simple structural applications from tubes to panels. Many of these criteria are being investigated through the work being performed by the World Wide Failure Exercise(WWFE) and are collated in [**?**] among other examples.

Counter to the many benefits composite materials exhibit they also suffer from a relatively low modulus and therefore there is a requirement for stiffeners to be utilised within the structure. Composite structures are often stiffened using a tophat approach, an application of a grillage of stiffeners is shown in the top left part of Figure 1. The top right part of Figure 1 shows the idealised representation of this grillage and at the bottom an idealisation of the stiffener geometery, which provides necessary flexural and axial stiffness to the structure.

Optimisation is often an approach that is taken to investigate how the different structures behave given a specific condition. There are many available papers discussing optimisation and more specifically genetic algorithms to solve more complex composite design problems. Genetic algorithms have been used over gradient based methods as this allowed a large, complex search space to be investigated without priming the result based on previous knowledge. Furthermore genetic algorithms have a unique ability to deal with alpha-numeric fields. A selection of literature for a range of applications follows. Satheesh et al. [?] looked at the use of multiple failure criteria with genetic algorithms for design optimisation of laminated plates. Kim and Kim[?] looked at the optimal design of stiffened panels from buckling. Kang and Kim [?] looked at minimum weight design of compressively loaded composite plates using nonlinear finite element analysis. Naik et al.[?] used a genetic algorithm to look at maximum stress and Tsai-Wu failure criteria for minimum weight design of composite plates. Lopez et al. [?] investigated the use of optimisation for composites considering maximum stress, Tsai-Wu and Puck failure criteria on a laminated plate. While this research has concentrated on composite structural optimisation and the use of more stringent failure criteria none have looked at the structural response of grillages.

To find the mechanical response of a grillage a number of theories can be used based on beam theories defined by Clarkson [?] which include orthotropic plate method and folded plate method. While these theories may lack some of the complexities of modern techniques they are rapid to solve and useful within highly computationally expensive algorithms, for example genetic algorithms. Previous research has been performed that has investigated using these different grillage theories on composite structures including Nagendra [?], Eksik [?] and Maneepan [?], but these have concentrated on more basic failure criteria. Whilst grillage analysis is an important method used for analysis of top-hat stiffened composite structures, the analysis of up to date failure criteria for these structures has not before been introduced and the effects of these criteria on the optimised structure must be investigated to determine the effectiveness of the method.

Research has been performed on the effects of complex failure criteria on simple structures but it is important to understand how these criteria affect more complex topologies. For the first time this paper investigates the effects of these failure criteria on more complex structures using tophat stiffened plates as an example. This paper determines a method for rapid assessment of composite tophat stiffened grillage panels, commonly used for increased torsional stability, and performs an optimisation on an example taken from boatbuilding. The structural model that has been developed uses Navier grillage with Third Order Shear Deformation theory to ensure that the panels in between the stiffeners do not fail. The optimisations have been performed using constraints taken from those developed for the WWFE, a buckling criterion and a deflection criterion where each has been used as a constraint independently allowing an analysis of the effects of using these as the overall limit state. Results from the analysis show that the Puck failure criterion appears to be the most conservative of the World Wide Failure Exercise but that this state is easier to satisfy than the deflection limit state.

2 Structural Modelling

2.1 Grillage method

The structural analysis is performed on a grillage under the loading condition shown in the top right part of Figure 1.

For the structural modelling of the stiffened plate Navier method grillage analysis has been adopted. This work is originally covered in Vedeler [?] and the has been shown in Maneepan et al. [?] to closely approximate the more accurate methods while being computationally more efficient. Navier theory is based upon the deflection of intersecting points found between longitudinal girders and transverse beamswith a pressure applied to the panels from the opposite side of the plate to the stiffeners, as shown in the top right part of Figure 1. From these deflections it is possible to determine the stresses within the stiffeners. This method has been used for many years and is combined with elastic stress analysis as covered in Datoo [?].

The grillage analysis uses the Navier summations of points within the grillage to develop the deflection of the stiffeners. This methodology has been performed based on a panel under simply supported boundary conditions as this will allow a conservative estimate of the stresses and deflections. The equation giving deflection of the stiffened plate is assumed to be

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \tag{1}$$

where the value of a_{mn} is a coefficient found from Eq.2. The coefficient a_{mn} is found based on the assumption that the change in potential energy from a small deflection will be a minimum. The coefficient a_{mn} is dependent on the flexural rigidities of the stiffeners ($D_{g,b}$).

$$a_{mn} = \frac{16PLB}{\pi^6 mn \left\{ m^4 (g+1) \frac{D_g}{L^3} + n^4 (b+1) \frac{D_b}{B^3} \right\}} \tag{2}$$

The moments can be found in the beams or girders ($M_{g,b}$) from Eq.3:

$$M_{g,b} = -D_{g,b} \frac{\partial^2 w}{\partial x^2} \tag{3}$$

Finally using the maximum moments in the grillage the maximum stress σ_{max} can be determined, where $E_{s(i)}$ is the Young's modulus of the element of a stiffener, either girder or beam, M_s is the moment created in the stiffener, $Z_s is$ the vertical distance of the centroid of an element to the neutral axis and D_s is the structural rigidity of a stiffener:

$$\sigma_{max} = \frac{E_{s(i)} M_s Z_s}{D_s} \tag{4}$$

The tophat stiffeners are idealised as shown in the bottom part of Figure 1 with each stiffener being made up of 4 elements labelled 1 to 4 where elements 1 to 3 are attached to a large base plate forming element 4. Each of these elements is made up of a number of different plies.

A grillage panel is then constructed of a number of these stiffeners on top of a flat base plate. The flexural rigidity can be found using these elastic equivalent properties method which can be found in Datoo [?]. The flexural rigidity can then be used to determine the stresses in the stiffeners using the Navier grillage method.

2.2 Third Order Shear Deformation Theory

The grillage method that has been used finds the maximum stresses in the stiffeners by assuming that the entire load is passed through to the stiffening members. It is also important to make sure that the plate of the hull is thick enough to withstand the expected loads between each of the stiffeners. This can be done computationally easily using classical laminate plate theory and first order shear deformation theory for thin structures with uncomplicated layups. As more complex layups are required it is necessary to use higher order shear deformation theories but these are computationally more expensive. Plate analysis has been calculated using third order shear deformation theory [?] to determine the properties required for the failure criteria as this will allow the full benefits of using different layups in the material to be used.

The forces at each point on the plate, q(x,y), are determined from Eq.5:

$$q(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Q_{mn} \sin \alpha x \sin \beta y \tag{5}$$

where Q_{mn} is the lateral loading on the plate and is given by:

$$Q_{mn}(z) = \frac{4}{LB} \int_0^L \int_0^B q(x, y) \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{B} \, dxdy \tag{6}$$

It is then possible to find the coefficients of the boundary conditions using the stiffness matrix [C] by substituting into the equations of motion the Eqs.5 to 6.

$$[C][\Delta] = \begin{vmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{vmatrix} \qquad [\Delta] = \begin{vmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \end{vmatrix} \tag{7}$$

The stresses and strains then allow the use of failure mechanisms to determine whether a given thickness of plate will fail. The maximum stress and deflection from the plate can therefore be used with the failure criteria to determine whether the plate is safe.

2.3 Failure Criteria

Having determined the stress and deflection within the panel it must be determined whether these values are applicable within a real structure both in terms of the strength of the structure and the serviceability, in this case deflection. Failure criteria have therefore been selected to determine a limit for the dimensions that will survive the given condition. Further to previous work reported by Sobey [**?**] extra failure criteria have been added to the model to more accurately model the behaviour of the composite materials. The failure criteria used came from the 'World Wide Failure Exercise' (WWFE) [**?**], [**?**] and [**?**]. The choice made for each failure type can be seen from Table 1 and was based upon the findings of the World Wide Failure Exercise.

The failure envelopes generated for maximum stress for a given material property are also shown in Figure 2(a) to 2(c).

The exercise concluded that in the case of buckling criteria that they 'did not address the prediction of buckling modes of failure' [**?**]. Buckling is a key part of failure in hull stiffeners and therefore an Euler based rule, seen in equation 8 and 9, where the crown and web are assumed to be taken as clamped at both ends has been used to constrain the model for both the crown and the webs and is taken from [**?**]. This criterion shows that the maximum stress in the web or crown must not increase beyond that defined using the material properties and the geometry.

$$\sigma_{cri,web} = \frac{6.97\pi^2 E_s}{12(1-\upsilon_{12}^2(d_s/c_s)^2)} \tag{8}$$

$$\sigma_{cri,crown} = \frac{6.97\pi^2 E_s}{12(1-\upsilon_{12}^2(a_s/b_s)^2)} \tag{9}$$

Furthermore an arbitrary deflection criterion of 10% of the length has been included to ensure that materials with a low stiffness and cost cannot be selected without creating a thicker topology. These failure criteria had require the inputs of material properties and maximum stresses and deflections provided for the stiffeners by the grillage theory and the panels in between the stiffeners by the TSDT theory reported in subsections 2.1 to 2.2.

2.3.1 Puck Failure Criteria

The Puck failure criterion is based upon 3-D phenomenological models where the development of the method is done through matching current theory to experimental results. The method is a composite laminate theory method which is nonlinear to solve. The Puck method is recommended by the World Wide Failure Exercise to be used for predicting strength of unidirectional laminae and this method has been used as it gives a more conservative view for the failure of the laminates. Puck's formulation is also used for predicting the initial strength of multidirectional laminates as other methods did not predict the failure well. Puck is further recommended to be used to predict final strength of multidirectional laminates. While this method is recommended for most cases there is a requirement to use other criteria at the same time to ensure that a conservative analysis is performed as this criteria is not conservative for all cases.

2.3.2 Zinoviev Failure Criteria

The Zinoviev failure criterion is based on the development of maximum stress theory. This method is based on composite laminate theory and has a linear solution. Zinoviev is recommended by the World Wide Failure Exercise to predict the deformation of laminates along with a non-linear method such as Puck.

2.3.3 Tsai Failure Criteria

The Tsai failure criterion is developed through an interactive progressive quadratic failure criterion. This method is also based on composite laminate theory and is linear in its solution. The Tsai failure criterion is used in conjunction with Puck to determine the response of lamina. The Tsai failure criterion is the best fit to the test data reported in Soden [**?**] for the behaviour of the laminates. This criterion underestimates the failure stress at given points and so the Puck failure criterion can be used to check that failure does not occur.

$$\left(\frac{\sigma_1}{X_T X_C}\right)^2 + \left(\frac{\sigma_2}{Y_T Y_C}\right)^2 + \left(\frac{1}{X_T} - \frac{1}{X_C}\right)\sigma_1 + \left(\frac{1}{Y_T} - \frac{1}{Y_C}\right)\sigma_2 + \left(\frac{2F_1 2\sigma_1\sigma_2}{\sqrt{X_T X_C Y_T Y_C}}\right) + \left(\frac{\tau_{12}}{S_{12}}\right)^2 = 1 \quad (10)$$

3 Optimisation

Due to the nature of modeling structures there is a compromise between many different input variables all of which must be manipulated correctly so that the optimum structure for the design criteria can be created. Design is also a process that will require rapid generation of results and for this reason genetic algorithms have been used due to the large search space available.

3.1 Genetic Algorithms

Genetic algorithms are a multiobjective optimisation method that will allow fast resolution of results while searching a large search space. Genetic algorithms work by copying the process of DNA transfer in living organisms. They then use the process of evolution to find the optimum solution for a given search space.

3.1.1 Introduction

Embedded genetic algorithms have been used in the optimisation process as shown in Figure 3 and developed from [**?**]. The main genetic algorithm is used to optimise the stiffener spacing in the longitudinal and transverse directions, the material type and layup angles. The embedded algorithm is used to determine the optimal geometry of the stiffeners through creation of the crown width and thickness as well as the web height and thickness. This means that the embedded algorithm will develop an optimal stiffener geometry, with respect to mass and cost, for each given stiffener spacing and material property generated in the main algorithm. The main algorithm will then determine the reaction for the total grillage and compare these grillages over many generations until the optimal topology is found.

The constraints for these different properties are shown in Table 4.

This leads to a description of the objective function and constraints in terms of the design variables used

$$\begin{aligned} min\psi(cost, mass) = \\ 1/cost(a_b, b_b, c_b, d_b, e_b, a_g, b_g, c_g, d_g, e_g, \\ N_{plies}, a, b, C_{Material}) \\ +1/mass(a_b, b_b, c_b, d_b, e_b, a_g, b_g, c_g, d_g, e_g, \\ N_{plies}, a, b, \rho_{Material}) \end{aligned} \quad (11)$$

subject to

$$\sigma < \sigma_{limit}, \sigma < \sigma_{cri} and \delta < \delta_{limit} \quad (12)$$

3.1.2 Initial population

The first step in generating a solution for the genetic algorithm is to develop the initial population of strings. The strings are made up of binary numbers, each section of which represents part of the geometry of the stiffener. This is done through a random number generator found in [**?**]. The first algorithm is made up of 100 strings of 60 numbers the second algorithm is created from 100 strings of

80 numbers. Each algorithm runs for 300 generations as this was found to be a value for which an adequate convergence occurred.

3.1.3 Exploiting operator

The exploiting operator is the selection process which chooses the strings to be used and those which will no longer be used. This is done using the criteria of a fitness function which will normally be based on a function of the inputs $f(x)$ for maximisation or $1/f(x)$ for minimisation problems. The main exploiting operators are outlined below. Tournament selection is carried out by using only the values with the best fitness. This is done by picking a tournament size and from this selection choosing the fittest selections to go through to the next round.

4 Production Modelling

To determine the cost accurately it is important to model the production route. For the example given later hand layup is used for the production technique as it is the method most prevalence used within boatbuilding community. Production modelling was originally performed using a parametric cost model taken from the SSA report by Shenoi et al. [**?**] as shown in Table 5.

This model has no cost for stiffeners and is for a sandwich plate. This has therefore meant that a stiffener cost model has been attached to the main model replacing the cutting and laying core section of the SSA production model for each longitudinal and transverse section and shown in Table 6.

The time for each action has been transformed into a cost by using a wage of £ 20/hour. To determine the raw material costs for the stiffeners cost per kg for each material has been used developed from a database of materials form Lloyds Register.

5 Applications

5.1 Structural Verification

Verification of both parts, Navier grillage analysis and TSDT, of the first principles structural analysis method was performed. The results from the grillage method have been compared to those found in Clarkson [**?**] for a panel with a length and width of 3810 mm. The panel consisted of 4 transverse beams and longitudinal girders with dimensions 254 mm deep 127 mm wide with 18.288 mm thick flanges and 9.144 mm thick webs and a pressure of 137.9 kPa was applied to each panel. The results are presented in Table 7.

These results were obtained with a wave number of 11, shown in eq. 1 as m and n, as it is this minimum value at which the deflection converges.

These values were found to be close to results found in Maneepan as can be seen in Table 7. Furthermore these values are similar to Clarkson, using the folded plate method, which has been compared to experimental results but also remain conservative. The grillage method was deemed valid for the stiffener modelling.

A validation of the shear stress has been made in comparison with a theoretical rectangular box beam found in Datoo [**?**]. The web height is 50 mm and the flange widths are 200 mm. The Young's modulus of the flanges are 54.1 kN/mm^2. The Young's modulus of the web is 17.7 kN/mm^2. A shear force of Q= 10.0 kN is found in the stiffeners. The thickness of the flanges are 1.0 mm and the thickness of the web is 0.5 mm. τ_1 is the shear stress at the corner of the crown element, τ_2 is the shear stress at the neutral axis of the cross section.

These values had no deviation from the results found in Maneepan and there is only a small deviation found compared to the results found in Datoo. It is therefore considered that the grillage theory is capable of calculating the shear stress.

Finally the elastic equivalent properties were compared to Datoo [**?**] using lamina properties E_1= 140 kN/mm^2, E_2= 10 kN/mm^2, G_{12}= 5 kN/mm^2, υ_{12}= 0.3 and a ply thickness= 0.125 mm for each of the 8 plies all having a 0° ply angle where the result was identical to Datoo's value of 140 GPa.

For the validation of third order shear deformation theory a layup of [0/90/90/0] has been used. The length to width ratio (L/B) of the plate is equal to 1.0 and the length to thickness ratio (L/t) is 100. The material properties are E_1 = 175 GPa, E_2 = 7 GPa, G_{12} = G_{13} = 3.5 GPa, G_{23} = 1.4 GPa, and υ_{12} = υ_{13} = 0.25. The load acting on the plate is q_0=50 kPa.

From the validation of the third order shear deformation theory it is possible to see that the results have at most a 1% deviation from those given in Reddy at a value for the wave numbers of eleven showing Third order Shear Deformation Theory has been modelled accurately.

5.2 Genetic Algorithm Verification

Genetic algorithms can be tested to determine if the optimisation that has been carried out reaches the optimum value. This is investigated by starting the algorithm at different points and determining if, at the finish, all the algorithms reach a similar fitness function. Genetic algorithms require that the best fitness value, after each generation, will gradually increase. This leads to a distinctive "handgun" shaped graph when fitness function is plotted against generation. If the same algorithm is started from different points this will lead to the optimisation reaching similar fitness functions as shown in Figure 4.

As can be seen from the examples in Figure 4 the graph follows the distinctive genetic algorithm shape where each of the individual strands reaches a similar final result from whichever starting position. This shows that the algorithm is working correctly therefore validating the optimisation.

6 Results

For each failure criterion reported a comparison has been made between the optimised structures that can be created for the constraints of those failure criteria. The grillage that is generated is developed for use within leisure boats, as an example. This optimisation used a genetic algorithm reducing both the mass and the cost of the developed structures. The cost was developed using a simple parametric model. The genetic algorithm used a weighting of 0.5 for cost and mass. The choice of structural models comprises of Navier Grillage theory for assessing the stiffeners and Third order Shear Deformation Theory for assessing the plates between the stiffeners. Each failure model has been used as the assessment for failure or success separately to determine how these failure criteria affect the optimal structure.

For each different failure criterion a simple study has been performed on a horizontal section of hull at the bottom of the boat. This has been carried out using a grillage panel length of 24m and width 2m. The structural analysis has been implemented using the pressure, 131.47kPa, from Lloyd's Register for Special Service Craft as this gives the most conservative estimate ensuring that the masses and costs used within the comparison are likely to be for the worst case scenario.

6.1 Puck

The first optimisation was undertaken using the Puck failure criterion of sub-section 2.3.1. The topology of the stiffened panel that was produced using the Puck failure criteria with the first principles method, shown in section 2.1 can be seen in Table 10. The optimised thickness of the stiffened plate and the spacing of the transverse and longitudinal stiffeners from the Puck criteria analysis are also reported in Table 10.

The web thickness of 0.86mm is small in comparison to those that would be expected in a real application. Due to the Puck failure criteria being stress-based the optimisation is attempting to reduce the maximum stress within the whole panel. This involves an increase in the moment of inertia in the stiffeners and for a low stress in the stiffener a high neutral axis is therefore required. The thickness of the web therefore does not affect the moment of inertia as much as the thickness of the crown and its distance from the plate. The panel topology has a wide stiffener spacing and a small plate thickness. Out of plane pressure on the panel develops a stress that is not as complex as a real life situation, lacking axial, torsional and shear forces, leading to thinner panel dimensions and a large stiffener spacing, than is realistic.

6.2 Tsai

The second optimisation was undertaken using the Tsai failure criterion of sub-section 2.3.3. The optimised topology produced using only the Tsai failure criterion can be seen in Table 11.

This failure criterion again produced a topology with a thin web thickness due to the nature of the optimisation attempting to reduce maximum stress but web thickness having little effect on the neutral axis and hence on this value. The stiffeners themselves are small in comparison to those from the Puck criterion. Furthermore the stiffener spacings are narrow and the panel thickness is thinner than the other cases. The criteria of the World Wide Failure Exercise are similar, being reliant on the maximum stress and having been produced to fit the same experimental data, it would be expected that the panel produced would be similar to that of the Puck criterion. The mass produced using the Tsai failure criterion is small and the cost quite large compared to the other plates, as seen in Figures 5 and 6, and therefore it is likely that the different shape of the failure envelope, shown in Figure 2(b), led the evolution of the genetic algorithm down a different route. This is shown from the small stiffener spacing. The extra stiffeners therefore allowed a reduction in the stiffener size but created extra cost. This shows that for a large number of stiffeners a high cost is incurred but a low mass is possible. Due to the weightings being equal between the mass and the cost it was possible to gain a similar fitness function through a plate that concentrated on low mass and high cost, high cost and low mass or a compromise.

6.3 Zinoviev

The third optimisation was run using the Zinoviev failure criterion of sub-section 2.3.2. The topology of the optimum stiffener plate using only the Zinoviev failure criterion can be seen in Table 12.

The Zinoviev criterion produced a similar panel to the other failure criteria selected from the World Wide Failure Exercise producing a high stiffener with a thin web and a thick crown. The Zinoviev criterion did show a large difference between the transverse and longitudinal stiffeners. The longitudinal stiffener shape did not appear to make much of a difference to the plate strength with the transverse stiffeners providing most of the strength which is why the longitudinal crown is small. Furthermore the stiffener spacings are a large distance apart with a thin panel. The stiffener topology was most similar to that produced using the Puck failure criteria. This result is to be expected as the envelope, shown in Figure 2(c), within in which the combined stresses would not cause a failure for the Zinoviev and Puck criteria are similar.

6.4 Deflection

The fourth optimisation was run using the arbitrary failure criterion of 10%. This failure criterion meant that the maximum deflection of the plate could be no more than 10% of its length. The resulting optimised stiffener topology can be seen from Table 13.

This optimised plate topology is similar to those found using the World Wide Failure Exercise criteria and this is due to the stress in the panel being based upon the deflection. This requirement means that to minimise one output, stress, a similar topology will be required to minimise the other, deflection. This therefore meant for a low deflection it was also important to have a high neutral axis. Further to this criterion, the material property would have been more important as the material's stiffness would have made a difference to the deflection. Since the material selected was E-glass, due to its low cost, the stiffening elements were required to be much larger due to the poor stiffness characteristics of the material or a larger number of stiffeners are required. This showed that in terms of the grillage plate, the failure criteria chosen and the predicted pressure that the constraints for deflection were more important than those of stress. The deflection criterion requires that the stiffener spacing is small as more stiffeners created a less flexible panel. Furthermore the thickness of the panel is small as this part of the topology did not affect the deflection of the plate.

6.5 Buckling

Finally buckling failure criteria have been applied to the stiffeners on the grillage. The buckling criterion was applied to the stiffeners only. The resulting optimised topology for the buckling criteria is given in Table 14.

The buckling criteria developed a stiffener topology different to those found using the other failure criteria. The main difference with this criterion was it developed a stiffener web thickness and crown height that was thicker than the corresponding dimensions found using the other criteria as seen in section 2.1. This is due to buckling being dependant on the equivalent thickness of the stiffening elements in comparison the length of those elements. For buckling not to occur there is still a requirement that the stress was low and therefore it can be seen that the stiffening elements produced during the optimisation were tall to increase the neutral axis. The panel topology developed a wide stiffener spacing and a thin panel thickness as these criteria did not affect the buckling of the stiffener.

7 Discussion

A method for first principles structural modelling has been developed and verified. The model has been attached to an optimisation algorithm and different failure models have been tested to determine the manner in which they affect the optimum structure. A review of these results for the mass and cost of this section are shown in Figures 5 and 6.

The models of the World Wide Failure Exercise, Puck, Tsai and Zinoviev, and maximum deflection criteria developed a similar topology with small web thickness and crown height. These results show a similarity between them except for the Tsai failure criterion where the genetic algorithm may have followed a different evolutionary route. This was due to the use of a 50/50 weighting between mass and cost. It was therefore possible to gain a similar fitness function result between those that used a small stiffener spacing, and therefore generated a lower mass and higher cost, and those that used a larger stiffener spacing for a lower cost but a higher mass. The buckling failure criterion ensured that the stiffener web and crown thicknesses were larger than for the other cases. This result could be improved through the introduction of more generations within the optimisation. This can be seen from the manner in which it varies from the other topologies with a higher cost but a lower mass than the other World Wide Failure Exercise criteria. A fault of the genetic algorithm is that it may find a close to optimum solution. A comparison of the fitness functions show that the Tsai failure criterion was slightly below that of the Puck failure criterion supporting this argument, as does Figure 4 which shows that the final fitness function was slightly different for optimisations that were run from different starting points. This behaviour can be reduced through an increase in generations.

The implications for these results are that the criteria used show a difference in the optimum geometry that was developed showing the method developed works to a reasonable level of complexity. As expected for the e-glass composite materials the serviceability limit state is most difficult to satisfy with a low cost and mass than that of ultimate strength. It is important to note, as none of the failure criteria developed match all failure modes, as can be seen in Table 1, that each of these failure criteria individually do not incorporate the entire strength limit state. It can be seen however from the WWFE [**?**], [**?**] and [**?**] that the Puck failure criteria in particular match many of these failure criteria well and therefore represents a criteria close to matching this limit state. This is due to its phenomelogical nature allowing effects noticed in experiments to be replicated within the theory allowing a more accurate representation of the real life phenomena. While the Puck criteria is the most conservative of the World Wide Failure Exercise criteria its use within structural design can be difficult due to the requirement for a large amount of material data. This requires estimates of the properties required to implement new materials or the use of previously used materials only.

Grillage theory has advantages in the speed with which the results can be processed while giving a good indication for the final structural topology. Though these results can be useful for initial design the complexities for the manner in which stiffener spacing and geometry affect the final limits is not fully taken into account. From the analysis performed it is possible to see that the failure criteria affected the optimised structures in a different manner. While the method used is not as complex as others that can be chosen the method was rapid allowing the use of the model without developing a metamodel. It will be important for future analysis to compare the final results to that done using a finite element analysis comparing and contrasting these different methods for use in rapid initial grillage design. While the results of the grillage analysis are not as complex of those used within FEA it can also be seen that changes to the failure criteria have a large effect on the design of the grillage.

8 Conclusion

A method for rapid analysis of composite structures has been proposed and validated. This method has been used for the first time to assess the effects of a number of failure criteria on complex tophat stiffened structures. While it can be seen that the deflection criteria produced the most constrained overall result it is possible to see that the Puck criteria was the most constraining of the world wide failure exercise criterion. The results show further indicate that this method of modelling could be utilised in concept design work allowing investigation of complex structures at an early stage. This method will allow rapid analysis of structures than that obtainable by combining FEA and genetic algorithms and allowing a less constrained analysis than using metamodels.

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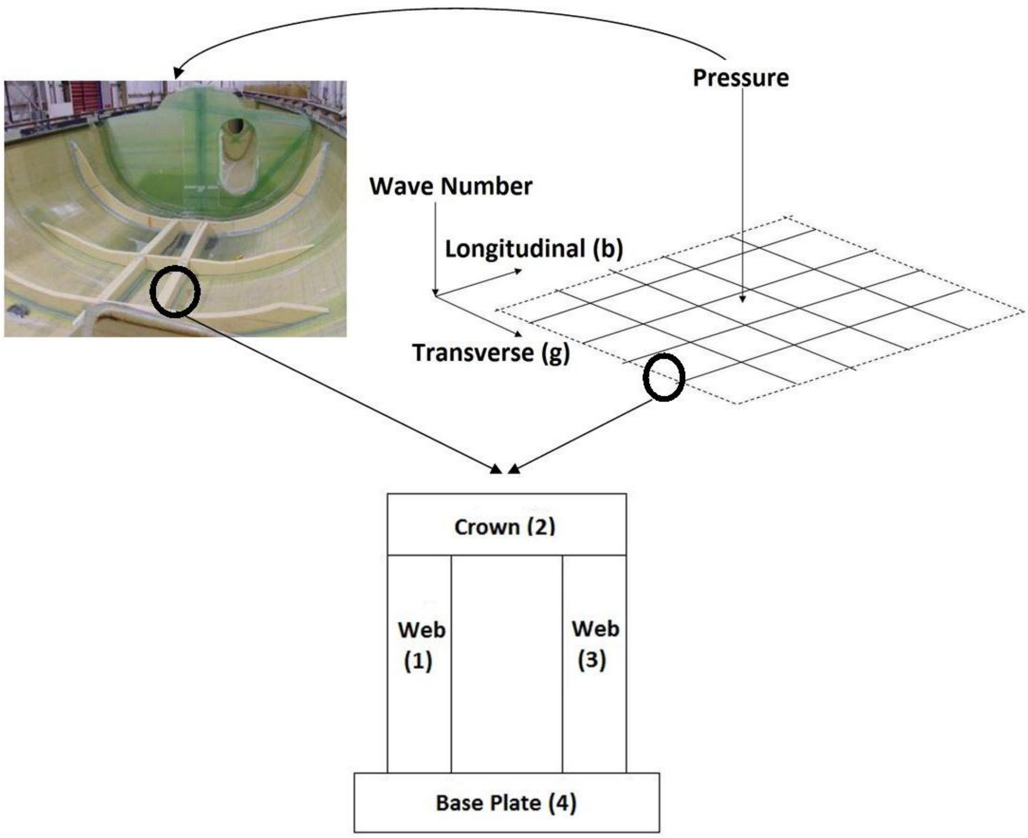
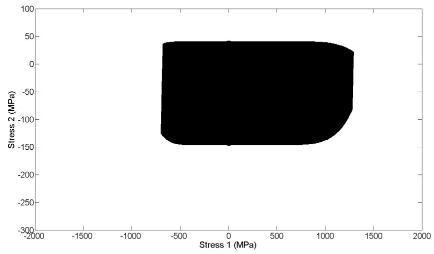
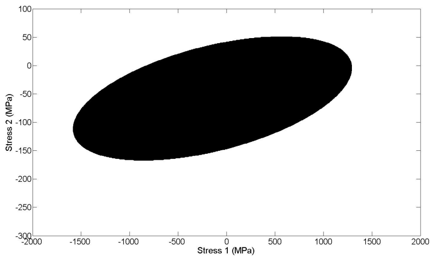


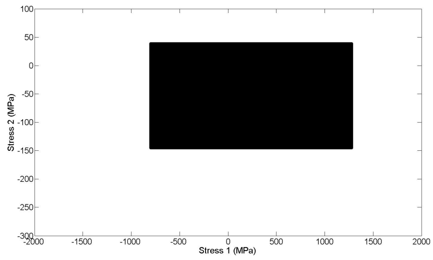
Fig. 1: Example of a Grillage Stiffened Plate in a Composite Boat



(a) Puck criterion



(b) Tsai criterion



(c) Zinoviev criterion

Fig. 2: Failure criteria stress envelopes

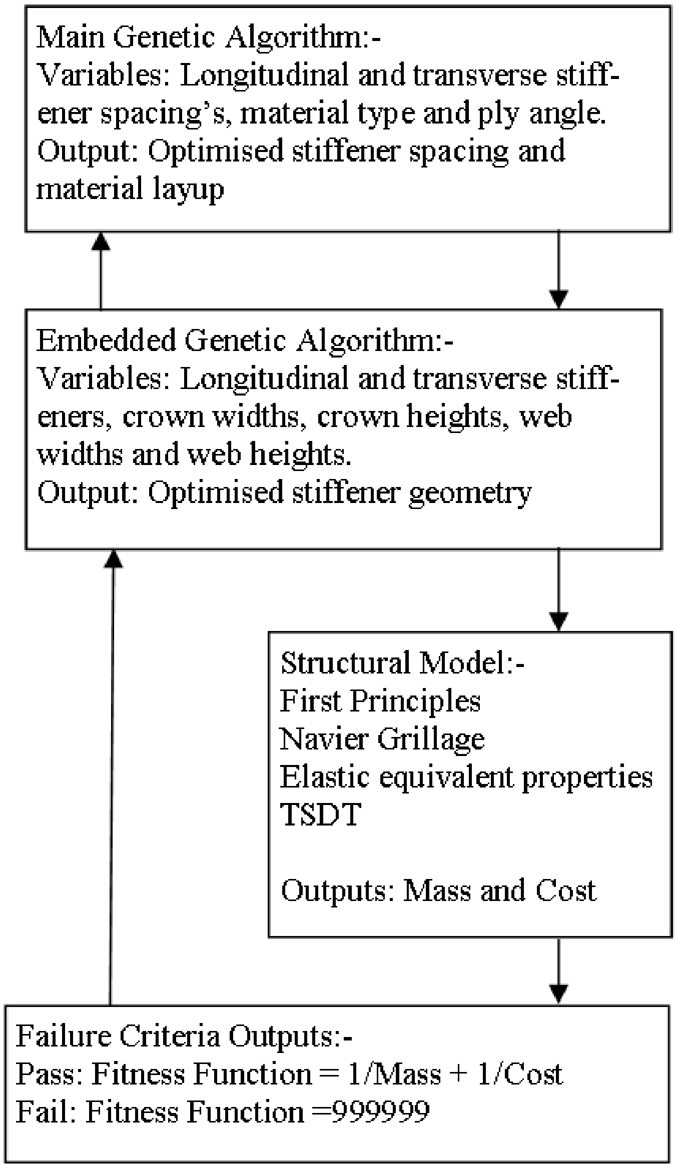


Fig. 3: Genetic Algorithm Flow Diagram

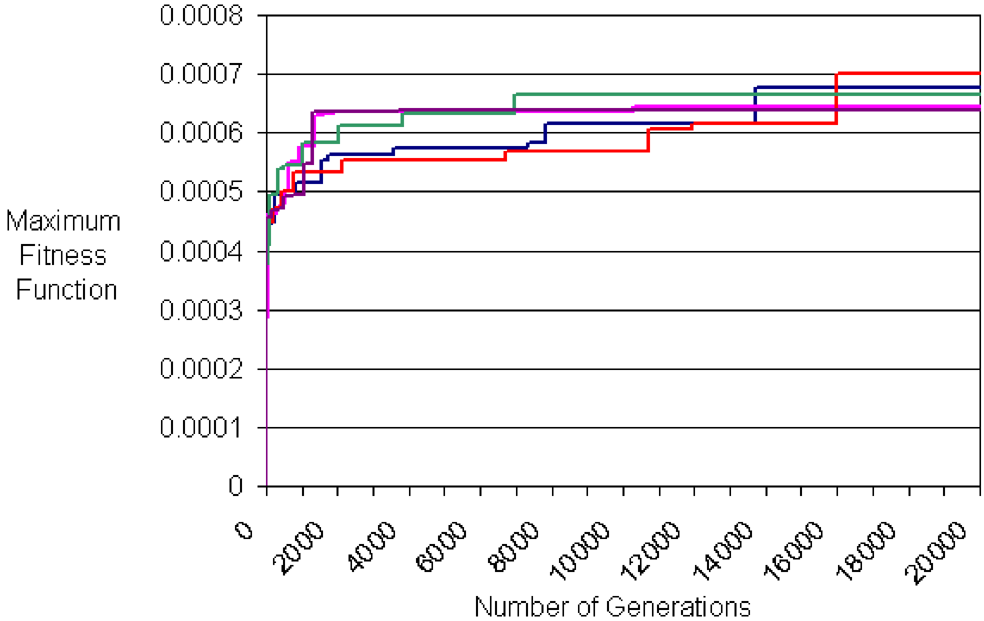


Fig. 4: Validation of genetic algorithm using different starting points

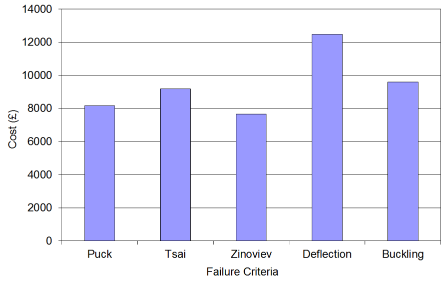


Fig. 5: Comparison of cost for failure criteria

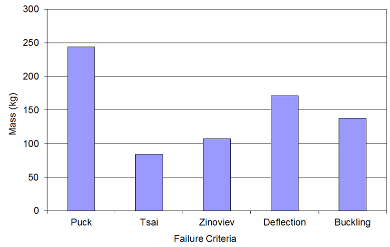


Fig. 6: Comparison of mass for failure criteria

a,b	Stiffener spacing
a_s	Crown width
a_{mn}	Coefficient for grillage analysis
$A_{i,j}$	Laminate stiffness terms
$A_{sx,sy}$	Axial rigidities of stiffeners
b_s	Crown thickness
b,g	Numbers of beams and girders
c_s	Web width
$D_{sx,sy}$	Stiffener rigidities
d_{na}	Cross sectional area to neutral axis distance
d_s	Web height
E	Young's modulus
E_{f1}	Young's modulus of fibre
G	Shear modulus
I	Second moment of area
I_{cx}	Moment of inertia
L,B	Length and breadth of plate
M_s	Moments of stiffeners
m,n	Wave numbers
$m_{\sigma f}$	Mean stress magnification factor
$n_{b,g}$	Number of beams or girders
P	Pressure
q(x,y)	Pressure at a given point on plate
S_{12}	Shear strength in the ply
t	Ply thickness
U, V, W, X, Y_{mn}	Coefficients for initial conditions
w	Deflection
$\bar{w}$	Non-dimensionalised deflection
X_C,X_T	Strength parallel to fibres
Y_C,Y_T	Strength transverse to fibres
ϵ, γ	Stiffness
ϵ_{1T}	Tensile failure strain
ϵ_{1C}	Compressive failure strain
$\rho_{\perp\parallel}$	Slope of the longitudinal fracture envelope
$\rho_{\perp\perp}$	Slope of the transverse fracture envelope
σ	Stress
σ_{cri}	Critical Stress
σ_{1D}	Stress value for linear degradation
τ	Shear stress
υ	Poisson's ratio

Table 1: Failure Criteria

Failure Type	Criteria
Predicting the response of lamina	Puck [?], [?] and Tsai [?], [?]
Predicting final strength of multidirectional laminates	Puck [?], [?]
Predicting the deformation of laminates	Zinoviev [?], [?] and Puck [?], [?]
Predicting final strength of multidirectional laminates deformation of laminates	Puck [?], Zinoviev [?] and Tsai [?], [?]

Table 2: Puck failure criteria

Fibre failure in tension	$\frac{1}{\epsilon_{1T}}\left(\epsilon_1 + \frac{\upsilon_{f12}}{E_{f1}} m_{\sigma f}\sigma_2\right) = 1$
Fibre failure in compression	$\frac{1}{\epsilon_{1C}}\left\lvert\left(\epsilon_1 + \frac{\upsilon_{f12}}{E_{f1}} m_{\sigma f}\sigma_2\right)\right\rvert = 1 - (10\gamma_{21})^2$
Inter-fibre failure mode A (for transverse tension)	$\sqrt{\left(\frac{\tau_{12}}{S_{12}}\right)^2 + \left(\rho^{(+)}_{\perp\parallel}\frac{Y_T}{S_{21}}\right)^2 + \left(\frac{\sigma_2}{Y_T}\right)^2} + \rho^{(+)}_{\perp\parallel}\frac{\sigma_2}{S_{12}} = 1 - \frac{\sigma_1}{\sigma_{1D}}$
Inter-fibre failure mode B (for moderate transverse compression)	$\frac{1}{S_{21}}\left(\sqrt{\tau_{21}^2 + \left(\rho^{(-)}_{\perp\parallel}\sigma_2\right)^2}\right) + \rho^{(-)}_{\perp\parallel}\sigma_2 = 1 - \frac{\sigma_1}{\sigma_{1D}}$
Inter-fibre failure mode C (for large transverse tension)	$\left[\left(\frac{\tau_{21}}{2\left(1+\rho^{(-)}_{\perp\perp}\right)S_{21}}\right)^2 + \left(\frac{\sigma_2}{Y_C}\right)^2\right]\frac{Y_C}{(-\sigma_2)} = 1 - \frac{\sigma_1}{\sigma_{1D}}$

Table 3: Zinoviev failure criteria

Longitudinal tension failure	$\sigma_1 = X_T$
Longitudinal compressive failure	$\sigma_1 = X_C$
Transverse tensile failure	$\sigma_2 = Y_T$
Transverse compressive failure	$\sigma_2 = X_C$
In-plane shear failure	$\tau_{12} = S_{12}$

Table 4: Genetic Algorithm Constraints

Property	Bounds	Property	Bounds
Long. Stiffener Spacing	0-10230mm	Ply Angles	0,90
Trans. Stiffener Spacing	0-2046mm	Ply Materials	E-glass, Aramid, Carbon, HM Carbon
Number of Plies	0-32	Long. Crown Width	0-102.3mm
Long. Crown Height	0-20.46mm	Long. Web Width	0-20.46mm
Long. Web Height	0-102.3mm	Trans. Crown Width	0-102.3mm
Trans. Crown Height	0-20.46mm	Trans. Web Width	0-20.46mm
Trans. Web Height	0-102.3mm	Plate Thickness	0-102.3mm
Stiffener Base Width	0-102.3mm	Stiffener Base Width	0-102.3mm

Table 5: SSA Sandwich Panel Production Model [**?**]

Action	Cost(mins)
Fairing Compound	10 minutes/m^2
Smoothing Fairing Compound	60 minutes/m^2
Apply Release Compound	10 minutes/m^2/ply
Cutting cloth	10 minutes/m^2/ply
Laying cloth	5 minutes/m^2/cloth
Cutting and laying core	60 minutes/m^2/core
Apply resin with brush or rollers	10 minutes/m^2
Remove the components from the mould	30 minutes/m^2
Quality Inspection	3 minutes/m^2
Trim	15 minutes/m/edge

Table 6: SSA Production Model

Action	Cost(mins)
Cutting cloth	10 minutes/m^2/ply
Laying cloth	5 minutes/m^2/cloth
Cutting and laying core	60 minutes/m^2/core
Apply resin with brush or roll	10 minutes/m^2

Table 7: Validation of Navier method grillage analysis - Stress

Property	Clarkson [**?**]	Maneepan [**?**]	Current
Deflection	9.63mm	9.93 mm	9.87 mm
Stress	165.52MPa	171.19 MPa	170.13 MPa

Table 8: Validation of Navier method grillage analysis - Shear Stress

Property	Datoo [?] (MPa)	Maneepan [?] (MPa)	Current (MPa)
τ_1	99	98.72	98.72
τ_2	101	102.76	102.76

Table 9: Validation of Third Order Shear Stress Deformation theory (TSDT)

L/t	Reddy($\bar{w} \times 10^2$)	TSDT($\bar{w} \times 10^2$)
10	1.0219	1.0102
20	0.7572	0.7546
100	0.6697	0.6696

Table 10: Panel Topology for Puck Failure Criteria

Stiffener Type	Web Height	Web Thickness	Crown Width	Crown Thickness	Stiffener Spacing	Plate Thickness	Ply Angles
Longitudinal	100.7mm	0.86mm	5.6mm	1.78mm	2200mm	1.2mm	0/0/90
Transverse	36.1mm	4.16mm	5.6mm	2.78mm	570mm	1.2mm	

Table 11: Panel Topology for Tsai failure criterion

Stiffener Type	Web Height	Web Thickness	Crown Width	Crown Thickness	Stiffener Spacing	Plate Thickness	Ply Angles
Longitudinal	38.3mm	0.02mm	1.1mm	6.28mm	430mm	0.5mm	0/90
Transverse	71.3mm	0.14mm	12.5mm	4.9mm	40mm	0.5mm	

Table 12: Panel Topology for Zinoviev failure criteria

Stiffener Type	Web Height	Web Thickness	Crown Width	Crown Thickness	Stiffener Spacing	Plate Thickness	Ply Angles
Longitudinal	91.9mm	0.06mm	0.2mm	17.22mm	1130mm	1.5mm	0/90
Transverse	95.3mm	0.02mm	22.4mm	2.02mm	2200mm	1.5mm	

Table 13: Panel Topology for Deflection

Stiffener Type	Web Height	Web Thickness	Crown Width	Crown Thickness	Stiffener Spacing	Plate Thickness	Ply Angles
Longitudinal	45.8mm	0.84mm	6.6mm	13.86mm	430mm	0.7mm	0/90
Transverse	83.5mm	0.52mm	23.5mm	1mm	170mm	0.7mm	

Table 14: Panel Topology for Buckling

Stiffener Type	Web Height	Web Thickness	Crown Width	Crown Thickness	Stiffener Spacing	Plate Thickness	Ply Angles
Longitudinal	37.2mm	2.34mm	33.9mm	6.76mm	2130mm	0.1mm	0/90/0/90
Transverse	45.9mm	2.94mm	82.7mm	2.1mm	2200mm	0.1mm	