The Application of Reliability Methods in the Design of Tophat Stiffened Composite Panels under In-plane Loading

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ABSTRACT:
Composite materials have been widely used in modern engineering fields such as aircraft, space and marine structures due to their high strength-to-weight and stiffness-to-weight ratios. However, structural efficiency gained through the adoption of composite materials can only be guaranteed by understanding the influence of production upon as-designed performance. In particular, topologies that are challenging to production including panels stiffened with *pi* or *tophat* stiffeners dominate many engineering applications and often observe complex loading. The design of stiffened composite panels against buckling is a key point of composite structures. While a growing number of studies are related to the reliability analysis of composites few of these relate to the local analysis of more complicated structures. Furthermore for the assessment of these structures in a design environment it is important to have models that allow the rapid assessment of the reliability of these local structures. This paper explores the use of a stochastic approach to the design of stiffened composite panels for which typical applications can be found in composite ship structures. A parametric study is conducted using Navier grillage theory and First-order Reliability Methods to investigate any detectable trend in the safety index with various design parameters. Finally, recommendations are made to provide guidance on applications.

Keywords: Tophat-Stiffened, Reliability Based Design, Sensitivity

1. INTRODUCTION

Composite materials have been widely used in modern engineering fields such as aircraft, space and marine structures due to their high strength-to-weight and stiffness-to-weight ratios. Stiffened panels, comprised of a plate, longitudinal stiffeners and transverse frames, are very important components in ship and offshore structures, which can be found in decks, bottoms, bulkheads, side shells and superstructures. The design of stiffened composite panels against buckling is a key point of composite structures, particularly in bottom shell or deck units subjected to compressive load by longitudinal wave-induced or explosion-induced bending of the ship hull.

The inherent uncertainties in geometry, materials, loads, and other aspects of any structure are unavoidable in structural responses. Because of the existence of such uncertainties, to ensure the structures can perform their intended function with desired confidence, these uncertainties or variabilities must be considered during structural design. Traditional methods of dealing with the uncertainties are to use conservative fixed values in equations to guard against the possibility of structural damage. Assumptions are made that all factors influencing the load, strength and other uncertainties are known, ignoring uncertainties that might occur such as variability of material properties or uncertainty in analysis models. The conventional deterministic design methods are simple but inflexible to adjust the prescribed safety margin and do not give a reliable indicator of satisfactory performance for the design of FRP structures. With the development of reliability technology, reliability methods have been used in reliability-based design for marine and offshore structures. Structural reliability methods allow the designers to limit the probability of undesirable events and lead to a balanced design. Reliability-based design is more flexible and consistent than corresponding deterministic analysis because it provides more rational safety levels over various types of structures and takes into account more information that is not considered properly by deterministic analysis.

Reliability techniques have been in development for a number of years. These methods first appeared in a mathematical form in the 1920's by Mayer [1] and further developed by Streletzki [2] and Wierzbieki [3]. Practical usage of these methods was not developed until the late 1960's with the development of a second moment reliability index by Cornell [4]. Cassenti [5] furthered deterministic design methods by developing the probabilistic static failure analysis procedure of unidirectional laminated composite structures. Yang [6] presented a reliability analysis of laminated plates based on the last-ply-failure analysis concept. Cederbaum [7] presented work related to in-plane loads using first ply-failure on symmetric angle-ply laminates. Gurvich [8],[9] developed a probabilistic failure model for the reliability of laminated composites subjected to combined lateral pressure and in-plane loads based on a ply group concept and this was further developed to include both a ply group and a laminated plate subjected to uni-axial tensile loads. Specific to a marine environment Jeong and Shenoi [10], [11] presented a simulation approach to assess the first-ply failure reliability of composite plates. Other marine specific studies have concentrated on global assessment of ship hulls including Chen *et al.*[12] and Zhang [13]. Finally Blake *et al.* [14] looked at a method for assessing the reliability of composite grillages utilising Navier grillage theory with simple limit states under out of plane load. This research showed that grillage theory was good for assessing more complex composite structures however it for a full analysis of composite structures there will be a requirement to analyse grillages with the addition of in-plane loads.

While these studies have progressed the status of reliability analysis of composite structures the analysis has been performed on simple structures, plates, cylindrical shells and others or in the case of the more complex analysis has concentrated on a global rather than local assessment of reliability. Furthermore this review shows that while there is a growing quantity of composite reliability literature this is generally not marine specific and it is important to perform reliability investigations using data similar to marine applications.

This paper therefore focuses on the reliability of composite grillage plates using Navier grillage theory for computationally inexpensive analysis under in-plane loading. The paper looks to incorporate reliability methods into the design of complex composite structures with rapid analysis techniques. Finally an assessment is made to detect any trend in the safety index with various design parameters and recommendations are made to provide guidance on applications.

2. RELIABILITY-BASED ANALYSIS METHODS

There are two types of design format that are normally used [15], namely direct reliability based design and Load and Resistance Factor Design (LRFD). Limit state equations are essential for conducting reliability analysis and the means by which a definition of failure is articulated mathematically. If the load applied to the structure is defined as *L* and the resistance of the structure to that load as *R*, the safety margin may be defined as

$$M = g(r,l) = R - L \tag{1}$$

Since *R* and *L* are random variants, *M* is also a random variant with corresponding probability density function f_M(m). In this case, failure is clearly the event ($M \le 0$) and thus the probability of failure is

$$P_f = P[M \le 0] = \int_{-\infty}^{0} f_M(m)dm \tag{2}$$

Cornell proposed a reliability index defined as

$$\beta = \mu_M / \sigma_M \tag{3}$$

where μ_M and σ_M are the mean value and the standard deviation of the safety margin, respectively.

In engineering practice, the safety index, β, instead of failure probability, P_f, is often used to represent the reliability level.

3. GENERAL MODEL OF STIFFENED COMPOSITE PANEL

A stiffened panel is a panel of plating bounded by, for example, transverse bulkheads, longitudinal bulkheads, side shell or large longitudinal girders. A typical stiffened panel configuration with the tophat-section stiffeners is shown in Figure 1. The stiffened panel is referred to x- and y- axis coinciding with its longitudinal and transverse edges, respectively, and a z-axis normal to its surface. The cross-section geometry is defined in terms of the six geometrical parameters b_1, b_2, b_3, b_4, b_5 and *d*. The length and breadth of the stiffened panel are denoted by *L* and *B*, respectively. The spacing of the stiffeners is denoted by *a* between longitudinal stiffeners and *b* between transverse stiffeners. The numbers of longitudinal and transverse stiffeners are N_g and N_s, respectively. The web (sides), table (top) and flange (base) structures forming a tophat-stiffener are made of FRP laminates .

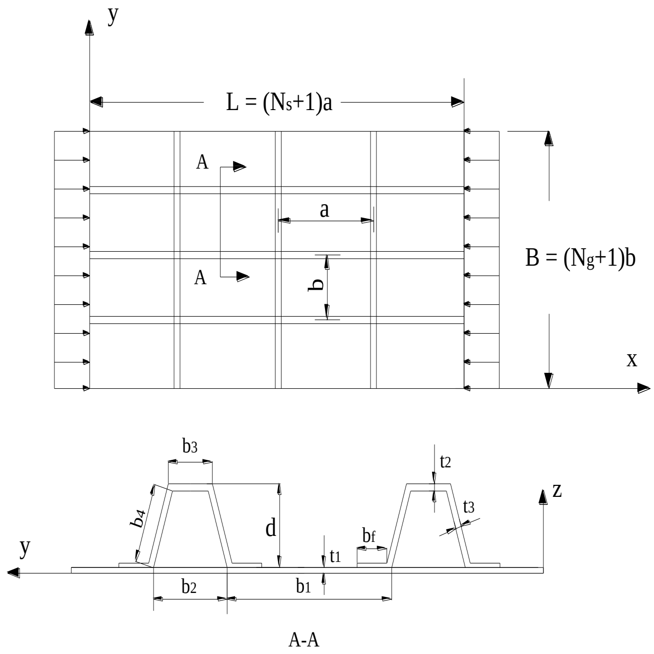

Figure 1 Stiffened panel configuration with the tophat-section stiffeners

During structural design and analysis, primary failure modes should be considered carefully as most forms of buckling will result sooner or later in complete collapse of the structure. According to [16, 17], the primary failure modes for a stiffened panel subject to compressive loads might arise in panels as follows:

- Local buckling of the plating between stiffeners

  Where the lowest initial buckling stress corresponds to local buckling of the plate between stiffeners, a substantial postbuckling reserve of strength may exist. Generally, local buckling of the shell is associated with loss of effective width, which may cause a reduction in the flexural rigidity of the cross-section.

- Column-like buckling

  This buckling mode indicates a failure pattern in which the collapse is reached by column or beam-column type collapse of the combination of the stiffener with the effective plate. Collapse is possibly caused by material tensile or compressive failure in the stiffeners.

- Tripping of stiffeners

  Tripping of a stiffener can occur when the ultimate strength is reached by lateral–torsional buckling of the stiffener. This form of instability is susceptible to open-section stiffeners. Tophat-section stiffeners which are usually used in composite ships have high torsional stiffness and this buckling mode can be prevented by using stiffeners with good proportions.

- Overall instability of the stiffened panel

  This failure mode refers to the buckling of the gross panel involving longitudinal and transverse frames between the major support members. Overall instability failure mode typically represents the collapse pattern when the stiffeners are relatively weak. This failure mode should be proportioned so that this form of failure is preceded by that interframe collapse mode because this failure involves a large portion of structure and is likely to be more catastrophic.

4. ANALYTICAL MODELS

Generally speaking, folded plate methods and numerical methods are capable of giving comprehensive and adequate results [17]. However, they are not computationally efficient from a design point of view for the considerable preparation and computational time involved, particularly if the repeated analyses are required at the preliminary design stage due to the involvement of a large number of variables. Simplified analytical methods provide a more time-effective means of calculating the strength of stiffened panels. For engineering practice, most of the necessary evaluation of stresses and deformations can be carried out by means of simple formulae based on beam and plate theory on idealized geometries and boundary conditions.

4.1 Column-like Buckling

As discussed above, initial buckling may be considered by the form of column-like instability of longitudinal stiffeners together with the effective plating so that they would behave as a beam-column when the stiffeners are dominant. Ignoring the torsional rigidity of the gross panel, the Poisson's ratio and the effect of the intersecting beams, assuming the panel is simply supported, column buckling strength (without consideration of initial imperfection and lateral pressure) is given using the Euler formula [17]

$$\sigma_E = \frac{\pi^2 D}{Aa^2} / (1 + \frac{\pi^2 D}{a^2 GA_s}) \tag{4}$$

in which $D = \sum_{i=1}^{N} (EI)_i$ is the flexural rigidity of a stiffener with associated effective width of plate;

i is the web (sides), table (top) and flange (base) structures; A is the total cross-sectional area of a stiffener with an attached strip of plate; a is the spacing between transverse frames; GA_s is the shear rigidity in which A_s may be taken as the area of the stiffener webs, G is the shear modulus of the web (sides)

In real applications, composite structures exhibit some unavoidable initial imperfections due to the manufacturing process or heavy load connected to the hull, these initial imperfections may trigger buckling or premature strength failure at a load far below those corresponding to elastic buckling. The initial deformation w_0 are assumed to be an idealization of initial deflection shapes for a one-dimensional member, which can be approximately expressed in mathematical form as

$$w_0 = \delta_0 \, sin \frac{\pi x}{a} \tag{5}$$

which takes the half sinusoidal wave pattern. The total (initial plus added) deflection ,w, may also take a similar shape to the initial deflection as shown in equation (6)

$$w = \delta \sin \frac{\pi x}{a} \qquad (6)$$

where δ_0 is the maximum initial imperfection and δ is amplitude of the total deflection.

The bending moment equilibrium is given by,

$$D\frac{d^2(w - w_0)}{dx^2} = -Pw \qquad (7)$$

The strain-energy-based approach is employed to determine the initially deflected column. The total potential energy can be given by,

$$\Pi = U + W \qquad (8)$$

The elastic strain energy U and the external potential energy W are calculated as,

$$U = \frac{D}{2}\int_0^a \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w_0}{\partial x^2}\right)^2 dx \qquad (9)$$

$$W = Pu = -\frac{P}{2}\int_0^a \left[\left(\frac{\partial w}{\partial x}\right)^2 - \left(\frac{\partial w_0}{\partial x}\right)^2\right] dx \qquad (10)$$

Applying the principle of minimum potential energy, the amplitude of the total deflection can be found as follows,

$$\delta = \frac{\delta_0}{1 - \sigma / \sigma_E} = \Phi \delta_0 \qquad (11)$$

where $\Phi=1/(1-\sigma/\sigma_E)$ is called the magnification factor.

The maximum stress σ_{max} at the outer fibre of the cross-section can therefore be obtained by the sum of axial stress and bending stress as follows,

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{W} \qquad (12)$$

where $M_{\max} = P\delta$; $W = \dfrac{D}{E_i y}$; E_i is the membrane equivalent Young's modulus of the element considered (Appendix A); y is the vertical distance from the neutral axis to the point in question.

For plate-beam under combined axial compression P and lateral line load q, the internal bending moment along the span can be obtained by the sum of the bending moment due to lateral load and geometric eccentricity, which may include lateral deflection caused by external load as well as an initial imperfection.

$$M_{max} = M_{q\,max} + P\phi(w_{q\,max} + \delta_0) \qquad (13)$$

where M_{qmax} and w_{qmax} are maximum bending moment and maximum deflection due to lateral load alone. The maximum stress σ_{max} at the outer fibre of the cross-section can therefore be obtained by substituting M_{max} from Eq.(13) into Eq.(12).

4.2 Overall Buckling

If support members are relatively weak, they will deflect together with the plate so that the stiffened panel can buckle together. This mode is termed overall buckling. The overall buckling of a stiffened composite plate is performed based on a modified grillage model, in which the stiffened panel is treated as a grillage through substituting equivalent elastic properties of its laminate components into the analysis (Appendix A)
The double series expression for the deflection of the stiffened panel can be assumed

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn} \sin\frac{m\pi x}{L}\sin\frac{n\pi y}{B} \tag{14}$$

which fulfil the end conditions when the plate is simply supported along all edges. The coefficient f_{mn} in the series for the deflection may be determined by the condition that the change in potential energy due to the assumed deflection is minimum.

The total potential energy of the longitudinal girders and transverse stiffeners are given by

$$V_g = \frac{1}{2}\sum_{i=1}^{N_g} D_{gi}\int_0^L \left(\frac{\partial^2 w}{\partial x^2}\right)^2_{y=y_i} dx \tag{15}$$

$$V_s = \frac{1}{2}\sum_{j=1}^{N_s} D_{sj}\int_0^B \left(\frac{\partial^2 w}{\partial y^2}\right)^2_{x=x_j} dy \tag{16}$$

where $y_i = \dfrac{iB}{N_g+1}$, $x_j = \dfrac{jL}{N_s+1}$ when all girders and stiffeners are arranged at equal distance; D_{gi} and D_{sj} are the flexural rigidity of the i^{th} girder and the j^{th} stiffener, respectively. The effect of the plate is to act as effective flange for girder and beams.

The total potential energy will be

$$V=V_g+V_b \tag{17}$$

When the stiffened panel is subjected to a uniform longitudinal compressive stress σ in the x-direction, the work of the external force will be

$$W_{gi} = -\frac{1}{2}\sigma_x A_g \int_0^l \left(\frac{\partial w}{\partial x}\right)^2_{y_i} dx \tag{18}$$

$$W_g = \sum_{i=1}^{Ng} W_{gi} \tag{19}$$

When the stiffened panel is subjected to a uniform pressure load q alone, the work of the external force will be

$$W_q = \int_0^L\int_0^B q\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}\, sin\frac{m\pi x}{L} sin\frac{n\pi y}{B} \tag{20}$$

where A_g is the cross-sectional area of a girder.

The critical load may be determined by the minimum potential energy

$$\frac{\partial(V-W)}{\partial f_{mn}}=0 \tag{21}$$

The coefficient f_{mn} can be expressed when the stiffened panel is subjected to a uniform pressure load alone

$$f_{mn}=\frac{16qLB}{\pi^6 mn\left[m^4(N_g+1)\frac{D_g}{L^3}+n^4(N_b+1)\frac{D_b}{B^3}\right]} \tag{22}$$

If the stiffened panel is subjected to a lateral load q as well as to axial compression σ, the deflection parameter f_{mn} are multiplied by the magnification factor

$$\phi=\frac{1}{1-\sigma/\sigma_E}. \tag{23}$$

where σ_E is the critical compression for the same m and n as the index of the parameter.

If the stiffened panel has the initial deflection w_0, it may approximately take a similar shape by equation (14) with the amplitude δ_0 . The maximum stress σ_{max} at the outer fibre of the cross-section of the girder can be obtained by Eq.(12) and Eq.(13).

For each element of the stiffened panel, the average direct stress value acting through the thickness of the element of a particular laminate can be predicted at any point in the section using Eq.(12). The corresponding direct force intensity per unit width N and moment intensity per unit width M of the laminate section about its own mid-plane can be obtained. Then the ply-to-ply stress analysis can be performed by Appendix B. Laminate composites may fail by fibre breakage, matrix cracking or shear failure depending on the geometry, stacking sequence and the load. In the present study, the maximum stress criterion is being used in the principal material direction of each layer, in which the individual stress components are compared with the corresponding material allowable strength values. Failure is defined as First-Ply-Failure (FPF): cracking or crazing of the surface resin, which is usually detected in marine structures, and should be avoided because such cracks allow ingress of water to the laminate, leading to degradation of strength and stiffness [18]. The ultimate strength is assumed to be reached when the maximum stress in any layer is reached: from this point onwards, rapid progressive collapse under compression is expected to ensue. This method requires an iterative procedure but usually only a few iterations are required.

4.3 Effective Width

The flange is usually not fully effectively induced by the plate buckling or shear lag, which results in a non-uniform stress distribution. Many different effective width equations for steel have been extensively studied since von Karman *et al.* [19] first introduced the concept. As a matter of fact, GRP reinforced plates have peculiar differences with respect to steel plates and not many formulae are available in literature except those presented by Classification Societies in which only simple relationships are provided. Boote [20] summarised the formulae for effective width calculation from different Classifications Societies. The formula from Bureau Veritas is chosen for this calculation where,

$$b_e = b \quad \text{or} \quad 0.2a + b_2 \tag{24}$$

and b_e is effective width between stiffeners, b is the physical width between longitudinal stiffeners, b_2 is the stiffener base width (no overlap) and a is the distance between the transverse stiffeners (see Figure 1). The choice of effective width is dependent on the consideration of either the transverse beam stiffeners or the longitudinal girders.

5. APPLICATION OF THE RELIABILITY METHOD

5.1 Introduction

In this section, a stochastic approach to the design of a stiffened composite panel under compressive load and the combination of compressive and lateral loads for ship structures is applied and the importance of different stochastic parameters on the reliability index and failure probability is investigated. The panel with rectangular tophat-sections consisting of webs, crown and base plate is shown in Table 1. This is typical of the topology in the bottom panel of composite ship structures. The shell and stiffener laminates are assumed to be reinforced by woven rovings, which are balanced laminates of the type commonly used in ship construction. Mechanical properties for a unidirectional layer are dependent on lay-up and fibre-volume fraction and calculated for practical application in Appendix C the material properties used within these equations for E-Glass and Epoxy are listed in Table 2.

Table 1. Geometric Properties (mm)

Panel thickness		Crown width	Crown height	Web width	Web height
15.68	Longitudinal	50	3.36	3.36	39
	Transverse	100	6.2	6.2	80
Thickness, t, of single layer = 0.56					

Table 2. Material properties of resin and fibre [17]

	Young's modulus E (GPa)	Poisson's ratio ν	Shear modulus G (GPa)	Tensile strength (MPa)	Compressive strength (MPa)	Tensile failure strain (%)
Epoxy	3.0	0.37	1.09	85	130	5.0
E-Glass	72.0	0.20	30	2400	-	3.0

5.2 Formulation of Limit States and Random Variables Definition

The following two limit states are generally considered in the design of the ship structures [22].

- Ultimate Limit States (ULS)
  The ULS generally considered the maximum load at which the structure collapses and can no longer serve its intended function.
- Serviceability Limit State (SLS)
  The SLS is usually related to failure modes leading to service interruptions or restrictions. Repair is usually required to return the structure to an acceptable state.

The failure due to instability or buckling of longitudinal stiffeners (flexural or tripping) or overall buckling is related to the ultimate limit state. The failure margin of structures can be evaluated when the applied compressive load reaches or exceeds its ultimate compressive strength as defined in Eq.(25).

$$g = X_u P_{ult} - P \tag{25}$$

where X_u is the model uncertainty of the strength prediction; P_{ult} is the ultimate compressive strength of a stiffened composite panel; P is the applied compressive load.

Table 3 Typical Distributions for Variables from DNV [22]

Variable	Distribution Type
Current – Long Term Speed (Pressure)	Weibull
Properties – Yield Strength (Steel)	Normal
Properties – Young's Modulus	Normal
Properties – Initial Deformation of Panels	Normal

The reliability is dependent upon the statistical distributions of the inputs. Different inputs are generally grouped together with statistical distributions as determined by structural codes e.g. CIRIA [21], DNV [22] or EUROCOMP [23]. Typical distributions for pressure and material definitions are Weibull distributions and Normal distributions respectively, as can be seen from Table 3 given by the DNV design rules and used for the analyses presented later. In general, the basic variables concerned with external load and geometric values have the largest and smallest coefficients of variation respectively. Therefore, the geometric properties such as dimension of panel a, b, b_3, b_4 and the thickness of laminae t, which may fluctuate in the vicinity of the given values depending on the manufacturing processes, are considered as random variables. All geometric properties are assumed to have a COV (Coefficient of Variation) of 3%. Initial imperfection is also taken into account as this problem can never be totally eliminated. The material properties of fibre and matrix, fibre volume fraction, which may affect the mechanical properties of the laminate, are treated as random variables with a 5% COV.

The modelling uncertainty is generally associated with idealizations in formulating mathematical models and on the like. The modelling uncertainty is usually incorporated into a reliability analysis by the ratio between the actual response and predicted modelling response. Faulkner *et al.*[24] suggested that a normal distribution is usually assumed, the mean value and coefficient of variance for strength parameters are assumed to be 1.0 and 10% for simplicity, respectively. It is assumed that a safety factor of 2 is applied to the analysis and therefore the mean value has been chosen as half of the failure load with the loads used in the analysis found from the resulting distribution. All these variables are assumed as independent variables and they are randomly generated according to their assumed probability distribution as shown in Table 4 where the values have been estimated based on experience in the marine industry and correlated against the values found in Sriramula and Chryssanthopoulos [25].

Table 4. Statistical properties of basic design variables

Symbol	Distribution	Mean value	C.O.V
a	Normal	550mm	0.03
b	Normal	500mm	0.03
t	Normal	0.56mm	0.03
b_3	Normal	50mm	0.03
b_4	Normal	39mm	0.03
δ_0	Normal	0.55mm	0.03
E_f	Normal	72.0GPa	0.05
E_m	Normal	3.0GPa	0.05
G_f	Normal	30.0GPa	0.05
G_m	Normal	1.09Gpa	0.05
V_f	Normal	0.55	0.05
X_u	Normal	1.0	0.10
P	Weibull	$0.5P_{ult}$	0.15

5.3 Sensitivity Analysis

In a practical structural design, knowing the most important design parameters and their impact on safety index enables the designers to know where to look to improve reliability. In a deterministic analysis, the sensitivities of design variables can only be computed by quantifying the change in the performance measure due to a change in the variable value. On the other hand, if a design is based on reliability theory, each random variable is defined by the mean value, coefficient of variance and distribution type. Once the probabilistic model is established, probabilistic analysis is run and then the sensitivity factors are obtained in order to determine the importance of a random variable. In addition, the complexity of the mathematical model is greatly influenced by the dimensionality of the space of variables in the analysis, therefore it is important to reduce the number of variables and thereby increase the efficiency of the reliability analysis. The variable having a small sensitivity factor might be assumed to be of fixed value rather than being a random variable in subsequent analyses.

The following three important factors were considered in this paper [26]. Sensitivity factor α is generally considered as a measure of the sensitivity of the reliability index β with respect to the standard normal variable u_i^*. It provides some insight into the relative weight that each one has in determining the final reliability of the structures. A larger α_i implies more sensitivity of reliability index β to the standard variate u_i^*

$$\alpha_i = \frac{\partial \beta}{\partial \mathrm{u}_\mathrm{i}^*} \tag{26}$$

This factor is usually providing an importance ranking of input variables. However, it is not useful for design purpose as they are dependent on mean value, standard deviation and distribution type of random variables. Another two sensitivity parameters δ and η scaled sensitivity of β with respect to the mean and the standard deviation of each basic random variable in question are more useful for design as defined in Eqs.(27)-(28).

$$\delta = \left\{ \sigma_i \frac{\partial \beta}{\partial \mu_i} \right\} \tag{27}$$

$$\eta = \left\{ \sigma_i \frac{\partial \beta}{\partial \sigma_i} \right\} \tag{28}$$

where μ and σ represent the mean value and standard deviation of basic random variables, respectively.

5.4 Results and Discussions

Table 5 shows the three sensitivity factors α, δ and η for the dominant variables. The important factors α for dominant variables are also shown in Figure 2. The safety index β = 3.67 and failure probability P_f = 1.227×10^{-4} are obtained via the proposed method together with the first order reliability method (FORM), calculated directly from the limit state equation.

From Figure 2, the importance of the dominant variables α, by order, is modelling uncertainty of the strength prediction X_u, applied load P, fibre volume V_f, the height of section b_4, the thickness of laminae t, the length a, Young's modulus of fibre E_f, and shear modulus of resin G_m. However, unrepresented in this figure are the sensitivities of other variables, which play such small roles in contributing to the probability of failure and can be replaced by deterministic values in the further analysis.

The sensitivity factor δ represents the sensitivity of β with respect to the mean values. The positive sensitivity factors δ such as geometric parameters t, b_4 and material properties of fibre and resin E_f, G_{m}, V_f are obtained and treated as strength parameters. That means the safety index increases with increasing mean value of the variables. The negative sensitivity factors δ are treated as load parameters such as the length of stiffener a and compressive load P. This indicates that the safety index decreases with increasing mean value.

The combination of in-plane and lateral loading is also considered because lateral loading from sea water pressure or cargo is always present on plates and stiffened plates elements. Pressure load of 131.47kPa with the uncertainty 10% is considered and a Weibull distribution is assumed in the reliability analysis. The direction of lateral pressure is assumed to be the same with the initial imperfection towards the stiffeners. By comparison of the panel with and without lateral pressure, the reliability index decreases from 3.67 to 2.50 and the probability of failure increases from 1.227×10^{-4} to 6.274×10^{-3}. The effect of lateral pressure on the stiffened plates is to lower the ultimate collapse load and therefore reduce the reliability index compared with the stiffened plate under in-plane loading alone.

Table 5 Sensitivity factors of basic variables

Random variable	α	δ	η	Random variable	α	δ	η
a	0.1786	-0.1786	-0.1167	G_m	-0.0829	0.0829	-0.0250
t	-0.2232	0.2232	-0.1814	V_f	-0.3157	0.3157	-0.3662
b_4	-0.2328	0.2328	-0.1980	X_u	-0.7226	0.7226	-1.9159
E_f	-0.1435	0.1435	-0.0748	P	0.4578	-0.6339	-1.0842

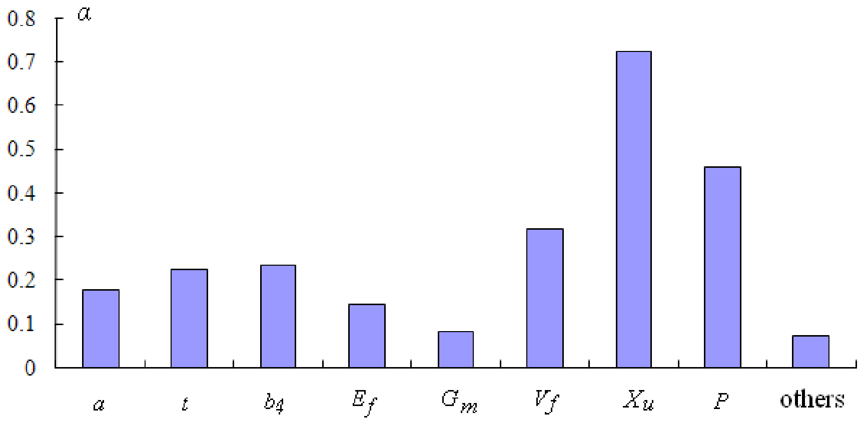

Figure 2 Important factor α

5.5 Parametric Study

Although the probabilistic method provides more information than the corresponding deterministic counterparts in the analysis, this method also requires more comprehensive information. Reliability analysis shows that not only the mean value but also COV of random variables play a significant role in determining the reliability or safety. However, such information is generally indeterminate. Furthermore this data can be used to inform designers or production engineers where effort can be best utilised for the greatest cost to benefit ratio. Therefore, it is worth studying the effects of the statistical distribution of the various variables having the largest sensitivity factors calculated in the previous section.

From Table 5, the modelling uncertainty $X_{u,}$ the fibre volume V_f, the thickness of laminae t, Young's modulus of fibre E_f and shear modulus of resin G_m are chosen to study the effect of coefficient of variation. The results are computed by varying each of the parameters in turn with other variables held the same as the previous analysis, shown in table 4. The results are presented in Figures 3-5. The model has also been analysed for a wider range of load uncertainties by ignoring the actual source of loading. Axial load COVs of 10%, 20% and 30% are considered. The results, in terms of P/P_{ult} versus the safety index β, are presented in Figure 6.

The reliability index of the model uncertainty and applied load reduces rapidly as the uncertainty increases. In other words, these two variables are very sensitive to the statistics of variables. Thus more precisely knowing the statistics of these two design variables will induce more meaningful results in the reliability analysis. For example, increasing the COV from 10% to 15% in the model uncertainty of the strength prediction leads to a reduction in reliability index of 22.8% and around a 20-fold increase in the failure probability.

The variations in component thickness and the fibre content within a component from design is caused by production defects, which may arise in the manufacture of composite structures and are difficult to eliminate. The influences of variation in the volume fraction V_f and thickness t on reliability are shown in Figure 4. It is evident that the reliability indices are strongly dependent on the variation of these two quantities. That is to say that the minor variation in these two variables has a major effect on the reliability. Therefore, if the uncertainties of such random variables can be reduced through appropriate care or stricter quality control, reliability can be increased.

Uncertainties in material properties E_f and G_m are also investigated as shown in Figure 5. It is evident that the influence of the variation in the fibre modulus E_f is stronger than the shear modulus G_m on reliability. This enables the designers to know where to look to improve reliability. For example, if the coefficient of variation of the G_m is reduced from 15% to 11%, the failure probability is reduced around 24%. However, the failure probability can then be reduced to almost half if the coefficient of variation of the E_f is reduced by the same percent. This indicates that it is better to reduce the uncertainty of E_f rather than G_m in terms of their relative importance with respect to variations in their standard deviations.

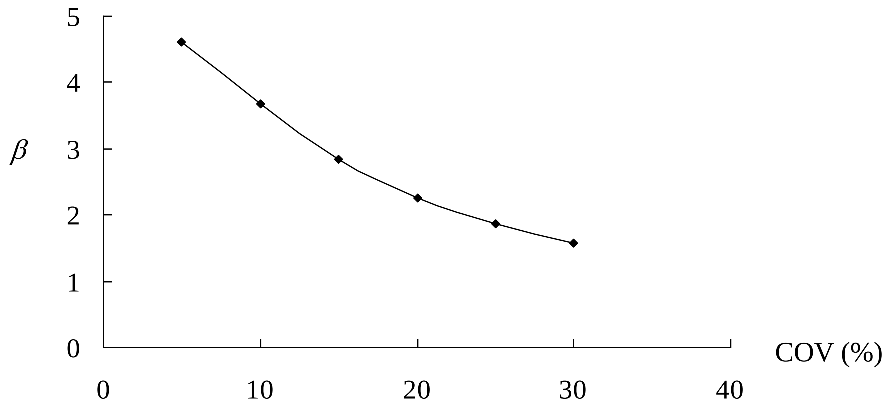

Figure 3 Variation of β with change of COV of X_u

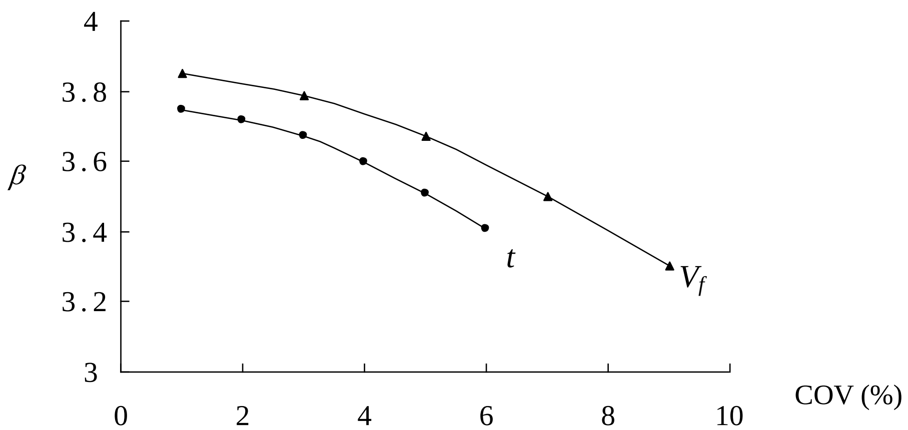

Figure 4 Variation of β with change of COV of V_f and t

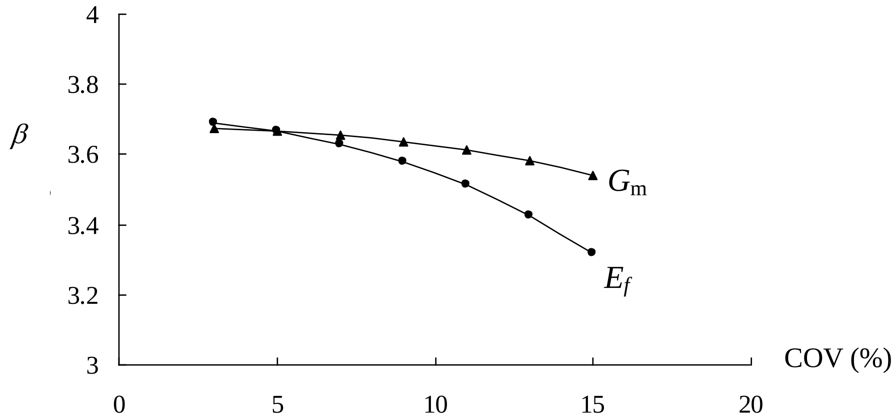

Figure 5 Variation of β with change of COV of E_f and G_m

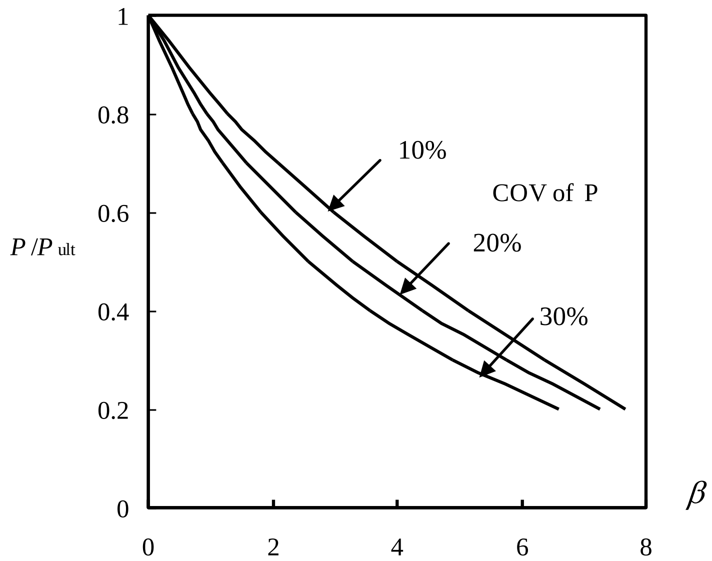

Figure 6 Variation of design axial load with safety index

6. CONCLUSIONS

In the present paper, a simplified analytical method was presented for the reliability assessment of the ultimate compressive strength of tophat-stiffened laminate panels under axial compression combined with uniform lateral pressure. A reliability-based design method is used for the practical design of composite stiffened panels. From the study the following specific conclusions can be drawn:

- A unique feature of fibre-reinforced polymers is the flexibility in their composition which enables a designer to design a structure to specifically meet constraints. The simplified analytical method provides a rapid means for the initial design purpose and explores quantitatively the influence of the various constituent properties on reliability. This has enabled the variables having the greatest influence on reliability to be identified and may allow the engineer to concentrate on these more important variables.

- The uncertainties of variables are generally caused by the lack of data, modelling simplifications, human errors or inadequate knowledge of physical phenomena and it is important to have an understanding for how these will affect a given structure. The parametric study provided an insight into COV of various parameters, provided by different sensitivity factors, to the effect on the reliability index allowing an understanding of how the variations, whatever their cause, affect this value.

- The results show that the model uncertainty and applied loads are sensitive to the distribution and variation of the variables. Thus more precisely knowing the statistics of these design variables will induce more meaningful results in the reliability analysis. Improving the accuracy of the analysis method, such as by applying more advanced methods can minimise the modelling error and its variation so as to increase the safety index. Furthermore in a practical structural design, knowing the most important design parameters and their impact on safety index enables the designers to know where to look to improve reliability.

- The variation in component thicknesses and the fibre content within a component has a large impact on the reliability. Therefore, if the uncertainties of such random variables can be reduced through appropriate care or stricter quality control, the reliability can be increased substantially. The analysis indicates that this area will give a good cost to benefit ratio in considering increasing the reliability either at the design or production stages."

- The influence of the variation in the fibre modulus E_f is stronger than in the shear modulus G_m on reliability. That indicates that it is better to reduce the uncertainty of E_f rather than G_m in terms of their relative importance with respect to variations in their standard deviations. More testing would be required to control the scatter of this significant variable.

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Appendix A

For stiffened plates, the section is made from an assembly of flat layered laminated composite element such as crown, web and flange, which will be referred to as elements of section. The material properties vary from element to element, depending on the laminate configuration in each element.

In order to perform the analysis of structures made from composite laminated plate using the methods mentioned above, the equivalent Young's modulus value is required for each element. Symmetric laminates are considered here only as they are the majority of laminate configurations used in practice. The coupling stiffness terms B_{ij} are zero for symmetric laminates: implying that there are no membrane-bending coupling effects. From Datoo [27], the membrane equivalent Young's modulus value of the laminate plate in the x-direction and the y-direction are

$$\begin{cases} E_x^m = (A_{11}A_{22} - A_{12}^2)/A_{22}t \\ E_y^m = (A_{11}A_{22} - A_{12}^2)/A_{11}t \end{cases} \qquad (A.1)$$

where A_{ij} are called extensional stiffness; t is the total thickness of the laminate element under consideration.

Appendix B

For each element, the average direct stress value acting through the thickness of the element of a particular laminate can be predicted at any point in the section using Eq.(12), then the corresponding direct force intensity per unit width N and moment intensity per unit width M of the laminate section about its own mid-plane can be obtained. Then the ply-to-ply stress analysis can be performed by using plate theory [28].

The force and moment resultants to the strains of a laminate are given in a compact form as

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\varepsilon^1\} \end{Bmatrix} \qquad (B.1)$$

$$\{\varepsilon^0\} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} \quad \{\varepsilon^1\} = \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \qquad (B.2)$$

where $\{\varepsilon^0\}$and $\{\varepsilon^1\}$ are vectors of the membrane and bending strains.

The extensional stiffnesses A_{ij}, the bending-extensional coupling stiffnesses B_{ij} and the bending stiffnesses D_{ij}, are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-t/2}^{t/2} \overline{Q}_{ij}(1, z, z^2)dz \tag{B.3}$$

Strains at any point in the plate can be computed

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \tag{B.4}$$

With these values, the strains $(\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy})$ through the laminate thickness can be determined and the ply-to-ply stresses can be calculated based on the constitutive relations and the failure criteria is used to determine whether the plate is failure.

Appendix C

Elastic properties for a unidirectional layer should be established ideally by tests, however, for initial design purpose, it may be obtained by several simple approximations to the elastic constants with reasonable accuracy. Semi-empirical equations of moduli derived by Halpin and Tsai are chosen here [17], i.e. Young's modulus E_1 in the fibre direction, Young's modulus E_2 in the transverse direction, major Poisson's ratio v_{12} and shear modulus G_{12} as follows

$$E_1 = E_f V_f + E_m V_m \tag{C.1}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \tag{C.2}$$

$$M = M_m \times \frac{1 + \xi \eta V_f}{1 - \eta V_f} \tag{C.3}$$

$$\eta = \frac{M_f / M_m - 1}{M_f / M_m + \xi} \tag{C.4}$$

where $M = E_2$ or G_{12}, $M_f = E_f$ or G_f, $M_m = E_m$ or G_m respectively. Reinforcing factor $\xi_G = 1$ for the prediction of shear G_{12} and $\xi_E = 2$ for the Young's modulus E_2 approximately.

The strengths of a unidirectional composite layer can be obtained using simple equations as follows
Longitudinal tensile strength

$$X_t = E_f V_f \varepsilon_f^* \qquad \text{When } \varepsilon_f^* > \varepsilon_m^* \tag{C.5}$$

$$X_t = (E_f V_f + E_m V_m)\varepsilon_f^* \qquad \text{When } \varepsilon_f^* < \varepsilon_m^* \tag{C.6}$$

Transverse tensile strength and in-plane shear strength

$$Y_t = \left(1 - (\sqrt{V_f} - V_f)\left(1 - \frac{E_m}{E_2}\right)\right)\sigma_m^* \tag{C.7}$$

$$S_t = \left(1 - (\sqrt{V_f} - V_f)\left(1 - \frac{E_m}{E_2}\right)\right)\tau_m^* \tag{C.8}$$

where ε_m^* and ε_f^* is strains to failure of the resin and fibres respectively. σ_m^* is the matrix tensile strength, τ_m^* is the matrix shear strength. The longitudinal and transverse compressive strength can be approximately estimated by the same equation.