Economics Division
School of Social Sciences
University of Southampton
Southampton SO17 1BJ
UK

**Discussion Papers in
Economics and Econometrics**

CREDIT RATIONING AND FIRMS IN OLIGOPOLY

Jian Tong

No. 0505

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Jian Tong[1]
University of Southampton

June 2005

[1]Correspondence address: Economics Division, School of Social Sciences, University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom. Tel: +44-23-8059 6801, Fax: +44-23-8059 3858, E-mail: j.tong@soton.ac.uk

Abstract

This paper develops a theory of the firm, and equilibrium credit rationing mechanisms in oligopoly with R&D-product market competition. Credit rationing arises from a hold-up problem between wealth-constrained entrepreneurs and external investors. Underinvestment occurs if entrepreneurial wealth constraint is binding, even though the equilibrium corporate governance structure addresses the hold-up problem optimally. In a symmetric equilibrium outcome all firms face equitable credit-size rationing. In contrast the asymmetric equilibrium outcome sees some firms (the 'preys') denied external credits entirely while the others (the 'predators') receiving more favorable finances, which turns out to increase market concentration and overall R&D investments.

Key words: credit rationing, oligopoly, hold-up, corporate governance, theory of the firm, market structure, predation

1 Introduction

There has been a growing concern among economists that the simplistic view of a firm as a profit-maximizing 'black box' may be inadequate for understanding corporate investment behavior. For example, long term and technically complex investments, such as research and development projects, are known to involve many contracting difficulties which can not be addressed by the 'black-box' theory of the firm. A gap in external finance for R&D has long been documented by economists. This finding can trace its origin back as far as Schumpeter (1942), and was more fully articulated by Arrow (1962) and subsequent researchers. It identifies a discrepancy between the private return to a firm and the cost of capital when the innovation investor and financier are different economic agents[1]. Small and start-up firms in R&D-intensive industries are found to suffer most from such a funding gap, relying heavily on internal funding for R&D investments (Himmelberg and Petersen (1994)). (See Hall (2002) for a recent survey of the literature on this topic.) The R&D funding gap literature is in tune with a larger empirical literature which provides widely spread evidence that corporate investments are positively correlated with internal finances (i.e., cash flows)[2], suggesting credit constraints from external capital markets, particularly for small start-up firms (see Fazzari, Hubbard and Petersen (1988), Hubbard (1998)). It seems fair to claim that the problem of credit constraints in relation to corporate investments has formed a prominent challenge to the theory of the firm.

While the empirical literature of 'funding gap' has a focus on evidencing the existence of credit constraints and the role of internal finance, its theoretical inspiration and underpinnings come broadly from the theories of equilibrium credit rationing, the central theme of which has been to rationalize quantity rationing of credits as a rational behavior (see, for example, Stiglitz and Weiss (1981), Allen (1983), Gale and Hellwig (1985), Williamson (1987)). It has now been established that when equilibrium quantity rationing occurs, the usual price mechanism does not work to the effect of equating demand and supply at the equilibrium price. Instead, suppliers impose limits on the quantities directly, which are short of the demand at the going market price, and are not to be revised in case of higher bidding prices. This kind of equilibrium is possible in the financial market if the higher bidding price is associated with higher default risk, hence, lower expected return. What remains open questions to economists are: What do corporations do (contractually and or organizationally) to address the problem of credit rationing? How does the market structure of the industry where the firms belong to respond to the credit rationing mechanisms? And how do the reactions at these two levels interact?

Although a comprehensive review of the theoretical credit rationing litera-

[1] This is independent of the argument that underinvestment in R&D occurs because of the existence of externality.

[2] The most robust finding from this literature is that there exists an excessive sensitivity of corporate investments to cash flows. Numerous auxiliary evidence suggests that cash flows should be seen as a reasonable proxy of internal finance.

ture is beyond the scope of the current paper, a few remarks would be helpful to motivate our research. The existing theories of endogenous credit rationing have been able to rationalize two types of credit rationing. The first type, which we call the 'Type I credit rationing', rations credit size, i.e., the amount of credit extended is positive but below the level that maximizes the borrowing firm's profit (Gale and Hellwig (1985)). The second type, which we refer to as the 'Type II credit rationing', denies credits to an unsuccessful fringe, i.e., only a fraction of the applicants can obtain a positive amount of credit (Stiglitz and Weiss (1981)). Gale and Hellwig (1985) considers a borrower-lender relationship which is subject to asymmetric information and costly state verification (monitoring) by lenders. The optimal (second-best) financial contract turns out to be the standard debt contract. Credit rationing (under-investment) arises in the optimal contract to the effect of saving on the default risk and monitoring cost. Stiglitz and Weiss (1981) study a potential adverse selection problem in the loan market. Randomized credit denial arises in equilibrium because the lenders who are less informed about the project qualities understand that charging high interest rate would attract more high-risk borrowers at the expense of driving away some low-risk borrowers, and result in higher expected default risk and lower expected returns. At the going market rate, the demand exceeds the supply of fund and a fraction of the loan applicants are denied credit entirely. Since Stiglitz and Weiss (1981) obtain the result of credit denial in a setting where the size of each investment is exogenously fixed, therefore it is not clear whether the credit denial thereof is just a disguised credit size rationing.

A general intuition about equilibrium credit rationing is that the risk of default on financial contract becomes endogenous as a result of informational asymmetry, moral hazard or other contracting problems[3]. Situations may occur where higher price is associated with higher default risk and lower expected return due to the various contracting problems, therefore in equilibrium the aforementioned two types of credit rationing mechanisms arise to replace the usual price mechanism in allocating capital among competing usages. As far as financial markets are concerned, quantity rationing becomes a fundamental part of the market mechanism. The effects of credit rationing on the real sectors of economies have long been a focus of interest for economists.[4] The most interesting micro implications of credit rationing concern the relationship between financial market and industry structure. One question, which receives much attention due to Bain (1956)'s controversial assertion that the disadvantage faced by entrants vis-a-vis incumbents in access to capital market formed a barrier to entry, is whether credit rationing contributes to market concentration. For example the 'long purse' theory of predation (see Fudenberg and Tirole (1985), Fudenberg and Tirole (1986), Bolton and Scharfstein (1990)) suggests that the endogenous credit rationing mechanism embedded in the relationship between a

[3]As will transpire in this article, the hold-up problem due to contract incompleteness is another source of the contracting problems.

[4]Macro implications of credit rationing have been studied in the areas of monetary policy, business cycle, and economic growth (see Stiglitz and Weiss (1992), Holmstrom and Tirole (1997), Kiyotaki and Moore (1997), Tong (2005b), Aghion, Howitt and Mayer-Foulkes (2004)).

firm and its external investors makes it particularly vulnerable to predation from its 'deep-pocket' rival(s), which jeopardizes its ability to invest and compete in the future, and results in higher market concentration in equilibrium. It is not clear, though, whether and how this example can be generalized.[5] The implications of credit rationing on market structure are not only interesting from an antitrust point of view, but also important for understanding the determinants of industrial R&D, innovation and technological progress. Schumpeter (1942) proposed that small firms were constrained in their ability to innovate because they had difficulties in access to capital market, therefore large corporations should be expected to play a major role in R&D and technological progress. Despite the Schumpeterian hypothesis about the relationship between firm size and innovation having inspired enormous research interest among economists (see Cohen and Levin (1989) for a survey on the empirical literature of testing the Schumpeterian Hypothesis), an integrated theory of equilibrium credit rationing, endogenous firm size distribution (market structure) and R&D is still absent after sixty three years since Schumpeter's seminal work.

This paper sets out to develop a theory of the firm and equilibrium credit rationing mechanisms in oligopoly, which features R&D-product market competition and endogenous industry structure. It blends two related literatures: one is the literature on technology and market structure (Dasgupta and Stiglitz (1980), Sutton (1991), Sutton (1998)); the other is the literature of incomplete contract theories of the firm and financial contracting (Grossman and Hart (1986), Hart and Moore (1988), Hart and Moore (1990), Hart and Moore (1994), Hart (1995), Hart and Moore (1998), Aghion and Bolton (1992)). Departing from the black-box view, we define a firm by its integrated business-corporate strategy. The business strategy side deals with how the firm is going to compete with rival firms, both in product market and in R&D. It determines the demand for long-term investment (e.g., R&D). The corporate strategy side deals with how to engage external investors in financing the long-term investment which the business strategy entails, and is beyond the reach of entrepreneurial own wealth. Whereas the former literature provides us a natural basis to understand the firms' business strategies in free R&D-product market competition, the latter helps us to gain insights on the firms' corporate strategies. It has now been well understood, that when long-term and technically complex investments (e.g., R&D projects[6]) are involved in a transaction, a complete (enforceable) ex ante contract that governs every possible aspect of the transaction is hardly feasible

[5] Two questions arise here. First, what will happen if the firms that enter the market all have the same depth (deep or shallow) of their pockets? Second, what fundamentally causes the distinction between the deep pockets and shallow pockets among the firms that enter the market?

[6] According to Hall (2002), "in practice 50 per cent or more of R&D spending is the wages and salaries of highly educated scientists and engineers. Their efforts create an intangible asset, the firm's knowledge base, from which profits in future years will be generated. To the extent that this knowledge is 'tacit' rather than codified, it is embedded in the human capital of the firm's employees, and is therefore lost if they leave or are fired." Using the terminology of Hart and Moore (1994), the outcome of R&D investments is, to a large extent, in the form of inalienable human capital.

or desirable[7] (Williamson (1975), Williamson (1985), Hart and Moore (1999), Hart and Moore (2004)); ex post negotiation (bargaining) is inevitable. The incomplete contract theory of the firm literature has had an extensive investigation on the so call 'hold-up' problem, and drawn two general conclusions. First, the 'hold-up' problem tends to result in under-investment. Second, the equilibrium organization forms (e.g., allocation of property rights) may be understood as an optimal (the second-best) solution to mitigate the 'hold-up' problem.

Our research contributes to the incomplete contract theory of financial contracting, which is concerned with the equilibrium allocation of controls within a firm that is founded by an entrepreneur who is wealth constrained (Aghion and Bolton (1992), Hart and Moore (1998)). The existing models usually consider fixed size investment projects, hence, do not address the issue of credit-size rationing. We model an industry where R&D investment is a continuous variable that increases a firm's market share and ex post profit, and there is a free entry of entrepreneurs who can start up businesses in this market. The founding entrepreneur of each firm is wealth-constrained, and has to seek the participation of external investors who themselves are not good at running the business. The investments by the external investors are essential only because the entrepreneur of the firm does not have sufficient internal funding. The outcome of an R&D project, to a large extent[8], resides in the entrepreneur's human capital, which can not be deprived by the external investors if the two parties terminate their relationship. The potential hold-up of the external investors by the entrepreneur may constitute a de facto strategic 'default' on the financial contract[9]. The inability of the 'firm' to pledge the entirety of the marginal return of investment to the external investors forces the external investors to ration the size of credit, which results in under-investment in R&D if the entrepreneurial wealth constraint is already binding.[10] To address the problem of underinvestment caused by credit size rationing, the equilibrium corporate governance structure of the firm, i.e., the allocation of controls (which determines the bargaining powers of the two parties), assigns maximal feasible external investor protection in exchange for maximal feasible external capital participation. This 'second best' solution alleviates, but does not eliminate, the underinvestment problem.[11]

[7] The issue of incompleteness of contracts is particularly salient when the transactions involve R&D, which is typically associated with complexity in technicality and a high degree of uncertainty.

[8] The degree to which R&D fruition is tacitly embedded in the entrepreneur's human capital is an endogenous variable, depending on the firm's governance of the intellectual properties, but bounded from above by some exogenous limit.

[9] If the financial contract takes the form of debt contract, then the hold-up literally constitutes a default on the debt service obligation. In case of equity financing, the hold-up may not literally constitute a default on a formal financial contract, but it is a default on an implicit contract.

[10] If the entrepreneurial wealth constraint is not binding, the investment deficit can be made up by entrepreneurial own wealth.

[11] If the entrepreneurial wealth constraint is not binding, the 'first best' result can be achieved in this model by a continuum of corporate governance structure and the associated level of external capital participation. In a sense, financial structure doesn't matter hereby, which echoes the familiar Modigliani-Miller (irrelevance) theorem.

In the equilibrium of our model, all firms' business-corporate strategies are jointly determined, which also determine the market structure. Therefore the equilibrium credit rationing mechanisms interact with the industry structure. One possible outcome of the interaction is a credit rationing mechanism that has a simple and equitable credit-size rationing rule. So that all firms are rationed and underinvest in a symmetric way, and free entry leads to a larger number of smaller firms who just break even. In this type of symmetric equilibrium outcome, competition in R&D is milder and competition in product market is more intense, which hamper the overall investments in R&D. This is the kind of equilibrium outcome that Schumpeter (1942) criticized heavily. There is another type of equilibrium outcome where the credit rationing mechanism comprises two sets of rules. One relates to credit denial to a certain fraction of firms, the other relates to credit-size rationing the firms who are not denied credits. It turns out that when there is credit denial, the credit-size rationing rule is more favorable than the one adopted in a symmetric equilibrium outcome. In this kind of asymmetric equilibrium outcome, free entry gives rise to a skew firm size distribution and a divide between winners ('predators') and losers ('preys'). The winners - the endogenous 'deep pocket' firms 'predate' upon the losers - the 'shallow pocket' firms whose vulnerable relationships with external investors break down as a result of the 'predation'[12].[13] Our theory shows that credit denial and the associated 'predatory' outcome arise as a 'market solution' to the contractual problems that cause credit rationing in the first place. In the context of large R&D-intensive industries, this market solution entails a selective allocation of credits and a competition for credits among a pool of excess entrepreneurial entrants. The external investors play an important role in selecting the winners. Although the selection process involves 'predation' among the firms, causing skewness in firm size distribution and raising market concentration, it nevertheless has an efficiency defence in terms of motivating more investments in R&D.[14]

The organization of the remainder of the paper is as follows. Section 2 constructs the model. Section 3 characterizes the symmetric equilibrium outcomes, particularly, those with binding credit constraints. In Section 4, we characterize asymmetric equilibrium outcomes where both types of credit rationing (credit size rationing and credit denial) coexist. Section 5 concludes.

[12] The 'deep-pocket' firms' behavior is 'predatory' in the sense that if the collapse of the 'shallow-pocket' firms' corporate strategy is not at stake, the former's integrated business-corporate strategies would have been much less aggressive.

[13] In a companion paper, Tong (2005a), develops a two-period model of predatory pricing, where the distinction between 'deep pocket' and 'shallow pocket' firms is entirely endogenous. The firms in the model can benefit from a learning effect in the second period if they produce in large quantities in the first period. The opportunity cost incurred in the form of forgone first-period profit constitutes an fixed investment cost. In the asymmetric equilibrium outcome, it is possible that in the first period the deep-pocket firms push the price to a predatory low level which jeopardizes the shallow-pocket firms' ability to learn in the first period and to survive in the second period.

[14] So this is the kind the equilibrium outcome one might be concerned with in light of Bain (1956), but would rather endorse following Schumpeter (1942) and in the new light of the current investigation.

2 The Model

We begin by describing the spot market where the industry sells its product. There are a number S of identical consumers in this market of a vertically differentiable product. Hence parameter S measures the size of the market. Each consumer's preference is given by the following quasi-linear utility function:

$$\begin{aligned} U &= \sum_{i=1}^{N} (u_i q_i) - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} q_i q_j + m \qquad (1) \\ s.t. &: \sum_{i=1}^{N} p_i q_i + m \leq I \end{aligned}$$

where N is the number of active firms that offer (a positive quantity of) the vertically differentiable product; u_i and q_i are the quality and quantity supplied by firm i; m is the numeraire; I is the income.

The first order necessary condition of the above maximization program implies the following individual inverse demand function: $p_i = u_i - \sum_{j=1}^{N} q_j$. Each firm has a constant marginal cost c_i (for firm i). When the number of firms and their respective levels of quality and marginal costs are fixed, the firms compete in the manner a la Cournot[15]. Hence, each firm's objective is given by the following maximization program:

$$\max \Pi_i = S \left(u_i - \sum_{j=1}^{N} q_j - c_i \right) q_i. \qquad (2)$$

The equilibrium output and price-cost margin at individual firm level are

$$q_i = p_i - c_i = \max \left(0, \kappa_i - \frac{1}{(N+1)} \sum_{j=1}^{N} \kappa_j \right) \qquad (3)$$

for all $i = 1, 2, \cdots, N$, where $\kappa_i \triangleq u_i - c_i$ is firm i's level of 'capability'. So a firm can have a high capability by possessing a high quality or a low marginal cost. In equilibrium, $q_i \geq 0$ and $p_i - c_i \geq 0$ are satisfied. The implied equilibrium profit of each firm is given by

$$\Pi_i (\kappa_i; \kappa_{-i}) = S\pi_i (\kappa_i; \kappa_{-i}) = \frac{S}{(N+1)^2} \left(\kappa_i + \sum_{j=1}^{N} (\kappa_i - \kappa_j) \right)^2, \qquad (4)$$

conditional on all firms: $i = 1, 2, \cdots, N$ remain active; where $\kappa_{-i} = (\kappa_j)_{N-1}$ for $j \neq i$ is the vector of all rivals' levels of capability; $\pi_i (\kappa_i; \kappa_{-i})$ is the normalized

[15] The main results of the paper does not rely on this specific assumption of firm conduct, e.g., a Bertrand competition with horizontal differentiation would not change them.

profit function (for unit market size). Most of the properties of $\Pi_i(\kappa_i;\kappa_{-i})$ are the properties of $\pi_i(\kappa_i;\kappa_{-i})$, which will be characterized in what follows.

To start with, $\pi_i(\kappa_i;\kappa_{-i})$ is twice continuously differentiable conditional on the composition of the set of active firms remains unchanged[16], non-decreasing in own capability, and non-increasing in any rival's capability, i.e., $\frac{\partial \pi_i}{\partial \kappa_i} \geq 0$, $\frac{\partial \pi_i}{\partial \kappa_j} \leq 0$. In general the elasticity of π_i with respect to κ_i is

$$\frac{\partial \pi_i}{\partial \kappa_i}\frac{\kappa_i}{\pi_i} = \frac{2N}{1+\sum_{j=1}^{N}\left(1-\frac{\kappa_j}{\kappa_i}\right)}. \tag{5}$$

It happens that when valued at a symmetric configuration, the elasticity is independent of the level of capability, but increases with the number of firms, more precisely,

$$\left.\frac{\partial \pi_i}{\partial \kappa_i}\frac{\kappa_i}{\pi_i}\right|_{(\kappa_i;\kappa_{-i})=\left(\kappa;(\kappa)_{N-1}\right)} = 2N > 0, \tag{6}$$

which means that the more fragmented the symmetric market structure, the higher the proportional marginal benefit of investing in capability. The property is driven by the 'business stealing' effect of increasing capability. Also a firm's proportional marginal benefit of increasing capability is non-increasing in its level of capability in an oligopoly, i.e.,

$$\frac{\partial\left(\frac{\partial \pi_i}{\partial \kappa_i}\frac{\kappa_i}{\pi}\right)}{\partial \kappa_i} < 0 \text{ for } N \geq 2 \text{ and } \pi_i > 0. \tag{7}$$

Having characterized a firm's (proportional) marginal benefit of investing in capability, now we look at the proportional marginal cost of investing in capability by considering the following simple increasing and convex fixed cost function:

$$F(\kappa) = \kappa^{\beta}, \ \ (\beta > 2), \tag{8}$$

which has a constant elasticity[17] or proportional marginal cost, i.e.,

$$\frac{\partial F}{\partial \kappa}\frac{\kappa}{F} = \beta. \tag{9}$$

In what follows we study how the levels of capability are determined in equilibrium when there are free entry and conflict of interest between the entrepreneurs and the external investors of the firms. The conflict of interest origins from the lack of commitment power by the entrepreneurs (corporate insiders) to pledge the entirety of profit to external investors. For simplicity, we model

[16] In case of any change in the composition of the set of active firms, additional active firms must be included in the expression, and inactive firms must be excluded from it. As a result, such a change may produce a 'kink' (i.e., continuous but non-differentiable point) in the profit function.

[17] As illustrated in Sutton (1998), what really matters is that the elasticity of the fixed cost function is bounded from above by a constant $\beta < \infty$. Assuming a constant elasticity makes the analysis much more tractable.

it as an ex post bargaining over the split of the profit: none of the two parities has an (ex post) outside option and the external investors entrepreneur's bargaining power is $s_i \in [0, \theta]$ where $\theta \in (0, 1)$. Then the pledgeable income[18] for the external investors is $s_i \Pi_i (\kappa_i; \kappa_{-i})$, and the entrepreneur takes the remaining $(1 - s_i) \Pi_i (\kappa_i; \kappa_{-i})$. Variable s_i is an endogenous variable that summarizes all the information about the corporate governance structure, including the allocation of property rights (hence, formal authority), delegation (hence, real authority) and decision procedure (hence, bargaining). As will transpire in the equilibrium analysis of the model, s_i will be optimally chosen to address the firm's demand for external investment and external investors' concern about the return of their investments.

The following three-stage game describes the environment where firms are incorporated, financed and run, and the industry structure is determined.

Stage 1: A (sufficiently large) number of potential entrepreneurs simultaneously decide whether to enter the market.

Stage 2: Upon entry, entrepreneurs simultaneously announce their target capability levels, and each proposes a take-it-or-leave-it offer to the external investors in the competitive capital market. So each entrepreneur's (integrated business-corporate) strategy is a vector $(\kappa_i, \tilde{\theta}_i, s_i)$, where κ_i is the capability level which summarizes the firm's business strategy, $\tilde{\theta}_i$ is the external investors' share of capital cost, which summarizes the firm's source of R&D finance, and $s_i \in [0, \theta]$ is the bargaining power assigned to the external investors by the endogenous corporate governance structure, which has an exogenous upper bound $\theta \in (0, 1)$. Each entrepreneur has to build the ability to commit to the corporate governance structure s_i, which requires the commitment of the insider equity[19]: $\left(1 - \tilde{\theta}_i\right) F(\kappa_i)$ at the time of the offering, which is funded from the entrepreneurial wealth ω.

- If the offer is accepted with the requested external investments, then it is implemented, namely, the corporate governance structure is ratified, the funds are collected from the external investors, the target capability is developed, and the investments are sunk.

[18] The term 'pledgeable income' is coined by Tirole (2001).

[19] The justification of this assumption is the following argument. The entrepreneur's commitment to the proposed corporate governance structure is investment outcome-specific. For example, the allocation of the control of certain intellectual property rights can only be arranged after the intellectual property rights have been created or defined as an outcome of the investment. Therefore the ability to commit to the proposed corporate governance structure depends on the extent to which the investment has been committed. Failure to commit the promised insider equity is a jeopardy to the ability to commit to the proposed corporate governance structure. For the sake of argument, suppose that a failure to commit insider equity is equivalent to choosing $s_i = 0$, which surely makes the contract unacceptable. Then, committing the proposed insider equity is a necessary condition for the contract to be accepted.

- If the offer is not accepted with the requested external investments, it is void; and the de facto investment level is $\left(1-\tilde{\theta}_i\right) F\left(\kappa_i\right)$, with no external investors involved.

Stage 3: After the investments are sunk, the entrepreneurs and external investors bargain over the sharing of ex post profits,[20] $\Pi_i\left(\kappa_i;\kappa_{-i}\right)$.

- The bargaining outcomes are determined by the Nash bargaining solution with external investors' bargaining power being s_i.
- The profit of a solely internally financed firm is accrued entirely to the entrepreneur.

We characterize the equilibrium of the game by backward induction. The equilibrium of the third stage subgame is self evident from the above description of the game. So we start with the second stage. Here we have to distinguish two types of equilibria. The first type does not involve any credit denial, i.e., the number of entrepreneurs and the offers they make are such that all the contracts are accepted. The second type of equilibrium sees some entrepreneurs are denied external credit hence relying on internal finance entirely. In this and the next section we focus on the first type of equilibrium. The second type is dealt with in Section 4.

At stage 2, conditional on the contract is accepted with certainty, each entrepreneur's program is to choose the integrated business-corporate-strategy $\left(\kappa_i,\tilde{\theta}_i,s_i\right)$ to maximize the entrepreneurial surplus, i.e.,

$$\max_{\left(\kappa_i,\tilde{\theta}_i,s_i\right)} \left(1-s_i\right) S\pi_i\left(\kappa_i;\kappa_{-i}\right)-\left(1-\tilde{\theta}_i\right) F\left(\kappa_i\right) \tag{10}$$

subject to:

$$s_i S\pi_i\left(\kappa_i;\kappa_{-i}\right)-\tilde{\theta}_i F\left(\kappa_i\right)\geq 0, \tag{11}$$

$$\left(1-\tilde{\theta}_i\right) F\left(\kappa_i\right)\leq \omega, \tag{12}$$

$$s_i\leq\theta \tag{13}$$

where the first constraint (11) is the external investors' participation constraint, which says the external investor surplus can not be negative; the second, (12), is the entrepreneurial wealth constraint; the third, is the corporate governance constraint, which says the maximal external investor protection is bounded from above by the exogenous limit θ.

[20] This is to implicitly assume that the sharing of profit can not be contracted ex ante, or the contracts will have to be renegotiated ex post. It is an assumption of incompleteness of contacts. Alternatively, we can assume that the sharing of profit is specified in the contract, but impose that only renegotiation-proof contracts are accepted in equilibrium.

The Lagrangian function of the program is

$$\begin{aligned}\mathcal{L} &= \left((1-s_i)\,S\pi_i(\kappa_i;\kappa_{-i}) - \left(1-\tilde{\theta}_i\right)F(\kappa_i)\right) \\ &\quad -\lambda_i\left(\tilde{\theta}_i F(\kappa_i) - s_i S\pi_i(\kappa_i;\kappa_{-i})\right) - \nu_i\left(\left(1-\tilde{\theta}_i\right)F(\kappa_i) - \omega\right) - \eta_i(s_i-\theta)\end{aligned}$$

where $\lambda_i \geq 0$, $\nu_i \geq 0$ and $\eta_i \geq 0$ are the Lagrangian multipliers, which are the 'shadow price' of one unit reduction of the external investor surplus, the marginal entrepreneurial surplus of the entrepreneurial wealth, and the 'shadow price' of one unit increase of the upper limit of the external investor bargaining power.

The first order necessary conditions for the solution of the program are:

$$\frac{\partial\mathcal{L}}{\partial\kappa_i} = (1-s_i)\,S\frac{\partial\pi_i}{\partial\kappa_i} - \left(1-\tilde{\theta}_i\right)F'(\kappa_i) - \lambda_i\left(\tilde{\theta}_i F'(\kappa_i) - s_i S\frac{\partial\pi_i}{\partial\kappa_i}\right) - \nu_i\left(1-\tilde{\theta}_i\right)F'(\kappa_i) = 0 \tag{14}$$

$$\frac{\partial\mathcal{L}}{\partial\tilde{\theta}_i} = F(\kappa_i) - \lambda_i F(\kappa_i) + \nu_i F(\kappa_i) = 0 \tag{15}$$

$$\frac{\partial\mathcal{L}}{\partial s_i} = -S\pi_i(\kappa_i;\kappa_{-i}) + \lambda_i S\pi_i(\kappa_i;\kappa_{-i}) - \eta_i = 0 \tag{16}$$

$$\begin{cases} s_i S\pi_i(\kappa_i;\kappa_{-i}) = \tilde{\theta}_i F(\kappa_i) & \text{if } \lambda_i > 0 \\ s_i S\pi_i(\kappa_i;\kappa_{-i}) > \tilde{\theta}_i F(\kappa_i) & \text{if } \lambda_i = 0 \end{cases} \tag{17}$$

$$\begin{cases} \omega - \left(1-\tilde{\theta}_i\right)F(\kappa_i) = 0 \text{ if } \nu_i > 0 \\ \omega - \left(1-\tilde{\theta}_i\right)F(\kappa_i) > 0 \text{ if } \nu_i = 0 \end{cases} \tag{18}$$

$$\begin{cases} \theta - s_i = 0 \text{ if } \eta_i > 0 \\ \theta - s_i > 0 \text{ if } \eta_i = 0 \end{cases} \tag{19}$$

Eq. (15) gives rise to

$$\lambda_i = \nu_i + 1, \tag{20}$$

which is used to simplify (14) and (16), yielding

$$\nu_i = \frac{S\frac{\partial\pi_i}{\partial\kappa_i} - F'(\kappa_i)}{F'(\kappa_i) - s_i S\frac{\partial\pi_i}{\partial\kappa_i}}, \tag{21}$$

$$\lambda_i = \frac{(1-s_i)\,S\frac{\partial\pi_i}{\partial\kappa_i}}{F'(\kappa_i) - s_i S\frac{\partial\pi_i}{\partial\kappa_i}}, \tag{22}$$

and

$$\eta_i = \nu_i S\pi_i(\kappa_i;\kappa_{-i}) = \frac{S\frac{\partial\pi_i}{\partial\kappa_i} - F'(\kappa_i)}{F'(\kappa_i) - s_i S\frac{\partial\pi_i}{\partial\kappa_i}} S\pi_i(\kappa_i;\kappa_{-i}). \tag{23}$$

Since $s_i \leq \theta < 1$ it follows form (21) that $\nu_i \geq 0$ if and only if

$$s_i S\frac{\partial\pi_i}{\partial\kappa_i} \leq F'(\kappa_i) \leq S\frac{\partial\pi_i}{\partial\kappa_i}, \tag{24}$$

which rules out the possibility of $\frac{\partial \pi_i}{\partial \kappa_i} = 0$ for $\kappa_i > 0$. And then (22) entails that $\lambda_i \neq 0$. As a result, (17) implies:

Lemma 1 *One necessary condition for the solution to program (10) is that the external investors' participation constraint (11) must be binding, i.e., $\lambda_i > 0$ and*

$$s_i S \pi_i (\kappa_i; \kappa_{-i}) = \tilde{\theta}_i F(\kappa_i). \tag{25}$$

The above lemma says that in equilibrium the external investor surplus is exactly zero. This result is not surprising given that there is perfect competition in the capital market and extracting more external investor surplus makes the entrepreneur better off.

Depending on the range of parameters, one (and only one) of the following two possible cases occurs in equilibrium.

Case 1: The entrepreneurial wealth constraint (12) is not binding, hence $\nu_i = 0$, $\lambda_i = 1$, $\eta_i = 0$, and

$$S \frac{\partial \pi_i}{\partial \kappa_i} = F'(\kappa_i). \tag{26}$$

i.e., the firm as a whole maximizes its net profit; there is neither "under-investment" nor "over-investment" from the firm's profit-maximizing point of view. In this case, the corporate governance constraint is not binding either, i.e., the entrepreneur sees no need to provide maximal feasible protection to external investors.

Case 2: The entrepreneurial wealth constraint (12) is binding, hence $\nu_i > 0$, and

$$\left(1 - \tilde{\theta}_i\right) F(\kappa_i) = \omega. \tag{27}$$

Consequently, condition (23) implies $\eta_i > 0$ and

$$s_i = \theta. \tag{28}$$

From (25), (27) and (28), it follows that

$$\theta S \pi_i (\kappa_i; \kappa_{-i}) - F(\kappa_i) + \omega = 0. \tag{29}$$

The left-hand side of the above equation is the external investor surplus function conditional on binding entrepreneurial wealth constraint. To emphasize its importance, we denote it by $Z_i(\kappa_i; \kappa_{-i})$, hence,

$$Z_i(\kappa_i; \kappa_{-i}) \triangleq \theta S \pi_i (\kappa_i; \kappa_{-i}) - F(\kappa_i) + \omega. \tag{30}$$

Then condition (29) can be reformulated as the following zero external investor surplus condition:

$$Z_i(\kappa_i; \kappa_{-i}) = 0. \tag{31}$$

Since the solution to this equation is not necessarily unique, the following lemma specifies which one of them should be selected in equilibrium.

Lemma 2 *When the entrepreneurial wealth constraint is binding, the global maximum of program (10) corresponds to the maximal solution to eq. (31).*

Proof. Suppose the equilibrium selects a sub-maximal solution to condition (31), say κ_i^*, and there exists $\tilde{\kappa}_i > \kappa_i^*$ which is the maximal solution to condition (31). Given that the entrepreneurial wealth constraint is binding, increasing κ_i from κ_i^* to $\tilde{\kappa}_i$ which is solely financed by the external investors is a Pareto improvement, i.e., it is acceptable to the external investors, and makes the entrepreneur better off. This constitutes a feasible and profitable deviation for the entrepreneur, which contradicts the assumption that κ_i^* is an equilibrium solution. ■

The following proposition states the role of entrepreneurial wealth in affecting the behavior of the firm.

Proposition 3 *If the entrepreneurial wealth constraint is not binding, the corporate governance constraint is not binding either and equilibrium level of investment maximizes the firm's net profit, i.e., condition (26) is satisfied. If the entrepreneurial wealth constraint is binding, then there is an under-investment in capability from a firm's profit-maximizing point of view, i.e.,*

$$S\frac{\partial \pi_i}{\partial \kappa_i} > F'(\kappa_i). \tag{32}$$

To address this problem optimally, the equilibrium corporate governance structure offers maximal feasible external investor protection, i.e., $s_i = \theta$.

Proof. When $\nu_i > 0$, condition (21) implies condition (32). ■

The hold-up problem generates a potential conflict of interest between two groups of stakeholders of a firm, i.e., the entrepreneur and the external investors. The above proposition says that when the entrepreneurial wealth constraint is not binding, the two parties can have an efficient (ex ante) bargaining, and achieve the 'first best' result from the firm's profit maximizing point of view. This is possible because in our model the entrepreneurial wealth and external investment are perfect substitutes. The efficient ex ante bargaining resolves around choosing the 'first best' level of total investment, dividing the total investment cost between the two parties, and allocating the ex post bargaining powers such that external investors get exactly zero surplus at this level. This maximizes the firm's net profit as well as the entrepreneurial surplus conditional on the external investors breaking even. In this case, obviously, the way to divide the total investment cost and allocating ex post bargaining powers to achieve the efficient (ex ante) bargaining outcome is not unique. In a sense, the sources of finance and the structure of corporate governance become irrelevant to the outcome. This result is reminiscent to the famous Modigliani and Miller (1958) theorem on the irrelevance of capital structure to corporate investment decisions. The result is reversed if the entrepreneurial wealth constraint is binding, where the two parties can not achieve efficient (ex ante) bargaining. Consequently, the firm, as a whole, underinvests in R&D. In an attempt to alleviate the underinvestment problem, the equilibrium corporate governance structure must offer the external investors the maximal (feasible) protection in order to encourage their capital contribution. Since the external investor participation constraint is always binding, if the entrepreneurial wealth constraint is also binding, it must be the case that firm's investment level is suboptimal. In the rest of paper we simply refer this (respectively, the opposite) situation as "with (respectively, without) binding credit constraints".

3 Equilibrium Outcome with Equal Firm Size Distribution

In what follows we characterize the equilibrium of the whole game. This allows us to see how the endogenous credit rationing mechanism interacts with the industry structure. To simplify the analysis at this stage, we delay the characterization of any asymmetric equilibrium outcomes to Section 4. In this section we characterize the equilibrium outcome with equal firm size distribution, i.e., $\kappa_i = \kappa^*$ for all $i = 1, 2, \cdots, N^*$, where N^* is the number of firms in equilibrium. Given our interest in the effect of competition, we only consider environments which warrant $N^* \geq 2$.

In a symmetric equilibrium outcome with free entry, the following entrepreneurial break-even condition arises:

$$(1 - s_i) S\pi(\kappa^*, N^*) = \left(1 - \tilde{\theta}_i\right) F(\kappa^*) . \tag{33}$$

Now the external investors' participation constraint takes a simpler form:

$$s_i S\pi(\kappa^*, N^*) = \tilde{\theta}_i F(\kappa^*) . \tag{34}$$

The above two conditions yield the following firm's break-even condition:

$$S\pi(\kappa^*, N^*) = F(\kappa^*) \tag{35}$$

and the condition

$$\tilde{\theta}_i = s_i \tag{36}$$

Proposition 4 *In a symmetric equilibrium outcome with free entry and external investments, the external investors' capital share equals to the external investors' ex post bargaining power, i.e.,* $\tilde{\theta}_i = s_i$, *and each firm breaks even.*

The above proposition says given the competition conditions of the R&D-product market and the corporate governance structure of each firm, the external investors choose the credit rationing mechanism precisely according to the protection they obtain from the corporate governance structure. A balanced composition of internal equity and external finance in relation to the corporate governance structure is required if external finance occurs.

3.1 Without binding credit constraints

The results presented in this subsection are useful benchmarks since they shed light on what would happen if there were no imperfection in the capital market. When the entrepreneurial wealth constraint (12) is not binding, conditions (26) and (35) both hold and jointly imply that

$$\left.\frac{\partial \pi_i}{\partial \kappa_i}\frac{\kappa_i}{\pi_i}\right|_{(\kappa_{nb},N_{nb})} = 2N_{nb} = \beta, \tag{37}$$

i.e.,

$$N_{nb} = \frac{\beta}{2} \tag{38}$$

where N_{nb} and κ_{nb} are the equilibrium number of firms and capability level in this case, and subscript $_{nb}$ indicates that the wealth constraint (12) is not binding. Notice that the number of firms (hence market structure) does not depend on the size of the market S. This is because if the market is large, the firms simply escalate their investments in capability, and the burden of the fixed cost prevents the number of firms from increasing. This can be shown by the following equilibrium level of capability:

$$\kappa_{nb} = \left(\frac{S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{1}{\beta-2}}. \tag{39}$$

Clearly, κ_{nb} increases with S. Also interestingly, κ_{nb} does not depend on the entrepreneurial wealth level. We know in this situation, the interests of the entrepreneur and external investors are aligned and the firm maximizes its profit, just as if the firm is solely financed by the entrepreneurial wealth and there is no conflict of interest. In this case, there is an indifference in the financial structure of a firm. An entrepreneur can assign a larger (smaller) share of capital cost to the external investors and at the same time offer them a higher (lower) degree of protection as long as the entrepreneurial wealth constraint is not binding, i.e.,

$$\omega \geq (1-s_i)\left(\frac{S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{\beta}{\beta-2}} \tag{40}$$

Proposition 5 *The symmetric market configuration* (κ_{nb}, N_{nb}) *occurs in an equilibrium if and only the entrepreneurial wealth level is sufficiently high, i.e.,* $\omega \geq (1-\theta)\left(\frac{S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{\beta}{\beta-2}}$, *or equivalently the market size is not too large, i.e.,*

$$S \leq \underset{\sim}{S}(\theta,\omega) \triangleq \left(\frac{\beta}{2}+1\right)^2\left(\frac{\omega}{1-\theta}\right)^{\frac{\beta-2}{\beta}}. \tag{41}$$

And κ_{nb} increases with market size S, i.e., $\frac{\partial \kappa_{nb}}{\partial S} > 0$, is independent of S, i.e., $\frac{\partial N_{nb}}{\partial S} = 0$.

Proof. First we show that given there are $(N_{nb} - 1)$ other firms of capability κ_{nb}, the integrated business-corporate strategy $\left(\kappa_{nb}, \tilde{\theta}_i, s_i\right)$ which satisfies condition (25) is a global maximum for entrepreneur i's the maximization program (10) if the entrepreneurial wealth constraint is not binding. Note that if the entrepreneurial wealth constraint is not binding, then the corporate governance constraint is not binding either; and we can substitute the binding external investors' participation constraint (25) into the objective function of program (10) to eliminate variable $\tilde{\theta}_i$ and s_i and get the following simplified the maximization program:

$$\max_{\kappa_i} S\pi_i\left(\kappa_i; \kappa_{-i}\right) - F\left(\kappa_i\right) \tag{42}$$

i.e., to choose κ_i to maximize the firm's net profit. We need to show the κ_{nb} is a global maximum of $S\pi_i\left(\kappa_i; (\kappa_{nb})_{N_{nb}-1}\right) - F\left(\kappa_i\right)$. It is known that $S\pi\left(\kappa_{nb}, N_{nb}\right) = F\left(\kappa_{nb}\right)$ and $\left.\frac{\partial \pi_i}{\partial \kappa_i}\frac{\kappa_i}{\pi_i}\right|_{(\kappa_{nb}, N_{nb})} = \left.\frac{dF(\kappa_i)}{d\kappa_i}\frac{\kappa_i}{F(\kappa_i)}\right|_{(\kappa_{nb})} = \beta$. These and property (7), i.e., $\frac{\partial \pi_i}{\partial \kappa_i}\frac{\kappa_i}{\pi_i}$ is decreasing in κ_i conditional on $\pi_i > 0$, jointly imply that

$$\begin{aligned}
S\pi_i\left(\kappa_i; (\kappa_{nb})_{N_{nb}-1}\right) &< F\left(\kappa_i\right) \text{ for } \kappa_i > \kappa_{nb}, \\
S\pi_i\left(\kappa_i; (\kappa_{nb})_{N_{nb}-1}\right) &< F\left(\kappa_i\right) \text{ for } \kappa_i < \kappa_{nb} \text{ and } \pi_i > 0,
\end{aligned}$$

i.e., κ_{nb} is the maximum for the range whereof $\pi_i > 0$. For the range whereof $\pi_i = 0$, the maximum is $\kappa_i = 0$. Overall, $\kappa_i = 0$ and κ_{nb} are the two global maximal points. Given the symmetry of the outcome, no firm should want to unilaterally deviate from the market configuration (κ_{nb}, N_{nb}) if the entrepreneurial wealth constraints are not binding. Next we show that condition (40) can be satisfied if condition (41) is satisfied; it is true for the example $s_i = \theta$. Conversely, if condition (40) is violated, supposing the entrepreneurial wealth constraint is not binding would cause a contradiction.
Finally, the comparative statics follow immediately from (38) and (39). ■

3.2 With binding credit constraints

The entrepreneurial wealth constraint is binding if

$$\omega < (1-\theta)\left(\frac{S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{\beta}{\beta-2}}, \tag{43}$$

or

$$S > \underline{S}\left(\theta, \omega\right), \tag{44}$$

which we assume are satisfied for the analysis of this subsection. As a result, (28) and (36) imply that

$$\tilde{\theta}_i = s_i = \theta. \tag{45}$$

Here the equilibrium credit rationing mechanism has a simple credit-size rationing rule. It is based on the expectation that each firm just breaks even. This is due to the free entry, which erodes any possible positive (or negative) net profits in a symmetric equilibrium outcome. To ensure that the external investors break even, their share of the capital cost must equal to their expected share of profit. And then (27) and (36) imply

$$\frac{S\left(\kappa_b\right)^2}{\left(N_b+1\right)^2} = \left(\kappa_b\right)^\beta = \frac{\omega}{1-\theta}, \tag{46}$$

where N_b and κ_b are the equilibrium number of firms and capability level in this case; and subscript $_b$ indicates that the entrepreneurial wealth constraint is binding. Consequently,

$$\kappa_b = \left(\frac{\omega}{1-\theta}\right)^{1/\beta} \tag{47}$$

and

$$N_b = \left(S\left(\frac{1-\theta}{\omega}\right)^{1-2/\beta}\right)^{1/2} - 1 \tag{48}$$

Proposition 6 *In the symmetric equilibrium outcome with binding credit constraints, the credit rationing rule is given by $\tilde{\theta}_i = \theta$, and the level of each firm's capability is increasing in the entrepreneurial wealth level, i.e., $\frac{\partial \kappa_b}{\partial \omega} > 0$; is increasing in the upper limit of the external investors' bargaining power, i.e., $\frac{\partial \kappa_b}{\partial \theta} > 0$; does not change with the size of the market, i.e., $\frac{\partial \kappa_b}{\partial S} = 0$. The number of firms, N_b, is decreasing in the entrepreneurial wealth level, i.e., $\frac{\partial N_b}{\partial \omega} < 0$; is decreasing in the upper limit of the external investors' bargaining power, i.e., $\frac{\partial N_b}{\partial \theta} < 0$; is increasing in the size of the market, i.e., $\frac{\partial N_b}{\partial S} > 0$.*

The symmetric equilibrium outcome without binding credit constraints can be used as a benchmark to measure the impacts of binding wealth constraints on the equilibrium market structure and capability level.

Proposition 7 *Compared with the symmetric equilibrium outcome without binding credit constraints, the symmetric equilibrium outcome with binding credit constraints has a lower level of capability and a larger number active firms in the market, i.e., $\kappa_b \leq \kappa_{nb}$ and $N_b \geq N_{nb}$.*

Proof. It follows from Proposition 6 that $\frac{\partial \kappa_b}{\partial \omega} > 0$, $\frac{\partial N_b}{\partial \omega} < 0$. It is also a fact that κ_{nb} and N_{nb} are independent of ω, and $\kappa_{nb} = \kappa_b$, $N_{nb} = N_b$ when $\omega = (1-\theta)\left(S/\left(\beta/2+1\right)^2\right)^{\beta/(\beta-2)}$. As a result, $\kappa_b \leq \kappa_{nb}$ and $N_b \geq N_{nb}$ as $\omega \leq (1-\theta)\left(S/\left(\beta/2+1\right)^2\right)^{\beta/(\beta-2)}$. ■

The above comparison reveals the consequences of credit size rationing (Type I credit rationing) in the symmetric equilibrium outcome with binding entrepreneurial wealth constraints, particularly about how it affects the level of capability and the market structure. Compared with the benchmark case, clearly there is an under-investment in capability at each individual firm level. Proposition 6 identifies the determinants and the consequences of the credit size rationing in the oligopoly market. The level of the entrepreneurial wealth and the upper limit of feasible external investor protection are the key determinants. More entrepreneurial wealth and higher external investor protection potential are shown to reduce the severity of the credit size rationing and under-investment in capability. The size of the market affects the relative (as opposed to the benchmark level), rather than the absolute, level of investment in capability. The consequences of the credit size rationing are an under-investment in R&D at the firm and industry levels (see the proposition below), and a rise in the number of active firms.

Proposition 8 *Compared with the symmetric equilibrium outcome without binding credit constraints, industry level of R&D expenditure in the symmetric equilibrium outcome with binding credit constraints is lower, it is increasing in the level of entrepreneurial wealth* ω *and the upper bound of the external investor protection* θ*.*

Proof. The industry level of R&D expenditure under symmetric binding credit constraints is given by $N_b (\kappa_b)^\beta$. The level without binding credit constraints is given by $N_{nb} (\kappa_{nb})^\beta$. It is known both (κ_b, N_b) and (κ_{nb}, N_{nb}) are solutions to eq. (35), more precisely, the following equation:

$$\frac{S\kappa^2}{(N+1)^2} = (\kappa)^\beta ,$$

which implies that the industry level of R&D expenditure is now given by

$$N (\kappa)^\beta = N \left(\frac{S}{(N+1)^2} \right)^{\frac{\beta}{\beta-2}} .$$

Differentiating the logarithm of both sides of the above equation w.r.t. N reveals that

$$\frac{d \ln \left(N (\kappa)^\beta \right)}{dN} = -\frac{\beta N - \beta + 2N + 2}{N (\beta - 2) (N+1)} < 0 \text{ since } N > 1 \text{ and } \beta > 2.$$

Given that $N_b > N_{nb}$, it follows that $N_b (\kappa_b)^\beta < N_{nb} (\kappa_{nb})^\beta$. Similarly, since N_{nb} is decreasing in ω and θ, it follows that $N_b (\kappa_b)^\beta$ should be increasing in ω and θ. ■

The result of under-investment in R&D at the industry level due to binding credit constraints has important implications for technological progress and economic growth. In a cross-section of countries, ceteris paribus, countries with higher values of θ have better long-term growth performance.[21] This result holds despite the fact that the under-investment at the individual firm level is associated with a larger number of active firms in the market. The finding that more entry of firms can not compensate the reduction of R&D investment at each individual firm level is consistent with Schumpeter (1942)'s assertion that small firms in near-perfectly competitive product market could contribute little to R&D and innovations.

One property of the symmetric equilibrium outcome with binding credit constraints revealed in Proposition 6 is that the number of firms increases with in the size of the market. This property resembles the relationship between market size and market structure in industries where the sunk (fixed) cost is exogenous. The difference between them though, is that while the lower bound of k-firm concentration measure approaches zero as the market size approaches infinity in the 'exogenous sunk cost' industries (Sutton (1991)), this 'convergence' result does not hold in the kind of markets modeled here. The reason for the difference is that as the size of market is sufficiently large, the symmetric outcome with binding credit constraints ceases to be equilibrium. To see why this is the case, in what follows we turn to a necessary condition for the symmetric market configuration (κ_b, N_b) to occur in an equilibrium.

A necessary condition for any equilibrium is condition (24), which in combination with eq. (35) and (45) implies a necessary condition for (κ_b, N_b) to occur in an equilibrium as follows:

$$\theta \left. \frac{\partial \pi_i}{\partial \kappa_i} \frac{\kappa_i}{\pi_i} \right|_{(\kappa_b, N_b)} \leq \left. \frac{F'(\kappa_i)\,\kappa_i}{F(\kappa_i)} \right|_{(\kappa_b, N_b)} . \tag{49}$$

Recall eq. (6) and (9) that $\left. \frac{\partial \pi_i}{\partial \kappa_i} \frac{\kappa_i}{\pi_i} \right|_{(\kappa_b, N_b)} = 2N_b$ and $\frac{F'(\kappa_i)\kappa_i}{F(\kappa_i)} = \beta$. The above condition implies

$$N_b \leq \frac{\beta}{2\theta}, \tag{50}$$

which also sets an upper bound to S given the fact that N_b increases in S. Define a threshold value of S, $\bar{S}(\theta, \omega)$, such that,

$$\bar{S}(\theta, \omega) \triangleq \left(\frac{\omega}{1-\theta} \right)^{1-2/\beta} \left(\frac{\beta}{2\theta} + 1 \right)^2 , \tag{51}$$

so when $S = \bar{S}(\theta, \omega)$,

$$N_b = \frac{\beta}{2\theta}. \tag{52}$$

Lemma 9 *A symmetric equilibrium outcome with binding credit constraints exists only if*

$$S \leq \bar{S}(\theta, \omega) . \tag{53}$$

[21] See Tong (2005b) for a formal discussion.

The intuition is that when the size of the market is sufficiently large, the number of firms N_b would be so large that each firm's (local) proportional marginal benefit of increasing capability at (κ_b, N_b) would be so high, that the proportional marginal benefit to the external investors would exceed their proportional marginal cost of increasing capability. There would be a profitable deviation for each entrepreneur. Hence no symmetric equilibrium outcome with binding credit constraints can exist.

4 Asymmetric Equilibrium Outcome

In this section, we characterize asymmetric equilibrium outcomes where the firm size distribution is uneven. We are interested in this kind of equilibrium outcomes not only because a symmetric equilibrium outcome does not always exist, but also because, as will transpire shortly, they have important economic and policy implications.

For tractability we consider the simplest asymmetric equilibrium outcome with symmetric integrated business-corporate strategies. The outcome is asymmetric because a strictly positive fraction of the start-up firms end up with being denied external credits completely in equilibrium. The only element of asymmetry that occurs is between the start-up firms who are successful in securing external investments and those who fail to do so. There is symmetry between those who succeed, and similarly between those who are not successful. From an ex ante point of view, the entrepreneurs who choose to enter the market in the first stage all seem identical. When they plan to propose the same offer to external investors at the second stage, for whatever ex ante expected selection strategy adopted by the external investors, they correctly expect each of them has an equal chance of being a winner.

In what follows we tell a more elaborate version of the story, which fits the above description well, and is a sufficient but not necessary story. Suppose among potential entrants of the market, there are two types of entrepreneurs: a good type, who are capable of adopting the best practice of corporate governance hence is bounded from above by the upper limit $\theta > 0$; a bad type, who are not capable of adopting the best practice of corporate governance, hence is bounded from above by a strictly smaller upper limit $\rho\theta$ $(0 \leq \rho < 1)$. Only a good entrepreneur is able to provide a signal (proof) of the upper limit being θ within a finite time interval (at stage 2). The speed of a good entrepreneur producing the signal (proof) is an independent random draw from an identical uniform distribution; a bad entrepreneur can never produce the signal. In equilibrium, entrepreneurs self-select: only the good entrepreneurs enter the market. External investors adopt the following selection strategy: first, request all entrepreneurial entrants to simultaneously race into proving proofs of their upper limits being θ; second, exclude all the entrepreneurs who fail to provide any proof in time, and rank the remaining entrepreneurs primarily by the expected levels of external investor surplus they offer, in case of a tie, break the tie by their race scores; third, accept and only accept the offers of the top-ranking

N_h entrepreneurs (where N_h is the equilibrium number of successful firms to be defined in the next paragraph). This outcome is an equilibrium because no player wants to unilaterally deviate from it. In this version of the story, such an equilibrium outcome constitutes an 'efficiency'-based selection process, which deters bad entrepreneurs from entry and gives entrepreneurs (who enter and hence are good) equal opportunities to win the competition for external credits.

Suppose in equilibrium there are N entrepreneurs that enter at stage 1, all propose the same integrated business-corporate strategy $\left(\kappa_h, \tilde{\theta}, s_h\right)$ in stage 2. Only a fraction $\phi \in (0,1)$ of the offers are accepted by external investors in equilibrium, giving rise to $N_h = \phi N$ successful, hence, high-capability firms with the implemented strategy $\left(\kappa_h, \tilde{\theta}, s_h\right)$. The other offers are rejected in equilibrium, giving rise to $N_l = (1-\phi)$ unsuccessful low-capability firms whose actual business-corporate status are $(\kappa_l, 0, 0)$, i.e., with no external finance. Each entrepreneur who enters in equilibrium has an ex ante equal chance of winning the selection with probability ϕ since their offers are identical in equilibrium. Off equilibrium path, if the offers differ, the external investors always select the entrepreneurs who offer the highest expected external investor surplus. By definition we have $\kappa_h > \kappa_l$. We have already assumed that as a necessary condition for an entrepreneurs to be able to commit to the proposed corporate governance structure, in stage 2 the entrepreneur has to commit the promised insider equity. Each entrepreneur maximizes the expected entrepreneurial surplus.

To characterize this equilibrium outcome, we begin by observing that the symmetric equilibrium strategy of a start-up firm must satisfy the binding entrepreneurial wealth constraint and the binding corporate governance constraint, i.e.,

$$\left(1-\tilde{\theta}\right)(\kappa_h)^{\beta} = \omega, \tag{54}$$

$$\kappa_l = \omega^{1/\beta} \tag{55}$$

and

$$s_h = \theta. \tag{56}$$

The proof is simple. Suppose otherwise, then an entrepreneur can always increase the entrepreneurial equity input by $\varepsilon_1 \to 0$, or increase the ex post external investor bargaining power by $\varepsilon_2 \to 0$, and therefore increase the probability of winning external finance by a finite positive number $(1-\phi)$, and hence increase the expected entrepreneurial surplus by a finite amount. This is a profitable deviation, which contracts the above supposition.

Since the capital market is perfectly competitive, in equilibrium the external investor surplus from each firm should be zero, i.e.,

$$\theta S\pi_h(\kappa_h; \kappa_{-h}) = \tilde{\theta}(\kappa_h)^{\beta} \tag{57}$$

or

$$\theta S\pi_h(\kappa_h; \kappa_{-h}) = (\kappa_h)^{\beta} - \omega \tag{58}$$

where κ_{-h} denotes $\left((\kappa_h)_{N_h-1}, (\kappa_l)_{N_l}\right)$. The binding entrepreneurial wealth constraint and the binding external investor participation constraint jointly imply that even the successful high-capability firms still face binding credit constraints and suffer from underinvestments from the firm-profit-maximization point of view. The unsuccessful low-capability firms obviously suffer an even worse underinvestment problem.

In equilibrium, free entry leads to the following ex ante entrepreneurial break-even condition:

$$\phi(1-\theta) S\pi_h(\kappa_h; \kappa_{-h}) + (1-\phi) S\pi_l(\kappa_l; \kappa_{-l}) = \omega. \tag{59}$$

Although on average each entrepreneurial entrant breaks even, the fact that all of them are willing to compete for obtaining external credits imply that the entrepreneurs of the high-capability firms must be better off than those of the low-capability firms in equilibrium, which further implies that the former must earn a strictly positive surplus and latter must earn a strictly negative surplus, i.e.,

$$(1-\theta) S\pi_h(\kappa_h; \kappa_{-h}) > \omega > S\pi_l(\kappa_l; \kappa_{-l}).$$

Given the fact that all external investors have zero surplus in equilibrium, it follows immediately that the high-capability firms earn strictly positive net profits and the low-capability firms make net losses. On average, each firm just breaks even.

In the asymmetric equilibrium outcome, the credit rationing mechanism comprises two sets of rules. The first relates to credit denial to certain fraction of firms. The second relates to credit-size rationing to the firms who are not denied credits. The credit-size rationing rule differs from the one adopted in the symmetric equilibrium outcome, as is stated in the following proposition.

Proposition 10 *In comparison with the symmetric outcome*[22] *with binding credit constraints, each high capability firm in the asymmetric equilibrium outcome obtains more credit and a higher level of capability, i.e.,*

$$\tilde{\theta}_h > \theta, \tag{60}$$

and

$$\kappa_h > \kappa_b. \tag{61}$$

Also, the number of high capability firms is smaller than the number of firms of a symmetric outcome with binding credit constraints, i.e.,

$$N_h < N_b. \tag{62}$$

Proof. The entrepreneur of a high capability firm earns a strictly positive surplus, i.e.,

$$(1-\theta) S\pi_h(\kappa_h; \kappa_{-h}) - \left(1-\tilde{\theta}_h\right)(\kappa_h)^\beta > 0. \tag{63}$$

[22] Which is not necessarily an equilibrium.

Using eq. (57) and (63) to eliminate $\pi_h(\kappa_h; \kappa_{-h})$ and $(\kappa_h)^\beta$ results in $\tilde{\theta}_h > \theta$. From (47) and (54) it follows that

$$\kappa_h = \left(\frac{\omega}{1-\tilde{\theta}_h}\right)^{\frac{1}{\beta}} > \kappa_b = \left(\frac{\omega}{1-\theta}\right)^{\frac{1}{\beta}}. \tag{64}$$

From the property that

$$S\pi\left(\kappa_h; (\kappa_h)_{N_h-1}\right) = S\frac{(\kappa_h)^2}{(N_h+1)^2} \geq S\pi(\kappa_h; \kappa_{-h}) = \frac{\tilde{\theta}_h}{\theta}(\kappa_h)^\beta > (\kappa_h)^\beta,$$

i.e., for given κ_h and N_h, if all the low-capability firms were removed from the product market competition exogenously, then each high-capability firm would be better off, it follows that

$$N_h < S^{\frac{1}{2}}(\kappa_h)^{\frac{2-\beta}{2}} - 1.$$

Since $\kappa_h > \kappa_b$ and $\beta > 2$, it is immediate that

$$N_h < S^{\frac{1}{2}}(\kappa_b)^{\frac{2-\beta}{2}} - 1 = N_b. \tag{65}$$

■

The credit-size rationing rule is based on the expectation that the high-capability firms can make positive net profits. Even when there is free entry, the high-capability firms' positive net profits are not eroded by competition because the marginal competitors are the low-capability firms whose ability to compete in R&D is constrained by credit denials. The external investors therefore are willing to take a bigger share of the cost of capital than the expected share of profit, i.e., $\tilde{\theta}_h > \theta$, up to the point that they just break even. This more favorable credit-size rationing rule enables the high-capability firms to invest more aggressively, i.e., $\kappa_h > \kappa_b$. In equilibrium, the credit denials to the low-capability firms and the more favorable credit-size rationing rule toward the high-capability firms reenforce each other. This asymmetric equilibrium credit rationing mechanism creates an endogenous distinction between 'deep-pocket' (high-capability) and 'shallow-pocket' (low-capability) firms, and makes them respectively the winners and losers of the R&D-product market competition. It turns the R&D-product market competition into a 'predation' in the sense that if the collapse of the 'preys', i.e., the low-capability firms' external finances were not expected (and intended), the 'predators', i.e., the high-capability firms' business-corporate strategies would have been less aggressive.[23] It is all high-capability firms' intention and the foundation of their business-corporate strategies that the low-capability firms' external finances should be destroyed (with or without the consequence of them being driven out of the product market). As a result, the high-capability firms can recoup the extra costs of their aggressive investments.

[23] See Tong (2005a) for an extension of the same idea in a model of predatory pricing rather than 'predatory' investment in R&D.

The asymmetric equilibrium credit rationing mechanism creates winners ('predators') and losers ('preys') among the entrepreneurs. Ex ante, upon deciding on the optimal integrated business-corporate strategy, an entrepreneur has to balance two elements: one is to maximize the entrepreneurial surplus conditional on the contract being acceptable; the other is to maximize the chance of the contract being selected. As a result of the competition for selective credit allocation and free entry, the equilibrium outcome must exhaust all the opportunities of pledging positive external investor surplus, i.e., the external investor surplus offered by the equilibrium business-corporate strategy is at the global maximum and equals zero. When this condition is satisfied, there is no profitable (unilateral) deviation for any players. First, for an entrepreneur, changing the asking size of external credit would only decrease the expected external investor surplus and hence destroy the chance of being selected, therefore is not a profitable deviation. Consequently, external credit must either be of this particular size, or zero. Second, it is not a profitable deviation for any group of external investors to change the number of high-capability firms. More elaborately, on the one hand, the external investors of a high capability firm would not find it more profitable to quit since the surplus is still zero. On the other hand, an outside group of external investors who have sufficient fund would find it unprofitable to turn a low capability firm into an additional high capability one, because the intensified competition would make all external investors in the market (including themselves) worse off, resulting in negative external investor surplus.

If the profit function π_i is continuously differentiable at the point $(\kappa_h; \kappa_{-h})$, then to rule out any profitable local deviation (i.e., positive external investor surplus), the following first order necessary condition must be satisfied:

$$\theta S \frac{\partial \pi_i}{\partial \kappa_i} = F'(\kappa_i). \tag{66}$$

Hence,

$$\theta \frac{\partial \pi_i}{\partial \kappa_i} \frac{\kappa_i}{\pi_i} = \frac{F'(\kappa_i)\,\kappa_i}{F(\kappa_i)} \frac{F(\kappa_i)}{S\pi_i} = \beta \frac{\theta}{\tilde{\theta}_i}$$

It follows that

$$\tilde{\theta}_h \left. \frac{\partial \pi_i}{\partial \kappa_i} \frac{\kappa_i}{\pi_i} \right|_{\kappa_i = \kappa_h, \kappa_{-i} = \kappa_{-h}} = \beta. \tag{67}$$

Using (54) and (67) to eliminate $\tilde{\theta}_h$ yields

$$\left(1 - \frac{\omega}{(\kappa_h)^{\beta}}\right) \left. \frac{\partial \pi_i}{\partial \kappa_i} \frac{\kappa_i}{\pi_i} \right|_{\kappa_i = \kappa_h, \kappa_{-i} = \kappa_{-h}} = \beta. \tag{68}$$

If the profit function π_i is not continuously differentiable at the point $(\kappa_h; \kappa_{-h})$, i.e., this point is a 'kink', then the slope to the immediate left (respectively, right) of this 'kink' must be non-negative (respectively, non-positive), to ensure it is a local maximum.

In the asymmetric equilibrium outcome, the low-capability firms' relationships with the external investors are the victims of the 'predation'. It is interesting to know whether as a consequence the low-capability firms are driven out of the product market competition. This hinges on whether it is optimal for a low-capability firm to remain active, i.e., to maintain a positive market share. It is possible that if the low capability is relatively too low, then the firm's optimal choice is not to produce at all at the third stage of the game.

Denote by $m(\kappa_i; \kappa_{-i})$ the equilibrium market share of firm i. Eq. (3) implies

$$m(\kappa_i; \kappa_{-i}) = \max\left(0, \frac{\kappa_i + \sum_{j=1}^{N}(\kappa_i - \kappa_j)}{\sum_{j=1}^{N}\kappa_j}\right). \tag{69}$$

We look for a threshold of the high capability, call it the 'limit capability', such that if the high capability firms all reach that level, a low capability firm will have zero market share. More precisely, define the normalized 'limit capability' $\bar{\kappa}(N_h)$ such that

$$m(\kappa_l; \kappa_{-l})|_{\kappa_l=\omega^{1/\beta}, \kappa_h=\omega^{1/\beta}\bar{\kappa}(N_h)} = 0 \tag{70}$$

and

$$m(\kappa_l; \kappa_{-l})|_{\kappa_l=\omega^{1/\beta}, \kappa_h=\omega^{1/\beta}\bar{\kappa}(N_h)-\varepsilon} > 0 \tag{71}$$

for any $\varepsilon > 0$. From the above definition, it follows that

$$\bar{\kappa}(N_h) = \left(1 + \frac{1}{N_h}\right). \tag{72}$$

There are three types of possible asymmetric equilibrium outcomes as far as κ_h and $m(\kappa_l; \kappa_{-l})$ are concerned:

Scenario 1: The low-capability firms are strictly active, i.e., $\kappa_h < \omega^{1/\beta}\bar{\kappa}(N_h)$ and $m(\kappa_l; \kappa_{-l}) > 0$.

Scenario 2: The low-capability firms are weakly inactive, i.e., $\kappa_h = \omega^{1/\beta}\bar{\kappa}(N_h)$ and $m(\kappa_l; \kappa_{-l}) = 0$.

Scenario 3: The low-capability firms are strictly inactive, i.e., $\kappa_h > \omega^{1/\beta}\bar{\kappa}(N_h)$ and $m(\kappa_l; \kappa_{-l}) = 0$.

In scenario 1, a high-capability firm's equilibrium strategy is featured in Figure 1. Where the thick solid curve is the external investor surplus from a high-capability firm as a function of its capability given other firms' position in the equilibrium outcome. The external investor surplus is at maximum, equal to zero and continuously differentiable at $\kappa_i = \kappa_h$ (for $\kappa_i > \omega^{\frac{1}{\beta}}$). The thin solid curve is the entrepreneurial surplus, which is increasing in κ_i (for $\kappa_i > \omega^{\frac{1}{\beta}}$), and is strictly positive at $\kappa_i = \kappa_h$. The equilibrium high capability level κ_h is below is the 'kink', which is the level that would make the low-capability firms weakly inactive. Obviously, in this equilibrium outcome all the low-capability firms remain strictly active.

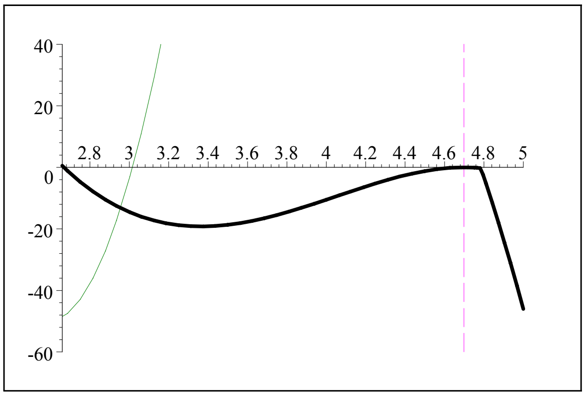

Figure 1: Asymmetric equilibrium outcome with strictly active low capability firms

In scenario 2, the external investor surplus, as shown in Figure 2, is maximal (for $\kappa_i > \omega^{\frac{1}{\beta}}$) at the 'kink', which is the level that makes the low capability firms weakly inactive. In this equilibrium outcome all the low-capability firms are weakly inactive.

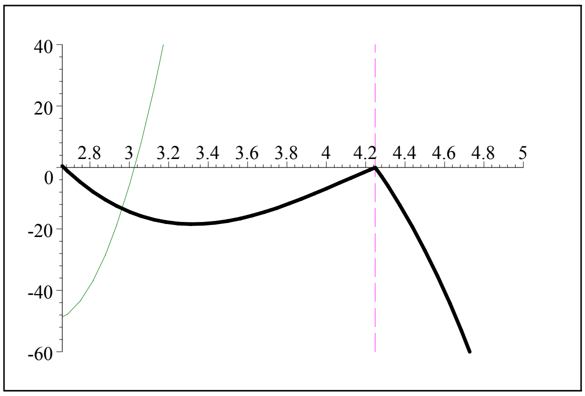

Figure 2: Asymmetric equilibrium outcome with weakly inactive low capability firms (maximal external investor surplus at the 'kink')

In scenario 3, the external investor surplus, as shown in Figure 3, is maximal (for $\kappa_i > \omega^{\frac{1}{\beta}}$) above the 'kink', which is threshold that would make the low-capability firms weakly inactive. Obviously, in this equilibrium outcome all the low-capability firms remain strictly inactive.

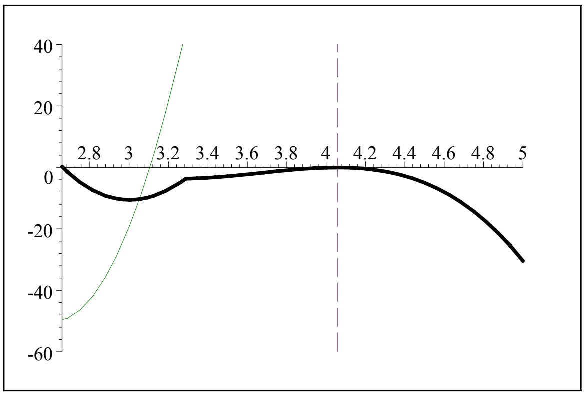

Figure 3: Asymmetric equilibrium outcome with strictly inactive low capability firms

4.1 Low-capability firms driven out of product market

In this subsection we analyze Scenario 3 in greater detail. This scenario is particularly interesting because here the 'preys', i.e., the low-capability firms, are literally driven out of the product market by the 'predation'. When the low capability firms are strictly inactive, the profit of a high-capability firm is given by

$$\Pi_h\left(\kappa_h;\kappa_{-h}\right)=\frac{S\left(\kappa_h\right)^2}{\left(N_h+1\right)^2}. \tag{73}$$

The ex ante entrepreneurial break-even condition (59) now takes the following form:

$$\phi\frac{(1-\theta)\,S\left(\kappa_h\right)^2}{\left(N_h+1\right)^2}=\omega. \tag{74}$$

The binding external investors' participation constraint (58) now has the following implementation:

$$\theta\frac{S\left(\kappa_h\right)^2}{\left(N_h+1\right)^2}=\left(\kappa_h\right)^\beta-\omega, \tag{75}$$

which implies

$$N_h=\left(\frac{\theta S\left(\kappa_h\right)^2}{\left(\kappa_h\right)^\beta-\omega}\right)^{\frac{1}{2}}-1. \tag{76}$$

Using eq. (74) and (75) to solve for ϕ yields

$$\phi=\frac{\theta}{1-\theta}\frac{\omega}{\left(\kappa_h\right)^\beta-\omega}, \tag{77}$$

which is the fraction of entrepreneurs who are the winners.

In this scenario, the high capability level strictly exceeds the limit capability level, i.e.,

$$\kappa_h > \bar{\kappa}(N_h)\,\omega^{1/\beta} = \left(1 + \frac{1}{N_h}\right)\omega^{1/\beta}. \tag{78}$$

The necessary condition (68) applies to this scenario and is implemented as follows:

$$\left(1 - \frac{\omega}{(\kappa_h)^{\beta}}\right) 2N_h = \beta. \tag{79}$$

It follows that

$$N_h = \frac{\beta\,(\kappa_h)^{\beta}}{2\left((\kappa_h)^{\beta} - \omega\right)} > \frac{\beta}{2} = N_{nb}. \tag{80}$$

The binding external investors' participation constraint (58) now takes the form as

$$\theta \frac{S\,(\kappa_h)^2}{(N_h + 1)^2} = (\kappa_h)^{\beta} - \omega. \tag{81}$$

Using Equations (79) and (81) to eliminate κ_h and N_h respectively results in

$$\frac{\theta S}{(N_h + 1)^2}\left(\frac{\omega N_h}{N_h - \frac{\beta}{2}}\right)^{2/\beta} = \left(\frac{\omega N_h}{N_h - \frac{\beta}{2}} - \omega\right), \tag{82}$$

and

$$\theta \frac{S\,(\kappa_h)^2}{\left(\frac{\beta(\kappa_h)^{\beta}}{2\left((\kappa_h)^{\beta} - \omega\right)} + 1\right)^2} = (\kappa_h)^{\beta} - \omega, \tag{83}$$

which imply the following set of comparative statics.

Proposition 11 *In the asymmetric equilibrium outcome with strictly inactive low-capability firms, the high capability* κ_h *increases with market size* S *and the upper limit of external investors' bargaining power* θ*, i.e.,* $\frac{\partial \kappa_h}{\partial S} > 0$ *and* $\frac{\partial \kappa_h}{\partial \theta} > 0$*. The number of high capability firms* N_h *decreases with market size* S *and the upper limit of external investors' bargaining power* θ*, i.e.,* $\frac{\partial N_h}{\partial S} < 0$ *and* $\frac{\partial N_h}{\partial \theta} < 0$*. The probability of being a high-capability firm,* ϕ*, decreases with market size* S*, i.e.,* $\frac{\partial \phi}{\partial S} < 0$*.*

Proof. Equations (82) and (83) imply the following comparative statics:

$$\frac{\partial N_h}{\partial \theta} = -\frac{1}{\frac{\theta}{N_h} A}, \frac{\partial N_h}{\partial S} = -\frac{1}{\frac{S}{N_h} A}, \frac{\partial \kappa_h}{\partial \theta} = \frac{1}{\frac{\theta}{\kappa_h} A\,(2N_h - \beta)}, \frac{\partial \kappa_h}{\partial \theta} = \frac{1}{\frac{\theta}{\kappa_h} A\,(2N_h - \beta)} \tag{84}$$

where $A \triangleq 1 + \frac{\beta - 2}{2N_h - \beta} - \frac{2N_h}{N_h + 1}$. The equilibrium entails the following second order necessary condition for maximization:

$$\theta \left.\frac{\partial^2 \pi_i}{\partial \kappa_i^2}\right|_{(\kappa_h, N_h)} - \beta(\beta - 1)(\kappa_h)^{(\beta - 2)} \leq 0, \tag{85}$$

which implies

$$\left.\frac{\partial^2 \pi_i}{\partial \kappa_i^2} \frac{\kappa_i}{\frac{\partial \pi_i}{\partial \kappa_i}}\right|_{(\kappa_h, N_h)} \leq \beta - 1. \tag{86}$$

Given the following property of the profit function:

$$\left.\frac{\partial^2 \pi_i}{\partial \kappa_i^2} \frac{\kappa_i}{\frac{\partial \pi_i}{\partial \kappa_i}}\right|_{(\kappa_h, N_h)} = N_h, \tag{87}$$

it follows that the number of high capability firms is bounded from above by:

$$N_h \leq \beta - 1, \tag{88}$$

and the inequality below is satisfied.

$$A = 1 + \frac{\beta - 2}{2N_h - \beta} - \frac{2N_h}{N_h + 1} > \frac{2}{N_h + 1} > 0. \tag{89}$$

As a result, we obtain definite signs for the comparative statics in (84) as follows:

$$\frac{\partial N_h}{\partial S} < 0, \; \frac{\partial N_h}{\partial \theta} < 0, \; \frac{\partial \kappa_h}{\partial S} > 0 \text{ and } \frac{\partial \kappa_h}{\partial \theta} > 0. \tag{90}$$

From (77) it follows that

$$\frac{\partial \phi}{\partial S} = \frac{\partial \phi}{\partial \kappa_h} \frac{\kappa_h}{\partial S} < 0. \tag{91}$$

■

Conditional on the low-capability firms are strictly inactive, the larger the market size, or the higher the upper limit of external investor protection, the more favorable is the credit-size rationing toward the winners, i.e., the larger is κ_h; and the smaller is the number of firms that receive external credits; hence the tougher is the 'predation'.

Now we look for the parametric condition under which the low-capability firms become strictly inactive. We look for the boundary between scenario 2 and 3, denoted by $S_I(\theta, \omega)$, above which inequality (78) is satisfied and below which this inequality is violated. Figure 4 depicts the the external investor surplus function for $S = S_I(\theta, \omega)$: there is a kink at the equilibrium level of high capability, particularly, the slope at the immediate right of the kink is flat, while it is upward at the immediate left of the kink.

When $S = S_I(\theta, \omega)$, there exists (κ_h, N_h) which satisfies conditions (82), (83) and $\kappa_h = \bar{\kappa}(N_h)\,\omega^{1/\beta}$. Define $r_I \triangleq \frac{\kappa_h}{\omega^{1/\beta}}$. It follows that

$$\frac{r_I^\beta - 1}{r_I - 1} = \frac{\beta}{2} r_I^\beta \tag{92}$$

The solution exists and is unique for $r_I > 1$. Consequently, $S_I(\theta, \omega)$ is given by

$$S_I(\theta, \omega) = \frac{\omega^{1-2/\beta}}{\theta} \frac{\left(\left(\frac{\beta}{2} + 1\right) r_I^\beta - 1\right)^2}{\left(r_I^\beta - 1\right) r_I^2} \tag{93}$$

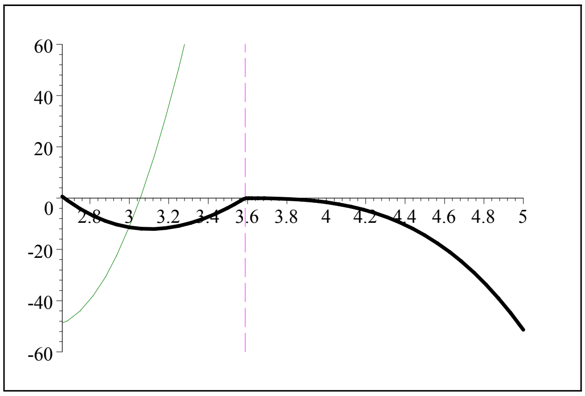

Figure 4: Asymmetric equilibrium outcome with (almost) strictly inactive low-capability firms (the threshold between scenarios 2 and 3)

Proposition 12 *There exists a threshold value of* S, $S_I(\theta,\omega)$, *such that the asymmetric equilibrium outcome with strictly inactive low-capability firms exists if and only if* $S > S_I(\theta,\omega)$.

Proof. (sketch) Let (N_h^*, κ_h^*) be the solution to eq. (82) and (83). When $S = S_I(\theta,\omega)$, in the (N_h, κ_h) space, the loci of (N_h^*, κ_h^*) intersects with the limit capability curve: $\kappa_h = \left(1 + \frac{1}{N_h}\right)\omega^{1/\beta}$. According to Proposition 11, when S increases, the loci of (N_h^*, κ_h^*) moves to the northwest. On the one hand, $\kappa_h^* \to \infty$ as $S \to \infty$. On the other hand the limit capability curve is bounded from above by $\left(1 + \frac{1}{N_{nb}}\right)\omega^{1/\beta}$ since $N_h \geq N_{nb}$. It follows that when $S \to \infty$ the loci of (N_h^*, κ_h^*) is above the limit capability curve. Given that the loci of (N_h^*, κ_h^*) is continuous and it intersects with the limit capability curve only once for $\kappa_h > \omega^{1/\beta}$ (otherwise the solution to eq. (92) would not be unique), then the loci of (N_h^*, κ_h^*) must stay above (below) the limit capability curve for $S > S_I(\theta,\omega)$ $(S < S_I(\theta,\omega))$. Therefore, inequality (78) is satisfied if and only if $S > S_I(\theta,\omega)$. ■

4.2 Very large market: the limiting case

In this subsection, we study the behavior of equilibrium when the market size S approaches infinity, hence, where the 'predation' is the toughest. We are particularly interested in the asymptotic behavior of variables: κ_h, N_h and ϕ.

Since κ_h increases with S, and approaches to infinity as S approaches infinity, it makes more sense if we compare κ_h with the benchmark capability, κ_{nb}, which also approaches infinity, i.e., we calculate $\lim_{S\to\infty} \frac{\kappa_h}{\kappa_{nb}}$ and relate it to parameter θ.

From eq. (83) it follows that

$$(\kappa_h)^{\beta-2} - \frac{\omega}{(\kappa_h)^2} = \frac{\theta S}{\left(\frac{\beta(\kappa_h)^\beta}{2\left((\kappa_h)^\beta-\omega\right)}+1\right)^2}, \tag{94}$$

which, as $\kappa_h \to \infty$, has the approximated solution:

$$\kappa_h \approx \left(\frac{\theta S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{1}{\beta-2}}. \tag{95}$$

Hence,

$$\frac{\kappa_h}{\kappa_{nb}} \approx \frac{\left(\frac{\theta S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{1}{\beta-2}}}{\left(\frac{S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{1}{\beta-2}}} \to \theta^{\frac{1}{\beta-2}} \text{ as } S \to \infty \tag{96}$$

From eq. (82), it follows

$$N_h = \frac{\beta}{2} + \frac{\omega N_h}{\frac{\theta S}{(N_h+1)^2}\left(\frac{\omega N_h}{N_h-\frac{\beta}{2}}\right)^{2/\beta}+\omega} \to N_{nb} = \frac{\beta}{2}. \tag{97}$$

This result is very interesting because it reenforces the 'non-convergence' result of Sutton (1991) and Sutton (1998), which says that in the 'endogenous sunk cost' industries the k-firm market concentration measure is bounded away from zero (even when the market size approaches infinity). Here we find that the 'non-convergence' result can be extended to environment where there are endogenous binding credit constraints on R&D investments. This result confirms that the 'escalation mechanism', as phrased by Sutton (1998), is indeed one of the most robust a few mechanisms that determine of market structure. What's surprising is the finding of the connection between the 'non-convergence' property and the 'predatory' nature of some 'endogenous sunk cost' industries, e.g., R&D intensive industries.

The probability for a start-up firm to become a high-capability firm approaches zero, i.e., $\phi \to 0$, as the market size approaches infinity.

When the low-capability firms are strictly inactive, the ex ante firm break-even condition entails that the total industry R&D expenditure equals the total ex post profit of all high-capability firms, which is given by

$$N_h \frac{S(\kappa_h)^2}{(N_h+1)^2} \to \theta^{\frac{2}{\beta-2}} \frac{\beta}{2}\left(\frac{S}{\left(\frac{\beta}{2}+1\right)^2}\right)^{\frac{\beta}{\beta-2}} = \theta^{\frac{2}{\beta-2}} N_{nb}(\kappa_{nb})^\beta, \tag{98}$$

which is a fraction $\theta^{\frac{2}{\beta-2}}$ of the level whereof there are no binding credit constraints. In contrast, the overall R&D investment would be much lower in a hypothetical symmetric outcome with binding credit constraints, as can be revealed by following calculation:

$$\lim_{S\to\infty}\frac{N_b\left(\kappa_b\right)^\beta}{N_{nb}\left(\kappa_{nb}\right)^\beta}=\lim_{S\to\infty}S^{-\frac{\beta+2}{2(\beta-2)}}\left(\frac{\omega}{1-\theta}\right)^{\frac{\beta+2}{2\beta}}\frac{2}{\beta}\left(\frac{\beta}{2}+1\right)^{\frac{\beta}{2(\beta-2)}}=0. \tag{99}$$

Therefore we can claim that, in the case of very large market, the 'predatory' asymmetric equilibrium outcome increases overall R&D investment as compared to the symmetric outcome. As for market structure, using eq. (77), the number of low-capability firms is given by

$$N_l=\left(\frac{1}{\phi}-1\right)N_h=\left(\frac{1-\theta}{\theta}\frac{\left(\kappa_h\right)^\beta-\omega}{\omega}-1\right)N_h. \tag{100}$$

Hence,

$$N_l\to\infty \text{ as } S\to\infty. \tag{101}$$

Nevertheless, since the low capability firms' collective market share is zero, the entry of many low-capability firms do not reduce market concentration. As market size goes to infinity, predation and the selective credit allocation fully restores market concentration to, but never exceeds, the level of the benchmark case with no binding credit constraints. It limits the number of major R&D investors, and allows the size of R&D investments (by the major R&D firms) to increase. When selective credit denial (Type II credit rationing) occurs in big markets, it impacts the market structure by generating 'a lot of start-ups, and a very few stars'. Its is an ingenious way of the markets to mitigate (but not yet to eliminate) the problem of under-investment in R&D.

Proposition 13 *When the size of market S approaches infinity, the number of high-capability firms approaches the benchmark number N_{nb}, and the probability of being a high-capability firm ϕ approaches zero, the high capability level κ_h approaches infinity, but still it is only a fraction $\theta^{\frac{1}{\beta-2}}$ of the benchmark level with no binding credit constraints; the number of high-capability firms approaches from above the benchmark numbers of firms with no binding credit constraints; the level of industry R&D approaches infinity, but is still only a fraction $\theta^{\frac{2}{\beta-2}}$ of the benchmark level with no binding credit constraints, and is higher than that in a hypothetical symmetric outcome with binding credit constraints.*

A consistent moral lesson we can learn from Propositions 8 and 13 is that the industry level of R&D investment increases with the level of upper limit of external investor protection provided by the corporate governance system. This suggests an important role played by the broader legal-political environment to improve contract enforcement and corporate governance in relation to R&D and technological progress.

5 Conclusion

In this paper we develop a theory of the firms and equilibrium credit rationing mechanism in oligopoly market with free entry. The theory departs from the simplistic 'black-box' view of the firm, and characterizes the relationship between wealth-constrained entrepreneur-manager and the external investors of the firm. It predicts that corporate investments may be subject to endogenous credit constraints due to incompleteness of the investment contracts. A firm is defined by its integrated business-corporate strategy. To address the problem of binding credit constraint, a firm's equilibrium corporate governance structure assigns maximal feasible external investor protection, which alleviates, but does not eliminate, the problem.

The theory predicts two types of interaction between equilibrium credit-rationing mechanism and industry structure. In a symmetric equilibrium outcome all firms receive equitable credit-size rationing and each of them just breaks even. In the asymmetric equilibrium outcome, the credit rationing mechanism involves both credit denials to an unsuccessful fringe of firms, and more favorable credit size rationing to the winners of the selection. It creates an endogenous distinction between the 'deep-pocket' and 'shallow-pocket' firms, turns the competition into a 'predation', and sustains positive net profits for the 'deep-pocket' firms. In comparison with the symmetric equilibrium outcome with binding credit constraints, the asymmetric equilibrium outcome increases the skewness of firm size distribution, market concentration, and overall R&D investments. It constitutes a 'market solution' to the 'funding gap' problem. This result suggests a novel interpretation for the stylized fact that some small high-tech start-up firms are denied credits while bigger R&D firms are treated more favorably by the capital market. According the our current theory, this kind of discrimination may have an efficiency justification: only by denying credits to an unsuccessful fringe, is the capital market able to relax the rationing of credit size for some major R&D competitors in equilibrium.

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