MATHPEN: IDENTIFYING AND SLOVING THE PROBLEMS OF ONLINE COLLABORATIVE LEARNING FOR MATHEMATICS

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Combining the interactive communication power of Web 2.0 and social-constructivist theory in education research, online collaborative learning (OCL) has now become an area of intensive research and has generated many favourable results. Yet, the term online collaborative learning, or any other related terms, are seldom seen in mathematics education journals. This paper will, after a brief overview of OCL theory, describe the problems associated with OCL in mathematics education and offer MathPen (an online handwriting recognition system) as a potential solution.

INTRODUCTION

Many researchers are well acquainted with the benefits of student group work. Based on the social-constructivist theories, studies have shown that students' learning can be enhanced as they explore different ideas, challenge various assumptions, justify and defend their understanding and finally draw a well-informed conclusion on the subject matter. It is argued that such interactions can uncover and challenge underlying misconceptions, thus widening the students' perspectives and deepening their understanding (Mercer, 1995). Therefore, it is increasingly common to see the use of group work being promoted in schools, colleges and other educational institutions (Edwards, 2009).

Online collaborative learning (OCL) is a term used to describe similar learning methods, but with an emphasis on internet-based collaboration between learners (Harasim, 2002). As with its offline counterparts, a clear distinction is drawn between *collaborative* and *cooperative* learning: the former refers to a collective effort of mutual engagement in the exploration of a given problem, while the latter refers to an organised manner of work division between students for task accomplishments (Roschelle & Teasley, 1995).

Research in many subject areas has shown that collaborative learning can be implemented online, and similar (if not better) results can be obtained through OCL (Allen & Seaman, 2010). The most commonly cited benefits include: improved reasoning skills, increased awareness of assumptions, enhanced understanding of scope and limitations and improved ability to apply new found knowledge in unfamiliar circumstances, all of which are desirable (even essential) for mathematics learning. Besides, with the increasingly easy access to the internet, successful implantation of OCL affords students extra learning opportunities/ flexibility beyond the physical constraints of time and space.

However, despite the many successful reports of OCL in text-based subjects, investigations into OCL for mathematics education has remained largely off the research radar. To understand the underlying reasons, this study investigates: a) the effectiveness of the internet as a platform for mathematical discussions, b) the evidence for suggesting the challenge of electronically formatting mathematical expressions is a significant barrier to OCL adoption, and c) the evidence for proposing MathPen (an online handwriting recognition system) as a solution.

METHODOLOGY

Since this study is primarily interested in the *interactive* nature of communication, an internet forum, where there is a large population of participants communicating mathematics in an interactive manner, is a natural choice for investigation. For the purpose of this study, the pre-university forums at www.mathhelpforum.com (MHF) were selected primarily due to their size, popularity and their administrator's kind approval for observing site activities for research purposes.

Adopting Sande's (2011) method of studying online communications, this study began with examining one hundred threads of mathematical discussions from each of the five different mathematical topics (algebra, trigonometry, geometry, pre-calculus and statistics) from MHF. Special care was taken to note the entry methods used to represent mathematics online and to identify cases that exemplifies the typical use, pros and cons of each entry method observed. The problems identified from these observations were then cross-examined and further investigated with an online questionnaire completed by eighty participants including internet forum members, practicing online tutors, UK-based qualified mathematics classroom teachers and university professors/ lecturers. To further verify the research findings and identify a possible way forward, a new revised concept of online handwriting recognition system (now called MathPen) was constructed especially to address the identified issues. To obtain reviews and comments from experts directly involved in mathematics education, the concept of MathPen is summarised in a short video and was sent to seven experts for professional feedback. Through this mixed method approach, the problems associated with OCL for mathematics learning were identified and MathPen is proposed as a potential solution.

RESULTS: EXAMINATION OF INTERNET FORUMS

The five hundred threads examined contained a total of 4819 mathematical statements or expressions, giving an average of 9-10 mathematical statements per thread/ discussion. With an average of 10-11 posts/ exchanges per discussions, it can be seen that these exchanges contain a mixture of mathematical statements and textual arguments/ explanations. Although the nature of these forums typically attracts a single standard textbook exercise per thread and therefore has little room for exploration, the potential for collaborative learning is strong.

		Topic					
		Algebra	Geometry	PreCalculus	Statistics	Trigonometry	Total
InputMethod	ASCII	580	348	580	336	288	2132
	Ext. Links	19	37	16	32	18	122
	Latex	767	418	574	224	432	2415
	Pictures	19	76	19	10	26	150
Total		1385	879	1189	602	764	4819

Table 1: Input method usage per mathematical topics on math help forum

The one hundred algebra-related threads contained a total of 1385 mathematical statements over just a mere 25 day period. Within the five topics studied (algebra, pre-calculus, statistics, trigonometry and geometry), 1385, 1189, 602, 764 and 879 mathematical statements were posted over a period of 25, 45, 49, 66 and 76 days respectively, giving an average of 55, 26, 12, 12 and 12 mathematical statements posted per day. The amount of mathematical statements posted within a short period of time prior to the start of the summer holiday further demonstrates the potential of the internet as a platform for online collaborative learning for mathematics.

The differing number of mathematical statements posted between each subject area is statistically significant (ANOVA, $\rho<0.001$), with Algebra being significantly more than Pre-Calculus, which is in turn significantly more than the rest. Although not significantly different from each other, both Trigonometry and Geometry are significantly more than Statistics. Possible reasons for the differences may include: A) higher interests in improving algebraic skills; B) some subject areas are more suitable to online communication; C) an imminent national exam affecting a significant number of students; and D) communication of algebra is less likely to be in text form.

Although most of the mathematical expressions were entered using Latex (50%) or ASCII (44%), establishing these as the main means of mathematical communications, the decision between using Latex or ASCII is subject dependent ($\chi^2(12)=228.385$, $\rho<0.001$). To highlight the inadequacy of ASCII, one of the threads studied shows a student asking for help with simplifying the fraction: "*(sin(n+1)A-sin(n-1)A)/(cos(n+1)A+2 cos(nA)+cos(n-1)A*". Figure 1a shows the students' subsequent attempt at solving the problem.

In order to help the student and work on the mathematics, one must first convert the mathematical statement into a recognisable form such as $\frac{\sin(n+1)A-\sin(n-1)A}{\cos(n+1)A+2\cos(nA)+\cos(n-1)A}$. Only then can one begin to interpret and comprehend the mathematics in the conventional way. Besides, did sin(n+1)A mean Asin(n+1) or sin{A(n+1)}? Studying the second statement $2\cos[\{(n+1)A+(n-1)A\}/2]\sin[\{(n+1)A-(n-1)A\}/2]$ as shown in Figure 1a, and converting it into $2\cos\left[\frac{\{(n+1)A+(n-1)A\}}{2}\right]\sin\left[\frac{\{(n+1)A-(n-1)A\}}{2}\right]$ indicated that sin{A(n+1)} was meant, a conclusion that is not immediately obvious in the ASCII format. Given that this was the student's 43rd post on this forum and having been a member for over a year, it is inferred that there is a certain measure of difficulty or reluctance to learn Latex. In this case, the complexity of the mathematics and the number of careful manipulations required to simplify the algebraic fraction shows that the use of ASCII would indeed distract the user from the mathematics itself, thus rendering its use unfit for online collaborative learning.

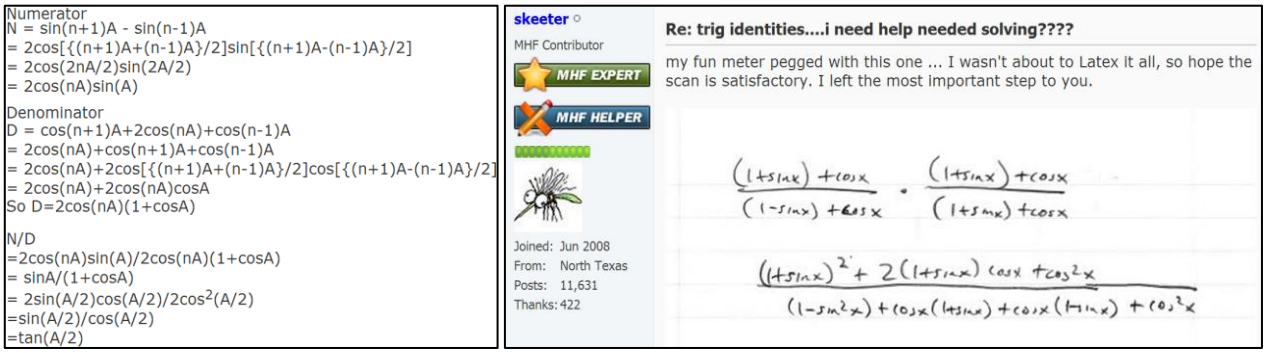

Figure 1: a) ASCII as an ineffective input method, b) Latex as an ineffective input method

That Latex can also be problematic is demonstrated in Figure 1b, which shows a post of an *expert helper*, despite his more than 11,000 posts, saying, "I wasn't about to Latex it all"! The helper subsequently uploaded a full page of scanned handwritten work instead. These Latex-replacement uses of scanned-pictures further indicate that forum members do rely on pen-and-paper to perform their calculations as opposed to calculating while writing online, thereby supporting the idea that the use of complex notations can be a barrier to online communication. Secondly, there is a cost associated with the use of Latex. Consider this: the helper would have performed the calculations on paper, confirmed its correctness, gone to the scanner, performed the scanning procedures, saved the file to a specified location on the computer, gone to the forum, located the pictorial file again and finally uploaded the file with a brief comment. Yet significantly, performing all these steps were considered to be faster and easier than having to "Latex it all", thus demonstrating Latex's weaknesses.

By studying a small sample of this vibrant forum, it can be seen that interactive communication of mathematics, and hence online collaborative learning, is possible. However, the two most commonly used input methods are inadequate for current needs. Users are still required to laboriously transfer their pen-and-paper-based calculations one line at a time and, when the tedious process becomes unbearable, users often resort to scanned-images.

RESULTS: ONLINE QUESTIONNAIRES

Of the 80 participants, 55% (44) are UK-based qualified mathematics teachers. There were also 18 university postgraduate students and 18 unqualified teachers. Amongst the qualified teachers, the average offline mathematics teaching experience is about 12 years, with the lower and upper quartile being 4 years and 19 years respectively.

		Confidence Level				Competence Level				Accuracy Level			
	N	min	max	mean	Std.dev	min	max	mean	Std.dev	min	max	mean	Std.dev
MS Math Editor	32	3	5	4.08	0.776	3	5	4.09	1.125	---	---	---	---
Latex	50	1	5	3.96	1.224	1	5	3.54	1.503	---	---	---	---
MathML	05	1	4	3.00	1.225	1	4	3.00	1.225	---	---	---	---
Handwriting Recognition	28	2	4	2.67	0.707	2	4	2.75	0.707	2	4	2.67	0.866

Table 2: Descriptive statistics of commonly used input method

The questionnaire (Table 2) reveals that Latex is the most commonly used input method. Though incompatible with the web, Microsoft's Mathematics Editor (MS-ME) also proves to be very popular and has the highest score for user confidence level (4.08 on a scale of 1-5). The confidence and competence levels also show that MS-ME is the most user-friendly, followed by Latex, MathML and current freely available handwriting recognition technology (such as Interactive Whiteboard and Windows 7 Math Input Panel). Unlike its coding/ programming based counterparts such as Latex and MathML, which requires a substantial amount of learning, MS-ME's mouse-based point-and-click operations make the user interface very easy and intuitive to learn. Similarly, since current freely available handwriting recognition technology suffers from poor accuracy (scoring only 2.67 on a scale of 1-5), it is not surprising to see it at the bottom of the preference list.

It is also noted that amongst those who hold a UK-based mathematics teaching qualification, there is a strong preference towards MS-ME over the use of Latex ($\rho<0.001$). One possible reason could be the simplicity of MS-ME. Interestingly, university postgraduates, who can be expected to overcome the challenges of learning Latex, have a strong preference towards Latex over any other technologies ($\rho=0.028$). These results further verify that Latex, although one of the most commonly used input methods for online communication, is non-intuitive to use.

Regarding the use of handwriting recognition technology as a solution to the problems, 72% of the participants believe that the technology will prove to be useful and 69% indicated that they are likely to use it should it become widely available. 56% of the participants have also volunteered personal contact details to be informed of future developments. Despite the small number of participants in this survey, these figures give a strong indication that an intuitive, user-friendly and accurate handwriting recognition technology would be greatly appreciated.

EXPERT REVIEWS

With the selection criteria of those who a) have been teaching mathematics for the last 10 years, b) have some experience of handwriting recognition technology and c) have not previously been known to the researcher, seven experts were identified and shown a video of MathPen. Their comments are:

> "*It would let me concentrate on substance rather than formatting*." --- Expert Helper of a Free Math Site

> "*If MathPen functions as shown in the video, then it will save me the time of having to look up LaTex codes and syntax. I would be able to tutor more students in less time, if I were able to easily and accurately transfer my writing into bulletin-board posts. I do have experience with web sites that offer LaTex symbol recognition by drawing the corresponding math symbol using the mouse. If MathPen works as shown in the video, then MathPen is a vast improvement over what I described above.*" --- Experienced Mathematics Teacher, Community College, USA.

> "*MathPen would speed up making worksheets. It would allow me more freedom over giving out worked solutions. I have used the free Microsoft Mathematics 4 on a tablet PC and on an interactive whiteboard, although it is not very accurate and it is only one line at a time. The ability to convert multiple lines is particularly attractive*." --- Experienced Mathematics Teacher, Comprehensive Secondary School, UK

"*For students, it would greatly facilitate their posing questions correctly. Less knowledge of math formatting is required.*" --- Head of Department, Comprehensive Secondary School, UK

"*Yes. A robust, reliable, scalable mathematical character recognition package compatible with the industry standard of Latex is long overdue and something that we have been saying should be developed for the past decade.*" --- Senior Lecturer in Mathematics, UK

"*It streamlines computer-mediated math communications; Learning LaTeX is tedious and will no longer be necessary.*" --- Professor in Mathematics Education, a university in Finland.

"*Students could present math expressions in their questions by simply writing it out. Student would not need to learn LaTex or texting conventions. Scanning to jpeg goes partway; Using MathType can be slow but is easily edited. If editable markup is available, MathPen would be a superior choice.*" --- Emeritus Professor of Mathematics Education, USA.

PROPOSING MATHPEN

As the forum activity shows, the desire to engage in mathematical conversations online can be so strong that people are willing to overcome the challenge of painstakingly converting each line of handwritten mathematics into electronic format. Figure 2a shows a student's attempt at seeking online help along with the Latex code required for such a post. Considering the complexity of the code, it is absolutely heart-warming and encouraging to see young school-aged children demonstrating such determination and passion. Yet, as the expert reviews suggested, should formatting mathematical expressions be a necessary evil? Should spontaneous responses to discussions (Figure 2a) be dampened by the strenuous efforts required to communicate? Should students not be able to write on tablet computers, as they would on a piece of paper? Should handwritten work (Figure 2b) not be automatically formatted into Latex (Figure 2c)?

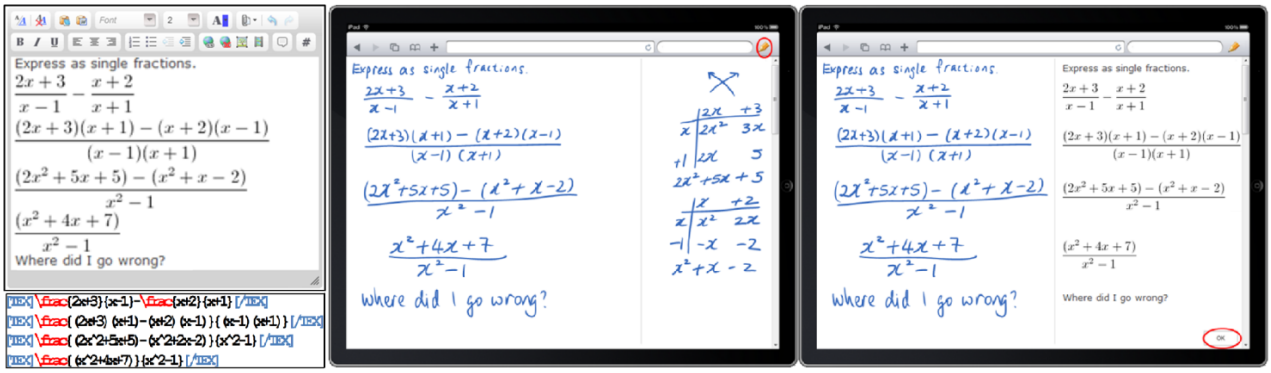

Figure 2: a) A forum message and its Latex code, b) handwritten work, c) automatic formatting

Although such recognition algorithms are freely accessible through research publications, commercial products are prohibitively expensive for many. Additionally, being *word-processing* orientated, every recognition system available assumes the user knows what each line of mathematics looks like before they start and that the users would be content with recognition one line at a time. In reality, however, each line of mathematical statements evolves as different pieces of information are processed. At the end of each line, new insight is gained thus sparking off another line of evolving mathematical statement. Therefore, the current user assumptions are invalid when it comes to the *doing* of mathematics (as opposed to the *word-processing* of mathematics) and the *interactive communications* of mathematics. Consequently, it is proposed that the published handwriting recognition algorithms should be repackaged with appropriate user interface to facilitate the *doing* of mathematics online and open the way for OCL in mathematics education. In fact, MathPen is currently under development and is intended to be a free, open-source online handwriting recognition system specifically designed for mathematics education.

SUMMARY

In agreement with other researchers (Catalin, Deyan, Kohlhase, & Corneli, 2010; Costello, 2010; Reba & Weaver, 2007), one of the problems associated with OCL in mathematics education is, as the forum analysis and online questionnaires showed, the lack of a natural and effective means of entering mathematical expressions online. Observations of forum discussions provided a glimpse of current practice and the challenges associated with entering mathematical expressions online. These were further verified with an online questionnaire, which provided further insight into the usability problems of current technologies. Based on these findings, the concept of MathPen was designed and sent to seven experts for professional feedback. All experts unanimously agreed on the potential benefits that MathPen could bring to mathematics education. Therefore, it is concluded that serious considerations should be given to online handwriting recognition systems as a means of opening the way to OCL for mathematics education.

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