Comparison of Correlation Dimension and Fractal Dimension in Estimating Level of Consciousness

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Abstract: *This paper compares the correlation dimension (D2) ,Higuchi fractal dimension (HFD),Katz fractal dimension(KFD)and Sevcik fractal dimension(SFD) approaches in estimating Depth of Anesthesia (DOA) based on of electroencephalogram (EEG). The single-channel EEG data was captured in both ICU and operating room and different anesthetic drugs, including propofol and isoflurane were used. For better analysis, application of adaptive segmentation on EEG signal for estimating DOA is evaluated and compared to fixed segmentation. Prediction probability (PK) is used as a measure of correlation between the predictors and BIS index to evaluate the proposed methods. The results show the ability of these algorithms in predicting DOA. Also, evolving fixed and adaptive windowing methods for segmentation of EEG reveals no meaningful difference in estimate DOA..*

Key words: Adaptive segmentation; Bispectral index; Correlation dimension; Depth of anesthesia; Fractal dimension.

1 Introduction

In the operating room, the anesthesiologist is essential in providing optimal working conditions to surgeons, and in ensuring patient safety and comfort. However, patient awareness during surgery with the rate of 1:1000 [1] and over dosing with anesthetic agents is of major clinical concerns of anesthesia. Therefore, the necessity to assess and monitor the depth of anesthesia (DOA) is obvious. In conventional methods, DOA is measured based on the monitoring of several physiological signals such as respiration pattern, blood pressure, body temperature, tearing, sweating and heart rate [2], even though these signals are affected indirectly by anesthetic agents. On the other hand, these agents have significant effects on the electroencephalogram (EEG) waveform. Hence, it is advocated that the EEG can provide a reliable basis for deriving a surrogate measurement of anesthesia.

In recent years, the anesthesia community has witnessed the development of a number of EEG-based algorithms of consciousness.

The earliest methods were based on the FFT analysis of EEG signals. These approaches tend to find parameters that describe spectrum characteristics. Peak power frequency (PPF), median power frequency (MPF), and spectral edge frequency (SEF) have been the first descriptors in this field. Another parameter extracted from spectrum was the ratio of power in two empirically derived frequency bands [3]. In a work presented by Traast et al. [4] the power of EEG in different frequency bands was determined and the results indicate pronounced changes in EEG during emergence from propofol/sufentanil total intravenous anesthesia.

Ferenets et al [5] analyzed the performance of several new measures based on the regularity and complexity of the EEG signal. These measures consist of spectral entropy (SpEn), approximate entropy (ApEn), and Higuchi fractal dimension (HFD) and Lempel-Ziv complexity (LZC). Their results show superior ability of the mentioned measures to predict DOA. In the case of their tested measures, they recommend window length of 20 s.

According to various mentioned methods, different EEG monitors have been developed. BIS™ (Aspect Medical Systems, Newton, MA), that is the first monitor in the marketplace and has become the benchmark comparator for all other monitors, introduces the BIS™ index (that is a unit-less number between 100 and 0) as a DOA indicator based on combination of spectral, bispectral and temporal analysis [3].

When a chaotic process (such as electrophysiological data obtained from electroencephalographic or electrocardiographic recordings) becomes more complex, it needs to be looked in higher dimensions. Grassberger and Procaccia [6], Denker and Keller [7], Theiler [8], and Cutler [9] developed the notion of time series such as physiological recordings [10] or technological applications [11]. Correlation dimension is by the most popular due to its computational efficiency relative to other fractal dimension such as the information dimension, capacity dimension, and pulse dimension [12].

Fractal dimension, on the other hand, is a measure of how ‘complicated’ a self-similar figure is [13]. In the particular case of curves in a plane, while a topological line is one-dimensional, a fractal curve has a fractal dimension D that is in the range $1 < D < 2$. In general, the compass fractal dimension represents a measure of the degree of shape complexity or roughness of the curve. Higuchi, Katz, Sevcik, Petrosian C,Petrocian D, Zero set,Adapted box, compass, Box Counting , variogram, Variance, Size Measure Relationship, FixedPoint, Correlogram, Periodgram, Whittle Estimator, Caspian, Capacity Dimension, Information Dimension, Lyapanov Dimension, Correlation Dimension, Box-Assisted are some methods to calculate fractal dimension of signals. The aim of this study is to introduce an efficient method with the application of adaptive segmentation, based on correlation dimension and fractal dimension to measure DOA.

2 Methodology

In this section, the experiment, the data acquisition, and the data analysis are described.

2.1 Patients

Following the approval of the ethical committee of the medical school, eight coronary artery bypass graft surgery candidates were selected (6 males, 2 females, of average age 56.2 years and the average weight of 68.3kg) and written informed consents were obtained from all selected subjects. Inclusion criteria were absent of neurological disorders such as cerebrovascular accidents and convulsions. Preoperative neurological complications (such as cerebral emboli and convulsion) caused exclusion from the study. The anesthesiologist performed Preoperative evaluation on the day before surgery. For anxiolysis, the patients were premedicated by intramuscular morphine 0.1 mg/kg and promethazine 0.5 mg/kg, 30 minutes before transfer to operating room. After arrival in operating room, electrocardiogram, pulse oxymetry, depth of anesthesia, and invasive blood pressure monitoring was established. The BIS-QUATTRO sensorTM (Aspect Medical Systems, Newton, MA) applied to the forehead of the patients before induction of anesthesia. Then 8 patients after preoxygenation with O2, were anesthetized in the same manner by intravenous thiopental sodium (5mg/kg), pancuronium bromide (0.1 mg/kg), fentanyl (5µg/kg), and lidocaine (1.5 mg/kg). After the induction of anesthesia and until cardiopulmonary bypass beginning, anesthesia continued by administration of isoflurane (1 MAC), morphine (0.2 mg/kg) and O2 (100%). During coronary artery bypass grafting under CPB, patients were anesthetized by propofol (50-150 µg/kg/min) under BIS control (40-60) and O2 (80%). For organ protection during CPB, patients were undergone mild hypothermia (31-33°C). After coronary artery bypass grafting and patients rewarming and obtaining standard CPB separation criteria, the patients gradually were weaned from CPB. After separation from CPB, anesthesia was continued by isoflurane (1 MAC) and O2 (100%) administration to the end of surgery. After surgery, patients were transported to ICU under portable monitoring and manual ventilation. In the ICU mechanical ventilation with 60% fractioned inspired oxygen and standard homodynamic monitoring were continued. In ICU and until complete recovery, the sedative regimen was intravenous morphine (2mg) if needed. In this study the raw EEG data and relative BIS index were collected during whole period of operation from operative room arrival to complete recovery in the intensive care unit.

2.2 Data acquisition

The EEG signal was collected by using a BIS-QUATTRO Sensor™ that was composed of self-adhering flexible bands holding four electrodes, applied to the forehead with a frontal-temporal montage.
The used EEG lead was Fpz-At1, and the reference lead was placed at FP1. The sensor was connected to a BIS-X-P Monitor and all binary data packets containing raw EEG data wave signals and BIS index which is converted to binary format using an A/D converter operating with 128 Hz sampling frequency were recorded via an RS232 interface on a laptop using a Bi-spectrum analyzer developed with C++ Builder by Satoshi Hagihira [14]. The algorithms that are presented in this study were tested on these raw EEG signals.

The sensor was attached to the patient's forehead at the beginning of anesthesia and the data were collected continuously until he/she awoke at ICU. Therefore, in this study a large amount of EEG data with their BIS index was collected for each patient. Some other events such as changes of anesthesia regimen, intubations and applying CPB and transferring to ICU were recorded. Because of short acting time of thiopental sodium (approximately 15-20 sec), this part of EEG data was not analyzed.

2.3 Correlation Dimension

The Grassberger and Proccacia algorithm using Theiler method [15 and 16] considers spatial correlation between pairs of points on a reconstructed attractor. For an E_m-dimensional phase space, the modified correlation integral $C(r)$ is defined as [17]if:

$$X(i) = (x(i), x(i+\tau), \ldots, x(i+(E_m-1)\tau)) \qquad (1)$$

$$C(r) = \frac{2}{(m+1)(m)} \sum_{i=1}^{m} \sum_{j=i}^{m-i} \Theta(r - \|X(i) - X(j)\|) \qquad (2)$$

Where $m = N-(E_m-1)\tau$ is the number of embedded points in an E_m-dimensional space, N is the length of data series, and r is the radius of sphere centered on $x(i)$ or its box size, and $\Theta(x)$ is the heaviside step function as follows

$$\Theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad (3)$$

If $C(r)$ scales like

$$C(r) = r^{v} \qquad (4)$$

Then, v is called the correlation dimension of the time series x or equivalently the slop of the log-log plot of $C(r)$ versus r . In this study correlation dimension is calculated while r is fixed to 10.

The delay time τ is commonly determined by using the autocorrelation function (ACF) method (by finding the place where ACF first attains zeros or below a small value, e.g. 0.2 or 0.1), or mutual Information (MI) method [18] (by finding the place where MI first attains a minimum). In this study, τ is changed manually from one to ten. The result shows that smaller τ s would yield in better correlation between D2 and BIS index. So, it is fixed to one. Finally, E_m is fixed to 10 and correlation dimension is calculated.

Consequently, EEG is divided in to fixed (20 seconds) segments and correlation dimension is calculated using the above parameters. The above parameter is named "fixed CD".

2.4 Fractal Dimension

2.4.1 Higuchi Fractal Dimension

Nonlinear methods that assess signal complexity matter if the signal itself is chaotic or deterministic.

Consider the time series $x(1), x(2), \ldots, x(N)$, where N is the total number of samples. The algorithm constructs k new time series as:

$$X_m(k) = \{x(m), x(m+k), x(m+2k), \ldots, x(m+\lfloor (N-m)/k \rfloor k)\}$$

Where $m = 1,2,\ldots,k$. The length, $L_m(k)$, is calculated as:

$$L_m(k) = \left[\left(\sum_{i=1}^{\left\lfloor \frac{N-m}{l} \right\rfloor} \left| x(m+ik) - x(m+(i-1)k) \right| \right) \cdot \frac{N-1}{\left\lfloor \frac{N-m}{k} \right\rfloor . k} \right] / k \qquad (5)$$

Higuchi algorithm calculates fractal dimension of a time series directly in the time domain. It is based on a measure of length, $L(k)$ of the curve. An average length is computed for all time series having the same scale k , as the mean of the k length $L_m(k)$ for $m = 1,2,3,\ldots,k$. If $L(k)$ scale like, $L(k) \propto k^{-D_f}$, then the curve is said to show fractal dimension D_f [19].

The fractal dimension of EEG signal is calculated via above method while fixed windowing method (with the window length of 20 seconds) which is named "fixed HFD".

2.4.2 Katz Fractal Dimension

The Katz algorithm [20] for obtaining a waveform FD is based on the time series and calculated as:

$$FD_{Katz-norm} = \frac{\log_{10}(n)}{\log_{10}(d/L) + \log_{10}(n)} \qquad (6)$$

Where d is the value of the maximum distance measured from the beginning of the first increment, l is the total length of the EEG time series ,and n is the number of samples.

The fractal dimension of EEG signal was calculated the same as "fixed HFD" which is named "fixed KFD".

2.4.3 Sevcik Fractal Dimension

In 1998 Carlos Sevcik [21] presented a variation of KFD method which involves normalization along the y_- and x_- axes before calculation of FD:

$$X_i^* = \frac{x_i}{x_{\max}} \quad (7)$$

$$y_i^* = \frac{y_i - y_{\max}}{y_{\max} - y_{\min}} \quad (8)$$

Where $x_{\max}$ is the maximum x_i and $y_{\min}$ and $y_{\max}$ are the minimum and maximum y_i from this values, the length l of the signal within the window of n samples is determined as mentioned above for the KFD.then,

$$FD_{sevcik} = 1 + \frac{\ln(l) + \ln(2)}{\ln(2n')} \qquad n' = n - 1 \quad (9)$$

In this part we calculated fractal dimension of EEG signal the same as other methods and named it "fixed SFD".

2.5 Adaptive Segmentation of EEG Data

It would be desirable to adapt the analysis window to changes in the given signal, allowing the window to be as long as possible while the signal remains stationary, and to start a new window at the exact instant when the signal changes its characteristics [24]. In order to perform the described approach, two adaptive windowing methods have been used, adaptive variance detection and ACF (Auto-Correlation Function) distance methods. In both approaches, a reference window is extracted at the beginning of each scan, and the given EEG signal is observed through a moving window. In each of the above algorithms, the performance of the reference window length L= 1 second has been evaluated.

In adaptive variance detection method, a segment boundary is drawn when the variance of the moving window becomes *m* times greater or larger than the variance of the reference window. By considering the changes of EEG variance, $k = 7$ and $k = 10$ are tested and finally $k = 10$ was considered better for applying the above procedure (The CD, HFD, KFD and SFD values obtained using this method of windowing are named "CD_var" , "HFD_var", "KFD_var" and "SFD_var", respectively).

On the other hand, for implementing ACF distance method, let $\phi_R(k)$ be the ACF of the reference window at the beginning of a new segmentation step, where *k* is the lag or delay. Let $\phi_T(n,k)$ be the ACF of the test and sliding window positioned at time instant *n*. A normalized power distance *d p* (*n*) between ACFs is computed as:

$$d_p(n) = \frac{\left|\sqrt{\phi_T(n,0)} - \sqrt{\phi_R(0)}\right|}{\min\{\sqrt{\phi_T(n,0)}, \sqrt{\phi_R(0)}\}} \quad (10)$$

The condition where *d p* (*n*) becomes larger than a specific threshold, *ThP*, is considered to represent a significant change in ACF, and used to mark a segment boundary [22]. Again, due to the variation of *d p* (*n*) values, three different *ThP* s (25000, 30000, and 35000), are tested and $Th_P = 30000$ is used to evaluate the described approach (The CD, HFD, KFD, SFD values obtained using this method of windowing are named "CD_ACF", "HFD_ACF", "KFD_ACF" and "SFD_ACF" respectively).

2.6 Statistical Analysis

The correlation between BIS index and the extracted sub-parameters was investigated with the model-independent Prediction Probability (Pk) [23]. As a nonparametric measure, the Pk is independent of scale units and does not require knowledge of underlying distributions or efforts to linearize or otherwise transform scales. A Pk value of 1 means that the predicting variables ("fixed HFD", "HFD_var", "HFD_ACF", "fixed CD", "CD_var", "CD_ACF", "fixed KFD", "KFD_var", "KFD_ACF" and "fixed SFD", "SFD_var", "SFD_ACF") always predict the value of the predicted variable (e.g., BIS index) correctly. Pk value of 0.5 means that predictors predict no better than only by chance. The Pk values were calculated on a spreadsheet using the Excel 2003 software program and the PKMACRO written by Warren Smith [23]. In the case of inverse proportionality between indicator and indicated parameters, the actual measured Pk value is 1- Pk. Another statistical analysis used in this study was ordinal logistic regression. This regression examines the relationship between one or more predictors and an ordinal response. The index that determines the efficiency of this regression model is called "Concordant", which shows the percentage of values predicted successfully with the model.

3 Results and Conclusion

Our results are presented in three different groups; during propofol and isoflurane anesthesia in operating room and in ICU under sedative regimens. Higuchi, katz, sevcik fractal dimension and correlation dimesion were calculated based on the algorithms described in the previous sections and different parameters were obtained. Correlation between our extracted parameters ("fixed HFD", "HFD_var", "HFD_ACF", "fixed CD", "CD_var", "CD_ACF", "fixed KFD", "KFD_var", "KFD_ACF", "fixed SFD", "SFD_var" and"SFD_ACF") and BIS index were calculated via proper statistical analysis. In order to apply logistic regression analysis the total BIS index range was divided into six non-overlapping groups: 0-25, 25-40, 40-50, 50-60, 60-80, and 80-100.Then, the best model was determined to identify how good a predictor could predict the BIS level.

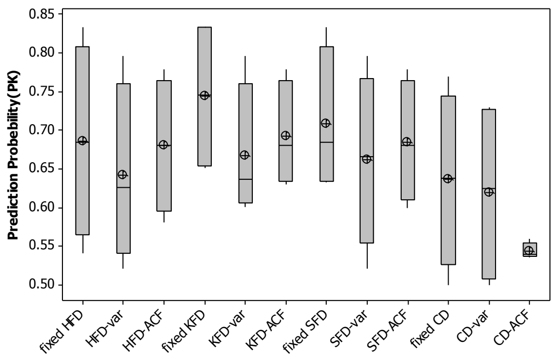

Figure 1. Comparison of different parameters in ICU group using Pk values

According to figure 1, KFD reveals higher Pk values than HFD ,SFD and CD. Thus, KFD could predict BIS index much more acceptable in comparison to the others. Also, it could be concluded from figure 1 that application of adaptive segmentation could improve the estimation of DOA neither based on KFD nor based on other methods.

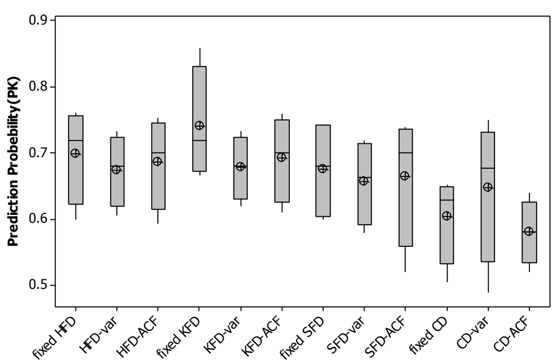

Figure 2. Comparison of different parameters in isoflurane group using Pk values

According to figure 2, KFD parameters ("fixed KFD", "KFD_var", and "KFD_ACF") are more capable of estimating DOA than HFD parameters ("fixed HFD", "HFD_var", and "HFD_ACF") in propofol group, due to their higher Pk values. HFD Pk values are more capable of estimating DOA than SFD and CD. Adaptive segmentation methods could improve the performance of CD in predicting DOA, especially using variance method. However, this is not true for none of the remaining parameters.

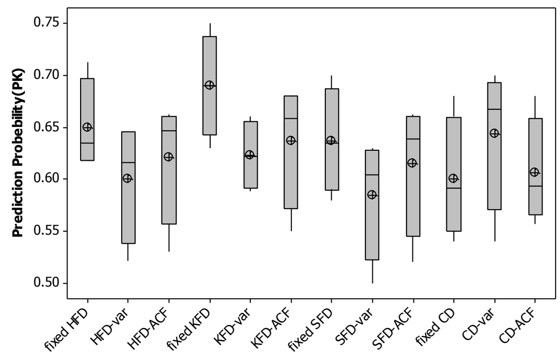

Figure 3. Comparison of different parameters in propofol group using Pk values

Based on figure 3, KFD parameters ("fixed KFD", "KFD_var", and "KFD_ACF") are approximately as capable as other parameters in estimating DOA in isoflurane group. Among different extracted parameters in this group, "CD_var" could predict BIS index better than other parameters. Although adaptive segmentation methods couldn't improve the performance of CD in predicting BIS index, they are able to improve estimating of BIS index based on correlation dimension calculation.

4 Discussion

Some methods based on Higuchi, Katz and Sevcik fractal dimension, correlation dimension and adaptive segmentation are proposed for estimating DOA. HFD, KFD, SFD and CD values are calculated using fixed and different adaptive windowing methods.

Due to the fact that HFD, KFD, SFD and CD are capable of quantifying signals and systems complexity, high Pk values were obtained while using these parameters as measures of DOA. Nevertheless, adaptive segmentation methods were not able to improve the assessment of DOA, except when CD was computed using variance method in isoflurane and propofol groups. The main result was the superiority of various HFD, KFD, SFD parameters to those of CDs in predicting the BIS index in most of the groups and methods which is evidenced by the high correlation of those parameters with BIS index.

The work reported is preliminary. Although the results are significant, wide patient population is necessary for better evaluation.

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