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AN ACTIVE VISCOELASTIC METAMATERIAL WITH ENHANCED BAND GAP PROPERTIES

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Metamaterials have been the subject of significant interest over the past decade due to their ability to produce novel acoustic behaviour beyond that seen in naturally occurring media. Of particular interest is the appearance of band gaps which lead to very high levels of attenuation across the material within narrow frequency ranges. Unlike traditional periodic materials which have been employed at high frequencies, the resonant elements within metamaterials allow band gaps to form within the long wavelength limit. It is at low frequencies where it is most difficult to design satisfactory passive isolation solutions, and hence metamaterials may provide a useful path to high performance, low frequency isolation. A locally resonant, periodic metamaterial is presented that could be employed as a high performance vibration isolator at low frequencies. The passively occurring band gap is enhanced using an active control architecture. The use of the active control system in conjunction with the natural passive behaviour of the metamaterial enables high levels of isolation across a broad frequency range. An eventual goal of the work is to produce such materials on a small scale, and as such the metamaterial developed has been designed for, and produced using, additive layer manufacturing techniques

1. Introduction

Over the last decade there has been significant research interest into the development of acoustic metamaterials and the novel behaviour they produce. Locally resonant periodic materials, called metamaterials, were first proposed in the electromagnetic domain by Smith[1] as a way of realising the 'left handed' behaviour postulated by Vesalago many years earlier,[2] where the implications of a material with simultaneously negative electromagnetic permeability and permittivity were presented. The analogy of acoustic and electromagnetic waves has led to a great deal of research into acoustic metamaterials,[3,4,5] where the analogues of permeability and permittivity are density and bulk modulus respectively. When both become negative simultaneously (where the material is said to be in its double negative (DNG) region), the refractive index has a negative sign so negative refraction occurs. In this region Snell's law still applies, however the path of the reflected wave lies to the opposite side of the incident normal than one would expect in a regular transmission medium, and the group and phase velocity vectors are anti-parallel. Band gaps appear in the dispersion characteristics of metamaterials, at high frequencies due to Bragg scattering effects[4] related to the periodic properties of the material and, in materials where low frequency resonances occur, at frequencies orders of magnitude lower.[5] This leads to high levels of attenuation in the transmission characteristics of the material at these frequencies. These novel properties mean that metamaterials are of particular interest and have been proposed as a potential solution to achieve acoustic cloaking,[6] transmission blocking[7] and subwavelength acoustic lenses.[5] More recently still, research has been published into the possibility of incorporating active control within a metamaterial structure to enhance and control this behaviour[8,9,10,11,12]

This paper presents experimental validation of an active 1-dimensional viscoelastic metamaterial consisting of a chain of transmission masses connected to each other and to resonator masses via spring elements. The material is based on that first proposed by Pope and Daley[10] and subsequently developed by Pope et al[13,14] and Reynolds et al.[11]The material proposed here achieves DNG behaviour through passive coupling of resonators to 2 transmission masses, as opposed to previous examples which had one resonator mass per transmission mass, and required active architecture to achieve double negativity. Since locally resonant designs produce materials with dispersive properties the beneficial behaviour is often limited to fixed and narrow frequency bands, active control has been employed in the material to broaden the region at which attenuation is produced using a Filtered-x Least Means Square (FxLMS) algorithm. Such a metamaterial would be useful in vibration isolation applications.

As metamaterials development continues towards a commercially viable solution it will be necessary to create a higher density of resonant elements and increase the degrees of freedom. As densities increase conventional manufacturing techniques are impractical on such scales, and additive layer manufacturing (3-dimensional printing) is likely to be a viable solution. Therefore the material presented here was designed for, and produced using, existing additive layer manufacturing techniques.

2. The Passively Coupled Metamaterial

The metamaterial that was created as part of this program was based on previous work carried out by Pope and Daley[10] and Reynolds et al.[11] Here, a metamaterial was designed based on a mechanical analogue of an array of Helmholtz resonators This original design was shown to be single negative (SNG), with a region of negative effective mass. It was shown that using an active control architecture to couple the resonator elements to adjacent transmission elements, DNG behaviour could be achieved and manipulated.The material presented in this paper achieves double negativity passively via additional viscoelastic connections, which are added to the resonator elements such that each resonator element is now connected to two adjacent mass elements. This passively couples the resonators to mass elements in the adjacent layer. The resulting material is shown in Fig. 1, where m_t, k_t, and c_t are the mass, stiffness and damping components respectively of what is considered the transmission elements of the material, and m_r, k_r and c_r are the mass, stiffness and damping elements of what are considered the resonator elements. To provide active control, unlike the point forces suggested in the previous work, reactive forces are applied between the resonator elements, denoted by f_c. The use of reactive actuators simplifies the design process of the control algorithm at the expense of freedom over the nature of control forces that can be applied.

By considering the material as an equivalent simple chain of effective masses connected by viscoelastic connections the equation of motion for the material can be written as

$$-\omega^2 M_e x_n + (i\omega C_e + K_e)(2x_n - x_{n+1} - x_{n-1}) = f_n \tag{1}$$

By formulating and rearranging the equations of motion for the explicit material model in Fig. 1, we

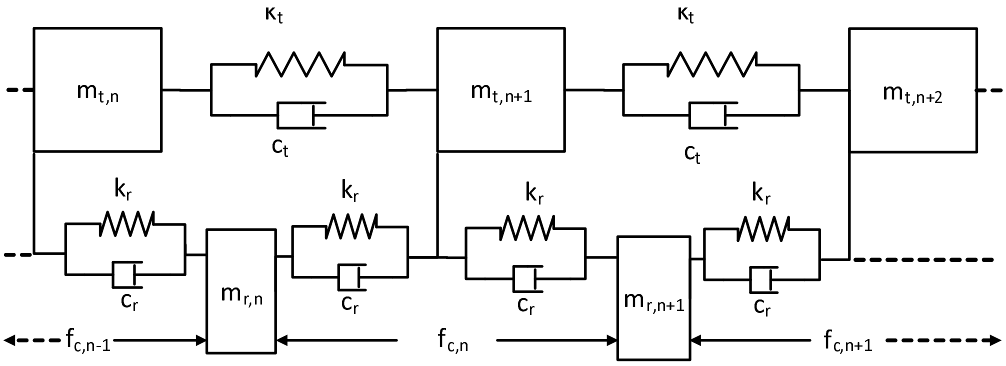


Figure 1: The metamaterial model

can write

$$-\omega^2\left(m+\frac{2m_r(k_r+j\omega c_r)}{-\omega^2 m_r+2(k_r+j\omega c_r)}\right)x_n\dots$$
$$+\left(k+j\omega c+\frac{(k_r+j\omega c_r)^2}{-\omega^2 m_r+2(k_r+j\omega c_r)}\right)(2x_n-x_{n+1}-x_{n-1})=f_n \qquad (2)$$

From this it is evident that the effective mass, M_e and stiffness and damping $K_e+j\omega C_e$ are given by

$$M_e=m+\frac{2m_r(k_r+j\omega c_r)}{-\omega^2 m_r+2(k_r+j\omega c_r)} \qquad (3)$$

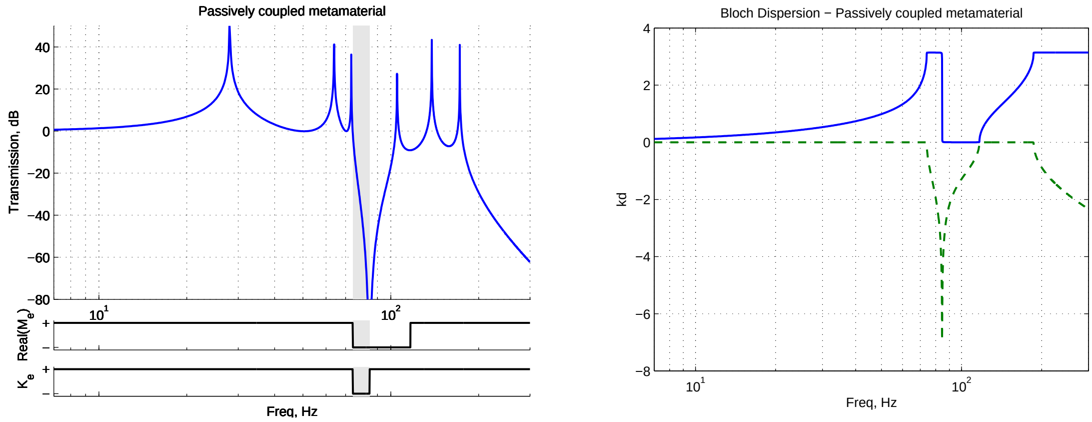
$$K_e+j\omega C_e=k+j\omega c+\frac{(k_r+j\omega c_r)^2}{-\omega^2 m_r+2(k_r+j\omega c_r)} \qquad (4)$$

These expressions are complex, where $\mathrm{Im}(M_e)$ is an additional dissipative term introduced by the presence of damping within the resonator elements, whilst $\mathrm{Re}(M_e)$ is the effective inertial mass of the system and the quantity of interest. Likewise the real part of $K_e+j\omega C_e$ is the effective stiffness of the system, with the imaginary part being the dissipative effective damping. Both terms for the effective stiffness and mass resonate at the same frequency, meaning that the regions of negative effective material parameters will coincide, and therefore the material will be double negative. A typical passive material transmission response (the motion of the last transmission mass related to the first), including the sign of each material parameter is shown in Fig. 2(a)

The double negative region also coincides with the region of attenuation known as the band gap. Previous studies have suggested that the resonant band gap occurs when the metamaterial becomes single negative, causing the wave vector, k, to become imaginary, leading to an evanescent rather than travelling wave solution to the wave equation. When the material is made to be double negative, the solution becomes negative real and a passband occurs in what is now a 'left-handed' material[15].[16] However, this explanation neglects dissipative losses within the material. Equation 5 states the Bloch dispersion relationship of a periodic metamaterial of this type of infinite length, where d is the lattice constant.

$$\cos kd=1-\frac{\omega^2 M_e}{K_e+j\omega C_e} \qquad (5)$$

Here it is clear that the solution will be complex for a DNG material if the effective material parameters are complex themselves, leading to a travelling (real) wave with an attenuation (imaginary) envelope. When the relationships governing the effective material parameters resonate, the magnitude of imaginary terms become very large, leading to high levels of attenuation. Whilst not a 'complete' band gap, as defined by Bloch-Floquet theory, attenuation levels are so high as to effectively behave as



(a) (b)

Figure 2: (a) The passive transmission response of a 4 layer coupled metamaterial, the double negative region is shown shaded. (b) The dispersion relationship of the material

a band gap. Solving Equation 5 for kd reveals the dispersion characteristics of the material, plotted in Fig. 2(b), and it is clear that the region where the magnitude of the imaginary part of the wave number becomes large coincides with the band gap of the material response. Note that due to the restricted domain of the $\cos^{-1}$ function, the calculated real part of the dispersion characteristic is restricted to $0 \leq kd \leq \pi$; in reality the wave number is not bound in this way.

The metamaterial consists of a number of periodic layers, therefore a convenient method for modelling vibration propagation through the material is to consider the material as a series of 4-pole linear ported networks making up a transmission line. Here, a transfer matrix is formulated to describe the transform from one state vector to another across the chain of masses (e.g. $q_n \rightarrow q_{n+1} \ldots \rightarrow q_N$). To formulate the transfer matrix the state vectors representing the force (f) and displacement (q) of each cell of the chain are considered, where in this case these state vectors are scalar values and each cell consists of a single set of equivalent mass-spring-damper elements. Now, the relationship between the force and displacement across a multiple layer material can be considered in terms of the transfer matrices, $\boldsymbol{T}$, such that

$$\begin{Bmatrix} q_N \\ f_N \end{Bmatrix} = \prod_0^N \boldsymbol{T}_n \begin{Bmatrix} q_0 \\ f_0 \end{Bmatrix} = \boldsymbol{\tau} \begin{Bmatrix} q_0 \\ f_0 \end{Bmatrix} \tag{6}$$

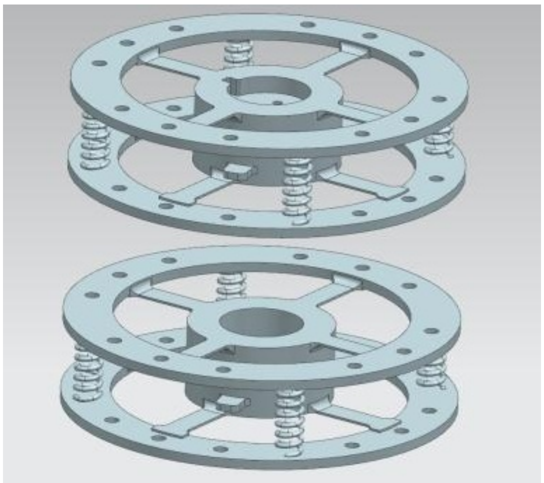
$$\boldsymbol{T}_n = \begin{bmatrix} 1 - \frac{\omega^2 M_e}{K_e + j\omega C_e} & -\frac{1}{K_e + j\omega C_e} \\ \\ \omega^2 M_e & 1 \end{bmatrix} \tag{7}$$

Note that if the multiple layers are identical, as is the case with a periodic metamaterial, the product of transfer matrices reduces to $\boldsymbol{\tau} = \boldsymbol{T}^N$. Also note that $|T| = 1$, which is a property of reciprocal systems, and more specifically a constraint of passive, linear, four-pole networks. By applying the initial condition $f_N = 0$ the transmission transfer function, H, of the material from the first to the last mass is given by Equation 8 where the subscript (p, q) denotes the pth and qth column of $\boldsymbol{T}^N$.

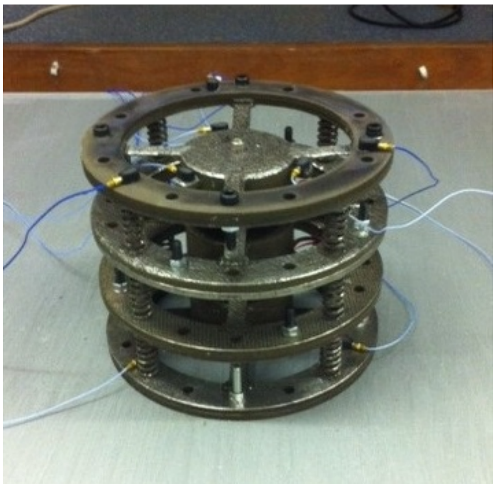
$$H_T = \frac{x_N}{x_0} = \left(\boldsymbol{T}_{1,1}^N - \frac{\boldsymbol{T}_{1,2}^N \boldsymbol{T}_{2,1}^N}{\boldsymbol{T}_{2,2}^N} \right) \tag{8}$$

3. Experimental Design

A 4-layer experimental realisation of the metamaterial was designed, consisting of titanium discs separated by coil springs acting as the transmission medium. Between each transmission layer is



(a)



(b)

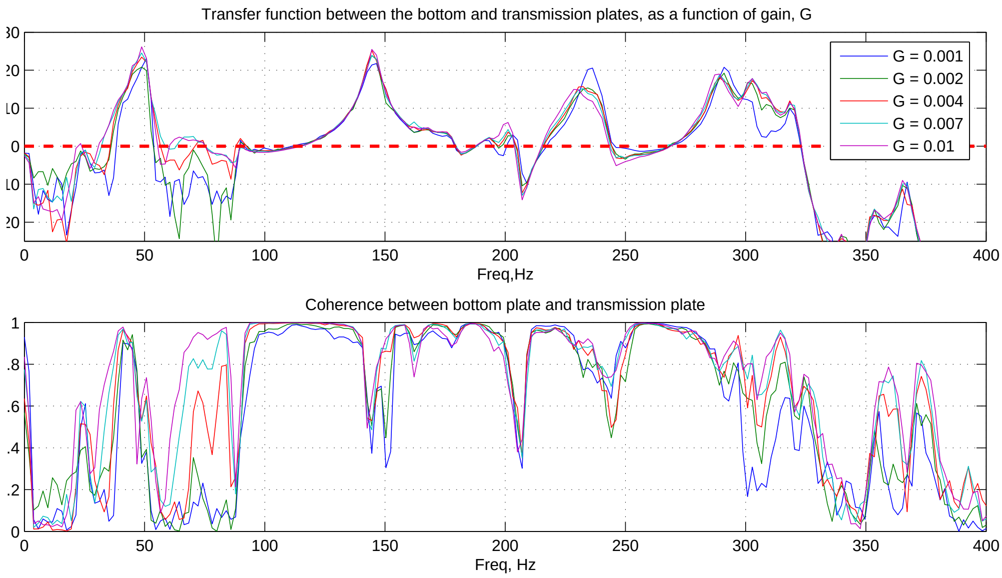
Figure 3: (a) A 3D CAD representation of the metamaterial construction. (b) A photo of the experimental metamaterial

a resonator mass, connected to each transmission layer using leaf springs. The material was produced as 3 cells using an additive layer, selective laser melting (SLM) technique, so each cell is a discrete titanium structure. The cells then bolt together to produce a continuous, periodic material. The resonator masses contain hollows to accommodate voice coil type reactive actuators, such that control forces can be applied as per Fig. 1. A CAD representation of the cell design is shown in Fig. 3(a), alongside a photo of the experimental setup in Fig. 3(b).

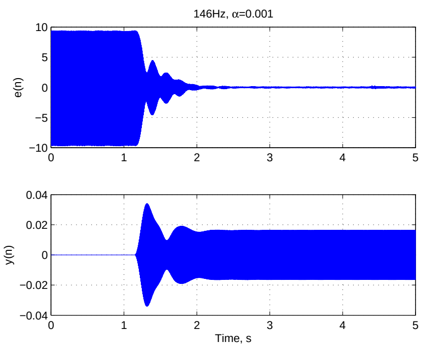
To measure the response of the complete metamaterial, the structure is bolted to a steel plate which is then excited using an inertial actuator and the motion of the top and bottom transmission plates is then measured using accelerometers, and the transfer function calculated. A dSpace rapid prototyping system is used to provide the excitation signal as well as record the accelerometer signals. Active control can also be applied using the dSpace system, which processes the accelerometer signals in real time using the appropriate algorithm to apply control forces to the reactive actuators. For experimental expediency, a single channel Filtered-x Least Mean Squared (FxLMS) algorithm was used as this is a well established, robust algorithm[17] that could be simply implemented to demonstrate the efficacy of the active approach. The algorithm was employed to minimise the acceleration of the top transmission plate using the acceleration of the bottom plate as a reference signal. Although the active material contains two actuators, in this instance only one is employed.

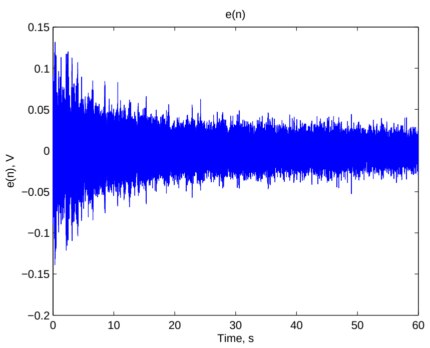
4. Experimental results

Figure 4(a) presents the passive response of the material for different excitation levels, as well as the coherence between the signals measured at the bottom and top plate. The material was designed to have a passive response as shown in Fig. 2(a), and initially it was believed that there was evidence of a band gap at approximately 80Hz as anticipated. However it can be seen that the attenuation in this region decreases as the excitation level increases.Therefore this behaviour appears to be the consequence of a non-linearity. This is thought to be due to the presence of stiction within the actuator assemblies due to misalignment, as well as a significant air-spring effect. However, it can be seen from the response that there are stable band gaps at higher frequencies - A deep, narrow gap at 207Hz, and a wider but shallow gap around 250Hz. It should be noted that it became clear during the manufacturing process that the additive layer manufacturing technique employed was being stretched to its limitations, and several problems had to be overcome. In light of this, the discrepancy in the predicted and achieved band gap frequencies is perhaps not surprising. Of particular consequence is

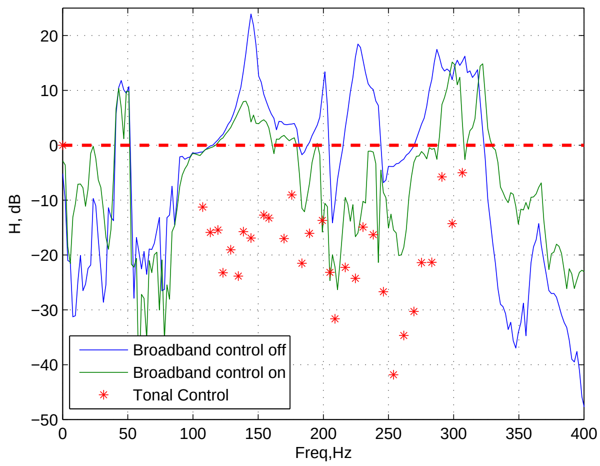


(a)





(b) (c)



(d)

Figure 4: (a) The transmission spectrum of the metamaterial at varying levels of input disturbance. (b-c) Time histories of the active metamaterial when control is applied after approximately 1 second, where (b) shows the top transmission plate acceleration (top) and control signal (bottom) for a 146Hz tonal disturbance and (c) the acceleration of the top transmission plate for a band limited broadband disturbance. (d) A comparison of the reduction in transmission achieved for a broadband input with the equivalent maximum reduction achieved to a tonal input at each frequency

the specification of the leaf springs, the stiffness of which is very sensitive to the thickness, t_b of the springs ($k_r \propto t_b^3$); manufacturing difficulties meant that the leaf springs produced were of varying thickness. In addition to these inconsistencies, several of these springs were warped and do not sit parallel with the transmission plate. This is likely to have had a large effect on the stiffness of the elements and means that the 2 resonator elements are not resonating at the designed frequency, nor necessarily at the same frequency as each other. Whilst the resulting response is not one that would be desirable in a passive isolation application, the material does indicate the potential of the approach, and provide a useful proof of concept of applying active control to enhance the attenuation properties of viscoelastic metamaterials.

The FXLMS algorithm was applied to control the acceleration of the top transmission plate initially for tonal disturbances, and later for a white noise disturbance that was band limited from 100Hz to 300Hz. Figures 4(b) and 4(c) show time histories for the top transmission plate after control has been applied for both tonal and broadband disturbances respectively, demonstrating that the controller is able to achieve a considerable reduction in the transmission relative to the passive performance. The tonal control is achieved using two filters of 1 coefficient, controlling the amplitude of an in-phase and a quadrature version of the reference signal to cancel out the disturbance. To control a broadband disturbance, a single control filter of 2048 coefficients is shaped by the FxLMS algorithm.

Figure 4(d) shows the transmission spectrum of the passive material alongside the performance achieved by the active material when subject to the band limited broadband disturbance. Also plotted on the graph are the equivalent reductions achieved by the tonal controller at each distinct frequency. The performance of the broadband controller is limited by the length of the control filter, and the tonal results can be considered a guide to the maximum performance that could be achieved using this set up, either using a longer filter or using more refined broadband control algorithm. The broadband results demonstrate that over the frequency span where excitation is present there is a significant improvement over the passive isolation performance. The naturally occurring band gap has been significantly broadened, and peak attenuation levels have been enhanced significantly. Detrimental out of band resonances have also been suppressed, notably at 146Hz.

The tonal results show that significant isolation performance can be achieved over a broad frequency range, in some cases up to 40dB. Of significance is the fact that the greatest levels of attenuation are achieved where the natural band gaps occur. This supports the argument for using active periodic materials in vibration isolation applications over more traditional active isolation mounts. A unified design process that matches the best features of both active and passive functionality can provide impressive levels of attenuation over wide band widths. Despite the fact that the experimental metamaterial did not match the original design specification, the results demonstrate a proof of concept that active architecture can be used to enhance the passive performance of metamaterials, and in particular the narrowband behaviour inherent in periodic, locally resonant designs.

5. Conclusion

A 1 dimensional viscoelastic metamaterial was designed that displays double negative behaviour passively. An active control architecture is incorporated into the design to facilitate the enhancement of the level and bandwidth of attenuation associated with the resonant band gap. Since an eventual goal of the work is to produce such materials on a small scale, the metamaterial was designed for manufacture using additive layer manufacturing techniques. An experimental demonstration of the material design was created from titanium using a selective laser melting additive layer technique. The limitations of this fledgling technology had a detrimental impact on the resulting dynamics of the material, however it was demonstrated that resonant band gaps appear in the passive transmission response. Furthermore it was shown that the level and bandwidth of attenuation of these passive band gaps can be enhanced through the use of the active architecture.

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