Benchmarking Capabilities of Evolutionary Algorithms in Joint Channel Estimation and Turbo Multi-User Detection/Decoding

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2013 IEEE Congress on Evolutionary Computation
Cancún, Mexico, June 20-23, 2013

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Outline

Motivations

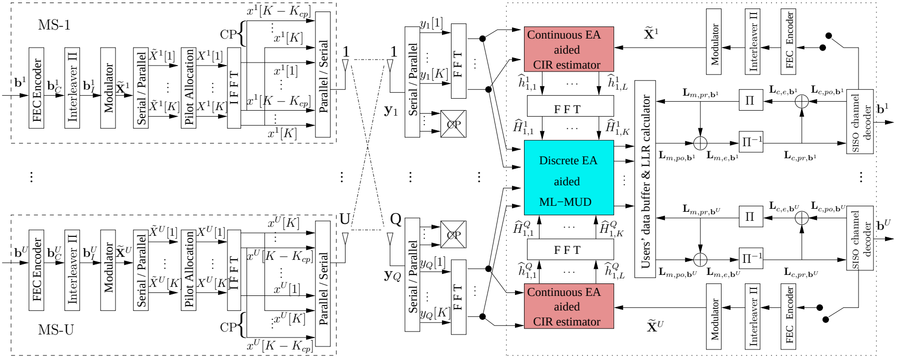
1. What critical to a communication signal processing application are **performance** and **complexity**
   - Optimal solutions are ofter NP-hard to obtain, with unaffordable cost
   - Traditionally, suboptimal solutions are sought, at lower complexity
2. Evolutionary algorithms are capable of offering near **optimal** performance with **affordable** cost
   - A well-tuned EA may solve a NP-hard problem with complexity at most polynomial in problem size
3. We evaluate several evolutionary algorithms in a very challenging application
   - Joint channel estimation and turbo multiuser detection-decoding for OFDM

Background

1. Joint channel estimation and turbo multiuser detection-decoding
   - Turbo MUD/decoding optimisation given CSI is NP-hard, and optimal ML solution is computationally **prohibitive**
2. Within joint optimisation of iterative CE and MUD/decoding
   - CE optimisation is defined on continuous space while MUD optimisation is defined on discrete space
3. We test both **discrete**-binary and **continuous**-valued
   - Genetic algorithm, repeated weighted boosting search, particle swarm optimisation, differential evolution algorithm
4. EA aided joint CE and turbo multiuser detector/decoder:
   - BER approaches ML bound associated with perfect CSI
   - CE accuracy attains optimal Cramér-Rao lower bound
   - Complexity is a **fraction** of NP-hard optimal ML complexity

Outline

System Schematic



- SDMA: U single-antenna users are spatially separated by user-specific CIRs
- OFDM: K subcarriers for combating dispersive channel
- BS: has Q antennas and performs soft-in soft-out iterative detection and decoding

Joint CE and MUD

Joint ML CE and MUD solution

$$(\widehat{\boldsymbol{h}}[s], \widehat{\boldsymbol{X}}[s]) = \arg \min_{\boldsymbol{h}[s], \boldsymbol{X}[s]} J(\boldsymbol{h}[s], \boldsymbol{X}[s])$$

with joint optimisation cost function

$$J(\boldsymbol{h}[s], \boldsymbol{X}[s]) = \sum_{q=1}^{Q} \|\mathbf{Y}_q[s] - \boldsymbol{X}^{\mathrm{T}}[s]\overline{\mathbf{F}}\mathbf{h}_q[s]\|^2$$

- $\mathbf{Y}_q[s] \in \mathbb{C}^{K\times 1}$: qth antenna **received** data over K subcarriers at sth OFDM symbol
- $\boldsymbol{X}[s] = \left[\mathbf{X}^1[s]\ \mathbf{X}^2[s] \cdots \mathbf{X}^U[s]\right]^{\mathrm{T}}$ with $\mathbf{X}^u[s] = \mathrm{diag}\{X^u[s,1], \cdots, X^u[s,K]\}$ being **transmitted** data of user u over K subcarriers at sth OFDM symbol
- $\overline{\mathbf{F}} = \mathrm{diag}\{\underbrace{\mathbf{F}, \mathbf{F}, \cdots \mathbf{F}}_{U}\}$ with $\mathbf{F} \in \mathbb{C}^{K\times L_{cir}}$ denoting FFT matrix
- $\mathbf{h}_q[s] = \left[(\mathbf{h}_q^1[s])^{\mathrm{T}}\ (\mathbf{h}_q^2[s])^{\mathrm{T}} \cdots (\mathbf{h}_q^U[s])^{\mathrm{T}}\right]^{\mathrm{T}}$ with $\mathbf{h}_q^u[s] \in \mathbb{C}^{L_{cir}\times 1}$ being **CIR** vector between uth user and qth receive antenna during sth OFDM symbol

Outline

ML MUD

1. Given CSI, ML MUD solution

$$\widehat{\mathbf{X}}[s,k] = \arg\min_{\mathbf{X}[s,k]\in\mathcal{S}^U} J_{mud}(\mathbf{X}[s,k]), 1 \le k \le K$$

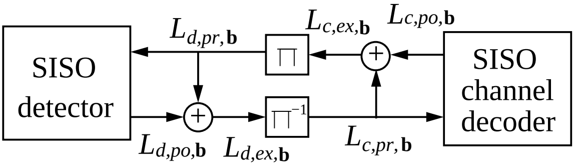
with symbol set $\mathcal{S}$ and MUD optimisation cost function

$$J_{mud}(\mathbf{X}[s,k]) = \|\mathbf{Y}[s,k] - \widehat{\mathbf{H}}[s,k]\mathbf{X}[s,k]\|^2$$

- $\mathbf{Y}[s,k] \in \mathbb{C}^{Q\times 1}$: **received** data of Q antennas at sth OFDM symbol and kth subcarrier
- $\widehat{\mathbf{H}}[s,k] \in \mathbb{C}^{Q\times U}$: **estimated** FDCHTF matrix (FFT transform of CIRs) at sth OFDM symbol and kth subcarrier
- $\mathbf{X}[s,k] \in \mathbb{C}^{U\times 1}$: U users' **transmitted** data at sth OFDM symbol and kth subcarrier

2. ML MUD is NP-hard, and we use **discrete-binary** evolutionary algorithm to solve this optimisation

Turbo SISO MUD-Decoder



1. Given estimated channel, EA assisted MUD detects data, which are converted into **soft** bits
2. SISO MUD and SISO channel decoder exchange **extrinsic** information I_{ite} times to enhance decoded bits
3. After convergence of turbo detection-decoding, detected bits are remodulated and passed to channel estimator

ML Channel Estimation

1. Given estimated data, ML CE solution

$$\widehat{\mathbf{h}}_q[s] = \arg\left\{\min_{\mathbf{h}_q[s]} J_{ce}\left(\mathbf{h}_q[s]\right)\right\}, \ 1 \leq q \leq Q$$

   with CE optimisation cost function

$$J_{ce}\left(\mathbf{h}_q[s]\right) = \|\mathbf{Y}_q[s] - \widehat{\boldsymbol{X}}^{\mathrm{T}}[s]\overline{\mathbf{F}}\mathbf{h}_q[s]\|^2$$

   As $\mathbf{h}_q[s] \in \mathbb{C}^{UL_{cir}\times 1}$, search space for each optimisation is a continuous $(2UL_{cir})$-dimensional space

2. We use a continuous EA to solve this optimisation

- Continuous EA assisted **channel estimator** and discrete-binary EA aided **turbo MUD-decoder** iterates I_{ce} times
- **Continuous** as well as **discrete-binary** GA, RWBS, PSO and DEA are detailed in the paper

Outline

Simulated Multiuser OFDM System

Simulation parameters of the multi-user OFDM system

Encoder	Type	RSC
	Code rate	1/2
	Constraint length	3
	Polynomial	$(g_0, g_1) = (7, 5)$
Channel	Number of paths L_{cir}	4
	Path delays	$\{0, 1, 2, 3\}$
	Average path gains	$\{0, -5, -10, -15\}$ (dB)
	Taps: frame to frame	Complex white Gaussian
	Taps: within frame	fading rate $F_D = 10^{-7}$
System	MSs U	4
	Receiver antennas Q	3
	Modulation	16-QAM
	Subcarriers K	64
	Cyclic prefix K_{cp}	16

EA Algorithmic Parameters

Algorithmic parameters for EA assisted joint CE and MUD-decoder

CE	Parameter	Value	MUD	Parameter	Value
CGA	Population size P_s	100	DBGA	Population size P_s	100
	Selection ratio r_s	0.5		Selection ratio r_s	0.5
	Mutation parameter γ	0.01		Mutation probabi. M_b	0.15
	Mutation probabi. M_b	0.2			
CRWBS	Population size P_s	100	DBRWBS	Population size P_s	100
	Mutation parameter γ	0.001		Mutation probabi. M_b	0.5
	WBS T_{wbs}	40		WBS T_{wbs}	40
CPSO	Population size P_s	100	DBPSO	Population size P_s	100
	Cognition learning c_1	2		Cognition learning c_1	0.1
	Social learning c_2	2		Social learning c_2	0.3
CDEA	Population size P_s	100	DBDEA	Population size P_s	100
	Greedy factor p	0.1		Greedy factor p	0.7
	Adaptive factor c	0.1		Adaptive factor c	0.8

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Evaluating Metrics

- CEA-based CE: given perfect data, and no channel noise ($N_o = 0$)
- Successful run: achieve target $J_{ce}\left(\widehat{\mathbf{h}}_{q,G^i_{\max},\text{best}}\right) < 10^{-4}$ within limit of CF-evaluations: $\overline{N}^{\text{lim}}_{CF-EVs} = P_s \cdot G^{\text{lim}}_{\max} = 100 \times 1000$
- Evaluate statistics:

  for run $= 1 : N_{\text{tot}}$ ($N_{\text{tot}} = 1000$)
  if $\left(G^{\text{run}}_{\max} \le G^{\text{lim}}_{\max}\right)$ and $\left(J_{ce}\left(\widehat{\mathbf{h}}_{q,G^{\text{run}}_{\max},\text{best}}\right) < 10^{-4}\right)$
  $N_{\text{suc}} = N_{\text{suc}} + 1$; $N^{\text{suc}}_{CF-EVs} = N^{\text{suc}}_{CF-EVs} + P_s \cdot G^{\text{run}}_{\max}$
  else
  $N_{\text{fai}} = N_{\text{fai}} + 1$; $N^{\text{fai}}_{CF-EVs} = N^{\text{fai}}_{CF-EVs} + P_s \cdot G^{\text{lim}}_{\max}$
  end if
  end for
- Average number of CF evaluations per run

$$\overline{N}^{\text{tot}}_{CF-EVs} = \left(N^{\text{suc}}_{CF-EVs} + N^{\text{fai}}_{CF-EVs}\right)/N_{\text{tot}}$$

  Average number of CF evaluations per successful run

$$\overline{N}^{\text{suc}}_{CF-EVs} = N^{\text{suc}}_{CF-EVs}/N_{\text{suc}}$$

Evaluating Metrics (continue)

- **Efficiency** is quantified by normalised average number of CF evaluations per run

$$\overline{R}^{\text{tot}}_{CF-EVs} = \overline{N}^{\text{tot}}_{CF-EVs} / \overline{N}^{\text{lim}}_{CF-EVs}$$

  or normalised average number of CF evaluations per successful run

$$\overline{R}^{\text{suc}}_{CF-EVs} = \overline{N}^{\text{suc}}_{CF-EVs} / \overline{N}^{\text{lim}}_{CF-EVs}$$

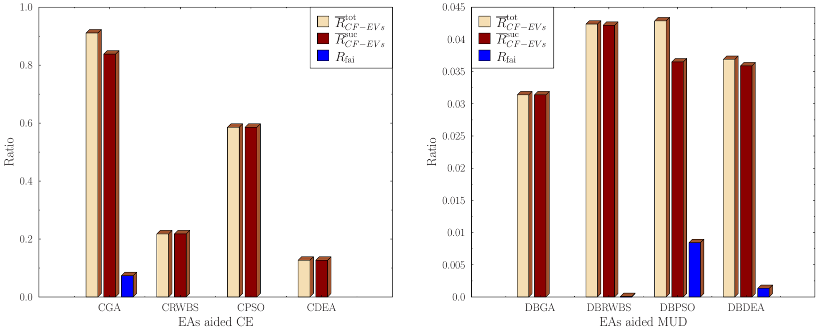
  Smaller $\overline{R}^{\text{tot}}_{CF-EVs}$ or $\overline{R}^{\text{suc}}_{CF-EVs}$, more efficient CEA-CE
- **Reliability** of CEA aided channel estimator is measured by failure ratio

$$R_{\text{fai}} = N_{\text{fai}} / N_{\text{tot}}$$

- Similar procedure evaluates efficiency and reliability of DBEA-based MUD, by setting $G^{\text{lim}}_{\text{max}} = 500$ and $\overline{N}^{\text{lim}}_{CF-EVs} = M^U = 16^4$
- Given perfect CSI, no turbo iterations ($I_{\text{ite}} = 1$), and a successful detection run:

$$\text{BER} \rightarrow 0 \text{ for } G^{\text{run}}_{\text{max}} \leq G^{\text{lim}}_{\text{max}}$$

Efficiency and Reliability



(a) (b)

(a) Histograms of efficiency and reliability measures, in terms of $\overline{R}^{\text{tot}}_{CF-EVs}$, $\overline{R}^{\text{suc}}_{CF-EVs}$ and R_{fai}, for four CEA assisted CE schemes

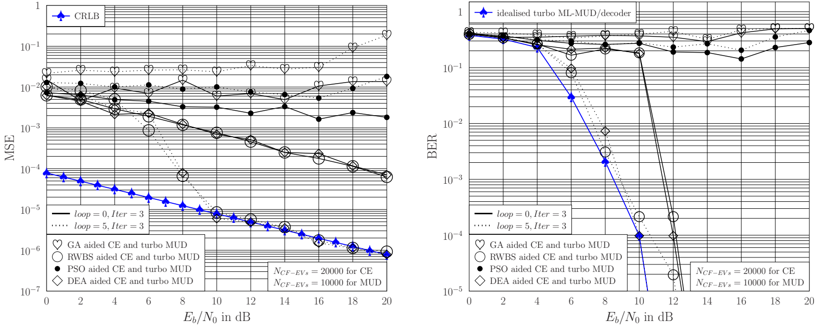
CDEA-CE is best, CRWBS-CE close second, and CGA-CE worst

(b) Histograms of efficiency and reliability measures, in terms of $\overline{R}^{\text{tot}}_{CF-EVs}$, $\overline{R}^{\text{suc}}_{CF-EVs}$ and R_{fai}, for four DBEA assisted MUDs

DBGA-MUD is best, DBDEA-MUD close second, and DBPSO-MUD worst

Outline

Performance



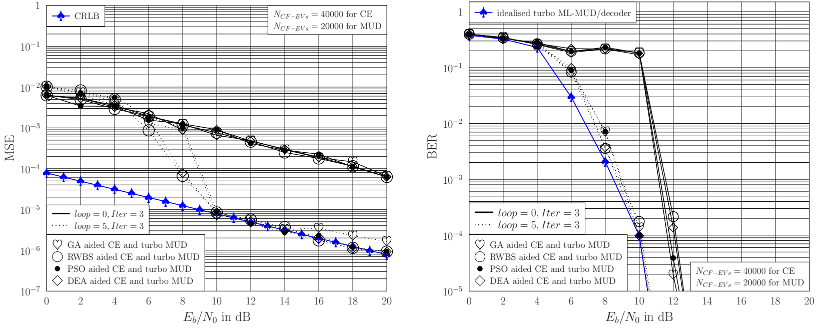
(a) (b)

(a) CEMSE as function of channel SNR for four EA assisted iterative CE and turbo MUD-decoder schemes

(b) BER as function of channel SNR for four EA assisted iterative CE and turbo MUD-decoder schemes

- $I_{\text{ite}} = 3$, $I_{\text{ce}} = 5$, number of CF evaluations for EA aided CE set to $N_{CF-EVs}^{ce} = 20000$ ($G_{\max} = 200$), and number of CF evaluations for EA aided MUD-decoder set to $N_{CF-EVs}^{mud} = 10000$ ($G_{\max} = 100$)
- GA based and PSO based schemes do not converge

Performance (continue)



(a) (b)

(a) CE MSE as function of channel SNR for four EA assisted iterative CE and turbo MUD-decoder schemes

(b) BER as function of channel SNR for four EA assisted iterative CE and turbo MUD-decoder schemes

- $I_{\text{ite}} = 3$, $I_{\text{ce}} = 5$, number of CF evaluations for EA aided CE set to $N^{ce}_{CF-EVs} = 40000$ ($G_{\max} = 400$), and number of CF evaluations for EA aided MUD-decoder set to $N^{mud}_{CF-EVs} = 20000$ ($G_{\max} = 200$)
- All four schemes converge to optimal solution

Complexity Comparison

Scheme	Operation	$C^{EA}_{\text{MUD}}/C^{ML}_{\text{MUD}}$	$C^{EA}_{\text{turbo}}/C^{ML}_{\text{turbo}}$	$C^{EA}_{\text{joint}}/C^{ML}_{\text{turbo}}$
GA aided joint CE and turbo MUD/decoder	multiplications	0.10%	5.69%	62.24%
	additions	0.10%	7.45%	91.41%
RWBS aided joint CE and turbo MUD/decoder	multiplications	0.10%	3.00%	31.27%
	additions	0.10%	3.88%	45.86%
PSO aided joint CE and turbo MUD/decoder	multiplications	0.10%	5.69%	62.24%
	additions	0.10%	7.45%	91.41%
DE aided joint CE and turbo MUD/decoder	multiplications	0.10%	3.00%	31.27%
	additions	0.10%	3.88%	45.86%

- C^{ML}_{MUD}: complexity of ML MUD given CSI
- C^{EA}_{MUD}: complexity of discrete-binary EA based MUD given CSI
- C^{ML}_{turbo}: complexity of turbo ML MUD-decoder given CSI
- C^{EA}_{turbo}: complexity of discrete-binary EA based turbo MUD-decoder given CSI
- C^{EA}_{joint}: complexity of EA assisted joint CE and turbo MUD-decoder

Outline

Summary

1. Joint channel estimation and turbo multiuser detection-decoding for OFDM communication offers a challenging application
   - to test capabilities of evolutionary algorithms
2. Our EA aided joint CE and turbo MUD-decoder is capable of
   - approaching CRLB of optimal channel estimate, and BER of turbo ML MUD-decoder associated with perfect CSI
   - only imposing a fraction of complexity of idealised turbo ML MUD-decoder
3. Our study has provided benchmark empirical results to support capabilities of EAs
   - for finding optimal or near optimal designs in challenging practical applications with affordable complexity
   - complimenting well current efforts to better understand EAs