

Transonic Aerodynamic Load Modeling of X-31 Aircraft Pitching Motions

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The generation of reduced-order models for computing the unsteady and nonlinear aerodynamic loads on an aircraft from pitching motions in the transonic speed range is described. The models considered are based on Duhamel's superposition integral using indicial (step) response functions, Volterra theory using nonlinear kernels, radial basis functions, and a surrogate-based recurrence framework, both using time-history simulations of a training maneuver(s). Results are reported for the X-31 configuration with a sharp leading-edge cranked delta wing geometry, including canard/wing vortex interactions. The validity of the various models studied was assessed by comparison of the model outputs with time-accurate computational-fluid-dynamics simulations of new maneuvers. Overall, the reduced-order models were found to produce accurate results, although a nonlinear model based on indicial functions yielded the best accuracy among all models. This model, along with a time-dependent surrogate approach, helped to produce accurate predictions for a wide range of motions in the transonic speed range.

Nomenclature

A	=	unit step function
a	=	acoustic speed, m/s
C_m	=	pitch-moment coefficient, $m/q_\infty Sc$
C_{m0}	=	zero angle of pitch-moment coefficient
$C_{m\alpha}$	=	pitch-moment coefficient with angle of attack, 1/rad
C_{mq}	=	pitch-moment coefficient with normalized pitch rate, 1/rad
c	=	mean aerodynamic chord, m
\mathbf{D}	=	input matrix
\mathbf{f}_i	=	regression functions
\mathbf{H}_i	=	Volterra kernels
k	=	reduced frequency, $\omega c/2V$
M	=	Mach number, V/a
m	=	pitch moment, N · m
p	=	pressure, Pa
q	=	normalized pitch rate, $\dot{q}c/V$, 1/rad
\dot{q}	=	time rate of pitch rate, rad/s ²
\ddot{q}	=	pitch rate, rad/s
q_∞	=	dynamic pressure, Pa
\mathbf{R}	=	correlation matrix
Re	=	Reynolds number, $\rho Vc/\mu$
S	=	reference area, m ²
s	=	normalized time, $2Vt/c$
T	=	temperature, K
t	=	time, s

t^*	=	nondimensional time step, Vt/c
t_0	=	start time, s
V	=	freestream velocity, m/s
u, v, w	=	velocity components in x, y , and z axes, m/s
x	=	input vector
y	=	output vector
\mathbf{Z}	=	output matrix
α	=	angle of attack, rad
$\dot{\alpha}$	=	normalized time rate of angle of attack, 1/rad
α_A	=	pitch amplitude, rad
α_0	=	mean angle of attack, rad
β_i	=	regression coefficients
ϵ	=	kriging random process
μ	=	air viscosity; kriging mean value
ρ	=	density, kg/m ³
σ^2	=	covariance
τ_{ij}	=	viscous stress tensor components, Pa
$\Phi_i(X)$	=	radial basis functions
ω	=	circular frequency, rad/s

I. Introduction

MODERN fighter aircraft often maneuver in extreme conditions of the flight envelope characterized by vortical flows and shock effects. Such conditions can easily lead to unsteady flow, which can cause flight dynamic and aeroelastic instabilities [1]. The linearized unsteady aerodynamic models no longer work at these conditions. Currently, the use of computational fluid dynamics (CFD) solutions is considered as the state of the art in modeling unsteady nonlinear flow physics [2] and offers an early and improved understanding and prediction of aircraft aerodynamic characteristics. It is well known that unsteady aerodynamic forces and moments not only depend on the instantaneous states but also on all of the states at times before the current state [3]. Unfortunately, this makes the solution of the aircraft equations of motion an infinite-dimensional problem, where the current states depend on the evolution of previous states at infinitely many points in time [4]. With advances in computing techniques, one straightforward way to solve this problem is to develop a full-order model based on direct solution of discretized Reynolds-averaged Navier–Stokes (RANS) equations coupled with the dynamic equations governing the aircraft motion [5–7].

Creating a full-order model for stability and control (S&C) analysis is a computationally expensive approach because such a model needs a large number of coupled computations for different

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values of motion frequency and amplitude [8]. An alternative approach to creating the full-order model is to develop a reduced-order model (ROM) that seeks to approximate CFD results by extracting information from a limited number of full-order simulations [9,10]. Ideally, the specified ROM can predict aircraft responses over a wide range of amplitude and frequency within a few seconds without the need of running CFD simulations again.

Recent efforts on the development of ROMs can be classified into two types: time-domain and frequency-domain approaches [11]. The frequency-domain models are obtained from matching transfer functions computed from the measured input–output data [12]. Examples of the frequency-domain ROMs are the indicial response method by Ballhaus and Goorjian [13] and Tobak et al. [14,15] and a frequency-domain approach based on proper orthogonal decomposition (POD) by Hall et al. [16]. Some examples of time-domain ROMs include the unit sample response by Gaitonde and Jones [17], Volterra theory by Silva and Bartels [18], radial basis functions (RBFs) [19], and state–space modeling [20]. These ROM techniques have been used extensively for flutter prediction, limit cycle oscillation, and gust-response modeling [21], but their application to S&C is still new. Also, only a few studies have been conducted for reduced-order modeling of aircraft configurations, mostly limited to the subsonic flow regime. The current paper aims to assess the accuracy of prediction and incurred computational cost of reduced-order models based on the indicial response method, Volterra theory, RBF, and a surrogate-based recurrence framework (SBRF) for X-31 aircraft pitching motions in the transonic speed range. These models can be grouped into two different categories: parametric and nonparametric. In this paper, parametric identification methods are presented for creating a Volterra model and an indicial response method. Also, RBF and SBRF are created with nonparametric identification techniques from the measured input–output behavior of the aircraft dynamics. Therefore, the main part of this work is accurate identification of the unknowns in parametric models and establishing the input–output mapping for nonparametric types.

The transient aerodynamic response due to a step change in a forcing parameter, such as angle of attack or pitch rate, is a so-called “indicial function”. Assuming that the indicial functions are known, the aerodynamic forces and moments induced in any maneuver can be estimated by using the well-known Duhamel superposition integral [22]. Tobak et al. [14,15], Reischel [23], and Reischel and Bettencourt [24] detailed the superposition process for the modeling of unsteady lift and pitch moment from angle of attack and pitch rate indicial functions. Ghoreyshi and Cummings [25] extended this model to include lateral forces and moments as well. However, this model is subject to the limitations of the identification methods of indicial functions. These functions can be derived from analytical, CFD, or experimental methods [26]. Limited analytical expressions of indicial functions exist for two-dimensional airfoils [27]. For incompressible flows, Wagner [28] was the first who detailed the analytical unsteady lift of a thin airfoil undergoing a plunging motion using a single indicial function (the so-called Wagner’s function), with its exact values defined in terms of Bessel functions. For unsteady, compressible flow past two-dimensional airfoils, Bisplinghoff et al. [29] also described an exponential approximation to the exact solutions of the indicial functions at different Mach numbers. However, these analytical expressions are not valid for aircraft configurations due to the three-dimensional tip vortices.

Direct and indirect methods are used to estimate indicial functions. Leishman [30] has presented an indirect technique for identifying indicial functions from aerodynamic responses due to harmonic motions. However, the derived indicial functions using indirect methods depend largely on the quality of motion (e.g., amplitude and frequency). Experimental tests are practically nonexistent for direct indicial function measurements due to wind-tunnel constraints. An alternative is to use CFD, but special considerations are required to simulate step responses in CFD. Singh and Baeder [31] used a surface transpiration approach to directly calculate the angle-of-attack indicial response using CFD. Ghoreyshi et al. [32] also described an approach based on a grid-motion technique for CFD-type calculation of linear and nonlinear indicial functions with respect to angle of

attack and pitch rate. Ghoreyshi and Cummings [25] later used this approach to generate longitudinal and lateral indicial functions for a generic unmanned combat air vehicle and used these functions for predicting the aerodynamic responses to aircraft six-degree-of-freedom maneuvers. In this paper, the transonic indicial functions of the X-31 aircraft are calculated using CFD and the grid-motion approach.

For motions at low angles of attack, and assuming incompressible flow, only a single indicial function with respect to each forcing parameter needs to be generated [33]. For flows at transonic speed, however, many indicial functions need to be generated for each combination of angle of attack and freestream Mach number. The generation of all these functions using CFD is expensive and makes the creation of a ROM very time-consuming. Note that these models are still cheaper than full-order simulations because the ROMs based on indicial functions eliminate the need to repeat calculations for each frequency. A cost-effective unsteady aerodynamic model needs a mathematical description of indicial functions as a function of angle of attack and freestream Mach number. However, this model is often unavailable for three-dimensional configurations. It is more common to use surrogate models, which are mathematical approximations of the true response of the system, built using some observed responses, and therefore the total cost of creating ROM is reduced. The most popular surrogate modeling techniques are artificial neural networks, support vector regression, radial basis function interpolation, and kriging (Gaussian process) [34]. In this paper, a surrogate model is proposed based on the kriging technique [35] to model indicial functions as a function of angle of attack and freestream Mach number.

Volterra’s functional mathematics is considered as one of the most important tools for the representation of nonlinear systems where the system output depends on the current and past values of the input. The approach was first introduced by Volterra in 1930 [36]. This work and further developments by others (for example Wiener [37], Barrett [38], and Kalman [39]) have been extensively used in electrical and biological systems engineering [40–44]. Recently, there is an increasing interest in using Volterra theory in the field of aerodynamic loads modeling [2,45], where the aerodynamic forces and moments are approximated by an infinite series of multidimensional convolution integrals of the inputs and increasing order kernels named the so-called Volterra kernels [46]. The main limitations of the Volterra model are that the model typically is accurate only for weak nonlinearities [47], and generally the Volterra kernels of a nonlinear dynamic system are difficult to compute. Silva [48] used the first and second terms of the Volterra functions and proposed a method based on the impulse functions to directly calculate first- and second-order kernels using CFD. In his approach, the first-order kernel is a combination of the response to unit and double unit impulses at time $t_1 = T$. The second-order kernel is a combination of two successive unit impulses at time $t_1 = T$ and $t_2 = T + \Delta t$ and two unit pulses, one at time T and a second at time $T + \Delta t$. Jirásek and Cummings [45] used this approach to find Volterra kernels for the X-31 aircraft at subsonic speeds. However, the CFD simulation of system impulse response at transonic speeds is very complicated because the impulse occurs over a very short period of time. Da Ronch et al. [49] proposed an approach of determining these functions from unsteady time-domain solutions. In this paper, the chirp and spiral training maneuvers are used to estimate the Volterra kernels.

The complex CFD system (that is flow, flow equations, and boundary conditions) can be represented by a simplified input–output relationship. The relationship can be learned using neural networks and surrogate models [46,50–52]. Marques and Anderson [46] used a temporal neural network to approximate the unsteady lift and pitch moment of a two-dimensional airfoil for the changes in angle of attack in the transonic regime. Although they found a good match for the lift, the predictions of pitch moment did not match as well. The network outputs in their work depended on time histories of angle of attack only and not previous values of lift and pitch moment. Fallor and Schreck [51] also used a neural network to predict the time-dependent surface pressures of a pitching wing. The network inputs

included the instantaneous pitch angle, angular velocity, and the initial surface pressure coefficients at t_0 . The network output then predicted the surface pressures at time $t_0 + \Delta t$. These predictions were fed back as the input to the network to predict surface pressures at the next time interval. This process continues in time until it reaches the final time. However, this model does not have the time histories of motion variables; therefore, it might not have sufficient information for modeling highly unsteady flows. Glaz et al. [52] developed a mapping between aerodynamic loads and time histories of both motion parameters and loads of a two-dimensional airfoil and then tried to learn this mapping using a surrogate model with the aid of design of experiments. They showed that SBRF can predict the strongly nonlinear effects of moving shocks on the unsteady pitching moments. Ghoreyshi et al. [53] extended this mapping to include both RANS and Euler calculations and used RBF neural networks (RBFNNs) trained from some special training maneuvers. Da Ronch et al. [49] also tried to establish this mapping using a SBRF model for a pitching airfoil at transonic speed. However, RBF and SBRF models could be limited by the computational cost if the size of the input space increases. Also, a key issue in these models is the effects of the training motion on identifying model parameters. This paper discusses the effects of three training motions of chirp, spiral, and Schroeder [19] for unsteady aerodynamic loads modeling of an aircraft.

The objective of this work is to generate cost-effective reduced-order models capable of predicting aerodynamic loads of an aircraft pitching within the frequency/amplitude/Mach space of interest. This forms the basis for the future studies of flight dynamics, where forces and moments are given by these models. Each reduced-order model used requires a different computational cost and is based on various simplifying assumptions pertaining to the flow physics. For example, the generation of linear indicial functions is relatively inexpensive, but the model is limited to small-amplitude motions at a fixed Mach number for which functions were calculated. A first-order linear Volterra model also has the same limitations. The nonlinear indicial functions include responses to different angles of attack and could be used for predicting responses to arbitrary motions at a fixed Mach number, but they have relatively high computational cost compared with a linear model. A Volterra model with second and higher-order kernels has the nonlinear dependencies of aerodynamic loads with amplitude. However, the suitability of the model for predicting responses to new motions depends on the type of training maneuver used to estimate kernels. Likewise, the accuracy of the RBF model depends to a large extent on the training maneuvers used. A model that includes Mach number effects significantly increases the computational cost because it requires many calculations for each combination of angle of attack and Mach number. SBRF and indicial functions are used to model dependencies of aerodynamic loads on Mach number. The SBRF model was generated using some pitch motion simulations in amplitude/Mach space; however, the model leads to a significant increase in the computational cost if the size of the input space increases. Nonlinear indicial functions, along with a developed time-dependent surrogate approach, are also used to predict responses for motions within the frequency/amplitude/Mach space. In this paper, all of these models are tested for predicting the unsteady loads induced in pitching motion at transonic speeds. The test case, geometry definition, and ROMs will first be detailed, and the paper continues by describing the generation of ROMs. Finally, the accuracy of the aerodynamic models will be compared with time-dependent CFD computations for new maneuvers not used to create the ROMs.

II. Formulation

A. Computational-Fluid-Dynamics Solver

The flow solver used for this study is the Cobalt code [54] that solves the unsteady, three-dimensional, and compressible Navier–Stokes equations in an inertial reference frame. These equations in integral form are [55]

$$\frac{\partial}{\partial t} \iiint_V \mathbf{Q} dV + \iint_S (\mathbf{f}\hat{i} + \mathbf{g}\hat{j} + \mathbf{h}\hat{k}) \cdot \hat{n} dS = \iint_S (\mathbf{r}\hat{i} + \mathbf{s}\hat{j} + \mathbf{t}\hat{k}) \cdot \hat{n} dS \quad (1)$$

where V is the fluid-element volume; S is the fluid-element surface area; \hat{n} is the unit normal to S ; \hat{i} , \hat{j} , and \hat{k} are the Cartesian unit vectors; $\mathbf{Q} = (\rho, \rho u, \rho v, \rho w, \rho e)^T$ is the vector of conserved variables, where ρ represents air density; u , v , and w are velocity components; and e is the specific energy per unit volume. The vectors of \mathbf{f} , \mathbf{g} , and \mathbf{h} represent the inviscid components and are detailed next:

$$\begin{aligned} \mathbf{f} &= (\rho u, \rho u^2 + p, \rho uv, \rho uw, u(\rho e + p))^T \\ \mathbf{g} &= (\rho v, \rho v^2 + p, \rho vu, \rho vw, v(\rho e + p))^T \\ \mathbf{h} &= (\rho w, \rho w^2 + p, \rho wu, \rho wv, w(\rho e + p))^T \end{aligned} \quad (2)$$

where the superscript T denotes the transpose operation. The vectors of \mathbf{r} , \mathbf{s} , and \mathbf{t} represent the viscous components are described as

$$\begin{aligned} \mathbf{r} &= (0, \tau_{xx}, \tau_{xy}, \tau_{xz}, u\tau_{xx} + v\tau_{xy} + w\tau_{xz} + kT_x)^T \\ \mathbf{s} &= (0, \tau_{xy}, \tau_{yy}, \tau_{yz}, u\tau_{xy} + v\tau_{yy} + w\tau_{yz} + kT_y)^T \\ \mathbf{t} &= (0, \tau_{xz}, \tau_{zy}, \tau_{zz}, u\tau_{xz} + v\tau_{zy} + w\tau_{zz} + kT_z)^T \end{aligned} \quad (3)$$

where τ_{ij} are the viscous stress tensor components, T is the temperature, and k is the thermal conductivity. The ideal gas law closes the system of equations, and the entire equation set is nondimensionalized by freestream density and speed of sound [54]. The Navier–Stokes equations are discretized on arbitrary grid topologies using a cell-centered finite-volume method. Second-order accuracy in space is achieved using the exact Riemann solver of Gottlieb and Groth [56] and least-squares gradient calculations using QR factorization. To accelerate the solution of the discretized system, a point-implicit method using analytic first-order inviscid and viscous Jacobians is used. A Newtonian subiteration method is used to improve time accuracy of the point-implicit method. Tomaro et al. [57] converted the code from explicit to implicit, enabling Courant–Friedrichs–Lewy numbers as high as 10^6 . The Cobalt solver has been used at the U.S. Air Force Seek Eagle Office and the U.S. Air Force Academy for a variety of unsteady nonlinear aerodynamic problems of maneuvering aircraft [58–62].

B. Reduced-Order Models Based on Volterra Theory

The X-31 aircraft pitch-moment modeling is considered in this paper, which is highly nonlinear in the transonic regime. The output function y is defined as $y = C_m(t)$. Using Volterra theory [36], the output of a continuous-time, casual, time-invariant, fading memory system in response to an input $x(t)$ can be modeled using the p th-order Volterra series:

$$y(t) = \Psi(x(t)) = \sum_{i=1}^p \mathbf{H}_i(x(t)) \quad (4)$$

where \mathbf{H}_i represents the i th-order Volterra operator, which is defined as an i -fold convolution between the input $x(t)$ and the i th-order Volterra kernel H_i , i.e.,

$$\begin{aligned} \mathbf{H}_i(x(t)) &= \int_{-\infty}^t \dots \int_{-\infty}^t H_i(t - \tau_1, t - \tau_2, \dots, t - \tau_i) \\ &\times \prod_{n=1}^i x(\tau_n) d\tau_n \end{aligned} \quad (5)$$

where, for forced oscillations about the pitch axis, the input vector includes

$$\mathbf{x}(t) = (\alpha(t), q(t), \dot{q}(t)) \quad (6)$$

representing the angle of attack, pitch rate, and the first derivative of pitch rate. A multi-input Volterra series is then formulated as

$$y(t) = \Psi(x_1(t), x_2(t), \dots, x_m(t)) = \sum_{i=1}^p \mathbf{H}_i^m \quad (7)$$

The term \mathbf{H}_i^m is the multi-input Volterra operator defined as a m^p -fold summation of p -fold convolution integrals between the inputs and the p th-order multi-input Volterra kernels [63]. The output response up to second order is rewritten as

$$\begin{aligned} y(t) = & \sum_{j=1}^m \int_{-\infty}^t H_1^{x_j}(t-\tau) x_j(\tau) d\tau \\ & + \sum_{j_1=1}^m \sum_{j_2=1}^m \int_{-\infty}^t \int_{-\infty}^t H_2^{x_{j_1}, x_{j_2}} \\ & \times (t-\tau_1, t-\tau_2) x_{j_1}(\tau_1) x_{j_2}(\tau_2) d\tau_1 d\tau_2 + \mathcal{O}(|x|^3) \end{aligned} \quad (8)$$

Note that the superscripts in Eq. (8) identify to which inputs the kernel corresponds; for example, the second-order kernel $H_2^{x_{j_1}, x_{j_2}}$ correlates the inputs x_{j_1} and x_{j_2} . Note that the second- and higher-order kernels are symmetric with respect to the arguments $H_2^{x_{j_1}, x_{j_2}} = H_2^{x_{j_2}, x_{j_1}}$. The determination of Volterra kernels using CFD simulations is discussed in Sec. II.G.

C. Reduced-Order Models Based on Indicial Functions

Linear and nonlinear models based on indicial functions are detailed here. Assuming a linear relationship between the input function and the output, a linear ROM is defined as the convolution (or Duhamel's superposition [30]) of the indicial response with the derivative of the forcing function [64]:

$$y(t) = \frac{d}{dt} \left[\int_0^t A(t-\tau) x(\tau) d\tau \right] \quad (9)$$

where A represents the unit response, or indicial response, of the system. Note that a linear Volterra system and the linear step-type ROM given in Eq. (9) are identical. For a linear system, H_1 is the impulse response function, and $H_1(t) = dA(t)/dt$ applies. The indicial response functions are used as a fundamental approach to represent the unsteady aerodynamic loads. The mathematical models are detailed by Tobak et al. [14,15], Reischel [23], and Reischel and Bettencourt [24]. The pitch moment is only considered in this paper because it is the more-difficult parameter to predict. For forced oscillations about the pitch axis, the input to the model includes angle of attack and pitch rate, i.e.,

$$\mathbf{x}(t) = (\alpha(t), q(t)) \quad (10)$$

Note that the response functions are time-dependent and have the effects of $\dot{\alpha}$ and \dot{q} ; therefore, these terms are not included in the input vector. If the time responses in aerodynamic coefficients due to the step changes in angle of attack α and angular velocity q are known, then the total produced pitch moment at time t can be obtained using Eq. (11):

$$C_m(t) = \frac{d}{dt} \left[\int_0^t C_{m\alpha}(t-\tau) \alpha(\tau) d\tau \right] + \frac{d}{dt} \left[\int_0^t C_{mq}(t-\tau) q(\tau) d\tau \right] \quad (11)$$

For nonlinear aerodynamic responses due to motions starting from different Mach numbers, the dependencies on the angle of attack and Mach number are added to the indicial functions, i.e.,

$$\begin{aligned} C_m(t) = & \frac{d}{dt} \left[\int_0^t C_{m\alpha}(t-\tau, \alpha, M) \alpha(\tau) d\tau \right] \\ & + \frac{d}{dt} \left[\int_0^t C_{mq}(t-\tau, \alpha, M) q(\tau) d\tau \right] \end{aligned} \quad (12)$$

where M denotes the freestream Mach number. The response function due to pitch rate, i.e., $C_{mq}(\alpha, M)$, can be estimated using a time-dependent interpolation scheme from the observed responses. This value is next used to estimate the second integral in Eq. (12); however, the estimation of the integral with respect to the angle of attack needs more explanation. Assuming a set of angle-of-attack samples of $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_n]$ at freestream Mach numbers of $M = [M_1, M_2, \dots, M_m]$, the pitch-moment response to each angle of $\alpha_i, i = 1, 2, \dots, n$ at Mach number of $M_j, j = 1, 2, \dots, m$ is denoted as $A_{\alpha}(t, \alpha_i, M_j)$. In these response simulations, $\alpha(t) = 0$ at $t = 0$ and is held constant at α_i for all $t > 0$. For a new angle of $\alpha^* > 0$ at a new freestream Mach number of M^* , the responses of $A_{\alpha}(t, \alpha_k, M^*)$ are being interpolated at $\alpha_k = [\alpha_1, \alpha_2, \dots, \alpha_s]$, such that $0 < \alpha_1 < \alpha_2 < \dots < \alpha_s$ and $\alpha_s = \alpha^*$. These angles can have a uniform or nonuniform spacing. The indicial functions of $C_{m\alpha_k}$ for $k = 1, \dots, s$ at each interval of $[\alpha_{k-1}, \alpha_k]$ are defined as

$$C_{m\alpha_1} = \frac{A_{\alpha}(t, \alpha_1, M^*) - C_{m0}}{\alpha_1} \quad (13)$$

$$C_{m\alpha_k} = \frac{A_{\alpha}(t, \alpha_k, M^*) - A_{\alpha}(t, \alpha_{k-1}, M^*)}{\alpha_k - \alpha_{k-1}} \quad (14)$$

where C_{m0} denotes the 0 deg angle-of-attack pitch-moment coefficient. The interval indicial functions are then used to estimate the values of first integral in Eq. (12). These steps can easily be followed for a negative angle of attack (i.e., $\alpha^* < 0$). Once this model is created, the aerodynamic response to a wide range of motions can be predicted on the order of few seconds; however, the previous model still requires a special time-dependent surrogate model to predict response functions at each α^* and M^* from some available samples.

D. Surrogate-Based Modeling of Indicial Functions

There has been some effort to extend reduced-order models to multiple state variables of highly nonlinear and unsteady transonic flows. For example, Lieu and Farhat [65] proposed a ROM based on POD for evaluating aeroelastic frequencies and damping ratio coefficients of F-16 aircraft for varying Mach number and angle-of-attack flight conditions. A multiple-state-variable model based on indicial functions, however, requires a special time-dependent surrogate-based modeling approach that predicts indicial responses from available (observed) responses. In this paper, these observed responses are viewed as a set of time-correlated spatial processes where the output is considered a time-dependent function. Romero et al. [66] developed a framework for multistage Bayesian surrogate models for the design of time-dependent systems and tested their model for free vibrations of a mass-spring-damper system assuming the input parameters of stiffness and damping factor at different initial conditions. This framework is examined for reduced-order modeling of nonlinear and unsteady aerodynamic loads. Assume an input vector of $\mathbf{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))$, where n represents the dimensionality of the input vector. To construct a surrogate model for fitting the input-output relationship, the unsteady aerodynamic responses corresponding to a limited number of input parameters (training parameters or samples) need to be generated. Design-of-experiment methods, for example, can be used to select m samples from the input space. The input matrix $\mathbf{D}(m \times n)$ is then defined as

Table 1 Special training maneuvers

Maneuver	Definition
Linear chirp	$\alpha(t) = \alpha_0 + \alpha_A \sin(\omega t^2)$
Spiral	$\alpha(t) = \alpha_0 + \alpha_A t \sin(\omega t)$
Schroeder	$\alpha(t) = \alpha_0 + \alpha_A \sum_{k=1}^n \sqrt{\frac{1}{2N}} \cos\left(\frac{2\pi k t}{T} - \frac{\pi k^2}{N}\right)$

$$\mathbf{D} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (15)$$

where rows correspond to different combinations of the design parameters. For each row in the input matrix, a time-dependent response was calculated at p discrete values of time, and this information is summarized in the output matrix of $\mathbf{Z}(m \times p)$ as

$$\mathbf{Z} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mp} \end{bmatrix} \quad (16)$$

where, for aerodynamic load modeling, p equals the number of iterations used in time-marching CFD calculations. The objective of surrogate modeling is to develop a model that allows predicting the aerodynamic response of $\mathbf{y}(\mathbf{x}_0) = (y_{01}, y_{02}, \dots, y_{0p})$ at a new combination of input parameter of \mathbf{x}_0 . To construct this surrogate model, the responses at each time step are assumed as a separate set, such that each column of the output matrix is a partial realization of the total response. In this sense, p surrogate models are created; they are denoted as $\mathbf{Z}_i(\mathbf{D})$ for $i = 1, 2, \dots, p$. A universal-type kriging function [35] is then used to approximate these models. For more details about creating kriging models, the reader is referred to Ghoreyshi et al. [67]. Having created kriging models for each $\mathbf{Z}_i(\mathbf{D})$ function, the total response at \mathbf{x}_0 is then combination of predicted values of each model, i.e.,

$$\tilde{\mathbf{Z}}(\mathbf{x}_0) = (\tilde{\mathbf{Z}}_1(\mathbf{x}_0), \tilde{\mathbf{Z}}_2(\mathbf{x}_0), \dots, \tilde{\mathbf{Z}}_p(\mathbf{x}_0)) \quad (17)$$

where the tilde shows that kriging model is an approximation of the actual function.

E. Reduced-Order Models Based on Surrogate-Based Recurrence-Framework

The nonlinear and unsteady aerodynamics can be viewed as a multi-input/multi-output discrete-time dynamic system with a mathematical model in state space given as

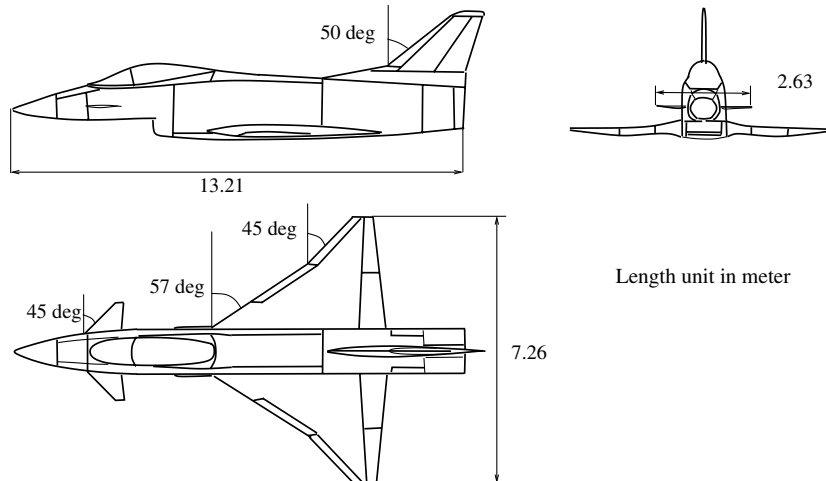
$$\mathbf{u}_{k+1} = f(\mathbf{u}_k, \mathbf{x}_k) \quad \mathbf{y}_k = h(\mathbf{u}_k) \quad \text{for } k = 0, 1, \dots \quad (18)$$

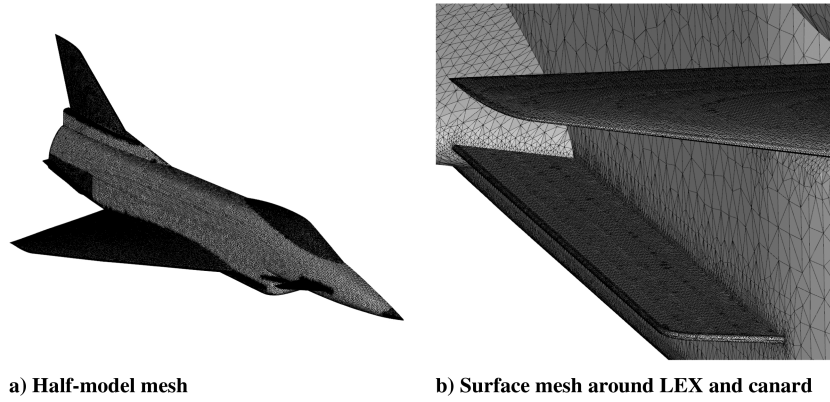
where k is an integer value showing discrete time values. The state function f is a smooth function that maps the current state \mathbf{u}_k and the input \mathbf{x}_k into a new state \mathbf{u}_{k+1} , and the output function h maps the state \mathbf{u}_k into the output \mathbf{y}_k . In this system, the outputs can be determined from the states at time instant k only; therefore, the past history of the system is irrelevant [68]. If the state variables are directly measured, the functions f and h can be approximated using neural networks or surrogate-based models. However, in many practical situations, measuring all state variables is limited. Referring to unsteady aerodynamic problems, the discretized governing equations of fluid dynamics serve as the state space functions with an internal state vector of (ρ, p, u, v, w, E) that corresponds to the values of density, pressure, velocity components, and energy at each grid point. This large amount of data makes the identification of Eq. (18) a very complex task. Fortunately, there are available methods that allow us to reconstruct the state-space model by mapping only the input and output data.

For a finite-time interval and a system described by Eq. (18), Levin and Narendra [69] used the implicit-function theorem to write the output vector at any instant as a function of the past m values of the inputs and the past n values of the outputs, i.e.,

$$\mathbf{y}(k) = \Phi(\mathbf{x}(k), \mathbf{x}(k-1), \dots, \mathbf{x}(k-m), \mathbf{y}(k-1), \dots, \mathbf{y}(k-n)) \quad (19)$$

where Φ is a vector-valued nonlinear function that maps the inputs to the outputs. The terms m and n represent the number of previous values of the external inputs and outputs, respectively, influencing the output at the current time instant. These parameters account for time-history effects and phase lag in the flow development. Equation (19) preserves the characteristics of the state-space model but no longer depends on system internal states [52]. Central to the generation of the reduced-order model is the computation of the function Φ . Without a closed-form analytical expression, a numerical approximation of Φ is constructed using a number of CFD solutions. The SBRF model in this paper is tested for a pitching aircraft within amplitude/Mach number space of interest at a fixed frequency. To generate a consistent set of unsteady aerodynamic loads in response to a given aircraft motion time history, the training cases at which CFD solutions are calculated should be representative of the expected flow conditions. Several design-of-experiment methods are available

**Fig. 1** X-31 aircraft geometry.



a) Half-model mesh

b) Surface mesh around LEX and canard

Fig. 2 X-31 aircraft mesh model (LEX — Leading Edge Extension).

in the literature. A description of the kriging-based framework used in this study is detailed in [70]. Let N_T be the number of training cases at which CFD solutions are available. Each training case consists of different combinations of the independent parameters,

$$\mathbf{x}_i = (\alpha_i(t), q_i(t), \dot{q}_i(t), M) \quad \text{for } i = 1, \dots, N_T \quad (20)$$

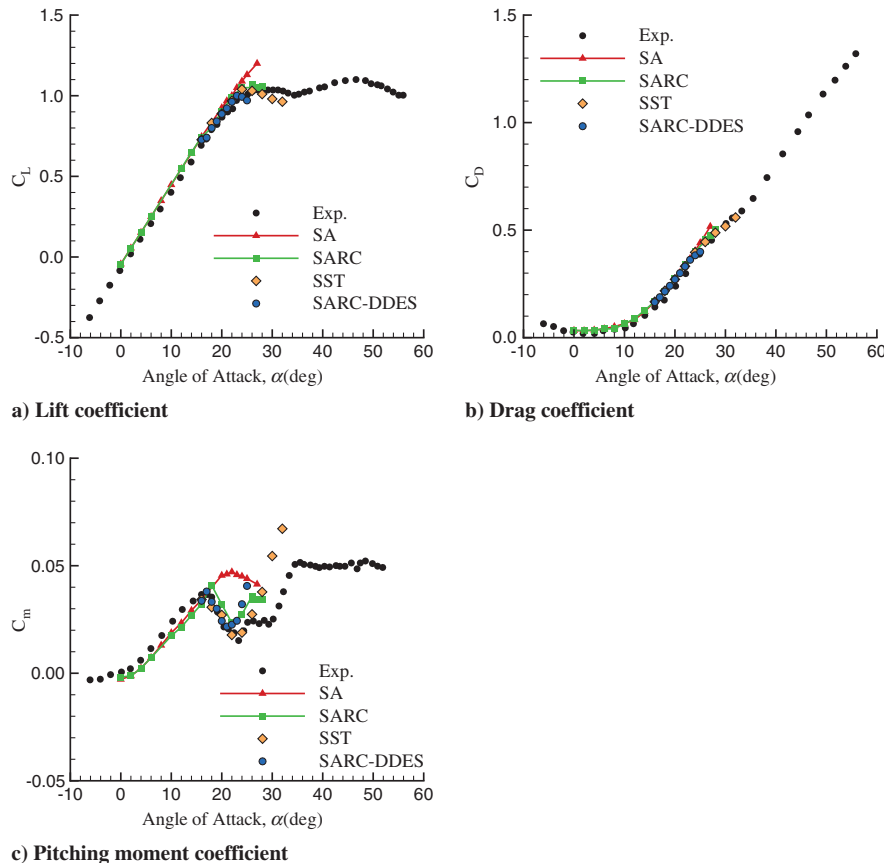
and the corresponding aerodynamic loads are indicated by $y_i(t)$. The approximation of the function Φ is obtained by interpolating the sampled data in the form of an input–output relationship. Several interpolation methods are available in the literature, and two of these have been used in the present study. Kriging interpolation is a common choice, but for increasing number of independent parameters, the problem can result to be ill-conditioned. An alternative approach is the multilinear interpolation technique, which is, in general, faster than the kriging interpolation.

F. Reduced-Order Models Based on Radial Basis Function Neural Network

The RBF model is similar to SBRF because they are both created by the input–output mapping function of Φ given in Eq. (19), except that the function Φ is now approximated through RBF neural networks from a set of training maneuvers with varying amplitude and frequency at a fixed Mach number. CFD simulation of these training maneuvers has the time histories of angle of attack and its first and second derivatives. These data are used as the input parameters to the model, i.e.,

$$\mathbf{x}(t) = (\alpha(t), q(t), \dot{q}(t)) \quad (21)$$

RBF network provides an approximation of the functions based on the location of data points and is generally much faster than multilayer feedforward neural networks [68]. Given an input vector



c) Pitching moment coefficient

Fig. 3 X-31 static loads validations; the static conditions are $M_\infty = 0.18$ and $Re = 2 \times 10^6$.

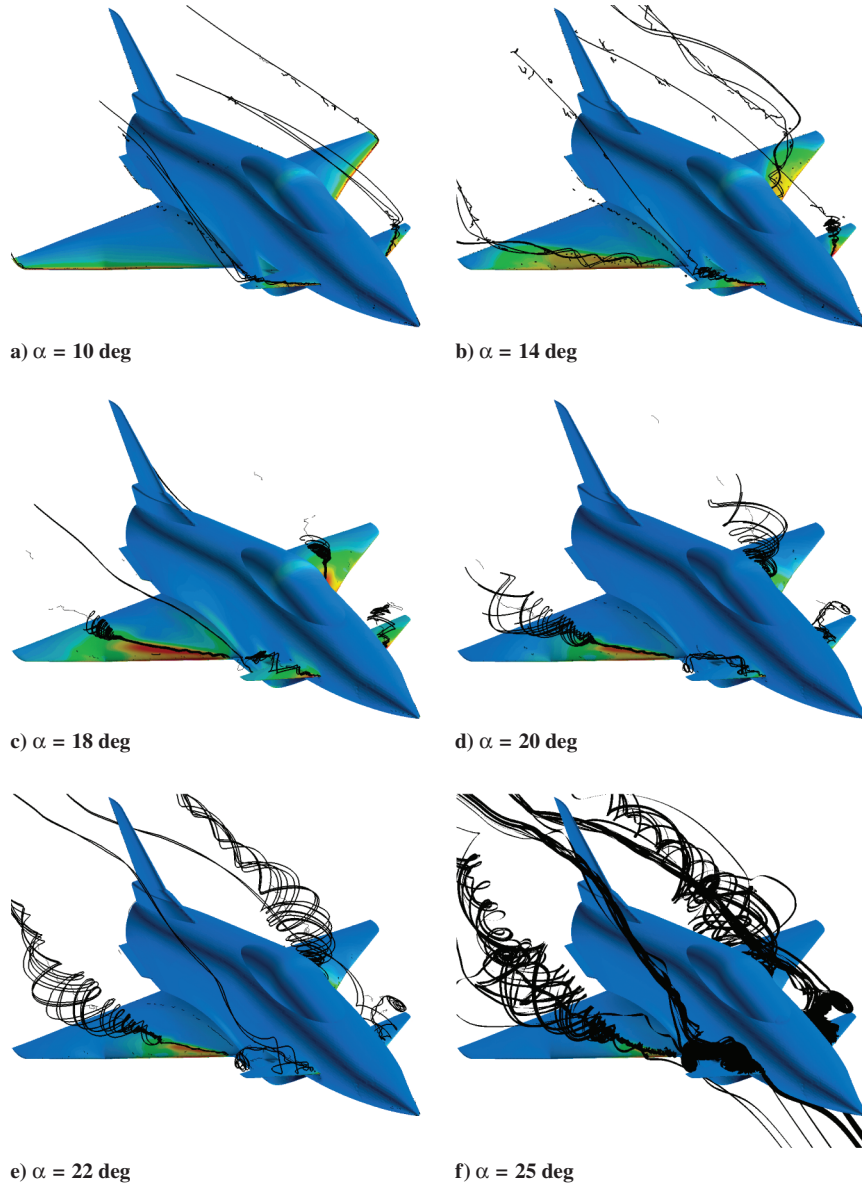


Fig. 4 X-31 vortical flows using SARC-DDES turbulence model. The conditions are : $M_\infty = 0.18$ and $Re = 2 \times 10^6$.

of $\{X_j^c: j = 1, \dots, p\}$, $X_j^c \in \mathbf{R}$ and a corresponding output vector of $\{Y_j^c: j = 1, \dots, p\}$, $Y_j^c \in \mathbf{R}$, the RBF approximates the output at a new given point as

$$\hat{Y}(X^*) = \sum_{k=1}^p \alpha_k \Phi_k(X) \quad (22)$$

such that

$$\hat{Y}(X_j^c) = Y_j^c, \quad \text{for } j = 1, 2, \dots, p \quad (23)$$

where α_k are the weights of the linear combiners. The functions Φ_i are named radial basis functions and are often described by a Gaussian basis function as

$$\Phi_i(X) = \exp\left(-\frac{\|X^* - X_j^c\|^2}{\beta^2}\right) \quad (24)$$

where β is a real variable to be chosen by the user, and $\|\cdot\|$ denotes the Euclidean norm such that the functions Φ_i will vanish at sufficiently large values of $\|X^* - X_j^c\|$. In terms of the network structure, the RBFNN is a two-layer processing structure with one

hidden layer that approximates Φ_i at each node. Then, the output layer is a set of linear combiners of approximation from hidden layer nodes. The network is then trained to minimize the error between the target (desired) values and the network predicted values. The performance and data-preparation process of RBFNN are detailed in [53].

G. System Identification

An indirect method that involves the resolution of an overdetermined system is used to estimate the Volterra kernels from time-accurate CFD simulations of the chirp and spiral maneuvers defined in Table 1. The CFD solution of these maneuvers are discrete in time with the time-step indicated by Δt^* . Denote the input vector at each time step of the simulation as $x[n] = x(n\Delta t^*) = x(t)$. The discrete-time representation of Eq. (8) is

$$y[n] = \sum_{j=1}^m \cdot \sum_{k=0}^n H_1^{x_j}[n-k]x_j[k] + \sum_{j_1=1}^m \sum_{j_2=1}^m \cdot \sum_{k_1=0}^n \sum_{k_2=0}^n H_2^{x_{j_1}, x_{j_2}} \times [n-k_1, n-k_2]x_{j_1}[k_1]x_{j_2}[k_2] + \mathcal{O}(|x|^3) \quad (25)$$

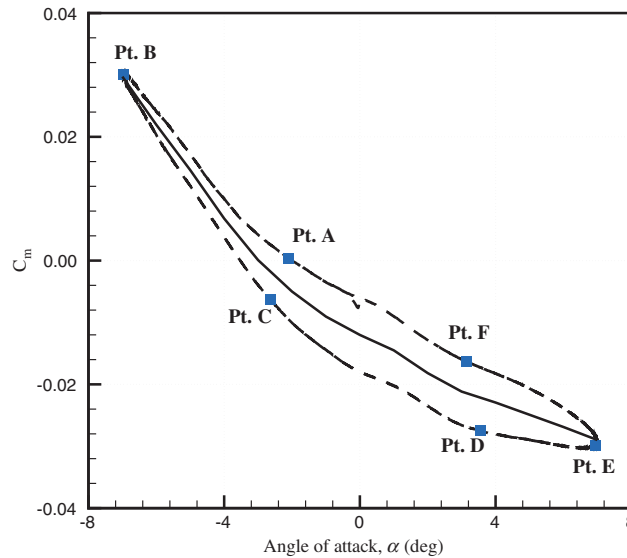
Values of aerodynamic coefficients and the time history of the motion variables are known from the CFD simulations of the chirp

and spiral maneuvers. Let $\mathbf{y} = (y[0], y[1], \dots, y[n])^T$ denote each aerodynamic load computed using CFD, and let \mathbf{A} contain the permutations of input parameters relevant to the unsteady motion. Equation (25) can be recast in the form

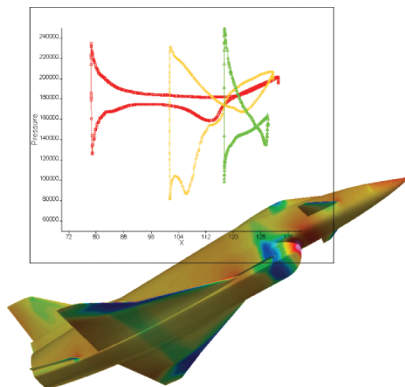
$$\mathbf{y} = \mathbf{A}\mathbf{b} \quad (26)$$

where the vector \mathbf{b} contains the unknown Volterra kernels. The matrix \mathbf{A} is in general nonsquare, with more rows than columns. Several

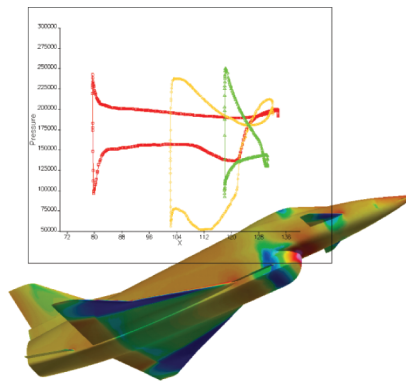
numerical methods are available to solve least-squares problems (e.g., direct inversion of $\mathbf{A}^T\mathbf{A}$, Gauss elimination, Moore–Penrose generalized inverse approach, and QR factorization). However, the Moore–Penrose approach and QR factorization are more accurate than the Gaussian elimination and the direct inversion solutions. The cost of QR factorization is $\mathcal{O}(n^2)$, and the Moore–Penrose inversion involves $\mathcal{O}(n^3)$ operations. Note that computational resources attributable to the identification of the Volterra kernels grow exponentially with order. Increasing the order of



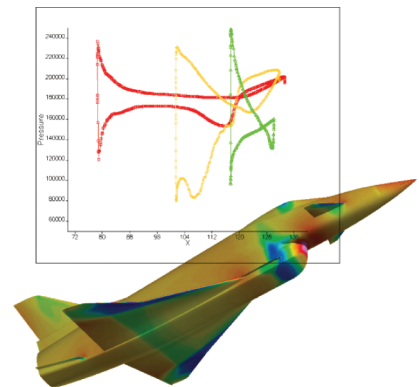
a) Target motion



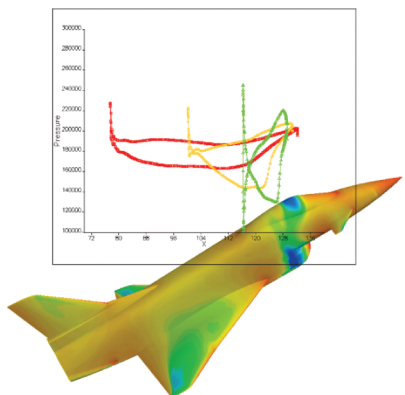
b) Point A



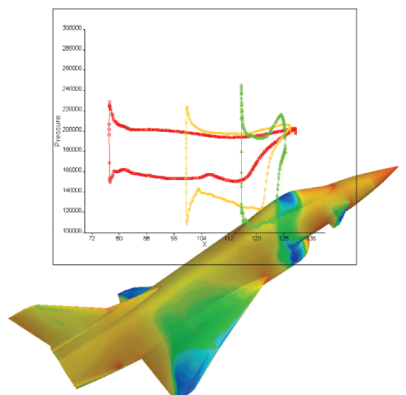
c) Point B



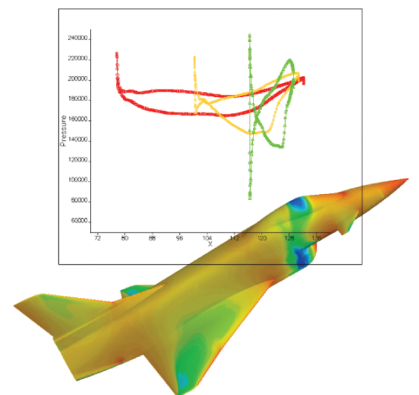
d) Point C



e) Point D



f) Point E



g) Point F

Fig. 5 Target motion. In Fig. 5a, static data are shown with a solid line. The chordwise pressure distributions are shown for the wing sections at $y/(b/2)$ of 0.3, 0.6, and 0.9 (shown with red, yellow, and green lines), where b is the wing span.

the Volterra series introduces a requirement for a training maneuver of sufficient duration. A remedy to this is the use of a simplified form of the kernel parametric structure. For example, Balajewicz et al. [71] proposed to set all off-diagonal terms of the kernel to zero, i.e.,

$$H_p^{x_{j_1} x_{j_2} \dots x_{j_p}} [n - k_1, n - k_2, \dots, n - k_p] = 0 \quad \text{for } k_1 \neq k_2 \neq \dots \neq k_p \quad (27)$$

The Volterra kernels are then identified from Eq. (26) solving for \mathbf{b} , with \mathbf{y} and \mathbf{A} being known from the simulation of chirp and spiral maneuvers. The matrix \mathbf{A} is then recomputed for a novel maneuver, and the low-order model in Eq. (26) is used to predict the resulting unsteady aerodynamic loads in place of the full-order system.

In this work, the indicial functions of $C_{m\alpha}$ and C_{mq} were calculated directly in CFD using a grid-motion approach. This approach allows the uncoupling of effects of angle of attack and pitch rate for the indicial functions. Cobalt uses an arbitrary Lagrangian–Eulerian formulation and hence allows all translational and rotational degrees of freedom [32]. The motion is specified from an input file that has the location of a reference point on the aircraft at each time step. In addition the rotation of the aircraft about this reference point is also defined using the rotation angles of yaw, pitch, and bank. The aircraft reference-point velocity \mathbf{v}_a in an inertial frame is calculated to achieve the required angles of attack and sideslip as well as the forward speed. The velocity is then used to calculate the location. The initial aircraft velocity \mathbf{v}_0 is specified in terms of Mach number, angle of attack, and sideslip angle in the main file. The instantaneous aircraft location for the motion file is then defined from the relative velocity vector $\mathbf{v}_a - \mathbf{v}_0$. For CFD-type calculation of a step change in the angle of attack, the grid immediately starts to move at $t = 0$ to the right and downward. The translation continues over time with a constant velocity vector, which changes the angle of attack. Because there is no rotation, all the effects in aerodynamic loads are from

changes in the angle of attack. For a unit step change in pitch rate, the grid moves and rotates simultaneously. The grid starts to rotate with a unit pitch rate at $t = 0$. To hold the angle of attack zero during the rotation, the grid moves right and upward.

The RBF modeling in this work is used for predicting aerodynamic loads on the aircraft due to pitching motions at a fixed Mach number. To build the RBF model, a training maneuver(s) is needed to provide enough information to learn the mapping between input and output given by Eq. (19). Previous studies to generate training maneuvers for aerodynamic characteristics [72–76] are limited by the range of the motion frequency content. A ROM identified from such a maneuver has limitations with respect to S&C applications. Thus, the basic requirement for a training maneuver to generate a reliable ROM in S&C applications is that it sufficiently covers the desired regressor space of state variables. A ROM built on data produced by such motions can then be used to predict the aircraft aerodynamic behavior within the regressor space. The systematic coverage of the regressor space can be, in general, treated as an optimization problem of filling the multidimensional space with strong constraints resulting from the fact that some axes of the regressor space do not represent an independent variable. For the present study, training maneuvers of linear chirp, spiral, and Schroeder were used to build the RBF model. These maneuvers are defined in Table 1.

These maneuvers were simulated at a fixed Mach number for which target motions were calculated. In Table 1, α_0 is the mean angle of attack, α_A is the amplitude, and ω is the angular velocity. The chirp maneuver used has a constant amplitude and linearly increasing frequency in time. In the spiral maneuver, the amplitude increases linearly in time, as does the angle of attack. The Schroeder maneuver is a multistage frequency sweep consisting of multiple cosine terms with a specified phasing. This maneuver has three parameters that enable direct control of the regressor space coverage; these are maneuver amplitude α_A , the maneuver length T , and the number of frequencies in the maneuver N .

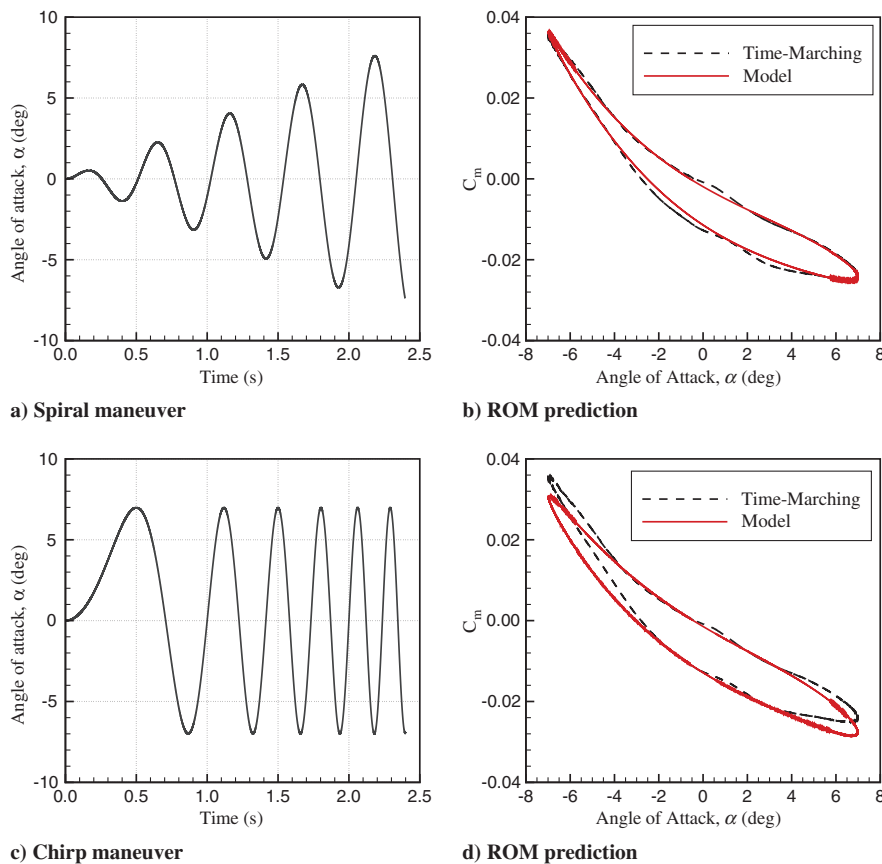


Fig. 6 Volterra reduced-order modeling using spiral and chirp training maneuvers. The flow conditions of training maneuvers are $M_\infty = 0.9$ and $Re = 2 \times 10^6$.

The SBRF model is tested for predicting aerodynamic loads of the X-31 aircraft pitching within amplitude/Mach number space of interest. The training maneuvers used are pitching motions with various combinations of amplitude and Mach number using design-of-experiment methods. The simulations were performed at a fixed frequency for which target motions were calculated.

III. Test Case

The X-31 aircraft is considered in this paper. The aircraft geometry and wind-tunnel data were provided to the participants in NATO Research and Technology Organisation Task Group AVT-161 (Assessment of Stability and Control prediction Methods for NATO Air and Sea Vehicles) [77]. The objective of this task group is to evaluate CFD codes against the wind-tunnel results. The vehicle is a supermaneuverable fighter that was built by the United States and West Germany in the 1990s. The test aircraft has been a subject of extensive flight, and wind-tunnel tests (see for example Canter and Groves [78], Alcorn et al. [79], Williams et al. [80], and Rein et al.

[81]) and CFD simulations (an example is the work of Schütte et al. [5]). A three-view drawing of the vehicle is shown in Fig. 1. The aircraft has a fuselage length of 13.21 m, a canard, and a double delta wing with total wing span of 7.26 m. The inner delta wing has a sweep angle of 57 deg, and the outer sweep is 45 deg. The inner wing sweep places the wing behind the supersonic shock wave, while the outer one improves the vehicle stability and control [82]. The canard is a cropped delta wing with a sweep angle of 45 deg. Additional characteristics of the model are the inner and outer leading-edge flaps, the trailing-edge flaps, the front wing, and the rear fuselage strakes.

The computational mesh was generated in two steps. In the first step, the inviscid tetrahedral mesh was generated using the ICEMCFD code. This mesh was then used as a background mesh by TRITET [83,84], which builds prism layers using a frontal technique. TRITET rebuilds the inviscid mesh while respecting the size of the original inviscid mesh from ICEMCFD. The mesh overview is shown in Fig. 2. The grid is a symmetric configuration and contains 4.9 million points and 11.7 million cells. Three boundary conditions

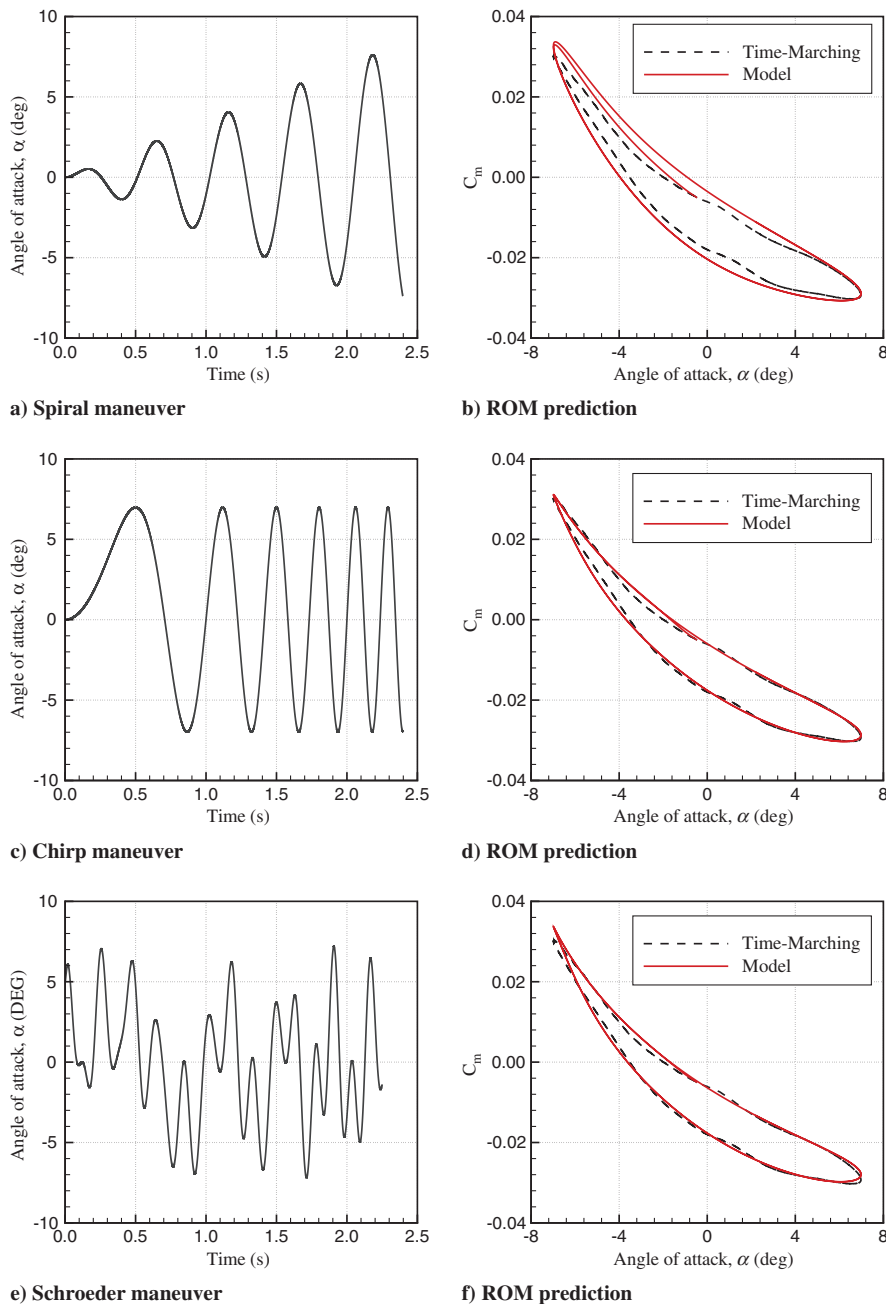


Fig. 7 RBF reduced-order modeling. Training maneuvers are spiral, chirp, and Schroeder motion. Flow conditions are $M_\infty = 0.9$ and $Re = 2 \times 10^6$.

were imposed to the surfaces: a far field, symmetry, and a solid wall. The low-speed experiments are available from DLR, German Aerospace Center [81]. The wind-tunnel model has a closed inlet and is fitted with moving lift and control surfaces. The experiments are composed of two setups. The first setup uses a belly-mounted sting attached to the model directly under the main wing. This setup allows six-degree-of-freedom motions. The second setup uses an aft mounted sting connected to an arm in the wind tunnel. The values of lift, drag, and pitch moment of the second setup are used to validate CFD predictions. CFD simulations were run on the Cray XE6 (open system) machine at the Engineering Research Development Center (the machine name is Chugach with 2.3 GHz core speed and 11,000 cores). Four turbulence models were tested: the Spalart–Allmaras (SA) [85], the SA with rotation correction (SARC) [86], Menter’s shear stress transport (SST) [87], and SARC with delayed detached-eddy simulations (SARC-DDES). Figure 3 compares the lift, drag, and pitch-moment coefficients obtained from each turbulence model with the available measurements. All of these models yield similar predictions at low angles of attack, but they result in a wide spread of predictions at moderate to high angles. For angles between $\alpha = 15$ and 23 deg, SARC-DDES and SST models perform quite well compared to SA and SARC. The DDES and SST models accurately predict unsteady separated flows occurring at these angles, but for angles above 23 deg, all models fail to predict accurately the massively separated flow.

Some features of aerodynamic characteristics from the SARC-DDES turbulence model predictions are explained. There is an emanating vortex from the canard tip at small angles of attack. This vortex is the source of the small nonlinearity in the pitch moment at low angles of attack. As angle of attack is increased, the canard vortex becomes stronger, resulting in a negative pressure on the upper surface and forward movement of the aerodynamic center. Therefore, the pitch-moment slope suddenly increases from the slope value at 0 deg angle of attack. This vortex is shown in Fig. 4a for 10 deg angle of attack. Around an angle of 14 deg, the canard vortex starts to breakdown, and the wing vortex is formed as shown in Fig. 4b. The

wing vortex helps to further forward movement of aerodynamic center and increase of pitch moment. At 18 deg angle of attack, the canard vortex breakdown point is nearly moved to the leading edge, and then the wing vortex starts to breakdown as shown in Fig. 4c. This results in an aftward movement of aerodynamic center and a change in the pitch-moment slope sign. The wing vortex breakdown point moves toward the leading edge by increasing angle of attack (Figs. 4d and 4e). The canard vortex is fully separated at these angles. As the vortex breakdown point becomes close to the wing leading edge (Fig. 4f), the pitch moment starts to rise again. Note that the purpose of this work is not to validate CFD codes or turbulence models, but rather to validate various reduced-order modeling approaches. Therefore, only one code (Cobalt) and only one turbulence model (SARC-DDES) have been used throughout the study based on our experience with these tools in predicting unsteady aerodynamics. Because the primary result of the work is to validate the modeling approaches, only comparisons will be made between the model results and the original CFD simulations.

IV. Results and Discussion

All ROMs are first evaluated to predict the unsteady pitch moment resulting from a sinusoidal pitch oscillation at freestream Mach number of 0.9 and reduced frequency of $k = 0.01$. The amplitude of oscillation is held constant at 7 deg, and the mean angle of attack is 0 deg. Figure 5a shows the computed pitch-moment coefficient C_m by solving the RANS and SARC-DDES turbulence model equations in a time-accurate fashion. This solution and other time-accurate CFD simulations are labeled as “time marching” in the ROM plots. The cost of simulating three pitch cycles is approximately 52 wall-clock hours using 256 processors (2.3 GHz). Figure 5a shows that the pitch-moment curve makes a nonlinear loop on the figure of moment versus angle of attack due to occurrence of shock waves and vortices. This figure also shows that the pitch-moment curve is not symmetric about 0 deg angle of attack. The moment curve shows a negative slope during the pitch cycle such that it has more negative slope

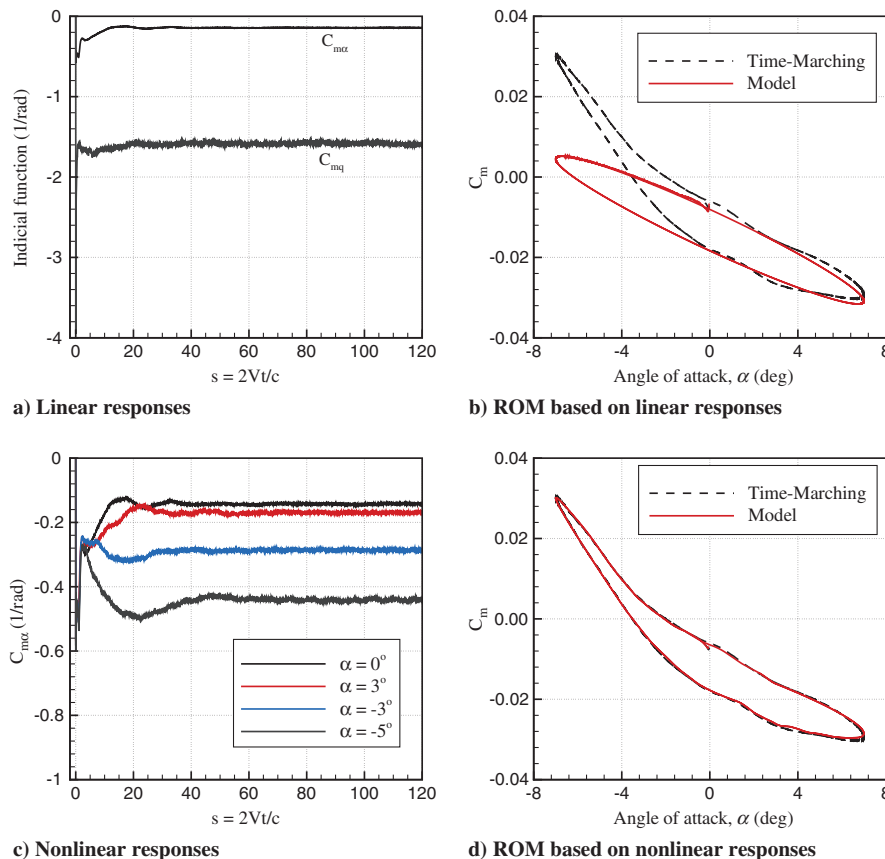


Fig. 8 ROM using indicial functions. The flow conditions are $M_\infty = 0.9$ and $Re = 2 \times 10^6$.

values at negative angles of attack compared with the slope values at positive angles of attack. Some flow features during the pitch oscillation are shown in Figs. 5b–5g and briefly discussed here. In Fig. 5b, the angle of attack is -2.1 deg, and a vortex can be seen emanating from the wing root on the lower surface, which spirals toward the wing tip. This vortex causes a sharp negative pressure peak to occur close to the wing leading edge, as shown in the surface pressure plots of Figs. 5b–5d. Figure 5b also shows that a shock wave is formed on the lower surface of the wing, which is nearly perpendicular to the fuselage before it interacts with the leading-edge vortex. At the minimum angle of attack in the pitch cycle (i.e., $\alpha = -7$ deg), the leading-edge vortex becomes much stronger, and the wing surface pressure close to the leading edge drops further, as shown in Fig. 5c. This figure also shows that, as the angle of attack becomes smaller, the shock moves downstream and therefore changes the pitch-moment curve slope. No vortices were observed on the wing during pitching at positive angles of attack, but a vortex was formed on the canard tip at the maximum angle of attack in the pitch

cycle (i.e., $\alpha = 7$ deg), as shown in Fig. 5f. Figures 5e–5g show that a shock wave is formed over the upper surface, which is no longer perpendicular to the fuselage and moves slowly with an increase in the angle of attack during upstroke.

For the identification of the Volterra kernels, the chirp and spiral training maneuvers were generated using CFD as the source of the data. The variation of angle of attack with time for these maneuvers is shown in Figs. 6a and 6c. Both maneuvers ran for 2.4 s of physical time and started from a steady-state solution. The chirp maneuver has an amplitude of 7 deg starting from 0 deg angle of attack and pitching with a frequency of 1 Hz at $t = 0$. Note that the chirp-motion frequency increases linearly with time. The spiral maneuver has an initial amplitude of 3.5 deg, starting from 0 deg angle of attack and pitching at constant frequency of $k = 0.01$. The oscillation amplitude in the spiral motion increases as time progresses. Note that the spiral maneuver is at the reduced frequency of the maneuver to be predicted. The cost of generating each training maneuver is approximately 84 wall-clock hours using 256 processors. The first- and second-order

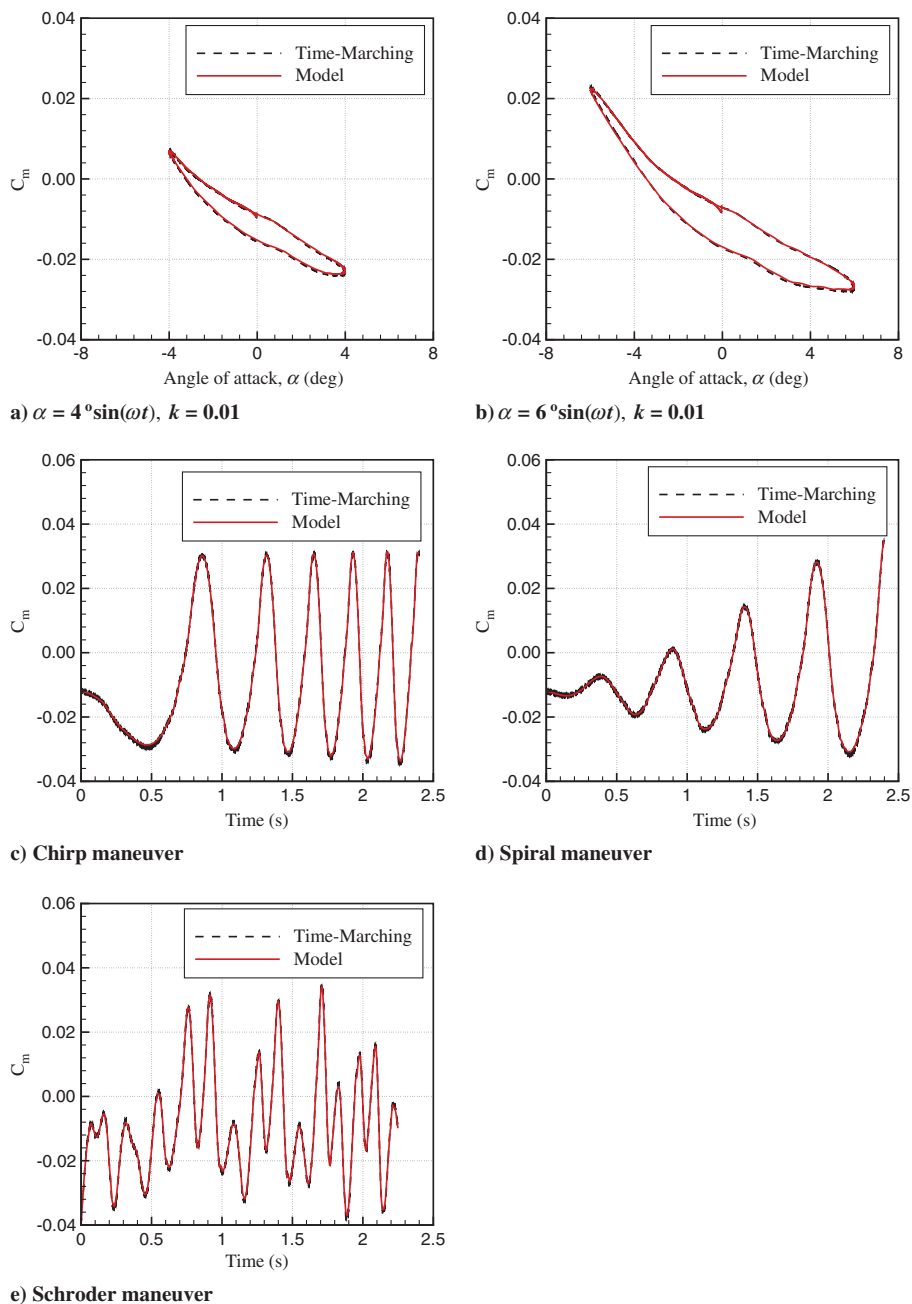


Fig. 9 ROM using nonlinear indicial functions for target motions at different angles of attack and frequency. The flow conditions are $M_\infty = 0.9$ and $Re = 2 \times 10^6$.

kernels of the Volterra model were estimated from time-history simulations of chirp and spiral training maneuvers. These estimations were used next to predict the unsteady pitch-moment data shown in Fig. 5a. The ROM predictions based on spiral and chirp training maneuvers are compared with the time-accurate solution data in Figs. 6b and 6d. The comparisons show a good agreement with CFD data for a ROM identified from spiral data, but the ROM identified from chirp data does not match well, in particular, around the maximum and minimum angles of attack. The instantaneous frequency in the chirp maneuver varies with time, and hence it might not have sufficient information to identify the Volterra kernels corresponding to a swept-amplitude motion at constant frequency. However, the ROM based on chirp data could be used for predicting aerodynamics responses from pitch oscillations at many other frequencies (those covered by the simulation of chirp training maneuver), but the ROM based on spiral is possibly valid for the motions at a fixed reduced frequency.

The generated chirp and spiral training maneuvers were also used to find a mapping between the pitch-moment coefficient and the instantaneous pitch motion variables. This mapping was next learned using a RBF neural network. Also, a Schroeder maneuver was defined by a multistage frequency sweep. This maneuver started from an initial angle of attack of 4.95 deg. The number of frequencies in the maneuver, N , was set to 20 with an initial amplitude of 7 deg. This maneuver ran for 2.4 s of physical time as well and is shown in Fig. 7e. The aircraft responses to these three maneuvers were generated using unsteady RANS equations. The training data were next normalized using the mean and standard deviation of each input. The data are then rearranged according to Eq. (19), and the RBF network performance is tested for different values of m and n , with a performance error threshold of 1×10^{-6} . All networks computed converged to the threshold error. The results showed that using $m = n = 4$ is sufficient for modeling the studied motions. The

trained networks were then tested against the target motion; the ROM predictions are shown in Figs. 7b, 7d, and 7f. These figures show that the predicted ROM values agree well with the time-marching solution, although the ROM based on the Schroeder maneuver showed better accuracy than models based on the chirp and spiral maneuvers.

The indicial pitch-moment responses of the X-31 aircraft with a unit step change in angle of attack and pitch rate are shown in Fig. 8a. These functions correspond to the fixed Mach number of 0.9. In C_{ma} simulations, the angle of attack is 0 deg at $t = 0$ and is held constant to 1 deg for all other times. In C_{mq} simulations, the grid starts to pitch up with a normalized pitch rate of $q = 1$ rad at $t = 0$, and the angle of attack is held to 0 deg during simulations with the aid of grid translation. All computations started from a steady-state solution and then advanced in time using second-order accuracy with five Newton subiterations. As shown in Fig. 8a, the pitch-moment responses have a negative peak at $t = 0$ followed by an increasing trend. As the steady flow around the vehicle is disturbed by the grid motion, a compression wave and an expansion wave are formed on the lower and upper surface of the vehicle that cause a sharp negative pitch-moment peak in the responses [32]. As the response time progresses, the waves begin to move away from the vehicle, and the pitch-moment responses start to increase and then asymptotically reach the steady-state values. The cost of generating each indicial function is around 1.5 wall-clock hours using 256 processors. A linear ROM was created using Eq. (11) and used for prediction of the target maneuver. The results are compared with the full-order model in Fig. 8b. The figure shows that linear ROM fails to accurately predict the pitch-moment values at all angles of attack. The functions of C_{ma} vary largely with angle of attack at transonic speed range, and thus a linear ROM cannot predict these effects. Next, the X-31 C_{ma} functions were simulated at different angles of attack and at a freestream Mach number of 0.9 and shown in Fig. 8c. Note that the pitch-moment slope is not symmetric with zero angle of attack, and hence the simulations included both positive and negative angle of attack responses. The total cost of generating a nonlinear ROM is now increased to approximately 21 wall-clock hours. Figure 8c shows that the responses at initial time are invariant with angle of attack, but the intermediate and final values change depending on the angle of attack. Figure 8c shows that the pitch-moment responses have more negative values than positive angles of attack due to vortex formation on the lower surface of the wing. A nonlinear ROM was created and then, using a linear interpolation scheme, the prediction of the target maneuver was evaluated. Figure 8d shows that the nonlinear ROM predictions agree very well with full-order simulation values. Note that such a nonlinear ROM could be used for computing the pitch-moment responses from many other motions with different amplitudes and frequencies. For example, the ROM was used to predict two pitch oscillations with 4 and 6 deg amplitude at $M = 0.9$ and $k = 0.01$. The predictions are compared with time-marching solutions in Figs. 9a and 9b. Again, a very good match was found. Also, the ROM predictions were evaluated for the chirp, spiral, and Schroeder maneuvers used in RBF work. Figures 9c–9e show that, even for this varying amplitude and frequency motions, the created ROM matches very well with CFD data.

For the generation of the SBRF model, time-accurate simulations were precomputed for various combinations of pitch amplitude and Mach number at a fixed reduced frequency. The two-dimensional parameter space was filled using design-of-experiment methods and is shown in Fig. 10a. Also, the pitch motion simulations of all samples are shown in Fig. 10b. The SBRF model was used for the prediction of the pitch-moment coefficient time history for sinusoidal forced motions about 0 deg angle of attack, amplitude of 7 deg, and values of Mach number of 0.78, 0.825, and 0.88. Model predictions are compared to time-accurate results in Fig. 11. Tests were performed to evaluate the dependency of the model predictions on the number of previous steps in the inputs (angle-of-attack time history, first and second time derivatives, and Mach number) and output (the prediction itself). No significant dependence was found for values of m and n up to 2. This may be attributed to the small time-step increments used in the time-accurate simulations. Although the

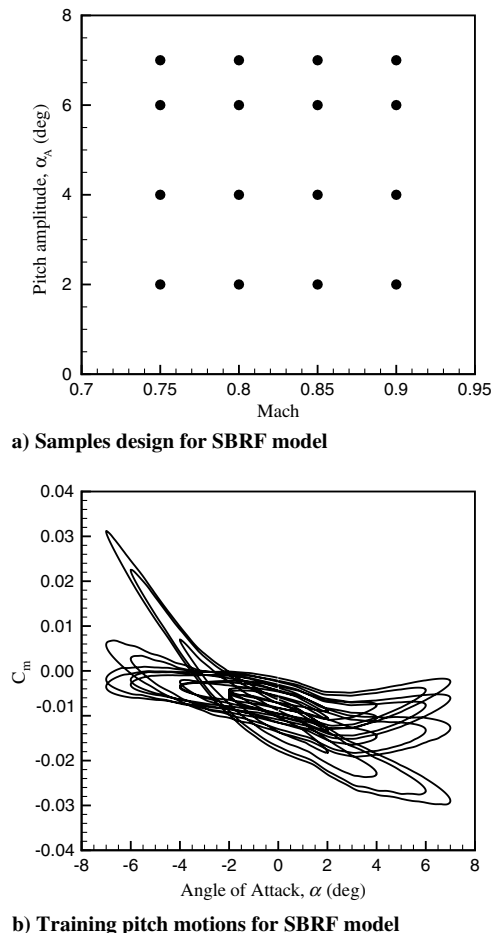


Fig. 10 Samples and training pitch motions for a SBRF model.

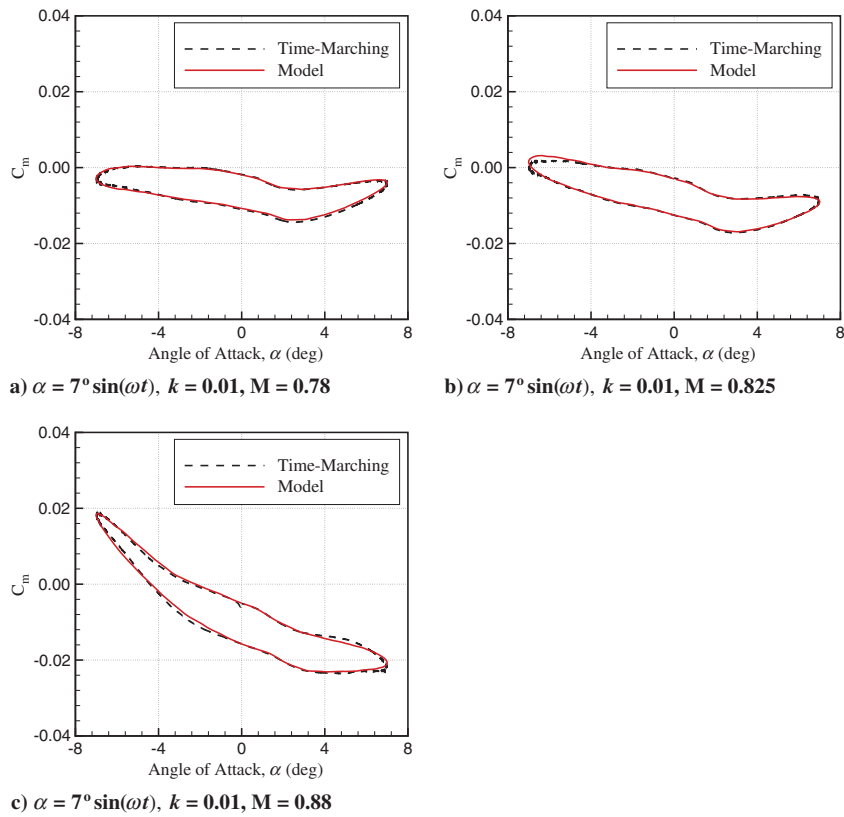
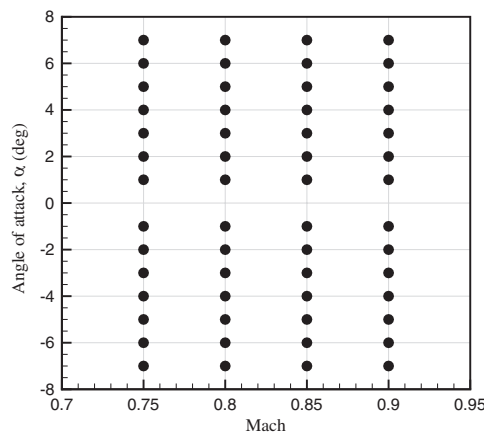


Fig. 11 Transonic load modeling in Mach-number/angle-of-attack space at fixed reduced frequency. The ROM is a SBRF model; ω is the angular velocity, and $k = \omega c / 2V$ is the reduced frequency.



a) Samples design for indicial functions

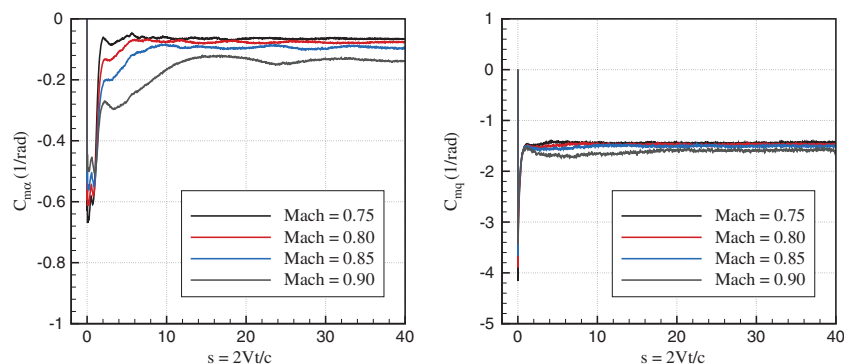


Fig. 12 Time-dependent surrogate modeling of indicial functions.

method robustness could degrade for higher values of m and n , it is considered relevant that the predictions are unaffected for a range of values. In this work, the kriging interpolation was used to approximate the mapping function between inputs and outputs. A good agreement is noted in Fig. 11 for all flight conditions, with the model predictions being generated in a few seconds.

An issue regarding the SBRF model is that the cost of simulating three pitch cycles for each sample shown in Fig. 10a is around 52 wall-clock hours, and the model still cannot predict the aerodynamic responses to motions at other frequencies. A ROM based on indicial functions, along with a time-dependent surrogate approach, is proposed for aerodynamics modeling in the angle-of-attack/Mach-number/frequency space. In this model, the indicial functions in the angle of attack and Mach number space are interpolated from some available samples. Note that the ROM based on these functions is still cheaper than the full-order model and SBRF model because the indicial functions eliminate the need of repeating calculations for each frequency. The samples could be generated using methods of factorial

design, Latin hypercube sampling, low-discrepancy sequences, and designs based on statistical optimality criteria [88]. Factorial designs are extremely simple to construct and have been used in this work. The X-31 motions considered encompass α and M values in the range of $[-7, 7]$ deg and $[0.75, 0.9]$, respectively. A set of samples including 56 points is defined on the α and M space using factorial design. These points are shown in Fig. 12a. The indicial functions with respect to angle of attack are calculated using the CFD and grid-motion approach for each sample condition. The pitch rate indicial functions are calculated for a unit step change in the pitch rate for each Mach number shown in Fig. 12a. The total cost of generating all functions is now approximately 90 h. The calculated indicial functions due to a unit-step change in angle of attack are shown in Fig. 12b for each Mach number in the sample design. This figure shows that the pitch-moment initial, intermediate, and final loadings are different at each Mach number. The initial peak in the pitch moment becomes smaller for higher Mach number. An explanation is given by Leishman [89]; this is due to the propagation of pressure disturbances at the speed of sound. Note that

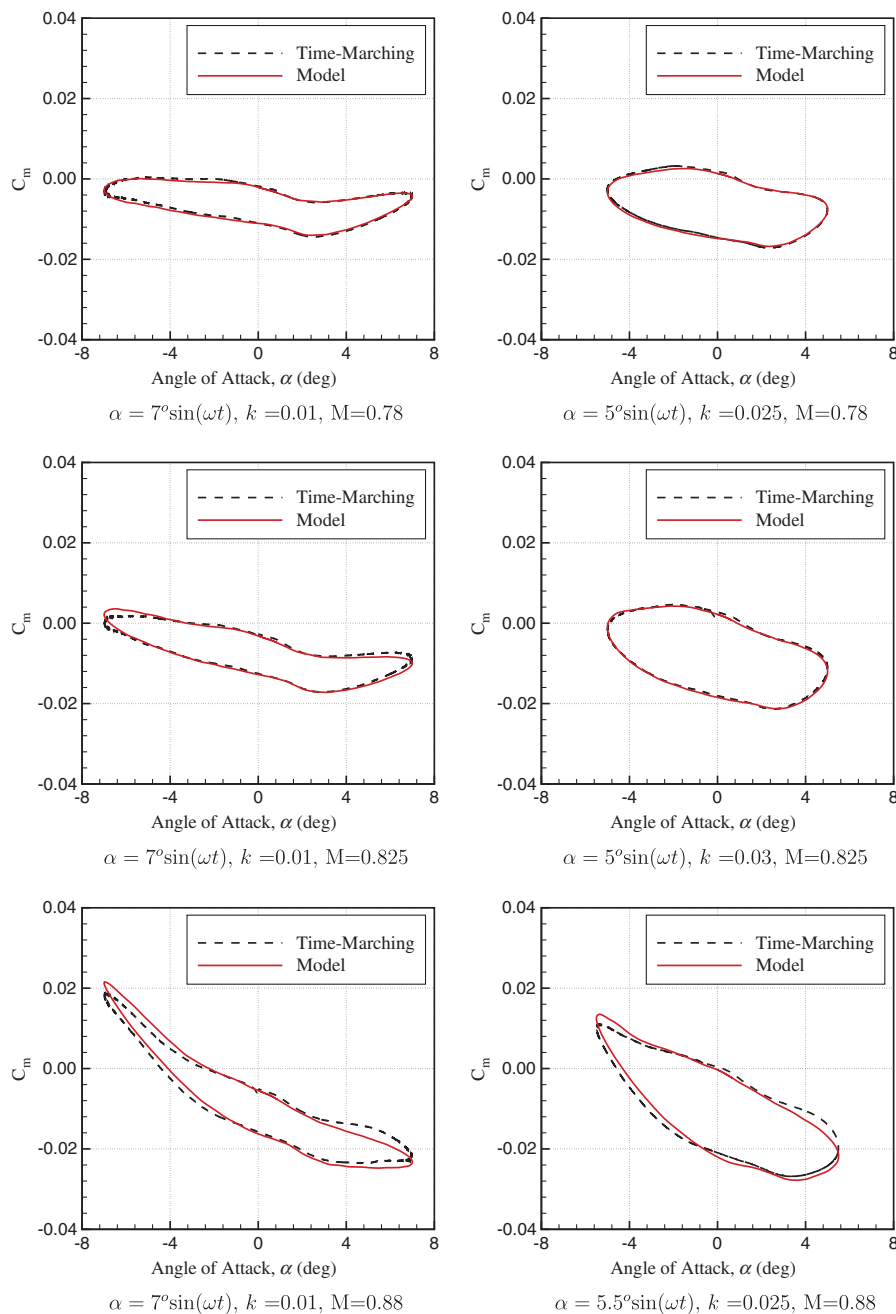


Fig. 13 Transonic load modeling in Mach-number/angle-of-attack/frequency space. The ROM is based on a time-dependent surrogate model that approximates the nonlinear indicial functions at different flight conditions.

the pitch-moment responses suddenly change for Mach numbers above 0.85 due to shock formation. Figure 12c also shows that pitch-rate indicial functions decrease as Mach number increases. A new ROM is now created along with a time-dependent surrogate model to determine the terms in Eq. (12) at each time step. The validity of the ROM is tested for several motions in the angle-of-attack/frequency/Mach-number space and compared with time-accurate CFD simulations in Fig. 13. This figure shows that the ROM predictions agree well with the CFD data. Small discrepancies are found in the high-speed motions. This is likely due to the sample design used with a uniform spacing and the fact that pitch moment changes suddenly at high speeds. More samples at high speeds could improve the model predictions. The future work extends the results to include different sampling strategies. Finally, the created ROM could predict many motions in the angle-of-attack/frequency/Mach-number space within a few seconds.

V. Conclusions

The aircraft stability and control analysis requires a very large number of computational fluid dynamics (CFD) simulations to determine appropriate forcing parameters within the frequency/amplitude/Mach-number space. Typically, the time-accurate CFD simulations start from a steady-state solution and are marched (iterated) in pseudotime within each physical time step using a dual-time-stepping scheme. Also, to have a free decay response to the initial grid perturbation, it is often necessary to march the time-accurate solution for several oscillations. Also, the configuration used in this work, the X-31 aircraft, has highly swept slender wings, resulting in complex vortical flow under various conditions. A highly refined mesh, small time step, and the use of hybrid turbulence models such as detached-eddy simulation and delayed detached-eddy simulation are required to accurately resolve the unsteady flow around the aircraft in space and time. Because of the combination of large grids, small time steps, hybrid turbulence models, and a large number of simulations, the full-order modeling approach is too expensive for stability and control analysis of aircraft. This paper investigates the use of reduced-order models that significantly reduce the CFD simulation time required to create a full aerodynamics database, making it possible to accurately model aircraft static and dynamic characteristics from a limited number of time-accurate CFD simulations.

The models considered were based on Duhamel's superposition integral using indicial (step) response functions, Volterra theory using nonlinear kernels, radial basis functions, and a surrogate-based recurrence framework, both using time-history simulations of a training maneuver(s). The indicial functions were directly calculated from unsteady Reynolds-averaged Navier-Stokes simulations starting from an initial steady-state condition with a prescribed grid motion. An important feature of this approach is uncoupling the effects of angle of attack and pitch rate into responses from pitching motions. A method to efficiently reduce the number of step function calculations within the angle-of-attack/Mach-number space was described. This method uses a time-dependent surrogate model to fit the relationship between flight conditions (Mach number and angle of attack) and step functions calculated for a limited number of samples. An indirect method was proposed to estimate the nonlinear Volterra kernels from time-accurate CFD simulations of chirp and spiral training maneuvers. These maneuvering simulations were also used to estimate the unknown parameters in a model based on radial basis functions. A design-of-experiment method was used to generate several pitching motions at different amplitudes and freestream Mach numbers. The model based on a surrogate-based recurrence framework then approximated the aerodynamic responses induced by pitching motions at new amplitudes and Mach numbers. Overall, the reduced-order models were found to produce accurate results, although a nonlinear model based on indicial functions yielded the best accuracy among all models. This model, along with a developed time-dependent surrogate approach, helped to produce accurate predictions for a wide range of motions in the transonic speed range. The cost of generating each motion using time-accurate

CFD simulations was approximately 52 wall-clock hours using 256 processors (2.3 GHz), but the reduced-order model predictions were obtained within a few seconds. The future work extends the results to include different sampling strategies and will include aerodynamic modeling of lateral forces and moments as well.

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