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5aEA2. Mutlichannel feedback control of interior road noise

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Active noise control systems offer a potential method of reducing the weight of passive acoustic treatments in vehicles and, therefore, increasing a its fuel efficiency. The active control of engine noise can be implemented cost-effectively by using the car audio loudspeakers as control sources and an array of low-cost microphones as error sensors. Such systems have been commercially implemented, but without also controlling road noise their subjective benefits may be limited. The active control of road noise using a feedforward control strategy has also been practically demonstrated, but these systems require a number of accelerometers to be mounted to the vehicle's structure to obtain a coherent reference signal and, therefore, lead to a significant implementation cost. This paper describes a multichannel feedback system for the active control of road noise, which uses an array of microphones and car audio loudspeakers that are common with ta feedforward engine noise control system. The design of the multichannel feedback controller is described and its performance is validated using offline simulations employing data measured in a small city car.

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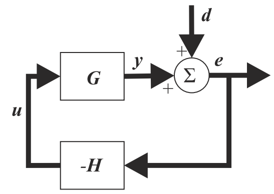
INTRODUCTION

Noise in road vehicles has been widely acknowledged as a key factor governing their commercial success [1]. Although passive acoustic treatments remain dominant in the reduction of both engine and road noise within the car cabin [2], there has been considerable interest in active noise control measures [3]. This interest has recently been driven by the need to improve the fuel efficiency of vehicles through the use of economical engine designs and by reducing the vehicle's weight. Economical engine designs such as variable displacement, which usually operates by deactivating a number of cylinders, often result in an increased low frequency noise, due to the use of a lower number of cylinders. Similarly, reducing the weight of a vehicle also results in increased low frequency noise. Low frequency noise is difficult to control using lightweight passive measures, and since active noise control systems are most effective at low frequencies and may be implemented within a car with relatively little increase in weight, they offer a convenient complementary solution. This is particularly true when the active noise control systems are integrated into the vehicle's electronic systems, for example, by employing the car audio loudspeakers [4].

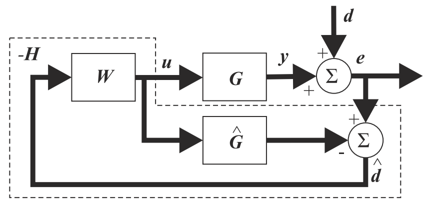
The increase in low frequency engine noise has been successfully controlled using feedforward control systems, employing an engine speed reference signal, low cost microphone error sensors and the car audio loudspeakers as control sources [5], and a number of commercial systems have since been implemented [6]. Reducing the weight of vehicles also increases the low frequency noise produced in the car cabin due to road-tyre interactions. Road noise has also been controlled using a feedforward control system [7], however, due to the random nature of road noise and the complex propagation path between the structural excitation of the tyre and the acoustic noise produced in the car cabin the implementation of a feedforward controller is significantly more demanding. Reference signals for a feedforward road noise control system have typically been obtained from accelerometers mounted to the vehicle's suspension and bodywork [7], however, in order to obtain sufficient coherence between the reference and disturbance signals, it is necessary to employ at least six accelerometers [7, 8]. Although a feedforward control system has been reported to achieve reductions of up to 7 dBA at the driver's ear position between 100 and 200 Hz [7], the need for multiple reference accelerometers means that the system is relatively expensive to implement commercially and, therefore, has seen limited commercial implementation.

As a result of the high cost of a feedforward road noise control system, interest has arisen in implementing road noise cancellation using a feedback system, as this avoids the need for separate reference sensors. Feedback control of road noise has been the focus of a body of work presented by Adachi and Sano, for example [9, 10], and this research culminated in a mass-production system implemented in the Honda Accord estate car which is presented by Sano *et al* in [4]. This single channel control system achieves a 10 dB reduction in a narrowband 40 Hz boom in the front seats, which corresponds to the first longitudinal enclosure mode, whilst avoiding enhancements in the rear seat positions. To achieve global control of booming noise a feedback controller based on modal control techniques, widely employed in structural control [11], has also been proposed [12, 13]. This single-input single-output (SISO) control system employs spatially weighted transducer arrays combined with temporal filtering and although it is shown to be effective when there is a single dominant resonance, as in the boom noise application, it is not able to achieve significant control when the response has significant contributions from a number of acoustic and structural resonances [13].

Although the SISO feedback control systems investigated in [4, 12, 13] may achieve significant control in the road noise application when there is a single dominant resonance, such resonances are only typical in vehicles that are significantly larger in one dimension, such as estate cars. Therefore, the application of these systems is rather specific. This paper builds on this previous work and investigates the design and performance of fully coupled multichannel feedback controllers for the attenuation of road noise in vehicles with the aim of providing a more general solution to the active road noise control problem.



(A) MIMO feedback controller.



(B) MIMO IMC feedback controller.

FIGURE 1: Block diagrams of the MIMO (a) and IMC MIMO (b) feedback controllers.

MULTI-INPUT, MULTI-OUTPUT FEEDBACK CONTROLLER DESIGN

Mutlichannel feedback control has been widely investigated and a wide variety of optimal design processes have been described [14]. The application of a multi-input, multi-output (MIMO) feedback controller to the road noise control problem has been suggested in [15], however, the performance of such a control system has not been presented.

Formulation

The MIMO feedback control system shown in Figure 1a consists of L error sensors and M control sources, which may also be employed by a feedforward engine noise control system. In the case of the MIMO system the sensitivity function which governs the closed-loop response is given by

$$\boldsymbol{S}(j\omega) = [\boldsymbol{I} + \boldsymbol{G}(j\omega)\boldsymbol{H}(j\omega)]^{-1} \tag{1}$$

where $\boldsymbol{G}(j\omega)$ is the M input, L output plant response and $\boldsymbol{H}(j\omega)$ is the L input, M output feedback controller. Although there are a large number of methods of designing MIMO feedback controllers, for example see [14], it is convenient to use the Internal Model Control (IMC) formulation of the MIMO controller shown in Figure 1b, since the design of the controller can then be achieved using a constrained convex optimisation, which may be solved using standard convex programming methods [16].

The response of the IMC feedback controller shown in Figure 1b, which is contained within the dashed lines, is given by

$$\boldsymbol{H}(j\omega) = -\left[\boldsymbol{I} + \boldsymbol{W}(j\omega)\hat{\boldsymbol{G}}(j\omega)\right]^{-1}\boldsymbol{W}(j\omega) \tag{2}$$

where $\boldsymbol{W}(j\omega)$ is the frequency response of the L input, M output control filter and $\hat{\boldsymbol{G}}(j\omega)$ is the model of the MIMO plant response. Assuming that the modelled plant response is perfect then the sensitivity function of the MIMO IMC controller is

$$\boldsymbol{S}(j\omega) = \boldsymbol{I} + \boldsymbol{G}(j\omega)\boldsymbol{W}(j\omega) \tag{3}$$

and the controller has an entirely feedfoward response.

Design Objectives

The aim of the MIMO feedback controller is to minimise the sum of the squared error signals, $\boldsymbol{e}(j\omega)$, which is given by

$$J(j\omega) = \text{trace}\left[E\left(\boldsymbol{e}(j\omega)\boldsymbol{e}^H(j\omega)\right)\right], \tag{4}$$

where E is the expectation operator and the vector of error signals is related to the vector of disturbance signals, $\boldsymbol{d}(j\omega)$, by the sensitivity function as

$$\boldsymbol{e}(j\omega) = \boldsymbol{S}(j\omega)\boldsymbol{d}(j\omega). \tag{5}$$

For the fully-coupled MIMO controller this cost function can be expressed using equation 3 as

$$\begin{aligned} J(j\omega) = \text{trace}\Big[&\boldsymbol{G}(j\omega)\boldsymbol{W}(j\omega)\boldsymbol{S}_{dd}(j\omega)\boldsymbol{W}^H(j\omega)\boldsymbol{G}^H(j\omega) + \cdots \\ &\boldsymbol{G}(j\omega)\boldsymbol{W}(j\omega)\boldsymbol{S}_{dd}(j\omega) + \boldsymbol{S}_{dd}(j\omega)\boldsymbol{W}^H(j\omega)\boldsymbol{G}^H(j\omega) + \boldsymbol{S}_{dd}(j\omega)\Big], \end{aligned} \tag{6}$$

where $\boldsymbol{S}_{dd}(j\omega)$ is the matrix of power and cross spectral densities of the primary disturbance. From equation 6 it can be seen that the IMC formulation leads to a quadratic cost function with respect to the control filter, $\boldsymbol{W}(j\omega)$, and the unconstrained, nominal solution can be obtained using the standard Wiener methods. However, in practice the plant response will not be perfectly modelled and this will result in a degree of feedback in the system. This leads to potential stability limitations and disturbance enhancements and, therefore, the need to enforce robust stability and enhancement constraints in the design process.

For the MIMO controller, if it is assumed that the plant uncertainty can be modelled as a multiplicative output uncertainty proportional to the scalar $B(j\omega)$, which has some limitations [14] but provides a realisable constraint, the condition for robust stability is given by

$$\bar{\sigma}\left(\boldsymbol{T}(j\omega)\right)B(j\omega) < 1 \quad \text{for all } \omega \tag{7}$$

where $\bar{\sigma}$ indicates the maximum singular value and $\boldsymbol{T}(j\omega)$ is the complementary sensitivity function given by

$$\boldsymbol{T}(j\omega) = -\boldsymbol{G}(j\omega)\boldsymbol{W}(j\omega). \tag{8}$$

In designing the MIMO feedback controller it is also desirable to enforce a constraint on the maximum out-of-band enhancement in the disturbance signal. In the case of the mutli-sensor system there are a number of possible constraints, which have been discussed in [6]. For the active noise control application, however, constraining the maximum enhancement in the individual error signals provides a more uniform reduction in the pressure by avoiding high levels of enhancements at some error sensors being balanced out by reductions at other sensors [6]. This constraint on the enhancement in the individual disturbance signals may be expressed as

$$\max\left[\text{diag}\left(\boldsymbol{D}(j\omega)\boldsymbol{S}(j\omega)\boldsymbol{S}_{dd}(j\omega)\boldsymbol{S}^H(j\omega)\right)\right]\frac{1}{A} < 1 \quad \text{for all } \omega, \tag{9}$$

where

$$\boldsymbol{D}(j\omega) = \begin{bmatrix} \frac{1}{E|d_1(j\omega)|^2} & 0 & 0 & 0 \\ 0 & \frac{1}{E|d_2(j\omega)|^2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \frac{1}{E|d_L(j\omega)|^2} \end{bmatrix}, \tag{10}$$

where $d_l(j\omega)$ is the disturbance at the l-th error sensor and the maximum enhancement in the L magnitude squared disturbance signals will be less than a maximum value defined by A.

Controller Optimisation

The design of the MIMO IMC feedback controller requires the minimisation of the cost function given by equation 6, whilst the robust stability and disturbance enhancement constraints are maintained. If the control filter matrix, $\boldsymbol{W}$, is implemented as an ML bank of FIR filters, $\boldsymbol{w}$, each with I coefficients and the design problem is discretised in the frequency domain at K linearly spaced frequencies then the optimisation can be expressed as

$$\begin{aligned}
\min_{\boldsymbol{w}} \quad & \frac{1}{K}\sum_{k=k_1}^{k_2} \operatorname{trace}\Big[\boldsymbol{G}(k)\boldsymbol{W}(k)\boldsymbol{S}_{dd}(k)\boldsymbol{W}^H(k)\boldsymbol{G}^H(k)\cdots \\
& \quad +\boldsymbol{G}(k)\boldsymbol{W}(j\omega)\boldsymbol{S}_{dd}(k)+\boldsymbol{S}_{dd}(k)\boldsymbol{W}^H(k)\boldsymbol{G}^H(k)+\boldsymbol{S}_{dd}(k)\Big] \qquad (11)\\
\text{subject to} \quad & \bar{\sigma}\left(\boldsymbol{T}(k)\right)B < 1 \quad \forall k, \\
& \max\Big[\operatorname{diag}\Big(\boldsymbol{D}(k)\boldsymbol{S}(k)\boldsymbol{S}_{dd}(k)\boldsymbol{S}^H(k)\Big)\Big]\frac{1}{A} < 1 \quad \forall k,
\end{aligned}$$

where k_1 and k_2 define the lower and upper bounds over which disturbance attenuation is desired. Since the cost function is convex with respect to the filter coefficients and the constraints are both affine functions of the filter coefficients, the optimal solution can be obtained using sequential quadratic programming [16]; this approach to designing a feedback active noise controller has previously been employed in [17].

To ensure that the solution to the discrete problem given by equation 11 approximates the desired solution to the continuous problem it is important that K is large enough such that the discretised frequency responses are accurately represented. This can be achieved by ensuring that the impulse responses of the discretised responses have negligible amplitude at the end of their responses [17]. It is also important to ensure that the FIR control filters are sufficiently long such that the obtained solution is optimal and this can be ensured by gradually increasing the length of $\boldsymbol{w}$ until there is no further improvement in performance [18].

Car Cabin Control System

To implement a cost-effective active noise control system it is preferable if significant levels of control could be achieved using the four standard car audio loudspeakers, thus avoiding additional weight and cost. Similarly, it is important for the road noise control system to be integrable with a complementary feedforward engine noise control system such as those described in [7, 6]. Therefore, the MIMO feedback controller investigated in the following section employs the four standard car audio low frequency loudspeakers and four error microphones, positioned at the four headrest positions, which may be common to a feedforward engine noise control system.

Controller Performance

To investigate the potential performance of the MIMO feedback controller a series of measurements have been conducted in a small city car. The plant response between the four car audio loudspeakers and four microphones positioned at the headrests have been measured for three different occupancy conditions, as detailed in Table 1. The primary disturbance, $\boldsymbol{d}$, has been measured when the car is driven at 50 km/h over a pave road surface.

The aim of the fully-coupled MIMO controller is to minimise the pressures at the four headrest microphones and, therefore, achieve a reduction in the sound pressure level for all of the car cabin occupants. The controller has therefore been designed according to the previous section to minimise the sum of the squared pressures at the four headrest error sensors whilst maintaining the robustness and enhancement constraints. The optimisation described by equation 11 has been discretised at $K = 198$ frequencies, with a sample rate of 2.56 kHz, and the bandwidth of

TABLE 1: Plant response measurement occupancy conditions

Condition	
O1	Driver and Front Passenger
O2	Driver
O3	Empty

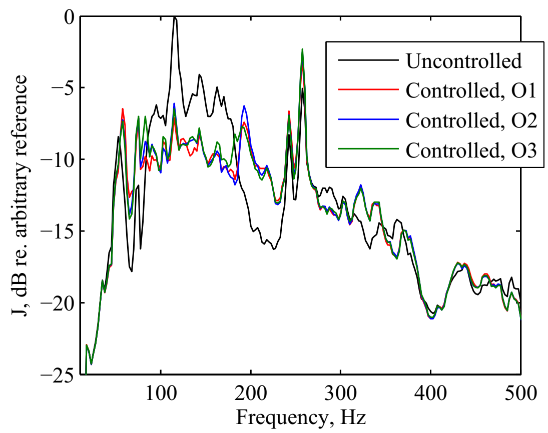
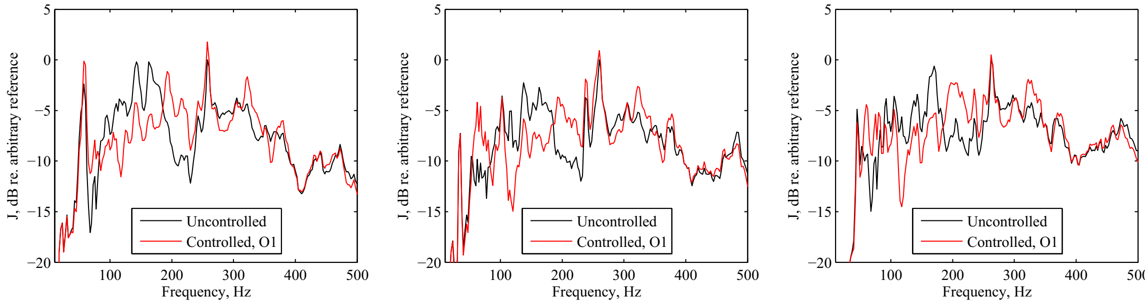


FIGURE 2: The sum of the squared pressures at the four headrest microphones before and after control using the MIMO IMC feedback controller in the small city car for the nominal plant response and two alternative occupancy conditions.

control has been defined between 80 and 185 Hz to target a broadband peak in the road noise disturbance spectrum. The robust stability and disturbance enhancement constraints have been defined as $B = 0.5$ and $A = 4$, so that enhancements of up to 6 dB are allowed at individual microphones, and the ML bank of filters have been implemented using $I = 64$ coefficients; for the (4×4) controller this results in a total of 1024 optimisation parameters. The controller has been optimised using the plant response measured under occupancy condition O1, which is detailed in Table 1, and its performance has also been calculated when the plant response has been varied by changing the number of car cabin occupants, as detailed in Table 1.

Figure 2 shows the predicted cost function, given by the sum of the squared pressures at the four headrest microphones, when there is no control and when the controller has been applied under the three different plant conditions. From this plot it can be seen that the cost function has been reduced by up to 8 dB and an average reduction of 3 dB has been achieved over the 80 to 185 Hz bandwidth for all three plant conditions. These results highlight both the potential performance of such a MIMO feedback controller and also the robustness to variations in the plant response. From the results presented in Figure 2 it can be seen that the most significant enhancements occur between around 185 and 240 Hz, where the cost function is enhanced by up to 5 dB. However, it can be seen that these enhancements occur at frequencies where the overall level is originally relatively low, and may therefore not significantly affect the subjective performance. To ensure that a road noise control system improved the subjective impression of the car cabin acoustic environment in practice it would be necessary to consider some subjective measure or constraint in the optimisation process, as has been considered in the context of engine noise control (for example see [19]).

Although it has been predicted that reduction of the broadband peak between 80 and 185 Hz is achievable using the proposed fixed MIMO feedback controller it is interesting to consider how the performance of the controller is affected by changes in the road surface and driving



(A) Rough road at 50 km/h **(B)** Rough road at 75 km/h **(C)** Rough road at 100 km/h

FIGURE 3: The sum of the squared pressures at the four headrest microphones before and after control using the MIMO IMC feedback controller in the small city car for the nominal plant response under three driving conditions.

speed. Therefore, the performance of the controller has been simulated when there is a change in the primary disturbance from that used during the optimisation. Figure 3 shows the results for simulations based on three different road speeds on a rough road surface of 50, 75 and 100 km/h. From the uncontrolled responses shown in each of these plots it can be seen that the broadband peak between 80 and 185 Hz produced on the pave road surface is no longer excited as significantly. However, it can be seen in these three rough road cases that some control over this bandwidth is still achieved, although with the enhancements that are also produced it is not expected not to make a significant change in the perceived quality of the acoustic environment.

CONCLUSIONS

This paper has proposed a method of designing a MIMO feedback controller for the attenuation of low frequency road noise in a car. The MIMO feedback controller is first formulated using an IMC architecture, which leads to the cost function – the sum of the squared error signals – being a quadratic function with respect to the control filters. Although the unconstrained minimisation of this cost function may be achieved using Wiener methods, in the feedback control context it is important to include constraints on the robustness of the controller and the out-of-band enhancements. A commonly used robust stability constraint is employed in conjunction with a novel enhancement constraint, which limits the maximum enhancement in the individual error signals. The control filters are then defined as FIR filters, and since the cost function is convex and the constraints are affine functions of the control filter coefficients to be optimised, the optimisation can be achieved in the discrete frequency domain using sequential quadratic programming.

Using the proposed MIMO feedback controller design method, the performance of a 4 loudspeaker, 4 error microphone control system implemented in a small city car has been evaluated through offline predictions, using measured plant responses and disturbance data. When the controller is optimised to control a broadband peak between 80 and 185 Hz, which is produced when the car is driven at 50 km/h over a pave road surface, attenuation in the sum of the squared pressures of up to 8 dB is achieved and an average reduction of 3 dB is achieved over the targeted bandwidth. The out-of-band enhancements are limited by the proposed constraint and they occur at frequencies where the primary disturbance is at a relatively low level.

In a practical application it is important that the control system is robust to variations in the plant response. To investigate this property the performance of the optimised controller is calculated for two plant responses modified by varying the number of occupants. These results indicate that the performance of the controller is maintained in the presence of plant response variations. It is also important to consider how the controller performs when the road and driving conditions change and, therefore, predictions of the performance of the optimised controller when there is a change in road surface and speed have been presented. These results have shown that although the characteristic of the uncontrolled road noise changes significantly, the MIMO feedback controller still achieves control, although its benefit is somewhat limited.

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