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ENERGY HARVESTING USING SEMI-ACTIVE NONLINEAR CONTROL

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ABSTRACT

This paper presents the application of semi-active control for power harvesting using an electro-mechanical energy harvester. Two semi-active control strategies are proposed in the form of a time-periodic damper and a nonlinear cubic damper. For the periodic time-varying damper the average harvested power and the throw are obtained based on the Fourier series. The semi-active periodic time-varying damper is optimised to maximise the harvested power. The performance of the optimum semi-active periodic damper is compared with the optimum passive and semi-active on-off model at a particular frequency. It is demonstrated that the periodic time-varying damper can significantly increase the harvested power at all frequencies of interest. For the nonlinear damper, the harvested power and the throw are derived using the concept of the describing function. The results are compared with the linear damper. It is demonstrated that the nonlinear damper can significantly increase the absorbed power despite having much lower displacement compared to the linear damper. This makes the semi-active nonlinear damper very attractive for mechanical energy harvesters.

1. INTRODUCTION

Semi-active control is achieved by altering a system parameter, such as damping or stiffness, in real-time to enhance the performance of the system through vibration isolation or energy harvesting [1]. Semi-active control has the advantages of showing similar vibration control performance to active control methods, while maintaining the advantages of passive methods such as simplicity and low cost implementation.

Vibration isolation systems in the form of semi-active control have been widely used in engineering applications. Examples include stay-cabled bridges [2, 3], buildings [4], automotive suspension systems [5, 6] and seat suspension [7].

Energy harvesting from the ambient vibration has also attracted significant attention in recent years [8]. Some interesting applications include low-power electronics, wireless sensors [9], electrostatic MEMS vibration energy harvester [10] and large-scale energy harvesters [11]. In many applications, the vibration amplitude is too low to be harvested efficiently. Hence, methods have been proposed to improve the energy harvesting rate by both mechanical and electrical approaches [12-16]. In order to increase the frequency range and the dynamic range of the excitation amplitude over which the vibration energy harvester operates, various nonlinear arrangements have been suggested, particularly using nonlinear springs [17, 18]. In this paper nonlinear damping will be investigated, which has previously been mainly used for vibration isolation [19, 20].

2. SEMI-ACTIVE TIME PERIODIC DAMPER

A single degree-of-freedom system (spring-mass-damper) shown in Figure 1 is considered, which is subjected to a base excitation, where m is the mass, k is the suspension stiffness, c is a semi-active damper, x is the mass displacement and y is the base displacement. The system is harmonically excited at frequency ω and the amplitude Y and the theory governing the mechanical behaviour of such system is examined here. The time-varying damping coefficient is assumed to harvest useful energy that there is no other mechanical dissipation.

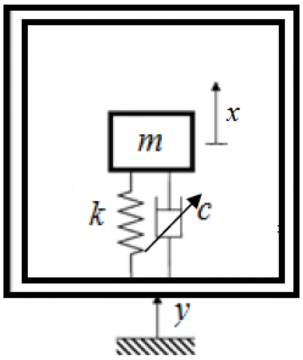


Figure 1. Single degree-of-freedom base excited system with semi-active damper

The governing dynamic equation can be written as:

$$m\ddot{x} + c(t)(\dot{x} - \dot{y}) + k(x - y) = 0. \tag{1}$$

For harmonic base excitation,

$$y = Y\cos(\omega t), \tag{2}$$

the fundamental component of the relative displacement is assumed to be,

$$z = x - y = Z\cos(\omega t - \phi). \tag{3}$$

The dynamic Eq.(1) can be rewritten as:

$$m\ddot{z} + c(t)\dot{z} + kz = -m\ddot{y} = m\omega^2 Y\cos(\omega t). \tag{4}$$

The instantaneous power absorbed at the fundamental frequency by the damper is,

$$P_{\text{ins}} = c(t)(\dot{x} - \dot{y})^2 = c(t)Z^2\omega^2\sin^2(\omega t - \phi) = \frac{c(t)Z^2}{2}\left(1 - \cos(2\omega t - 2\phi)\right) \tag{5}$$

It can be seen that the instantaneous power is composed of a constant part and a time dependent part having frequency of 2ω.
The average absorbed power can be obtained from the energy harvested per cycle:

$$P_{\text{ave}} = \frac{1}{T}\int_0^T c(t)\dot{z}(t)^2\,\mathrm{d}t. \tag{6}$$

For semi-active control, the time-varying damper is assumed to have the form of,

$$c(t) = \sum_{q=0}^{\infty} c_q\cos(q\omega t) + \sum_{q=1}^{\infty} d_q\sin(q\omega t). \tag{7}$$

To obtain an analytical solution for the average absorbed power, we use the harmonic balance method. Thus the solution to Eq.(3) now takes the form of the Fourier series:

$$z(t)=\sum_{n=0}^{\infty}A_n\cos(n\omega t)+\sum_{n=1}^{\infty}B_n\sin(n\omega t). \tag{8}$$

Taking derivatives of Eq.(8) and substituting into Eq.(1) yields the approximate analytical solution. The derivation of the response is given in more detail in [21].

The approximate analytical response of the system can be found from solving the coefficients in Eq.(8) for A_n and B_n:

$$\begin{bmatrix} k & -\frac{d_1\omega}{2} & -d_2\omega & \frac{c_1\omega}{2} & c_2\omega \\ 0 & k-m\omega^2-\frac{d_2\omega}{2} & -d_1\omega & \left(c_0+\frac{c_2}{2}+c_{\mathrm{m}}\right)\omega & c_1\omega \\ 0 & \frac{d_1\omega}{2} & k-4m\omega^2 & \frac{c_1\omega}{2} & 2(c_0+c_{\mathrm{m}})\omega \\ 0 & -\left(c_0-\frac{c_2}{2}+c_{\mathrm{m}}\right)\omega & -c_1\omega & k-m\omega^2+\frac{d_2\omega}{2} & -d_1\omega \\ 0 & -\frac{c_1\omega}{2} & -2(c_0+c_{\mathrm{m}})\omega & \frac{d_1\omega}{2} & k-4m\omega^2 \end{bmatrix}\begin{pmatrix} A_0 \\ A_1 \\ A_2 \\ B_1 \\ B_2 \end{pmatrix}=\begin{pmatrix} 0 \\ m\omega^2 Y \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

Substitution of A_n, B_n in $\dot{z}(t)$ allows the calculation of the average absorbed power using Eq.(6).

3. NONLINEAR DAMPING

The dynamic equation with nonlinear damping in the form of cubic damping is written as:

$$m\ddot{z}(t)+c\dot{z}(t)+c_n\dot{z}^3(t)+kz=-m\ddot{y}(t)=m\omega^2Y\cos(\omega t-\varphi). \tag{10}$$

We use the method of harmonic balance to approximate the response with the fundamental frequency of ω.

$$z(t)=Z\cos(\omega t), \tag{11}$$

Substitute $z(t)$ and its derivatives into Eq.(11), partitioning Sine and Cosine and ignoring higher order harmonics yields,

$$-m\omega^2Z+kZ=m\omega^2Y\cos\varphi \tag{12}$$

$$-\frac{3}{4}c_n\omega^3Z^3-c\omega Z=+m\omega^2Y\sin\varphi, \tag{13}$$

and the throw can be obtained analytically as,

$$\left|\frac{Z}{Y}\right|=\frac{m\omega^2}{\sqrt{\left(k-m\omega^2\right)^2+\left(c\omega+\frac{3}{4}c_n\omega^3Z^2\right)^2}}. \tag{14}$$

The linearized equivalent damping is thus,

$$c_{eq}=\frac{3}{4}c_n\omega^2Z^2. \tag{15}$$

The instantaneous power absorbed by the damper is,

$$P_{\mathrm{ins}}=\left(c+c_{eq}\right)\dot{z}^2=\left(c+\frac{3}{4}c_n\omega^2Z^2\right)Z^2\omega^2\sin^2(\omega t)=\frac{\left(c+\frac{3}{4}c_n\omega^2Z^2\right)Z^2}{2}\omega^2\left(1-\cos(2\omega t)\right). \tag{16}$$

The average absorbed power can be obtained as,

$$P_{\mathrm{ave}} = \frac{1}{T}\int_0^T P_{\mathrm{ins}}\mathrm{d}t = \frac{c}{2}Z^2\omega^2 + \frac{3c_n}{8}Z^4\omega^4. \tag{17}$$

4. SIMULATION RESULTS

The simulation parameters for the single degree-of-freedom system are:

$$m = 0.75\times10^{-3}\,\mathrm{kg} \quad k = 107.4\,\mathrm{N/m}$$

so that the natural frequency of the system is 60.2 Hz. These values are realistic for a linear inductive generator of about 1 cm^3 as described in [14]. In this case, for simplicity, the stiffness is assumed to be linear.

4.1 Periodic time-varying damper

A periodic time-varying damper having a constant term c_0 and time-varying terms as a function of ωt and $2\omega t$ are initially considered, so that,

$$c(t) = c_0 + c_1\cos(\omega t) + c_2\cos(2\omega t) + d_1\sin(\omega t) + d_2\sin(2\omega t) \tag{18}$$

Numerical simulations are conducted with a sweep sine excitation with linearly increasing frequency in the range of 20 - 120 Hz with $Y = 10^{-4}\,\mathrm{m}$ to construct the frequency response of the system.

Figure 2 shows that expanding the Fourier series of the response $z(t)$ for $n = 2$ is sufficient to represent the dynamic response accurately. This could be justified by the fact that the 2ω terms are present in the damping Eq.(18). In this example $c_0 = 0.0710\mathrm{Ns/m}$, while $c_1 = c_2 = \text{-}0.0355\mathrm{Ns/m}$, and $d_1 = d_2 = 0$.

The average absorbed power and the throw are obtained using both the approximated analytical approach above and a time-domain numerical method (using ode45) and the results are also shown in Figure 2. The throw is defined using the amplitude response of the fundamental frequency. When including two terms in the response of the system, the approximated analytical harvested power and throw are in good agreement with the ones obtained from the numerical approach. Further increase of the number of terms in the response, for example to $n = 3$, does not significantly improve the accuracy of the results.

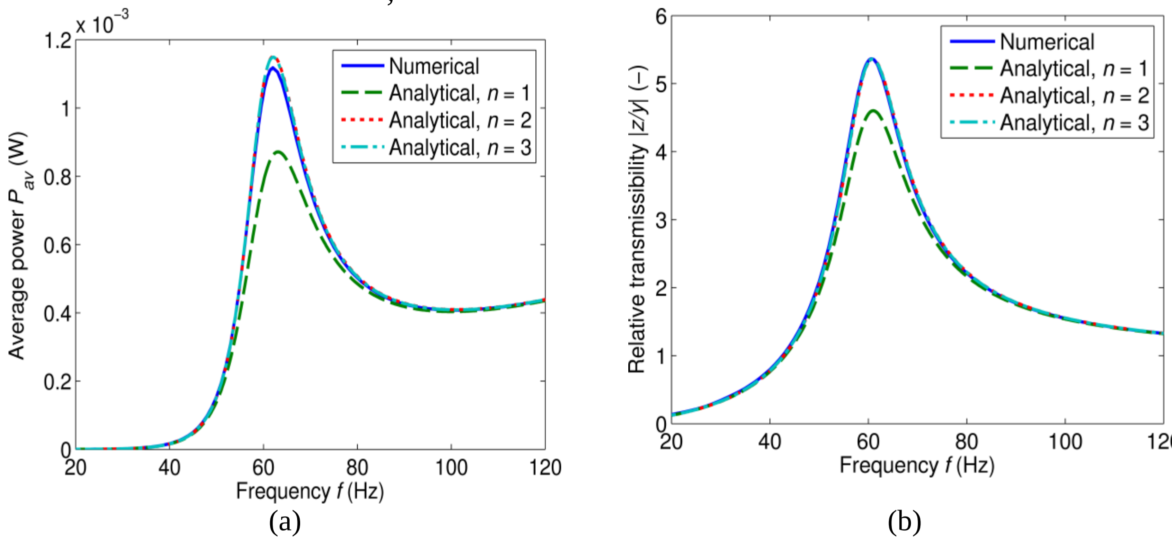


Figure 2. Time-periodic damper: (a) average absorbed power and (b) throw

An optimisation is now considered in order to obtain the parameters of the time-varying damper in Eq.(18), by maximising the average absorbed power. In this example, optimisation is carried out at a single frequency of 80 Hz; however, we can run the optimisation over the frequency range of interest. A genetic algorithm from Matlab optimisation toolbox is used to find the five unknown damping parameters c_0, c_1, c_2, d_1 and d_2. In the optimisation procedure, solutions, which give negative damping, are excluded. The optimum coefficients for the time-varying damper in Eq.(18) are shown in Figure 3 at 80 Hz.

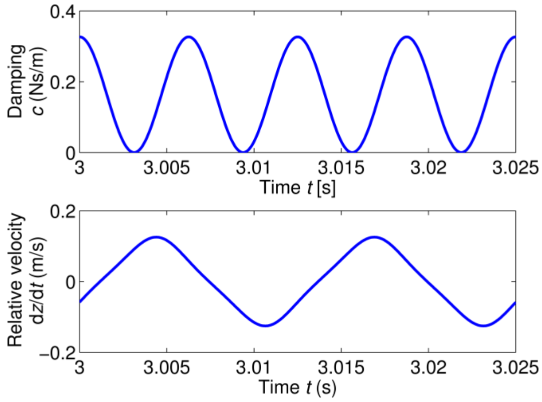


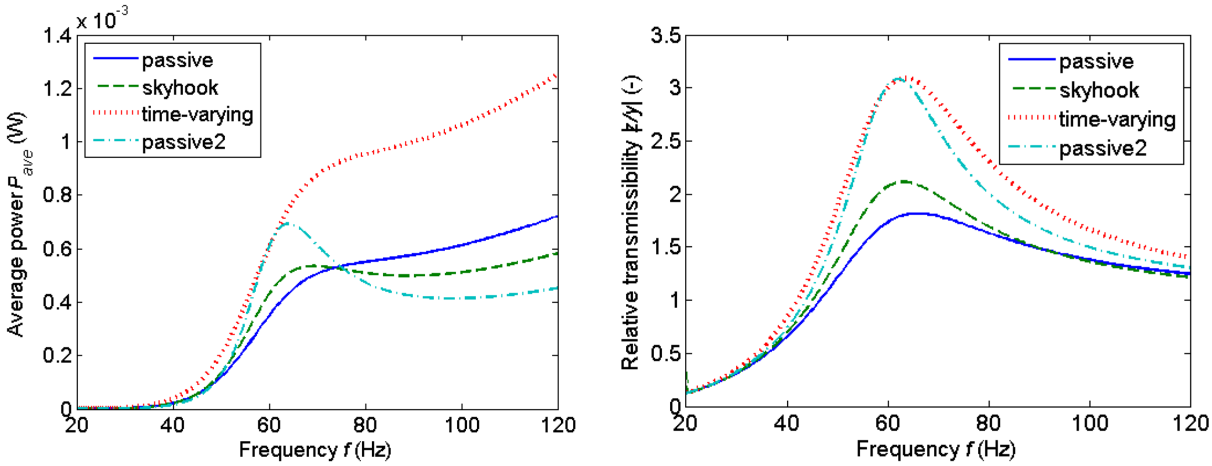
Figure 3. Optimum damping $c(t) = 0.1886(1 - \cos 2\omega t)$ and velocity waveforms at 80Hz

The performance of the optimum time periodic damper is compared with the optimum passive damper $c^* = 0.1633\text{Ns/m}$, with the passive damper having the same throw as the optimum time periodic damper at resonance $c = 0.093375\text{Ns/m}$ and the on-off skyhook semi-active models. The optimum damping coefficient for the passive model and the on-off switch models are provided in Table1. For the on-off skyhook semi-active model, the switch occurs between the two levels of damping when the sign of the product of the relative velocity and the absolute velocity changes.

Damper	Optimum Damper Coefficients	Average Harvested Power
Passive	$c^* = 0.1633\text{Ns/m}$	0.55 mW
Passive 2	$c = 0.093375\text{Ns/m}$	0.47 mW
On-off skyhook	$c^* = \begin{cases} c_{\max} = 0.13935\text{Ns/m} & \text{if } \dot{x}(t)\dot{z}(t) \geq 0 \\ c_{\min} = 0\text{Ns/m} & \text{if } \dot{x}(t)\dot{z}(t) < 0 \end{cases}$	0.51 mW
Periodic Time-varying	$c_0^* = 0.1886\text{Ns/m}$ $c_2^* = -c_0$ $c_1^* = d_1^* = d_2^* = 0\text{Ns/m}$	0.95 mW

Table 1. Optimal damping parameters for three different damping strategies at 80 Hz

Figure 4 compares the average harvested power and the throw for the four different optimised dampers. At 80 Hz, the average harvested power for these four strategies is also shown in Table 1. Although the on-off semi-active model is useful for minimizing the absolute displacement it is not suitable for energy harvesting. The time-varying damper has increased the average absorbed power more than 70% with respect to the optimum passive one. Although the optimisation is carried out at 80 Hz, the average absorbed power and the throw are increased over the whole range of frequency.

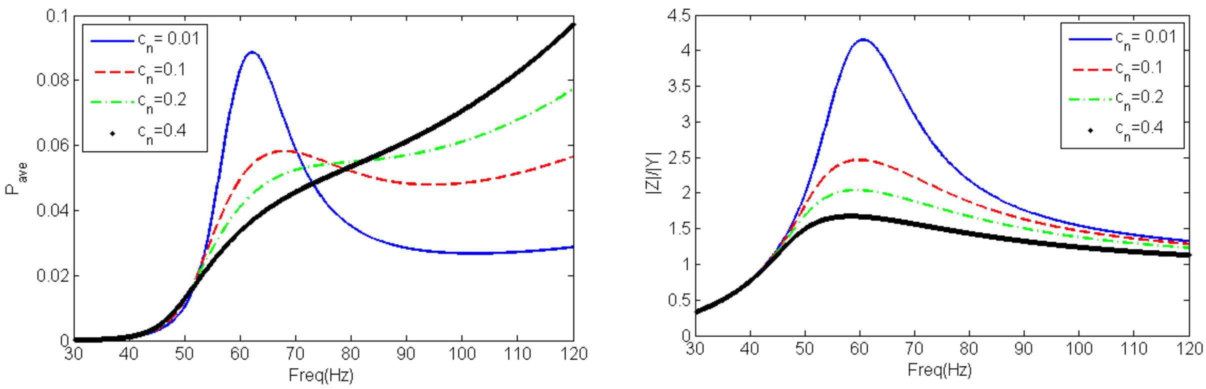


(a) (b)

Figure 4. The average absorbed power (a) and throw (b) for the optimum passive, passive with skyhook and time-varying damping models optimised at 80Hz as a function of excitation frequency

4.2 Nonlinear damper

The average absorbed power and the throw are plotted in Figure 5 for a harvester with base excitation amplitude of $Y = 10^{-3}\text{m}$, a linear damper $\text{c} = 0.05\text{Ns/m}$ and various values of the cubic dampers c_n .

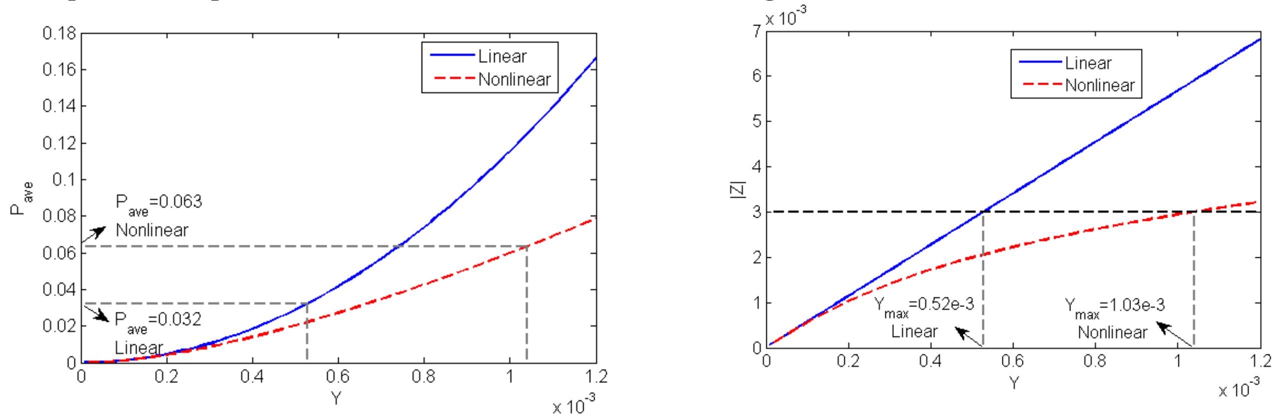


(a) (b)

Figure 5. The average absorbed power (a) and throw (b) as a function of excitation frequencies for the system with $\text{c} = 0.05\text{Ns/m}$ and various values of nonlinear damper c_n

The amplitude of the throw reduces when the damping is increased, as expected. At low frequencies, the increase of damping does not have much effect on the average harvested power. At resonance, the average harvested power reduces when the damping increases. At frequencies much higher from the resonance, the increase of damping however increases the harvested power, since the mass is almost stationary, and the average power harvested increases with the square of frequency.

Figure 6 shows the average absorbed power and the relative displacement magnitude as a function of the base excitation amplitude for the linear system, when $c_\text{n} = 0\text{Ns/m}$, and the nonlinear system, when $c = 0.05\text{Ns/m}$ $c_\text{n} = 0.05\text{Ns}^3/\text{m}^3$, at resonance. For a limited throw of $Z = 3\text{mm}$, the maximum base excitation amplitude for the linear system is 0.52mm. This is a larger value (1.03mm) for the nonlinear system. For the maximum base excitation amplitude in which the two devices are operational, the nonlinear harvester can harvest much more power compared to the linear harvester as shown in Figure 6.



(a) (b)

Figure 6. The average absorbed power (a) and relative displacement (b) for the system with linear and nonlinear damping when excited at resonance.

5. CONCLUSIONS

This paper has investigated a periodic time-varying damper for idealised energy harvester. When the system is harmonically excited, the semi-active control determines the damping of the system. An analytical solution is presented for the average absorbed power using the harmonic balance method. The optimal value of damping is obtained based on maximising the average absorbed power at a specific frequency. The average harvested power is also increased at other frequencies, even though the optimisation is considered for a single frequency, although the throw is also increased.

A nonlinear damper is also considered and the harvested power and the throw are derived using the describing function. It is demonstrated that for a limited throw, the nonlinear harvester can have higher amplitude of base excitation as well as can harvest much more power compared to the linear harvester.

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