A Low-Complexity Cross-Layer Algorithm for Coordinated Downlink Scheduling and Robust Beamforming Under a Limited Feedback Constraint

Yingxiang Yang, Bo Bai, *Member, IEEE*, Wei Chen, *Senior Member, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—Coordinated scheduling/beamforming (CS/CB) substantially mitigates the intercell interference (ICI), hence increasing the cell-edge throughput on the downlink (DL) of coordinated multipoint (CoMP) systems. To maximize the DL throughput, the cooperating base stations (BSs) jointly select the best set of users for DL scheduling and then jointly design a set of beamforming (BF) vectors to approach the throughput limit. However, finding the optimal BF vectors requires an exhaustive search and substantial channel state information (CSI) feedback, hence resulting in high algorithmic complexity and heavy uplink traffic load. Hence, we conceive a new cross-layer algorithm to achieve high performance at a lower feedback amount and at lower algorithmic complexity. Based on the fact that different BSs usually have different traffic loads, we divide the BSs into two different types, i.e., the master BSs (MBSs) and the slave BSs (SBSs), where MBSs have a higher transmission priority than SBSs. The scheduling relies on an interference threshold, whereas our robust BF scheme exploits both the channel direction information (CDI), which is quantized using the technique of limited feedback, and the channel quality information (CQI), which is assumed to be fed back accurately. Our numerical results show that the proposed algorithm does not lose much performance compared with that achieved by an exhaustive search, whereas the algorithmic complexity is as low as that of the solutions operating without CoMP.**

***Index Terms*—Author, please supply index terms/keywords for your paper. To download the IEEE Taxonomy go to http://www.ieee.org/documents/taxonomy_v101.pdf .**

AQ1

I. Introduction

COORDINATED multipoint (CoMP) transmission is a key feature in the Long-Term Evolution system [1]–[3], which promises performance improvements for the cell-edge users by allowing several base stations (BSs) to cooperate. On the uplink side, the cooperating BSs share the information and jointly process the data received from the mobile stations (MSs). On the downlink (DL) side, two commonly implemented methods are the joint processing and coordinated scheduling/beamforming (CS/CB) schemes, where CS/CB allows the BSs to cooperatively schedule their DL transmissions to a set of users and then cooperatively design a set of efficient beamforming (BF) vectors. Under the assumption that the users' channel state information (CSI) is perfectly known at the BSs' DL transmitters, the throughput performance of coordinated BF varies for different sets of scheduled users. Thus, the ultimate task for the BSs is to schedule their DL transmissions to the optimal set of users that are capable of approaching the maximum DL-transmission throughput and then to design the particular set of BF vectors that can approach this limit.

However, the problem described earlier has the following obstacles.

1) *High algorithmic complexity*. The algorithmic complexity imposed by finding the optimal set of MSs for which the DL transmissions should be scheduled is high since an exhaustive search is required for optimal scheduling. Assuming that there are M BSs and that the user set of the ith BS (BSi) is denoted by $\mathcal{U}_i$, the complexity of the scheduling algorithm is on the order of $\mathcal{O}(\prod_{i=1}^{M}|\mathcal{U}_i|)$. This complexity becomes excessive, when the number of BSs and MSs in each cell increases.
2) *High feedback load*. Assuming that the feedback "budget" of each MS's CSI is B bits for the Channel Direction Information (CDI) and b bits for the Channel Quality Information (CQI), the feedback traffic load can be expressed as $(\sum_{i=1}^{M}|\mathcal{U}_i|)(MB+b)$, where the MB bits of the CDI feedback are related to M channels, i.e., one for the specific channel receiving the desired signal and the remaining $(M-1)$ for the channel receiving the interfering signal.
3) *High backhaul traffic*. To calculate the set of optimal BF vectors, at least one of the BSs has to know the CSI of all the MSs. Thus, the backhaul traffic load is at least $(\sum_{i=1}^{M}|\mathcal{U}_i| - \max_i\{|\mathcal{U}_i|\})(MB+b)$.
4) *Inaccuracy caused by imperfect CSI feedback*. Since the CSI feedback introduces both quantization errors and latency, the CSI acquired at the BSs is inaccurate. Thus, it is possible that the DL-scheduling decision will be inaccurate when the quantization error is high.

Manuscript received August 7, 2012; revised February 5, 2013 and May 17, 2013; accepted June 22, 2013. This paper was supported in part by the National Basic Research Program of China (973 Program) under Grant 2013CB336600 and Grant 2012CB316000, by the U.K.–China Bridge Fellowship, by Chuanxin Funding, by the Beijing Nova Program, and by the Chinese Ministry of Education New Century Talent Program. The work of L. Hanzo was supported by the European Research Council under their Advanced Fellow Grant. The review of this paper was coordinated by Dr. J.-C. Chen.

Y. Yang was with Tsinghua University, Beijing 100084, China. He is currently with the Department of Electrical and Computer Engineering, Rutgers University, New Brunswick, NJ 08901 USA (e-mail: yangyx.thu@gmail.com).

B. Bai and W. Chen are with the Department of Electronic Engineering, Tsinghua University, Beijing, 100084, China (e-mail:eebobai@tsinghua.edu.cn; wchen@tsinghua.edu.cn).

L. Hanzo is with the School of Electronic and Computer Science, University of Southampton, Southampton, SO17 1BJ U.K. (e-mail: lh@ecs.soton.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2013.2271876

There has been a plethora of contributions related to CoMP [1], [4]–[15], exploring possible solutions and finding remedies to the impediments aforementioned. Although the original contributions relied on the assumption of perfect CSI [4], more realistic recent contributions assumed imperfect CSI feedback, where the channel vectors are quantized to a codeword stored in a codebook designed offline and the index of the codeword is fed back instead of the actual quantized values of the channel vectors. Hence, the amount of feedback per user can be substantially reduced. A comprehensive introduction to the topic of limited feedback aid communications can be found in [5], where the authors discussed the feedback design in a broad range of scenarios, employing methods used in industrial standards and protocols.

One of the common issues in realistic limited-feedback-aided systems is the inaccuracy of the CSI feedback both due to the delay encountered and by the transmission errors imposed by the feedback channels [6]–[10]. In [6], Wu and Lau proposed a feedback design for spatial-division multiple-access (SDMA) systems, demonstrating that their scheme is robust against feedback channel errors and characterized the system's goodput. Another contribution of Wu and Lau [7] provided two robust designs for multiple-input–multiple-output precoder adaptation under the scenario of potentially error-prone limited feedback and showed that both frameworks provided significant gains compared with the idealized designs assuming no feedback errors. In [8] and [9], the performances of equal gain transmission and precoded broadcast transmission were studied, respectively, under the scenario of error-prone limited feedback. Finally, in [10], Housfater and Lim derived a Cramér–Rao-type lower bound for linear precoders. These contributions provided insights into the mitigation of the detrimental impact of the imperfect CSI feedback channel.

Another common issue that arises when limited feedback is applied to a typical CoMP system is the codebook design problem. Although the design of codebooks conceived for limited-feedback-aided systems has been extensively studied [1], [11], the number of BSs in a CoMP cluster may vary over time, hence requiring a specific design. Thus, it is a challenge to design a codebook imposing low overhead when the number of cooperating BSs is high. A promising solution is based on the per-cell codebook design philosophy of [12]–[15], which separately quantizes the channel associated with each cell within a CoMP channel matrix to avoid a large codebook and to circumvent frequent updates of the codebook owing to either user mobility or due to the different clustering of the BSs. To elaborate a little further, Cheng *et al.* [12] presented a limited-feedback-based per-cell codebook design and showed that its performance is close to that of the conventional joint-cell codebook design having high overhead. In [13], attention is focussed on the problem of optimal per-cell codebook designs and derived a closed-form solution for the codebook size that minimizes the quantization error on average. In [14], a method of reconstructing the CoMP channel's CDI was first proposed and then, the performance of different codebook generation techniques and per-cell codeword selection methods was compared.

In contrast with the insightful contributions listed earlier, we pursue a different approach in reducing both the algorithmic complexity and the CSI feedback overhead for a scenario where CS/CB is employed.

1) We conceive a low-complexity noniterative cross-layer algorithm, which is based on the fact that, in multicell systems, all BSs tend to have different DL-transmission rate requirements and traffic loads. We commence by classifying the BSs into two types. The BSs having higher transmission rate requirements are referred to as master BSs (MBSs), which benefit from a higher priority. The remaining BSs having lower transmission rate requirements are referred to as slave BSs (SBSs), which have a lower priority.
2) We propose a low-complexity interference-threshold-based algorithm for scheduling, which is combined with appropriately adjusting the BF vectors of the cooperating BSs. As shown in Section III, this part of the algorithm only relies on the CSI at the user's side; thus, it is capable of effectively reducing the CSI feedback load while mitigating the inaccuracy of CSI feedback imposed by the error-prone feedback channel.
3) Furthermore, we propose a new robust BF vector design for the scenario, where the CDI and CQI are fed back separately. More explicitly, the CDI is quantized before being fed back, whereas the CQI is assumed to be perfectly fed back to the BS.
4) We will demonstrate both with the aid of our theoretical derivation and by numerical simulations that our algorithm has similar algorithmic complexity as the noncooperative algorithms. It imposes low backhaul traffic and circumvents the dynamic channel-matrix clustering of CoMP.
5) We also show that the performance of the proposed algorithm is not overly compromised and that its performance is similar to that of the iterative algorithm proposed in [16], as far as the MBS is concerned. Our algorithmic philosophy is highlighted in a simple scenario, where only two BSs are involved, but it may be readily extended to more general scenarios supporting multiple BSs without a dramatic increase in complexity and feedback requirements.

The remainder of this paper is organized as follows. In Section II, our system model is introduced, along with some of our basic assumptions. In Section III, both the proposed scheduling and our BF algorithm are detailed. In Section IV, we present a comparison of the algorithmic complexity and the signaling overhead of different algorithms. Our performance analysis is provided in Section V, whereas Section VI offers our conclusions. The proof of some of the theorems is provided in the Appendix.

II. System Model

Consider the two-cell network of Fig. 1, where each cell has a BS at its center and multiple MSs scattered within the cell. BS1 on the left of Fig.1 is assumed to be the MBS, and BS2 on the right is an SBS. Both BS1 and BS2 are equipped with K transmit antennas, whereas each MS has a single receive antenna. We assume that, each time, each BS schedules its DL

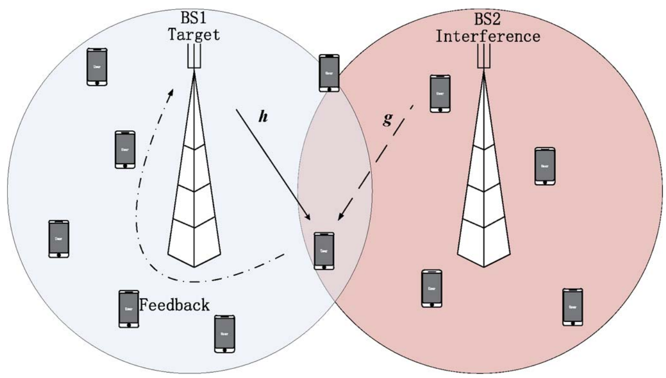

Fig. 1. System model.

transmission to a single MS within its cell, and different BSs use the same frequency; hence, the MSs suffer from intercell interference (ICI) imposed by the neighboring BS. The received signal power is related to the location of the MS. There are a number of studies on the fairness control issues of scheduling algorithms [17], [18], but in this paper, we focus our attention on the scenario where the MSs are located at the cell edge, and we assume that all the MSs have the same large-scale fading factor. In other words, the fairness effect of different large-scale fading factors is not considered here.

Again, we denote the user set of cell i ($i \in \{1, 2\}$) as $\mathcal{U}_i$. Then, the signal received by user j ($j \in \{1, 2, \ldots, |\mathcal{U}_1|\}$) in cell 1 and user $k (k \in \{1, 2, \ldots, |\mathcal{U}_2|\})$ in cell 2 can be expressed as

$$\begin{cases} y_{1,\,j} = p_{11,\,j}\langle \mathbf{w}_1, \mathbf{h}_{1,\,j}\rangle u_{1,\,j} + p_{21,\,j}\langle \mathbf{w}_2, \mathbf{g}_{1,\,j}\rangle u_{2,\,k} + n_j \\ y_{2,\,k} = p_{22,\,k}\langle \mathbf{w}_2, \mathbf{h}_{2,\,k}\rangle u_{2,\,k} + p_{12,\,k}\langle \mathbf{w}_1, \mathbf{g}_{2,\,k}\rangle u_{1,\,j} + n_k \end{cases} \tag{1}$$

where $\langle \mathbf{x}, \mathbf{y}\rangle$ represents the inner product of the vectors $\mathbf{x}$ and $\mathbf{y}$. Variable $y_{i,\,j}$ represents the signal received by user j in cell i, where i equals either 1 or 2. At the receiver of the jth user in cell i_1, the signal strength received from cell i_2 can be represented as $p_{i_1 i_2,\,j}$. The power of the symbol transmitted from BSi to its jth user is denoted $u_{i,\,j}$, which is normalized as $E\{|u_{i,\,j}|^2\} = 1$. The random variables n_j and n_k represent the normalized Gaussian noise, with $E\{|n_k|^2\} = E\{|n_j|^2\} = 1$, whereas vector $\mathbf{h}_{i,\,j} \in \mathbb{C}^{K\times 1}$ represents the DL channel conditions between BSi and its jth user, which can be viewed as the target channel condition. Furthermore, vector $\mathbf{g}_{i,\,j} \in \mathbb{C}^{K\times 1}$ denotes the DL channel condition between the jth user of BSi and the neighboring BS, which is the interferring channel. The target channel vectors and the interfering channel vectors are independent and identically distributed in terms of their statistics, and they both follow a probability distribution of $CN(0, \mathbf{I}_K)$. Finally, vector $\mathbf{w}_i$ is the BF vector adopted by BSi.

The goal of the proposed algorithm is to increase the DL throughput, which is quantified in terms of the channel capacity of

$$R_i = \log(1 + \text{SINR}_i) \tag{2}$$

where SINR_i is the signal-to-interference-plus-noise ratio at the scheduled user's terminal of BSi. We assume that each BS schedules its DL transmission to a single MS each time. Thus, to simplify the notation, we denote the target channel condition and the interfering channel condition of the scheduled MS located in cell i by $\mathbf{h}_i$ and $\mathbf{g}_i$, respectively, yielding

$$\text{SINR}_i = \frac{p_{ii}\|\mathbf{h}_i\|^2 \left|\langle \mathbf{w}_i, \tilde{\mathbf{h}}_i\rangle\right|^2}{1 + p_{ji}\|\mathbf{g}_i\|^2 \left|\langle \mathbf{w}_j, \tilde{\mathbf{g}}_i\rangle\right|^2}. \tag{3}$$

Note that, in the given equation, the norms of vectors $\mathbf{h}_i$ and $\mathbf{g}_i$ are separated from their directions so that we have $\|\tilde{\mathbf{h}}_i\| = \|\tilde{\mathbf{g}}_i\| = 1$. We use p_{ii} to denote the signal strength received by the selected user in cell i and p_{ji} to denote the strength of the interfering signal arriving from cell j contaminating the desired user's signal in cell i. We simplified the subscript since we only consider the case where each BS schedules its DL transmission to a single user at a time.

A. *Schemes Operating Without CoMP*

Transmission schemes operating without CoMP typically have lower complexity than those relying on CoMP. Here, we simply consider the classic time-division multiple-access (TDMA) and frequency-division multiple-access (FDMA) schemes. In the TDMA scheme, the M BSs transmit sequentially so that each BS transmits in $1/M$ fraction of the time, without imposing any ICI. In the FDMA scheme, on the other hand, the M BSs share the transmission bandwidth so that each BS transmits in a separate subband without being interfered by the neighboring BSs.

We compare the performance of the TDMA and FDMA schemes to that of our proposed algorithm and to the exhaustive search algorithm. In TDMA and FDMA schemes, all the MSs feed back both their CQI and their target channel conditions using limited feedback, so that the BSs can design BF vectors accordingly. The BSs, on the other hand, schedule their DL transmission for the specific MS having the highest CQI. The transmission throughput for BSi can be expressed as

$$R_i = \frac{1}{M}\log\left(1 + \max_{j\in\mathcal{U}_i} \text{SNR}_j\right) \tag{4}$$

where $M = 2$ in our scenario.

B. *Iterative Scheduling and Beamforming*

Iterative algorithms are capable of reducing the algorithmic complexity while maintaining a similar performance to their exhaustive-search-algorithm-based counterparts. In this paper, we adopt the iterative approach proposed in [16] as a benchmark for our performance evaluation, which relies on the following three steps.

1) Fix the power allocation and scheduled users; then, find the best combination of BF vectors.
2) Fix the BF vectors and power allocation; then, find the best set of users for scheduling.
3) Fix the BF vectors and the scheduled users; then, update the power allocation among the scheduled users.

Since the scenario that we study assumes fixed power allocation, the given three-step algorithm reduces to two steps in

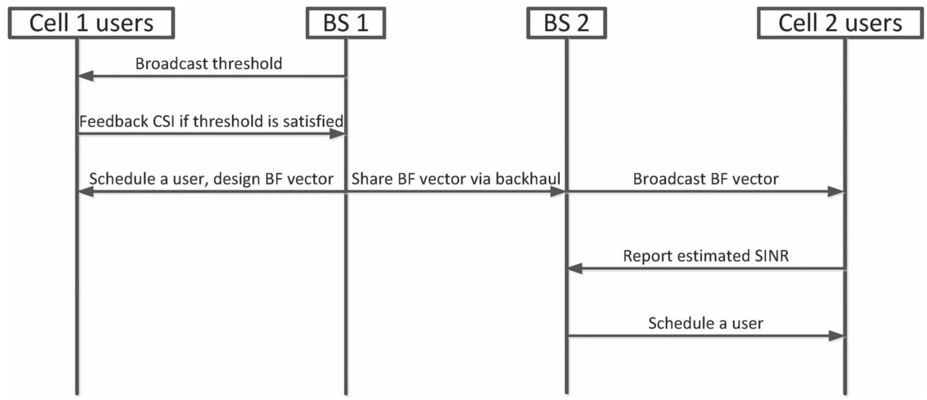

Fig. 2. Algorithm outline.

which the combination of BF vectors and the set of scheduled users is updated in an iterative fashion as follows.

1) Fix the set of scheduled users; then, find the optimal set of BF vectors correspondingly.
2) Fix the set of BF vectors; then, find the set of users that yield optimal throughput performance correspondingly.

The performance of the algorithm is characterized in Section V.

III. Cross-Layer Algorithm Under Limited Feedback Constraint

The algorithmic steps are shown in Fig. 2. In the first step, the MBS, i.e., BS1, broadcasts a threshold value constraining the channel directions of the desired users with respect to the interfering BS to maintain the target integrity. Explicitly, the directions of the desired user's target channel and of the same user's interfering channel should be perpendicular to each other, which corresponds to the absolute value of their inner product being close to zero. The users within cell 1 receive the threshold value and decide whether or not to feed their CSI back to their anchor BS. If a desired user's target channel and interfering channel satisfy the required angular separation threshold constraint, he/she feeds back the CDI of both the target channel and the interfering channel, as well as the CQI of both channels. Here, the CQI is defined as the product of $p_{ii}\|\mathbf{h}_i\|^2$ for the target channel and $p_{ji}\|\mathbf{g}_i\|^2$ for the interfering channel, when the user is located in cell i. Once the CSI of all the users that satisfy the angular separation constraint has been fed back to the MBS, the MBS decides which particular user to schedule for its DL transmission and then designs the BF vector, following our robust BF vector design method to be presented later. Once the BF vector is determined, it is shared with the SBS, i.e., BS2, via a backhaul link, the SBS broadcasts the BF vector, and all the users within cell 2 feed back their expected SINR values computed with the aid of the BF vector and their local CSI. In the final step, the SBS schedules its DL transmissions to the user having the highest "reported" SINR value.

The algorithm introduced relies on a few important assumptions, which are based on the following motivation.

1) *Introduction of the constraint* $\mathbf{w}_1 = \mathbf{w}_2$. In the original CS/CB problem, the BF vectors of different BSs do not necessarily have the same direction. In fact, if not shared via the backhaul link, the BF vector of a BS can be regarded as a random vector both for the other BSs and for all the MSs of the neighboring cells. There is no simple yet elegant way of effectively reducing the size of the candidate user set, but introducing the given constraint brings us obvious benefits. First, when we have $\mathbf{w}_1 = \mathbf{w}_2$, the local CSI can be directly used to compute the level of interference, which is now $\langle\tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_2\rangle$ for cell 1 and $\langle\tilde{\mathbf{h}}_2, \tilde{\mathbf{g}}_1\rangle$ for cell 2. This can assists us in exploiting the CSI at the MSs for naturally ruling out DL transmission to the MSs suffering from severe interference. As will be shown in Section IV, the feedback load is substantially reduced. Second, both the scheduling and BF parts of the algorithm can be implemented in each cell by relying merely on local CSI, which means that the backhaul traffic is effectively reduced. Additionally, the algorithmic complexity is also significantly cut down since only a small portion of the MBS's users perform CSI feedback, whereas the users of the SBS only feed their CQI back to the BS. These complexity and feedback requirements are similar to or even lower than those of some standard non-CoMP solutions, such as those of the TDMA and FDMA schemes.
2) *Introduction of the MBS and the SBS*. The related assumptions are based on the fact that, at each moment, it is likely that some BSs have a higher transmission rate requirement than the others; hence, they should be granted a higher priority and, ultimately, a higher transmission rate. Hence, an MBS can schedule its DL transmission to a user and design the BF vector with a higher priority, whereas the SBS cannot. The performance loss imposed by this unbalanced priority can be partially recovered when the number of users in the cells is high.

A. Threshold-Based User Scheduling

As shown in Fig. 2, the BSs schedule their DL transmissions according to a thresholding algorithm based on a carefully designed threshold. In our proposed algorithm, the MBS and the SBS have the same BF directions. Here, we focus on scheduling the DL transmissions to the specific MS, whose target channel direction $\tilde{\mathbf{h}}$ is "most different" from its interfering

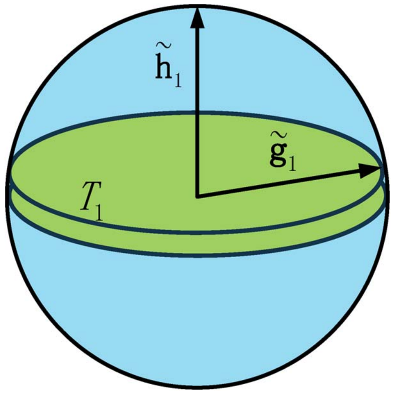

Fig. 3. Threshold.

channel direction $\tilde{\mathbf{g}}$ among all the users. More explicitly, we have to find the specific user, whose target channel direction and interfering channel direction have the smallest inner product absolute value across the entire set of users served by the MBS. However, it should be noted that the user to be scheduled for DL transmission in CoMP relying on an exhaustive search might not have an $\tilde{\mathbf{h}}$ perpendicular to $\tilde{\mathbf{g}}$. The thresholds can be defined as follows.

The MBS schedules the user whose channel conditions satisfy

$$\left|\langle \tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1 \rangle\right| \leq T_1 \tag{5}$$

where T_1 is the relevant threshold. An intuitive interpretation of the threshold is shown in Fig. 3, which shows (5) with the aid of T_1. Since the norms of both the BF vectors and of the channel directions of the MSs are 1, these vectors can be placed on a globe-like unit-radius hypersphere in $\mathbb{C}^K$, with one end at the origin and the other on the surface of the hypersphere. By assuming that the target channel direction $\tilde{\mathbf{h}}_1$ of the scheduled user points to the "north pole" of the globe, the interfering channel direction $\tilde{\mathbf{g}}_1$ will fall within the area bounded by the "Tropic," which is characterized by the value of T_1.

Intuitively, when the threshold T_1 becomes looser, i.e., when it approaches 1, more users will satisfy (5), and the complexity of the algorithm is increased. In particular, when we have $T_1 = 1$, all the users feed back their CSI, and the algorithm becomes identical to the exhaustive search for the MBS. By contrast, when T_1 approaches 0, the scheduling part of the algorithm will guarantee a higher transmission rate for the scheduled users, but it also comes more likely that no users satisfy (5), which leads to lower algorithmic complexity and reduced feedback load. Thus, the threshold controls the tradeoff between the algorithmic complexity and the desired performance; hence, it should be determined under the constraint of ensuring a minimum probability of at least one successful DL scheduling for the entire set of users. The selection of the threshold based upon the given principle can be achieved with the aid of the following theorem.

Theorem 1: Let us assume that there are N_1 users in cell 1. Then, for the MBS, the probability of a successful DL scheduling action can be expressed as

$$P_{\text{suc1}} = 1 - \left(1 - T_1^2\right)^{N_1(K-1)}. \tag{6}$$

Proof: See Appendix A. ■

The performance achieved with the aid of this threshold is characterized in Section IV.

B. Robust Beamforming Under Limited Feedback for the Channel Direction Information

According to Fig. 2, upon scheduling the DL transmission to a user whose channel conditions satisfy (5), the MBS adopts a BF vector that further improves the throughput. Since the MSs perform limited feedback of their channel conditions, when quantizing both $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ using a preset codebook and when transmitting the index of a codeword, the quantization error imposes inaccuracy on the design of BF vectors. We mitigate this impact using a robust BF technique, which maximizes the lowest possible SINR of the specific user selected. Numerous studies have been dedicated to robust BF [19]–[22]. Although the scenarios of these contributions are different, they all model the quantization error as an additive noise vector. For example, the target channel's channel vector of the selected user in cell 1 would be modeled as

$$\tilde{\mathbf{h}}_1 = \hat{\mathbf{h}}_1 + \mathbf{e} \tag{7}$$

where $\tilde{\mathbf{h}}_1$ represents the actual target channel direction, whereas $\hat{\mathbf{h}}_1$ is its quantized version, which is acquired from the user's feedback. Vector $\mathbf{e}$ in (7) represents the quantization error, which satisfies the ellipsoid constraint $\|\mathbf{e}\| \leq \varepsilon$. The quantization error for interfering channels can be defined similarly. The problem is then solved using convex optimization, and this technique is assumed to be known in this paper. This traditional way of designing robust BF vectors does not meet the assumptions stipulated in this paper. Earlier, we assumed that the norms of the channel directions are 1 both before and after quantization. This imposes more complex constraints on the description of the quantization error. Hence, we conceive a new technique of designing robust BF vectors for the scenario when the CDI and CQI are fed back separately.

We adopt the random-vector-quantization codebook concept [23] and use the model of [24] to analyze the quantization error, where the quantization codebook index of B bits is linked with the quantization error by

$$\left|\langle \hat{\mathbf{h}}_1, \tilde{\mathbf{h}}_1 \rangle\right| \geq 1 - 2^{\frac{-B}{K-1}}. \tag{8}$$

When $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_1$ are given, the problem of designing the robust BF vector can be broken down into two separate parts.

1) For each BF vector $\mathbf{w}$, find the set of $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ minimizing the SINR (denoted by SINR_1) for this $\mathbf{w}$.
2) Find the BF vector $\mathbf{w}$, which ensures that this minimized value of SINR_1 is maximized.

The whole idea can be formulated as

$$\mathbf{w}_{\text{opt}} = \arg\max_{\mathbf{w}} \left\{ \min_{\tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1} \left\{ \text{SINR}_1(\tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1 | \hat{\mathbf{h}}_1, \hat{\mathbf{g}}_1, \mathbf{w}) \right\} \right\}. \tag{9}$$

Again, an intuitive illustration is given in Fig. 4. Since $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ are independent of each other, the problem of finding the specific $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ that minimize the value of SINR for a given

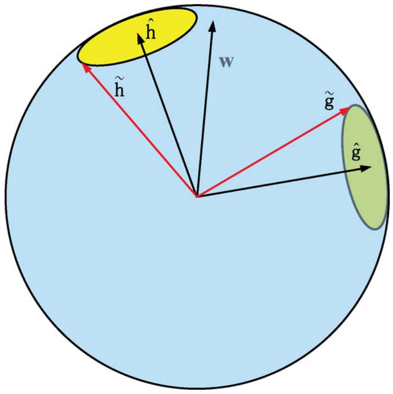

Fig. 4. Robust BF.

$\mathbf{w}$ is further simplified to finding $\tilde{\mathbf{h}}_1$ that minimizes $|\langle \mathbf{w}, \tilde{\mathbf{h}}_1\rangle|^2$ and finding $\tilde{\mathbf{g}}_1$ that maximizes $|\langle \mathbf{w}, \tilde{\mathbf{g}}_1\rangle|^2$. The following lemmas provide a way of finding these $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ vectors.

Lemma 1: Upon assuming that $|\langle \mathbf{w}, \hat{\mathbf{g}}_1\rangle| = \lambda_1$ and that the quantization error satisfies $|\langle \tilde{\mathbf{g}}_1, \hat{\mathbf{g}}_1\rangle| = \lambda_2 \geq \lambda_{2\min} \geq \lambda_1$, we have

$$|\langle \mathbf{w}, \tilde{\mathbf{g}}_1\rangle| \leq \lambda_1\lambda_{2\min} + \sqrt{(1-\lambda_1^2)\,(1-\lambda_{2\min}^2)}. \tag{10}$$

Proof: See Appendix B. ■

Lemma 2: Assuming that $|\langle \mathbf{w}, \hat{\mathbf{h}}_1\rangle| = \lambda_1$ and that the quantization error satisfies $|\langle \tilde{\mathbf{h}}_1, \hat{\mathbf{h}}_1\rangle| = \lambda_2 \geq \lambda_{2\min} \geq \sqrt{1-\lambda_1^2}$, we have

$$\left|\langle \mathbf{w}, \tilde{\mathbf{h}}_1\rangle\right| \geq \lambda_1\lambda_{2\min} - \sqrt{(1-\lambda_1^2)\,(1-\lambda_{2\min}^2)}. \tag{11}$$

Proof: See Appendix C. ■

In the given lemmas, $\lambda_{2\min} = 1 - 2^{(-B/K-1)}$ represents the maximum quantization error. With the aid of the given lemmas, we have the following theorem.

Theorem 2: Upon introducing the notation of

$$\mathbf{h}_1^{\dagger} = \frac{\hat{\mathbf{h}}_1 - \langle \hat{\mathbf{h}}_1, \hat{\mathbf{g}}_1\rangle \hat{\mathbf{g}}_1}{\left\|\hat{\mathbf{h}}_1 - \langle \hat{\mathbf{h}}_1, \hat{\mathbf{g}}_1\rangle \hat{\mathbf{g}}_1\right\|} \tag{12}$$

and $\hat{\mathbf{h}}_1 = \delta_1 \mathbf{h}_1^{\dagger} + \delta_2 \hat{\mathbf{g}}_1$, the optimal BF vector can be expressed as a linear combination of vectors $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_1$, which is formulated as

$$\mathbf{w}_{\text{opt}} = \xi_1 \mathbf{h}_1^{\dagger} + \xi_2 \hat{\mathbf{g}}_1 \tag{13}$$

where ξ_1 and ξ_2 are appropriately chosen so that their norms maximize the SINR in (14), shown at the bottom of the page, whereas the arguments of ξ_1 and ξ_2 satisfy the constraint $\arg(\delta_1^*\xi_1) = \arg(\delta_2^*\xi_2)$.

Proof: See Appendix D. ■

The given theorem provides a numerical technique of designing the optimal robust BF vector by finding the optimal value of $|\xi_2|$, which falls in the real-valued interval of [0, 1].

C. Extending the Algorithm to Multiple-BS Scenarios

Earlier, we developed a low-complexity cross-layer algorithm based on a scenario considering only two BSs. Let us now show that this algorithm can be readily extended to more general scenarios with multiple BSs involved.

Let us assume that there are M BSs. In this multi-BS extension of the algorithm, we still assume that there is a single MBS, which cooperates with multiple SBSs, and that all the BSs use the same BF vector. Additionally, since it is reasonable to assume that different BSs have to transmit independent messages to their scheduled users, we can treat the interference at the jth user of cell k received from multiple BSs as interference arriving from a single source associated with a channel vector of

$$\sum_{i=1,\, i\neq k}^{M} \mathbf{g}_{i,\,k,\,j} u_i \tag{15}$$

where $\mathbf{g}_{i,\,k,\,j}$ is the channel vector of the link spanning from the ith BS to the jth user in cell k, and u_i is the symbol transmitted to the scheduled user in cell i. Since the linear combination of isotropic random vectors is also isotropic, the threshold-based scheduling of Section III-A remains unchanged.

IV. Algorithmic Complexity and Signaling Overhead

Here, we characterize the complexity of four algorithms: the exhaustive search algorithm, the TDMA/FDMA scheme, the iterative algorithm of [16], and our proposed algorithm.

Consider the scenario associated with M BSs. The search for the BF vector of a single BS is based on a searching through an L-entry codebook, as implied in (14). Therefore, the exhaustive search algorithm has complexity of $\mathcal{O}(L^M \prod_{i=1}^{M} |\mathcal{U}_i|)$, where $|\mathcal{U}_i|$ is the supported number of users in the ith BS, because the algorithm will consider all possible user combinations to find the one that achieves the optimal performance, which is estimated based on an exhaustive search through the codebook of each BS for an optimal combination of BF vectors for each possible user combination.

The complexity of the iterative algorithm proposed in [16] cannot be readily determined without knowing the number of iterations needed for the convergence of the scheduling process. Here, we introduce parameter S, which represents

$$\text{SINR}_1 = \frac{p_{11}\|\mathbf{h}_1\|^2\left(\lambda_{2\min}\left(|\delta_1\xi_1| + |\delta_2\xi_2|\right) - \sqrt{(1-\lambda_{2\min}^2)\left(1-\left(|\delta_1\xi_1|+|\delta_2\xi_2|\right)^2\right)}\right)^2}{1 + p_{21}\|\mathbf{g}_1\|^2\left(|\lambda_{2\min}\xi_2| + \sqrt{(1-\lambda_{2\min}^2)\,(1-|\xi_2|^2)}\right)^2} \tag{14}$$

TABLE I
ALGORITHMIC COMPLEXITY

Algorithm	Exhaustive Search	Iterative	TDMA/FDMA	Proposed
Time(s)	1.34e3	1.50e1	2.15e-1	6.04e-1
Complexity	$\mathcal{O}\left(L^M \prod_{i=1}^{M} \lvert\mathcal{U}_i\rvert\right)$	$\mathcal{O}\left(S\left(L^M + \prod_{i=1}^{M} \lvert\mathcal{U}_i\rvert\right)\right)$	$\mathcal{O}\left(\sum_{i=1}^{M} \lvert\mathcal{U}_i\rvert\right)$	$\mathcal{O}\left(\lvert\mathcal{U}_1\rvert L + \sum_{i=2}^{M} \lvert\mathcal{U}_i\rvert\right)$

TABLE II
SIGNALING OVERHEAD

Algorithm		Exhaustive search	Iterative	TDMA/FDMA	Proposed	
					BS1 (MBS)	BSi (SBS)
CDI feedback per user		$2M$	$2M$	0	2	0
CQI feedback per user		$2M$	$2M$	1	2	1
Number of feedback users		$\lvert\mathcal{U}_i\rvert$	$\lvert\mathcal{U}_i\rvert$	$\lvert\mathcal{U}_i\rvert$	Greatly reduced	$\lvert\mathcal{U}_i\rvert$
Backhaul traffic	CDI exchange	$\sum_{i=1}^{M} M \lvert\mathcal{U}_i\rvert$	M^2	0	0	
	CQI exchange	$\sum_{i=1}^{M} M \lvert\mathcal{U}_i\rvert$	M^2	0	0	
	BF vector exchange	M	M	0	$M-1$	
	User identity exchange	M	M	0	0	

the maximum affordable number of iterations for the algorithm. Then, the resultant complexity may be expressed as $\mathcal{O}(S(L^M + \prod_{i=1}^{M} \lvert\mathcal{U}_i\rvert))$, which follows from the fact that, each time the BF vectors are updated, an exhaustive search of the codebooks is conducted, whereas an exhaustive search for the optimal user combination is carried out every time, when the combination is updated.

The complexity of the TDMA/FDMA algorithms is $\mathcal{O}(\sum_{i=1}^{M} \lvert\mathcal{U}_i\rvert)$, which follows from the fact that each BS operates on its own and only has to set the BF vector to the direction of the target channel of the scheduled user.

For the proposed algorithm, if we assume that BS1 is the MBS, the upper bound of the complexity is $\mathcal{O}(\lvert\mathcal{U}_1\rvert L + \sum_{i=2}^{M} \lvert\mathcal{U}_i\rvert)$. This is because the MBS only has a portion of its users performing feedback, and the SBSs only have to find the best users on their own by comparing their CQI feedback values.

In the following, we characterize the algorithmic complexity by monitoring the simulation time required for a single trial. We conducted 1000 trials on all the algorithms aforementioned, and in Table I, we listed the time required for each single trial for CDI codebook sizes spanning from 2 to 15. As we can see, the exhaustive search algorithm and the iterative algorithm require significantly more time than the noncooperative algorithm and the proposed algorithm.

Finally, we compare the signaling overhead of the four given algorithms in Table II, which includes the overhead generated from users' feedback and the backhaul traffic. It is observed in Table II that the exhaustive search algorithm and the iterative algorithm require the same amount of feedback overhead, except that the iterative algorithm generates less backhaul traffic when the number of iterations is small. The overhead generated, particularly the backhaul traffic, is enormous compared with the noncooperative TDMA/FDMA schemes.

The proposed algorithm, on the other hand, effectively controls the overhead generated from the CSI feedback and backhaul traffic. Since the BF vectors are set to the same direction, each user can view its received signals as if they were sent from only two sources, i.e., the destination and a single interfering source. As a benefit, each user only has to feed back two sets of channel information. Furthermore, the number of users generating feedback for the MBS is substantially reduced. A theoretical expression for this number is hard to derive. Nonetheless, we can determine with the aid of numerical simulations that, when there are 20 users in each cell and the successful transmission ratio is set to 99.9%, the number of feedback users in the MBS is, on average, 5.7. Meanwhile, the proposed algorithm only requires the sharing of the BF vector selected by the MBS, which is almost negligible compared with the backhaul traffic generated by the iterative algorithm.

V. NUMERICAL RESULTS

Here, we first characterize the properties of threshold T_1, and then quantify the performance of our proposed algorithm.

A. Threshold Value

According to (5) and (6), the "Tropic" T_1 controls the trade-off between the feedback load, algorithmic complexity, and the probability of a successful DL scheduling action. From (6), T_1 is also a function of the number of transmit antennas and the number of active users within the cell.

Fig. 5 shows the relation between the number of users and the value of T_1, where the number of transmit antennas was set to 2. Parameters p_{ii} and p_{ji} of both the MBS and the SBS are set to 30 dB, whereas the probability of success ranged from 10% to 99.9%. We can see in Fig. 5 that, when the probability of success decreases, the threshold becomes stricter, and when the number of users increases, the value of T_1 approaches 0. This is a manifestation of multiuser diversity since we are more likely to have a user with better channel conditions when the number of users becomes larger.

Fig. 6 shows the relationship between the probability of successful DL scheduling and the percentage of reduction in the CDI feedback per user. The numbers of users in both cells range from 10 to 40, whereas the number of transmit antennas

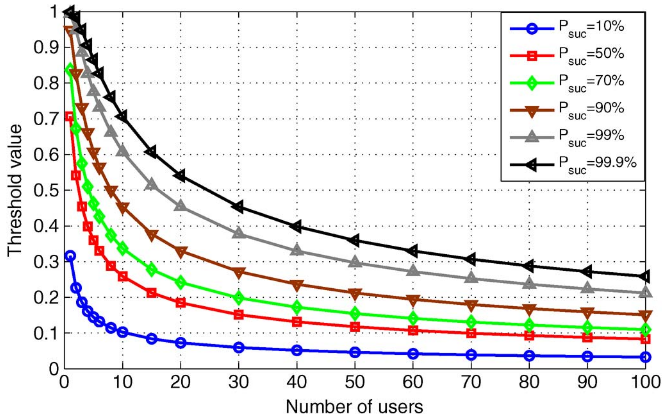

Fig. 5. Threshold value.

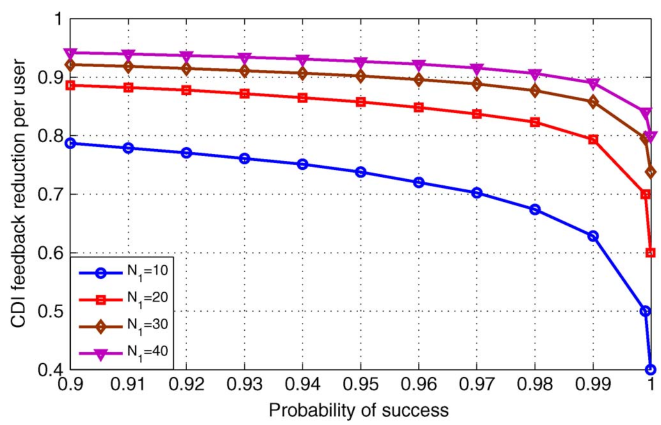

Fig. 6. Feedback load reduction per user.

remains 2, and the values p_{ii} and p_{ji} of both the MBS and the SBS remain 30 dB. Note that, in traditional TDMA and FDMA schemes, all the users feed back their CSI; hence, the feedback load per user is one CDI, where the number of bits is determined by the codebook size. In CS/CB relying on exhaustive search, the feedback load per user is two CDI times the number of codebook index bits. In our proposed algorithm, however, the feedback load per user is only 0.3 CDI, when the number of users is 20 in both cells and the probability of success is 99.9%. This implies that, compared with TDMA and FDMA schemes, the threshold T_1 allows us to rule out 70% of feedback without undue degradation of the probability of success. Additionally, it is possible to achieve an even lower feedback load by reducing the probability of a successful DL scheduling, when the number of users is given, but an excessive degradation of the probability of success may ultimately impose throughput degradation.

B. Throughput Performance

Fig. 7 quantifies the throughput of the MBS relying on the proposed algorithm in comparison to both the exhaustive search scheme and the traditional TDMA and FDMA schemes. The iterative algorithm proposed in [16] is used as a benchmark. *The number of users in both cells was set to 20, whereas the parameters* p_{ii} and p_{ji} *of both the target BS* and *of the interfering BS were set to 30 dB, which was attenuated by the path loss*. The number of transmit antennas was two, and the probability of DL scheduling success was set to 99.9%. As shown in Fig. 7, the performance of the MBS does not substantially degrade, and it is similar to the performance of the iterative algorithm proposed in [16]. The discrepancy between the exhaustive search algorithm and our proposed algorithm is a result of striking a tradeoff between the algorithmic complexity, overhead, and performance. Again, the discrepancy between the iterative algorithm and the exhaustive search algorithm is contributed by both the imperfect CSI feedback and by the fact that the iterative algorithm cannot guarantee consistent convergence to the global optimum. We also note that the left-hand part of the curves is not as smooth as their right-hand part. When the size of the codebook is 2 bits, the iterative algorithm even yields a slightly better performance than the exhaustive search algorithm. This, however, does not mean that the iterative algorithm performs in general better than the exhaustive search algorithm. This phenomenon is due to the large quantization errors of the CDI feedback. While the exhaustive search algorithm did guarantee the maximization of the minimum throughput of the scheduled users, the maximization of the actual performance is evaluated with the aid of the actual channel conditions encountered by the scheduled users. Hence, we conclude that the better performance of the iterative algorithm is a result of the large quantization errors imposed by the limited codebook size of the CDI feedback. This phenomenon does not occur when the codebook size is large.

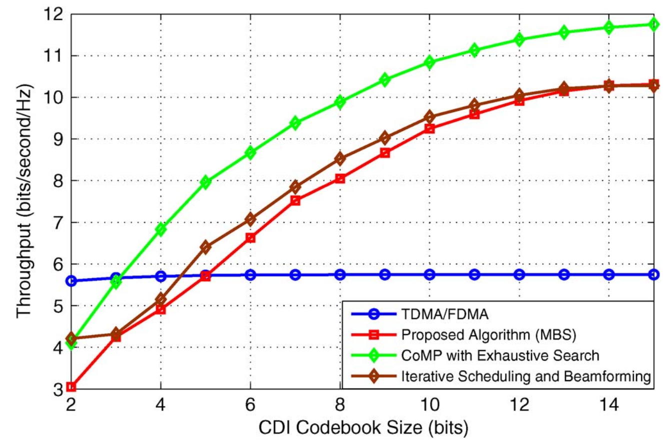

Fig. 7. DL throughput performance of the MBS.

Furthermore, observe in Fig. 7 that when the CDI codebook size is small, the performance of cooperative BF is similar to or even lower than that of the noncooperative TDMA and FDMA schemes. This phenomenon raises the question as to what is the minimum amount of feedback required by cooperative transmission and scheduling schemes to outperform their traditional noncooperative counterparts. This question is studied in detail in [15].

Fig. 8 shows the performance comparison of an SBS and the performance of a regular BS running under TDMA/FDMA schemes. The codebook size is set to 15 in this case, whereas the performances with other codebook sizes are similar. It can be easily seen that the performance of the SBS is sometimes not as good as the regular BS but becomes better as the number of users increases. This is caused by the multiuser diversity effect. It can be also observed that the intersection of the

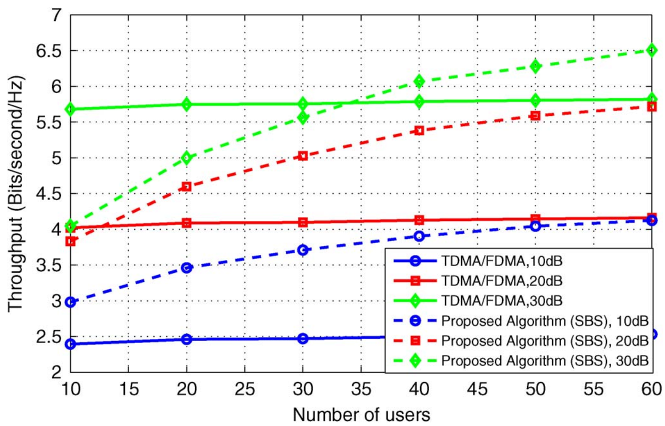

Fig. 8. Performance comparison between an SBS and a BS running under TDMA/FDMA schemes.

curves of the performances of the SBS and the regular BS shifts toward right as the SNR increases, which means that at high SNR levels, the proposed scheme will need more users to beat the performance of the TDMA/FDMA schemes. This phenomenon can be interpreted by the suboptimality of the BF vector. For a small amount of the given users, the effect of BF is dominating when SNR level is low. However, as SNR increases, the suboptimality of the direction of the BF vector becomes a major constraint to improve the performance, and the proposed algorithm is surpassed by the TDMA/FDMA schemes gradually.

VI. Conclusion

In this paper, we have conceived a low-complexity cross-layer algorithm for DL CoMP, which promises a good performance for the MBS while significantly reducing both the amount of feedback and the algorithmic complexity. The scheduling scheme of the proposed algorithm efficiently exploited the knowledge of CSI at the receiver. For the BF part of the algorithm, we provided a new technique of designing robust BF vectors, when the CDI and CQI of the users are fed back to the BSs separately. Our numerical results demonstrated that our solution only moderately degraded the performance of the potentially excessive-complexity exhaustive search technique, despite having complexity as low as that of a conventional system operating without CoMP. We next present the proofs of the theorems stated earlier.

Appendix A
Proof of Theorem 1[1]

For the MBS, denote the channel directions of the scheduled user as $\tilde{\mathbf{h}}_1 = [h_1, h_2, \ldots, h_K]^T, \tilde{\mathbf{g}}_1 = [g_1, g_2, \ldots, g_K]^T$. Since $\mathbf{h}_1 \sim CN(0, \mathbf{I}_K)$, it follows that $[\mathrm{Re}(h_i), \mathrm{Im}(h_i)]^T \sim N(0, (1/2\mathbf{I}_2)$ and $[\mathrm{Re}(g_i), \mathrm{Im}(g_i)]^T \sim N(0, (1/2)\mathbf{I}_2)$. Thus, the random vector $\tilde{\mathbf{h}}_1$ spanning the complex space $\mathbb{C}^K$ equals a random vector confined to the real space, which can be formulated as $\bar{\mathbf{h}}_1 \in \mathbb{R}^{2K}$. The real random vector $\bar{\mathbf{h}}_1$ obeys the normal distribution of $N(0, (1/2)\mathbf{I}_{2K})$.

According to Section III-A, the goal of the scheduling algorithm is to find the specific user, whose channel directions $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ are "most different" from each other. Assuming $|\langle \tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1 \rangle| = |\cos\theta|$, the probability density function of $|\cos\theta|$ can be expressed as

$$f\left(|\cos\theta|\right) = f\left(\left|\sum_{i=1}^{K} h_i g_i^*\right|\right) = f\left(\sqrt{A_3^2 + A_4^2}\right) \tag{16}$$

where $f(\cdot)$ denotes the probability density function of any random variable or random vector, and

$$A_3 = \sum_{i=1}^{K} \left[\mathrm{Re}(h_i)\mathrm{Re}(g_i) + \mathrm{Im}(h_i)\mathrm{Im}(g_i)\right] \tag{17}$$

$$A_4 = \sum_{i=1}^{K} \left[\mathrm{Re}(h_i)\mathrm{Im}(g_i) - \mathrm{Re}(g_i)\mathrm{Im}(h_i)\right]. \tag{18}$$

Now, if we define three new random vectors in the set of $\mathbb{R}^{2K}$

$$\bar{\mathbf{h}}^{(1)} = [\mathrm{Re}(h_1), \ldots, \mathrm{Re}(h_K), \mathrm{Im}(h_1), \ldots, \mathrm{Im}(h_K)]^T$$
$$\bar{\mathbf{g}}^{(1)} = [\mathrm{Re}(g_1), \ldots, \mathrm{Re}(g_K), \mathrm{Im}(g_1), \ldots, \mathrm{Im}(g_K)]^T$$
$$\bar{\mathbf{g}}^{(2)} = [\mathrm{Im}(g_1), \ldots, \mathrm{Im}(g_K), -\mathrm{Re}(g_1), \ldots, -\mathrm{Re}(g_K)]$$

then, (16) can be further simplified as

$$f\left(|\cos\theta|\right) = f\left(\sqrt{\left|(\bar{\mathbf{h}}^{(1)})^T\bar{\mathbf{g}}^{(1)}\right|^2 + \left|(\bar{\mathbf{h}}^{(1)})^T\bar{\mathbf{g}}^{(2)}\right|^2}\right). \tag{19}$$

Since we have $|(\bar{\mathbf{g}}^{(1)})^T\bar{\mathbf{g}}^{(2)}| = 0$, there exists $(2K-2)$ real-valued vectors with unit norms of $\bar{\mathbf{g}}^{(3)}, \ldots, \bar{\mathbf{g}}^{(2K)}$, which are orthogonal to each other, including $\bar{\mathbf{g}}^{(1)}$ and $\bar{\mathbf{g}}^{(2)}$. Thus, by letting $\mathbf{M} = [\bar{\mathbf{g}}^{(1)}, \bar{\mathbf{g}}^{(2)}, \ldots, \bar{\mathbf{g}}^{(2K)}]$, we have $|(\bar{\mathbf{h}}^{(1)})^T\mathbf{M}\mathbf{M}^T\bar{\mathbf{g}}^{(1)}|^2 = \mathrm{Re}(h_1)^2$ and $|(\bar{\mathbf{h}}^{(1)})^T\mathbf{M}\mathbf{M}^T\bar{\mathbf{g}}^{(2)}|^2 = \mathrm{Im}(h_1)^2$; thus

$$f\left(|\cos\theta|\right) = f\left(\sqrt{\mathrm{Re}(h_1)^2 + \mathrm{Im}(h_1)^2}|\bar{\mathbf{g}}^{(1)}\right) \ f\left(\bar{\mathbf{g}}^{(1)}\right). \tag{20}$$

This means that the value of $|\cos\theta|^2$ equals the sum of the squares of the two coordinates of $\bar{\mathbf{h}}^{(1)}$ along the two orthogonal dimensions. Additionally, note that the direction of the random vector $\bar{\mathbf{h}}^{(1)}$ is isotropic [26], which implies that the probability density function of $\tilde{\mathbf{h}}_1$ on the surface of the $2K$-D hypersphere with unit radius is $1/S_{2K}$, where we have $S_{2K} = 2K\pi^K/\Gamma(1+K)$. If we define $S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2 \le T_1^2}$ and $S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2 = r^2}$ to be the surface area of the hypersphere satisfying the constraint described in the subscript, the probability that a specific user's channel directions satisfy the threshold constraint denoted by P_1 can be expressed as

$$P_1 = P\left(\sqrt{\mathrm{Re}(\mathrm{h}_1)^2 + \mathrm{Im}(\mathrm{h}_1)^2} \le T_1\right) = \frac{S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2 \le T_1^2}}{S_{2K}}. \tag{21}$$

[1]We discovered that similar results are derived in [25], where the authors directly computed the surface area of the unit hypersphere and spherical cap in complex space. The absolute surface area is not the same when considered in a K-D complex space and a $2K$-D real space, but the resulting probability is the same.

By exploiting

$$\begin{aligned} &S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2\le T_1^2} \\ &= \int_0^{T_1} S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2=r^2} dr \\ &= \int_0^{\arcsin T_1} \left[\frac{2(K-1)\pi^{K-1} r^{2K-3}}{\Gamma(K)}\right]_{r=\cos\theta} 2\pi \sin\theta d\theta \\ &= \frac{2\pi^K \left(1-\left(1-T_1^2\right)^{K-1}\right)}{\Gamma(K)} \end{aligned} \tag{22}$$

we arrive at

$$P_1 = 1-\left(1-T_1^2\right)^{K-1}. \tag{23}$$

Since there are N_1 users in the cell, the probability that there is at least one user that satisfies the threshold constraint can be expressed as

$$P_{\mathrm{suc}1} = 1-(1-P_1)^{N_1}. \tag{24}$$

With the aid of (23), we finally have

$$P_{\mathrm{suc}1} = 1-\left(1-T_1^2\right)^{N_1(K-1)}. \tag{25}$$

Appendix B
Proof of Lemma 1

Let

$$\mathbf{w}_1 = \frac{\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{g}}_1\rangle\hat{\mathbf{g}}_1}{\|\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{g}}_1\rangle\hat{\mathbf{g}}_1\|}. \tag{26}$$

Then, $\mathbf{w}_1$ is orthogonal to $\hat{\mathbf{g}}_1$, and it can be expressed as a linear combination of $\hat{\mathbf{g}}_1$ and $\mathbf{w}$. Thus, we can assume that

$$\mathbf{w} = \varphi_1\mathbf{w}_1+\varphi_2\hat{\mathbf{g}}_1 \tag{27}$$

$$\tilde{\mathbf{g}}_1 = \theta_1\mathbf{w}_1+\theta_2\hat{\mathbf{g}}_1+\theta_3\mathbf{u} \tag{28}$$

where vector $\mathbf{u}$ is an arbitrary vector with unit norm and orthogonal to both $\hat{\mathbf{g}}_1$ and $\mathbf{w}$. Additionally, we should also note that, in the given equations

$$|\varphi_1|^2+|\varphi_2|^2 = |\theta_1|^2+|\theta_2|^2+|\theta_3|^2 = 1. \tag{29}$$

Thus, we have

$$|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle| = |\varphi_1\theta_1^*+\varphi_2\theta_2^*| \le |\varphi_1\theta_1|+|\varphi_2\theta_2| \tag{30}$$

where the equality on the right side holds if and only if we have

$$\arg\left(\varphi_1\theta_1^*\right) = \arg\left(\varphi_2\theta_2^*\right). \tag{31}$$

According to (28), we have

$$|\langle\tilde{\mathbf{g}}_1,\hat{\mathbf{g}}_1\rangle| = |\theta_2| = \lambda_2 \ge \lambda_{2\min} \ge \lambda_1 = |\varphi_2|. \tag{32}$$

We can see from the given equation that the maximum value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ is achieved when θ_3 is zero. If not, we can force θ_3 to zero and multiply both θ_1 and θ_2 by a factor of $1/\sqrt{1-|\theta_3|^2}$. According to (30), the value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ increases. Since $|\theta_2|$ is always larger than $|\varphi_2|$, according to Chebyshev's inequality, we can see that increasing the value of $|\varphi_2|$ will result in a monotonic increase in the value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$. Thus, the maximum value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ is achieved, when $\lambda_2=\lambda_{2\min}$. This means that the maximum value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ is achieved, when the actual channel direction falls on the quantization cell boundary between the BF vector and the unit vector representing the quantized channel direction.

Appendix C
Proof of Lemma 2

Lemma 2 can be proven in the same way as Lemma 1. Let

$$\mathbf{w}_1 = \frac{\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{h}}_1\rangle\hat{\mathbf{h}}_1}{\|\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{h}}_1\rangle\hat{\mathbf{h}}_1\|}. \tag{33}$$

Then, vector $\mathbf{w}_1$ is orthogonal to $\hat{\mathbf{h}}_1$, and it can be expressed as a linear combination of $\mathbf{w}$ and $\hat{\mathbf{h}}_1$. Thus, we can assume that

$$\mathbf{w} = \varphi_3\mathbf{w}_1+\varphi_4\hat{\mathbf{h}}_1 \tag{34}$$

$$\tilde{\mathbf{h}}_1 = \theta_4\mathbf{w}_1+\theta_5\hat{\mathbf{h}}_1+\theta_6\mathbf{u} \tag{35}$$

where vector $\mathbf{u}$ is an arbitrary vector with a unit norm and orthogonal to both $\hat{\mathbf{h}}_1$ and $\mathbf{w}$. Since the norms of both $\mathbf{w}$ and $\tilde{\mathbf{h}}_1$ are 1, it easily follows that

$$|\varphi_3|^2+|\varphi_4|^2 = |\theta_4|^2+|\theta_5|^2+|\theta_6|^2 = 1. \tag{36}$$

Thus, we have

$$|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle| = |\varphi_3\theta_4^*+\varphi_4\theta_5^*| \ge \|\varphi_3\theta_4|-|\varphi_4\theta_5\|. \tag{37}$$

It is clear from (34) and (35) that $|\varphi_4|=|\lambda_1|$ and $|\theta_5|=\lambda_2$. Since $\lambda_2\ge\sqrt{1-\lambda_1^2}$, we have $\lambda_1\ge\sqrt{1-\lambda_2^2}$. Thus

$$\|\varphi_3\theta_4|-|\varphi_4\theta_5\| = \lambda_1\lambda_2-\sqrt{\left(1-\lambda_1^2\right)\left(1-\lambda_2^2-|\theta_6|^2\right)}. \tag{38}$$

In the given equation, we first observe that when $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is minimized, $|\theta_6|$ has to be zero. This is obvious since, for a given λ_1, if we hold λ_2 at a fixed value smaller than 1, increasing the value of $|\theta_6|$ will result in a decrease in $\sqrt{(1-\lambda_1^2)(1-\lambda_2^2-|\theta_6|^2)}$, and if the value of $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is minimized, $|\theta_6|$ increases. Additionally, when $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is minimized, $|\theta_6|$ is minimized, and $\lambda_2=\lambda_{2\min}$. This is because when λ_2 decreases, $\lambda_1\lambda_2$ decreases, and $\sqrt{(1-\lambda_1^2)(1-\lambda_2^2-|\theta_6|^2)}$ increases at the same time.

With the given observations, the lemma is proven. Note that this lemma tells us that the minimum of $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is achieved when the actual channel direction $\tilde{\mathbf{h}}_1$ is away from both the BF vector $\mathbf{w}$ and the quantized channel direction $\hat{\mathbf{h}}_1$ and falls on the quantization cell boundary.

$$\mathrm{SINR}_1 = \frac{p_{11}\|\mathbf{h}_1\|^2\left(\lambda_{2\min}|\xi_5| - \sqrt{\left(1-\lambda_{2\min}^2\right)\left(1-|\xi_5|^2\right)}\right)^2}{1 + p_{21}\|\mathbf{g}_1\|^2\left(|\lambda_{2\min}\xi_2| + \sqrt{\left(1-\lambda_{2\min}^2\right)\left(1-|\xi_2|^2\right)}\right)^2} \tag{45}$$

APPENDIX D
PROOF OF THEOREM 2

Let

$$\mathbf{g}_1^\dagger = \frac{\hat{\mathbf{g}}_1 - \langle \hat{\mathbf{g}}_1, \hat{\mathbf{h}}_1\rangle \hat{\mathbf{h}}_1}{\|\hat{\mathbf{g}}_1 - \langle \hat{\mathbf{g}}_1, \hat{\mathbf{h}}_1\rangle \hat{\mathbf{h}}_1\|} \tag{39}$$

where $\mathbf{h}_1^\dagger$ is orthogonal to $\hat{\mathbf{g}}_1$ and can be expressed as a linear combination of $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$. Vector $\mathbf{g}_1^\dagger$ is orthogonal to $\hat{\mathbf{h}}_1$ and can be also expressed as a linear combination of $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$. Assuming that vector $\mathbf{u}_1$ is an arbitrary vector with a unit norm and orthogonal to both $\mathbf{h}_1^\dagger$ and $\hat{\mathbf{h}}_1$ and that vector $\mathbf{u}_2$ is an arbitrary vector with a unit norm and orthogonal to both $\mathbf{g}_1^\dagger$ and $\hat{\mathbf{g}}_1$, we have

$$\mathbf{w} = \xi_1\mathbf{h}_1^\dagger + \xi_2\hat{\mathbf{g}}_1 + \xi_3\mathbf{u}_1 = \xi_4\mathbf{g}_1^\dagger + \xi_5\hat{\mathbf{h}}_1 + \xi_6\mathbf{u}_2 \tag{40}$$
$$\hat{\mathbf{h}}_1 = \delta_1\mathbf{h}_1^\dagger + \delta_2\hat{\mathbf{g}}_1 \tag{41}$$
$$\hat{\mathbf{g}}_1 = \delta_3\mathbf{g}_1^\dagger + \delta_4\hat{\mathbf{h}}_1. \tag{42}$$

In the given equations, since the norms of $\mathbf{w}$, $\hat{\mathbf{h}}_1$, and $\hat{\mathbf{g}}_1$ are all 1, it follows that

$$\sum_{i=1}^{3}|\xi_i|^2 = \sum_{i=4}^{6}|\xi_i|^2 = 1 \tag{43}$$
$$|\delta_1|^2 + |\delta_2|^2 = |\delta_3|^2 + |\delta_4|^2 = 1. \tag{44}$$

Thus, with the aid of (40) to (44) and the two lemmas, for a given set of $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$, the minimum of SINR_1 takes the form of (45), shown at the top of the page. Additionally, we also have

$$|\xi_5| = \left|\langle \mathbf{w}, \hat{\mathbf{h}}_1\rangle\right| = |\xi_1\delta_1^* + \xi_2\delta_2^*| \le |\xi_1\delta_1| + |\xi_2\delta_2|. \tag{46}$$

When the minimum SINR in (45) is maximized, the equality on the right side of (46) holds. This is because, when $|\xi_5|$ increases, the numerator of (45) increases, and increasing the value of $|\xi_5|$ can be achieved by changing only the principles of ξ_1 and ξ_2, which will not affect the value of the denominator in the equation. Additionally, we observe that, when the minimum SINR in (45) is maximized, we have $|\xi_3| = 0$. The proof exploits that, if the maximum value of this SINR_1 is achieved when the BF vector $\mathbf{w}$ is not on the same complex plane with both $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$, we have $|\xi_3\xi_6| \ne 0$. In this case, we can hold $|\xi_2|$ at a fixed value, and set ξ_3 to 0. This will result in an increase in $|\xi_1|$, and since

$$|\xi_5| = |\delta_1\xi_1| + |\delta_2\xi_2| \tag{47}$$

the value of $|\xi_5|$ increases, resulting in an increase in the value of SINR_1. Upon combining (45) and (47), we arrive at (14); hence, the theorem is proven.

REFERENCES

[1] M. Sawahashi, Y. Kishiyama, A. Morimoto, D. Nishikawa, and M. Tanno, "Coordinated multipoint transmission/reception techniques for LTE-advanced," *IEEE Wireless Commun.*, vol. 17, no. 3, pp. 26–34, Jun. 2010.

[2] R. Irmer, H. Droste, P. Marsch, M. Grieger, G. Fettweis, S. Brueck, H. P. Mayer, L. Thiele, and V. Jungnickel, "Coordinated multipoint: Concepts, performance, and field trial results," *IEEE Commun. Mag.*, vol. 49, no. 2, pp. 102–111, Feb. 2011.

[3] D. Choi, D. Lee, and J. H. Lee, "Resource allocation for CoMP with multiuser MIMO-OFDMA," *IEEE Trans. Veh. Technol.*, vol. 60, no. 9, pp. 4626–4632, Nov. 2011.

[4] R. Bhagavatula and R. W. Heath, "Adaptive limited feedback for sum-rate maximizing beamforming in cooperative multicell systems," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 800–811, Feb. 2011.

[5] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[6] T. Wu and V. Lau, "Design and analysis of multi-user SDMA systems with noisy limited CSIT feedback," *IEEE Trans. Wireless Commun*, vol. 9, no. 4, pp. 1446–1450, Apr. 2010.

[7] T. Wu and V. Lau, "Robust precoder adaptation for MIMO links with noisy limited feedback," *IEEE Trans. Inf. Theory*, vol. 55, no. 4, pp. 1640–1649, Apr. 2009.

[8] C. Murthy, J. Zheng, and B. Rao, "Performance of quantized equal gain transmission with noisy feedback channels," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2451–2460, Jun. 2008.

[9] A. Housfater and T. J. Lim, "Throughput of precoded broadcast transmission with noisy feedback," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2009, pp. 398–402.

[10] A. Housfater and T. J. Lim, "Noisy feedback linear precoding: A bayesian cramér-rao bound," in *Proc. Int. Symp. Inf. Theory*, Jul. 2009, pp. 1689–1693.

[11] I. S. Dhillon, R. W. Heath, T. Strohmer, and J. A. Tropp, "Constructing packings in Grassmannian manifolds via alternating projection," *Exp. Math.*, vol. 17, no. 7, pp. 9–35, 2008.

[12] Y. Cheng, V. Lau, and Y. Long, "A scalable limited feedback design for network MIMO using per-cell product codebook," *IEEE Trans. Wireless Commun.*, vol. 9, no. 10, pp. 3093–3099, Oct. 2010.

[13] X. Hou and C. Yang, "Codebook design and selection for multi-cell cooperative transmission limited feedback systems," in *Proc. IEEE 73rd VTC*, May 2011, pp. 1–5.

[14] D. Su, X. Hou, and C. Yang, "Quantization based on per-cell codebook in cooperative multi-cell systems," in *Proc. IEEE WCNC*, Mar. 2011, pp. 1753–1758.

[15] X. Hou and C. Yang, "How much feedback overhead is required for base station cooperative transmission to outperform non-cooperative transmission?" in *Proc. IEEE ICASSP*, May 2011, pp. 3416–3419.

[16] W. Yu, T. Kwon, and C. Shin, "Multicell coordination via joint scheduling, beamforming and power spectrum adaptation," in *Proc. IEEE INFOCOM*, Apr. 2011, pp. 2570–2578.

[17] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, Jun. 2002.

[18] D. Park and G. Caire, "Hard fairness versus proportional fairness in wireless communications: The multiple-cell case," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2008, pp. 2036–2040.

[19] S. A. Vorobyov, A. B. Gershman, and Z.-Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 313–324, Feb. 2003.

[20] R. Lorenz and S. Boyd, "Robust minimum variance beamforming," *IEEE Trans. Signal Process*, vol. 53, no. 5, pp. 1684–1696, May 2005.

[21] A. Abdel-Samad, A. B. Gershman, and T. N. Davidson, "Robust transmit beamforming based on imperfect channel feedback," in *Proc. IEEE 60th VTC-Fall*, Sep. 2004, vol. 3, pp. 2049–2053.

[22] E. Gharavol, Y.-C. Liang, and K. Mouthaan, "Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information," *IEEE Trans. Veh. Technol.*, vol. 59, no. 6, pp. 2852–2860, Jul. 2010.

[23] W. Santipach and M. Honig, "Asymptotic capacity of beamforming with limited feedback," in *Proc. ISIT*, Jul. 2004, p. 290.

[24] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sep. 2007.

[25] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.

[26] G. Marsaglia, "Choosing a point from the surface of a sphere," *Ann. Math. Statist.*, vol. 43, no. 2, pp. 645–646, 1972.

Yingxiang Yang received the B.E. degree from Tsinghua University, Beijing, China, in 2012. He is currently working toward the Ph.D. degree with Rutgers University, New Brunswick, NJ, USA.

His research interests include communications theory, optimization theory, and game theory.

Mr. Yang has been a Peer Reviewer for the IEEE Vehicular Technology Conference since 2011.

Bo Bai (S'09–M'11) received the B.S. degree (first-class honors) from Xidian University, Xi'an, China, in 2004 and the Ph.D. degree from Tsinghua University, Beijing, China, in 2010.

From 2009 to 2012, he was a Visiting Research Staff (Research Assistant from April 2009 to September 2010 and Research Associate from October 2010 to April 2012) with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong. He is currently an Assistant Professor with the Department of Electronic Engineering, Tsinghua University. His research interests include hot topics in wireless communications, information theory, random graph, and combinatorial design.

Dr. Bai was a Session Chair of the IEEE Global Communications Conference (IEEE GLOBECOM) in 2008 and has served as a member of the Technical Program Committee of the IEEE International Communications Conference in 2010 and 2012 and the IEEE International Conference on Communications in China and the IEEE International Conference on Connected Vehicles and Expo in 2012. He was invited as a Young Scientist Speaker to the IEEE Technology Time Machine in 2011. He has also served as a Reviewer for a number of major IEEE journals and conferences. He received a Student Travel Grant at the IEEE GLOBECOM in 2009 and support from the Backbone Talents Supporting Project of Tsinghua University. He also received the Young Academic Talent of Electronic Engineering Award from Tsinghua University and the Outstanding Graduates of Shaanxi Province Award.

Wei Chen (SM'13) received the B.S. and Ph.D. degrees in electronic engineering (first-class honors) from Tsinghua University, Beijing, China, in 2002, and 2007, respectively.

From 2005 to 2007, he was a Visiting Research Staff with the Hong Kong University of Science and Technology, Hong Kong. Since July 2007, he has been with the Department of Electronic Engineering, Tsinghua University, where he is currently a Full Professor and the Vice-Director of the Institute of Communications. From June 2010 to September 2010, he was a Visiting Researcher with the University of Southampton, Southampton, U.K. He is a 973 Youth Project Chief Scientist and is supported by the New Century Talent Program of the Chinese Ministry of Education, the Beijing Nova Program, and 100 fundamental research talents program of Tsinghua University (also known as the 221 Talents Program). His research interests include wireless communications, information theory, and applied optimization.

Dr. Chen served as a Vice-Director of the Youth Committee of the China Institute of Communications, a Tutorial Cochair of the 2013 IEEE International Conference on Communications, a track Cochair of the Wireless Track of the IEEE Consumer Communications and Networking Conference in 2013, a Cochair of the Technical Program Committee (TPC) of the IEEE Vehicular Technology Conference in Spring 2011, the Publication Chair of the IEEE International Conference on Communications in China in 2012, a TPC Cochair of the Wireless Communication Symposium at the 2010 IEEE International Conference on Communications (ICC), and a Student Travel Grant Chair of ICC 2008. He also served as an Editor for IEEE WIRELESS COMMUNICATIONS LETTERS. He received the 2010 IEEE Communication Society Asia–Pacific Board Best Young Researcher Award; the 2009 IEEE Marconi Prize Paper Award; the Best Paper Awards at the IEEE ICC in 2006, the First IEEE International Workshop on Cross Layer Design in 2007, and the IEEE Smart Gird Commmunications Conference in 2012; the 2011 Tsinghua Rising Academic Star Award; the 2012 Tsinghua Young Faculty Teaching Excellence Award; the First Prize at the First National Young Faculty Teaching Competition; the First Prize at the Seventh Beijing Young Faculty Teaching Competition; the First Prize at the Fifth Tsinghua University Young Faculty Teaching Competition; and the National May 1st Labor Medal.

Lajos Hanzo (M'91–SM'92–F'04) received the M.S. degree (with first-class honors) in electronics and the Ph.D. degree from the Technical University of Budapest, Budapest, Hungary, in 1976 and 1983, respectively, the D.Sc. degree from the University of Southampton, Southampton, U.K., in 2004, and the "Doctor Honoris Causa" degree from the Technical University of Budapest in 2009.

During his 35-year career in telecommunications, he has held various research and academic posts in Hungary, Germany, and the U.K. Since 1986, he has been with the School of Electronics and Computer Science, University of Southampton, Southampton, U.K., where he holds the Chair for Telecommunications. Since 2009, he has been a Chaired Professor with Tsinghua University, Beijing China. He is currently directing a 100-strong academic research team, working on a range of research projects in the field of wireless multimedia communications sponsored by industry; the Engineering and Physical Sciences Research Council, U.K.; the European IST Programme; and the Mobile Virtual Centre of Excellence, U.K. He is an enthusiastic supporter of industrial and academic liaison and offers a range of industrial courses. He has successfully supervised 80 Ph.D. students, coauthored 20 John Wiley/IEEE Press books on mobile radio communications totaling in excess of 10 000 pages, published more than 1250 research entries on IEEE Xplore, and presented keynote lectures. (For further information on research in progress and associated publications, please refer to http://www-mobile.ecs.soton.ac.uk.)

Dr. Hanzo is Fellow of the Royal Academy of Engineering, U.K., a Fellow of the Institution of Electrical Engineers, and a Governor of the IEEE Vehicular Technology Society. He has been a Technical Program Committee Chair and a General Chair for IEEE conferences. During 2008–2012, he was the Editor-in-Chief of the IEEE Press. He has received a number of distinctions.

AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = Please provide keywords

END OF ALL QUERIES

A Low-Complexity Cross-Layer Algorithm for Coordinated Downlink Scheduling and Robust Beamforming Under a Limited Feedback Constraint

Yingxiang Yang, Bo Bai, *Member, IEEE*, Wei Chen, *Senior Member, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

***Abstract*—Coordinated scheduling/beamforming (CS/CB) substantially mitigates the intercell interference (ICI), hence increasing the cell-edge throughput on the downlink (DL) of coordinated multipoint (CoMP) systems. To maximize the DL throughput, the cooperating base stations (BSs) jointly select the best set of users for DL scheduling and then jointly design a set of beamforming (BF) vectors to approach the throughput limit. However, finding the optimal BF vectors requires an exhaustive search and substantial channel state information (CSI) feedback, hence resulting in high algorithmic complexity and heavy uplink traffic load. Hence, we conceive a new cross-layer algorithm to achieve high performance at a lower feedback amount and at lower algorithmic complexity. Based on the fact that different BSs usually have different traffic loads, we divide the BSs into two different types, i.e., the master BSs (MBSs) and the slave BSs (SBSs), where MBSs have a higher transmission priority than SBSs. The scheduling relies on an interference threshold, whereas our robust BF scheme exploits both the channel direction information (CDI), which is quantized using the technique of limited feedback, and the channel quality information (CQI), which is assumed to be fed back accurately. Our numerical results show that the proposed algorithm does not lose much performance compared with that achieved by an exhaustive search, whereas the algorithmic complexity is as low as that of the solutions operating without CoMP.**

***Index Terms*—Author, please supply index terms/keywords for your paper. To download the IEEE Taxonomy go to http://www.ieee.org/documents/taxonomy_v101.pdf .**

I. Introduction

COORDINATED multipoint (CoMP) transmission is a key feature in the Long-Term Evolution system [1]–[3], which promises performance improvements for the cell-edge users by allowing several base stations (BSs) to cooperate. On the uplink side, the cooperating BSs share the information and jointly process the data received from the mobile stations (MSs). On the downlink (DL) side, two commonly implemented methods are the joint processing and coordinated scheduling/beamforming (CS/CB) schemes, where CS/CB allows the BSs to cooperatively schedule their DL transmissions to a set of users and then cooperatively design a set of efficient beamforming (BF) vectors. Under the assumption that the users' channel state information (CSI) is perfectly known at the BSs' DL transmitters, the throughput performance of coordinated BF varies for different sets of scheduled users. Thus, the ultimate task for the BSs is to schedule their DL transmissions to the optimal set of users that are capable of approaching the maximum DL-transmission throughput and then to design the particular set of BF vectors that can approach this limit.

However, the problem described earlier has the following obstacles.

1) *High algorithmic complexity*. The algorithmic complexity imposed by finding the optimal set of MSs for which the DL transmissions should be scheduled is high since an exhaustive search is required for optimal scheduling. Assuming that there are M BSs and that the user set of the ith BS (BSi) is denoted by $\mathcal{U}_i$, the complexity of the scheduling algorithm is on the order of $\mathcal{O}(\prod_{i=1}^{M}|\mathcal{U}_i|)$. This complexity becomes excessive, when the number of BSs and MSs in each cell increases.
2) *High feedback load*. Assuming that the feedback "budget" of each MS's CSI is B bits for the Channel Direction Information (CDI) and b bits for the Channel Quality Information (CQI), the feedback traffic load can be expressed as $(\sum_{i=1}^{M}|\mathcal{U}_i|)(MB+b)$, where the MB bits of the CDI feedback are related to M channels, i.e., one for the specific channel receiving the desired signal and the remaining $(M-1)$ for the channel receiving the interfering signal.
3) *High backhaul traffic*. To calculate the set of optimal BF vectors, at least one of the BSs has to know the CSI of all the MSs. Thus, the backhaul traffic load is at least $(\sum_{i=1}^{M}|\mathcal{U}_i| - \max_i\{|\mathcal{U}_i|\})(MB+b)$.
4) *Inaccuracy caused by imperfect CSI feedback*. Since the CSI feedback introduces both quantization errors and latency, the CSI acquired at the BSs is inaccurate. Thus, it is possible that the DL-scheduling decision will be inaccurate when the quantization error is high.

Manuscript received August 7, 2012; revised February 5, 2013 and May 17, 2013; accepted June 22, 2013. This paper was supported in part by the National Basic Research Program of China (973 Program) under Grant 2013CB336600 and Grant 2012CB316000, by the U.K.–China Bridge Fellowship, by Chuanxin Funding, by the Beijing Nova Program, and by the Chinese Ministry of Education New Century Talent Program. The work of L. Hanzo was supported by the European Research Council under their Advanced Fellow Grant. The review of this paper was coordinated by Dr. J.-C. Chen.

Y. Yang was with Tsinghua University, Beijing 100084, China. He is currently with the Department of Electrical and Computer Engineering, Rutgers University, New Brunswick, NJ 08901 USA (e-mail: yangyx.thu@gmail.com).

B. Bai and W. Chen are with the Department of Electronic Engineering, Tsinghua University, Beijing, 100084, China (e-mail:eebobai@tsinghua.edu.cn; wchen@tsinghua.edu.cn).

L. Hanzo is with the School of Electronic and Computer Science, University of Southampton, Southampton, SO17 1BJ U.K. (e-mail: lh@ecs.soton.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TVT.2013.2271876

There has been a plethora of contributions related to CoMP [1], [4]–[15], exploring possible solutions and finding remedies to the impediments aforementioned. Although the original contributions relied on the assumption of perfect CSI [4], more realistic recent contributions assumed imperfect CSI feedback, where the channel vectors are quantized to a codeword stored in a codebook designed offline and the index of the codeword is fed back instead of the actual quantized values of the channel vectors. Hence, the amount of feedback per user can be substantially reduced. A comprehensive introduction to the topic of limited feedback aid communications can be found in [5], where the authors discussed the feedback design in a broad range of scenarios, employing methods used in industrial standards and protocols.

One of the common issues in realistic limited-feedback-aided systems is the inaccuracy of the CSI feedback both due to the delay encountered and by the transmission errors imposed by the feedback channels [6]–[10]. In [6], Wu and Lau proposed a feedback design for spatial-division multiple-access (SDMA) systems, demonstrating that their scheme is robust against feedback channel errors and characterized the system's goodput. Another contribution of Wu and Lau [7] provided two robust designs for multiple-input–multiple-output precoder adaptation under the scenario of potentially error-prone limited feedback and showed that both frameworks provided significant gains compared with the idealized designs assuming no feedback errors. In [8] and [9], the performances of equal gain transmission and precoded broadcast transmission were studied, respectively, under the scenario of error-prone limited feedback. Finally, in [10], Housfater and Lim derived a Cramér–Rao-type lower bound for linear precoders. These contributions provided insights into the mitigation of the detrimental impact of the imperfect CSI feedback channel.

Another common issue that arises when limited feedback is applied to a typical CoMP system is the codebook design problem. Although the design of codebooks conceived for limited-feedback-aided systems has been extensively studied [1], [11], the number of BSs in a CoMP cluster may vary over time, hence requiring a specific design. Thus, it is a challenge to design a codebook imposing low overhead when the number of cooperating BSs is high. A promising solution is based on the per-cell codebook design philosophy of [12]–[15], which separately quantizes the channel associated with each cell within a CoMP channel matrix to avoid a large codebook and to circumvent frequent updates of the codebook owing to either user mobility or due to the different clustering of the BSs. To elaborate a little further, Cheng *et al.* [12] presented a limited-feedback-based per-cell codebook design and showed that its performance is close to that of the conventional joint-cell codebook design having high overhead. In [13], attention is focussed on the problem of optimal per-cell codebook designs and derived a closed-form solution for the codebook size that minimizes the quantization error on average. In [14], a method of reconstructing the CoMP channel's CDI was first proposed and then, the performance of different codebook generation techniques and per-cell codeword selection methods was compared.

In contrast with the insightful contributions listed earlier, we pursue a different approach in reducing both the algorithmic complexity and the CSI feedback overhead for a scenario where CS/CB is employed.

1) We conceive a low-complexity noniterative cross-layer algorithm, which is based on the fact that, in multicell systems, all BSs tend to have different DL-transmission rate requirements and traffic loads. We commence by classifying the BSs into two types. The BSs having higher transmission rate requirements are referred to as master BSs (MBSs), which benefit from a higher priority. The remaining BSs having lower transmission rate requirements are referred to as slave BSs (SBSs), which have a lower priority.
2) We propose a low-complexity interference-threshold-based algorithm for scheduling, which is combined with appropriately adjusting the BF vectors of the cooperating BSs. As shown in Section III, this part of the algorithm only relies on the CSI at the user's side; thus, it is capable of effectively reducing the CSI feedback load while mitigating the inaccuracy of CSI feedback imposed by the error-prone feedback channel.
3) Furthermore, we propose a new robust BF vector design for the scenario, where the CDI and CQI are fed back separately. More explicitly, the CDI is quantized before being fed back, whereas the CQI is assumed to be perfectly fed back to the BS.
4) We will demonstrate both with the aid of our theoretical derivation and by numerical simulations that our algorithm has similar algorithmic complexity as the noncooperative algorithms. It imposes low backhaul traffic and circumvents the dynamic channel-matrix clustering of CoMP.
5) We also show that the performance of the proposed algorithm is not overly compromised and that its performance is similar to that of the iterative algorithm proposed in [16], as far as the MBS is concerned. Our algorithmic philosophy is highlighted in a simple scenario, where only two BSs are involved, but it may be readily extended to more general scenarios supporting multiple BSs without a dramatic increase in complexity and feedback requirements.

The remainder of this paper is organized as follows. In Section II, our system model is introduced, along with some of our basic assumptions. In Section III, both the proposed scheduling and our BF algorithm are detailed. In Section IV, we present a comparison of the algorithmic complexity and the signaling overhead of different algorithms. Our performance analysis is provided in Section V, whereas Section VI offers our conclusions. The proof of some of the theorems is provided in the Appendix.

II. System Model

Consider the two-cell network of Fig. 1, where each cell has a BS at its center and multiple MSs scattered within the cell. BS1 on the left of Fig.1 is assumed to be the MBS, and BS2 on the right is an SBS. Both BS1 and BS2 are equipped with K transmit antennas, whereas each MS has a single receive antenna. We assume that, each time, each BS schedules its DL

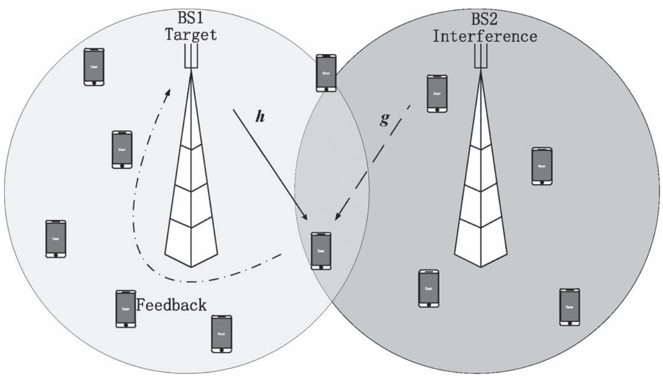

Fig. 1. System model.

transmission to a single MS within its cell, and different BSs use the same frequency; hence, the MSs suffer from intercell interference (ICI) imposed by the neighboring BS. The received signal power is related to the location of the MS. There are a number of studies on the fairness control issues of scheduling algorithms [17], [18], but in this paper, we focus our attention on the scenario where the MSs are located at the cell edge, and we assume that all the MSs have the same large-scale fading factor. In other words, the fairness effect of different large-scale fading factors is not considered here.

Again, we denote the user set of cell i $(i \in \{1, 2\})$ as $\mathcal{U}_i$. Then, the signal received by user j $(j \in \{1, 2, \ldots, |\mathcal{U}_1|\})$ in cell 1 and user $k (k \in \{1, 2, \ldots, |\mathcal{U}_2|\})$ in cell 2 can be expressed as

$$\begin{cases} y_{1,j} = p_{11,j} \langle \mathbf{w}_1, \mathbf{h}_{1,j} \rangle u_{1,j} + p_{21,j} \langle \mathbf{w}_2, \mathbf{g}_{1,j} \rangle u_{2,k} + n_j \\ y_{2,k} = p_{22,k} \langle \mathbf{w}_2, \mathbf{h}_{2,k} \rangle u_{2,k} + p_{12,k} \langle \mathbf{w}_1, \mathbf{g}_{2,k} \rangle u_{1,j} + n_k \end{cases} \tag{1}$$

where $\langle \mathbf{x}, \mathbf{y} \rangle$ represents the inner product of the vectors $\mathbf{x}$ and $\mathbf{y}$. Variable $y_{i,j}$ represents the signal received by user j in cell i, where i equals either 1 or 2. At the receiver of the jth user in cell i_1, the signal strength received from cell i_2 can be represented as $p_{i_1 i_2, j}$. The power of the symbol transmitted from BSi to its jth user is denoted $u_{i,j}$, which is normalized as $E\{|u_{i,j}|^2\} = 1$. The random variables n_j and n_k represent the normalized Gaussian noise, with $E\{|n_k|^2\} = E\{|n_j|^2\} = 1$, whereas vector $\mathbf{h}_{i,j} \in \mathbb{C}^{K \times 1}$ represents the DL channel conditions between BSi and its jth user, which can be viewed as the target channel condition. Furthermore, vector $\mathbf{g}_{i,j} \in \mathbb{C}^{K \times 1}$ denotes the DL channel condition between the jth user of BSi and the neighboring BS, which is the interferring channel. The target channel vectors and the interfering channel vectors are independent and identically distributed in terms of their statistics, and they both follow a probability distribution of $CN(0, \mathbf{I}_K)$. Finally, vector $\mathbf{w}_i$ is the BF vector adopted by BSi.

The goal of the proposed algorithm is to increase the DL throughput, which is quantified in terms of the channel capacity of

$$R_i = \log(1 + \mathrm{SINR}_i) \tag{2}$$

where SINR_i is the signal-to-interference-plus-noise ratio at the scheduled user's terminal of BSi. We assume that each BS schedules its DL transmission to a single MS each time. Thus, to simplify the notation, we denote the target channel condition and the interfering channel condition of the scheduled MS located in cell i by $\mathbf{h}_i$ and $\mathbf{g}_i$, respectively, yielding

$$\mathrm{SINR}_i = \frac{p_{ii} \|\mathbf{h}_i\|^2 \left| \langle \mathbf{w}_i, \tilde{\mathbf{h}}_i \rangle \right|^2}{1 + p_{ji} \|\mathbf{g}_i\|^2 \left| \langle \mathbf{w}_j, \tilde{\mathbf{g}}_i \rangle \right|^2}. \tag{3}$$

Note that, in the given equation, the norms of vectors $\mathbf{h}_i$ and $\mathbf{g}_i$ are separated from their directions so that we have $\|\tilde{\mathbf{h}}_i\| = \|\tilde{\mathbf{g}}_i\| = 1$. We use p_{ii} to denote the signal strength received by the selected user in cell i and p_{ji} to denote the strength of the interfering signal arriving from cell j contaminating the desired user's signal in cell i. We simplified the subscript since we only consider the case where each BS schedules its DL transmission to a single user at a time.

A. Schemes Operating Without CoMP

Transmission schemes operating without CoMP typically have lower complexity than those relying on CoMP. Here, we simply consider the classic time-division multiple-access (TDMA) and frequency-division multiple-access (FDMA) schemes. In the TDMA scheme, the M BSs transmit sequentially so that each BS transmits in $1/M$ fraction of the time, without imposing any ICI. In the FDMA scheme, on the other hand, the M BSs share the transmission bandwidth so that each BS transmits in a separate subband without being interfered by the neighboring BSs.

We compare the performance of the TDMA and FDMA schemes to that of our proposed algorithm and to the exhaustive search algorithm. In TDMA and FDMA schemes, all the MSs feed back both their CQI and their target channel conditions using limited feedback, so that the BSs can design BF vectors accordingly. The BSs, on the other hand, schedule their DL transmission for the specific MS having the highest CQI. The transmission throughput for BSi can be expressed as

$$R_i = \frac{1}{M} \log \left(1 + \max_{j \in \mathcal{U}_i} \mathrm{SNR}_j \right) \tag{4}$$

where $M = 2$ in our scenario.

B. Iterative Scheduling and Beamforming

Iterative algorithms are capable of reducing the algorithmic complexity while maintaining a similar performance to their exhaustive-search-algorithm-based counterparts. In this paper, we adopt the iterative approach proposed in [16] as a benchmark for our performance evaluation, which relies on the following three steps.

1) Fix the power allocation and scheduled users; then, find the best combination of BF vectors.
2) Fix the BF vectors and power allocation; then, find the best set of users for scheduling.
3) Fix the BF vectors and the scheduled users; then, update the power allocation among the scheduled users.

Since the scenario that we study assumes fixed power allocation, the given three-step algorithm reduces to two steps in

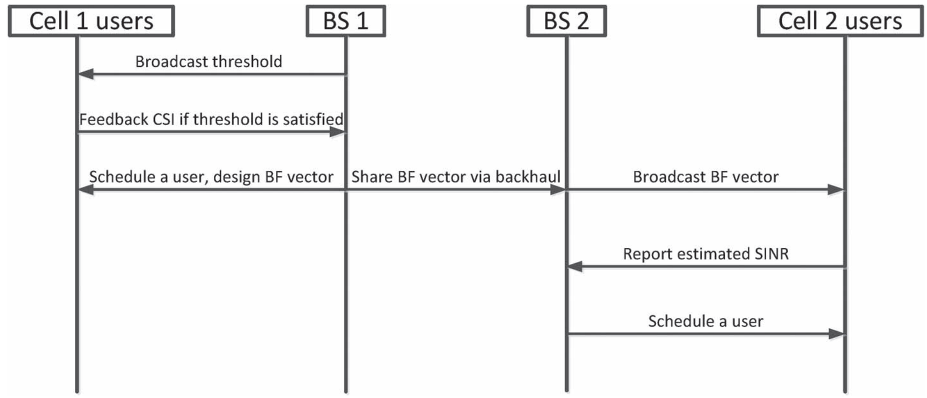

Fig. 2. Algorithm outline.

which the combination of BF vectors and the set of scheduled users is updated in an iterative fashion as follows.

1) Fix the set of scheduled users; then, find the optimal set of BF vectors correspondingly.
2) Fix the set of BF vectors; then, find the set of users that yield optimal throughput performance correspondingly.

The performance of the algorithm is characterized in Section V.

III. Cross-Layer Algorithm Under Limited Feedback Constraint

The algorithmic steps are shown in Fig. 2. In the first step, the MBS, i.e., BS1, broadcasts a threshold value constraining the channel directions of the desired users with respect to the interfering BS to maintain the target integrity. Explicitly, the directions of the desired user's target channel and of the same user's interfering channel should be perpendicular to each other, which corresponds to the absolute value of their inner product being close to zero. The users within cell 1 receive the threshold value and decide whether or not to feed their CSI back to their anchor BS. If a desired user's target channel and interfering channel satisfy the required angular separation threshold constraint, he/she feeds back the CDI of both the target channel and the interfering channel, as well as the CQI of both channels. Here, the CQI is defined as the product of $p_{ii}\|\mathbf{h}_i\|^2$ for the target channel and $p_{ji}\|\mathbf{g}_i\|^2$ for the interfering channel, when the user is located in cell i. Once the CSI of all the users that satisfy the angular separation constraint has been fed back to the MBS, the MBS decides which particular user to schedule for its DL transmission and then designs the BF vector, following our robust BF vector design method to be presented later. Once the BF vector is determined, it is shared with the SBS, i.e., BS2, via a backhaul link, the SBS broadcasts the BF vector, and all the users within cell 2 feed back their expected SINR values computed with the aid of the BF vector and their local CSI. In the final step, the SBS schedules its DL transmissions to the user having the highest "reported" SINR value.

The algorithm introduced relies on a few important assumptions, which are based on the following motivation.

1) *Introduction of the constraint* $\mathbf{w}_1 = \mathbf{w}_2$. In the original CS/CB problem, the BF vectors of different BSs do not necessarily have the same direction. In fact, if not shared via the backhaul link, the BF vector of a BS can be regarded as a random vector both for the other BSs and for all the MSs of the neighboring cells. There is no simple yet elegant way of effectively reducing the size of the candidate user set, but introducing the given constraint brings us obvious benefits. First, when we have $\mathbf{w}_1 = \mathbf{w}_2$, the local CSI can be directly used to compute the level of interference, which is now $\langle \tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_2 \rangle$ for cell 1 and $\langle \tilde{\mathbf{h}}_2, \tilde{\mathbf{g}}_1 \rangle$ for cell 2. This can assists us in exploiting the CSI at the MSs for naturally ruling out DL transmission to the MSs suffering from severe interference. As will be shown in Section IV, the feedback load is substantially reduced. Second, both the scheduling and BF parts of the algorithm can be implemented in each cell by relying merely on local CSI, which means that the backhaul traffic is effectively reduced. Additionally, the algorithmic complexity is also significantly cut down since only a small portion of the MBS's users perform CSI feedback, whereas the users of the SBS only feed their CQI back to the BS. These complexity and feedback requirements are similar to or even lower than those of some standard non-CoMP solutions, such as those of the TDMA and FDMA schemes.
2) *Introduction of the MBS and the SBS*. The related assumptions are based on the fact that, at each moment, it is likely that some BSs have a higher transmission rate requirement than the others; hence, they should be granted a higher priority and, ultimately, a higher transmission rate. Hence, an MBS can schedule its DL transmission to a user and design the BF vector with a higher priority, whereas the SBS cannot. The performance loss imposed by this unbalanced priority can be partially recovered when the number of users in the cells is high.

A. Threshold-Based User Scheduling

As shown in Fig. 2, the BSs schedule their DL transmissions according to a thresholding algorithm based on a carefully designed threshold. In our proposed algorithm, the MBS and the SBS have the same BF directions. Here, we focus on scheduling the DL transmissions to the specific MS, whose target channel direction $\tilde{\mathbf{h}}$ is "most different" from its interfering

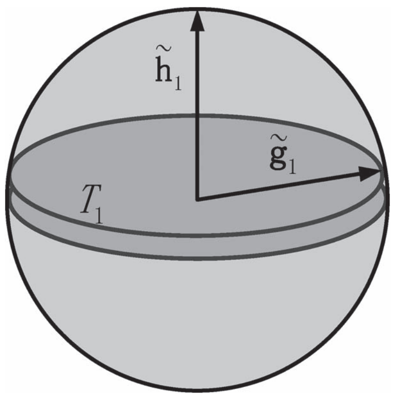

Fig. 3. Threshold.

channel direction $\tilde{\mathbf{g}}$ among all the users. More explicitly, we have to find the specific user, whose target channel direction and interfering channel direction have the smallest inner product absolute value across the entire set of users served by the MBS. However, it should be noted that the user to be scheduled for DL transmission in CoMP relying on an exhaustive search might not have an $\tilde{\mathbf{h}}$ perpendicular to $\tilde{\mathbf{g}}$. The thresholds can be defined as follows.

The MBS schedules the user whose channel conditions satisfy

$$\left|\langle \tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1 \rangle\right| \leq T_1 \tag{5}$$

where T_1 is the relevant threshold. An intuitive interpretation of the threshold is shown in Fig. 3, which shows (5) with the aid of T_1. Since the norms of both the BF vectors and of the channel directions of the MSs are 1, these vectors can be placed on a globe-like unit-radius hypersphere in $\mathbb{C}^K$, with one end at the origin and the other on the surface of the hypersphere. By assuming that the target channel direction $\tilde{\mathbf{h}}_1$ of the scheduled user points to the "north pole" of the globe, the interfering channel direction $\tilde{\mathbf{g}}_1$ will fall within the area bounded by the "Tropic," which is characterized by the value of T_1.

Intuitively, when the threshold T_1 becomes looser, i.e., when it approaches 1, more users will satisfy (5), and the complexity of the algorithm is increased. In particular, when we have $T_1 = 1$, all the users feed back their CSI, and the algorithm becomes identical to the exhaustive search for the MBS. By contrast, when T_1 approaches 0, the scheduling part of the algorithm will guarantee a higher transmission rate for the scheduled users, but it also comes more likely that no users satisfy (5), which leads to lower algorithmic complexity and reduced feedback load. Thus, the threshold controls the tradeoff between the algorithmic complexity and the desired performance; hence, it should be determined under the constraint of ensuring a minimum probability of at least one successful DL scheduling for the entire set of users. The selection of the threshold based upon the given principle can be achieved with the aid of the following theorem.

Theorem 1: Let us assume that there are N_1 users in cell 1. Then, for the MBS, the probability of a successful DL scheduling action can be expressed as

$$P_{\mathrm{suc}1} = 1 - \left(1 - T_1^2\right)^{N_1(K-1)}. \tag{6}$$

Proof: See Appendix A. ■

The performance achieved with the aid of this threshold is characterized in Section IV.

B. Robust Beamforming Under Limited Feedback for the Channel Direction Information

According to Fig. 2, upon scheduling the DL transmission to a user whose channel conditions satisfy (5), the MBS adopts a BF vector that further improves the throughput. Since the MSs perform limited feedback of their channel conditions, when quantizing both $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ using a preset codebook and when transmitting the index of a codeword, the quantization error imposes inaccuracy on the design of BF vectors. We mitigate this impact using a robust BF technique, which maximizes the lowest possible SINR of the specific user selected. Numerous studies have been dedicated to robust BF [19]–[22]. Although the scenarios of these contributions are different, they all model the quantization error as an additive noise vector. For example, the target channel's channel vector of the selected user in cell 1 would be modeled as

$$\tilde{\mathbf{h}}_1 = \hat{\mathbf{h}}_1 + \mathbf{e} \tag{7}$$

where $\tilde{\mathbf{h}}_1$ represents the actual target channel direction, whereas $\hat{\mathbf{h}}_1$ is its quantized version, which is acquired from the user's feedback. Vector $\mathbf{e}$ in (7) represents the quantization error, which satisfies the ellipsoid constraint $\|\mathbf{e}\| \leq \varepsilon$. The quantization error for interfering channels can be defined similarly. The problem is then solved using convex optimization, and this technique is assumed to be known in this paper. This traditional way of designing robust BF vectors does not meet the assumptions stipulated in this paper. Earlier, we assumed that the norms of the channel directions are 1 both before and after quantization. This imposes more complex constraints on the description of the quantization error. Hence, we conceive a new technique of designing robust BF vectors for the scenario when the CDI and CQI are fed back separately.

We adopt the random-vector-quantization codebook concept [23] and use the model of [24] to analyze the quantization error, where the quantization codebook index of B bits is linked with the quantization error by

$$\left|\langle \hat{\mathbf{h}}_1, \tilde{\mathbf{h}}_1 \rangle\right| \geq 1 - 2^{\frac{-B}{K-1}}. \tag{8}$$

When $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_1$ are given, the problem of designing the robust BF vector can be broken down into two separate parts.

1) For each BF vector $\mathbf{w}$, find the set of $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ minimizing the SINR (denoted by SINR_1) for this $\mathbf{w}$.
2) Find the BF vector $\mathbf{w}$, which ensures that this minimized value of SINR_1 is maximized.

The whole idea can be formulated as

$$\mathbf{w}_{\mathrm{opt}} = \arg\max_{\mathbf{w}} \left\{ \min_{\tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1} \left\{ \mathrm{SINR}_1(\tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1 | \hat{\mathbf{h}}_1, \hat{\mathbf{g}}_1, \mathbf{w}) \right\} \right\}. \tag{9}$$

Again, an intuitive illustration is given in Fig. 4. Since $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ are independent of each other, the problem of finding the specific $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ that minimize the value of SINR for a given

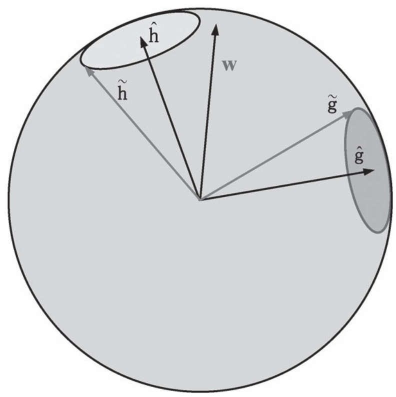

Fig. 4. Robust BF.

$\mathbf{w}$ is further simplified to finding $\tilde{\mathbf{h}}_1$ that minimizes $|\langle \mathbf{w}, \tilde{\mathbf{h}}_1 \rangle|^2$ and finding $\tilde{\mathbf{g}}_1$ that maximizes $|\langle \mathbf{w}, \tilde{\mathbf{g}}_1 \rangle|^2$. The following lemmas provide a way of finding these $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ vectors.

Lemma 1: Upon assuming that $|\langle \mathbf{w}, \hat{\mathbf{g}}_1 \rangle| = \lambda_1$ and that the quantization error satisfies $|\langle \tilde{\mathbf{g}}_1, \hat{\mathbf{g}}_1 \rangle| = \lambda_2 \geq \lambda_{2\min} \geq \lambda_1$, we have

$$|\langle \mathbf{w}, \tilde{\mathbf{g}}_1 \rangle| \leq \lambda_1 \lambda_{2\min} + \sqrt{(1-\lambda_1^2)\left(1-\lambda_{2\min}^2\right)}. \tag{10}$$

Proof: See Appendix B. ■

Lemma 2: Assuming that $|\langle \mathbf{w}, \hat{\mathbf{h}}_1 \rangle| = \lambda_1$ and that the quantization error satisfies $|\langle \tilde{\mathbf{h}}_1, \hat{\mathbf{h}}_1 \rangle| = \lambda_2 \geq \lambda_{2\min} \geq \sqrt{1-\lambda_1^2}$, we have

$$\left|\langle \mathbf{w}, \tilde{\mathbf{h}}_1 \rangle\right| \geq \lambda_1 \lambda_{2\min} - \sqrt{(1-\lambda_1^2)\left(1-\lambda_{2\min}^2\right)}. \tag{11}$$

Proof: See Appendix C. ■

In the given lemmas, $\lambda_{2\min} = 1 - 2^{(-B/K-1)}$ represents the maximum quantization error. With the aid of the given lemmas, we have the following theorem.

Theorem 2: Upon introducing the notation of

$$\mathbf{h}_1^{\dagger} = \frac{\hat{\mathbf{h}}_1 - \langle \hat{\mathbf{h}}_1, \hat{\mathbf{g}}_1 \rangle \hat{\mathbf{g}}_1}{\left\| \hat{\mathbf{h}}_1 - \langle \hat{\mathbf{h}}_1, \hat{\mathbf{g}}_1 \rangle \hat{\mathbf{g}}_1 \right\|} \tag{12}$$

and $\hat{\mathbf{h}}_1 = \delta_1 \mathbf{h}_1^{\dagger} + \delta_2 \hat{\mathbf{g}}_1$, the optimal BF vector can be expressed as a linear combination of vectors $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{g}}_1$, which is formulated as

$$\mathbf{w}_{\text{opt}} = \xi_1 \mathbf{h}_1^{\dagger} + \xi_2 \hat{\mathbf{g}}_1 \tag{13}$$

where ξ_1 and ξ_2 are appropriately chosen so that their norms maximize the SINR in (14), shown at the bottom of the page, whereas the arguments of ξ_1 and ξ_2 satisfy the constraint $\arg(\delta_1^* \xi_1) = \arg(\delta_2^* \xi_2)$.

Proof: See Appendix D. ■

The given theorem provides a numerical technique of designing the optimal robust BF vector by finding the optimal value of $|\xi_2|$, which falls in the real-valued interval of [0, 1].

C. Extending the Algorithm to Multiple-BS Scenarios

Earlier, we developed a low-complexity cross-layer algorithm based on a scenario considering only two BSs. Let us now show that this algorithm can be readily extended to more general scenarios with multiple BSs involved.

Let us assume that there are M BSs. In this multi-BS extension of the algorithm, we still assume that there is a single MBS, which cooperates with multiple SBSs, and that all the BSs use the same BF vector. Additionally, since it is reasonable to assume that different BSs have to transmit independent messages to their scheduled users, we can treat the interference at the jth user of cell k received from multiple BSs as interference arriving from a single source associated with a channel vector of

$$\sum_{i=1,\, i\neq k}^{M} \mathbf{g}_{i,\,k,\,j} u_i \tag{15}$$

where $\mathbf{g}_{i,\,k,\,j}$ is the channel vector of the link spanning from the ith BS to the jth user in cell k, and u_i is the symbol transmitted to the scheduled user in cell i. Since the linear combination of isotropic random vectors is also isotropic, the threshold-based scheduling of Section III-A remains unchanged.

IV. Algorithmic Complexity and Signaling Overhead

Here, we characterize the complexity of four algorithms: the exhaustive search algorithm, the TDMA/FDMA scheme, the iterative algorithm of [16], and our proposed algorithm.

Consider the scenario associated with M BSs. The search for the BF vector of a single BS is based on a searching through an L-entry codebook, as implied in (14). Therefore, the exhaustive search algorithm has complexity of $\mathcal{O}(L^M \prod_{i=1}^{M} |\mathcal{U}_i|)$, where $|\mathcal{U}_i|$ is the supported number of users in the ith BS, because the algorithm will consider all possible user combinations to find the one that achieves the optimal performance, which is estimated based on an exhaustive search through the codebook of each BS for an optimal combination of BF vectors for each possible user combination.

The complexity of the iterative algorithm proposed in [16] cannot be readily determined without knowing the number of iterations needed for the convergence of the scheduling process. Here, we introduce parameter S, which represents

$$\text{SINR}_1 = \frac{p_{11}\|\mathbf{h}_1\|^2 \left(\lambda_{2\min}\left(|\delta_1\xi_1| + |\delta_2\xi_2|\right) - \sqrt{\left(1-\lambda_{2\min}^2\right)\left(1 - \left(|\delta_1\xi_1| + |\delta_2\xi_2|\right)^2\right)}\right)^2}{1 + p_{21}\|\mathbf{g}_1\|^2 \left(|\lambda_{2\min}\xi_2| + \sqrt{\left(1-\lambda_{2\min}^2\right)\left(1-|\xi_2|^2\right)}\right)^2} \tag{14}$$

TABLE I
ALGORITHMIC COMPLEXITY

Algorithm	Exhaustive Search	Iterative	TDMA/FDMA	Proposed
Time(s)	1.34e3	1.50e1	2.15e-1	6.04e-1
Complexity	$\mathcal{O}\left(L^M \prod_{i=1}^{M} \lvert\mathcal{U}_i\rvert\right)$	$\mathcal{O}\left(S\left(L^M + \prod_{i=1}^{M} \lvert\mathcal{U}_i\rvert\right)\right)$	$\mathcal{O}\left(\sum_{i=1}^{M} \lvert\mathcal{U}_i\rvert\right)$	$\mathcal{O}\left(\lvert\mathcal{U}_1\rvert L + \sum_{i=2}^{M} \lvert\mathcal{U}_i\rvert\right)$

TABLE II
SIGNALING OVERHEAD

Algorithm		Exhaustive search	Iterative	TDMA/FDMA	Proposed	
					BS1 (MBS)	BSi (SBS)
CDI feedback per user		$2M$	$2M$	0	2	0
CQI feedback per user		$2M$	$2M$	1	2	1
Number of feedback users		$\lvert\mathcal{U}_i\rvert$	$\lvert\mathcal{U}_i\rvert$	$\lvert\mathcal{U}_i\rvert$	Greatly reduced	$\lvert\mathcal{U}_i\rvert$
Backhaul traffic	CDI exchange	$\sum_{i=1}^{M} M \lvert\mathcal{U}_i\rvert$	M^2	0	0	
	CQI exchange	$\sum_{i=1}^{M} M \lvert\mathcal{U}_i\rvert$	M^2	0	0	
	BF vector exchange	M	M	0	$M-1$	
	User identity exchange	M	M	0	0	

the maximum affordable number of iterations for the algorithm. Then, the resultant complexity may be expressed as $\mathcal{O}(S(L^M + \prod_{i=1}^{M} \lvert\mathcal{U}_i\rvert))$, which follows from the fact that, each time the BF vectors are updated, an exhaustive search of the codebooks is conducted, whereas an exhaustive search for the optimal user combination is carried out every time, when the combination is updated.

The complexity of the TDMA/FDMA algorithms is $\mathcal{O}(\sum_{i=1}^{M} \lvert\mathcal{U}_i\rvert)$, which follows from the fact that each BS operates on its own and only has to set the BF vector to the direction of the target channel of the scheduled user.

For the proposed algorithm, if we assume that BS1 is the MBS, the upper bound of the complexity is $\mathcal{O}(\lvert\mathcal{U}_1\rvert L + \sum_{i=2}^{M} \lvert\mathcal{U}_i\rvert)$. This is because the MBS only has a portion of its users performing feedback, and the SBSs only have to find the best users on their own by comparing their CQI feedback values.

In the following, we characterize the algorithmic complexity by monitoring the simulation time required for a single trial. We conducted 1000 trials on all the algorithms aforementioned, and in Table I, we listed the time required for each single trial for CDI codebook sizes spanning from 2 to 15. As we can see, the exhaustive search algorithm and the iterative algorithm require significantly more time than the noncooperative algorithm and the proposed algorithm.

Finally, we compare the signaling overhead of the four given algorithms in Table II, which includes the overhead generated from users' feedback and the backhaul traffic. It is observed in Table II that the exhaustive search algorithm and the iterative algorithm require the same amount of feedback overhead, except that the iterative algorithm generates less backhaul traffic when the number of iterations is small. The overhead generated, particularly the backhaul traffic, is enormous compared with the noncooperative TDMA/FDMA schemes.

The proposed algorithm, on the other hand, effectively controls the overhead generated from the CSI feedback and backhaul traffic. Since the BF vectors are set to the same direction, each user can view its received signals as if they were sent from only two sources, i.e., the destination and a single interfering source. As a benefit, each user only has to feed back two sets of channel information. Furthermore, the number of users generating feedback for the MBS is substantially reduced. A theoretical expression for this number is hard to derive. Nonetheless, we can determine with the aid of numerical simulations that, when there are 20 users in each cell and the successful transmission ratio is set to 99.9%, the number of feedback users in the MBS is, on average, 5.7. Meanwhile, the proposed algorithm only requires the sharing of the BF vector selected by the MBS, which is almost negligible compared with the backhaul traffic generated by the iterative algorithm.

V. Numerical Results

Here, we first characterize the properties of threshold T_1, and then quantify the performance of our proposed algorithm.

A. Threshold Value

According to (5) and (6), the "Tropic" T_1 controls the trade-off between the feedback load, algorithmic complexity, and the probability of a successful DL scheduling action. From (6), T_1 is also a function of the number of transmit antennas and the number of active users within the cell.

Fig. 5 shows the relation between the number of users and the value of T_1, where the number of transmit antennas was set to 2. Parameters p_{ii} and p_{ji} of both the MBS and the SBS are set to 30 dB, whereas the probability of success ranged from 10% to 99.9%. We can see in Fig. 5 that, when the probability of success decreases, the threshold becomes stricter, and when the number of users increases, the value of T_1 approaches 0. This is a manifestation of multiuser diversity since we are more likely to have a user with better channel conditions when the number of users becomes larger.

Fig. 6 shows the relationship between the probability of successful DL scheduling and the percentage of reduction in the CDI feedback per user. The numbers of users in both cells range from 10 to 40, whereas the number of transmit antennas

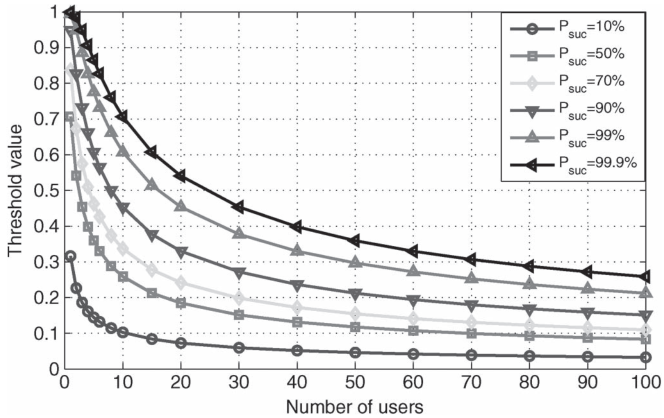

Fig. 5. Threshold value.

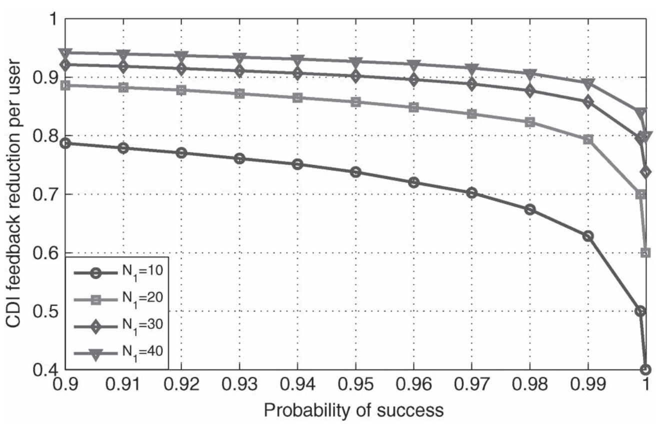

Fig. 6. Feedback load reduction per user.

remains 2, and the values p_{ii} and p_{ji} of both the MBS and the SBS remain 30 dB. Note that, in traditional TDMA and FDMA schemes, all the users feed back their CSI; hence, the feedback load per user is one CDI, where the number of bits is determined by the codebook size. In CS/CB relying on exhaustive search, the feedback load per user is two CDI times the number of codebook index bits. In our proposed algorithm, however, the feedback load per user is only 0.3 CDI, when the number of users is 20 in both cells and the probability of success is 99.9%. This implies that, compared with TDMA and FDMA schemes, the threshold T_1 allows us to rule out 70% of feedback without undue degradation of the probability of success. Additionally, it is possible to achieve an even lower feedback load by reducing the probability of a successful DL scheduling, when the number of users is given, but an excessive degradation of the probability of success may ultimately impose throughput degradation.

B. Throughput Performance

Fig. 7 quantifies the throughput of the MBS relying on the proposed algorithm in comparison to both the exhaustive search scheme and the traditional TDMA and FDMA schemes. The iterative algorithm proposed in [16] is used as a benchmark. *The number of users in both cells was set to 20, whereas the parameters* p_{ii} and p_{ji} *of both the target BS* and *of the interfering BS were set to 30 dB, which was attenuated by the path loss*. The number of transmit antennas was two, and

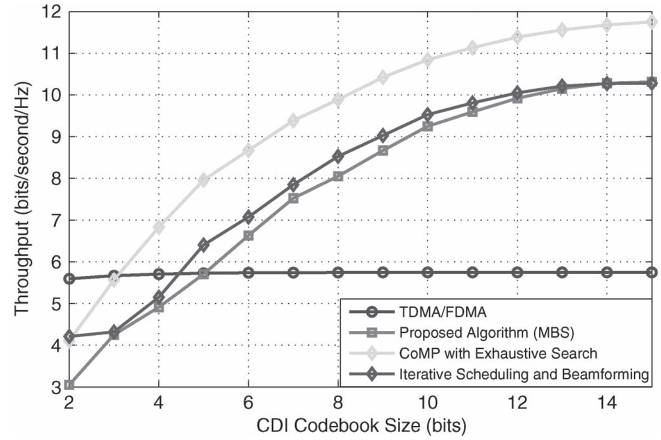

Fig. 7. DL throughput performance of the MBS.

the probability of DL scheduling success was set to 99.9%. As shown in Fig. 7, the performance of the MBS does not substantially degrade, and it is similar to the performance of the iterative algorithm proposed in [16]. The discrepancy between the exhaustive search algorithm and our proposed algorithm is a result of striking a tradeoff between the algorithmic complexity, overhead, and performance. Again, the discrepancy between the iterative algorithm and the exhaustive search algorithm is contributed by both the imperfect CSI feedback and by the fact that the iterative algorithm cannot guarantee consistent convergence to the global optimum. We also note that the left-hand part of the curves is not as smooth as their right-hand part. When the size of the codebook is 2 bits, the iterative algorithm even yields a slightly better performance than the exhaustive search algorithm. This, however, does not mean that the iterative algorithm performs in general better than the exhaustive search algorithm. This phenomenon is due to the large quantization errors of the CDI feedback. While the exhaustive search algorithm did guarantee the maximization of the minimum throughput of the scheduled users, the maximization of the actual performance is evaluated with the aid of the actual channel conditions encountered by the scheduled users. Hence, we conclude that the better performance of the iterative algorithm is a result of the large quantization errors imposed by the limited codebook size of the CDI feedback. This phenomenon does not occur when the codebook size is large.

Furthermore, observe in Fig. 7 that when the CDI codebook size is small, the performance of cooperative BF is similar to or even lower than that of the noncooperative TDMA and FDMA schemes. This phenomenon raises the question as to what is the minimum amount of feedback required by cooperative transmission and scheduling schemes to outperform their traditional noncooperative counterparts. This question is studied in detail in [15].

Fig. 8 shows the performance comparison of an SBS and the performance of a regular BS running under TDMA/FDMA schemes. The codebook size is set to 15 in this case, whereas the performances with other codebook sizes are similar. It can be easily seen that the performance of the SBS is sometimes not as good as the regular BS but becomes better as the number of users increases. This is caused by the multiuser diversity effect. It can be also observed that the intersection of the

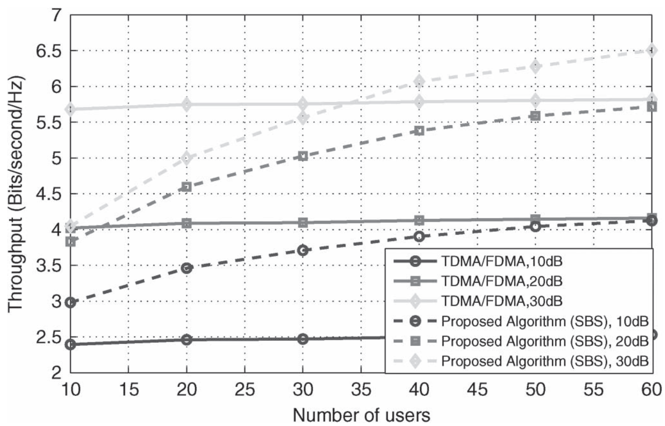

Fig. 8. Performance comparison between an SBS and a BS running under TDMA/FDMA schemes.

curves of the performances of the SBS and the regular BS shifts toward right as the SNR increases, which means that at high SNR levels, the proposed scheme will need more users to beat the performance of the TDMA/FDMA schemes. This phenomenon can be interpreted by the suboptimality of the BF vector. For a small amount of the given users, the effect of BF is dominating when SNR level is low. However, as SNR increases, the suboptimality of the direction of the BF vector becomes a major constraint to improve the performance, and the proposed algorithm is surpassed by the TDMA/FDMA schemes gradually.

VI. Conclusion

In this paper, we have conceived a low-complexity cross-layer algorithm for DL CoMP, which promises a good performance for the MBS while significantly reducing both the amount of feedback and the algorithmic complexity. The scheduling scheme of the proposed algorithm efficiently exploited the knowledge of CSI at the receiver. For the BF part of the algorithm, we provided a new technique of designing robust BF vectors, when the CDI and CQI of the users are fed back to the BSs separately. Our numerical results demonstrated that our solution only moderately degraded the performance of the potentially excessive-complexity exhaustive search technique, despite having complexity as low as that of a conventional system operating without CoMP. We next present the proofs of the theorems stated earlier.

Appendix A
Proof of Theorem 1[1]

For the MBS, denote the channel directions of the scheduled user as $\tilde{\mathbf{h}}_1 = [h_1, h_2, \ldots, h_K]^T, \tilde{\mathbf{g}}_1 = [g_1, g_2, \ldots, g_K]^T$. Since $\mathbf{h}_1 \sim CN(0, \mathbf{I}_K)$, it follows that $[\mathrm{Re}(h_i), \mathrm{Im}(h_i)]^T \sim N(0, (1/2\mathbf{I}_2)$ and $[\mathrm{Re}(g_i), \mathrm{Im}(g_i)]^T \sim N(0, (1/2)\mathbf{I}_2)$. Thus, the random vector $\tilde{\mathbf{h}}_1$ spanning the complex space $\mathbb{C}^K$ equals a random vector confined to the real space, which can be formulated as $\bar{\mathbf{h}}_1 \in \mathbb{R}^{2K}$. The real random vector $\bar{\mathbf{h}}_1$ obeys the normal distribution of $N(0, (1/2)\mathbf{I}_{2K})$.

According to Section III-A, the goal of the scheduling algorithm is to find the specific user, whose channel directions $\tilde{\mathbf{h}}_1$ and $\tilde{\mathbf{g}}_1$ are "most different" from each other. Assuming $|\langle \tilde{\mathbf{h}}_1, \tilde{\mathbf{g}}_1 \rangle| = |\cos\theta|$, the probability density function of $|\cos\theta|$ can be expressed as

$$f(|\cos\theta|) = f\left(\left|\sum_{i=1}^{K} h_i g_i^*\right|\right) = f\left(\sqrt{A_3^2 + A_4^2}\right) \quad (16)$$

where $f(\cdot)$ denotes the probability density function of any random variable or random vector, and

$$A_3 = \sum_{i=1}^{K} [\mathrm{Re}(h_i)\mathrm{Re}(g_i) + \mathrm{Im}(h_i)\mathrm{Im}(g_i)] \quad (17)$$

$$A_4 = \sum_{i=1}^{K} [\mathrm{Re}(h_i)\mathrm{Im}(g_i) - \mathrm{Re}(g_i)\mathrm{Im}(h_i)]. \quad (18)$$

Now, if we define three new random vectors in the set of $\mathbb{R}^{2K}$

$$\bar{\mathbf{h}}^{(1)} = [\mathrm{Re}(h_1), \ldots, \mathrm{Re}(h_K), \mathrm{Im}(h_1), \ldots, \mathrm{Im}(h_K)]^T$$
$$\bar{\mathbf{g}}^{(1)} = [\mathrm{Re}(g_1), \ldots, \mathrm{Re}(g_K), \mathrm{Im}(g_1), \ldots, \mathrm{Im}(g_K)]^T$$
$$\bar{\mathbf{g}}^{(2)} = [\mathrm{Im}(g_1), \ldots, \mathrm{Im}(g_K), -\mathrm{Re}(g_1), \ldots, -\mathrm{Re}(g_K)]$$

then, (16) can be further simplified as

$$f(|\cos\theta|) = f\left(\sqrt{\left|(\bar{\mathbf{h}}^{(1)})^T\bar{\mathbf{g}}^{(1)}\right|^2 + \left|(\bar{\mathbf{h}}^{(1)})^T\bar{\mathbf{g}}^{(2)}\right|^2}\right). \quad (19)$$

Since we have $|(\bar{\mathbf{g}}^{(1)})^T\bar{\mathbf{g}}^{(2)}| = 0$, there exists $(2K-2)$ real-valued vectors with unit norms of $\bar{\mathbf{g}}^{(3)}, \ldots, \bar{\mathbf{g}}^{(2K)}$, which are orthogonal to each other, including $\bar{\mathbf{g}}^{(1)}$ and $\bar{\mathbf{g}}^{(2)}$. Thus, by letting $\mathbf{M} = [\bar{\mathbf{g}}^{(1)}, \bar{\mathbf{g}}^{(2)}, \ldots, \bar{\mathbf{g}}^{(2K)}]$, we have $|(\bar{\mathbf{h}}^{(1)})^T\mathbf{M}\mathbf{M}^T\bar{\mathbf{g}}^{(1)}|^2 = \mathrm{Re}(h_1)^2$ and $|(\bar{\mathbf{h}}^{(1)})^T\mathbf{M}\mathbf{M}^T\bar{\mathbf{g}}^{(2)}|^2 = \mathrm{Im}(h_1)^2$; thus

$$f(|\cos\theta|) = f\left(\sqrt{\mathrm{Re}(h_1)^2 + \mathrm{Im}(h_1)^2}|\bar{\mathbf{g}}^{(1)}\right) f\left(\bar{\mathbf{g}}^{(1)}\right). \quad (20)$$

This means that the value of $|\cos\theta|^2$ equals the sum of the squares of the two coordinates of $\bar{\mathbf{h}}^{(1)}$ along the two orthogonal dimensions. Additionally, note that the direction of the random vector $\bar{\mathbf{h}}^{(1)}$ is isotropic [26], which implies that the probability density function of $\tilde{\mathbf{h}}_1$ on the surface of the $2K$-D hypersphere with unit radius is $1/S_{2K}$, where we have $S_{2K} = 2K\pi^K/\Gamma(1+K)$. If we define $S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2 \le T_1^2}$ and $S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2 = r^2}$ to be the surface area of the hypersphere satisfying the constraint described in the subscript, the probability that a specific user's channel directions satisfy the threshold constraint denoted by P_1 can be expressed as

$$P_1 = P\left(\sqrt{\mathrm{Re}(\mathrm{h}_1)^2 + \mathrm{Im}(\mathrm{h}_1)^2} \le T_1\right)$$
$$= \frac{S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2 \le T_1^2}}{S_{2K}}. \quad (21)$$

[1]We discovered that similar results are derived in [25], where the authors directly computed the surface area of the unit hypersphere and spherical cap in complex space. The absolute surface area is not the same when considered in a K-D complex space and a $2K$-D real space, but the resulting probability is the same.

By exploiting

$$\begin{aligned}
&S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2\le T_1^2}\\
&= \int_0^{T_1} S_{\mathrm{Re}(h_1)^2+\mathrm{Im}(h_1)^2=r^2}\,dr\\
&= \int_0^{\arcsin T_1} \left[\frac{2(K-1)\pi^{K-1}r^{2K-3}}{\Gamma(K)}\right]_{r=\cos\theta} 2\pi\sin\theta d\theta\\
&= \frac{2\pi^K\left(1-\left(1-T_1^2\right)^{K-1}\right)}{\Gamma(K)}
\end{aligned} \tag{22}$$

we arrive at

$$P_1 = 1-\left(1-T_1^2\right)^{K-1}. \tag{23}$$

Since there are N_1 users in the cell, the probability that there is at least one user that satisfies the threshold constraint can be expressed as

$$P_{\mathrm{suc1}} = 1-(1-P_1)^{N_1}. \tag{24}$$

With the aid of (23), we finally have

$$P_{\mathrm{suc1}} = 1-\left(1-T_1^2\right)^{N_1(K-1)}. \tag{25}$$

Appendix B
Proof of Lemma 1

Let

$$\mathbf{w}_1 = \frac{\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{g}}_1\rangle\hat{\mathbf{g}}_1}{\|\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{g}}_1\rangle\,\hat{\mathbf{g}}_1\|}. \tag{26}$$

Then, $\mathbf{w}_1$ is orthogonal to $\hat{\mathbf{g}}_1$, and it can be expressed as a linear combination of $\hat{\mathbf{g}}_1$ and $\mathbf{w}$. Thus, we can assume that

$$\mathbf{w} = \varphi_1\mathbf{w}_1+\varphi_2\hat{\mathbf{g}}_1 \tag{27}$$

$$\tilde{\mathbf{g}}_1 = \theta_1\mathbf{w}_1+\theta_2\hat{\mathbf{g}}_1+\theta_3\mathbf{u} \tag{28}$$

where vector $\mathbf{u}$ is an arbitrary vector with unit norm and orthogonal to both $\hat{\mathbf{g}}_1$ and $\mathbf{w}$. Additionally, we should also note that, in the given equations

$$|\varphi_1|^2+|\varphi_2|^2 = |\theta_1|^2+|\theta_2|^2+|\theta_3|^2 = 1. \tag{29}$$

Thus, we have

$$|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle| = |\varphi_1\theta_1^*+\varphi_2\theta_2^*| \le |\varphi_1\theta_1|+|\varphi_2\theta_2| \tag{30}$$

where the equality on the right side holds if and only if we have

$$\arg\left(\varphi_1\theta_1^*\right) = \arg\left(\varphi_2\theta_2^*\right). \tag{31}$$

According to (28), we have

$$|\langle\tilde{\mathbf{g}}_1,\hat{\mathbf{g}}_1\rangle| = |\theta_2| = \lambda_2 \ge \lambda_{2\,\min} \ge \lambda_1 = |\varphi_2|. \tag{32}$$

We can see from the given equation that the maximum value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ is achieved when θ_3 is zero. If not, we can force θ_3 to zero and multiply both θ_1 and θ_2 by a factor of $1/\sqrt{1-|\theta_3|^2}$. According to (30), the value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ increases. Since $|\theta_2|$ is always larger than $|\varphi_2|$, according to Chebyshev's inequality, we can see that increasing the value of $|\varphi_2|$ will result in a monotonic increase in the value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$. Thus, the maximum value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ is achieved, when $\lambda_2 = \lambda_{2\,\min}$. This means that the maximum value of $|\langle\mathbf{w},\tilde{\mathbf{g}}_1\rangle|$ is achieved, when the actual channel direction falls on the quantization cell boundary between the BF vector and the unit vector representing the quantized channel direction.

Appendix C
Proof of Lemma 2

Lemma 2 can be proven in the same way as Lemma 1. Let

$$\mathbf{w}_1 = \frac{\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{h}}_1\rangle\hat{\mathbf{h}}_1}{\|\mathbf{w}-\langle\mathbf{w},\hat{\mathbf{h}}_1\rangle\hat{\mathbf{h}}_1\|}. \tag{33}$$

Then, vector $\mathbf{w}_1$ is orthogonal to $\hat{\mathbf{h}}_1$, and it can be expressed as a linear combination of $\mathbf{w}$ and $\hat{\mathbf{h}}_1$. Thus, we can assume that

$$\mathbf{w} = \varphi_3\mathbf{w}_1+\varphi_4\hat{\mathbf{h}}_1 \tag{34}$$

$$\tilde{\mathbf{h}}_1 = \theta_4\mathbf{w}_1+\theta_5\hat{\mathbf{h}}_1+\theta_6\mathbf{u} \tag{35}$$

where vector $\mathbf{u}$ is an arbitrary vector with a unit norm and orthogonal to both $\hat{\mathbf{h}}_1$ and $\mathbf{w}$. Since the norms of both $\mathbf{w}$ and $\tilde{\mathbf{h}}_1$ are 1, it easily follows that

$$|\varphi_3|^2+|\varphi_4|^2 = |\theta_4|^2+|\theta_5|^2+|\theta_6|^2 = 1. \tag{36}$$

Thus, we have

$$|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle| = |\varphi_3\theta_4^*+\varphi_4\theta_5^*| \ge \|\varphi_3\theta_4|-|\varphi_4\theta_5\|. \tag{37}$$

It is clear from (34) and (35) that $|\varphi_4| = |\lambda_1|$ and $|\theta_5| = \lambda_2$. Since $\lambda_2 \ge \sqrt{1-\lambda_1^2}$, we have $\lambda_1 \ge \sqrt{1-\lambda_2^2}$. Thus

$$\|\varphi_3\theta_4|-|\varphi_4\theta_5\| = \lambda_1\lambda_2-\sqrt{\left(1-\lambda_1^2\right)\left(1-\lambda_2^2-|\theta_6|^2\right)}. \tag{38}$$

In the given equation, we first observe that when $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is minimized, $|\theta_6|$ has to be zero. This is obvious since, for a given λ_1, if we hold λ_2 at a fixed value smaller than 1, increasing the value of $|\theta_6|$ will result in a decrease in $\sqrt{(1-\lambda_1^2)(1-\lambda_2^2-|\theta_6|^2)}$, and if the value of $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is minimized, $|\theta_6|$ increases. Additionally, when $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is minimized, $|\theta_6|$ is minimized, and $\lambda_2 = \lambda_{2\,\min}$. This is because when λ_2 decreases, $\lambda_1\lambda_2$ decreases, and $\sqrt{(1-\lambda_1^2)(1-\lambda_2^2-|\theta_6|^2)}$ increases at the same time.

With the given observations, the lemma is proven. Note that this lemma tells us that the minimum of $|\langle\mathbf{w},\tilde{\mathbf{h}}_1\rangle|$ is achieved when the actual channel direction $\tilde{\mathbf{h}}_1$ is away from both the BF vector $\mathbf{w}$ and the quantized channel direction $\hat{\mathbf{h}}_1$ and falls on the quantization cell boundary.

$$\mathrm{SINR}_1 = \frac{p_{11}\|\mathbf{h}_1\|^2\left(\lambda_{2\min}|\xi_5| - \sqrt{\left(1-\lambda_{2\min}^2\right)\left(1-|\xi_5|^2\right)}\right)^2}{1 + p_{21}\|\mathbf{g}_1\|^2\left(|\lambda_{2\min}\xi_2| + \sqrt{\left(1-\lambda_{2\min}^2\right)\left(1-|\xi_2|^2\right)}\right)^2} \tag{45}$$

APPENDIX D
PROOF OF THEOREM 2

Let

$$\mathbf{g}_1^\dagger = \frac{\hat{\mathbf{g}}_1 - \langle\hat{\mathbf{g}}_1, \hat{\mathbf{h}}_1\rangle\hat{\mathbf{h}}_1}{\|\hat{\mathbf{g}}_1 - \langle\hat{\mathbf{g}}_1, \hat{\mathbf{h}}_1\rangle\hat{\mathbf{h}}_1\|} \tag{39}$$

where $\mathbf{h}_1^\dagger$ is orthogonal to $\hat{\mathbf{g}}_1$ and can be expressed as a linear combination of $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$. Vector $\mathbf{g}_1^\dagger$ is orthogonal to $\hat{\mathbf{h}}_1$ and can be also expressed as a linear combination of $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$. Assuming that vector $\mathbf{u}_1$ is an arbitrary vector with a unit norm and orthogonal to both $\mathbf{h}_1^\dagger$ and $\hat{\mathbf{h}}_1$ and that vector $\mathbf{u}_2$ is an arbitrary vector with a unit norm and orthogonal to both $\mathbf{g}_1^\dagger$ and $\hat{\mathbf{g}}_1$, we have

$$\mathbf{w} = \xi_1\mathbf{h}_1^\dagger + \xi_2\hat{\mathbf{g}}_1 + \xi_3\mathbf{u}_1 = \xi_4\mathbf{g}_1^\dagger + \xi_5\hat{\mathbf{h}}_1 + \xi_6\mathbf{u}_2 \tag{40}$$

$$\hat{\mathbf{h}}_1 = \delta_1\mathbf{h}_1^\dagger + \delta_2\hat{\mathbf{g}}_1 \tag{41}$$

$$\hat{\mathbf{g}}_1 = \delta_3\mathbf{g}_1^\dagger + \delta_4\hat{\mathbf{h}}_1. \tag{42}$$

In the given equations, since the norms of $\mathbf{w}$, $\hat{\mathbf{h}}_1$, and $\hat{\mathbf{g}}_1$ are all 1, it follows that

$$\sum_{i=1}^{3}|\xi_i|^2 = \sum_{i=4}^{6}|\xi_i|^2 = 1 \tag{43}$$

$$|\delta_1|^2 + |\delta_2|^2 = |\delta_3|^2 + |\delta_4|^2 = 1. \tag{44}$$

Thus, with the aid of (40) to (44) and the two lemmas, for a given set of $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$, the minimum of SINR_1 takes the form of (45), shown at the top of the page. Additionally, we also have

$$|\xi_5| = \left|\langle\mathbf{w}, \hat{\mathbf{h}}_1\rangle\right| = |\xi_1\delta_1^* + \xi_2\delta_2^*| \le |\xi_1\delta_1| + |\xi_2\delta_2|. \tag{46}$$

When the minimum SINR in (45) is maximized, the equality on the right side of (46) holds. This is because, when $|\xi_5|$ increases, the numerator of (45) increases, and increasing the value of $|\xi_5|$ can be achieved by changing only the principles of ξ_1 and ξ_2, which will not affect the value of the denominator in the equation. Additionally, we observe that, when the minimum SINR in (45) is maximized, we have $|\xi_3| = 0$. The proof exploits that, if the maximum value of this SINR_1 is achieved when the BF vector $\mathbf{w}$ is not on the same complex plane with both $\hat{\mathbf{g}}_1$ and $\hat{\mathbf{h}}_1$, we have $|\xi_3\xi_6| \neq 0$. In this case, we can hold $|\xi_2|$ at a fixed value, and set ξ_3 to 0. This will result in an increase in $|\xi_1|$, and since

$$|\xi_5| = |\delta_1\xi_1| + |\delta_2\xi_2| \tag{47}$$

the value of $|\xi_5|$ increases, resulting in an increase in the value of SINR_1. Upon combining (45) and (47), we arrive at (14); hence, the theorem is proven.

REFERENCES

[1] M. Sawahashi, Y. Kishiyama, A. Morimoto, D. Nishikawa, and M. Tanno, "Coordinated multipoint transmission/reception techniques for LTE-advanced," *IEEE Wireless Commun.*, vol. 17, no. 3, pp. 26–34, Jun. 2010.

[2] R. Irmer, H. Droste, P. Marsch, M. Grieger, G. Fettweis, S. Brueck, H. P. Mayer, L. Thiele, and V. Jungnickel, "Coordinated multipoint: Concepts, performance, and field trial results," *IEEE Commun. Mag.*, vol. 49, no. 2, pp. 102–111, Feb. 2011.

[3] D. Choi, D. Lee, and J. H. Lee, "Resource allocation for CoMP with multiuser MIMO-OFDMA," *IEEE Trans. Veh. Technol.*, vol. 60, no. 9, pp. 4626–4632, Nov. 2011.

[4] R. Bhagavatula and R. W. Heath, "Adaptive limited feedback for sum-rate maximizing beamforming in cooperative multicell systems," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 800–811, Feb. 2011.

[5] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[6] T. Wu and V. Lau, "Design and analysis of multi-user SDMA systems with noisy limited CSIT feedback," *IEEE Trans. Wireless Commun*, vol. 9, no. 4, pp. 1446–1450, Apr. 2010.

[7] T. Wu and V. Lau, "Robust precoder adaptation for MIMO links with noisy limited feedback," *IEEE Trans. Inf. Theory*, vol. 55, no. 4, pp. 1640–1649, Apr. 2009.

[8] C. Murthy, J. Zheng, and B. Rao, "Performance of quantized equal gain transmission with noisy feedback channels," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2451–2460, Jun. 2008.

[9] A. Housfater and T. J. Lim, "Throughput of precoded broadcast transmission with noisy feedback," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2009, pp. 398–402.

[10] A. Housfater and T. J. Lim, "Noisy feedback linear precoding: A bayesian cramér-rao bound," in *Proc. Int. Symp. Inf. Theory*, Jul. 2009, pp. 1689–1693.

[11] I. S. Dhillon, R. W. Heath, T. Strohmer, and J. A. Tropp, "Constructing packings in Grassmannian manifolds via alternating projection," *Exp. Math.*, vol. 17, no. 7, pp. 9–35, 2008.

[12] Y. Cheng, V. Lau, and Y. Long, "A scalable limited feedback design for network MIMO using per-cell product codebook," *IEEE Trans. Wireless Commun.*, vol. 9, no. 10, pp. 3093–3099, Oct. 2010.

[13] X. Hou and C. Yang, "Codebook design and selection for multi-cell cooperative transmission limited feedback systems," in *Proc. IEEE 73rd VTC*, May 2011, pp. 1–5.

[14] D. Su, X. Hou, and C. Yang, "Quantization based on per-cell codebook in cooperative multi-cell systems," in *Proc. IEEE WCNC*, Mar. 2011, pp. 1753–1758.

[15] X. Hou and C. Yang, "How much feedback overhead is required for base station cooperative transmission to outperform non-cooperative transmission?" in *Proc. IEEE ICASSP*, May 2011, pp. 3416–3419.

[16] W. Yu, T. Kwon, and C. Shin, "Multicell coordination via joint scheduling, beamforming and power spectrum adaptation," in *Proc. IEEE INFOCOM*, Apr. 2011, pp. 2570–2578.

[17] P. Viswanath, D. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, Jun. 2002.

[18] D. Park and G. Caire, "Hard fairness versus proportional fairness in wireless communications: The multiple-cell case," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2008, pp. 2036–2040.

[19] S. A. Vorobyov, A. B. Gershman, and Z.-Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 313–324, Feb. 2003.

[20] R. Lorenz and S. Boyd, "Robust minimum variance beamforming," *IEEE Trans. Signal Process*, vol. 53, no. 5, pp. 1684–1696, May 2005.

[21] A. Abdel-Samad, A. B. Gershman, and T. N. Davidson, "Robust transmit beamforming based on imperfect channel feedback," in *Proc. IEEE 60th VTC-Fall*, Sep. 2004, vol. 3, pp. 2049–2053.

[22] E. Gharavol, Y.-C. Liang, and K. Mouthaan, “Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 6, pp. 2852–2860, Jul. 2010.

[23] W. Santipach and M. Honig, “Asymptotic capacity of beamforming with limited feedback,” in *Proc. ISIT*, Jul. 2004, p. 290.

[24] T. Yoo, N. Jindal, and A. Goldsmith, “Multi-antenna downlink channels with limited feedback and user selection,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sep. 2007.

[25] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, “On beamforming with finite rate feedback in multiple-antenna systems,” *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2562–2579, Oct. 2003.

[26] G. Marsaglia, “Choosing a point from the surface of a sphere,” *Ann. Math. Statist.*, vol. 43, no. 2, pp. 645–646, 1972.

Yingxiang Yang received the B.E. degree from Tsinghua University, Beijing, China, in 2012. He is currently working toward the Ph.D. degree with Rutgers University, New Brunswick, NJ, USA.

His research interests include communications theory, optimization theory, and game theory.

Mr. Yang has been a Peer Reviewer for the IEEE Vehicular Technology Conference since 2011.

Bo Bai (S’09–M’11) received the B.S. degree (first-class honors) from Xidian University, Xi’an, China, in 2004 and the Ph.D. degree from Tsinghua University, Beijing, China, in 2010.

From 2009 to 2012, he was a Visiting Research Staff (Research Assistant from April 2009 to September 2010 and Research Associate from October 2010 to April 2012) with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong. He is currently an Assistant Professor with the Department of Electronic Engineering, Tsinghua University. His research interests include hot topics in wireless communications, information theory, random graph, and combinatorial design.

Dr. Bai was a Session Chair of the IEEE Global Communications Conference (IEEE GLOBECOM) in 2008 and has served as a member of the Technical Program Committee of the IEEE International Communications Conference in 2010 and 2012 and the IEEE International Conference on Communications in China and the IEEE International Conference on Connected Vehicles and Expo in 2012. He was invited as a Young Scientist Speaker to the IEEE Technology Time Machine in 2011. He has also served as a Reviewer for a number of major IEEE journals and conferences. He received a Student Travel Grant at the IEEE GLOBECOM in 2009 and support from the Backbone Talents Supporting Project of Tsinghua University. He also received the Young Academic Talent of Electronic Engineering Award from Tsinghua University and the Outstanding Graduates of Shaanxi Province Award.

Wei Chen (SM’13) received the B.S. and Ph.D. degrees in electronic engineering (first-class honors) from Tsinghua University, Beijing, China, in 2002, and 2007, respectively.

From 2005 to 2007, he was a Visiting Research Staff with the Hong Kong University of Science and Technology, Hong Kong. Since July 2007, he has been with the Department of Electronic Engineering, Tsinghua University, where he is currently a Full Professor and the Vice-Director of the Institute of Communications. From June 2010 to September 2010, he was a Visiting Researcher with the University of Southampton, Southampton, U.K. He is a 973 Youth Project Chief Scientist and is supported by the New Century Talent Program of the Chinese Ministry of Education, the Beijing Nova Program, and 100 fundamental research talents program of Tsinghua University (also known as the 221 Talents Program). His research interests include wireless communications, information theory, and applied optimization.

Dr. Chen served as a Vice-Director of the Youth Committee of the China Institute of Communications, a Tutorial Cochair of the 2013 IEEE International Conference on Communications, a track Cochair of the Wireless Track of the IEEE Consumer Communications and Networking Conference in 2013, a Cochair of the Technical Program Committee (TPC) of the IEEE Vehicular Technology Conference in Spring 2011, the Publication Chair of the IEEE International Conference on Communications in China in 2012, a TPC Cochair of the Wireless Communication Symposium at the 2010 IEEE International Conference on Communications (ICC), and a Student Travel Grant Chair of ICC 2008. He also served as an Editor for IEEE WIRELESS COMMUNICATIONS LETTERS. He received the 2010 IEEE Communication Society Asia–Pacific Board Best Young Researcher Award; the 2009 IEEE Marconi Prize Paper Award; the Best Paper Awards at the IEEE ICC in 2006, the First IEEE International Workshop on Cross Layer Design in 2007, and the IEEE Smart Gird Commmunications Conference in 2012; the 2011 Tsinghua Rising Academic Star Award; the 2012 Tsinghua Young Faculty Teaching Excellence Award; the First Prize at the First National Young Faculty Teaching Competition; the First Prize at the Seventh Beijing Young Faculty Teaching Competition; the First Prize at the Fifth Tsinghua University Young Faculty Teaching Competition; and the National May 1st Labor Medal.

Lajos Hanzo (M’91–SM’92–F’04) received the M.S. degree (with first-class honors) in electronics and the Ph.D. degree from the Technical University of Budapest, Budapest, Hungary, in 1976 and 1983, respectively, the D.Sc. degree from the University of Southampton, Southampton, U.K., in 2004, and the “Doctor Honoris Causa” degree from the Technical University of Budapest in 2009.

During his 35-year career in telecommunications, he has held various research and academic posts in Hungary, Germany, and the U.K. Since 1986, he has been with the School of Electronics and Computer Science, University of Southampton, Southampton, U.K., where he holds the Chair for Telecommunications. Since 2009, he has been a Chaired Professor with Tsinghua University, Beijing China. He is currently directing a 100-strong academic research team, working on a range of research projects in the field of wireless multimedia communications sponsored by industry; the Engineering and Physical Sciences Research Council, U.K.; the European IST Programme; and the Mobile Virtual Centre of Excellence, U.K. He is an enthusiastic supporter of industrial and academic liaison and offers a range of industrial courses. He has successfully supervised 80 Ph.D. students, coauthored 20 John Wiley/IEEE Press books on mobile radio communications totaling in excess of 10 000 pages, published more than 1250 research entries on IEEE Xplore, and presented keynote lectures. (For further information on research in progress and associated publications, please refer to http://www-mobile.ecs.soton.ac.uk.)

Dr. Hanzo is Fellow of the Royal Academy of Engineering, U.K., a Fellow of the Institution of Electrical Engineers, and a Governor of the IEEE Vehicular Technology Society. He has been a Technical Program Committee Chair and a General Chair for IEEE conferences. During 2008–2012, he was the Editor-in-Chief of the IEEE Press. He has received a number of distinctions.

AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES

AQ1 = Please provide keywords

END OF ALL QUERIES