Nonlinearity in the vertical transmissibility of seating: the role of human body apparent mass and seat dynamic stiffness

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Abstract

The efficiency of a seat in reducing vibration depends on the characteristics of the vibration, the dynamic characteristics of the seat, and the dynamic characteristics of the person sitting on the seat. However, it is not known whether seat cushions influence the dynamic response of the human body, whether the human body influences the dynamic response of seat cushions, or the relative importance of human body nonlinearity and seat nonlinearity in causing nonlinearity in measures of seat transmissibility. This study was designed to investigate the nonlinearity of coupled seat and human body systems and compare the apparent mass of the human body supported on rigid and foam seats. A frequency domain model was used to identify the dynamic parameters of seat foams and investigate their dependence on subject sitting weight and hip breadth. With 15 subjects, the force and acceleration at the seat base and acceleration at the subject interface were measured during random vertical vibration excitation (0.25 to 25 Hz) at each of five vibration magnitudes, (0.25 to 1.6 ms^{-2} r.m.s.) with four seating conditions (rigid flat seat and three foam cushions). The measurements are presented in terms of the subject apparent mass on the rigid and foam seat surfaces, and the transmissibility and dynamic stiffness of each of the foam cushions. Both the human body and the foams showed nonlinear softening behaviour, which resulted in nonlinear cushion transmissibility. The apparent masses of subjects sitting on the rigid seat and on foam cushions were similar, but with an apparent increase in damping when sitting on the foams. The foam dynamic stiffness showed complex correlations with characteristics of the human body, which differed between foams. The nonlinearities in cushion transmissibilities, expressed in terms of changes in resonance frequencies and moduli, were more dependent on human body nonlinearity than on cushion nonlinearity.

Keywords: seat transmissibility; nonlinearity; apparent mass; polyurethane foam; dynamic stiffness

1. Introduction

The extent to which a seat modifies the vibration on the floor in any form of transport depends on the characteristics of the vibration, the characteristics of the seat, and the characteristics of the person sitting on the seat. Since the vibration on a seat can influence human comfort, the performance of activities, and human health, it is appropriate to select vehicle seats for their efficiency in isolating the vibration that has the greatest adverse influence [1].

The laboratory measurement of the vibration transmissibility of a seat usually involves the exposure of human subjects to vibration using specialised simulation facilities with the need for safety precautions. Differences between people lead to the requirement to test a group of representative subjects and consequent costs in both time and money. For this reason research has investigated methods of predicting seat transmissibility without involving human subjects. The vertical transmissibility of a seat can be determined without human subjects by: (i) using anthropodynamic dummies [2]; (ii) predicting the transmissibility from the measured seat dynamic stiffness and the measured apparent mass of the human body [3]; (iii) using mathematical models of the dynamic response of the seat and the human body [4]. Predictions of vertical seat transmissibility show differences from the transmissibility measured with human subjects, suggesting there are limitations to the models or the associated assumptions.

It is commonly assumed that the apparent mass of the body and the dynamic stiffness of a seat do not change when they are coupled together. The apparent mass of the human body is usually measured on rigid flat surfaces (e.g. [5]): different pressure distributions and contact areas may change the responses of soft tissues to vibration [6][7]. The dynamic stiffness of a seat is measured using rigid indenters of specific size and shape [4][8], or derived from curve fitting of transmissibility data [5], with little study of the effect of contact area, contact shape, vibration magnitude and vibration spectral content. The dynamic stiffness of foam also varies with changes in the weight it supports [3][4], and it is not clear how vibration magnitude, subject weight, and contact area combine to determine the dynamic stiffness of seating foam [5][8][9][10].

Although linear behaviour is commonly assumed, both the human body [11][12] and polyurethane foams used in seat construction [5][8] are nonlinear systems that soften with increasing magnitude of vibration. Consequently, the vertical transmissibilities of seats are nonlinear, with the resonance frequencies reducing as the vibration magnitude increases [3][5]. While the effect of vibration magnitude [14] and spectral content [15] on human

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apparent mass has been studied, it is not yet possible to predict how the apparent mass of the human body varies with the vibration input. The apparent mass of the body and the dynamic stiffness of seats may be measured using broadband random vibration with equal energy over a specified frequency range, so as to ensure generality and allow repeatability, but such a spectrum is not representative of the vibration in transport [16]. The nonlinearity in both systems results in the measured apparent mass of the body and the measured dynamic stiffness of the seat not being appropriate for the conditions in which the seat is used. When using a specific apparent mass for the human body and a specific dynamic stiffness for a seat, the linear prediction of seat transmissibility will only apply to a limited range of vibration conditions.

Models representing the vertical dynamic responses of the human body and seats could be developed with model parameters that are dependent on the magnitude and spectra of the input vibration [13][17][18]. However, the non-linearity of the human body is also influenced by vibration in other directions [19], and so useful non-linear models of the body that are generally applicable are not yet available.

The study reported in this paper was designed to determine experimentally the human body apparent mass and the seat dynamic stiffness by measuring them simultaneously on a range of different conditions (hard and soft seats, five vibration levels, fifteen subjects). The paper seeks to clarify some common assumptions about factors affecting the transmission of vertical vibration through seats, especially: (i) the influence of soft seating of human body apparent mass, (ii) the factors influencing seat dynamic response, and (iii) the relative influence of human body nonlinearity and seat nonlinearity on the nonlinearity seen in measures of seat transmissibility.

2. Method

2.1 Apparatus
Vertical dynamic force was measured using a Kistler force platform (model 9281B) with an aluminium top plate secured to a rigid seat attached to a 1-m stroke vertical hydraulic vibrator in the laboratories of the Human Factors Research Unit at the Institute of Sound and Vibration Research (Figure 1, left). The charge signal from vertical force cells at the four corners of the force platform were summed and amplified by a Kistler 5001 amplifier.

FIGURE 1 ABOUT HERE

Vertical acceleration was measured using an Endevco 2265/10 accelerometer at the centre of the force plate and a 2265/20 accelerometer in a SIT-bar placed between the seat cushion and the subject buttocks [20].

The force and acceleration signals were amplified and low-pass filtered (3-pole Butterworth, 50-Hz cut-off frequency) by Fylde signal conditioning before being acquired to computer at 256 samples/second via a National Instrument NI-USB-6251 DAQ, controlled by *HVLab* Matlab Toolbox software.

Three blocks of polyurethane foams suitable for automotive seats were selected from a larger sample so as to represent a broad range of dynamic characteristics (Table 1). The upper flat surface of each foam block was 400 mm wide by 450 mm thick. The lower surfaces of the foams were flat, apart from a 40-mm reduction in thickness over 60-mm wide strips on both sides. The foams (with no covers) were supported on a wooden base (weighing 2.1 kg) resting on the aluminium plate secured to the force platform

TABLE 1 ABOUT HERE

2.2 Experimental design
Fifteen male subjects participated in the study. Their ages, standing weights, statures, and hip breadths were measured as described by Pheasant [21] (see Table 2). The sitting weight shown in Table 2 for each subject is the median measured apparent mass at 0.625 Hz, as determined when sitting on the seat foams and measured as described below. The ratio of the sitting weight to the standing weight had a median of 72%.

TABLE 2 ABOUT HERE

The apparent masses of the subjects were determined in four conditions: while they sat on the force plate without a cushion (i.e. rigid seat condition) and while they sat on each of the three blocks of foam placed on the force plate. The subjects were instructed to sit in an erect posture with no backrest contact, with their lower legs vertical and their hands on their laps. A footrest supported on the moving platform of the vibrator was adjusted in height so as to maintain the uncompressed cushion surface 300 mm above the feet. So as to reduce the influence of foam relaxation on seat properties, the subjects sat on the foam blocks for at least 3 minutes before starting the dynamic tests.

For each of the four seating conditions, the force plate was excited for 60 seconds using 0.25 to 25 Hz Gaussian random vibration at each of five magnitudes: 0.25, 0.4, 0.63, 1.0, and 1.6 ms^{-2} r.m.s.

The order of presenting the four seating conditions was randomised, as well as the order of presenting the vibration magnitudes with each seat.

2.3 Human-seat model

Using the signals from the two accelerometers and the force transducer, it was possible to calculate: the foam transmissibility, $T(f)$, between the seat base acceleration, $a_1(f)$, and the seat pan acceleration, $a_2(t)$; the apparent mass of the subject, $AM(f)$; and the dynamic stiffness of the foam, $S(f)$. As derived from Figure 1 (right), $AM(f)$ is the complex ratio (i.e. the transfer function) of $F_3(f)$ to $a_2(f)$. While the acceleration at the human-seat interface, $a_2(t)$, was directly measured by the accelerometer in the SIT-bar; the force, $F_3(t)$, at the human-seat interface was derived (see below).

The foam was supported on the force platform, so $F_1(t)$ was the gross force measured by force transducers. The foam was assumed to be a pure complex stiffness element, with its mass m_{seat} added to the mass of the plate m_{plate} for mass cancellation. It follows that [3]:

$$F_2(t) = F_1(t) - \left(m_{plate} + m_{seat}\right) a_1(t) = F_3(t) + m_{SIT-bar} a_2(t)$$

$$\Delta a(t) = a_2(t) - a_1(t)$$

$$S(f) = \frac{F_2(f)}{\Delta a(f)} (2\pi f)^2 = M(f)(2\pi f)^2$$

$$AM(f) = \frac{F_3(f)}{a_2(f)}$$

$$T(f) = \frac{a_2(f)}{a_1(f)} = \frac{M(f)}{M(f) + AM(f) + m_{SIT-bar}} \tag{1}$$

Mass cancellation for the mass of the force platform above the force sensors and the mass of the seat and wooden frame was performed in the time domain, while $S(f)$ and $AM(f)$ are frequency domain response functions and were determined from the ratio of the input-output cross spectrum to the power spectral density of the input [22]:

$$H_{io}(f) = \frac{G_{io}(f)}{G_{ii}(f)}.$$

For each quantity, the coherence function was calculated as [22]:

$$\gamma_{io}^{2}(f) = \frac{\left|G_{io}(f)\right|^{2}}{G_{ii}(f)G_{oo}(f)}$$

The transfer functions were determined with a resolution of 0.125 Hz and 32 statistically independent samples (degrees-of-freedom).

The normalized apparent masses of the subjects were determined from their apparent mass function divided per their quasi-static sitting weights calculated from their apparent mass at 0.625 Hz.

2.4 Evaluation of nonlinearity

For each subject sitting on each seat foam, the resonance frequencies of both the apparent mass and the seat transmissibility were determined from the maxima of the absolute values of apparent mass and transmissibility, respectively, with each of the five vibration magnitudes (0.25, 0.4, 0.63, 1.0, and 1.6 ms^{-2} r.m.s.). The maximum difference in the resonance frequency across the five magnitudes was used as the measure of nonlinearity. Equation (1) was used to estimate the separate influences of the nonlinearity in the human body and the nonlinearity of the seat foam on the seat transmissibility. To evaluate the influence of the human body nonlinearity, the subject apparent mass at each of the five magnitudes was substituted in Equation (1), with the dynamic stiffness held constant at the appropriate reference magnitude (0.25, 0.4, 0.63, 1.0, or 1.6 ms^{-2} r.m.s.). As result, five estimated transmissibility functions were obtained for each vibration magnitude. To evaluate the influence of seat nonlinearity on transmissibility, the subject apparent mass was held constant at the value measured with each vibration magnitude and the five dynamic stiffness functions were substituted in Equation (1). The maximum difference in the resonance frequency over the five vibration magnitudes in each case (either the dynamic stiffness held constant or the apparent mass held constant) was used as the measure of nonlinearity.

2.5 Seat Dynamic Stiffness model

Previous studies [9, 2] and preliminary data indicated that the real and imaginary stiffness of foam tend to increase with increasing frequency of vibration. It was therefore considered appropriate to model the dynamic stiffness of the foam using a linear model composed of pure stiffness, *k*, and viscous damping, c, with the addition of a linear frequency dependence in the stiffness, *k'*, and an hysteretic component, *c'* [23]:

$$S(f) = k + \omega k' + i(\omega c + c') \tag{2}$$

where $\omega = 2\pi f$ is the frequency in radians per second. The foam stiffness, k, results in a force in-phase with the foam displacement. The frequency-dependence in the stiffness, k', may be explained by reduced airflow through the foam at increased deformation rates: the air is trapped in the foam and contributes to the stiffening. The damping, c, represents the energy loss resulting from the movement of air through the polymer cellular matrix, while the viscous equivalent hysteretic damping, c', is related to the characteristics of the base polymer [10].

Curve fitting was performed by minimising the least square error function between the measurements and Equation (2) at frequencies over the range 4 to 20 Hz.

3. Results

3.1 Apparent Mass – Rigid and soft seat

The effects of vibration magnitude on the median apparent mass, median normalised apparent mass, and median phase over the 15 subjects are shown for each of the four conditions (sitting on the rigid seat and sitting on the three foams) in Figure 2. The apparent mass had a main resonance peak at about 5 Hz, with some subjects showing a second resonance with lower apparent mass in the range 8 to 17 Hz (see individual data in Figure 7). An increase in the modulus of the apparent mass was occasionally apparent at about 2 Hz. Median curves show a small phase lead, with a maximum at about 1 Hz, in the rigid seat condition for all magnitudes and in the hard foam condition at the lowest magnitude. In the rigid seat condition, the magnitude of the phase lead decreases with increasing vibration magnitude.

FIGURE 2 ABOUT HERE

With all four seating conditions there was a significant overall effect of vibration magnitude on the apparent mass resonance frequency (Friedman two way analysis of variance, $p < 0.05$, where the p-value indicates the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true [24]). The frequency of the main resonance decreased with each incremental increase in the vibration magnitude (Wilcoxon matched pairs signed rank test [24], $p < 0.05$), except for foam 3 from 0.25 to 0.4 ms^{-2} r.m.s., consistent with a softening system. There was no significant effect of vibration magnitude on the modulus of the apparent mass at resonance (Friedman, $p > 0.05$). Increases in vibration magnitude decreased the apparent mass at 20 Hz (Wilcoxon, $p < 0.05$), except for foam 1 (from 0.25 to 0.4 ms^{-2} r.m.s.), foam 2 (from 0.25 to 0.4 ms^{-2} r.m.s.), and foam 3 (from 0.25 to 0.4 ms^{-2} r.m.s., and from 0.4 to 0.63 ms^{-2} r.m.s.).

The effects of seating condition on the median apparent mass, median normalised apparent mass, and median phase over the 15 subjects are shown for each magnitude of vibration in Figure 3. Although the footrest height was the same for each seating condition, the quasi-static mass (i.e. the apparent mass at 0.625 Hz) was slightly less (by about 4 kg) when sitting on the rigid seat than when sitting on any of the foams. There were no significant differences in the resonance frequencies of the apparent mass between the four seats at any vibration magnitude (Friedman, $p > 0.05$). However, at all five vibration magnitudes, the apparent mass at resonance was about 9 kg greater on the rigid seat (Wilcoxon, $p < 0.05$). It may be seen that the apparent mass at 20 Hz was slightly reduced on the rigid seat and that the phase lag was greater for the rigid seat at frequencies from 5 to 20 Hz. Comparing Figures 2 and 3, it can be seen that subject apparent mass was more influenced by changes in vibration magnitude than changes in seating condition (i.e. sitting on a rigid seat or sitting on a block of foam).

FIGURE 3 ABOUT HERE

At 20 Hz, the normalised apparent mass at all vibration magnitudes was significantly less on the rigid seat than on the foam (Wilcoxon, $p < 0.01$)

3.2 Dynamic Stiffness – Effect of Vibration Magnitude

The measured dynamic stiffness, $S(f)$, seemed to be well represented by the model using the four parameters in Equation (2) (Figure 4). The coefficient of determination R^2 of the linear regression had a mean of 0.924 (range 0.736 to 0.988).

FIGURE 4 ABOUT HERE

There were significant effects of vibration magnitude on the stiffness parameters k and k' (Friedman, $p < 0.05$). As the vibration magnitude increased, the dynamic stiffness of each of the three foams showed significant overall trends for a decrease in both the stiffness, k, and the stiffness frequency-dependence, k' ($p < 0.05$, Wilcoxon; Figure 5). However, some increases in the magnitude of vibration did not produce statistically significant changes in dynamic stiffness, k (foam 1: 0.25 to 0.4, and 0.4 to 0.63 ms^{-2} r.m.s.; foam 2: 0.25 to 0.4, and 0.4 to 0.63 ms^{-2} r.m.s.; foam 3: 0.25 to 0.4, 0.25 to 0.4 and 0.4 to 0.63, and 0.63 to 1.0 ms^{-2} r.m.s.; $p > 0.05$, Wilcoxon) or k' (foam 1: 0.4 to 0.63 ms^{-2} r.m.s.; foam 2: 0.25 to 0.4 and 0.4 to 0.63 ms^{-2} r.m.s.; foam 3: 0.4 to 0.63 ms^{-2} r.m.s.; $p > 0.05$, Wilcoxon). The magnitude of the vibration had no effect on the viscous damping, c, ($p > 0.05$, Friedman). The hysteretic damping, c', was significantly dependent on vibration magnitude for foam 3 ($p < 0.05$,

Friedman), with a significantly lower value with the lowest magnitude of vibration (0.25 ms^{-2} r.m.s.) compared to the other magnitudes ($p < 0.05$, Wilcoxon).

FIGURE 5 ABOUT HERE

3.3 Dynamic Stiffness – Effect of subject characteristics
Associations between the foam dynamic stiffness and subject characteristics were investigated using the Spearman rank-order correlation coefficient [24] at each vibration magnitude. The stiffness, *k*, of foam 2 and foam 3 generally increased with increasing sitting weight and increasing hip breadth (Table 3a). The stiffness frequency-dependence, *k'*, was insensitive to subject weight and hip-breadth ($p > 0.05$). The damping, *c*, generally increased with increasing hip breadth and increasing subject weight with all three foams (Table 3b). The hysteretic damping, *c'*, was independent of both subject weight ($p > 0.05$) and hip breadth ($p > 0.05$).

TABLE 3 ABOUT HERE

The weights and hip breadths of subjects were highly correlated ($r = 0.859$, $p < 0.001$), so Spearman partial correlation analysis (i.e. Pearson partial correlation on ranked data) was used to identify which of these parameters was influencing the stiffness and damping of the foam. For foam 1 and foam 2, the stiffness, *k*, was generally correlated with hip breadth when controlling for the effect of weight ($p < 0.05$, Table 3c). Only for foam 2, the stiffness, *k*, was correlated with subject weight after controlling hip breadth ($p < 0.05$, Table 3c). When controlling for weight none of the foams showed a significant correlation between damping, *c*, and hip breadth ($p > 0.05$, Table 3d). When controlling for hip breadth, the damping, *c*, showed a significant correlation with weight in some conditions (Table 3d).

3.4 Seat transmissibility
With increasing magnitude of vibration, there were significant reductions in the resonance frequency of the transmissibility of all three foams ($p < 0.05$, Wilcoxon; Figure 6). Similarly, the modulus of the transmissibility at resonance decreased with increasing magnitude of vibration ($p < 0.05$, Wilcoxon).

FIGURE 6 ABOUT HERE

For the foam 1, Figure 7 shows the inter-subject variability in the measured apparent mass, measured dynamic stiffness, measured foam transmissibility and predicted foam transmissibility, for the intermediate vibration magnitude (0.63 ms^{-2} r.m.s.).

FIGURES 7 ABOUT HERE

As described in Section 2.4, the transmissibility resonance frequencies and the transmissibility moduli at resonance were calculated at each of the five vibration magnitudes when either: (i) substituting the five apparent masses with a fixed foam dynamic stiffness, or (ii) substituting the five foam dynamic stiffnesses with a fixed apparent mass. In each case (i.e. 15 subjects, 3 foams, and 5 reference vibration magnitudes), the maximum changes in the resonance frequency and in modulus at resonance when substituting the apparent mass at each magnitude were greater than those obtained by substituting the seat dynamic stiffness at each magnitude (Tables 4 and 5). In each case, the nonlinearity in the seat transmissibility (i.e. reduction in seat resonance frequency and reduction in transmissibility modulus at the resonance frequency) due to the nonlinearity in the apparent mass was greater than that due to the nonlinearity in the foam (Wilcoxon, $p < 0.001$). An example is illustrated in Figure 8.

FIGURE 8 ABOUT HERE

TABLES 4 AND 5 ABOUT HERE

The associations between foam transmissibility and both body weight and hip breadth were investigated using the non-parametric Spearman correlation coefficient. The resonance frequency evident in the foam transmissibility was negatively correlated with sitting weight at all magnitudes of vibration for foam 1 ($p < 0.05$), at the three highest magnitudes of vibration for foam 2 ($p < 0.05$ at 0.63, 1.0, and 1.6 ms^{-2} r.m.s.), but at only one magnitude of vibration for foam 3 ($p < 0.05$ at 0.4 ms^{-2} r.m.s.). The resonance frequency was also negatively correlated with hip breadth at three magnitudes of vibration for foam 1 ($p < 0.05$ at 0.4, 1.0, and 1.6 ms^{-2} r.m.s.), at one magnitude for foam 2 ($p < 0.05$ at 0.63 ms^{-2} r.m.s.) and at two magnitudes for foam 3 ($p < 0.05$ at 0.4 and 1.6 ms^{-2} r.m.s.). The negative correlations mean that an increase in weight or hip breadth decreased the transmissibility resonance frequency. There were no significant correlations between the modulus of the foam transmissibility at resonance and either subject weight or hip breadth ($p > 0.05$).

3.5 Coherence functions

Ordinary coherence functions were greater than 0.75 in the frequency range 0 to 20 Hz for each computed transfer function.

4. Discussion

4.1 Apparent Mass – Rigid and soft seat

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The apparent masses of subjects measured on the rigid seat have similar shapes to those reported in previous studies [11][12], and with increasing magnitude of vibration previous studies have reported similar reductions in the apparent mass resonance frequency [12][14]. The absence of an effect of vibration magnitude on the modulus of apparent mass at resonance is consistent with some previous measurements [7][12][15], but not others [14][19].

A device for measuring pressure distributions has been used to estimate apparent mass on a soft seat [25]. Consistent with the present study, two peaks in the modulus of the apparent mass were found, with the same dependence of the resonance frequency of the first peak with respect to vibration magnitude. There was a similar resonance frequency when measuring apparent mass on a rigid and soft seat, but the modulus of the apparent mass on the rigid seat was remarkably greater than on the soft seat. Although the contact conditions different between the seats (e.g. there was a backrest on the soft seat) and the vibration spectra reaching the subjects will have differed between seats, it is doubtful whether these factors can sufficiently explain why subject apparent mass showed a large difference between the soft and rigid seats, unlike the present study. Fairley and Griffin [3] estimated the apparent mass of the body sitting on a soft seat using a method similar to the present study, but with a conventional car seat without a backrest and a flat broadband spectrum on the surface of the soft seat. Similarly to the present study, a SIT-bar was used to place the accelerometer at the seat-human interface. Although no statistical tests were reported, the apparent masses on the soft and rigid seats were very similar, except at high frequencies (12.25 to 18.25 Hz), consistent with the present findings. The effect of vibration magnitude on apparent mass when sitting on the soft seat was investigated with one subject and showed a reduction in apparent mass resonance frequency with increasing magnitude of vibration, consistent with the present research. Apart from these studies, there are no known previous reports of the directly measured apparent mass of the human body sitting on a soft seat.

The differences in the apparent mass of the body sitting on the rigid seat and the soft seat suggest that the soft seat condition decreases the internal damping of the human body if the body is represented by a 1- or 2-degree-of-freedom lumped parameter model ([5][11][26]). This change in damping might be attributed to differences in the contact area and pressure distribution: tissues around the ischial tuberosities are more compressed when sitting on a flat rigid seat because the weight of the body is supported on a smaller contact area than when sitting on foam [6][25]. Although the subjects were sitting on a SIT-bar, the compliance of the

foam resulted in a greater contact area when sitting on the soft seat and a more uniform pressure distribution. There is some evidence that increased contact area (obtained by sitting on a 'bead cushion') may slightly increase body internal damping whereas increased contact area may tend to decrease the apparent mass resonance frequency (with excitation at 1.0 and 2.0 ms^{-2} r.m.s. in [7]). Another cause for the changes in damping may be the difference in the vibration experienced by the subject when seating on a soft seat: the effect of the foam transmissibility was to amplify low frequency vibration and attenuate high frequency vibration, giving an overall reduction in the r.m.s. acceleration magnitude to about 74%. The overall decrease in the vibration magnitude on the soft seat might be expected to increase the resonance frequency, due to the non-linearity in the body apparent mass, so offsetting any decrease associated with the greater contact area with the soft seat. However, the overall reduction in vibration magnitude was due to attenuation of high frequencies, and the foam amplified the low frequencies that may have had a greater influence on the nonlinearity in the apparent mass [15].

4.2 Dynamic Stiffness – Effect of Vibration Magnitude

The use of simple blocks of foam on a rigid flat surface removed the effects that cushion and frame geometry and seat covers can have on the vibration transmissibility of a seat. This made it possible to focus on the influence of the properties of the foam materials. The use of a four parameter model to describe the response of the foam, based on the observation of the frequency response functions, resulted in better curve fitting than the $S(f) = k + i2\pi fc$ or $S(f) = k(1 + id)$ models [3][4][5][8], and assisted the interpretation of the data. In this study, the dynamic stiffness was similar to that reported by Lewis and Griffin [2], showing a dependence on frequency, unlike the response of a conventional seat ('foam supported on wire springing') with a cover reported by Fairley and Griffin [3]. As suggested by Hilyard [9] and Patten and Pang [13], polyurethane foams have a nonlinear softening behaviour. The present findings show that the nonlinearity affects not only k, but also the stiffness rate of increase with frequency, k'. In the conditions of the present study, with relatively low magnitude vibration and a spectral content influenced by the human body impedance the softening of the foam with increasing vibration magnitude was not dramatic. Measuring seat transmissibility with a simple rigid mass (e.g. [13][18]) is likely to enhance the influence of seat nonlinearities as greater seat dynamic deflection is likely than when a seat supports the human body [16]. The conditions in which there was no statistically significant change in foam dynamic stiffness (foam 1: and 0.25 to 0.4 and 0.4 to 0.63 ms^{-2} r.m.s.; foam 2: 0.25 to

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0.4 and 0.4 to 0.63 ms^{-2} r.m.s.; foam 3: 0.25 to 0.4, 0.4 to 0.63, and 0.63 to 1.0 ms^{-2} r.m.s) suggest that the harder the seat the greater the change in acceleration magnitude required to trigger nonlinearity.

The energy loss parameters were generally not affected by the magnitude of vibration, consistent with [5] and [8].

4.3 Dynamic Stiffness – Effect of Subject Build

The present findings are not easily compared with either the literature on cellular polymers, that mainly presents fully compressive stress-strain static curves (e.g. [9]) or dynamic transmissibility curves obtained with rigid masses (e.g. [13][18]), or studies of seat dynamics that have used different models to fit the experimental data (e.g. [3][4][5][8]). Deflection measures were not obtained in this study, but it is possible to correlate seat characteristics with the static force to which the foam was subjected (i.e. subject sitting weight) and the dimensions of the 'indenter' (i.e. subject hip breadth). The dynamic stiffness parameter k had a clear dependence on hip breadth, showing that increased dimensions of the 'indenter' (and so of the foam supporting area) increased the stiffness of the foam. This differs from the findings in [8], where, for a given static load, there was no correlation between indenter area and foam stiffness. The significant stiffening of the foam stiffness, k, with increasing subject weight (for foams 2 and 3) is consistent with previous research [3][4][5][10]. However the behaviour of the softest foam (foam 1) demonstrates that stiffening may not always occur. Furthermore, the stiffness of foam 3 showed no dependence on subject weight after controlling for subject hip breadth, demonstrating that the stiffening of the foam may depend on the contact dimensions more than the supported weight. However, in this study, the range of sitting weights resulted in only 240 N variation in the force applied to the foam, whereas previous studies varied the static indentation force applied to foam by up to 600 N and found increased stiffness with increased force [3][4][8].

The positive correlation between the viscous damping, c, of the foam and the static weight on the foam is consistent with Wei and Griffin [8] but not with Toward and Griffin [5]. This may be because Toward and Griffin [5] fitted the seat parameters from transmissibility data, fixing the body parameters previously measured on a rigid plate. The correlation between viscous damping, c, and hip breadth dropped when controlling for weight, while some correlation remained between c and weight when controlling for hip breadth (Table 3d), consistent with the damping, c, being more strongly associated with the weight of subjects than the breadth of their hips.

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The absence of a correlation between either subject weight or hip breadth and the hysteretic damping coefficient, c', was expected, since c' has been reported to depend on the cellular geometry and on the viscoelastic behaviour of the base polymer [10].

4.4 Seat Vertical Transmissibility

As reported in the literature [3][5][16], vertical seat transmissibility is nonlinear with acceleration magnitude. The prediction of the transmissibility from the apparent mass and the dynamic stiffness measured at different magnitudes gave definitive evidence that the human body contributed most to the nonlinearity, as previously assumed [3][8]. Although variations in the vibration magnitude resulted in variations in the seat dynamic stiffness (in some cases k varied by up to 30%), this had relatively little effect on seat transmissibility.

The correlations between foam transmissibility and subject weight and hip breadth are not easily interpreted. Whether the statistical tests were significant or not, the correlation coefficients were negative, indicating that increased subject weight or increased hip breadth decreased the transmissibility resonance frequency. For foam 1, this implies that the increase in the mass supported by the foam was not fully compensated by an increase in the stiffness of the foam. For foams 2 and 3, the non-significant statistical tests imply that increased weight or increased hip breadth was compensated by increased seat stiffness, so that the transmissibility resonance frequency remained constant, consistent with the findings of Toward and Griffin [5].

4.5 Implication of the results and future research

Foam dynamic properties are better modelled with four-parameter frequency-domain stiffness than with simple stiffness and damping. Foam dynamic parameters show only weak dependence on vibration magnitude and, compared to the influence of the nonlinearity of the human body, do not have a strong influence on the transmission of vibration through a foam cushion. Future research may focus on improving understanding of the dependence of foam dynamic response on the supported load and contact area rather than focusing on the dependence of foam dynamic properties on vibration magnitude.

Although nonlinearity in the human body can be the dominant cause of nonlinearity in the transmissibility of seat foam, the causes and characteristics of the biodynamic nonlinearity are poorly understood. Future experimental research should seek the information needed to model the body nonlinearity and thereby allow improvements in the modelling of seat transmissibility.

5. Conclusions

The apparent mass of the human body supported on a foam cushion has a similar resonance frequency as when supported on a rigid surface. The apparent mass at resonance was slightly less when supported by foam, suggesting increased body damping. An increase in the apparent mass of the body at 20 Hz when sitting on the foam was also consistent with increased body damping.

The resonance frequency of the foam transmissibility and the modulus of the foam transmissibility were generally negatively correlated with subject weight and hip breadth. It seems that subject weight and contact area can alter the dynamic stiffness of seat foam. However, correlations between the dynamic properties of the foam and subject weight and hip breadth suggest complex foam behaviour and differences between foams.

Changes in vibration magnitude revealed nonlinear softening behaviour of both the human body and the foam, resulting in changes in foam transmissibility. The foam elastic stiffness was dependent on vibration magnitude but energy loss parameters were not.

The principal contribution to the nonlinearity in foam transmissibility can be ascribed to nonlinearity in the human body, with only a minor contribution from the nonlinearity of the foam.

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Table 1 Foam properties.

	Dynamic stiffness	Composition	Density (kg/m^3)	Maximum thickness (mm)	Weight (kg)
foam 1	soft	MDI	75	110	1.35
foam 2	medium	TDI	50	110	0.89
foam 3	hard	MDI	75	80	0.93

Table 2 Subject characteristics [21].

	Age (years)	Stature (cm)	Weight (kg)	Sitting weight (kg)	Hip breadth (cm)
Median	27	178	76	55	36
Minimum	21	169	56	41	31
Maximum	41	186	93	65	41
Interquartile range	8	10	21	12	5

Table 3 Statistical significance of correlations (*p*-values) involving foam stiffness, *k*, and foam damping, *c*.

a)   Spearman correlations between foam stiffness, *k*, and subject weight and hip breadth

	Vibration magnitude (ms^{-2} r.m.s.)	foam 1	foam 2	foam 3
Weight	0.25	**0.274**	**0.064**	0.001
	0.4	**0.305**	0.043	0.000
	0.63	**0.308**	0.030	0.000
	1.0	**0.292**	0.022	0.000
	1.6	**0.245**	0.044	0.000
Hip breadth	0.25	0.030	0.001	0.001
	0.4	**0.056**	0.000	0.000
	0.63	**0.060**	0.000	0.000
	1.0	**0.059**	0.000	0.000
	1.6	0.045	0.000	0.000

b)   Spearman correlations between foam damping, *c*, and subject weight and hip breadth

	Vibration magnitude (ms^{-2} r.m.s.)	foam 1	foam 2	foam 3
Weight	0.25	0.000	**0.052**	0.004
	0.4	0.000	**0.056**	0.010
	0.63	0.000	0.014	0.001
	1.0	0.000	0.000	0.008
	1.6	0.001	0.000	0.007
Hip breadth	0.25	0.004	0.047	0.039
	0.4	0.000	0.029	**0.081**
	0.63	0.001	0.013	0.004
	1.0	0.003	0.000	0.010
	1.6	0.002	0.001	0.011

c)   Spearman partial correlations between foam stiffness, *k*, and subject weight and hip breadth

	Vibration magnitude (ms^{-2} r.m.s.)	foam 1	foam 2	foam 3
Weight controlling hip breadth	0.25	**0.136**	**0.066**	**0.275**
	0.4	**0.243**	0.038	**0.139**
	0.63	**0.261**	0.023	**0.171**
	1.0	**0.287**	0.033	**0.326**
	1.6	**0.305**	**0.065**	**0.182**
Hip breadth controlling weight	0.25	0.030	0.001	**0.237**
	0.4	**0.058**	0.000	**0.093**
	0.63	0.031	0.000	0.047
	1.0	**0.063**	0.000	**0.087**
	1.6	0.032	0.001	**0.109**

d)   Spearman partial correlations between foam damping, *c*, and subject weight and hip breadth

	Vibration magnitude (ms^{-2} r.m.s.)	foam 1	foam 2	foam 3
Weight controlling hip breadth	0.25	0.028	**0.614**	0.039
	0.4	**0.053**	**0.871**	**0.051**
	0.63	0.005	**0.470**	**0.084**
	1.0	**0.050**	0.039	**0.365**
	1.6	**0.131**	0.031	**0.287**
Hip breadth controlling weight	0.25	**0.990**	**0.530**	**0.544**
	0.4	**0.348**	**0.306**	**0.460**
	0.63	**0.922**	**0.433**	**0.729**
	1.0	**0.781**	**0.288**	**0.476**
	1.6	**0.465**	**0.673**	**0.579**

Note: All correlation coefficients are positive; *p* = 0.000 is equivalent to *p* < 0.001.

Table 4 Median, maximum and minimum changes in foam transmissibility resonance frequency, f_r, with the fifteen subjects when substituting the foam dynamic stiffness, $S(f)$, at each vibration magnitude or when substituting the human body apparent mass, $AM(f)$, at each vibration magnitude (spectral resolution: 0.125 Hz).

	foam 1					
	effect of $S(f)$ nonlinearity			effect of $AM(f)$ nonlinearity		
Reference vibration magnitude (ms^{-2} r.m.s.)	Median f_r change (Hz)	Max f_r change (Hz)	Min f_r change (Hz)	Median f_r change (Hz)	Max f_r change (Hz)	Min f_r change (Hz)
0.25	0.250	0.500	0	0.750	1.125	0.500
0.4	0.125	0.375	0	0.750	1.125	0.500
0.63	0.125	0.375	0	0.750	1.000	0.500
1.0	0.125	0.375	0	0.625	1.000	0.375
1.6	0.125	0.250	0	0.625	0.875	0.375

	foam 2					
	effect of $S(f)$ nonlinearity			effect of $AM(f)$ nonlinearity		
Reference vibration magnitude (ms^{-2} r.m.s.)	Median f_r change (Hz)	Max f_r change (Hz)	Min f_r change (Hz)	Median f_r change (Hz)	Max f_r change (Hz)	Min f_r change (Hz)
0.25	0.125	0.500	0	0.875	1.250	0.625
0.4	0.125	0.375	0	0.875	1.125	0.625
0.63	0.125	0.500	0	0.875	1.125	0.625
1.0	0.125	0.375	0	0.750	1.125	0.500
1.6	0.125	0.375	0	0.875	1.000	0.625

	foam 3					
	effect of $S(f)$ nonlinearity			effect of $AM(f)$ nonlinearity		
Reference vibration magnitude (ms^{-2} r.m.s.)	Median f_r change (Hz)	Max f_r change (Hz)	Min f_r change (Hz)	Median f_r change (Hz)	Max f_r change (Hz)	Min f_r change (Hz)
0.25	0.125	0.375	0	0.750	1.125	0.625
0.4	0.125	0.500	0	0.750	1.000	0.500
0.63	0.250	0.625	0	0.875	1.125	0.625
1.0	0.250	0.500	0	0.750	1.000	0.500
1.6	0.125	0.375	0	0.750	1.000	0.625

Table 5 Median, maximum, and minimum changes in the modulus of foam transmissibility at resonance, $H(f_r)$, with the fifteen subjects when substituting the foam dynamic stiffness, $S(f)$, at each vibration magnitude or when substituting the human body apparent mass, $AM(f)$, at each vibration magnitude (spectral resolution: 0.125 Hz)

	foam 1					
	effect of $S(f)$ nonlinearity			effect of $AM(f)$ nonlinearity		
Reference vibration magnitude (ms^{-2} r.m.s.)	Median $H(f_r)$ change	Max $H(f_r)$ change	Min $H(f_r)$ change	Median $H(f_r)$ change	Max $H(f_r)$ change	Min $H(f_r)$ change
0.25	0.15	0.32	0,04	0.41	0.77	0.12
0.4	0.12	0.23	0.02	0.42	0.94	0.21
0.63	0.12	0.30	0.03	0.44	0.84	0.12
1.0	0.14	0.27	0.04	0.43	0.95	0.18
1.6	0.10	0.32	0.03	0.48	0.85	0.19
	foam 2					
	effect of $S(f)$ nonlinearity			effect of $AM(f)$ nonlinearity		
Reference vibration magnitude (ms^{-2} r.m.s.)	Median $H(f_r)$ change	Max $H(f_r)$ change	Min $H(f_r)$ change	Median $H(f_r)$ change	Max $H(f_r)$ change	Min $H(f_r)$ change
0.25	0.11	0.33	0.03	0.35	0.75	0.12
0.4	0.11	0.30	0.04	0.35	0.60	0.21
0.63	0.12	0.22	0.04	0.36	0.61	0.14
1.0	0.13	0.20	0.04	0.36	0.63	0.07
1.6	0.10	0.18	0.07	0.40	0.71	0.14
	foam 3					
	effect of $S(f)$ nonlinearity			effect of $AM(f)$ nonlinearity		
Reference vibration magnitude (ms^{-2} r.m.s.)	Median $H(f_r)$ change	Max $H(f_r)$ change	Min $H(f_r)$ change	Median $H(f_r)$ change	Max $H(f_r)$ change	Min $H(f_r)$ change
0.25	0.08	0.20	0.04	0.31	0.58	0.13
0.4	0.09	0.17	0.01	0.36	0.55	0.20
0.63	0.06	0.14	0.03	0.31	0.53	0.16
1.0	0.10	0.15	0.02	0.36	0.57	0.16
1.6	0.09	0.22	0.03	0.34	0.48	0.18

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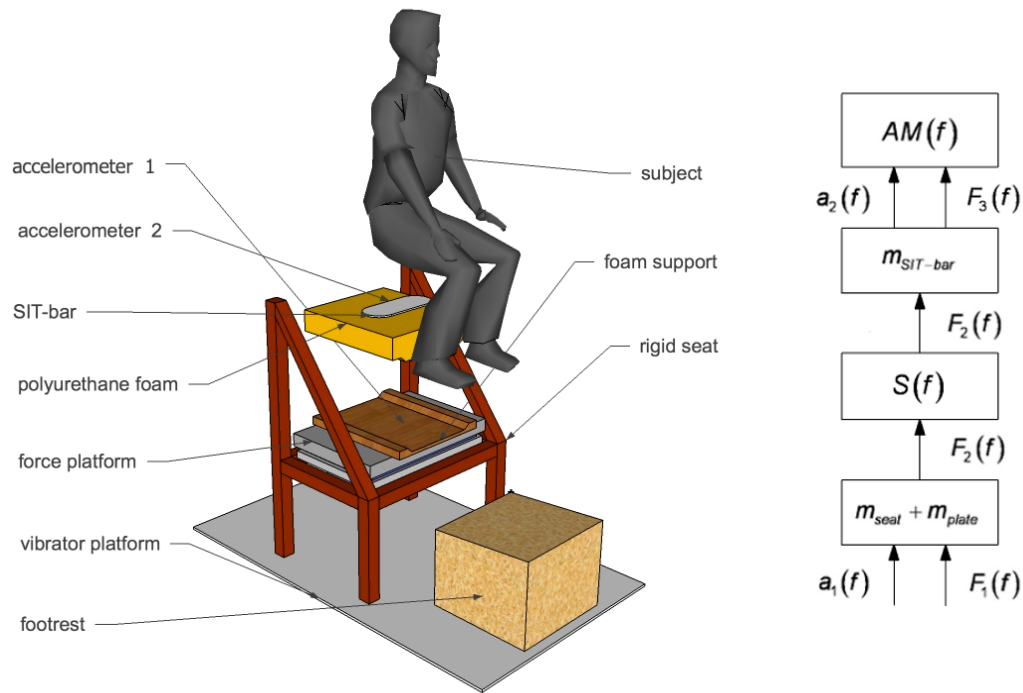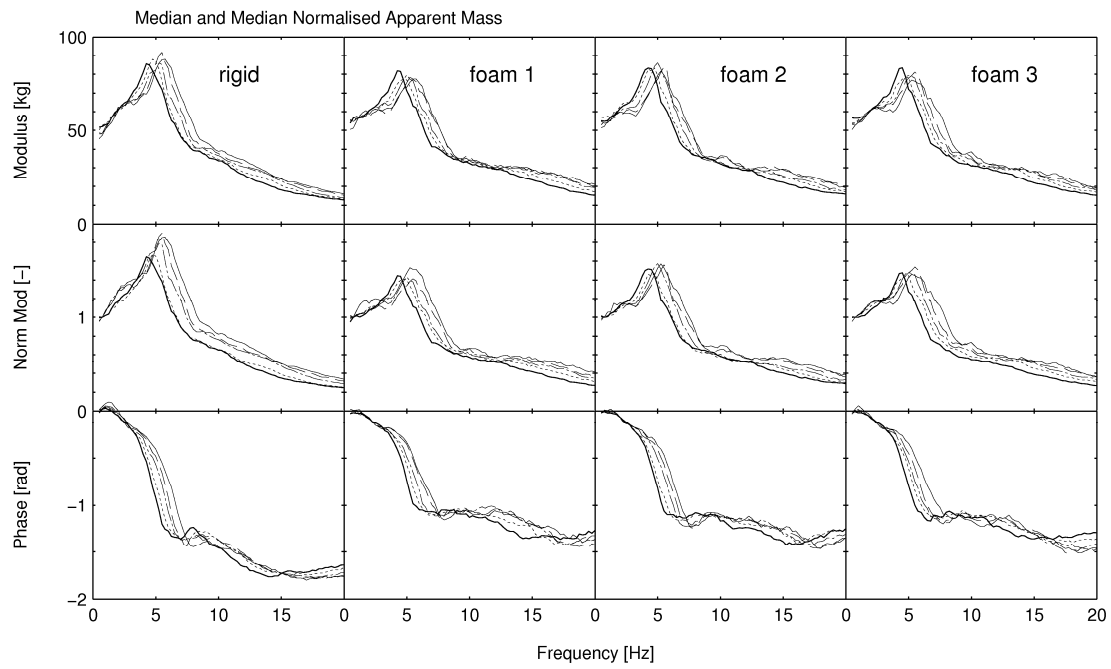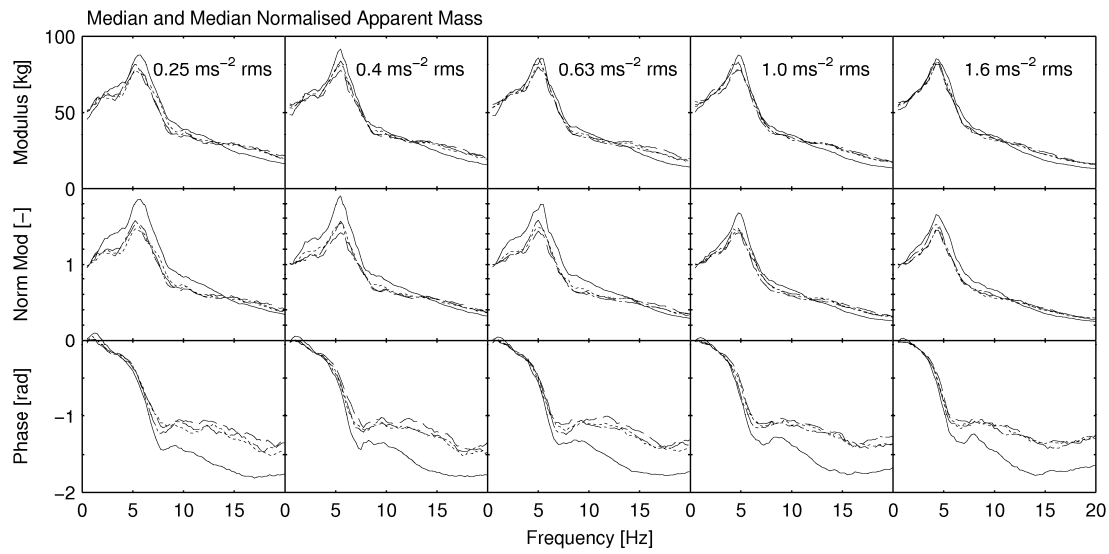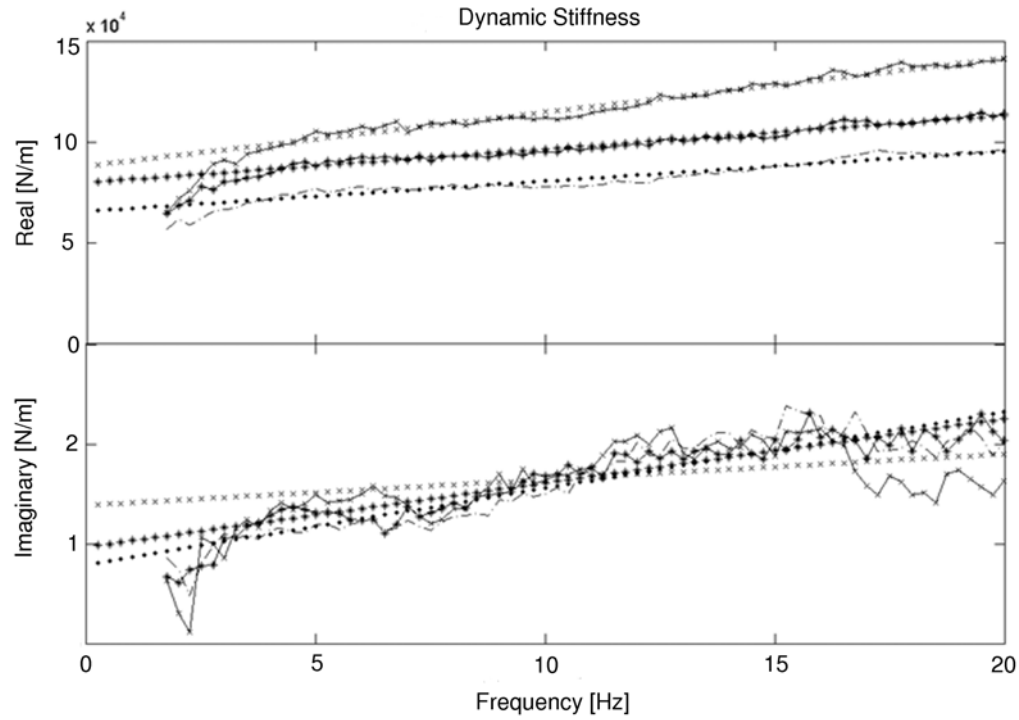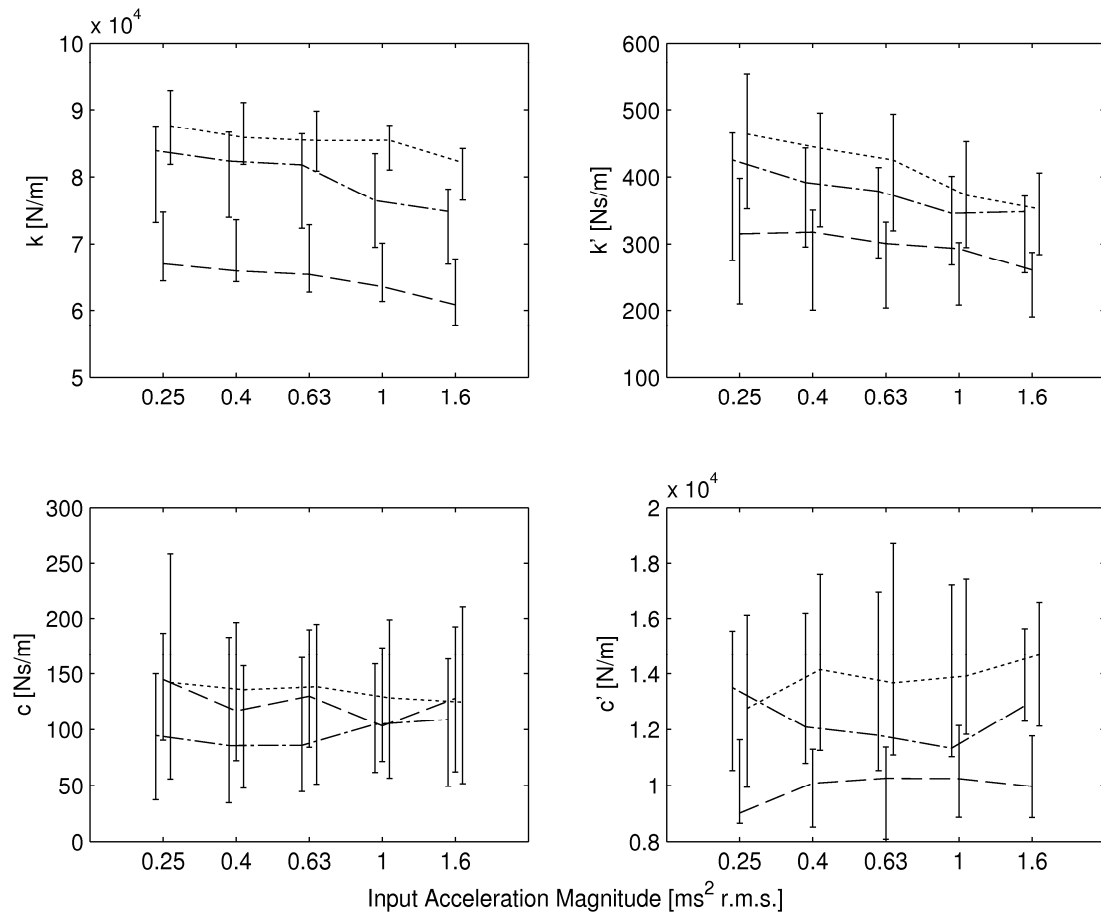
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Figure 4. Example of dynamic stiffness curve fitting (subject 1, 0.63 ms^{-2} r.m.s., —+— foam 1 measured data, —*— foam 2 measured data, —*— foam 3 measured data, ◆ ◆ ◆ ◆ foam 1 fitted, +++ + foam 2 fitted, × × × × foam 3 fitted).

Figure 5. Effect of vibration magnitude on foam dynamic stiffness for each of the three foams (medians and inter-quartile ranges for 15 subjects) (– – foam 1; –·– foam 2; ···· foam 3).
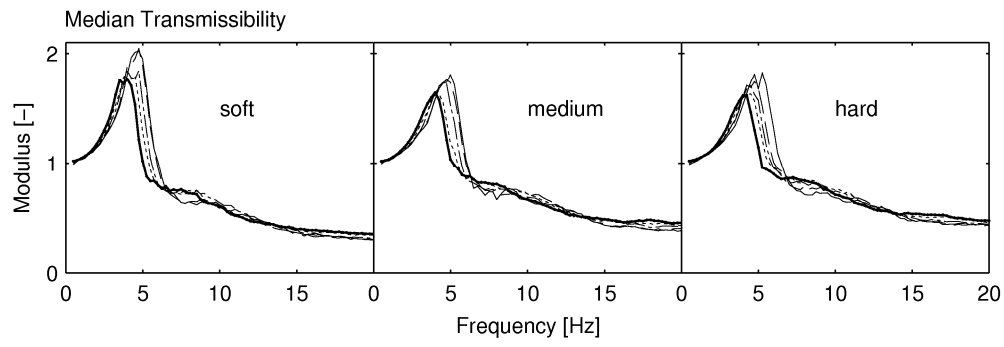
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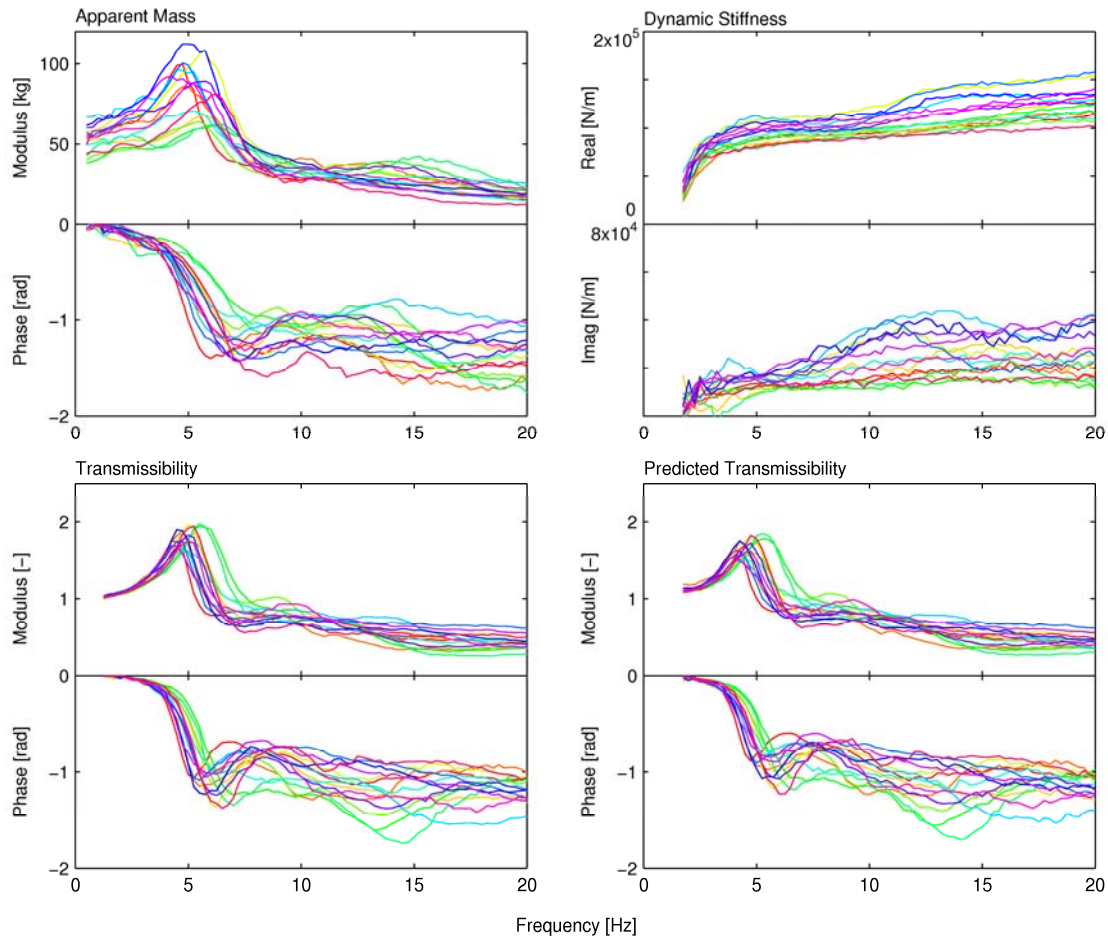
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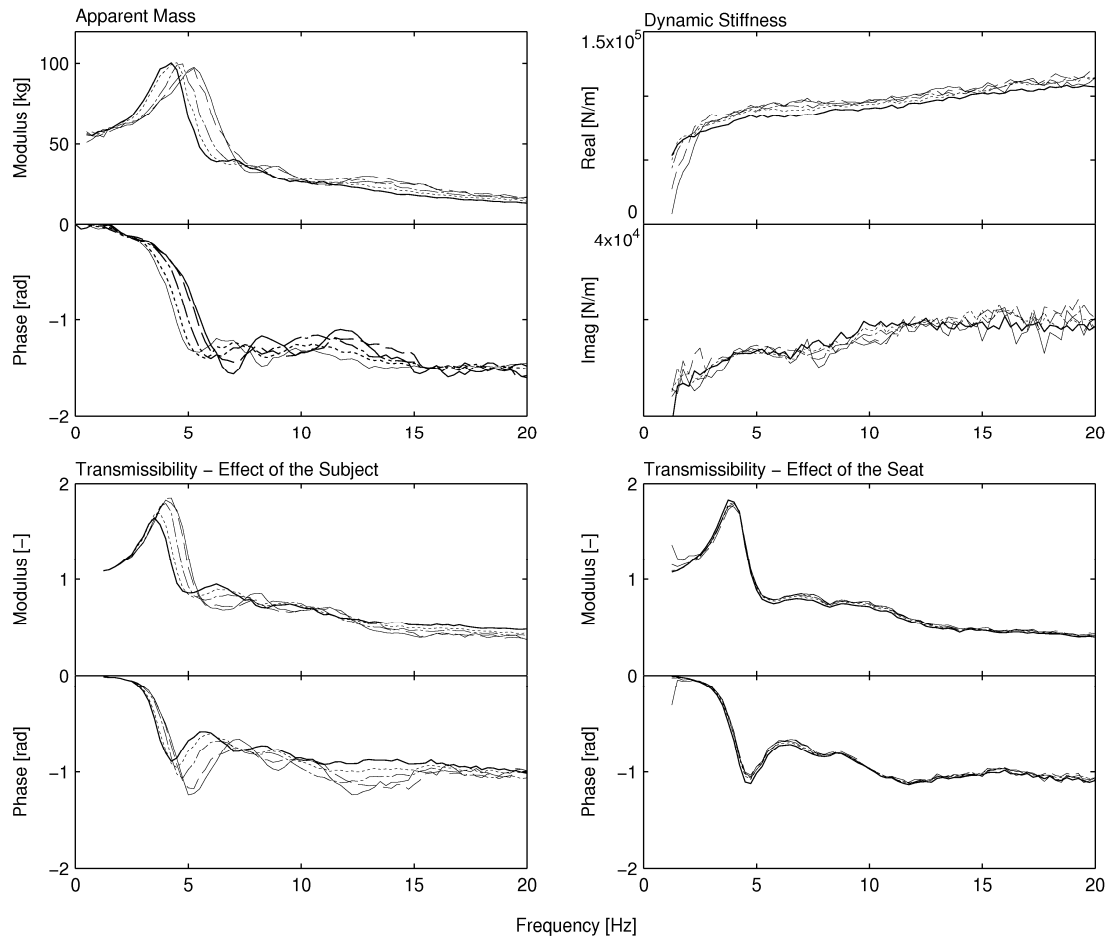
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