# Correspondence

## QRD-Assisted Adaptive Modulation-Aided MIMO Systems

Ping Yang, Yue Xiao, Shaoqian Li, and Lajos Hanzo

***Abstract*—In this paper, we propose $QR$-decomposition (QRD)-based adaptive modulation (AM)-aided multiple-input–multiple-output (MIMO) systems. The proposed algorithm yields a tight lower bound of the free distance (FD), which determines the error probability of the detector in the high-signal-to-noise-ratio (SNR) region. Thus, this QRD-based AM algorithm is capable of achieving near-optimal performance at low complexity because the full QRD, which imposes high complexity, is performed only once for each channel realization, regardless of the number of AM modes. Our simulation results show that the proposed algorithm exhibits a better bit-error-rate (BER) performance and reduced complexity compared with the existing algorithms.**

***Index Terms*—Adaptive modulation (AM), free distance (FD), multiple-input multiple-output (MIMO), $QR$ decomposition (QRD).**

## I. Introduction

Multiple-input–multiple-output (MIMO) systems are capable of achieving a capacity gain and/or increased link robustness [1]–[3]. Hence, they have been adopted in most of the recent communication standards such as IEEE 802.11n, IEEE 802.16e, and Third-Generation Partnership Project Long-Term Evolution [4]. They may be also beneficially combined with adaptive modulation (AM) for adjusting the transmission parameters for the sake of accommodating time-varying channels [5]. Therefore, the effective combination of AM and MIMO techniques is a promising design alternative for high-rate wireless transmission systems [5], [6].

In MIMO systems, several AM-based link adaptation schemes [5]–[10] have been proposed for improving the achievable system performance. For example, the performance of AM-aided MIMO systems has been analyzed under both continuous- and discrete-rate scenarios in [7]. Moreover, adaptive MIMO architectures utilizing different combinations of modulation/coding schemes have been proposed in [8], which aimed for the maximization of the channel capacity at a predefined target bit error rate (BER), rather than for optimizing the BER. Moreover, these schemes were optimized for linear receivers. However, the AM candidate selection criterion designed for specific linear receivers in [5]–[8] may be unsuitable for the powerful nonlinear maximum-likelihood (ML) detector, which has an optimal performance.

To find an efficient AM algorithm for the MIMO ML detector, an exhaustive-search algorithm may be invoked to maximize the minimum Euclidean distance among the signal constellation points. However, this algorithm may be impractical owing to its potentially excessive complexity. To reduce the complexity, Ham *et al.* [11] invoked the singular value decomposition (SVD) to avoid the direct calculation of the Euclidean free distance (FD). Although this SVD-based algorithm is capable of achieving a lower bound of the FD, this bound may be loose; hence, the resultant solution is suboptimal. Furthermore, the computational complexity of the SVD-based algorithm is high for large search spaces associated with a high number of AM modes and transmit antennas $N_t$. In [12], $QR$ decomposition (QRD) was applied for antenna selection in MIMO systems, and it was demonstrated that the lower bound of the FD associated with the QRD can be further tightened compared with that of the SVD [12]. In [13], the equal-diagonal-based QRD was invoked for both the associated precoder's design at the transmitter and the successive-cancelation-based detection at the receiver. However, it has not been considered whether the QRD methods in [12] and [13] can be directly applied to AM candidate selection.

Against this background, these are the novel contributions of this paper.

- We apply the QRD to AM-based MIMO systems and propose a novel AM-mode assignment algorithm to improve the attainable BER performance. Specifically, we first formulate the FD by including the diagonal matrix $\mathbf{D}$ corresponding to different AM modes. Then, equivalent channel matrix $\tilde{\mathbf{H}}$, which combines the effects of both the channel conditions and of the modulation parameters, is decomposed by invoking the QRD algorithm [12], [13] for further tightening the lower bound of the given FD determined in [11].
- As a further benefit, we demonstrate that, regardless of the number of AM-mode candidates, the full QRD of channel matrix $\mathbf{H}$ in our proposed AM-based MIMO only has to be performed once for each channel realization, which dramatically reduces the search complexity. Thus, the proposed algorithm exhibits an attractive system performance at low complexity. Furthermore, our simulation results indicate that the algorithm is more robust to imperfect channel state information (CSI) than the SVD-based algorithm in [11].

The organization of this paper is as follows. Section II presents the system model of the AM-based MIMO systems. Our simulation results and performance comparisons are presented in Section III. Finally, Section IV concludes this paper.

## II. System Model and AM Algorithms

Consider an AM-aided MIMO system having $N_t$ transmit and $N_r$ receive antennas, as shown in Fig. 1. The $(N_t \times 1)$-element transmit symbol vector $\mathbf{x}$ is assumed to satisfy $E[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_{N_t}$, where $\mathbf{I}_{N_t}$ denotes an $(N_t \times N_t)$-element identity matrix. At the receiver, the corresponding $(N_r \times 1)$ received signal is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

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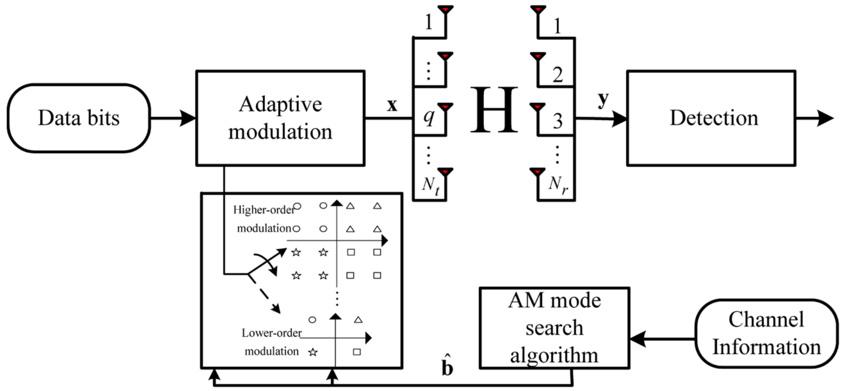


Fig. 1. System model of an AM-aided MIMO system.

where $\mathbf{H}$ is an $(N_r \times N_t)$ channel matrix, and the elements of the $N_r$-element noise vector are Gaussian random variables obeying $\mathcal{CN}(0, N_0)$. Given channel matrix $\mathbf{H}$, the pairwise error probability (PEP) between two arbitrary transmit vectors $\mathbf{x}_i$ and $\mathbf{x}_j (\mathbf{x}_i \neq \mathbf{x}_j)$ may be estimated as [10]

$$P(\mathbf{x}_i \to \mathbf{x}_j|\mathbf{H}) \approx \lambda \cdot Q\left(\sqrt{\frac{1}{2N_0} d_{\min}^2(\mathbf{H})}\right) \tag{2}$$

where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-y^2/2} dy$, and $\lambda$ is the number of neighboring constellation points [10], with FD $d_{\min}(\mathbf{H})$ defined as

$$d_{\min}(\mathbf{H}) = \min_{\substack{\mathbf{x}_i, \mathbf{x}_j \in \Lambda, \\ \mathbf{x}_i \neq \mathbf{x}_j}} \|\mathbf{H}(\mathbf{x}_i - \mathbf{x}_j)\| \tag{3}$$

where $\Lambda$ is the set of all possible transmit symbols, whereas $\mathbf{x}_i$ and $\mathbf{x}_j$ are two distinct transmitted symbols in $\Lambda$. In (2), the conditioned PEP is a monotonically decreasing function of $d_{\min}(\mathbf{H})$. Hence, the system's performance may be improved by maximizing the distance $d_{\min}(\mathbf{H})$ upon adapting the transmit parameters.

### A. Exhaustive-Search-Based AM Algorithm

Let $\mathbf{B}$ be a set of AM candidates given as $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_j, \ldots, \mathbf{b}_J\}$, where $\mathbf{b}_j = [b_j^1, \ldots, b_j^n \ldots, b_j^{N_t}]$, and $b_j^n$ denotes the modulation order for the $n$th transmit antenna of the $j$th AM candidate. These candidates satisfy the throughput constraint as

$$R = \sum_{n=1}^{N_t} \log_2\left(b_j^n\right), \qquad j = 1, 2, \ldots, J \tag{4}$$

where $J$ is the total number of possible modulation order combinations for the throughput of $R$ bits/symbol. To achieve the optimum performance, an exhaustive-search-based AM-mode search algorithm, which is referred to as the max–min AM algorithm, is formulated as

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_j \in \mathbf{B}} d_{\min}(\mathbf{H}). \tag{5}$$

Then, the corresponding index of the optimal AM-mode candidate $\hat{\mathbf{b}}$ is signaled back to the transmitter, which transmits the modulated symbols accordingly. Indeed, since the error events mainly arise from the nearest neighbors, the maximization of the FD in (3) directly reduces the probability of error, particularly at high SNRs [5], [9]. Deriving a closed-form solution to (5) remains an open challenge since the FD depends both on the constellation and on the channel realization [9]. Hence, a numerical search is adopted. Unfortunately, finding the optimal modulation order vector satisfying (5) has to tentatively invoke all possible candidates in (4). Moreover, for each AM-mode candidate, all possible error vectors, as given by $\mathbf{x}_e = \mathbf{x}_i - \mathbf{x}_j \; (\mathbf{x}_i, \mathbf{x}_j \in \Lambda)$, have to be considered in the calculation of (3). As a result, this exhaustive search has potentially prohibitive complexity.

### B. Conventional SVD-Based AM Algorithm

Here, we present the AM candidate selection criterion for the SVD-based algorithm [11]. Let us define the symbol constellation set of $x_k$ as $X$ and the minimum distance of the set $X$ as

$$d_{\min}(X) = \min_{\substack{x_i, x_j \in X, \\ x_i \neq x_j}} \|x_i - x_j\|^2. \tag{6}$$

The corresponding error symbol may be expressed as

$$e_{ij} = x_i - x_j = \bar{d}_{\min} \cdot \tilde{e}_{ij} \tag{7}$$

where $\bar{d}_{\min}$ denotes the minimum distance of the signal constellation according to the modulation order. For example, we have $\bar{d}_{\min} = 2$ for BPSK, $\bar{d}_{\min} = 2/\sqrt{2}$ for 4-ary quadrature amplitude modulation (4-QAM), $\bar{d}_{\min} = 2/\sqrt{6}$ for 8-QAM, $\bar{d}_{\min} = 2/\sqrt{10}$ for 16-QAM, and $\bar{d}_{\min} = 2/\sqrt{42}$ for 64-QAM [16]. Hence, $e_{ij}$ can be expressed as a multiple of $\bar{d}_{\min}$, and $\tilde{e}_{ij}$ is a version of $e_{ij}$ normalized by $\bar{d}_{\min}$. Let us now consider the case of $(N_t \times 1)$-element transmit vector $\mathbf{x}$, where the minimum distance of the symbol vector constellation may be expressed as

$$d_{\min}(\Lambda) = \min_{\substack{\mathbf{x}_i, \mathbf{x}_j \in \Lambda, \\ \mathbf{x}_i \neq \mathbf{x}_j}} \|\mathbf{x}_i - \mathbf{x}_j\| = \min_{\mathbf{e}_{ij} \in E} \|\mathbf{e}_{ij}\| \tag{8}$$

where $E$ is a set of error vectors, $\mathbf{e}_{ij}$ is the error vector, and

$$\mathbf{x}_i - \mathbf{x}_j = \begin{bmatrix} x_{i,1} - x_{j,1} \\ x_{i,2} - x_{j,2} \\ \vdots \\ x_{i,N_t} - x_{j,N_t} \end{bmatrix} = \begin{bmatrix} d_1 . \tilde{e}_{ij,1} \\ d_2 . \tilde{e}_{ij,2} \\ \vdots \\ d_{N_t} . \tilde{e}_{ij,N_t} \end{bmatrix} = \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_{N_t} \end{bmatrix} \begin{bmatrix} \tilde{e}_{ij,1} \\ \tilde{e}_{ij,2} \\ \vdots \\ \tilde{e}_{ij,N_t} \end{bmatrix} = \mathbf{D} \cdot \tilde{\mathbf{e}}_{ij} \tag{9}$$

where $\mathbf{D}$ is a diagonal matrix with $d_k$ being its diagonal elements, and normalized error vector $\tilde{\mathbf{e}}_{ij}$ is composed of normalized symbols $\tilde{e}_{ij,k} \; (k = 1, 2, \ldots, N_t)$. Here, $d_k$ is the minimum distance $\bar{d}_{\min}$ according to the modulation order assigned to each transmit antenna. Thus, based on (7)–(9), the FD of (3) can be reformulated as

$$d_{\min}(\mathbf{H}) = \min_{\tilde{\mathbf{e}}_{ij} \in \tilde{E}} \|\mathbf{H}\mathbf{D}\tilde{\mathbf{e}}_{ij}\| = \min_{\tilde{\mathbf{e}}_{ij} \in \tilde{E}} \|\tilde{\mathbf{H}}\tilde{\mathbf{e}}_{ij}\| \tag{10}$$

where $\tilde{\mathbf{H}} = \mathbf{HD}$ is the equivalent channel matrix. Using the Rayleigh–Ritz theorem in [8], the lower bound of (10) is expressed as

$$d_{\min}(\mathbf{H}) \geq \sqrt{\lambda_{\min}(\tilde{\mathbf{H}})} d_{\min}(\tilde{\Lambda}) \tag{11}$$

where $\lambda_{\min}(\tilde{\mathbf{H}})$ is the nonzero minimum eigenvalue of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, and $d_{\min}(\tilde{\Lambda}) = \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\tilde{\mathbf{e}}_{ij}\|_F$ is the minimum distance of the normalized error vector. In a spatial multiplexing MIMO system, the elements $\tilde{e}_{ij,k}$ $(k = 1, 2, \ldots, N_t)$ of $\tilde{\mathbf{e}}_{ij}$ are usually uncorrelated. Thus, according to (7), this minimum distance can be expressed as

$$d_{\min}(\tilde{\Lambda}) = \min\left\{d_{\min}(\tilde{\Lambda}_1), d_{\min}(\tilde{\Lambda}_2), \ldots, d_{\min}(\tilde{\Lambda}_{N_t})\right\} = 1 \tag{12}$$

where $d_{\min}(\tilde{\Lambda}_k)$ denotes the minimum distance from all legitimate signal constellation points $\tilde{e}_{ij,k}$. The lower bound formulated in (11) indicates that the FD can be evaluated with the aid of $\lambda_{\min}(\tilde{\mathbf{H}})$. As shown in (12), the value of $d_{\min}(\tilde{\Lambda})$ is constant, whereas the value of $\lambda_{\min}(\tilde{\mathbf{H}})$ is changed for the different AM candidates. Then, the SVD-based AM-mode search algorithm can be formulated as

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_j\in\mathbf{B}} \lambda_{\min}(\tilde{\mathbf{H}}). \tag{13}$$

In the SVD-based AM algorithm, only the minimum nonzero singular value of each $\tilde{\mathbf{H}}$ has to be calculated; thus, the complexity is reduced compared to the max–min AM algorithm formulated in (5). However, the computational complexity of this SVD-based algorithm is high when the number of AM-mode candidates and $N_t$ is large. For example, assuming that the AM-modes of no transmission, BPSK, and $M$-ary QAM ($M = 4$, 8, 16, 32, and 64) are available for all the $N_t = 8$ transmit antennas, for an 8-bit/symbol transmission, we need an exhaustive search over a set of 6371 candidates. In (13), we have to carry out the SVD of the equivalent channel matrix $\tilde{\mathbf{H}}$ for each candidate; hence, the resultant complexity is high. More importantly, the lower bound of the FD achieved in (11) may not be tight enough; hence, the resultant solution becomes suboptimal.

### C. Proposed QRD-Based AM Algorithm

Here, we apply the QRD algorithm to formulate our AM candidate selection criterion. According to (3), channel matrix $\mathbf{H}$ is first subject to the QRD [12], [13], yielding $\mathbf{H} = \mathbf{QR}$, where $\mathbf{Q}$ is an $(N_r \times N_t)$ column-wise orthonormal matrix, and $\mathbf{R}$ is an $(N_t \times N_t)$ upper triangular matrix with positive real-valued diagonal entries as

$$\mathbf{R} = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,N_t} \\ 0 & R_{2,2} & \cdots & R_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{N_t,N_t} \end{bmatrix}.$$

Let $[\mathbf{R}]_k$ denote the $k$th diagonal entry of $\mathbf{R}$. The FD expression of (10) can be equivalently written as

$$d_{\min}(\mathbf{H}) = \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\mathbf{QRD}\tilde{\mathbf{e}}_{ij}\| = \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\mathbf{Q}\tilde{\mathbf{R}}\tilde{\mathbf{e}}_{ij}\| \tag{14}$$

where $\tilde{\mathbf{R}} = \mathbf{RD}$ is also an upper triangular matrix with positive real-valued diagonal entries. From (14), we note that the QRD of the equivalent channel matrix $\tilde{\mathbf{H}}$ in (10) can be expressed as $\tilde{\mathbf{H}} = \mathbf{Q}\tilde{\mathbf{R}}$. The $k$th diagonal entry of $\tilde{\mathbf{R}}$ is $[\tilde{\mathbf{R}}]_k = [\mathbf{R}]_k \times [\mathbf{D}]_k$, where $[\mathbf{D}]_k$ is the $k$th diagonal entry of $\mathbf{D}$. Assuming that $\mathbf{H}$ is a full column rank matrix, its diagonal entries may be expressed as

$$[\mathbf{R}]_k = R_{k,k} = \sqrt{\frac{\det\left(\mathbf{H}_k^H\mathbf{H}_k\right)}{\det\left(\mathbf{H}_{k-1}^H\mathbf{H}_{k-1}\right)}} \tag{15}$$

where $\mathbf{H}_k = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_k]$ denotes a matrix consisting of the first $k$ columns of $\mathbf{H}$. Then, the diagonal entries of $\tilde{\mathbf{R}}$ become

$$[\tilde{\mathbf{R}}]_k = R_{k,k} \times d_k. \tag{16}$$

Now, another lower bound of the FD can be formulated via this QRD algorithm [13] as AQ1

$$d_{\min}(\mathbf{H}) = \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\mathbf{Q}\tilde{\mathbf{R}}\tilde{\mathbf{e}}_{ij}\| \geq \left(\min_{1\leq k\leq N_t}\left([\tilde{\mathbf{R}}]_k\right)\right) d_{\min}(\tilde{\Lambda})$$
$$= \min_{1\leq k\leq N_t}(R_{k,k} \times d_k) = [\tilde{\mathbf{R}}]_{\min}. \tag{17}$$

Here, $[\tilde{\mathbf{R}}]_{\min}$ is the minimum nonzero diagonal entry of the upper matrix $\tilde{\mathbf{R}}$, which varies depending on the different AM candidates. To maximize the FD of (17), the QRD-based AM-mode search algorithm is formulated as AQ2

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_j\in\mathbf{B}} [\tilde{\mathbf{R}}]_{\min}. \tag{18}$$

For the $(N_r \times N_t)$ equivalent channel matrix $\tilde{\mathbf{H}}$, it was shown in [12] that inequality $\sqrt{\lambda_{\min}(\tilde{\mathbf{H}})} \leq [\tilde{\mathbf{R}}]_{\min}$ holds for all channel realizations. Thus, the lower bound of (17) achieved by our QRD-based algorithm is tighter than that of the SVD-based method in (11).[1] On the other hand, it may be observed by utilizing (16) and (17) that the complexity imposed is low since only a single QRD is required for all AM-mode candidates in each channel realization. More specifically, only the diagonal entries of $\mathbf{R}$ have to be calculated by using QRD. Then, for each AM-mode candidate, we use the corresponding modulation order parameters $d_k$ $(k = 1, 2, \ldots, N_t)$ to calculate the diagonal entries $[\tilde{\mathbf{R}}]_k$ in (16). Note that computing $[\tilde{\mathbf{R}}]_k$ of our QRD-based AM-mode search algorithm only imposes simple scalar multiplications. The AM-mode optimization is completed by selecting the optimum AM-mode candidate having the largest minimum diagonal entry in (18).

## III. Simulation Results

Here, we compare the performance of the proposed QRD-based AM-mode search algorithm with that of conventional AM-based algorithms. The simulation setup is based on transmissions of 4–6 bits/MIMO symbol over MIMO channels exhibiting frequency-flat block Rayleigh fading. Note that a frequency-selective fading channel can be decomposed into orthogonal nondispersive subbands by the orthogonal frequency-division multiplexing (OFDM) technique, and the proposed AM-mode search algorithm can be then applied to each OFDM subband. The AM modes of no transmission, BPSK, and $M$-QAM ($M = 4$, 8, 16, 32, and 64) [16] are available for each transmit antenna. The optimal ML detector is adopted. To reduce the computational complexity, some lattice-based ML decoding algorithms, such as the classic sphere decoding and its diverse variants, can be considered, as detailed in [1], [17].

Fig. 2 shows the BER performance of various AM-based schemes having different throughputs. For completeness, we included the simulation-based BER curves and the theoretical union bounds [10] for the identical-throughput nonadaptive MIMO schemes. Moreover, we also included the conventional nonadaptive MIMO schemes. As expected, the proposed QRD-based AM-mode search algorithm has

[1] The QRD-based lower bound of (17) can be further tightened by using the basis transformation method of [12]. However, this method imposes high complexity, which becomes a major obstacle of its practical implementation. As shown in Section III, our proposed QRD algorithm still attains a near-optimal performance.

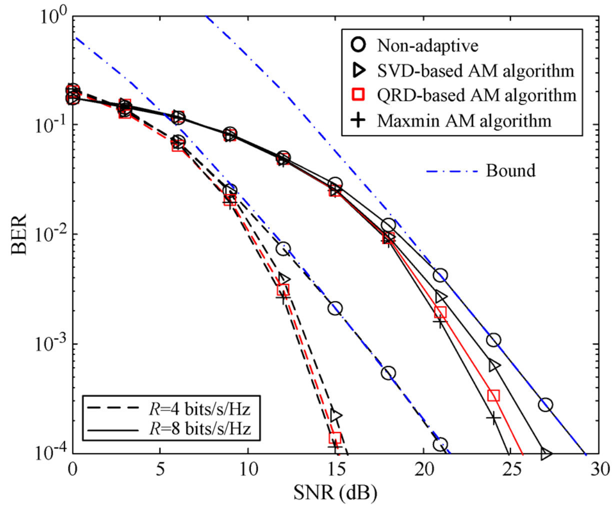


Fig. 2. BER performance comparison of different AM algorithms with $R =$ 4- and 8-bit/MIMO symbols for $(2 \times 2)$ MIMO channels. For completeness, the 4-QAM-assisted and 16-QAM-assisted $(2 \times 2)$ nonadaptive MIMO schemes are also employed.

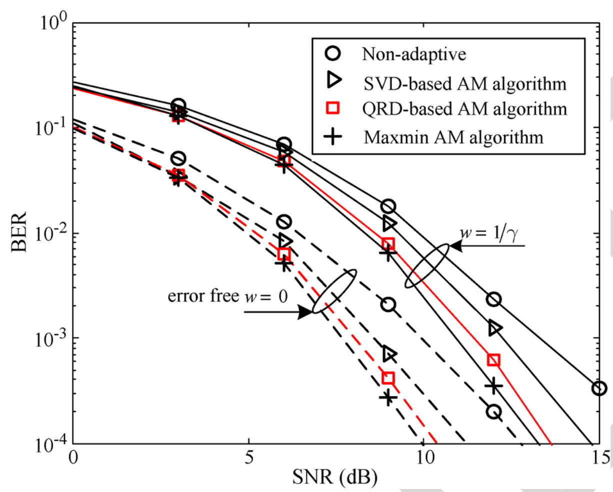


Fig. 3. BER performance of different AM algorithms for transmitting 4-bit/MIMO symbol over $(4 \times 4)$ MIMO channels. We also considered the effects of CSI error associated with a channel estimation noise variance of $1/\gamma$, assuming that the estimation error is Gaussian.

a tighter lower bound and performs better than the SVD-based AM algorithm. Explicitly, this scheme provides an SNR gain of about 3.9–6.1 dB over its nonadaptive MIMO counterparts at the BER of $10^{-4}$. Moreover, the performance gap between the QRD-based AM algorithm and the exhaustive-search-based max–min AM algorithm is only about 0.1–0.4 dB.

Fig. 3 compares the achievable BER performance of these AM algorithms in the presence of Gaussian-distributed CSI errors obeying $\mathcal{CN}(0, w)$ [14] associated with $w = 1/\gamma$, where $\gamma$ is the average CSI-estimation SNR[2] at each receiver antenna. Observe in Fig. 3 that, as expected, the BER performances of all schemes are degraded upon introducing CSI estimation errors. However, the AM-based algorithms still provide a considerable performance improvement over their nonadaptive counterparts having an identical throughput.

[2]For example, when the CSI-estimation error variance is 10 dB below the received signal variance, the CSI-estimation SNR is 10 dB.

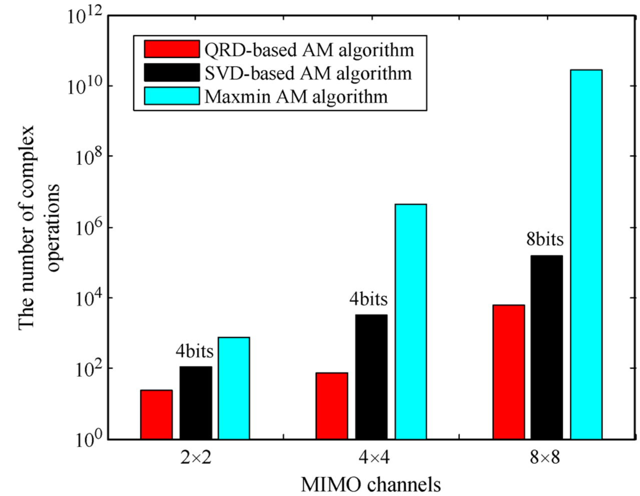


Fig. 4. Complexity comparison of various AM-based schemes under different MIMO channels.

Moreover, observe in Fig. 3 that the performance degradation imposed by the proposed QRD-based algorithm is lower than that by the SVD-based algorithm. As detailed in [11]–[13], the SVD-based AM-selection algorithm depends on $\sqrt{\lambda_k(\tilde{\mathbf{H}})}$, whereas our proposed QRD-based AM-selection algorithm relies on $[\tilde{\mathbf{R}}]_k$ $(k = 1, \ldots, N_t)$, where $\lambda_k(\tilde{\mathbf{H}})$ denotes the $k$th eigenvalue of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$. Let $\lambda_{\max}(\tilde{\mathbf{H}})$ and $\lambda_{\min}(\tilde{\mathbf{H}})$ denote the maximum and minimum nonzero eigenvalues of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, respectively, and $[\tilde{\mathbf{R}}]_{\max}$ and $[\tilde{\mathbf{R}}]_{\min}$ denote the maximum and minimum nonzero diagonal entries of upper matrix $\tilde{\mathbf{R}}$, respectively. It was shown in [12] that inequalities $\sqrt{\lambda_{\min}(\tilde{\mathbf{H}})} \leq [\tilde{\mathbf{R}}]_{\min}$ and $\sqrt{\lambda_{\max}(\tilde{\mathbf{H}})} \geq [\tilde{\mathbf{R}}]_{\max}$ hold for all channel realizations. As a result, inequality $\sqrt{\lambda_{\max}(\tilde{\mathbf{H}})}/\sqrt{\lambda_{\min}(\tilde{\mathbf{H}})} \geq [\tilde{\mathbf{R}}]_{\max}/[\tilde{\mathbf{R}}]_{\min}$ holds, which indicates that the range of variations is generally larger for $\sqrt{\lambda_k(\tilde{\mathbf{H}})}$ than that for $[\tilde{\mathbf{R}}]_k$. Hence, the channel matrix inaccuracies may result in larger variation on the eigenvalues of the SVD than those of $[\tilde{\mathbf{R}}]_k$ in the QRD. Therefore, our QRD-based algorithm may be expected to be more resilient to CSI estimation errors than the SVD-based algorithm. Nonetheless, the rigorous theoretical proof of the fact that the proposed QRD-based method's improved robustness to channel estimation errors remains an open challenge, which will be investigated in our further studies.

Finally, Fig. 4 shows the complexity comparison of the various AM-based algorithms, where only the multiplications and additions of complex numbers are considered operations. The max–min AM algorithm has to evaluate all legitimate candidates in (4), and all possible error vectors should be considered for each candidate. Hence, according to (3) and (4), the complexity $C$ of the max–min AM algorithm is

$$C = J \cdot \binom{2}{2^R}(N_t + 2N_rN_t) \tag{19}$$

where $\binom{\cdot}{\cdot}$ is the binomial coefficient, and $J$ is the number of the AM modes. For the QRD and SVD, the complexity is dominated by applying the Householder reflections [15] ,[3] which require $2N_t^2(N_r - N_t/3)$ complex-valued operations. Moreover, for the SVD-based AM algorithm, multiplying the diagonal matrix $\mathbf{D}$ in (10) requires $(N_t +$

[3]A Householder reflection is a linear transformation that describes a reflection about a plane or hyperplane containing the origin [15]. Householder transformations are widely used in numerical linear algebra to perform QR decompositions and in the first step of the SVD algorithm.

$N_r$) operations. Considering all possible candidates, the complexity $C$ of the SVD-based algorithm is calculated as

$$C = J \cdot \left[2N_t^2(N_r - N_t/3) + (N_t + N_r)\right]. \tag{20}$$

As shown in Section II-B, the complexity of the proposed QRD-based algorithm is approximately $J$ times lower than that of the SVD-based algorithm. In Fig. 4, we consider only the dominant complexity contributions of these algorithms. As expected, the QRD-based algorithm further reduced the complexity with respect to the SVD-based algorithm, particularly for a high number of transmit antennas.

## IV. Conclusion

In this paper, a QRD-based AM-mode search algorithm has been proposed to improve the attainable performance of limited-feedback-aided MIMO systems. The proposed algorithm exploits the nonzero minimum diagonal entry of the upper matrix generated by the QRD of the equivalent channel matrix for adjusting the modulation order. It has been shown that the proposed algorithm improved the achievable performance, despite its reduced complexity, when compared with the SVD-based algorithm in [11]

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AQ1 = Please check if subscript “k.k” should be “k, k”.
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AQ3 = Note that references [1] and [18] are the same. Therefore, reference [18] was deleted from the list. Citations were renumbered accordingly. Please check.

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# Correspondence

## QRD-Assisted Adaptive Modulation-Aided MIMO Systems

Ping Yang, Yue Xiao, Shaoqian Li, and Lajos Hanzo

***Abstract*—In this paper, we propose $QR$-decomposition (QRD)-based adaptive modulation (AM)-aided multiple-input–multiple-output (MIMO) systems. The proposed algorithm yields a tight lower bound of the free distance (FD), which determines the error probability of the detector in the high-signal-to-noise-ratio (SNR) region. Thus, this QRD-based AM algorithm is capable of achieving near-optimal performance at low complexity because the full QRD, which imposes high complexity, is performed only once for each channel realization, regardless of the number of AM modes. Our simulation results show that the proposed algorithm exhibits a better bit-error-rate (BER) performance and reduced complexity compared with the existing algorithms.**

***Index Terms*—Adaptive modulation (AM), free distance (FD), multiple-input multiple-output (MIMO), $QR$ decomposition (QRD).**

### I. Introduction

Multiple-input–multiple-output (MIMO) systems are capable of achieving a capacity gain and/or increased link robustness [1]–[3]. Hence, they have been adopted in most of the recent communication standards such as IEEE 802.11n, IEEE 802.16e, and Third-Generation Partnership Project Long-Term Evolution [4]. They may be also beneficially combined with adaptive modulation (AM) for adjusting the transmission parameters for the sake of accommodating time-varying channels [5]. Therefore, the effective combination of AM and MIMO techniques is a promising design alternative for high-rate wireless transmission systems [5], [6].

In MIMO systems, several AM-based link adaptation schemes [5]–[10] have been proposed for improving the achievable system performance. For example, the performance of AM-aided MIMO systems has been analyzed under both continuous- and discrete-rate scenarios in [7]. Moreover, adaptive MIMO architectures utilizing different combinations of modulation/coding schemes have been proposed in [8], which aimed for the maximization of the channel capacity at a predefined target bit error rate (BER), rather than for optimizing the BER. Moreover, these schemes were optimized for linear receivers. However, the AM candidate selection criterion designed for specific linear receivers in [5]–[8] may be unsuitable for the powerful nonlinear maximum-likelihood (ML) detector, which has an optimal performance.

To find an efficient AM algorithm for the MIMO ML detector, an exhaustive-search algorithm may be invoked to maximize the minimum Euclidean distance among the signal constellation points. However, this algorithm may be impractical owing to its potentially excessive complexity. To reduce the complexity, Ham *et al.* [11] invoked the singular value decomposition (SVD) to avoid the direct calculation of the Euclidean free distance (FD). Although this SVD-based algorithm is capable of achieving a lower bound of the FD, this bound may be loose; hence, the resultant solution is suboptimal. Furthermore, the computational complexity of the SVD-based algorithm is high for large search spaces associated with a high number of AM modes and transmit antennas $N_t$. In [12], $QR$ decomposition (QRD) was applied for antenna selection in MIMO systems, and it was demonstrated that the lower bound of the FD associated with the QRD can be further tightened compared with that of the SVD [12]. In [13], the equal-diagonal-based QRD was invoked for both the associated precoder's design at the transmitter and the successive-cancelation-based detection at the receiver. However, it has not been considered whether the QRD methods in [12] and [13] can be directly applied to AM candidate selection.

Against this background, these are the novel contributions of this paper.

- We apply the QRD to AM-based MIMO systems and propose a novel AM-mode assignment algorithm to improve the attainable BER performance. Specifically, we first formulate the FD by including the diagonal matrix $\mathbf{D}$ corresponding to different AM modes. Then, equivalent channel matrix $\tilde{\mathbf{H}}$, which combines the effects of both the channel conditions and of the modulation parameters, is decomposed by invoking the QRD algorithm [12], [13] for further tightening the lower bound of the given FD determined in [11].
- As a further benefit, we demonstrate that, regardless of the number of AM-mode candidates, the full QRD of channel matrix $\mathbf{H}$ in our proposed AM-based MIMO only has to be performed once for each channel realization, which dramatically reduces the search complexity. Thus, the proposed algorithm exhibits an attractive system performance at low complexity. Furthermore, our simulation results indicate that the algorithm is more robust to imperfect channel state information (CSI) than the SVD-based algorithm in [11].

The organization of this paper is as follows. Section II presents the system model of the AM-based MIMO systems. Our simulation results and performance comparisons are presented in Section III. Finally, Section IV concludes this paper.

### II. System Model and AM Algorithms

Consider an AM-aided MIMO system having $N_t$ transmit and $N_r$ receive antennas, as shown in Fig. 1. The $(N_t \times 1)$-element transmit symbol vector $\mathbf{x}$ is assumed to satisfy $E[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_{N_t}$, where $\mathbf{I}_{N_t}$ denotes an $(N_t \times N_t)$-element identity matrix. At the receiver, the corresponding $(N_r \times 1)$ received signal is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

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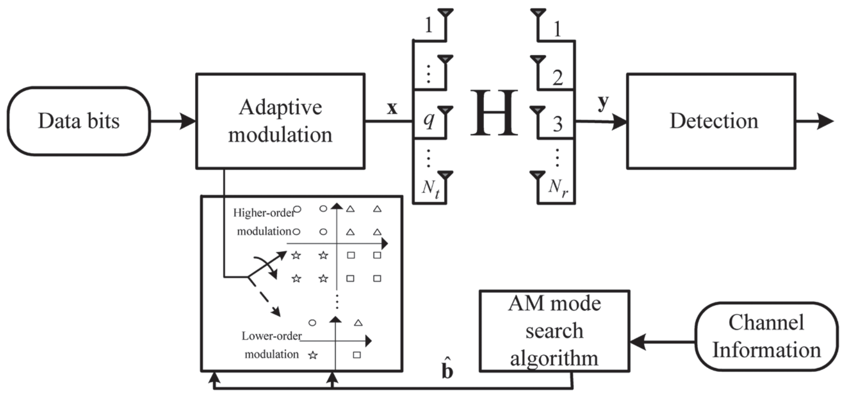


Fig. 1. System model of an AM-aided MIMO system.

where $\mathbf{H}$ is an $(N_r \times N_t)$ channel matrix, and the elements of the $N_r$-element noise vector are Gaussian random variables obeying $\mathcal{CN}(0, N_0)$. Given channel matrix $\mathbf{H}$, the pairwise error probability (PEP) between two arbitrary transmit vectors $\mathbf{x}_i$ and $\mathbf{x}_j (\mathbf{x}_i \neq \mathbf{x}_j)$ may be estimated as [10]

$$P(\mathbf{x}_i \to \mathbf{x}_j | \mathbf{H}) \approx \lambda \cdot Q\left(\sqrt{\frac{1}{2N_0} d^2_{\min}(\mathbf{H})}\right) \tag{2}$$

where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-y^2/2} dy$, and $\lambda$ is the number of neighboring constellation points [10], with FD $d_{\min}(\mathbf{H})$ defined as

$$d_{\min}(\mathbf{H}) = \min_{\substack{\mathbf{x}_i, \mathbf{x}_j \in \Lambda, \\ \mathbf{x}_i \neq \mathbf{x}_j}} \|\mathbf{H}(\mathbf{x}_i - \mathbf{x}_j)\| \tag{3}$$

where $\Lambda$ is the set of all possible transmit symbols, whereas $\mathbf{x}_i$ and $\mathbf{x}_j$ are two distinct transmitted symbols in $\Lambda$. In (2), the conditioned PEP is a monotonically decreasing function of $d_{\min}(\mathbf{H})$. Hence, the system's performance may be improved by maximizing the distance $d_{\min}(\mathbf{H})$ upon adapting the transmit parameters.

### A. Exhaustive-Search-Based AM Algorithm

Let $\mathbf{B}$ be a set of AM candidates given as $\mathbf{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_j, \ldots, \mathbf{b}_J\}$, where $\mathbf{b}_j = [b_j^1, \ldots, b_j^n \ldots, b_j^{N_t}]$, and $b_j^n$ denotes the modulation order for the $n$th transmit antenna of the $j$th AM candidate. These candidates satisfy the throughput constraint as

$$R = \sum_{n=1}^{N_t} \log_2\left(b_j^n\right), \qquad j = 1, 2, \ldots, J \tag{4}$$

where $J$ is the total number of possible modulation order combinations for the throughput of $R$ bits/symbol. To achieve the optimum performance, an exhaustive-search-based AM-mode search algorithm, which is referred to as the max–min AM algorithm, is formulated as

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_j \in \mathbf{B}} d_{\min}(\mathbf{H}). \tag{5}$$

Then, the corresponding index of the optimal AM-mode candidate $\hat{\mathbf{b}}$ is signaled back to the transmitter, which transmits the modulated symbols accordingly. Indeed, since the error events mainly arise from the nearest neighbors, the maximization of the FD in (3) directly reduces the probability of error, particularly at high SNRs [5], [9]. Deriving a closed-form solution to (5) remains an open challenge since the FD depends both on the constellation and on the channel realization [9]. Hence, a numerical search is adopted. Unfortunately, finding the optimal modulation order vector satisfying (5) has to tentatively invoke all possible candidates in (4). Moreover, for each AM-mode candidate, all possible error vectors, as given by $\mathbf{x}_e = \mathbf{x}_i - \mathbf{x}_j \ (\mathbf{x}_i, \mathbf{x}_j \in \Lambda)$, have to be considered in the calculation of (3). As a result, this exhaustive search has potentially prohibitive complexity.

### B. Conventional SVD-Based AM Algorithm

Here, we present the AM candidate selection criterion for the SVD-based algorithm [11]. Let us define the symbol constellation set of $x_k$ as $X$ and the minimum distance of the set $X$ as

$$d_{\min}(X) = \min_{\substack{x_i, x_j \in X, \\ x_i \neq x_j}} \|x_i - x_j\|^2. \tag{6}$$

The corresponding error symbol may be expressed as

$$e_{ij} = x_i - x_j = \bar{d}_{\min} \cdot \tilde{e}_{ij} \tag{7}$$

where $\bar{d}_{\min}$ denotes the minimum distance of the signal constellation according to the modulation order. For example, we have $\bar{d}_{\min} = 2$ for BPSK, $\bar{d}_{\min} = 2/\sqrt{2}$ for 4-ary quadrature amplitude modulation (4-QAM), $\bar{d}_{\min} = 2/\sqrt{6}$ for 8-QAM, $\bar{d}_{\min} = 2/\sqrt{10}$ for 16-QAM, and $\bar{d}_{\min} = 2/\sqrt{42}$ for 64-QAM [16]. Hence, $e_{ij}$ can be expressed as a multiple of $\bar{d}_{\min}$, and $\tilde{e}_{ij}$ is a version of $e_{ij}$ normalized by $\bar{d}_{\min}$. Let us now consider the case of $(N_t \times 1)$-element transmit vector $\mathbf{x}$, where the minimum distance of the symbol vector constellation may be expressed as

$$d_{\min}(\Lambda) = \min_{\substack{\mathbf{x}_i, \mathbf{x}_j \in \Lambda, \\ \mathbf{x}_i \neq \mathbf{x}_j}} \|\mathbf{x}_i - \mathbf{x}_j\| = \min_{\mathbf{e}_{ij} \in E} \|\mathbf{e}_{ij}\| \tag{8}$$

where $E$ is a set of error vectors, $\mathbf{e}_{ij}$ is the error vector, and

$$\mathbf{x}_i - \mathbf{x}_j = \begin{bmatrix} x_{i,1} - x_{j,1} \\ x_{i,2} - x_{j,2} \\ \vdots \\ x_{i,N_t} - x_{j,N_t} \end{bmatrix} = \begin{bmatrix} d_1 . \tilde{e}_{ij,1} \\ d_2 . \tilde{e}_{ij,2} \\ \vdots \\ d_{N_t} . \tilde{e}_{ij,N_t} \end{bmatrix} = \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_{N_t} \end{bmatrix} \begin{bmatrix} \tilde{e}_{ij,1} \\ \tilde{e}_{ij,2} \\ \vdots \\ \tilde{e}_{ij,N_t} \end{bmatrix} = \mathbf{D} \cdot \tilde{\mathbf{e}}_{ij} \tag{9}$$

where $\mathbf{D}$ is a diagonal matrix with $d_k$ being its diagonal elements, and normalized error vector $\tilde{\mathbf{e}}_{ij}$ is composed of normalized symbols $\tilde{e}_{ij,k} \ (k = 1, 2, \ldots, N_t)$. Here, $d_k$ is the minimum distance $\bar{d}_{\min}$ according to the modulation order assigned to each transmit antenna. Thus, based on (7)–(9), the FD of (3) can be reformulated as

$$d_{\min}(\mathbf{H}) = \min_{\tilde{\mathbf{e}}_{ij} \in \tilde{E}} \|\mathbf{H}\mathbf{D}\tilde{\mathbf{e}}_{ij}\| = \min_{\tilde{\mathbf{e}}_{ij} \in \tilde{E}} \|\tilde{\mathbf{H}}\tilde{\mathbf{e}}_{ij}\| \tag{10}$$

where $\tilde{\mathbf{H}} = \mathbf{HD}$ is the equivalent channel matrix. Using the Rayleigh–Ritz theorem in [8], the lower bound of (10) is expressed as

$$d_{\min}(\mathbf{H}) \geq \sqrt{\lambda_{\min}(\tilde{\mathbf{H}})}\, d_{\min}(\tilde{\Lambda}) \tag{11}$$

where $\lambda_{\min}(\tilde{\mathbf{H}})$ is the nonzero minimum eigenvalue of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, and $d_{\min}(\tilde{\Lambda}) = \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\tilde{\mathbf{e}}_{ij}\|_F$ is the minimum distance of the normalized error vector. In a spatial multiplexing MIMO system, the elements $\tilde{e}_{ij,k}$ $(k = 1, 2, \ldots, N_t)$ of $\tilde{\mathbf{e}}_{ij}$ are usually uncorrelated. Thus, according to (7), this minimum distance can be expressed as

$$d_{\min}(\tilde{\Lambda}) = \min\left\{d_{\min}(\tilde{\Lambda}_1), d_{\min}(\tilde{\Lambda}_2), \ldots, d_{\min}(\tilde{\Lambda}_{N_t})\right\} = 1 \tag{12}$$

where $d_{\min}(\tilde{\Lambda}_k)$ denotes the minimum distance from all legitimate signal constellation points $\tilde{e}_{ij,k}$. The lower bound formulated in (11) indicates that the FD can be evaluated with the aid of $\lambda_{\min}(\tilde{\mathbf{H}})$. As shown in (12), the value of $d_{\min}(\tilde{\Lambda})$ is constant, whereas the value of $\lambda_{\min}(\tilde{\mathbf{H}})$ is changed for the different AM candidates. Then, the SVD-based AM-mode search algorithm can be formulated as

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_j\in\mathbf{B}} \lambda_{\min}(\tilde{\mathbf{H}}). \tag{13}$$

In the SVD-based AM algorithm, only the minimum nonzero singular value of each $\tilde{\mathbf{H}}$ has to be calculated; thus, the complexity is reduced compared to the max–min AM algorithm formulated in (5). However, the computational complexity of this SVD-based algorithm is high when the number of AM-mode candidates and $N_t$ is large. For example, assuming that the AM-modes of no transmission, BPSK, and $M$-ary QAM ($M =$ 4, 8, 16, 32, and 64) are available for all the $N_t = 8$ transmit antennas, for an 8-bit/symbol transmission, we need an exhaustive search over a set of 6371 candidates. In (13), we have to carry out the SVD of the equivalent channel matrix $\tilde{\mathbf{H}}$ for each candidate; hence, the resultant complexity is high. More importantly, the lower bound of the FD achieved in (11) may not be tight enough; hence, the resultant solution becomes suboptimal.

### C. Proposed QRD-Based AM Algorithm

Here, we apply the QRD algorithm to formulate our AM candidate selection criterion. According to (3), channel matrix $\mathbf{H}$ is first subject to the QRD [12], [13], yielding $\mathbf{H} = \mathbf{QR}$, where $\mathbf{Q}$ is an $(N_r \times N_t)$ column-wise orthonormal matrix, and $\mathbf{R}$ is an $(N_t \times N_t)$ upper triangular matrix with positive real-valued diagonal entries as

$$\mathbf{R} = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,N_t} \\ 0 & R_{2,2} & \cdots & R_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{N_t,N_t} \end{bmatrix}.$$

Let $[\mathbf{R}]_k$ denote the $k$th diagonal entry of $\mathbf{R}$. The FD expression of (10) can be equivalently written as

$$d_{\min}(\mathbf{H}) = \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\mathbf{QRD}\tilde{\mathbf{e}}_{ij}\| = \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\mathbf{Q}\tilde{\mathbf{R}}\tilde{\mathbf{e}}_{ij}\| \tag{14}$$

where $\tilde{\mathbf{R}} = \mathbf{RD}$ is also an upper triangular matrix with positive real-valued diagonal entries. From (14), we note that the QRD of the equivalent channel matrix $\tilde{\mathbf{H}}$ in (10) can be expressed as $\tilde{\mathbf{H}} = \mathbf{Q}\tilde{\mathbf{R}}$. The $k$th diagonal entry of $\tilde{\mathbf{R}}$ is $[\tilde{\mathbf{R}}]_k = [\mathbf{R}]_k \times [\mathbf{D}]_k$, where $[\mathbf{D}]_k$ is the $k$th diagonal entry of $\mathbf{D}$. Assuming that $\mathbf{H}$ is a full column rank matrix, its diagonal entries may be expressed as

$$[\mathbf{R}]_k = R_{k,k} = \sqrt{\frac{\det\left(\mathbf{H}_k^H\mathbf{H}_k\right)}{\det\left(\mathbf{H}_{k-1}^H\mathbf{H}_{k-1}\right)}} \tag{15}$$

where $\mathbf{H}_k = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_k]$ denotes a matrix consisting of the first $k$ columns of $\mathbf{H}$. Then, the diagonal entries of $\tilde{\mathbf{R}}$ become

$$[\tilde{\mathbf{R}}]_k = R_{k,k} \times d_k. \tag{16}$$

Now, another lower bound of the FD can be formulated via this QRD algorithm [13] as AQ1

$$\begin{aligned} d_{\min}(\mathbf{H}) &= \min_{\tilde{\mathbf{e}}_{ij}\in\tilde{E}} \|\mathbf{Q}\tilde{\mathbf{R}}\tilde{\mathbf{e}}_{ij}\| \geq \left(\min_{1\leq k\leq N_t}\left([\tilde{\mathbf{R}}]_k\right)\right) d_{\min}(\tilde{\Lambda}) \\ &= \min_{1\leq k\leq N_t}(R_{k,k}\times d_k) = [\tilde{\mathbf{R}}]_{\min}. \end{aligned} \tag{17}$$

Here, $[\tilde{\mathbf{R}}]_{\min}$ is the minimum nonzero diagonal entry of the upper matrix $\tilde{\mathbf{R}}$, which varies depending on the different AM candidates. To maximize the FD of (17), the QRD-based AM-mode search algorithm is formulated as AQ2

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_j\in\mathbf{B}} [\tilde{\mathbf{R}}]_{\min}. \tag{18}$$

For the $(N_r \times N_t)$ equivalent channel matrix $\tilde{\mathbf{H}}$, it was shown in [12] that inequality $\sqrt{\lambda_{\min}(\tilde{\mathbf{H}})} \leq [\tilde{\mathbf{R}}]_{\min}$ holds for all channel realizations. Thus, the lower bound of (17) achieved by our QRD-based algorithm is tighter than that of the SVD-based method in (11).[1] On the other hand, it may be observed by utilizing (16) and (17) that the complexity imposed is low since only a single QRD is required for all AM-mode candidates in each channel realization. More specifically, only the diagonal entries of $\mathbf{R}$ have to be calculated by using QRD. Then, for each AM-mode candidate, we use the corresponding modulation order parameters $d_k$ $(k = 1, 2, \ldots, N_t)$ to calculate the diagonal entries $[\tilde{\mathbf{R}}]_k$ in (16). Note that computing $[\tilde{\mathbf{R}}]_k$ of our QRD-based AM-mode search algorithm only imposes simple scalar multiplications. The AM-mode optimization is completed by selecting the optimum AM-mode candidate having the largest minimum diagonal entry in (18).

## III. Simulation Results

Here, we compare the performance of the proposed QRD-based AM-mode search algorithm with that of conventional AM-based algorithms. The simulation setup is based on transmissions of 4–6 bits/MIMO symbol over MIMO channels exhibiting frequency-flat block Rayleigh fading. Note that a frequency-selective fading channel can be decomposed into orthogonal nondispersive subbands by the orthogonal frequency-division multiplexing (OFDM) technique, and the proposed AM-mode search algorithm can be then applied to each OFDM subband. The AM modes of no transmission, BPSK, and $M$-QAM ($M =$ 4, 8, 16, 32, and 64) [16] are available for each transmit antenna. The optimal ML detector is adopted. To reduce the computational complexity, some lattice-based ML decoding algorithms, such as the classic sphere decoding and its diverse variants, can be considered, as detailed in [1], [17].

Fig. 2 shows the BER performance of various AM-based schemes having different throughputs. For completeness, we included the simulation-based BER curves and the theoretical union bounds [10] for the identical-throughput nonadaptive MIMO schemes. Moreover, we also included the conventional nonadaptive MIMO schemes. As expected, the proposed QRD-based AM-mode search algorithm has

[1]The QRD-based lower bound of (17) can be further tightened by using the basis transformation method of [12]. However, this method imposes high complexity, which becomes a major obstacle of its practical implementation. As shown in Section III, our proposed QRD algorithm still attains a near-optimal performance.

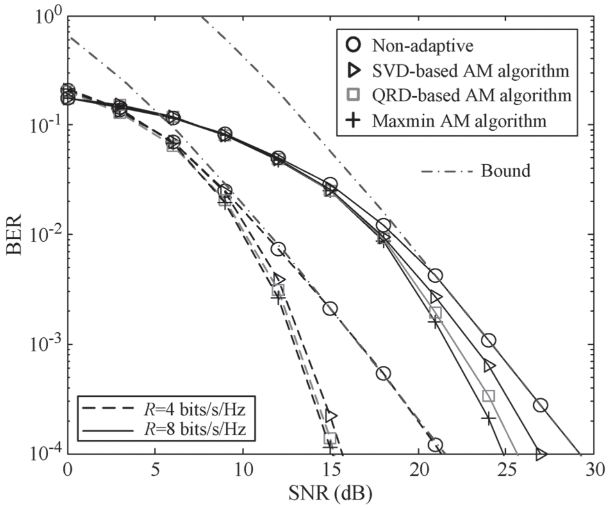


Fig. 2. BER performance comparison of different AM algorithms with $R =$ 4- and 8-bit/MIMO symbols for $(2 \times 2)$ MIMO channels. For completeness, the 4-QAM-assisted and 16-QAM-assisted $(2 \times 2)$ nonadaptive MIMO schemes are also employed.

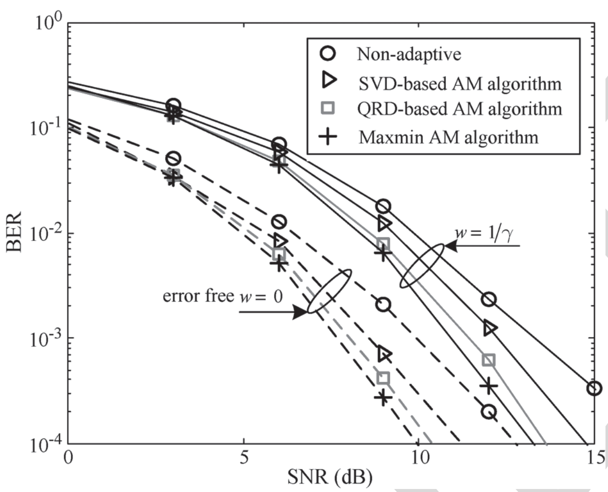


Fig. 3. BER performance of different AM algorithms for transmitting 4-bit/MIMO symbol over $(4 \times 4)$ MIMO channels. We also considered the effects of CSI error associated with a channel estimation noise variance of $1/\gamma$, assuming that the estimation error is Gaussian.

a tighter lower bound and performs better than the SVD-based AM algorithm. Explicitly, this scheme provides an SNR gain of about 3.9–6.1 dB over its nonadaptive MIMO counterparts at the BER of $10^{-4}$. Moreover, the performance gap between the QRD-based AM algorithm and the exhaustive-search-based max–min AM algorithm is only about 0.1–0.4 dB.

Fig. 3 compares the achievable BER performance of these AM algorithms in the presence of Gaussian-distributed CSI errors obeying $\mathcal{CN}(0, w)$ [14] associated with $w = 1/\gamma$, where $\gamma$ is the average CSI-estimation SNR[2] at each receiver antenna. Observe in Fig. 3 that, as expected, the BER performances of all schemes are degraded upon introducing CSI estimation errors. However, the AM-based algorithms still provide a considerable performance improvement over their nonadaptive counterparts having an identical throughput.

[2]For example, when the CSI-estimation error variance is 10 dB below the received signal variance, the CSI-estimation SNR is 10 dB.

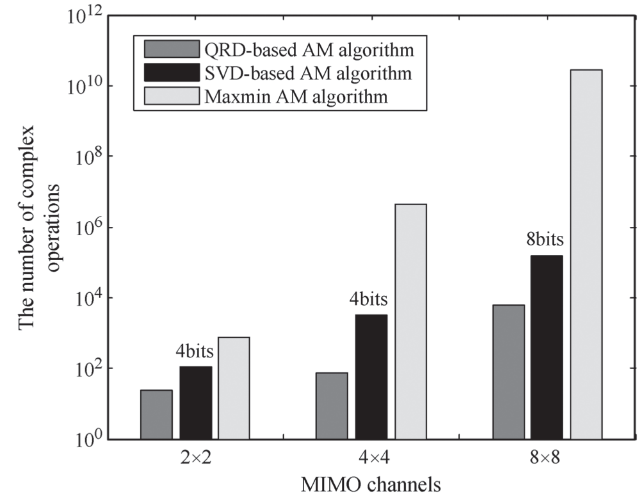


Fig. 4. Complexity comparison of various AM-based schemes under different MIMO channels.

Moreover, observe in Fig. 3 that the performance degradation imposed by the proposed QRD-based algorithm is lower than that by the SVD-based algorithm. As detailed in [11]–[13], the SVD-based AM-selection algorithm depends on $\sqrt{\lambda_k(\tilde{\mathbf{H}})}$, whereas our proposed QRD-based AM-selection algorithm relies on $[\tilde{\mathbf{R}}]_k$ $(k = 1, \ldots, N_t)$, where $\lambda_k(\tilde{\mathbf{H}})$ denotes the $k$th eigenvalue of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$. Let $\lambda_{\max}(\tilde{\mathbf{H}})$ and $\lambda_{\min}(\tilde{\mathbf{H}})$ denote the maximum and minimum nonzero eigenvalues of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, respectively, and $[\tilde{\mathbf{R}}]_{\max}$ and $[\tilde{\mathbf{R}}]_{\min}$ denote the maximum and minimum nonzero diagonal entries of upper matrix $\tilde{\mathbf{R}}$, respectively. It was shown in [12] that inequalities $\sqrt{\lambda_{\min}(\tilde{\mathbf{H}})} \leq [\tilde{\mathbf{R}}]_{\min}$ and $\sqrt{\lambda_{\max}(\tilde{\mathbf{H}})} \geq [\tilde{\mathbf{R}}]_{\max}$ hold for all channel realizations. As a result, inequality $\sqrt{\lambda_{\max}(\tilde{\mathbf{H}})}/\sqrt{\lambda_{\min}(\tilde{\mathbf{H}})} \geq [\tilde{\mathbf{R}}]_{\max}/[\tilde{\mathbf{R}}]_{\min}$ holds, which indicates that the range of variations is generally larger for $\sqrt{\lambda_k(\tilde{\mathbf{H}})}$ than that for $[\tilde{\mathbf{R}}]_k$. Hence, the channel matrix inaccuracies may result in larger variation on the eigenvalues of the SVD than those of $[\tilde{\mathbf{R}}]_k$ in the QRD. Therefore, our QRD-based algorithm may be expected to be more resilient to CSI estimation errors than the SVD-based algorithm. Nonetheless, the rigorous theoretical proof of the fact that the proposed QRD-based method's improved robustness to channel estimation errors remains an open challenge, which will be investigated in our further studies.

Finally, Fig. 4 shows the complexity comparison of the various AM-based algorithms, where only the multiplications and additions of complex numbers are considered operations. The max–min AM algorithm has to evaluate all legitimate candidates in (4), and all possible error vectors should be considered for each candidate. Hence, according to (3) and (4), the complexity $C$ of the max–min AM algorithm is

$$C = J \cdot \binom{2}{2^R} (N_t + 2N_r N_t) \tag{19}$$

where $\binom{\cdot}{\cdot}$ is the binomial coefficient, and $J$ is the number of the AM modes. For the QRD and SVD, the complexity is dominated by applying the Householder reflections [15] ,[3] which require $2N_t^2(N_r - N_t/3)$ complex-valued operations. Moreover, for the SVD-based AM algorithm, multiplying the diagonal matrix $\mathbf{D}$ in (10) requires $(N_t +$

[3]A Householder reflection is a linear transformation that describes a reflection about a plane or hyperplane containing the origin [15]. Householder transformations are widely used in numerical linear algebra to perform QR decompositions and in the first step of the SVD algorithm.

$N_r$) operations. Considering all possible candidates, the complexity $C$ of the SVD-based algorithm is calculated as

$$C = J \cdot \left[2N_t^2(N_r - N_t/3) + (N_t + N_r)\right]. \quad (20)$$

As shown in Section II-B, the complexity of the proposed QRD-based algorithm is approximately $J$ times lower than that of the SVD-based algorithm. In Fig. 4, we consider only the dominant complexity contributions of these algorithms. As expected, the QRD-based algorithm further reduced the complexity with respect to the SVD-based algorithm, particularly for a high number of transmit antennas.

## IV. Conclusion

In this paper, a QRD-based AM-mode search algorithm has been proposed to improve the attainable performance of limited-feedback-aided MIMO systems. The proposed algorithm exploits the nonzero minimum diagonal entry of the upper matrix generated by the QRD of the equivalent channel matrix for adjusting the modulation order. It has been shown that the proposed algorithm improved the achievable performance, despite its reduced complexity, when compared with the SVD-based algorithm in [11]

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