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ANALYSIS AND OPTIMISATION OF TUNED MASS DAMPERS FOR IMPULSIVE EXCITATION

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ABSTRACT

Tuned Mass Damper (TMD) devices have been widely studied and optimised in the framework of persistent dynamic loadings, such as harmonic and white noise excitations, in order to reduce as much as possible the steady-state response of an assigned primary structure. In this sense, the present paper arises as a complementary study on the topic, since here shock input is assumed as dynamic loading, so as to investigate the effectiveness of TMDs in reducing the transient structural response. In particular, an impulsive loading has been considered, acting as a base displacement, which is a situation that may occur in real applications. First, a comprehensive dynamic analysis of the system comprising of a primary structure and an attached passive TMD is carried out in the time domain. Focus is placed on the relationships between the load input and the system properties, in order to explore the dynamic behaviour of such system and to identify the main response trends, mostly as a function of the free TMD parameters, namely mass, frequency and TMD damping ratios. Subsequently, a hybrid TMD has been considered, by adding a feedback controller to the previously optimised passive TMD, so as to improve the performance of such a device, especially in reducing the peak response of the primary structure. The contents of the present work have the final aim of identifying the potential level of effectiveness of the TMD devices and to supply important guidelines towards their optimal design in reducing the structural response also to shock excitations. This should display significant relevance in different practical applications, including in the field of earthquake engineering.

1. INTRODUCTION

The present work concerns the optimisation of passive and hybrid Tuned Mass Damper devices for structural systems subjected to impulse loading, and is placed within a wider ongoing research project at the University of Bergamo [1–4] and at the University of Southampton [5–8].

Structural systems can be easily subjected to a wide range of harmful dynamic actions of different nature, especially from the point of view of duration and intensity. Within this context, the reduction and the control of the dynamic response due to pulse loading is doubtless an important research topic, mostly for its potential contribution in several engineering applications, such as those in earthquake engineering and in the automotive field.

In the framework of vibration control, the Tuned Mass Damper is certainly one of the most studied devices. Indeed, despite that its introduction could be dated back to more than a century ago, many studies on Tuned Mass Dampers are still currently under development, especially on the optimal design, usually referred to as tuning, of its structural parameters. The documented introduction of Tuned Mass Dampers is probably represented by the patent of Frahm [9], and has been followed by different fundamental studies [10–12], which determined first the theoretical bases and formulas for the optimal design of TMD devices, assuming as external loading a harmonic excitation, for a force acting on an undamped primary structure.

Subsequently, many works have focused on the optimal tuning of Tuned Mass Dampers, in order to deepen the knowledge for different response indices and dynamic excitations. In particular, the usual framework of such studies considered a primary single-degree-of-freedom (SDOF) damped structure with an added TMD subjected to either harmonic [13–15] or white noise [16, 17] excitations, acting as a force on the primary structure or as base motion [4, 18–20].

In recent years, semi-active and active Tuned Mass Dampers have been also thoroughly investigated as complementary or alternative control devices, with respect to passive TMDs [21–23], for their inherent efficiency limits, mostly due to the operational narrow band and the sensitivity to parameter variations, also called de-tuning. Also for these studies, persistent signals of different characteristics have been considered, such as generic harmonic excitation [24] or earthquake input [25, 26].

In the case of shock excitation, the passive Tuned Mass Damper is generally considered as not significantly effective in reducing the structural response [27]. However, it appears from the literature that this field has not been thoroughly investigated yet. The present paper considers this problem and deals with the study of the optimal tuning of a Tuned Mass Damper when the structural system is subjected to shock excitation. A structural system composed of a damped SDOF primary structure and a Tuned Mass Damper added on top, when subjected to a unit impulse base displacement has been considered.

The paper is organised as follows. Firstly, the structural context and the dynamic response are explained in detail, then the numerical optimisation of passive Tuned Mass Dampers is developed, showing the potential application benefits. Then, the hybrid configuration of the TMD is also considered, by the introduction of a feedback controller between the primary structure and the TMD, in order to investigate the possible improvement in terms of efficiency in reducing the dynamic response of the primary structure with respect to the passive case.

The optimisation process has been studied in detail, and an overview of the obtained results, in terms of optimal parameters of the Tuned Mass Damper and the corresponding obtained reduction of the dynamic response, have been presented, together with first significant considerations for the design of TMDs within the considered framework.

2. STRUCTURAL CONTEXT AND DYNAMIC RESPONSE

2.1 Dynamic response to unit impulse base displacement

The structural system comprising of a single-degree-of-freedom primary structure and an added Tuned Mass Damper, subjected to a generic base displacement x_g, is represented in Fig. 1.

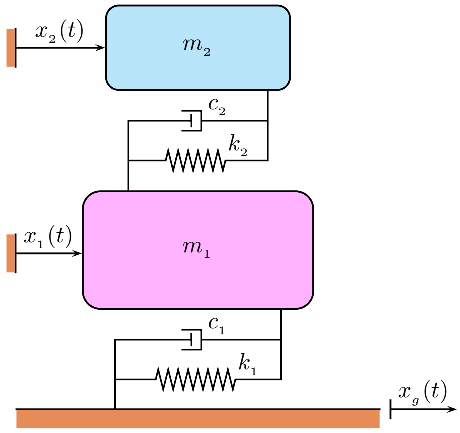


Figure 1: Structural parameters and absolute dynamic degrees of freedom of a 2DOF mechanical system comprised of a SDOF primary structure (1), equipped with an added passive TMD (2), subjected to generic base displacement.

The structural parameters which characterise the primary structure (1) are the mass m_1, the elastic stiffness k_1 and the viscous damping coefficient c_1. In a similar way, the Tuned Mass Damper is represented by the mass m_2, the elastic stiffness constant k_2 and the viscous damping coefficient c_2. The equations of motion of the system described above take the following form, in terms of the absolute degrees of freedom $x_1(t)$ and $x_2(t)$:

$$\begin{cases} m_1\ddot{x}_1(t) + (c_1 + c_2)\dot{x}_1(t) - c_2\dot{x}_2(t) + (k_1 + k_2)x_1(t) - k_2x_2(t) = c_1\dot{x}_g(t) + k_1x_g(t) \\ m_2\ddot{x}_2(t) - c_2\dot{x}_1(t) + c_2\dot{x}_2(t) - k_2x_1(t) + k_2x_2(t) = 0 \end{cases} \tag{1}$$

Such a system is assumed to be initially at rest and subjected to a unit impulse excitation at $t = 0$, which may be ideally defined by a Dirac delta function, characterised by the property:

$$\int_{-\infty}^{+\infty} \delta(t)\mathrm{d}t = 1 \tag{2}$$

Due to computational reasons, such loading has been modelled in real terms as a frequency-variable versed-sine pulse [28], defined as follows:

$$x_g(t) = \begin{cases} A_p[1 - \cos(\omega_p\, t)]\,, & 0 \le t \le T_p \\ 0\,, & \text{elsewhere} \end{cases} \tag{3}$$

where A_p, T_p and $\omega_p = 2\pi/T_p$ define the amplitude, the duration and the angular frequency of the versed-sine pulse, respectively.

In order to emulate the unit impulse, the versed-sine pulse $x_g(t)$ must fulfil the condition expressed by Eq. (2). Thus, since

$$\int_{-\infty}^{+\infty} [1 - \cos(\omega_p t)]\mathrm{d}t = \int_{-\frac{T_p}{2}}^{+\frac{T_p}{2}} \left[1 - \cos\left(\frac{2\pi}{T_p}t\right)\right]\mathrm{d}t = T_p \tag{4}$$

one obtains the link:

$$A_p = \frac{1}{T_p} = \frac{\omega_p}{2\pi} \tag{5}$$

Despite that the ideal case of a Dirac delta function may be represented only in the limit for $\omega_p \to \infty$, a sample study on the response of a SDOF system revealed that values not that high of the versed-sine pulse frequency allow for a good approximation of an impulse signal, suitable for dynamical analyses. In this sense, for the present study ω_p = 1000 rad/s has been assumed.

By substituting Eq. (3) into Eq. (1), in the case of versed-sine pulse base displacement, one obtains:

$$\begin{cases} m_1 \ddot{x}_1(t) + (c_1 + c_2)\dot{x}_1(t) - c_2 \dot{x}_2(t) + (k_1 + k_2)x_1(t) - k_2 x_2(t) = \\ \qquad\qquad = c_1 \dfrac{\omega_p^2}{2\pi} \sin(\omega_p t) + k_1 \dfrac{\omega_p}{2\pi}[1 - \cos(\omega_p t)] \\ m_2 \ddot{x}_2(t) - c_2 \dot{x}_1(t) + c_2 \dot{x}_2(t) - k_2 x_1(t) + k_2 x_2(t) = 0 \end{cases} \tag{6}$$

The primary structure natural angular frequency ω_1 and damping ratio ζ_1 are defined as follows:

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \qquad \zeta_1 = \frac{c_1}{c_{1,cr}} = \frac{c_1}{2\sqrt{k_1 m_1}} = \frac{c_1}{2\,\omega_1 m_1} \tag{7}$$

and, likewise, the TMD natural frequency ω_2 and damping ratio ζ_2 are defined as:

$$\omega_2 = \sqrt{\frac{k_2}{m_2}}, \qquad \zeta_2 = \frac{c_2}{c_{2,cr}} = \frac{c_2}{2\sqrt{k_2 m_2}} = \frac{c_2}{2\,\omega_2 m_2} \tag{8}$$

Besides the TMD damping ratio ζ_2, two further parameters are introduced for tuning purposes, namely the mass ratio μ and the frequency ratio f:

$$\mu = \frac{m_2}{m_1}, \qquad f = \frac{\omega_2}{\omega_1} \tag{9}$$

The structural response in the time domain has been obtained indirectly through a pair of Laplace transforms of Eq. (6), with relevant procedure and analytical expressions reported in Appendix A.

3. PASSIVE TMD

3.1 Tuning process

The optimal tuning of the TMD parameters, i.e. mass ratio μ, frequency ratio f and TMD damping ratio ζ_2, which have been defined in Eqs. (8)–(9), can be easily turned into a classical optimisation problem, where an assigned multi-variable objective (or cost) function is minimised:

$$\min_{\mathbf{v}} \mathrm{F}(\mathbf{v}), \qquad \mathbf{l_b} \leq \mathbf{v} \leq \mathbf{u_b} \tag{10}$$

where $\mathbf{v}$ is the vector of the tuning variables, $\mathrm{F}(\mathbf{v})$ is the objective function, $\mathbf{l_b}$ and $\mathbf{u_b}$ are the lower and upper bound vectors of the tuning variables, respectively. In this work, the scalar objective function to be minimised is assumed to be a norm of the displacement of the primary structure in the time domain $x_1(t)$, denoted in Eq. (A.7) with index $n = 1$.

The typical approach in the literature will be adopted here as well, towards seeking the optimal TMD parameters, i.e. by assuming the value of the mass ratio μ as given within some reasonable typical range suitable for engineering applications, while the frequency ratio f and the TMD damping ratio ζ_2 are taken as the optimisation variables. A reliable range of values of the primary structure damping ratio ζ_1 has been also considered, so that one may study the influence of this parameter on the best TMD parameters and relevant efficiency. The optimisation problem related to the response quantity (given in Eq. (A.7)) is quite complex to be solved analytically, therefore recourse to numerical optimisation methods become suitable, if not necessary. In this sense, in the present work the optimisation has been developed within a *MATLAB* environment [1–4], by means of the *fmincon* function, which allows for constrained non-linear optimisation. In a preliminary stage of numerical simulations, different norms of the displacement of the primary structure have been considered. The H_i norm of a general response quantity x is defined as follows [31]:

$$\|x\|_i = \left(\sum_{n=1}^{N} x_n^i \right)^{\frac{1}{i}} \tag{11}$$

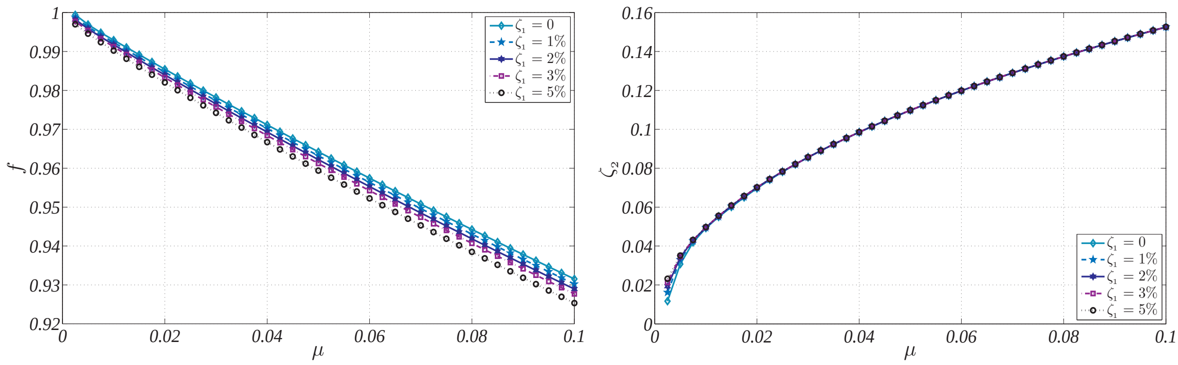
where N is the total number of time samples of x in the assumed time interval. The present tests considered H_2 and H_∞ norms, leading to the control of the overall and the maximum displacement, respectively. The values of the structural parameters considered in this trial are $m_1 = 100$ kg, $k_1 = 10000$ N/m, $\zeta_1 = 0.05$ and $\mu = 0.05$. These trials identified that the optimisation of the H_∞ norm leads to disappointing results, since the reduction of the peak (H_∞ norm) displacement turns out to be negligible and the overall (H_2 norm) displacement of the primary structure may even appear to be increased. On the other hand, the optimisation of the H_2 norm allows for a significant decrease in the overall response, even if the peak of displacement is again not significantly reduced. Thus, this latter norm has been assumed as the objective function in the following optimisation process. The parameters (assumed by considering engineering applications) and bounds adopted within the optimisation process, have been shown in Table 1.

Tuning variables	$\mathbf{v} = [f; \zeta_2]$
Lower bounds	$\mathbf{l_b} = [10^{-3}; 10^{-3}]$
Upper bounds	$\mathbf{u_b} = [5; 1]$
Mass ratio range	$\boldsymbol{\mu} = [0.0025 : 0.0025 : 0.1]$
Primary structure damping ratio range	$\boldsymbol{\zeta_1} = [0, 0.01, 0.02, 0.03, 0.05]$
Tolerance on variable parameter	10^{-6}
Tolerance on constraint violation	10^{-6}
Tolerance on objective function	10^{-6}
Max. number of iterations	300
Max. number of function evaluation	300

Table 1: Chosen parameter values for the optimisation process.

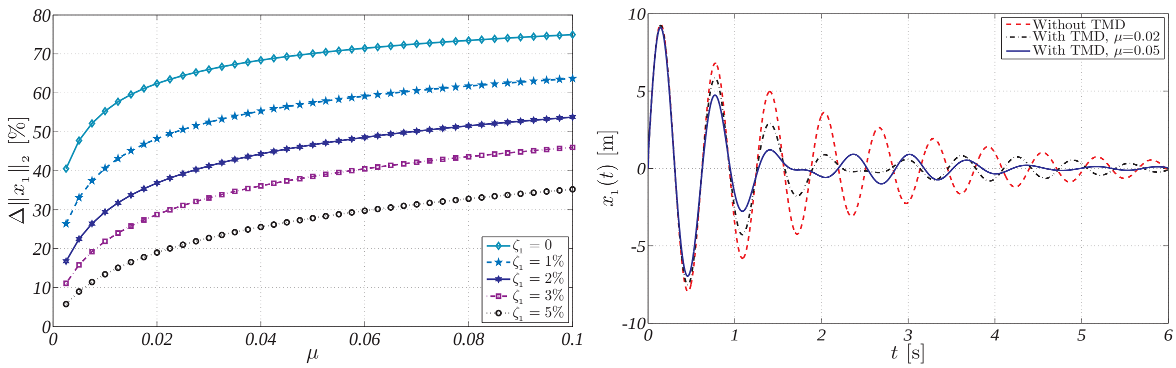
3.2 Results

A significant extract of the results achieved from the optimisation process is presented in Figs. 2–3, where the optimal TMD parameters, the percent reduction of the cost function and a sample of the time response in terms of the displacement of the primary structure are represented. Such results lead to the following considerations. Firstly, the trends assumed by the optimal TMD parameters f^{opt} and ζ_2^{opt} as a function of the mass ratio μ, displayed in Fig. 2, are quite similar to those obtained in the case of persistent input usually assumed in TMD analysis, e.g. harmonic or white noise loading [4]. Indeed, for increasing mass ratio μ and primary structure damping ratio ζ_1, the frequency ratio f decreases and the TMD damping ratio ζ_2 increases, with typical trends (Fig. 2a). In fact, it is worth noting the insensitivity of the TMD damping ratio ζ_2 with respect to the variation of the primary structure damping ratio ζ_1 (Fig. 2b). In Fig. 3a, from the achieved percentage response reduction it can be noted that the efficiency of the optimal TMD increases as the mass ratio μ increases, and decreases significantly at increasing ζ_1. For instance, by considering an assigned mass ratio $\mu = 0.05$, one obtains a reduction of the primary structure displacement of about 70% for $\zeta_1 = 0$ and of 30% for $\zeta_1 = 0.05$. The increase in the Tuned Mass Damper efficiency is remarkable, specifically in the range of values of mass ratio $\mu < 0.05$, beyond which any further improvement in the TMD performance is less noticeable. These results demonstrate that, in principle, optimal passive Tuned Mass Dampers may reduce significantly the overall dynamic response of structural systems even in the case of shock loadings, such as the impulsive excitation considered here. The time history of the primary structure displacement (Fig. 3b), where $\zeta_1 = 0.05$ is assumed, indicates that a TMD characterised by a mass ratio $\mu = 0.02$ allows for a considerable reduction of the primary structure displacement in the whole time window, especially after the first peaks in the response. With an increase in the mass ratio to $\mu = 0.05$, a noticeable improvement of TMD efficiency is obtained also for the initial part of the response, just after the transient input has been applied. However, the maximum response, which in case of shock loading for a system initially at rest occurs at the first peak of the dynamic response, seems not to be affected by the insertion of a passive TMD.



(a) (b)

Figure 2: Optimal TMD parameters at variable μ for different values of the primary structure damping ratio ζ_1: (a) frequency ratio f; (b) TMD damping ratio ζ_2.



(a) (b)

Figure 3: (a) Percentage reduction of the H_2 norm of the displacement of the primary structure at variable μ for different values of ζ_1 and (b) displacement time history with $\zeta_1 = 0.05$.

4. HYBRID TMD

4.1 Statement of feedback control

From the results presented in the previous section, the passive Tuned Mass Damper appears not to be effective in reducing significantly the peak displacement, which occurs during the early dynamic response for the case of impulsive loading, probably due to its own inertia. It is therefore meaningful to attempt upgrading such a control device from the previously analysed merely passive version to a hybrid form, by the introduction of a feedback controller, apt to supply an external relative control force $f_c(t)$. The control force is taken as a linear combination of terms of the dynamic response, where the constant coefficient values of the combination are called gains. Such a structural system, subjected to a generic base displacement, is represented in Fig. 4 and its dynamic behaviour is described by the following equations of motion, in terms of the absolute degrees of freedom $x_1(t)$ and $x_2(t)$:

$$\begin{cases} m_1\ddot{x}_1(t) + (c_1 + c_2)\dot{x}_1(t) - c_2\dot{x}_2(t) + (k_1 + k_2)x_1(t) - k_2 x_2(t) + f_C(t) = c_1\dot{x}_g(t) + k_1 x_g(t) \\ m_2\ddot{x}_2(t) - c_2\dot{x}_1(t) + c_2\dot{x}_2(t) - k_2 x_1(t) + k_2 x_2(t) - f_c(t) = 0 \end{cases} \tag{12}$$

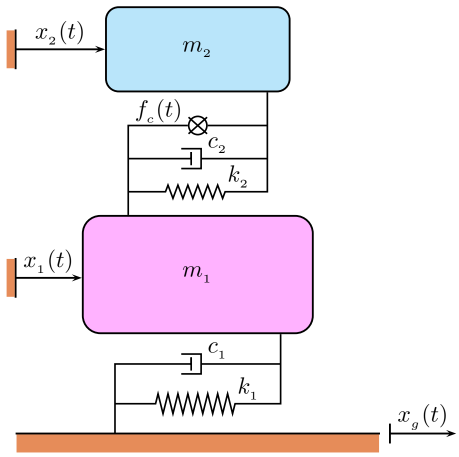


Figure 4: Structural parameters and absolute dynamic degrees of freedom of a 2DOF mechanical system composed of a SDOF primary structure (1), equipped with an added hybrid TMD (2), subjected to generic base displacement.

The feedback strategy assumed here is based on a function for the active relative control force exploiting the acceleration of the primary structure and the relative velocity between primary structure and TMD, by means of acceleration and velocity gains g_a and g_v, respectively:

$$f_c(t) = g_a\ddot{x}_1(t) + g_v(\dot{x}_1(t) - \dot{x}_2(t)) \tag{13}$$

By substituting Eq. (13) into Eq. (12), in the case of versed-sine pulse base displacement, Eqs. (3)-(5), one obtains:

$$\begin{cases} (m_1 + g_a)\ddot{x}_1(t) + (c_1 + c_2 + g_v)\dot{x}_1(t) - (c_2 + g_v)\dot{x}_2(t) + (k_1 + k_2)x_1(t) - k_2 x_2(t) = \\ \qquad = c_1 \dfrac{\omega_p^2}{2\pi}\sin(\omega_p\, t) + k_1 \dfrac{\omega_p}{2\pi}[1 - \cos(\omega_p\, t)] \\ -g_a\ddot{x}_1(t) + m_2\ddot{x}_2(t) - (c_2 + g_v)\dot{x}_1(t) + (c_2 + g_v)\dot{x}_2(t) - k_2 x_1(t) + k_2 x_2(t) = 0 \end{cases} \tag{14}$$

The dynamic response has been obtained again through the Laplace transform of Eq. (14), and relevant expressions are reported in Appendix A.

4.2 Stability analysis

Before proceeding to the optimisation of the feedback controller parameters, a preliminary stability analysis has been developed, so as to establish the bounds on the values that gains g_a, g_v may assume (which will be further considered in the optimisation process), in order to ensure *a priori* a limited magnitude of the dynamic response in time for a given bounded input signal (BIBO, Bounded-Input-Bounded-Output stability), such as the versed-sine pulse considered in this study.

In this sense, the necessary and sufficient condition required for the stability of the system is negative real parts of the closed-loop poles [30], which for the considered structural system and feedback strategy are the roots of the following characteristic equation:

$$\begin{aligned} D_{CL}(s) = & s^4(m_1 m_2 + g_a m_2) + s^3(c_1 m_2 + c_2 m_1 + c_2 m_2 + g_v(m_1 + m_2)) + \\ & + s^2(c_1 c_2 + k_2 m_1 + k_1 m_2 + k_2 m_2 + g_v c_1) + s(c_1 k_2 + c_2 k_1 + g_v k_1) + k_1 k_2 = 0 \end{aligned} \tag{15}$$

In particular, for each pair of given values of the gains, g_a, g_v, the sign of the less negative (or more positive) real part of the closed-loop poles of the system has been investigated, in order to establish a sort of instability threshold, by adopting the structural parameters of the system described in Section 3.1. In this sense, further analyses with different pairs of values of (μ, ζ_1) have been carried out, by showing however negligible changes with respect to the trends presented here, which therefore shall be considered as a suitable reference.

The results of this analysis are represented in Fig. 5, where the stability region within a range of the acceleration gain g_a, for given values of the velocity gain g_v, is displayed.

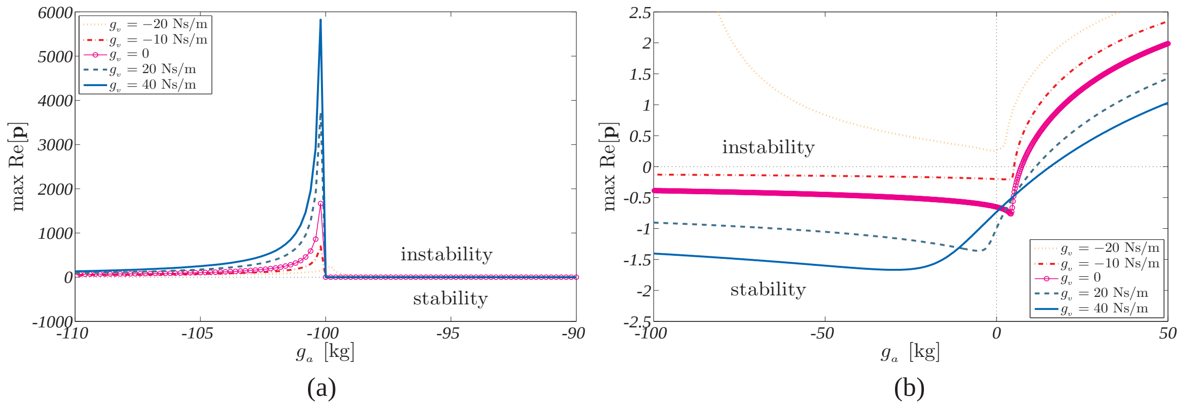


Figure 5: Maximum value of the real part of the system poles **p** as a function of the gains g_a, g_v for primary structure damping ratio $\zeta_1 = 0.05$ and mass ratio $\mu = 0.05$, with focus (a) on values of g_a around $g_a = -m_1$ and (b) on values of g_a around 0.

As can be inspected from the plots, the stability of the system may be discussed from the point of view of both gains. Considering firstly the velocity gain g_v, it essentially ensures a dynamically stable system mostly for positive values, while even for slightly negative values it may lead to system instability. On the other hand, the suitable range for the acceleration gain g_a is characterised by a lower and an upper bound. The former is reached for the value $g_a = -m_1$, beyond which the system becomes unstable, whilst the latter takes place for slightly positive values of g_a, which increase a small amount for increasing values of g_v.

Hence, as a general indication, it can be pointed out that the stability of the system is assured mostly for positive values of the velocity gain g_v and for negative values of the acceleration gain g_a larger than $-m_1$.

4.3 Optimal gains

Once the stability range for the controller gains has been defined, such gains have been optimised, starting from the structural system composed of a primary structure and an optimised passive Tuned Mass Damper, as defined in Section 3.1. This is achieved by means of the same numerical algorithm for nonlinear constrained optimisation, and by assuming as new objective function in this phase the following combination of the displacement of the primary structure $x_1(t)$ and the supplied active control force $f_c(t)$:

$$\mathrm{F}(\mathbf{v}) = ||x_1(t)||_\infty + \alpha||f_c(t)||_\infty \tag{16}$$

where $\mathbf{v} = [g_a; g_v]$ is now the vector of the gains, which play the role of optimisation variables, and α is a weight factor, which allows for balancing the optimisation, in order to reduce the effect of either the structural response or the supplied controller force and to even out the magnitude and the measure units of the two components. The parameters and bounds adopted within the optimisation process have been shown in Table 2. It can be noted from Eq. (16) that the objective function considers the H_∞ norm of the displacement of the primary structure, instead of the H_2 norm previously considered for the tuning of the passive TMD. This choice is due to the main task behind the introduction of the feedback controller, that is, the reduction of the peak response of the primary structure.

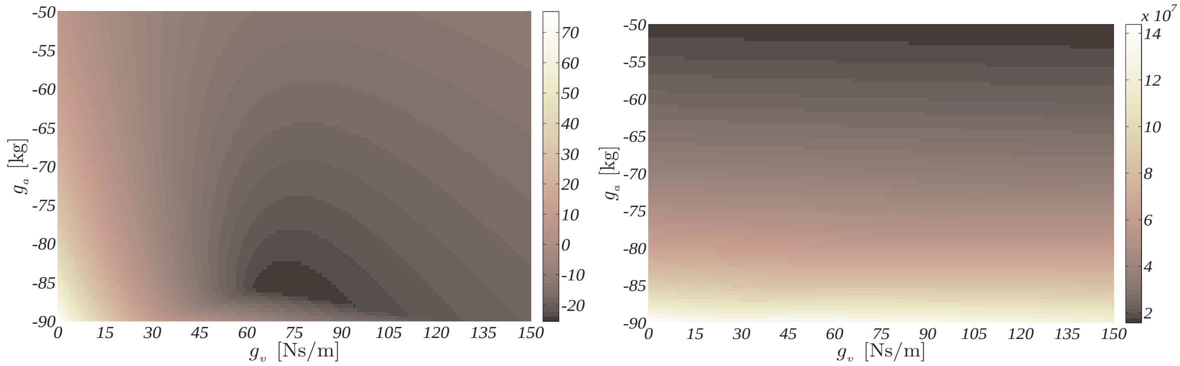
Tuning variables	$\mathbf{v} = [g_a;\ g_v]$
Lower bounds	$\mathbf{l_b} = [-90; 0]$
Upper bounds	$\mathbf{u_b} = [0; 150]$
Tolerance on variable parameter	10^{-10}
Tolerance on constraint violation	10^{-10}
Tolerance on objective function	10^{-10}
Max. number of iterations	1000
Max. number of function evaluation	1000

Table 2: Chosen parameter values for the controller gains optimisation process.

A study based on a wide range of values of α has been carried out, which pointed out that there is a sort of threshold value of α which strongly separates the case of an optimisation devoted to the structural response, obtained for values lower than the threshold one, with respect to the case dedicated to the supplied controller force, which takes place for higher values. Further analyses on this framework revealed that this sharp change of the optimisation task is probably due to the inherent sudden nature of the impulsive excitation. Indeed, it could be possible to demonstrate that, for less sudden excitations, the switch between the two types of optimisation should be more gradual, allowing for an intermediate range of values of α which lead to partial optimisation, where both dynamic response and supplied force are minimised. A further interesting outcome of this study is the high sensitivity of the obtained optimal gains with respect to the variation of the given bounds and optimisation parameters.

The trends of the two contributions in the objective function, as a function of the gains g_a, g_v, are represented in Fig. 6. The minimum values of the peak displacement of the primary structure (Fig. 6a) are obtained correspondingly to the threshold value $g_a = -m_1$. However, this region is close to that of the maximum values, which occur for values of g_a just smaller than $-m_1$. The proximity of the region of maxima to the region of minima may easily cause some problems in the optimisation process, since the latter may not be found by the algorithm. It is interesting to note that all the considerations above are in fact a function of

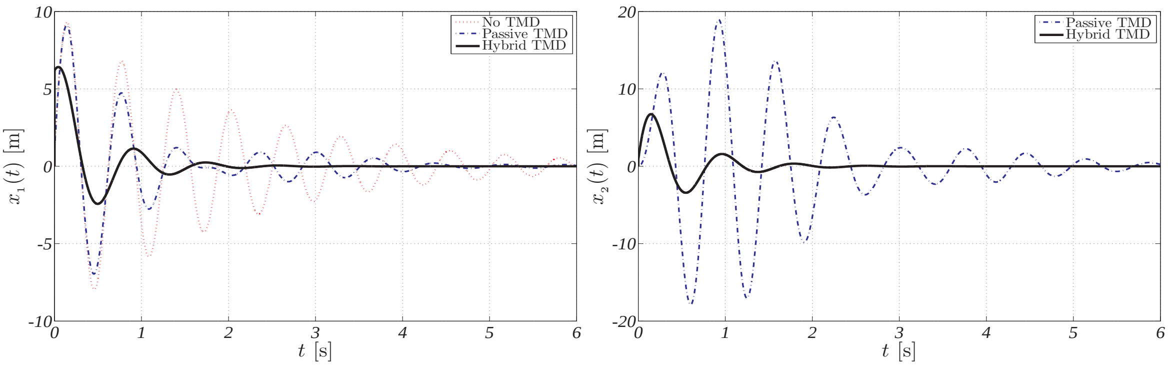
g_a only, since the behaviour with respect to g_v is almost unaffected. Another region where the two components exhibit very low values, close to the minimum ones, is found for values of $g_a > -m_1$ and for a small range of values of g_v. Such areas doubtless represent a suitable alternative to the actual optimal region, since they allow for a significant reduction of the peak response, together with the introduction of a smaller controller force. On the other hand, the peak of the supplied controller force (Fig. 6b) takes values almost constant within the overall considered range of values of g_v, while the magnitude of such peaks increases for increasing g_a.



(a) (b)

Figure 6: (a) Percent reduction of peak displacement of the primary structure [%] and (b) supplied controller force [N] as a function of the feedback gains g_a, g_v.

Several trials have been developed, in order to fully explore the efficiency of the hybrid Tuned Mass Damper, in terms of response reduction (for $\alpha = 0$). A significant sample of such studies is represented by the time history of the displacement of both the primary structure and the Tuned Mass Damper, reported in Fig. 7. For this case, the obtained optimal values for the feedback gains are $g_a = -86.1549$ kg and $g_v = 72.4089$ Ns/m. This physically means, respectively, that the controller attempts to counteract as much as possible the inertia force due to the primary structure and, at the same time, to amplify the relative damping force, so that to reduce the effect of the TMD movement, which in particular conditions may lead to an increase in the displacement of the primary structure instead of reducing it.



(a) (b)

Figure 7: (a) Displacement of the primary structure and (b) of the Tuned Mass Damper, for primary structure damping ratio $\zeta_1 = 0.05$ and mass ratio $\mu = 0.05$.

The plot indicates that, despite that the aim of the optimisation process was that of reducing the peak response, the overall displacement of the primary structure has also been remarkably decreased. Indeed, the peak response has been reduced by about 25%, while the overall response has been decreased by about 54%. The presence of a supplied force implies the initial introduction of a peak displacement, which occurs before the peak response in the case of passive system. A further benefit due to the action of the feedback controller is obtained for the TMD displacement. Indeed, the peak response and the total stroke of the passive device have been reduced of about 62% and 77% respectively, when the supplied control force is introduced. This latter feature should be of great interest in view of practical applications, where the allowable movement of the control device may be quite limited. It is worth noting that the results presented above have been obtained by neglecting possible design bounds, which should likely limit the allowable amount of supplied controller force. However, even a low contribution of the feedback controller should imply a remarkable reduction in the dynamic response.

5. CONCLUSIONS

In the present work, the optimal tuning of the free parameters of Tuned Mass Dampers has been analysed for the case of impulsive base displacement. The analytical expression of the dynamic response in terms of displacement of a structural system composed by a SDOF primary structure and an added TMD has been obtained first, by means of a Laplace transform of the equations of motion. The tuning process, developed within a numerical optimisation method, has been firstly carried out by assuming a passive Tuned Mass Damper and by considering norms of the displacement of the primary structure as the objective function.

The results achieved exhibit trends quite similar to those obtained in previously reported literature for the case of persistent input such as harmonic or white noise excitations. In terms of optimal TMD parameters, at increasing mass ratio μ the frequency ratio f decreases while the TMD damping ratio ζ_2 increases, with characteristic trends. Moreover, both tuned parameters seem to be quite insensitive to variations of the primary structure damping ratio ζ_1. On the other hand, the level of effectiveness of the Tuned Mass Damper is fairly dependent on ζ_1, since for lower values of this parameter the level of the reduction in the response is significantly higher, and, as expected, it increases also for increasing mass ratio μ.

In general, the introduction of a passive Tuned Mass Damper allows for significant decrease in the overall dynamic response, therefore this may support the potential presence of such a device in structural systems. In the case of pulse loading and a system initially at rest, a peak occurs at the very beginning of the dynamic response and does not appear to be substantially reduced, probably due to the intrinsic inertia of the passive system, which needs some time before becoming fully operative.

It is mainly for this reason that the upgrading of the TMD from passive to hybrid may become useful, by addition of a feedback controller, which supplies a force based on a selected control strategy. The strategy assumed in this study has been taken as a linear combination of the acceleration of the primary structure and of the relative velocity between the primary structure and secondary mass. A preliminary stability analysis has been performed, so as to define the range of possible values for the gain coefficients involved in the control strategy. The results obtained have clearly shown that the system turns out more stable as the velocity gain g_v increases. On the other hand, even small positive values of the acceleration gain g_a lead to an unstable system, whilst negative values imply a stable system, until a limit value, which is found to be equal to $-m_1$; beyond this bound, the system becomes unstable again. The optimisation of the controller parameters has been developed so as to take into account the minimisation of both the structural response and supplied controller force, through a weight factor α.

When only the structural response is minimised within the optimisation process, the feedback controller has been proved to be quite efficient in improving the performance of the passive Tuned Mass Damper, with a remarkable reduction of both peak and overall responses. The obtained results indicate that the optimal parameters for the gains might lead to a theoretical amount of supplied force which could be excessive for practical engineering applications. However, even a smaller magnitude of control force should provide a significant response reduction. The introduction of the controller force also allows for significant reduction in the TMD displacement and stroke, which in the case of the passive device may result quite large. Such a fact might have important consequences in practical applications, which usually place strict limits on the displacement of the control devices.

In conclusion, the results obtained in this study indicate that remarkable benefits come from the insertion of a Tuned Mass Damper in a structural system. The performance of the device allows for a substantial reduction of the dynamic response also under pulse excitation. Moreover, being the optimal TMD parameters similar to those evaluated for persistent excitations, a general and comprehensive tuning of TMDs is possible, with potentially relevant consequences towards engineering applications. Further improvement of such a device, where necessary or required by applications, may be obtained by the addition of a feedback controller, i.e. by switching from a passive TMD to a hybrid TMD.

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APPENDIX A. RESPONSE FUNCTIONS

Appendix A.1 SDOF primary structure with passive TMD

In this section, the procedure for obtaining the dynamic response of the structural system in time domain, through a pair of Laplace transforms [28, 29], is explained and reported in detail. Firstly, the Laplace transform, in terms of the complex variable s, of the versed-sine pulse expressed by Eq. (3) takes the form:

$$X_g(s) = \mathcal{L}[x_g(t)] = A_p \frac{\omega_p^2}{s(s^2+\omega_p^2)}(1 - \mathrm{e}^{-T_p\, s}) \tag{A.1}$$

Then, Eq. (6) are transformed as well as follows, by substituting Eq. (A.1) and considering the amplitude A_p in Eq. (5), with zero initial conditions:

$$\begin{cases} [s^2 m_1 + s(c_1+c_2) + (k_1+k_2)]X_1(s) + [-s\,c_2 - k_2]X_2(s) = [s\,c_1 + k_1]\dfrac{\omega_p^3}{2\pi\, s(s^2+\omega_p^2)}(1-\mathrm{e}^{-T_p\, s}) \\ [-s\,c_2 - k_2]X_1(s) + [s^2 m_2 + s\,c_2 + k_2]X_2(s) = 0 \end{cases} \tag{A.2}$$

Such a system of equations can be represented in the following matrix form:

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{A.3}$$

Then, the transfer functions may be easily worked out through Cramer's rule:

$$X_1(s) = \frac{\begin{vmatrix} f_1 & z_{12} \\ f_2 & z_{22} \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{N_1(s)}{D(s)}, \qquad X_2(s) = \frac{\begin{vmatrix} z_{11} & f_1 \\ z_{21} & f_2 \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{N_2(s)}{D(s)} \tag{A.4}$$

where:

$$\begin{aligned} N_1(s) &= \omega_p^3(1 - \mathrm{e}^{-T_p s})[s^3(c_1 m_2) + s^2(k_1 m_2 + c_1 c_2) + s(k_1 c_2 + c_1 k_2) + k_1 k_2] \\ N_2(s) &= \omega_p^3(1 - \mathrm{e}^{-T_p s})[s^2(c_1 c_2) + s(k_1 c_2 + c_1 k_2) + k_1 k_2] \\ D(s) &= 2\pi\, s(s^2 + \omega_p^2)[s^4(m_1 m_2) + s^3(c_1 m_2 + c_2 m_1 + c_2 m_2) + \\ &\quad + s^2(c_1 c_2 + k_2 m_1 + k_1 m_2 + k_2 m_2) + s(c_1 k_2 + c_2 k_1) + k_1 k_2] \end{aligned} \tag{A.5}$$

The transfer functions may be rewritten in factorised form [29]:

$$X_n(s) = \frac{\omega_p}{2\pi}\left[\frac{K_{n,1}}{s} + \frac{K_{n,2}}{s - \mathrm{i}\,\omega_p} + \frac{K_{n,3}}{s + \mathrm{i}\,\omega_p} + \frac{K_{n,4}}{s - a_1 - \mathrm{i}\,b_1} + \frac{K_{n,5}}{s - a_1 + \mathrm{i}\,b_1} + \frac{K_{n,6}}{s - a_2 - \mathrm{i}\,b_2} + \frac{K_{n,7}}{s - a_2 + \mathrm{i}\,b_2}\right] \tag{A.6}$$

where the denominator of each partial fraction is in the form $s - p_j$, p_j being the j-th system's pole, and the constants $K_{n,j}$ are the system's residues, where the indices $n = 1, 2$ and $j = 1, ..., 7$ mark the degrees of freedom of the structural system and the roots of $D(s)$ respectively. It is worthy to note that, despite that it being possible to determine analytical expressions for these constants, their complexity, even in the case of a relatively simple mechanical system as that considered here, is such that the recourse to numerical methods becomes suitable.

Once the residues have been evaluated, the inverse Laplace transform of Eq. (A.6) gives the expression of the response in the time domain:

$$x_n(t) = \frac{\omega_p}{2\pi}\left[K_{n,1} + K_{n,2}\mathrm{e}^{\mathrm{i}\,\omega_p t} + K_{n,3}\mathrm{e}^{-\mathrm{i}\,\omega_p t} + K_{n,4}\mathrm{e}^{(a_1 + \mathrm{i}\,b_1)t} + K_{n,5}\mathrm{e}^{(a_1 - \mathrm{i}\,b_1)t} + K_{n,6}\mathrm{e}^{(a_2 + \mathrm{i}\,b_2)t} + K_{n,7}\mathrm{e}^{(a_2 - \mathrm{i}\,b_2)t}\right] \tag{A.7}$$

Appendix A.2 SDOF primary structure with hybrid TMD

The Laplace transform of Eq. (14), for zero initial conditions, takes the form:

$$\begin{cases} [s^2(m_1 + g_a) + s(c_1 + c_2 + g_v) + (k_1 + k_2)]X_1(s) + [-s(c_2 + g_v) - k_2]X_2(s) = \\ \qquad = [s\,c_1 + k_1]\dfrac{\omega_p^3}{2\pi\, s(s^2 + \omega_p^2)}(1 - \mathrm{e}^{-T_p s}) \\ [-s^2 g_a - s(c_2 + g_v) - k_2]X_1(s) + [s^2 m_2 + s(c_2 + g_v) + k_2]X_2(s) = 0 \end{cases} \tag{A.8}$$

Following the procedure explained in the previous section, the transfer functions relevant to the displacement of the primary structure and the Tuned Mass Damper take in this case the form:

$$\begin{aligned} N_1(s) &= \omega_p^3(1 - \mathrm{e}^{-T_p s})[s^3(c_1 m_2) + s^2(m_2 k_1 + c_1 c_2 + c_1 g_v) + s(c_1 k_2 + c_2 k_1 + g_v k_1) + k_1 k_2] \\ N_2(s) &= \omega_p^3(1 - \mathrm{e}^{-T_p s})[s^3(g_a c_1) + s^2(c_1 c_2 + c_1 g_v + g_a k_1) + s(c_1 k_2 + c_2 k_1 + g_v k_1) + k_1 k_2] \\ D(s) &= 2\pi\, s(s^2 + \omega_p^2)[s^4(m_1 m_2 + g_a m_2) + s^3(c_1 m_2 + c_2 m_1 + c_2 m_2 + g_v(m_1 + m_2)) + \\ &\quad + s^2(c_1 c_2 + k_2 m_1 + k_1 m_2 + k_2 m_2 + g_v c_1) + s(c_1 k_2 + c_2 k_1 + g_v k_1) + k_1 k_2] \end{aligned} \tag{A.9}$$

The response in time domain may be then obtained likewise in Eqs. (A.6)–(A.7).