30 May - 1 June 2006 Tampere, Finland

STOCHASTIC ANALYSIS OF THE NON-LINEAR ROAD/TYRE EXCITATION

E Rustighi[1], M A Sek[2], and S J Elliott[1]

[1]Institute of Sound and Vibration Research, University of Southampton
Highfield, Southampton, SO17 1BJ, UK
er@isvr.soton.ac.uk

[2]Victoria University, Melbourne Victoria, Australia

ABSTRACT

The correlation between the road profile and the waveform of interior and exterior noise in a car is generally poor. The non-linearity due to the contact between the asphalt and the tyre has been suggested as the main reason, as demonstrated by some hybrid road noise analyses. This paper focuses on the analysis of the non-linear contact in order to investigate the relationship between the spectrum of the road profile and that of the tyre excitation. A Winkler model has been used to describe the contact of a two-dimensional idealised tyre with the ground, using a deterministic non-linear analysis in the time domain. The power spectral density of the displacements on the tyre has then been compared with that of the road profile. Moreover, the influence of the contact stiffness on the Winkler model approximation has been investigated.

1 INTRODUCTION

Sandberg and Descornet [1] provided strong argument that the low-frequency car exterior noise, below 800 Hz, is related to the tyre vibrations. However, the authors could not find a linear relationship including the macrotexture to predict the pavement noisiness because of the non-linearity between sound and profile amplitudes. Anfosso-Lédée and Do [2] tried to use more advanced geometric road descriptors, but still found poor correlation between geometric parameters and noise levels. Regarding interior noise a study by Jha [3] showed that, for the tested vehicle, about 36% of the interior noise could be accounted for considering interior mechanical paths only.

Fong [4] showed that the transfer function between contact pressure, computed by a half-space approximation, and the near-field tyre noise are independent of the road texture. This suggests the non-linearity due to the contact between the asphalt and the tyre as the main reason for the poor correlation between the road profile and the interior or exterior car noise.

This paper focuses on the analysis of the non-linear contact description in order to find a spectral relationship between the road profile and the tyre excitation. After the introduction, the second section introduces a stochastic description of the road profile. In the third section a Winkler model has been considered in order to describe the non-linear contact of a two-dimensional tyre with the ground, using a deterministic non-linear analysis. The next section shows the results of the stochastic analysis carried out on the time domain where two different contact stiffnesses have been considered.

2 ROAD PROFILE

The road profile has been assumed to be a random stationary process, hence it can be described in the wavelength domain by its power spectral distribution (PSD) $G(n)$. In particular the following description, suggested by Robson [5], has been adopted:

$$G(n)=\begin{cases} cn_a^{-2.5} & \text{for} \quad 0<|n|\le n_a \\ cn^{-2.5} & \text{for} \quad n_a<|n|\le n_b \\ 0 & \text{for} \quad |n|>n_b \end{cases} \tag{1}$$

where n is the wavenumber expressed in cycles/m, n_a and n_b are constants and c has the value suggested in [5]. In particular, the parameters have been set as n_a = 40 cycles/m, n_b = 600 cycles/m and $c = 50\times10^{-6}$. Applying the inverse Fourier transform and assuming a random phase to the magnitude of the PSD, the road profile in the spatial domain has then been obtained. The road has been sampled with a sampling displacement of 0.625 mm.

3 NON-LINEAR CONTACT

The contact is usually non-linear since not all the tyre contact surface is in contact with the road at all times. A Winkler model [6] has been used to represents the non-linear contact problem. Such contact model is a discrete bedding of springs lying on a rigid round body that represents the tread surface of the tyre. The springs in the bedding have been distributed with the same interval which the road has been sampled at. The stiffness of each spring is the same, then the non-linearity in the contact is due to the fact that not all the springs are in contact at all times. When the tyre is pressed against the bedding the equilibrium point is searched by an algorithm that find out how many springs are in contact and if their compression is enough to equilibrate the force applied to the tyre, that is the average weight of a quarter of a passenger car. Therefore, the analysis carried out is quasi static and at this point no dynamics has been taken into account. As discussed in details later, two bedding stiffnesses have been considered, k_{soft} = 1600 kN/m and k_{hard} = 16000 kN/m.

Fig. 1 shows both the spring bedding at the bottom of a rigid tyre and the road profile underneath. The road is moved under the tyre with steps equal to the road sampling and for each point the displacement of each spring has been recorded. At the end of the process a vector of displacements x_i is obtained for each spring i in the contact patch. The displacement of the spring in the middle of the contact patch would be the most similar to the road profile since such a spring is the most likely to always be in contact. On the contrary, moving away from the centre of the contact patch the springs in the bedding are less likely to stay in contact with the road.

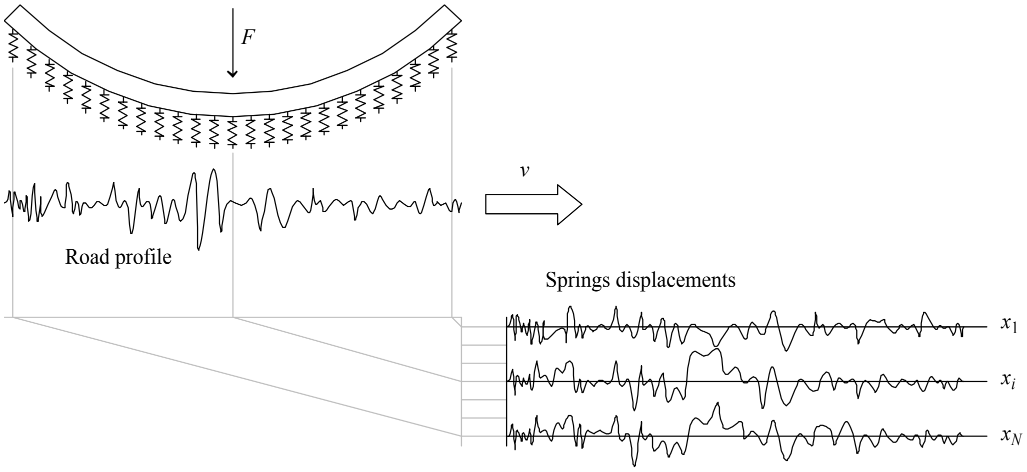

Fig. 1. The Winkler contact model.

Fig. 2 shows the profile of 1 m of road, the spring displacement at the middle point in the contact patch when the contact stiffness is 1600 N/m and the global displacement of the tyre in order to reach the equilibrium position. On the right of each graph the probability density (pdf) of these quantities is shown, evaluated over a length of 10 m. The road profile has been assumed to be a normal distributed stochastic variable with an rms value of 0.56 mm. In the case of linear process the spring displacements should be stochastic variables with a normal distribution too. As shown in the figure, the spring displacements have an almost Gaussian distribution so that the non-linearity seems to be negligible for this contact stiffness. The global tyre displacement, that seems to be obtained by filtering the tyre profile, appears to have a normal distribution as well, but the coherence between the two signals is very poor, proving the relationship between the two to be nonlinear.

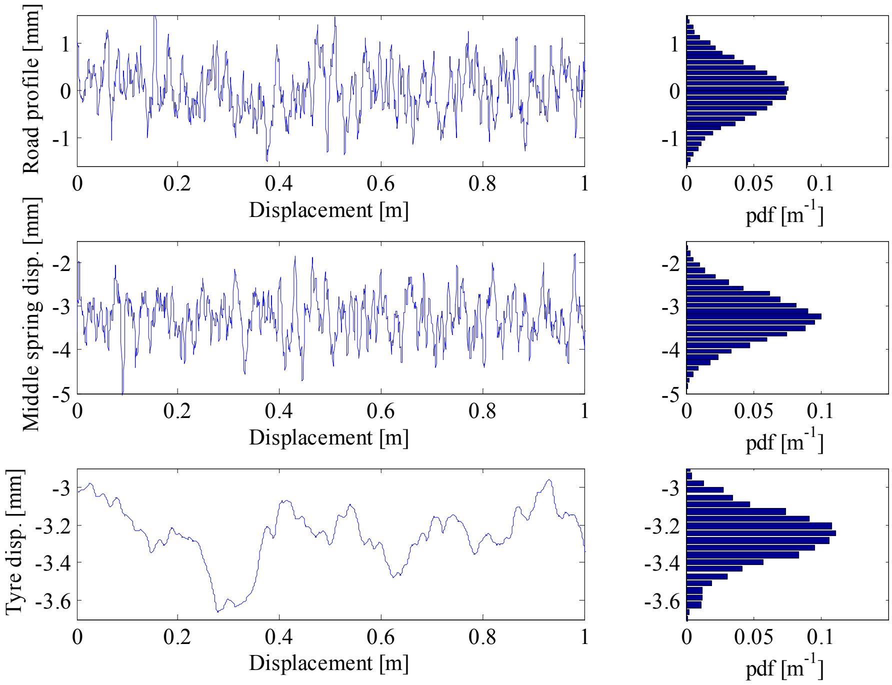

Fig. 2. Time domain displacements of the road profile, the spring in the middle of the contact patch and the rigid tyre hub. On the left side of the picture the probability density function (pdf) of the corresponding quantity is shown.

4 SPECTRAL ANALYSIS

A profile of 10 m of road surface has been generated and used to find the displacements of the springs in the Winkler bedding for two different contact stiffnesses, that is k_{soft} = 1600 kN/m and k_{hard} = 16000 kN/m. With k_{soft} the tyre is nearly always in contact with the road. On the contrary, with k_{hard} a lot of air voids are visible in the contact patch.

Firstly, a check of the spectral characteristic of the generated road profile can be carried out, as shown in the black line of Fig. 3. The figure shows that the PSD of the generated road approximates quite well the desired PSD described by Eq. 1.

Next the analysis of the spring displacements can be carried out. In order to illustrate such analysis the displacement of the spring in the middle of the contact patch and that of a spring out of the centre have been calculated. Fig. 3 shows the PSD of the displacements of these two springs in the case of soft and hard contact stiffness. When soft tyres tread are considered the tread surface is nearly always in contact with the road and the tyre PSD displacement is similar to that of the road profile, at least above 40 cycles/m. When a harder tyre tread is considered, the PSD of the tyre displacement has a lower magnitude over all the frequency range since the springs are sometimes detached from the road profile and their displacements are reduced with respect to the road profile. For harder contact stiffness, the PSD change from the centre to the ends of the contact patch is more evident. In fact the probability to get in touch reduces for the springs at the ends of the contact patch.

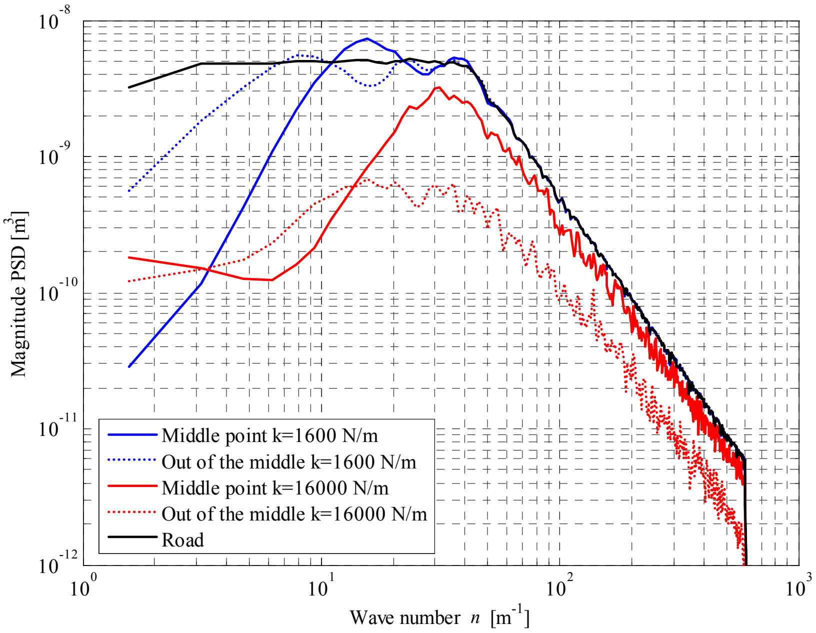

Fig. 3. Comparison between spectra for simulations of soft and hard Winkler beddings: k_{soft} = 1600 kN/m (blue lines) and k_{hard} = 16000 kN/m (red lines). The continuous lines represent the auto-spectral densities evaluated for the spring in the middle of the contact patch whereas the dotted line represent the auto-spectral density for a spring out of the middle point. The continuous black line shows the PSD of the road.

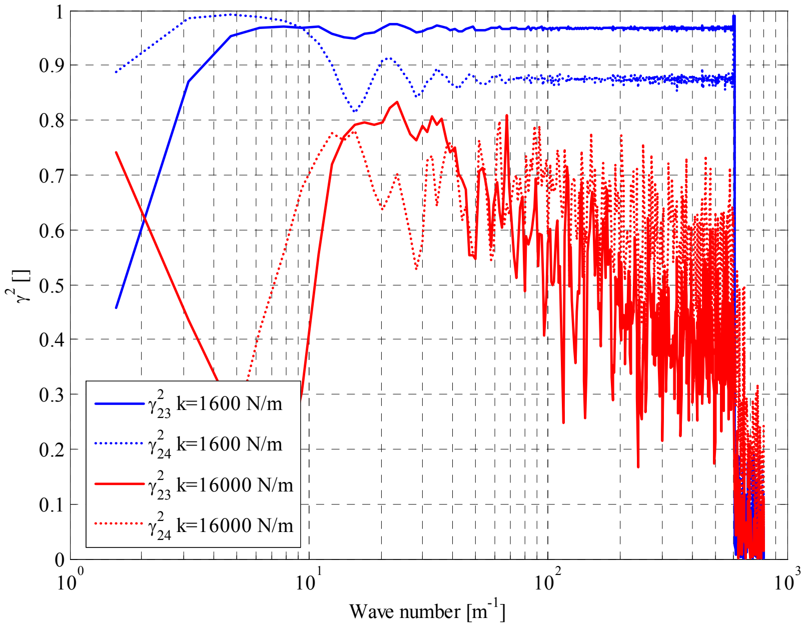

Fig. 4. Coherence γ^2 between the displacement x_2 (on the right of the middle point), x_3 (middle point of the contact patch) and x_4 (on the left of the middle point). Blue lines refer to soft Winkler bedding (k_{soft} = 1600 kN/m) and red lines refer to the hard one (k_{hard} = 16000 kN/m). γ^2_{23} are shown in continuous line while γ^2_{24} are represented by dotted lines.

Each spring experiences the same road profile but just delayed in space. In the case of full contact between tyre and road the cross-spectrum between two points in the contact patch is given by the auto-spectrum delayed according to the distance between the two springs. In order to quantify the linearity relationship between displacements of different springs the coherence between them have been then taken into account. Fig. 4 shows the coherence $\gamma^2{}_{23}$ between the displacement x_2 (on the right of the middle point) and x_3 (middle point of the contact patch) and the coherence $\gamma^2{}_{24}$ between the point x_2 and x_4 (on the left of the middle point), in order to consider a delay that is the double of the previous one. Such coherences have been computed in the case of soft and hard road. From the figure, it's clear that a reduction in the contact stiffness results in higher coherence values while an increase in the distance of the points considered brings to a reduction in the coherence.

In conclusion to this preliminary study, a linear approximation results to be reasonable instead of a nonlinear Winkler model when the contact patch stiffness is soft enough. More advanced studies are needed to quantify this threshold. The nonlinearities introduced by the Winkler model could perhaps be taken into account in a linear model where the road profile PSD has been suitably modified, but a more extensive investigation is needed to validate this.

5 CONCLUSIONS

The displacement of the tyre tread when in contact with the road has been found by applying non-linear contact algorithm to a Winkler bedding. The spectral density distribution of the road profile and that of the tyre tread displacement have been compared.

It has been shown that, when the contact stiffness is sufficiently low, the non-linearity of the contact does not need to be accounted for at the points in the middle of the contact patch. When the contact stiffness is increased the PSD of the tread differs from that of the road with a degree that will be evaluated in further work. Eventually, this will permit to use a fully linear stochastic model to predict interior and exterior car noise.

REFERENCES

[1] U. Sandberg and G. Descornet, "Road surface influence on tyre/road noise". Inter-noise, 259–272, Miami, Florida, USA, 1980.

[2] F. Anfosso-Lédée and M.-T. Do, "Geometric descriptor of road surface texture in relation to tire-road noise". *Transport Research Record* 1806, 160-167, 2002

[3] S. Jha, "Identification of road/tyre induced noise transmission paths in a vehicle", *Int. J. of Vehicle Design* 5(1/2), 143–158, 1984.

[4] S. Fong, "Tyre induced predictions from computed road surface texture induced contact pressure". *Inter-noise*, Christchurch, New Zealand, 1998.

[5] J. Robson, "Road surface description and vehicle response", *Int. J. of Vehicle Design* 1(1), 25–35, 1979.

[6] K. L. Johnson, *Contact mechanics*. Cambridge University Press, 1985.