Investigation into Noise Emitted by Bluff Bodies with Large Roughness

by

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A set of wind tunnel experiments were performed to study the effect of large surface roughness on circular cylinder noise, with the goal of improving landing gear noise predictions. Roughness increases vortex shedding noise levels, and shifts the peak to a lower Strouhal number. The noise levels in the fall-off range also increase, but no significant change in the fall-off rate is observed. The decrease of the vortex shedding peak frequency has been associated with early detachment caused by the effect of roughness on the TBLs, which is in agreement with previous experimental studies with smaller roughness. The high frequency range of the spectrum revealed a broadband, Strouhal-based peak, which is caused by roughness noise generated on the upstream face of the cylinder. The peak Strouhal number is well predicted by Howe’s model using the maximum outer velocity around the cylinder. Cylindrical roughness presents a weaker roughness noise peak, but higher noise levels for higher frequencies, and is thought to be caused by sharp edge separation. A bluff body roughness noise model has been developed based on the model of Howe and a Green’s function tailored to the bluff body geometry, calculated using the Boundary Element Method. The application to rough circular cylinders using a flat wall (ZPG) TBL model shows good agreement with experiments for downstream observers, but the model overpredicts the levels in over-head observers. The disagreement is thought to be due to inaccuracy of the flat wall TBL model. The transition from smooth regime to rough regime was studied experimentally by partially covering the cylinder with distributed roughness in spanwise uniform configurations. Transition regarding vortex shedding happens mainly when roughness is added or removed around the separation region. The results agree with the fact that roughness changes the separation location by perturbing the TBL close to separation. Sparse and dense two-dimensional roughness on a circular cylinder, studied using CFD, have similar effects than distributed roughness regarding the vortex shedding peak level and frequency.
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Declaration of Authorship

I, Antoni Alomar

declare that the thesis entitled

Investigation into Noise Emitted by Bluff Bodies with Large Roughness

and the work presented in the thesis are both my own, and have been generated by me
as the result of my own original research. I confirm that:

• this work was done wholly or mainly while in candidature for a research degree at
  this University;

• where any part of this thesis has previously been submitted for a degree or any
  other qualification at this University or any other institution, this has been clearly
  stated;

• where I have consulted the published work of others, this is always clearly at-
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• where I have quoted from the work of others, the source is always given. With the
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• I have acknowledged all main sources of help;

• where the thesis is based on work done by myself jointly with others, I have made
  clear exactly what was done by others and what I have contributed myself;

• parts of this work have been published as: a conference paper [44]

Signed:

Date:
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<td>roughness element diameter</td>
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Symbols

\( R_{e_x} \) Reynolds number based on streamwise length, \( U_ox_1/\nu \) [-]

\( S \) surface area \( m^2 \)

\( St \) Strouhal number, \( fD/U_o \) [-]

\( St_b \) bar fetch Strouhal number, \( fD_b/U_o \) [-]

\( St_r \) roughness element Strouhal number, \( fh_r/U_r \) [-]

\( t \) observer time \( s \)

\( t_{convection} \) convection time, \( D/U_o \) \( s \)

\( t_r \) retarded time, \( t - |x-y|/c_o \) \( s \)

\( T_{ij} \) Lighthill stress tensor \( N\cdot m^{-2} \)

\( T_o \) free stream temperature \( K \)

\( U_o \) free stream velocity \( m\cdot s^{-1} \)

\( U_c \) eddy convection velocity \( m\cdot s^{-1} \)

\( U_o(\theta') \) TBL outer velocity \( m\cdot s^{-1} \)

\( U_r \) velocity impinging on the roughness elements \( m\cdot s^{-1} \)

\( U_{ref} \) reference velocity \( m\cdot s^{-1} \)

\( u_r \) friction velocity \( m\cdot s^{-1} \)

\( u_1, u_2, u_3 \) flow velocity components \( m\cdot s^{-1} \)

\( V \) volume outside of \( \Sigma \) [-]

\( w(x_2/\delta) \) wake function [-]

\( w_h(h_r) \) PDF of the roughness height [-]

\( w_{hu}(U_r, h_r) \) PDF of the random variables \( U_r \) and \( h_r, w_h(h)w_u(U_r) \) [-]

\( w_u(U_r) \) PDF of the velocity impinging on the roughness elements [-]

\( w_o(\omega) \) PDF of the noise frequency [-]

\( W_R \) acoustic power \( N\cdot m\cdot s^{-1} \)

\( x_1, x_2, x_3 \) observer cartesian coordinates \( m \)

\( x_2^+ \) wall distance in wall units [-]

\( x_{1e} \) distance from plate leading edge to rough surface \( m \)

\( x_{mn} \) coordinates of the \( m \)th roughness element [55, 56] \( m \)

\( x_n \) local coordinate normal to the surface \( m \)

\( x_o \) correction for the rough surface length \( m \)

\( x_\eta \) local coordinate parallel to the surface \( m \)

\( y_1, y_2, y_3 \) source cartesian coordinates \( m \)

\( Y_\eta \) velocity potential of an ideal incompressible flow \( m^2\cdot s^{-1} \)
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<td>$\sigma$</td>
<td>surface density of roughness elements [-]</td>
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<td>$\tau_\omega$</td>
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<td>point pressure frequency spectrum</td>
<td>N²m⁻⁴s</td>
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<td>Ŧ</td>
<td>non-dimensional wavenumber-frequency spectrum</td>
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<tr>
<td>Φₛ</td>
<td>function in Smol’yakov [59] model</td>
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<td>Ω</td>
<td>inner solid angle</td>
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<tr>
<td>ω</td>
<td>angular frequency</td>
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### Acronyms

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<td>Adverse Pressure Gradient</td>
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<td>BEM</td>
<td>Boundary Element Method</td>
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<td>CFD</td>
<td>Computational Fluid Dynamics</td>
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<td>DDES</td>
<td>Delayed Detached Eddy Simulation</td>
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<td>ISVR</td>
<td>Institute of Sound and Vibration Research</td>
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<td>Large Eddy Simulation</td>
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Chapter 1

Introduction

1.1 Landing gear noise

Modern commercial aircraft are important sources of acoustic emission, especially during the flight phases where they operate at low altitudes, near the airports of origin and destination. Noise emitted is due to either the flow through the engines, and the flow around the airframe. Engine noise is dominant when engines are operating at medium to high power, which is the case during take-off. Airframe noise is generated in the regions of high flow instability close to the body, which happens especially around the high-lift devices and landing gears (when they are deployed). During take-off high-lift devices are deployed, as well as the undercarriage (although it is stowed shortly after the aircraft loses contact with the ground), but these two airframe noise sources are much weaker than the engines. During approach to landing, however, the engines operate at low power. In this phase, the high-lift devices and the undercarriage are deployed, and measurements have shown that their acoustic power is of the same order as the engines. For this reason, to reduce the overall noise of modern aircraft during approach to landing both the engine noise and the airframe noise must be reduced [1].

Landing gears of modern aircraft are composed of a large set of bluff-body-type components. i.e. their shape is not streamlined so that the aerodynamic forces that appear (and the radiated noise) are associated with flow structures due to early detachment (such as vortex shedding on cylinders). The flow structures and forces associated with bluff body shedding at high Reynolds numbers are very complex (the Reynolds number
sets the smallest dynamical length scales in the flow. Moreover, the various components have a wide range of characteristic lengths and are arranged in complex geometries that cause complicated flows around them, not only due to the shedding of individual components, but also due to interactions between them. The large elements such as the wheels and the main struts determine the size of the large scales of the flow. In addition, there are also smaller elements which cover the large ones, as seen in figure 1.1. At high Reynolds numbers, the geometrical complexity makes the computational simulations very expensive (even using turbulence models). Many simplified landing gear models have been tested computationally [2, 3], and these studies have shed light on the main characteristics of the flow and noise generation associated with the large and medium components. Experimental tests of simplified and complete landing gears have also been performed in flight [4, 5] and in wind tunnels [6, 7, 8], which have been the main instruments used to develop empirical and semi-empirical noise prediction models.

\textbf{Figure 1.1:} Fully dressed landing gears of commercial aircraft.
They also have shown that the influence of the small scale details on the high-frequency range of the noise spectrum is very important. The fact that this range contributes significantly to the most used noise metrics such as EPNL (the A-frequency weighting peaks within the range 2 - 3 kHz) makes their environmental impact more relevant.

During the last decade, a large amount of devices have been considered to reduce the emitted noise. Some examples are streamlining of the bottom face of the bogey [8], streamlined fairings around the gear boxes and between the wheels, and a splitter plate downstream of the oleo [2], or a redistribution of some of the components to avoid high-speed impingement and flow interactions between them [3]. Also, two other fairing-type mechanisms have been tested which aim to reduce the noise emitted by single bluff bodies [9, 10].

Landing gears are formed of many elements of various dimensions, which implies that many different characteristic lengths of velocity fluctuations around these elements will be present, and thus a wide frequency range of sound waves (associated to those velocity fluctuations) will be generated. This explains the fact that noise emitted by entire landing gears is mainly broadband. Figure 1.2 shows a spectrum of a simplified landing gear. This results correspond to the experimental work by Heller and Dobrzynski [4]. They tested a simplified two-wheel landing gear and four-wheel landing gear in the wind tunnel. The spectrum is normalised with the sixth power of a flow velocity ratio $U_o/U_{ref}$, being $U_{ref}$ = 100 m/s, and the square of a distance ratio $r/D_{ref}$, being $D_{ref}$ the wheel diameter. The frequency has been normalised on a Strouhal number basis $fD_{ref}/U_o$.

The different curves correspond to a) complete configuration, b) main strut, c) drag brace and actuator, d) wheel, and e) door. The spectrum contains a significant hump in the low- and mid-frequency range. This hump is associated to the biggest elements of the landing gear, i.e. the wheels, bogey and main struts. In the low and medium frequency range, noise intensity scales approximately with the sixth power of the velocity, which is related to dipole noise due to unsteady forces acting on compact elements. They suggest that the elements responsible for noise at low and medium frequencies are small compared to the wavelength of the sound they emit and can therefore be considered compact.
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Figure 1.2: Far field sound pressure level spectrum corresponding to a simplified landing gear [4].

Figure 1.3: 1/3-octave noise spectra predicted by Guo’s component-based model [13]. The total noise (solid line) is the sum of low frequency components (dotted line), medium frequency components (dashed line) and high frequency components (dash-dotted line).

1.2 Effect of small components

Current prediction schemes of landing gear noise are not successful in the high frequency range of the spectrum. The first prediction schemes did not consider the small details, and resulted in underprediction of EPNL of up to 8 dB [12]. More recent models include the effects of small details, but introducing huge simplifications, such as the reduction...
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Figure 1.4: Fully dressed and clean configurations [7].

of the small scale geometry to a characteristic length and a ‘complexity factor’ [13], or the simplification that all small elements are of cylindrical shape [14]. Figure 1.3 shows the predicted far field noise spectrum by Guo [13], as the sum of low, medium and high frequency components. It can be appreciated that the high frequency range is dominated by the small components. Other more accurate models require the input of all relevant elements loading and surface pressures [15], which is not possible nowadays due to CFD limitations.

The experimental work by Guo [7] showed the effect that some of the small elements had in the noise emitted by a Boeing 737 nose landing gear. In particular the effect of adding the hydraulic hoses and the brake braces attached behind the main strut and the torque link was analysed (figure 1.4). The far field measurements showed that for frequencies $f > 500$ Hz the fully dressed configuration was significantly noisier than the clean configuration (figure 1.5). For both configurations the far field noise scaled well
with the eighth power of the velocity for frequencies $f > 2$ kHz. The comparison of the near field measurements of both configurations revealed that the effect of the hoses and brake braces was to increase the intensity of the dominant noise sources, located at the back of the main strut.

The small elements of landing gears are thought to be significant noise contributors in the high frequency range of the noise spectrum, and therefore, if the predictive accuracy in this range is to be increased, the modelling of the small components needs improvement.

### 1.3 Noise generation mechanisms

Prior to the analysis of our particular case, the general laws that govern aerodynamic generation of sound will be presented. This is important in computational approaches since the far field noise is calculated using acoustic analogies, but also in experimental approaches, because understanding the noise generation mechanisms is fundamental to devise possible ways to reduce the noise radiation as well as to interpret the results.
1.3.1 Lighthill’s equation

The equations governing aerodynamic flows without external forces are the conservation of mass:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0,
\]

(1.1)

and the conservation of momentum:

\[
\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_j}{\partial x_i \partial x_j} = -\frac{\partial}{\partial x_j} (p \delta_{ij} - \sigma_{ij}), \quad \text{for } i = 1, 2, 3. \tag{1.2}
\]

together with the state equations and appropriate initial and boundary conditions. Repeated indices appearing in the same term imply a summation over that index (Einstein summation convention). From the above two equations only, Lighthill \cite{16} derived an expression with the form of an inhomogeneous wave equation for the density. Multiplying the mass conservation equation by \( u_i \) and adding it to the momentum equation it yields:

\[
\frac{\partial (\rho u_i)}{\partial t} = -\frac{\partial \Pi_{ij}}{\partial x_j},
\]

(1.3)

where \( \Pi_{ij} = \rho u_i u_j + (p - p_o) \delta_{ij} - \sigma_{ij} \). Now the mass conservation equation can be modified to:

\[
\frac{\partial}{\partial t} (\rho - \rho_o) + \frac{\partial (\rho u_i)}{\partial x_i} = 0. \tag{1.4}
\]

Now, eliminating the momentum density \( \rho u_i \) between (1.3) and (1.4) it reaches:

\[
\frac{\partial^2 (\rho - \rho_o)}{\partial t^2} = \frac{\partial^2 \Pi_{ij}}{\partial x_i \partial x_j},
\]

(1.5)

and subtracting \( \frac{\partial^2}{\partial x_i \partial x_j} c^2 (\rho - \rho_o) \) from both sides it yields Lighthill’s wave equation:

\[
\left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x_j \partial x_j} \right) (\rho - \rho_o) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}.
\]

(1.6)
Lighthill’s stress tensor $T_{ij}$ is:

$$T_{ij} = \rho u_i u_j - \sigma_{ij} + ((p - p_o) - c^2(\rho - \rho_o))\delta_{ij}. \quad (1.7)$$

$T_{ij}$ contains the unknown fields $\rho$ and $p$ so in principle this equation is not strictly a wave equation since the source term contains the unknown fields. However, Lighthill showed that, under certain circumstances which apply in common engineering flows, the feedback of the acoustic field on the background hydrodynamic flow is neglectable. The equation, then, corresponds to the wave equation, with the term $\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$ as the sound sources. Since the source has the form of a double divergence, by definition it corresponds to a spatial distribution of quadrupoles. Lighthill’s analogy states that a region of turbulence is equivalent to a volume distribution of quadrupoles in a medium at rest. The phenomenon of convection of sound by the background velocity fluctuations and the refraction due to changes of sound speed are taken into account by $T_{ij}$.

It can be shown that for the case of high Reynolds number and low Mach number flows we can neglect the viscous effects, the non-linear acoustic effects and the density changes in the term $\rho u_i u_j$, so Lighthill’s stress tensor can be simplified to:

$$T_{ij} \approx \rho_o u_i u_j \quad (1.8)$$

He also showed that this tensor is only significant in the regions of intense turbulent fluctuations, so that far from these regions, Lighthill’s equation recovers the non-forced wave equation, allowing only acoustic waves to be present. The radiated sound can then be calculated if the background hydrodynamic flow, which constitutes the sound sources, is known. It can be shown using an alternative formulation of Lighthill’s theory, that Lighthill’s stress tensor is dominated by the presence of vorticity in the flow [18].

Under Lighthill’s acoustic analogy, the solution of the inhomogeneous wave equation (1.6) is:

$$(\rho - \rho_o)(x, t) = \frac{1}{4\pi c^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial^2 T_{ij}(y, \tau)}{\partial y_i \partial y_j} G_o(x, y, t - \tau) d^3y d\tau, \quad (1.9)$$

where $G_o(x, y, t - \tau)$ is the free field Green’s function of the wave equation:
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\[ G_o(x, y, t - \tau) = \frac{\delta \left( t - \tau - \frac{|x - y|}{c} \right)}{4\pi|x - y|}, \quad (1.10) \]

and interchanging the space derivatives from the source space to the observer space it yields:

\[ (\rho - \rho_o)(x, t) = \frac{1}{4\pi c^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T_{ij}(y, \tau) G_o(x, y, t - \tau) d^3y d\tau, \quad (1.11) \]

Now, performing the time integration it yields:

\[ (\rho - \rho_o)(x, t) = \frac{1}{4\pi c^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T_{ij}(y, t - |x - y|/c) \frac{d^3y}{|x - y|}. \quad (1.12) \]

### 1.3.2 Curle’s equation

A way of taking into account the presence of bodies was developed by Curle [19]. He extended Lighthill’s equation to yield a wave equation with two additional source terms associated with the presence of the body. The following formulation using generalised functions is due to Ffowcs-Williams and Hawking [20].

Let \( V \) be a closed volume that contains the body. \( V \) is enclosed by the surface \( \Sigma: F(x) = 0 \) (the body is assumed to be at rest, so \( \partial F/\partial t = 0 \)). It is \( F(x) > 0 \) for \( x \in V \), and \( F(x) < 0 \) for \( x \notin V \). Then, using the function \( F \) as independent variable of the Heaviside function \( H \), we have that \( H(F) = 1 \) for \( x \in V \) and \( H(F) = 0 \) for \( x \notin V \). Multiplying Lighthill’s wave equation by \( H(F) \) and performing some manipulations (and using \( \nabla H = \nabla F \delta(F) \)) the following wave equation is reached for \( H(\rho - \rho_o): \)

\[
\left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x_i \partial x_j} \right) [H(\rho - \rho_o)] = \frac{\partial^2}{\partial x_i \partial x_j} (HT_{ij}) \\
- \frac{\partial}{\partial x_i} \left( (\rho u_i u_j + (p - p_o)\delta_{ij} - \sigma_{ij}) \frac{\partial H}{\partial x_j} \right) \\
+ \frac{\partial}{\partial t} \left( \rho u_j \frac{\partial H}{\partial x_j} \right). \quad (1.13)
\]
The second term corresponds to the volume pulsations of the surface $\Sigma$ and has the form of surface monopoles. The function $F(x)$ can be chosen to coincide with the body surface or to be outside of it. In the latter case the additional terms comprise also the effect of the turbulent sources between the body and $\Sigma$. Integrating Curle’s equation using the free field Green’s function yields:

$$H(\rho - \rho_o)(x, t) = \frac{1}{4\pi c^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \int \int_V \frac{[T_{ij}]}{|x - y|} d^3y - \frac{1}{4\pi c^2} \frac{\partial}{\partial x_i} \int \int \Sigma \left[\rho u_i u_j + (p - p_o)\delta_{ij} - \sigma_{ij}\right] \frac{n_j d^2y}{|x - y|} + \frac{1}{4\pi c^2} \frac{\partial}{\partial t} \int \int \Sigma \left[\rho u_j n_j\right] \frac{n_j d^2y}{|x - y|},$$

(1.14)

where the integration over $\tau$ has been already performed and the functions in square brackets are evaluated in the retarded time $t_r = t - |x - y| / c$. In the case that the body is rigid and $\Sigma$ is taken as the body surface, the velocity on its surface is zero and Curle’s equation simplifies to:

$$(\rho - \rho_o)(x, t) = \frac{1}{4\pi c^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \int \int_V \frac{[T_{ij}]}{|x - y|} d^3y - \frac{1}{4\pi c^2} \frac{\partial}{\partial x_i} \int \int \Sigma \left[(p - p_o)\delta_{ij} - \sigma_{ij}\right] \frac{n_j d^2y}{|x - y|}.$$  

(1.15)

Furthermore, in high Reynolds number flows in contact with a surface it can be shown that the viscous term in the surface integral is much smaller than the pressure term and it can be neglected [18].

1.3.3 Far field expansion

An observer is located in the far field when $|x| \to \infty$ and $|x| >> |y|$. In this case the expression of the acoustic field can be greatly simplified. From the previous relations we have:

$$|x - y| \approx |x| - \frac{x \cdot y}{|x|},$$

(1.16)
\[ \frac{1}{|x-y|} \approx \frac{1}{|x|}. \] (1.17)

Using these approximations and the following rule, to interchange the space derivatives and the time derivatives in the far field:

\[ \frac{\partial}{\partial x_i} \leftrightarrow -\frac{1}{c} \frac{x_i}{|x|} \frac{\partial}{\partial t}, \] (1.18)

\[
(p - p_o)(x, t) = \frac{1}{4\pi c^3} \frac{x_i x_j}{|x|^3} \frac{\partial^2}{\partial t^2} \int \int \int_V \rho_o u_i u_j \left( y, t - \frac{|x|}{c} + \frac{x \cdot y}{c |x|^3} \right) d^3y \\
+ \frac{1}{4\pi c^3} \frac{x_i}{|x|^2} \frac{\partial}{\partial t} \int \int \int_\Sigma (p u_i u_j + (p - p_o) \delta_{ij} - \sigma_{ij}) \left( y, t - \frac{|x|}{c} + \frac{x \cdot y}{c |x|^3} \right) n_j d^2y \\
+ \frac{1}{4\pi c^2} \frac{\partial}{\partial t} \int \int \Sigma p u_j \left( y, t - \frac{|x|}{c} + \frac{x \cdot y}{c |x|^3} \right) n_j d^2y. \] (1.19)

### 1.3.4 Compact sources

A source region is acoustically compact at a frequency \( f \) if the sound wavelength \( \lambda = c/f \) is much larger than the size of that region, \( \lambda >> L_c \). Compactness allows to neglect the retarded time variations within the source region, and the retarded time is equal across the source region:

\[ t_r = t - \frac{|x-y|}{c} \approx t - \frac{|x|}{c}. \] (1.20)

It can be shown that the turbulent eddies responsible for quadrupole noise are each acoustically compact. For the case of a body of size \( L_{ref} \), moving through air at a speed \( U_o \), the compact condition is equivalent to \( M \ll 1 \), assuming that the emitted sound frequency scales like \( f \propto U_o/L_{ref} \), as is the case in Strouhal-based flow mechanisms (assuming weak dependence on the Reynolds number).

In the case of a compact rigid body immersed in high Reynolds number flow, the dipole term in the far field approximation of Curle’s equation is equivalent to an unsteady force that the body exerts on the surrounding fluid:
\[(\rho - \rho_o)(\mathbf{x}, t) \approx \frac{1}{4\pi c^3} \frac{x_i}{|\mathbf{x}|^2} \frac{d}{dt} \int \int_{\Sigma} \left( (p - p_o) \delta_{ij} \right) \left( \mathbf{y}, t - \frac{|\mathbf{x}|}{c} \right) n_j d^2\mathbf{y} \]
\[= \frac{1}{4\pi c^3} \frac{x_i}{|\mathbf{x}|^2} \frac{dF_i}{dt} \left( t - \frac{|\mathbf{x}|}{c} \right), \quad (1.21)\]

where \(F_i\) is the instantaneous total force that the body exerts on the fluid.

### 1.3.5 Scaling laws

From the far field noise expressions the scaling with the incoming velocity can be determined. For quadrupole noise it is:

\[\rho - \rho_o \propto \frac{1}{c^4} \frac{1}{|\mathbf{x}|} \left( \frac{U_o}{L_c} \right)^2 (\rho_o U_o^2) L_c^3 = \rho_o \frac{L_c}{|\mathbf{x}|} \left( \frac{U_o}{c} \right)^4, \quad (1.22)\]

and the acoustic power, which is proportional to \((\rho - \rho_o)^2\) scales with \(U_o^8\). The dipole term can be scaled in an analogous way:

\[\rho - \rho_o \propto \frac{1}{c^3} \frac{1}{|\mathbf{x}|} \frac{U_o}{L_c} (\rho_o U_o^2) L_c^2 = \rho_o \frac{L_c}{|\mathbf{x}|} \left( \frac{U_o}{c} \right)^3. \quad (1.23)\]

The dipole acoustic power scales with \(U_o^6\). Finally, for the monopole term:

\[\rho - \rho_o \propto \frac{1}{c^2} \frac{1}{|\mathbf{x}|} \frac{U_o}{L_c} (\rho_o U_o) L_c^2 = \rho_o \frac{L_c}{|\mathbf{x}|} \left( \frac{U_o}{c} \right)^2, \quad (1.24)\]

and the acoustic power scales with \(U_o^4\).

### 1.3.6 Extension for moving sources

Lighthill’s and Curle’s wave equations describe the far field noise in a quiescent medium, i.e. in a medium at rest. This is because the sources are described in a fixed reference frame \(\mathbf{y}\). Ffowcs Williams and Hawkings \[20\] extended Curle’s equation to include the effect of moving sources with respect to the stationary medium. To label the sources they introduced a reference frame \(\mathbf{z}\), which moves with the sources:
\[ y = z + \int_{\tau} \mathbf{cM}(z, \tau')d\tau', \quad (1.25) \]

where \( \mathbf{M} \) is the Mach number vector of the sources (and the surface \( \Sigma : F = 0 \)).

Introducing this change of coordinates into equation \( (1.14) \), and after performing a series of manipulations the following expression for the acoustic field is reached:

\[
(r - \rho_0)(x, t) = \frac{1}{4\pi c^2} \frac{\partial^2}{\partial x_i \partial x_j} \int \int \int_{V} \left[ \frac{T_{ij}J}{r |1 - M_r|} \right] d^3z - \frac{1}{4\pi c^2} \frac{\partial}{\partial x_i} \int \int \int_{\Sigma} \left[ \frac{(\rho u_i (u_j - cM_j) + (p - p_o)\delta_{ij} - \sigma_{ij}n_jA)}{r |1 - M_r|} \right] d^2z + \frac{1}{4\pi c^2} \frac{\partial}{\partial t} \int \int \int_{\Sigma} \left[ \frac{(\rho(u_j - cM_j) + p_o cM_j)n_jA}{r |1 - M_r|} \right] d^2z. \quad (1.26) \]

\( J \) is the Jacobian of the coordinate transformation (ratio of volume elements in \( y \)-space and \( z \)-space), \( A = J |\nabla_y f| |\nabla_z f| \) (ratio of surface area elements in \( y \)-space and \( z \)-space), \( r = |x - y(z, \tau')| \), \( M_r \) is the component of the Mach number vector in the direction of the sound radiation vector \( x - y(z, \tau') \). In the regions where the source is approaching the medium \( M_r > 0 \) and \( |1 - M_r|^{-1} > 1 \) causing an enhancement of the received sound intensity. On the other hand, where the source is moving away from the quiescent medium \( M_r < 0 \) and \( |1 - M_r|^{-1} < 1 \), causing a decrease of the measured sound intensity.

### 1.4 Approach to the problem of modelling the small components

An investigation of bluff body noise due to large roughness elements will be performed in this project, with the goal of improving the prediction of landing gear noise associated with the small components. A priori roughness can affect all the flow scales, and therefore all the noise spectral range. The large scales of the flow have a size of the order of the bluff body size, and the turbulent stresses associated with them, when scattered around the body (which is compact for low mach number flows), give rise to the low frequency noise. The large scale structures are greatly affected by the separation location, and this depends on the TBL state prior to separation. For example, as will be seen
in the next chapter, the strength of the vortex shedding velocity fluctuations around a circular cylinder change significantly depending on the state of the (T)BL prior to separation, since that determines the separation location. The forces on the cylinder and the variation in radiated noise levels is large between low and large separation angles.

If the small components are small enough and the Reynolds number is large enough a TBL remains attached on the upstream face. It detaches when the adverse pressure gradient reaches a certain value, which depends on the TBL state. Sound is generated from the turbulent shear flow covering the upstream face of the large elements interacting with the small elements. Given the high number of details and their geometric variability, these small scale elements may be considered as roughness elements which cover the large and medium components of the landing gear. The noise generated by a TBL over rough walls is described by roughness noise, and the development of a model for the case of bluff bodies is one of the goals of this work. Two main requirements are imposed for the model. Firstly, it needs to be practical, i.e. input information has to be available without the need for tests or computer simulations. Secondly, the noise sources are to be described physically, so the dependence on the geometry, observer location (directivity) and free stream velocity should appear naturally in the model. This will provide predictive power, as oposed to empirical approaches which are limited to the particular geometries used to calibrate the model. These two requirements are fulfilled by the roughness noise models on flat walls developed by Howe [45, 55] and Smol’yakov [59]. They physically describe the noise sources, with certain assumptions. These models are the starting point to develop a bluff body roughness noise model, and they will be presented in the following chapter.

1.4.1 Bluff body

There are large and medium components located partially or completely in the wake of other upstream components. For example the flow detached from the articulation links impinges on the bottom region of the main strut, or the downstream wheels are in the wake of the upstream wheels. In these situations the incoming flow is not clean, and it must be accounted for to correctly determine the flow around them. On the other hand, a large portion of the main strut is located in a free stream (with a higher velocity due
to installation effects). As a preliminary study the case of a single bluff body in free stream has been considered.

Cylinders are often used as fundamental components in landing gear noise prediction models, so that all medium and large components are modelled as cylinders of a specific length, diameter, yaw angle, etc. As has been mentioned, one of the large/medium components of landing gears is the main strut. It is oriented approximately normal to the flow, as seen in figure 1.1. It typically has a circular cross section and is covered with elements such as cables, protrusions, joints, etc. A circular cylinder has been chosen in the present work.

1.4.2 Modelling of the small components

Now the small components that will cover the circular cylinder need to be specified. A physical description of the radiated noise with predictive capability should include dependence on the geometry (as well as on the observer location and the free stream velocity). In order to discern the noise emitted by particular small element geometries, sizes and surface distributions, simple configurations of roughness elements have to be considered. The geometry of the roughness configuration is described by:

1. Shape
2. Size
3. Surface density
4. Location

All of these are in general needed to describe (parameterise) the roughness geometry and therefore the flow and radiated noise. However, a simplification can be used for high Reynolds numbers if the noise sources are mostly due to a TBL covering the roughness elements. Roughness noise describes the noise generated by a TBL interacting with wall roughness.

It is wished here to extend roughness noise models, which are developed for flat walls, to the case of bluff bodies. In the roughness noise models certain features of the roughness geometry are specified since the noise generation mechanisms depend on them.
1.5 Roughness configurations

A number of geometries for the small elements have been considered. In the first place, a set of dense distributed roughness configurations were tested both on flat wall and on a circular cylinder (figures 1.7 and 1.6). For these configurations there exist roughness noise models corresponding to ZPG TBLs. Fully covered cylinders, as well as cylinders only partially covered have been considered.

Due to computational limitations it is not possible to simulate circular cylinder flow with dense distributed roughness with the required span cylinder length. Therefore different roughness configurations were chosen, computationally feasible, but still relevant for landing gear application. A series of two-dimensional roughness configurations were considered. Here two-dimensional roughness means essentially a cable-type geometry, i.e. surface bars/cable on the cylinder surface. Also, the computational studies are useful to assess the capability of the CFD codes, and of the turbulence models to resolve the flow surrounding a bluff body with small components on its surface. The turbulence models based on modelling near wall turbulence using RANS (S-A, kω-SST, DDES) are based on smooth wall TBLs. Roughness shifts the TBL further from the wall, the maximum Reynolds stresses being located above the roughness elements. The performance of the models in this situation is unknown. Especially the S-A model, whose only length scale is the wall distance, can potentially miss-predict the TBL state. A detailed study of the grid sensitivity surrounding the roughness elements has not been performed (here the computational resources have limited the grid sizes), but it is a recommendable future study to assess the capability of the turbulence models regarding rough walls.

In the first place surface cables parallel to the cylinder axis were considered, with three different surface densities. An additional configuration of surface parallel bars on a flat wall was tested experimentally. The goal was to study the effect of dense and sparse roughness using computational means, which can provide further insight in the flow structure, difficult to obtain by experimental means. They are shown in figures 1.8 and 1.9.

Secondly, configurations of surface helicoidal cable were considered, also with various surface densities (figure 1.10). The parallel and helicoidal cable configurations are expected to have very different effects on the circular cylinder flow and radiated noise.
To study a more specific geometry, typical of the main strut of landing gears, a configuration resembling the hoses that cover the main strut of real landing gears (figure 1.11) was considered. They consist of off-surface bar fetches parallel to the cylinder axis, and located at various positions around the cylinder. They were tested computationally and experimentally: upstream, downstream and at the cylinder side (figure 1.12).

**Figure 1.6:** Circular cylinder covered with dense distributed roughness.

**Figure 1.7:** Dense distributed roughness.

### 1.6 Novel aspects of the present work

- Previous aerodynamic experiments have shown that roughness has a significant effect on the mean and unsteady flow in the supercritical and postcritical regimes, and therefore must have important effects in the far field noise. However, the
acoustic radiation has not been studied, and several questions remain to be answered in the present work. Firstly, how relevant is vortex shedding noise when the cylinder is covered with large distributed roughness? If it is relevant, through what mechanisms does it modify the flow (spanwise correlation length, separation angle,...)?

- Regarding roughness noise, i.e. noise generated directly by roughness through its interaction with the TBL, its relevance in bluff body noise will be studied experimentally, for the case of relatively large distributed roughness, and for various roughness shapes.

- The applicability of rough flat wall prediction models to the case of a bluff body will be analysed. Are the roughness noise models applicable to the case of bluff bodies? An extension of roughness noise models to account for the cylindrical geometry is one of the goals of this work.

- Current landing gear noise models account for the small components empirically, and their predictive accuracy is very limited. This work is a first step for improve the prediction of noise due to the small components, both by including their effect in the low frequency noise associated with the large components, and describing the high frequency noise radiated by themselves.

- The capacity of conventional CFD codes (together with turbulence models), to resolve the flow features around cylinders with large roughness at high Reynolds number.
numbers has not been studied. It is one of the goals of this work to assess this question.

1.7 Contents of the thesis

The second chapter contains the literature review. It includes, firstly, a review of smooth and rough circular cylinder flow at high Reynolds numbers. The characteristics of noise emitted by a smooth cylinder is described, which is the baseline configuration. Secondly, the TBL flow over smooth and rough walls is addressed, and the main results in the literature regarding roughness noise are detailed. Thirdly, the two roughness noise models considered are described. Finally, BEM is introduced, together with tailored Green’s functions, and the derivation of the integral equation to be solved.

The third chapter contains the experimental methods. Firstly, the facilities and the arrangement of the flat plate and the circular cylinder are described. Next, the experimental techniques are described, together with the measurement uncertainty and
repeatability tests. Finally, the roughness elements used in the tests are described.

The fourth chapter contains the numerical methods used in the simulations: RANS equations, turbulence models, flow domain, space and time discretisation, solver, initial conditions and boundary conditions, grid topology, and (maximum) wall-$y^+$. 

The fifth chapter is the first results chapter. It contains the results and discussion of
the radiated noise by flat walls and circular cylinders with distributed roughness. The flat wall roughness noise models are compared with the flat wall measurements, and they are tentatively used to identify roughness noise in the circular cylinder case. For the latter, a comparison with the smooth configurations is performed, and the additional noise sources due to roughness are analysed, using the directivity, velocity scaling and spectral features.

The sixth chapter is the second chapter of results. It contains the experimental results of the noise radiated by circular cylinders partially covered with distributed roughness. The transition from smooth regime to fully rough regime regarding the noise spectra is studied, with rough surface increasing from upstream, and from downstream.

The seventh chapter describes the circular cylinder roughness noise model, which includes the algorithm used to solve the BEM integral equation, the TBL model, and the
coupling of the tailored Green's function with the flat wall roughness noise model. It is validated with the experimental results of fifth chapter.

Finally, the eighth chapter contains the results of circular cylinders with two-dimensional roughness, mainly obtained from CFD simulations.
Chapter 2

Literature Review

The goal of this research project is to gain understanding of the effects of surface roughness elements on the noise radiated by bluff bodies, and in particular a single circular cylinder in cross-flow. The small components size in landing gears are much smaller than the large components they cover, but their shapes and surface distributions have significant variability.

The case of uniform size distributed roughness (approximately constant number of roughness elements per unit area) will be investigated primarily. There is special interest in these configurations since in high Reynolds number flows and roughness much smaller than the cylinder diameter, \( h/D \ll 1 \), there is a TBL flow developing over the roughness elements. In absence of a pressure gradient, turbulent stresses in a fully developed TBL are approximately independent of the particular roughness geometry, in the limit of high Reynolds number (Townsend’s similarity hypothesis [47]). This similarity is wished to be extended to the case of TBLs on the upstream face of rough bluff bodies (FPG TBLs).

In addition to distributed roughness, other configurations consisting of two-dimensional roughness and bar fetches will also be considered.

The review of the relevant literature background is divided into three parts. Firstly, the effects of distributed roughness on the flow and acoustics of circular cylinders is revised. The second part includes previous work done on noise emitted by TBL flow on rough flat walls. Finally, the third part describes the most relevant roughness noise models existing in the literature.
2.1 Aerodynamics and acoustics of circular cylinder flow

2.1.1 Smooth cylinder

Before examining the effects of roughness it is necessary to introduce the main characteristics of the radiated noise around smooth circular cylinders, since the effects of roughness, at least for $h \ll D$, are analysed in terms of variations with respect to the flow around smooth cylinders. At low Mach numbers, the only parameter needed to describe it is the Reynolds number based on the inflow velocity $U_o$ and the cylinder diameter $D$, $Re = \rho U_o D/\mu = U_o D/\nu$.

The following table, taken from Zdravkovich [21], contains the observed flow regimes around a circular cylinder as a function of the Reynolds number $Re$ (upper Reynolds number range):

1. **TrSL. Transition-In-Shear-Layers State.** Transition to turbulence occurs in the free shear layers.
   - **TrSL1. Lower Subcritical Regime.** Transition Waves appear along the free shear layers and stabilise the near wake. $375 < Re < 1.5 \times 10^3$.
   - **TrSL2. Intermediate Subcritical Regime.** Progressive movement of transition towards the separation points with increasing Reynolds Number. $1.5 \times 10^3 < Re < 3 \times 10^4$.
   - **TrSL3. Upper Subcritical Regime.** Transition to turbulence immediately after the separation points. $3 \times 10^4 < Re < 1.5 \times 10^5$.

2. **TrBL. Transition-In-Boundary-Layers State.** Transition to turbulence occurs in the boundary layer before separation.
   - **TrBL0. Precritical Regime.** $1.5 \times 10^5 < Re < 3.2 \times 10^5$. Decay of the drag coefficient and the alternating lift coefficient
   - **TrBL1. Single Bubble Regime.** $3.2 \times 10^5 < Re < 3.9 \times 10^5$.
   - **TrBL2. Two-Bubble Regime.** $3.9 \times 10^5 < Re < 7 \times 10^5$.
   - **TrBL3. Supercritical Regime.** $7 \times 10^5 < Re < 4.7 \times 10^6$.
   - **TrBL4. Postcritical Regime.** $4.7 \times 10^6 < Re < (?)$. 
3. **T. Fully Turbulent State.** The three main flow zones around the cylinder, i.e. the boundary layer, the free shear layers and the wake, are fully turbulent. \(?\) to \(\infty\).

The flow feature that characterises the division of the subcritical, critical and supercritical regimes is the location of the transition to turbulence point, with respect to the separation point. In the subcritical regime there is laminar separation, and in the supercritical regime the separation is turbulent. Within the critical regime the separation point is in the vicinity of the separation point. The regimes relevant for our purposes of application to flow around landing gears are supercritical, postcritical and fully turbulent.

As was mentioned earlier, vortex shedding is a two-dimensional instability. In three-dimensional cylinders vortex shedding decorrelates along the span, and the correlation length is defined as the span length where the vortex shedding is correlated. The decorrelation causes the total unsteady force to be smaller than in the two-dimensional case, where the shedding is perfectly correlated along the span. Different Reynolds number regimes present different correlation lengths: in the subcritical regime the correlation length is large compared with the cylinder diameter \((L_c \gg D)\), whereas in the supercritical and postcritical regimes it is of the order of the cylinder diameter \((L_c \approx D)\). Our interest is in the latter case, \(Re \gtrsim 10^6\).

At low Mach numbers and under Lighthill’s acoustic analogy, the noise sources are essentially incompressible, i.e. they are the same as the ones obtained from the incompressible Navier-Stokes equations. The smooth circular cylinder noise spectrum is characterised, in the supercritical and postcritical regimes, by a low frequency peak and a spectral fall-off. The low frequency peak is associated with vortex shedding, i.e. a two-dimensional flow instability (Hopf bifurcation) which causes a tonal force oscillation on cylindrical bluff bodies. The noise peak frequency is equal to the vortex shedding peak frequency as measured, for example, by a hot-wire probe in the wake, or by a surface microphone. The peak level is directly related with the intensity of the force fluctuations on the cylinder. The non-dimensional parameter used to specify the shedding frequency is the Strouhal number \(St = fD/U_o\). For typical diameters and inflow velocities the shedding frequency is low within the human hearing range. This relation between vortex shedding and the observed tonal noise was made clear in previous experimental work.
and more recent computational work has successfully been able to obtain the far field noise using FW-H expansions \[30\] as well as a tailored Green’s function \[31\]. The spectral fall-off at higher frequencies is of broadband nature and is due to the noise associated with smaller flow scales.

It is also known that in the supercritical and postcritical regimes (TrBL and T) the spanwise correlation length of the shedding is significantly lower than in the subcritical regime (TrSL), which causes the shedding peak level to diminish and the peak to get broader. The effect of lower spanwise correlation length in the fluctuating forces and noise is manifested by the overprediction of two-dimensional simulations, with respect to three-dimensional ones. Two-dimensional simulations assume there is no spanwise decorrelation of vortex shedding. In addition to the peak level decrease, in the supercritical regime background noise levels due to the turbulent shear layers and turbulent wake increase, and the relative relevance of the vortex shedding peaks diminishes.

2.1.2 Rough cylinder

In the case of uniformly distributed roughness with approximately uniform roughness size, the simplest way to characterise roughness effects is to include in the analysis the roughness size with respect to the cylinder diameter, $h/D$. Therefore, the flow is defined now by two non-dimensional governing parameters: the Reynolds number $Re = U_o D/\nu$, and the ratio of roughness size and cylinder diameter, $h/D$.

Achenbach \[33\] performed experimental measurements on smooth and rough cylinders with roughness sizes up to $h/D = 0.017$, at Reynolds numbers ranging from the subcritical to the postcritical regime. The roughness consisted of emery paper and glued spheres. The mean surface pressure and skin friction at a cylinder section were measured, and the effect of roughness on the location of separation and transition to turbulence could be analysed. From the surface pressures the drag force was calculated. The conclusions were that increasing roughness size had very significant effects on the mean flow in the critical, supercritical and postcritical regimes, directly related to the effect of roughness on the state of the boundary layer. Particularly, higher values of $h/D$ had two main effects: firstly, it decreased the critical Reynolds number, eventually causing its elimination, and reaching the postcritical regime at lower Reynolds numbers. And secondly,
increasing $h/D$ caused a significant increase of the drag coefficient in the postcritical regime.

Guven et al. \cite{34} revised the previous work on rough circular cylinder flow and performed mean surface pressure measurements as well as boundary layer mean velocity profiles. Reynolds numbers up to $Re = 5.5 \times 10^5$ were tested, and roughness sizes up to $h/D = 0.006$. From their results and the previous work the observed dependence of the separation point, and therefore the drag coefficient, on the roughness size, in terms of the effect of roughness on the TBL were explained. Higher roughness sizes causes an increase of the TBL thickness, with a higher momentum deficit, and able to reach a lower pressure rise before separation. This lower pressure recovery causes the increase of drag coefficient.

The following work included the measurement of unsteady flow features, especially the effect of roughness on the existence and intensity of vortex shedding. Achenbach and Heinecke \cite{35} used a hot-wire probe to determine the vortex shedding frequency of smooth and rough cylinders. The Reynolds numbers ranged up to the postcritical regime, and the roughness consisted of pyramidal elements, with sizes as high as $h/D = 0.03$. It was observed that roughness diminished the Reynolds number at which there is a sudden increase of Strouhal number, and caused that increase to be smaller, for higher roughness size (figure 2.1). They measured the steady drag force on the cylinders as well (figure 2.2). Regarding the smooth cylinder case they observed that the aspect ratio of the model has an impact on the presence of regular vortex shedding in the critical regime: for $L/D < 4$ vortex shedding was not detected, whereas for larger aspect ratios it was. The rough cylinders didn’t suffer this interruption. Even for the lower aspect ratio, regular vortex shedding was observed in the critical regime.

Buresti \cite{37} also measured the vortex shedding frequency of rough cylinders in cross flow using a hot-wire probe, for Reynolds numbers up to $Re = 2.8 \times 10^5$, and with roughness sizes up to $h/D = 0.01$. Results confirmed the previous findings. Furthermore, the possibility of correlating all the data in terms of a single non-dimensional parameter, $Re_h = hU_o/\nu$, was analysed. This attempt failed, concluding that both the Reynolds number $Re = DU_o/\nu$ and the non-dimensional roughness size $h/D$ were needed to characterise the flow.
Nakamura and Tomonari [36] tested smooth and rough cylinder flows at Reynolds numbers up to the postcritical regime, with roughness sizes \( h/D < 0.02 \). Simulation of supercritical flow around a smooth cylinder using roughness strips located at \( \theta = 50^\circ \) (from the upstream stagnation line) was achieved, for Reynolds numbers as low as \( Re = 3 \times 10^5 \). This technique is very useful when trying to obtain high Reynolds number flows in wind tunnels of limited test section dimensions and flow speeds.

Circular cylinders with dense surface grooves (normal to the cylinder axis) present similar values of drag and shedding peak Strouhal number as cylinders with dense distributed
roughness, indicating that this type of roughness has the same effect as distributed roughness regarding vortex shedding [22].

Regarding the acoustic radiation from rough circular cylinder flow, a set of experimental work was performed by Hutcheson and Brooks [38] to study, among other topics, the effect that surface roughness and various types of surface protrusions had on the aeolian tone, i.e. the tonal noise associated with vortex shedding (figure 2.3). The Reynolds number range was $3.8 \times 10^3 < Re < 10^5$ and the roughness consisted of sandpaper of size $0.007 < h/D < 0.07$. The vortex shedding frequencies measured agree with the previously studies ([35, 37]). The goal of the tests with the collars and the helicoidal cable was to study its capability to prevent vortex shedding. The collars were such that $h/D = 0.5$ and had an individual length $l_{collar}/D = 3$, and the wire had a diameter corresponding to $h/D = 0.25$. The angle that the cable formed with the cylinder axis was $32^\circ$. The peak level of the cylinder with collars diminished about 30 dB, and the one with helicoidal cable about 38 dB. In the spectrum corresponding to the helicoidal cable appeared a broadband peak centred at a Strouhal number of about 0.7. They observed that when scaling the peak frequency with the cable diameter instead of the cylinder diameter they obtained a Strouhal number close to 0.2, which corresponds to the vortex shedding Strouhal number from an isolated cable, and they related the peak with the shedding from the cable.
<table>
<thead>
<tr>
<th>Author</th>
<th>Material</th>
<th>(Re) Range</th>
<th>(\delta/D) Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achenbach [33]</td>
<td>Glued spheres and Emery paper</td>
<td>(6.5 \times 10^6 &lt; Re &lt; 10^7)</td>
<td>(0.0001 &lt; \delta/D &lt; 0.012)</td>
</tr>
<tr>
<td>Achenbach and Heinemecke [35]</td>
<td>Pyramidal</td>
<td>(6 \times 10^3 &lt; Re &lt; 5 \times 10^6)</td>
<td>(0.00075 &lt; \delta/D &lt; 0.03)</td>
</tr>
<tr>
<td>Buresti [37]</td>
<td>Sandpaper</td>
<td>(2.6 \times 10^4 &lt; Re &lt; 2.8 \times 10^5)</td>
<td>(0.001 &lt; \delta/D &lt; 0.012)</td>
</tr>
<tr>
<td>Guven et al. [34]</td>
<td>Glued spherical beads</td>
<td>(7 \times 10^4 &lt; Re &lt; 5 \times 10^5)</td>
<td>(0.002 &lt; \delta/D &lt; 0.006)</td>
</tr>
<tr>
<td>Nakamura and Tomonari [36]</td>
<td>Glued spheres and sandpaper</td>
<td>(9.6 \times 10^4 \times 3 \times 10^6 &lt; Re &lt; 10^7)</td>
<td>(0 &lt; \delta/D &lt; 0.07)</td>
</tr>
</tbody>
</table>
2.2 Aerodynamics and acoustics of ZPG TBLs over rough walls

In the flow around rough circular cylinders the TBLs are attached on the front face until separation. They are FPG TBLs, with an outer velocity that changes from 0 m/s, in the vicinity of the stagnation line, to approximately $2U_o$, near the separation line. It is known that turbulence in the vicinity of acoustically compact bodies is a potential source of noise. The mechanism based on the scattering of turbulent noise depends crucially on the flow structure of the TBL, and the latter is affected by the presence of roughness. As will be seen, previous researchers have studied the problem of noise emission by TBL flow over rough walls. The structure of smooth ZPG TBLs has been intensely studied, as well as the effects of roughness. Before considering TBLs over rough walls, the structure of TBLs over smooth walls will be briefly presented.

The space coordinates used are the streamwise direction $x_1$, $y_1$, the wall-normal direction $x_2$, $y_2$ (with origin in the plane of the wall), and the wall-parallel transverse direction $x_3$, $y_3$.

### 2.2.1 Smooth wall TBL

A ZPG TBL over a smooth wall is divided in four different zones depending on the distance to the wall, as shown in table 2.2. There is a region near the wall (inner layer) where the effects of the wall affect the mean velocity profile, and an exterior region (outer layer) where the mean velocity profile is determined by the exterior flow.

| Inner layer $x_2/\delta < 0.1$ | Viscous sublayer $x_2^+ < 5$ | linear velocity profile |
| Buffer layer $5 < x_2^+ < 30$ | Logarithmic region $x_2^+ > 30$, $x_2/\delta < 0.3$ | logarithmic velocity profile |
| Outer layer $x_2^+ > 50$ | $x_2/\delta > 0.3$ |

Table 2.2: Regions of smooth flat surface TBL [41].

The TBL thickness ($\delta(x_1)$) grows along the streamwise direction, and the skin friction coefficient ($c_f(x_1)$) diminishes. Simple models can be derived from the integral equations of the TBL together with empirical measurements. $\delta(x_1)$ is defined as the distance where
the mean velocity is $0.99U_o$. $c_f(x_1)$ is the wall shear stress normalised with the outer dynamic pressure:

$$c_f(x_1) = \frac{\tau_\omega(x_1)}{1/2\rho_o U_o^2} = \frac{\rho_o u_\tau(x_1)^2}{1/2\rho_o U_o^2} = 2 \left( \frac{u_\tau(x_1)}{U_o} \right)^2,$$

where the friction velocity has been defined by $\tau_\omega = \rho_o u_\tau^2$, and it can be expressed in terms of the friction coefficient:

$$u_\tau(x_1) = U_o \sqrt{\frac{c_f(x_1)}{2}}.$$

The streamwise length normalised with the viscous length appears naturally as the non-dimensional streamwise distance:

$$Re_x(x_1) = \frac{U_o x_1}{\nu}.$$

The skin friction coefficient diminishes along the TBL like:

$$c_f(x_1) = 0.0257 Re_x^{-1/7}.$$

The mean velocity profile is:

$$\bar{u}(x_2) = \frac{u_\tau}{u_\tau} \ln \left( \frac{u_\tau x_2}{\nu} \right) + A + \frac{2\Pi_o}{\kappa_o} w \left( \frac{x_2}{\delta} \right),$$

where $A \approx 5$, $\Pi_o \approx 0.45$ and $\kappa_o \approx 0.41$ are empirical constants. The function $w \left( \frac{x_2}{\delta} \right)$ is the wake function:

$$w \left( \frac{x_2}{\delta} \right) = \frac{1}{2\Pi_o} \left[ (1 + 6\Pi_o) - (1 + 4\Pi_o) \left( \frac{x_2}{\delta} \right) \right] \left( \frac{x_2}{\delta} \right)^2.$$

From the above relations, particularisation for $x_2 = \delta$ provides a direct relationship between the outer velocity $U_o$, the friction velocity $u_\tau$ and the TBL thickness $\delta$:

$$\frac{U_o}{u_\tau} = \frac{1}{\kappa_o} \ln \left( \frac{\delta u_\tau}{\nu} \right) + A + \frac{2\Pi_o}{\kappa_o},$$
and the TBL thickness is:

$$\delta(x_1) = \frac{\nu}{u_r} \exp \left( \kappa \left( \frac{2}{c_f(x_1)} \right)^{1/2} - \kappa A - 2\Pi_0 \right).$$ \quad (2.8)$$

Until here, only the mean velocity field has been considered. The unsteady flow is relevant to sound generation because the sources of sound are the turbulent stresses:

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x_i \partial x_j} = \frac{\partial^2 \rho_o u_i u_j}{\partial x_i \partial x_j},$$ \quad (2.9)

The Fourier transform of the source term is:

$$\hat{S}(k, \omega, x_2) = \int \int \int \rho_o \frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} e^{-i(k \cdot x - \omega t)} d^2 x dt.$$ \quad (2.10)

Howe \cite{55} determined a direct relation between the above source spectrum and the spectrum of the pressure field on any flat surface $x_2 = C$. This is helpful since the latter can be determined experimentally \cite{58}. In particular, on a flat wall the pressure wavenumber-frequency spectrum $P_s(k_1, k_3, \omega)$ is defined as the power spectral density of the wall pressure field $p_s(x_1, 0, x_3, t)$:

$$\langle \hat{p}_s(k_1, k_3, \omega), \hat{p}_s^*(k_1', k_3', \omega') \rangle = (2\pi)^3 P_s(k_1, k_3, \omega) \delta(k_1 - k_1') \delta(k_3 - k_3') \delta(\omega - \omega'),$$ \quad (2.11)

where $\hat{p}_s(k_1, k_3, \omega)$ is the Fourier transform of $p_s(x_1, 0, x_3, t)$. There are various semi-empirical models of $P_s(k_1, k_3, \omega)$ in the literature \cite{45, 75}.

### 2.2.2 Rough wall TBL

The presence of roughness modifies the TBL growth and structure. Jimenez \cite{50} analysed the effects of roughness using the available experimental data. He concluded that roughness effects can be parameterised in terms of: the roughness Reynolds number
Smooth regime $h_s^+ < 5$

Transitionally rough regime $5 < h_s^+ < 80$

Fully rough regime $h_s^+ > 80$, $\delta/h_s < 50$

<table>
<thead>
<tr>
<th>Smooth regime</th>
<th>$h_s^+ &lt; 5$</th>
<th>TBL mean velocity profile as on a smooth surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitionally rough regime</td>
<td>$5 &lt; h_s^+ &lt; 80$</td>
<td></td>
</tr>
<tr>
<td>Fully rough regime</td>
<td>$h_s^+ &gt; 80$, $\delta/h_s &lt; 50$</td>
<td>roughness effects extend across the TBL</td>
</tr>
</tbody>
</table>

Table 2.3: Regimes of rough flat wall TBL [50].

$h_s^+ = h_s u_\tau/\nu$, and the ratio of the TBL thickness to the effective roughness height $h_s/\delta$ (table 2.3). He concluded that for $\delta/h_s \lesssim 50$ roughness affects the mean and turbulent fields through all the TBL.

Krogstad et al. [51] proposed the following model to describe the mean velocity profile on rough walls, which is normally used to define the effective roughness height $h_s$:

$$\bar{u}(x_2) = \frac{1}{\kappa_o} \ln \left( \frac{x_2}{h_s} \right) + B + \frac{2\Pi_o}{\kappa_o} w \left( \frac{x_2}{\delta} \right), \quad (2.12)$$

where $B \approx 8.5$. In the same way as for a smooth wall, its particularisation for $x_2 = \delta$ provides a direct relationship between $U_o$, $u_\tau$ and $\delta$:

$$\frac{U_o}{u_\tau} = \frac{1}{\kappa_o} \ln \left( \frac{\delta}{h_s} \right) + B + \frac{2\Pi_o}{\kappa_o}, \quad (2.13)$$

and the TBL thickness $\delta(x_1)$ can be determined:

$$\delta(x_1) = h_s \exp \left( \kappa_o \sqrt{\frac{2}{c_f(x_1)}} - \kappa_o B - 2\Pi_o \right). \quad (2.14)$$

The following function $c_f(x_1)$ proposed by Mills and Hang [52] contains corrections for the case of walls with dense distributed roughness:

$$c_f(x_1) = \left( 3.476 + 0.707 \ln \left( \frac{x_1}{h_s} \right) \right)^{2.46}. \quad (2.15)$$

Finally, the eddy convection velocity $U_c$ (average velocity of eddies in the vicinity of the ‘convective ridge’) is assumed to be a constant (it has a weak frequency dependence [53]) in the range $0.5 < U_c/U_o < 0.7$. 


As Jimenez [50] states, in the fully rough regime and in the case of ‘K’-type roughness ($\lambda_s \lesssim 0.15$, where $\lambda_s$ is the total projected frontal roughness area per unit wall-parallel projected area), $h_s$ should be proportional to the size of the roughness elements $h$, the proportionality constant depending on the roughness geometry and surface density. For distributed roughness it is $h_s \sim h$, whereas for mesh type roughness $h_s \sim 3h$ and for surface bars in cross-flow values in the range $6 < h_s/h < 14$ have been observed [48].

There has been recent experimental evidence [48, 49] that the Reynolds stresses profiles of fully rough TBLs with distributed roughness, in inner scales, are equal to smooth walls, outside of the roughness sublayer ($x_2/h_s \gtrsim 3$). This similarity of the turbulent stresses between smooth and rough walls is Townsend’s similarity hypothesis. Assuming its validity, the turbulent stresses of rough walls can be calculated if the scaling parameters ($u_\tau, \delta$) are known. According to Townsend’s similarity hypothesis, the normalised wall pressure wavenumber-frequency spectrum defined before is also applicable for rough walls.

As concluded by Jimenez [50], the outer layer universality requires a low enough value of $h/\delta$. Since the TBL grows, this parameter decreases along the TBL, and a low enough value for outer layer similarity is eventually reached downstream. Therefore, close to the leading edge, or of the first rows of roughness, the TBL will have values of $h/\delta$ not small enough, and therefore Townsend’s similarity hypothesis is not applicable. Indeed, in the present application we deal with TBLs that are potentially not fully developed, and therefore the TBL models presented are potential sources of error.

The relevance of these errors on the radiated noise is, however, not known, and actually, as will be shown in the next section, it has been found good agreement between roughness noise prediction and experiments for values of $h/\delta$ as high as 0.5 (at the upstream roughness elements).

### 2.2.3 TBL roughness noise

Hersh [54] studied experimentally the far field noise emitted by a TBL on the interior of a rough circular duct. The roughness size range covered was $0.005 < h/\delta < 0.5$ and $14.6 < h^+ < 4200$. The far field noise for various mean flow velocities and various roughness configurations consisting of sand paper and wire mesh was measured. Results
showed that roughness noise (noise due to the presence of roughness) was important for frequencies $\omega h/u_{*} \leq 5$. It was observed that roughness noise scaled like $p^2 \propto U_o^6$, i.e. like dipole noise. Also, for a given surface roughness the noise generation efficiency increased for decreasing TBL thickness.

Howe [55] developed a theoretical model for the noise generation due to hemispherical roughness. The dominant mechanism for noise generation was assumed to be the scattering of the turbulence sources present in the TBL, on the roughness elements. The model predicts the acoustic dipole character of the sound sources found by Hersh [54]. In a second work, Howe [56] showed that the diffracted component of the flow field (field component diffracted/scattered on the roughness elements) dominated the subconvective and acoustic domains. The experimental measurements of Farabee and Geib [58] confirmed the theoretical results of Howe.

Liu and Dowling [45] reviewed the model of Howe [55] and performed an improvement to it. They substituted the perturbative approximation used by Howe to estimate the integral over the wavenumber space that defined the scattered sound pressure field, for a more accurate numerical integration. In order to test their method, experiments were performed in an open jet of a low speed wind tunnel with non-anechoic conditions. Hemispherical roughness configurations were tested, with surface density $\sigma \approx 0.5$ and roughness heights in the range $0.09 < h/\delta < 0.7$ and $190 < h^+ < 380$. Figure 2.4 shows that Liu and Dowling’s scheme was capable of accurately predicting the absolute noise levels. They attributed the disagreement at high frequencies to the fact that the acoustic wavelength was not much larger than the TBL thickness in that range.

Figure 2.4: Predicted and measured noise spectra for $U_o = 30$ m/s. a) $h = 4$ mm, $\sigma = 0.5$. b) $h = 3$ mm, $\sigma = 0.44$ [45].
Liu et al. [46] continued their study with phased microphone array measurements of the roughness noise emitted by flat rough surfaces, and compared the sound source map with a predicted sound source map derived using Liu and Dowling’s [45] scheme. The experiments were done in the same open jet facility as their previous work, and the roughness configurations tested were the same. The results from the phased array measurements showed a decrease of the source intensity with the streamwise distance. This is explained by the fact that the ratio $h/\delta$ diminishes in the downstream direction and the sound emission efficiency decreases. The next step was applying Liu and Dowling’s [45] prediction method to compare the predictions with the measurements. The beamforming algorithm assumes the sound sources are monopoles, but in reality they are dipoles, which prevents a meaningful comparison of the measured and predicted source maps. To solve this problem they calculated the monopole source map associated with the prediction model. They could then compare it directly to the measured map. The measured and predicted source strengths at $f = 1.25$ kHz and $f = 1.6$ kHz agree within $\pm 1$ dB, which is remarkable and provides a further validation of Howe’s model. However, at $f = 2$ kHz the results show a difference of about 3 dB. This discrepancy was attributed to the lack of prediction accuracy at high frequencies (figure 2.5).

Howe’s roughness noise model [55] as implemented using Liu and Dowling’s numerical integration [45] is considered in the present work to study bluff body roughness noise. The model formulation will be exposed in the next section.

Smol’yakov [59] developed a different model of roughness noise. He argued that for sufficiently large roughness flow detaches at their back faces, resulting in large scale
shedding, and thus emitting noise as individual bluff bodies. In this model, each rough-
ness element emits noise at a particular frequency which depends on the size of the
element and the local velocity that impinges on it. The model includes two empirical
parameters, namely the Strouhal number at which all the elements shed vortices, and
a proportionality constant. He also expressed the roughness noise as the sum of the
turbulence quadrupole sound emitted by the enhanced Reynolds stresses and the noise
emitted directly from the roughness elements. Due to the fact that turbulence noise
is only dominant at low frequencies, where the background noise of the wind tunnel is
higher, its presence is sometimes obviated and the term ‘roughness noise’ refers to the
noise emitted by the roughness elements exclusively. Here we will use this terminology.
Smol’yaakov prediction model does not include the directivity of the emitted noise: the
unit of roughness area is assumed to radiate sound perpendicularly to the surface. In
the same work, Smol’yaakov performed experiments to validate his theory and determine
the value of the two empirical parameters. On-surface pressure fluctuations were mea-
sured. Roughness consisted of abrasive paper with roughness dimensions in the range
$80 < h^+ < 160$. Smol’yaakov’s model is also considered in the present work, and the
formulation will be exposed in the next section.

Grissom et al. [64] measured the roughness noise characteristics of flat walls with dense
distributed roughness in order to test the existing roughness noise models. They used
aluminium oxide and floor sanding with roughness heights $0.003 < h/\delta < 0.04$, and
$2 < h^+ < 75$. The amplitude scaling revealed the presence of two regions: a low
frequency region controlled by the scattering mechanism ($\omega\delta/U_o < 95$), and a high
frequency region dominated by a quadrupole source due to the increased turbulence
intensity due to roughness ($\omega\delta/U_o > 95$).

The experimental work performed in Virginia Tech [61, 63] was followed by Smith et al.
[65]. With the same experimental configuration as their predecessors, they performed
far field noise measurements as well as on-surface point pressure measurements. They
reached the conclusion that the far field noise spectra collapsed to a single curve when
normalised with the point pressure spectrum, the frequency squared and the mean-
square roughness height, for all roughness sizes and free stream velocities. However,
for the largest roughness sizes the normalised spectra diverged from this pattern at
large frequencies. They interpreted the experimental results using the theoretical model
of Glegg et al. [62], from which the observed scaling is derived assuming that the wavenumber spectrum of the surface roughness slope is constant.

Glegg and Devenport [66] culminated their theoretical work with a roughness noise prediction model based on a smooth wall Green’s function coupled with a first order perturbation expansion of the wall boundary condition. The predicted scaling of the radiated noise with roughness height and free stream velocity is the one observed by Smith et al. [65], which was in good agreement with all measurements on moderately rough surfaces \((h^+ < 98)\) at frequencies lower than a certain ‘break frequency’. The model provides a distinction between the effect of roughness geometry, confined in a ‘filter function’, and the acoustic sources, determined by the on-surface pressure spectrum. However, as was recognised by Howe [56], as well as the authors, the perturbation expansion of the rough surface above the underlying flat wall is strictly valid only for frequencies \(\omega h/U_c \ll 1\), which is equivalent to the dominant eddies being much larger than the roughness size. This limits the validity to low frequencies and small roughness sizes. For example, in Liu and Dowling’s roughness configuration and flow speed \((h = 4 \text{ mm}, U_o = 30 \text{ m/s})\) this condition would limit the predictions to \(f \ll 720 \text{ Hz}\), which is below the measured roughness noise peak. For our purpose of application to large roughness this model was discarded.

Due to computational limitations, simulations that resolve dense distributed roughness in a fully rough TBL are not plausible. However, such simulations would be useful to identify the dominant noise generation mechanisms. Yang and Wang [69, 70, 71] have studied computationally the radiated noise of a fully rough TBL flow over sparse distributed roughness patches of various roughness geometries, and using Lighthill’s acoustic analogy. The roughness height, surface density and TBL thickness were such that \(h^+ \approx 170\), \(\sigma = 0.1\) and \(h/\delta \approx 0.12\). They calculated the far field noise assuming that the roughness elements were compact, and therefore each of them was equivalent to an acoustic dipole, whose strength is determined by the force oscillations on the element. The quadrupole terms were neglected with respect to the dipole source terms. Three roughness shapes were tested: hemispheres, cylinders (normal to the wall) and cuboids, all with the same geometric height. The streamwise and spanwise dipole strength were observed to be of the same order, and the streamwise dipole spectra peaked at \(fh/U_o \approx 0.17\) (close to the prediction of Howe’s model), for all configurations. The higher wall pressure fluctuations were located on the upstream face of the roughness elements for
all the roughness geometries and all the streamwise locations. This indicates that the shedding from individual elements is not a significant sound source. They also observed significant differences between the three roughness shapes tested. The strongest pressure fluctuations in the case of the hemispheres appeared in the second row of elements, due to the impingement of the eddies shed by the first row of elements. The cylindrical roughness showed a streamwise continuous increase of dipole strength, the last row of elements being the strongest dipoles. Cuboidal roughness presented a streamwise decrease of the dipole strengths, the first row of elements being the strongest dipoles. In contrast to hemispherical roughness, the higher wall pressure fluctuations of cylindrical and cuboidal roughness appeared in the surroundings of the sharp edges, indicating turbulent stresses caused by sharp edge separation. Also, the noise spectra of cylindrical and cuboidal roughness presented higher levels than hemispherical roughness, especially at frequencies higher than the peak frequency. Sharp edges appeared to be responsible for the high noise levels at high frequencies. All roughness elements radiated as independent dipoles from each other, and the turbulent stresses responsible for dipole noise were caused directly by their presence, i.e. the larger eddies present in the TBL did not appear to be relevant.

Table 2.4 summarises the most relevant experimental and computational work performed on roughness noise.

2.3 Roughness noise generation mechanisms

The previous theoretical and experimental investigations of roughness noise have observed several mechanisms of noise generation.

1. The noise sources do not interact with the roughness elements, behaving therefore as free turbulence noise sources, of quadrupole nature. The presence of roughness is known to enhance the turbulence intensity in the TBL, thus increasing the emitted noise levels, but not its quadrupole nature.

2. Dominant noise sources are the turbulent stresses characteristic of a smooth TBL (enhanced by roughness), which are scattered/diffracted on the roughness elements, thus becoming dipole sources with a higher radiation efficiency, and dipole axes parallel to the wall.
<table>
<thead>
<tr>
<th>Author</th>
<th>Approach</th>
<th>$\sigma$</th>
<th>$h/\delta$</th>
<th>$h^+$</th>
<th>Roughness elements</th>
<th>Flow features studied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hersh [51]</td>
<td>Experimental</td>
<td>$\approx 0.5$</td>
<td>0.005 - 0.5</td>
<td>14.6 - 4200</td>
<td>Sandpaper, wire mesh</td>
<td>TBL mean velocity profile, and far field noise.</td>
</tr>
<tr>
<td>Grissom et al. [64]</td>
<td>Experimental</td>
<td>$\approx 0.35$</td>
<td>0.003 - 0.04</td>
<td>2 - 75</td>
<td>Sandpaper</td>
<td>TBL mean velocity profile and Reynolds stresses, and noise far field</td>
</tr>
<tr>
<td>Smith et al. [65]</td>
<td>Experimental</td>
<td>$\approx 0.35$</td>
<td>0.003 - 0.04</td>
<td>2 - 75</td>
<td>Sandpaper</td>
<td>On-surface pressures and far field noise</td>
</tr>
<tr>
<td>Alexander et al. [68]</td>
<td>Experimental</td>
<td>$\approx 0.1$</td>
<td>0.14 - 0.19</td>
<td>73 - 197</td>
<td>Hemispheres and cuboids</td>
<td>On-surface pressures and far field noise</td>
</tr>
<tr>
<td>Devenport et al. [67]</td>
<td>Experimental</td>
<td>$\approx 0.5$</td>
<td>$\approx 0.1$</td>
<td>0 - 98</td>
<td>Sandpaper</td>
<td>On-surface pressures and far field noise</td>
</tr>
<tr>
<td>Liu and Dowling [45]</td>
<td>Experimental</td>
<td>0.44, 0.5</td>
<td>0.09 - 0.7</td>
<td>190 - 380</td>
<td>Hemispheres</td>
<td>Far field noise</td>
</tr>
<tr>
<td>Liu et al. [46]</td>
<td>Experimental</td>
<td>0.44, 0.5</td>
<td>0.09 - 0.7</td>
<td>190 - 380</td>
<td>Hemispheres</td>
<td>Noise source strength maps and far field noise</td>
</tr>
<tr>
<td>Farabee and Geib [58]</td>
<td>Experimental</td>
<td>?</td>
<td>0.01 - 0.2</td>
<td>19 - 2580</td>
<td>Sandpaper, Gravel, Barnacle</td>
<td>TBL mean velocity profiles, and on-surface pressures</td>
</tr>
<tr>
<td>Yang and Wang [69]</td>
<td>Computational</td>
<td>-</td>
<td>0.036</td>
<td>95</td>
<td>Hemispheres</td>
<td>Complete incompressible flow field and far field noise</td>
</tr>
<tr>
<td>Yang and Wang [70]</td>
<td>Computational</td>
<td>$\approx 0.1$</td>
<td>0.12</td>
<td>168</td>
<td>Hemispheres and cuboids</td>
<td>Complete incompressible flow field and far field noise</td>
</tr>
<tr>
<td>Yang and Wang [71]</td>
<td>Computational</td>
<td>$\approx 0.1$</td>
<td>0.12</td>
<td>168</td>
<td>Hemispheres and cuboids</td>
<td>Complete incompressible flow field and far field noise</td>
</tr>
</tbody>
</table>

Table 2.4: Previous experimental and computational work.
3. Dominant noise sources are directly created by the roughness elements themselves, or by their interaction with incoming eddies. These noise sources include: vortex shedding from single roughness elements, unsteady field on a downstream element when the wake of an upstream element impinges on it, unsteadiness due to sharp edge separation (elements with sharp edges), flow interactions between side-by-side roughness elements, etc. Howe called ‘interstitial flow’ the flow structures directly affected or caused by the roughness elements. It is highly dependent on the roughness geometry. Since in the frequencies of interest and typical roughness sizes the roughness elements are compact, they radiate as dipoles with its axes parallel to the wall.

All the above mechanisms are present simultaneously and the total radiated noise is the sum of all of them. However the results in the literature show that some mechanisms might be dominant over the others, depending on the roughness geometry and the TBL characteristics. In these cases only the dominant ones are needed to estimate the radiated noise. Both Howe and Smol’yakov predicted roughness noise to be the sum of two different mechanisms from the ones listed above. Howe: mechanisms (1) and (2). Smol’yakov: mechanisms (1) and (3). To estimate the sound due to mechanism (1) they both assumed that the TBL structure is the same as if the surface was smooth, but the turbulent pressure fluctuations were appropriately increased according to the effects of roughness. Neither of them considered mechanism (3) with interaction between neighbouring elements. Glegg and Devenport have developed a semi-empirical model which is claimed to apply to all mechanisms. However, strictly it is limited to small roughness height and low acoustic frequencies.

Turbulence noise (mechanism (1)) is not going to be considered in this work since it is much lower than the scattering noise at low Mach numbers.

2.4 Roughness noise models

2.4.1 Howe’s model

The experimental measurements performed by Hersh showed that roughness noise scales approximately with \( U_o^6 \) (for dense distributed roughness), which indicated that
the dominant sound sources are not free turbulence sources, as in the case of a smooth surface. Howe [55, 56] proposed a sound generation mechanism based on the scattering of the turbulent pressure fluctuations present in a TBL on the roughness elements. The geometry and coordinates are shown in figure 2.6.

Under Lighthill’s acoustic analogy, Lighthill’s wave equation together with the wall boundary condition describe the acoustic emission. The rigid wall boundary condition is:

$$\frac{\partial p}{\partial x_n} = 0, \text{ on the rough surface,}$$

where $x_n$ is a local coordinate normal to the wall including the roughness elements. Howe solved the above system introducing a Green’s function that fulfills approximately both the differential equation and the boundary condition:

$$G(x, y, t, \tau) = \frac{\delta(t - \tau - |x - Y|/c)}{2\pi |x - Y|},$$

$$Y_2 = y_2$$

$$Y_\eta = y_\eta + \sum_m \frac{\mu_H h^2 (y_\eta - x_{m\eta})}{2 |y - x_m|^2}.$$

$x_m = (x_{m1}, 0, x_{m3})$ is the centre of the $m$th hemispherical element, and $Y_\eta$ is identical with the velocity potential describing an ideal incompressible flow in the $\eta$ direction over the rough wall. $\mu_H$ is a parameter introduced by Howe in the model to take partial
account of the interaction between elements. The value of $\mu_H$ depends on the roughness surface density, $\sigma$, and is approximately equal to $1/(1 + \sigma/4)$. Since both $p(x, t)$ and $G(x, y, t, \tau)$ both satisfy the boundary condition, the resultant acoustic field can be obtained from:

$$p(x, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x, y, t, \tau) \frac{\partial^2 (\rho_0 u_i u_j)}{\partial x_i \partial x_j} d^3 y d\tau,$$

and to determine the acoustic pressure field it is thus necessary to estimate the turbulent stresses through the TBL. The following derivation corresponds to Liu and Dowling [45], and is based on the one by Howe [55]. The Fourier transforms are defined:

$$\hat{f}(x_2, k, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, t) e^{i(\omega t - k \cdot x)} dx_1 dx_3 dt$$

$$k = (k_1, 0, k_3),$$

and

$$f(x, t) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(x_2, k, \omega) e^{i(k \cdot x - \omega t)} d^2 k d\omega.$$ 

The far field acoustic power emitted per unit area of rough surface is $\overline{pu_2^2}$, where $p$ and $u_2$ obey the linearised momentum equation:

$$\rho_0 \frac{\partial u_2}{\partial t} = - \frac{\partial p}{\partial x_2}.$$ 

This equation determines $u_2(x, t)$ in the far field since $p(x, t)$ is known from equation (2.19). The acoustic power spectrum is defined as the Fourier transform of the cross-correlation function:

$$\Pi(\omega) = \int_{-\infty}^{\infty} \frac{p(x, \tau) u_2(x, \tau + t)}{p(x, t)} e^{i\omega t} dt.$$ 

Performing the Fourier transform of (2.19) and using the above relation, the far field acoustic power spectrum yields:
\[ \Pi(\omega) = \frac{N \pi^2 h^6 \omega^2}{6 \rho_o c^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi(k)e^{-2|k|h} \int_{0}^{\infty} \int_{-\infty}^{\infty} S(y_2, y_2', k, \omega)e^{i(\gamma y_2 - \gamma^* y_2')} dy_2 dy_2' d^2k, \]  
\[ (2.24) \]

where the following identity has been used:
\[ \frac{e^{i k_o |x-Y|}}{|x-Y|} = \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{e^{i(k \cdot (x-Y) + \gamma(k)x_2)}}{\gamma(k)} d^2k. \]  
\[ (2.25) \]

\( \gamma(k) = (k_o^2 - |k|^2)^{1/2} \) and the branch cuts are the ones that make \( \text{sgn}(\gamma) = \text{sgn}(k_o) \) if \( \gamma \) is real, and \( \gamma \to +i |k| \) for \( |k| \to \infty \) on the real axis. The function \( S(y_2, y_2', k, \omega) \) is the cross-power spectral density of the turbulent stresses:
\[ S(y_2, y_2', k, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{Q}(y_2, k, \omega)\hat{Q}^*(y_2', k', \omega')d^2k'd\omega'. \]  
\[ (2.26) \]

\( \hat{Q}(y_2, k, \omega) \) is the Fourier transform of \( Q(y, t) \). The function \( \Psi(|k|) \), which depends on the number of roughness elements per unit surface, was estimated by Howe:
\[ \Psi(|k|) = \frac{[1 - \sigma J_1(2|k|h)/|k|h^3]}{[1 + \sigma J_1(2|k|h)/|k|h]}, \]  
\[ (2.27) \]

\( J_1 \) being the first order Bessel function. The power spectral density of the acoustic pressure, \( P_R(x, \omega) \), is defined by Liu and Dowling:
\[ \overline{p^2(x, t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_R(x, \omega)d\omega, \]  
\[ (2.28) \]

which corresponds to a double-sided PSD. Since in our experiments we use a one-sided PSD, the final expression of \( P_R(x, \omega) \) will be multiplied by 2. The observation angles are defined by the unit vector \( \hat{n} = x/|x| \). \( P_R(x, \omega) \) can be obtained:
\[ P_R(x, \omega) = \frac{NS \pi^2 h^6 k_o^2}{4 |x|^2} \int_{-\infty}^{\infty} \frac{(k \cdot \hat{n})^2 \Psi(|k|)e^{-2|k|h}}{|k|^2} \times \int_{0}^{\infty} \int_{0}^{\infty} S(y_2, y_2', k, \omega)e^{i(\gamma y_2 - \gamma^* y_2')} dy_2 dy_2' d^2k, \]  
\[ (2.29) \]
where the origin of $x$ is the centre of the surface $S$. A polar expression of the far field acoustic spectrum can be obtained if the angles $\theta, \phi$ are defined such that $\mathbf{n} = (\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$, with $0 \leq \theta \leq \pi$, $-\pi/2 \leq \phi \leq \pi/2$ (figure 2.7).

![Figure 2.7: Observation angles](image)

To determine the function $S(y_2, y_2', k, \omega)$, a smooth wall TBL is considered. On it, the wall pressure wavenumber-frequency spectrum is:

$$P_S(k, \omega) = \int_0^\infty \int_0^\infty S(y_2, y_2', k, \omega) e^{i(\gamma y_2 - \gamma^* y_2')} \frac{|\gamma(k)|^2}{|\gamma(k)|^2} dy_2 dy_2'. \quad (2.30)$$

$P_R(x, \omega)$ can then be expressed in terms of $P_S(k, \omega)$:

$$P_R(x, \omega) = \frac{NS\pi \mu^2 H h^6 \kappa_0^2}{4 |x|^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (k \cdot \mathbf{n})^2 P_S(k, \omega) |\gamma(k)|^2 \Psi(|k|) e^{-2|k|h} d^2k. \quad (2.31)$$

Following Graham [75] the wall pressure wavenumber-frequency spectrum can be normalised in the following way:
\[ P_S(k, \omega) = \frac{U_c^2 \Phi(\omega)}{\omega^2} \hat{\Phi}(k, \omega), \quad (2.32) \]

where \( \hat{\Phi}(k, \omega) \) is the non-dimensional wall pressure wavenumber-frequency spectrum.

The function \( \Phi(\omega) \) is the point pressure frequency spectrum, i.e. the spectrum of the pressure signal measured by a single microphone flush to the wall. Using the non-dimensional form \( \hat{\Phi}(k, \omega) \) the far field noise spectrum is:

\[ P_R(x, \omega) = \frac{NS\pi \mu^2_H h^6 U_c^2}{4|x|^2 e^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (k \cdot \hat{n})^2 \hat{\Phi}(k, \omega) \gamma(k)|\Psi(|k|)|e^{-2|k|h} |k| d^2 |k|. \quad (2.33) \]

There are various existing models in the literature for \( \hat{\Phi}(k, \omega) \). These models contain the information of the TBL state: thickness, skin friction coefficient and friction velocity. As was previously mentioned, the experimental results of Schultz and Flack \[48\] showed the same scaling of the Reynolds stresses with \( u_\tau \) in smooth and rough TBLs. The presence of roughness will therefore be accounted in this prediction model by the increased value of the friction velocity. At this point the integration over wavenumber space needs to be performed. Howe used an asymptotic approximation based on the fact that the function \( S(y_2, y'_2, k, \omega) \) is highly peaked in the vicinity of the ’convective ridge’, \( k_1 = \omega/c, k_3 = 0 \). He used Chase \[57\] model of \( P_S(k, \omega) \).

In the case of finite rough surfaces both the distance to the observer and the observation angle depend on the particular source point. Furthermore, the boundary layer thickness and friction velocity depend on the distance to the plate leading edge. To calculate the far field noise spectrum, an integration over the rough surface must be performed. Liu and Dowling \[45\] considered the simplest numerical integration scheme, which consists of dividing the surface in a number of small areas, and assuming the integrand to be constant in each of them: \( \iint_S f(x_1, x_3)dx_1dx_3 \approx \sum_{i,j} f(x_{i1}, x_{3j}) \Delta x_{i1} \Delta x_{3j} \).

### 2.4.1.1 Liu and Dowling’s method

Liu and Dowling \[45\] suggested that the asymptotic expansion Howe performed to evaluate the integral over wavenumber space may not be accurate enough, and developed a scheme to calculate the integral numerically.
A wavenumber polar representation is used via the transformation $k_1 = |k| \cos \eta$, $k_3 = |k| \sin \eta$, and therefore:

$$\mathbf{k} \cdot \hat{n} = |k| (\cos \eta \cos \theta + \sin \eta \sin \theta \sin \phi). \quad (2.34)$$

The far field noise spectrum can be expressed as:

$$P_R(x, \omega) = \frac{S \sigma \mu_H^2 h^4 U_c^2}{4|x|^2 \delta^* \phi(\omega) D(\theta, \phi)}, \quad (2.35)$$

where $\delta^*$ is the boundary layer displacement thickness, defined as $\delta^* = \int_0^\infty (1 - \pi / U_o) dx_2$, and the function $D(\theta, \phi)$ contains the directivity pattern of the emitted sound, which includes the integrals:

$$D(\theta, \phi) = Z_1(\omega) \cos^2 \theta + Z_2(\omega) \sin^2 \theta \sin^2 \phi + 2Z_3(\omega) \cos \theta \sin \theta \sin \phi, \quad (2.36)$$

$$Z_1(\omega) = \int_0^\infty \int_0^{2\pi} \Gamma \cos^2 \eta d(|k| \delta^*) d\eta, \quad (2.37)$$

$$Z_2(\omega) = \int_0^\infty \int_0^{2\pi} \Gamma \sin^2 \eta d(|k| \delta^*) d\eta, \quad (2.38)$$

$$Z_3(\omega) = \int_0^\infty \int_0^{2\pi} \Gamma \sin \eta \cos \eta d(|k| \delta^*) d\eta, \quad (2.39)$$

$$\Gamma = |\gamma(k)|^2 \delta^* \phi(k, \omega) \Psi(|k|) e^{-2|k|h} (|k| \delta^*). \quad (2.40)$$

The ($|k| \delta^*$)-integral upper limit must be specified. Liu and Dowling argue that the weighting factor $e^{-2|k|h}$ assures that $|k| \delta^* < 25\delta^*/h$ is large enough for practical purposes.

The point pressure spectrum $\Phi(\omega)$ used by Liu and Dowling is the one proposed by Ahn [77], which best fits the experimental data in the frequency range of interest:
\[ \Phi(\omega) = \frac{1}{4} \rho_0^2 U_o^3 \delta^* \left( \frac{2\pi 8.28(\omega \delta^* / U_o)^{0.8}}{1 + 4.1(\omega \delta^* / U_o)^{1.7} + 4.4 \cdot 10^{-4}(\omega \delta^* / U_o)^{5.9}} \right). \] (2.41)

Liu and Dowling tested the prediction model using various forms of the surface pressure wavenumber-frequency spectrum \( \hat{\Phi}(k, \omega) \) present in the literature, and observed very little differences between their corresponding noise spectra. This is why it has been decided to restrict to only one model for \( \hat{\Phi}(k, \omega) \). The function \( \hat{\Phi}(k, \omega) \) used in the present work corresponds to Corcos [78]:

\[ \hat{\Phi}(k, \omega) = \frac{4\alpha_1 \alpha_2}{\left[ \alpha_1^2 + (U_c k_1 / \omega - 1)^2 \right] \left[ \alpha_2^2 + (U_c k_3 / \omega)^2 \right]} \] (2.42)

The boundary layer scaling parameters \( \delta \) and \( u_\tau \), needed to calculate the far field noise, are determined using the equations (2.1), (2.14) and (2.15). In these equations, \( h_s \) is the equivalent roughness height, which depends on the particular roughness geometry and surface density. However, for this case \( h_s \) is of the order of the geometrical roughness height \( h \). Using HWA they measured the TBL thickness evolution corresponding to their roughness configurations, and using the geometrical height \( h \) as \( h_s \), they determined the value of \( \Pi_o \) that best fitted the predictions with the measurements.

### 2.4.1.2 Assumptions and limitations

Firstly, Howe’s model assumes noise sources associated with interstitial flow are much weaker than the ones present above the roughness elements. This was shown not to be true for sparse (\( \sigma \ll 1 \)) distributed roughness at low values of \( h/\delta \) [68, 70, 71]. Also, it is expected that, even for dense roughness, if \( h/\delta \) is not small enough or there is a transition smooth-to-rough, interstitial flow may a priori be as important as the turbulent flow present above the roughness elements.

Secondly, both Howe and Liu and Dowling used a smooth wall model of the wall pressure wavenumber-frequency spectrum \( \hat{\Phi}(k, \omega) \) and the surface point pressure spectrum \( \Phi(\omega) \), extended to rough walls through the scaling variables \( u_\tau, \delta (\delta^*) \). The use of a smooth wall model is justified by Townsend’s similarity hypothesis [47], which has received recent experimental support for walls with dense roughness [48, 49]. Additionally, Howe assumed that the integral over the wavenumber space contained only the contribution
from the vicinity of the ‘convective ridge’ $k_1 = \omega/U_c$, $k_3 = 0$, where $\Phi(k, \omega)$ peaks. Liu and Dowling’s method performs the integral numerically over all wavenumber space.

Finally, for the assumption of compactness of the roughness elements to be correct, the wavelength of the emitted sound must be much larger than the TBL thickness (and the roughness height). This imposes a limitation on the valid frequency range of the predictions.

Basically, this model describes roughness noise generation mechanism 2.

### 2.4.1.3 Strength of the dipole sources

Howe’s model describes the roughness noise sources exactly as surface dipoles. Liu and Dowling used this fact to calculate exactly the dipole strengths for an arbitrary given state of the TBL on a flat wall. The far field radiated by a small rough surface element of area $S$ is given by equation (2.35). The noise radiated by a finite rough area is obtained by surface integration. The radiated sound by an individual roughness element is comprised of a streamwise dipole and a transverse dipole, of strengths determined by the local TBL properties. The acoustic field radiated by an individual dipole, composed of a pair of monopoles out of phase separated by a distance $l_{dip}$ much smaller than the wavelength of the waves they emit, is:

$$p_{dip}(x, \omega) = p_1(x, \omega) + p_2(x, \omega) = \frac{a(\omega)}{4\pi r_1} e^{ik_o r_1} - \frac{a(\omega)}{4\pi r_2} e^{ik_o r_2} \approx \frac{a(\omega)}{4\pi r} e^{ik_o r} \Phi_{dip} \cos \theta,$$

(2.43)

The PSD is easily obtained, since the function multiplying the source strength is deterministic:

$$P_{dip}(x, \omega) = \Lambda(\omega) \left| \frac{ik_o r \cos \theta}{2\pi r} e^{ik_o r} \right|^2 = \frac{\Lambda(\omega) k_o^2 r_{dip}^2}{4\pi^2 r^2} \cos^2 \theta.$$

(2.44)

$\Lambda(\omega)$ is the PSD of the source strength. The above expression for a number $N$ of dipoles, together with equation (2.35) determine the strength of the streamwise dipoles $\Lambda(\omega)$:

$$\frac{N\Lambda(\omega) k_o^2 r_{dip}^2}{4\pi^2 r^2} \cos^2 \theta = \frac{S \sigma_H^2}{4 |x|^2} \frac{h^4}{\delta^4} \frac{U_c^2}{c^2} \Phi(\omega) Z_1(\omega) \cos^2 \theta,$$

(2.45)
\[ \Lambda(\omega) = \pi^3 \mu_H^2 \left( \frac{h}{l_{dip}} \right)^2 \left( \frac{h}{\delta H} \right)^4 \left( \frac{U_o}{\omega} \right)^2 \Phi(\omega) Z_1(\omega). \] (2.46)

The relation \( \sigma = N \pi h^2 / S \) has been used. An equivalent derivation leads to the strength of the transverse dipoles. However, for large roughness Reynolds number \( (h^+ > 1000) \) the contribution of the transverse dipoles in the far field is negligible compared with the streamwise dipoles [15]. This was also verified in the present experiments on rough flat walls (chapter 5), for the same roughness configurations considered here. Therefore, only the streamwise dipoles are considered.

Equation (2.46) has been obtained from the flat wall roughness noise expression (2.35), but it is valid for any TBL. The dipole strength \( \Lambda(\omega) \) depends on the TBL skin friction \( c_f(\theta) \), thickness \( \delta(\theta) \), and the ratio of convection velocity to outer velocity \( U_c(\theta)/U_o(\theta) \).

### 2.4.2 Smol’yakov’s model

Another model used to describe roughness noise is the one developed by Smol’yakov [59], which assumes that each roughness element acts as an individual, acoustically compact bluff body, protruding into the inertial region of the TBL. Each roughness element emits noise like a bluff body in free stream, which is tonal at a frequency \( f = St_r U_r/d_r \), with \( St_r \approx 0.2 \) in the range \( 4 \times 10^2 < Re < 4 \times 10^5 \). \( U_r \) is an effective velocity impinging on the element and \( d_r \) is the element diameter. The dipole acoustic power emitted by the roughness element scales like:

\[ W_R \propto \rho_o U_r^{16} e^{-3} h_r d_r. \] (2.47)

According to Schlichting and Gersten [79], \( U_r \) is approximately equal to \( 8.5 u_r \). Assuming also that the roughness elements width and height are proportional to each other \( h_r \propto d_r \), and that the number of roughness elements per unit surface, \( N \), is inversely proportional to the elements cross section \( d_r^2 \):

\[ N \propto \frac{1}{d_r^2} \propto \frac{1}{h_r^2}. \] (2.48)

The emitted sound power per unit surface is then:
where \( k_r \) is a proportionality constant, \( I_r \) is the acoustic intensity in the direction normal to the surface, and the relation \( u_r = U_o \sqrt{\epsilon_f/2} \) has been used. \( k_r \) accounts for the roughness surface density \( \sigma \), which does not appear explicitly in the model.

It is assumed that the noise is radiated perpendicularly to the rough surface. The sound intensity perceived by an observer located above the surface, at a distance \( |x| \) from it, is calculated simply by dividing \( I_r \) over \( |x|^2 \).

The flow velocity impinging on the roughness elements, \( U_r \), is treated statistically as a normal distribution with mean equal to 8.5\( u_r \) and variance equal to \( u_r^2 \). Its PDF is:

\[
w_u(U_r) = \frac{1}{u_r \sqrt{2\pi}} e^{-\frac{(U_r-8.5 u_r)^2}{2u_r^2}}.
\]  

Roughness height is also assumed to follow a statistical distribution (\( \chi^2 \)-distribution):

\[
w_h(h_r) = \frac{6}{h_r \Gamma(6)} \left( \frac{6h}{h_r} \right)^5 e^{-\frac{6h}{h_r}},
\]  

where \( \Gamma(x) \) is the Gamma function, \( h_r \) is the roughness height of each roughness element, and \( h = \int_0^\infty h_r w_h(h_r) dh_r \) is the mean roughness height. A two-dimensional PDF of the independent variables \( U_r \) and \( h_r \) is obtained multiplying the single PDFs:

\[
w_{hu}(U_r, h_r) = w_u(U_r)w_h(h_r).
\]

Smol’yakov argues that roughness noise is Strouhal-based, but with an unknown constant value of the Strouhal number:

\[
St_r = \frac{\omega h_r}{2\pi U_r} = C.
\]

So the frequency of the emitted sound can be described by a random variable from the equation \( \omega = 2\pi St_r U_r / h_r \). The PDF of the frequency can be calculated:
\[
\omega_{\omega}(\omega) = \int_{0}^{\infty} w_{hu}(U_{r}, h_{r}) \left( \omega h_{r} \right) \frac{dh_{r}}{2\pi St_{r}} \left( \frac{\nu_{r}}{\pi} \right) = \frac{1}{2\pi St_{r}} \int_{0}^{\infty} h_{r} w_{hu}(\omega h_{r}, h_{r}) dh_{r}. \tag{2.54}
\]

Using the expressions of \( w_{u}(U_{r}) \) and \( w_{h}(h_{r}) \), the above equation leads to:

\[
w_{\omega}(\omega) = \frac{1.604 \times 10^{-19}}{2\pi St_{r}} \frac{h}{U_{o} \sqrt{c_{f}}} \Phi_{S}\left( \frac{\omega h_{r}}{12\pi U_{o} \sqrt{c_{f}} S t_{r}} \right), \tag{2.55}
\]

where the function \( \Phi_{S} \) is defined by:

\[
\Phi_{S}(\xi) = \int_{0}^{\infty} z^{6} e^{-\xi^{2}z^{2}+(8.5\sqrt{2}\xi-1)z} dz, \tag{2.56}
\]

and \( z = 6h_{r}/h \). The function \( w_{\omega}(\omega) \) fulfills the normalisation condition for PDFs: \( \int_{0}^{\infty} w_{\omega}(\omega)d\omega = 1 \). Smol'yakov argued that the frequency PDF must be proportional to the spectrum \( P_{R}(x, \omega) \), and determined their relationship using the reasoning that follows. The far field noise intensity is:

\[
I_{r} = \frac{W_{R}}{S} = k_{r} \rho_{o} c_{f}^{3} U_{o}^{5} M^{3}. \tag{2.57}
\]

It is related to the RMS sound pressure:

\[
\overline{p^{2}}(x, t) = \frac{1}{2\pi} \int_{0}^{\infty} P_{R}(x, \omega)d\omega = I_{r} \rho_{o} c \frac{S}{|x|^{2}} = k_{r} \rho_{o}^{2} c_{f}^{3} U_{o}^{4} M^{2} \frac{S}{|x|^{2}}, \tag{2.58}
\]

which is equivalent to:

\[
\int_{0}^{\infty} \frac{1}{2\pi k_{r} \rho_{o}^{2} c_{f}^{3} U_{o}^{4} M^{2}} \frac{P_{R}(x, \omega)}{S} |x|^{2} d\omega = 1. \tag{2.59}
\]

Comparing this equation with the normalisation condition of \( w_{\omega}(\omega) \) it yields:

\[
P_{R}(x, \omega) = K_{R} \frac{S}{|x|^{2}} c_{f}^{5/2} h_{r} \rho_{o}^{2} U_{o}^{2} M^{2} \Phi_{S}\left( \frac{\omega h_{r}}{U_{o} \sqrt{c_{f}} 12\pi S t_{r}} \right). \tag{2.60}
\]
$K_R$ is a proportionality constant which, together with $St$, must be determined by comparison with experimental results. In the experiments performed by Smol’yakov he determined $K_R = 1.59 \times 10^{-19}$ and $St_r = 0.61$, for roughness Reynolds numbers in the range $80 < h^+ < 160$. It must be remarked that the measurements consisted of single point wall pressure, using a microphone flush to the wall, and not of the acoustic far field.

2.4.2.1 Assumptions and limitations

Firstly, this model describes roughness elements as dipole sources emitting noise at a single Strouhal number. According to Smol’yakov, roughness elements can be interpreted as a set of cylindrical bodies emitting noise due to vortex shedding, which is approximately tonal noise at $St_r$. It is known that other bluff bodies have vortex shedding at different Strouhal numbers, and also that $St_r$ changes with Reynolds number.

Secondly, noise directivity is not described by the model. The derivation assumes normal radiation from the wall. This is a strong limitation if noise radiation is significantly directional.

Finally, the model contains two empirical parameters which a priori need to be fixed by comparison with experiments, and limit the predictive capabilities.

This model aims at accounting for roughness noise generation mechanism 3., with no interactions between neighbouring roughness elements, and without accounting for source directivity.

2.5 Limitations in the application of existing roughness noise models to bluff bodies

To be able to apply the above models to bluff bodies it must be noticed that:

- Roughness noise is scattered on the bluff bodies, therefore modifying the original directivity.
• Roughness elements sufficiently large compared with the TBL thickness \( \frac{h}{\delta} \lesssim 1 \) may significantly affect the structure of the TBL, making the smooth wall wavenumber-frequency spectrum models inappropriate.

• Large components cannot be modelled as flat surfaces since there are strong pressure gradients along them. A first approximation for single large components could be to obtain the exterior flow velocity \( U_o(x_1) \) from a potential flow solution and determine the TBL state using flat wall models.

• The flow separates at the back of the large components.

• In the case of multiple bluff bodies, some of them may be immersed in the wake/s of other components.

2.6 BEM

BEM is, in general, a method used to solve linear partial differential equations with certain boundary conditions, using Green’s identities to transform the volume integrals to surface boundary integrals \[42\]. The derivation of the BEM integral equation is going to be performed for the specific case of the Helmholtz equation with wall boundary conditions.

Once the noise is generated by the roughness noise mechanism, it is propagated to the far field. However the sound waves are scattered on the bluff body changing the far field sound spectra in possibly all directions. There are several possible ways to calculate the scattered field around a body. If the on-surface pressure over all the surface was known we could use Helmholtz equation or a FW-H expansion to determine the acoustic far field. However in this case what is known is information of the sources: its location and strength. In addition the sources are not defined as volume distributions, but as point sources. Therefore all we need to calculate the far field propagation is the system Green’s function corresponding to the right boundary conditions, i.e. normal pressure gradient on the body wall. The Green’s function tailored to the bluff body is the response of the system to a point monopole in the presence of the body. Our source distribution is equivalent to a set of incoherent dipoles. But a dipole is equivalent to a pair of monopoles of the same strength and out of phase, close to each other. Therefore the tailored Green’s function can be used to calculate the response to our sources.
2.6.1 Tailored Green’s functions

Here tailored Green’s functions are introduced and linked to Curle’s acoustic analogy. For convenience the derivation is performed in the frequency domain. Lighthill’s wave equation for the pressure field \( p(x,t) \) is transformed to the inhomogeneous Helmholtz equation by taking it’s (time) Fourier transform:

\[
(\nabla^2 + k^2_o) \hat{p}(x,\omega) = -\frac{\partial^2 \hat{T}_{ij}}{\partial x_i \partial x_j} = \hat{Q}(x,\omega),
\]  

(2.61)

where \( k_o = \omega/c \) is the acoustic wavenumber, and \( \hat{p} \) is the Fourier transform:

\[
\hat{p}(x,\omega) = \int_{-\infty}^{\infty} p(x,t) e^{-i\omega t} dt.
\]  

(2.62)

Now consider the same volume \( V \) and function \( F(x) \) used previously, which is negative outside \( V \), zero on \( \Sigma \) and positive inside \( V \). Multiplying the Heaviside function \( H(F) \) by the above equation and rearranging terms it yields the Helmholtz equation for \( H\hat{p}: \)

\[
(\nabla^2 + k^2_o) H\hat{p}(x,\omega) = H\hat{Q} + \nabla H \cdot \nabla \hat{p} + \nabla \cdot (\hat{p} \nabla H).
\]  

(2.63)

The solution of this equation using an arbitrary Green’s function \( \hat{G}(x,y,\omega) \) such that

\[
(\nabla^2 + k^2_o)\hat{G}(x,y,\omega) = \delta(x - y)
\]  

(2.64)

is:

\[
H\hat{p}(x,\omega) = \int \int_{-\infty}^{\infty} H\hat{Q}(y,\omega)\hat{G}(x,y,\omega)d^3y \\
+ \int \int_{-\infty}^{\infty} \hat{G}(x,y,\omega)(\nabla H \cdot \nabla \hat{p} + \nabla \cdot (\hat{p} \nabla H))d^3y.
\]  

(2.65)

The first integral reduces to the volume integral outside of the body (\( F > 0 \) in \( V \)), and the second integral reduces to a surface integral on \( \Sigma \) after some manipulations:
\[
H\hat{p}(x, \omega) = \int \int_V \hat{Q}(y, \omega) \hat{G}(x, y, \omega) d^3y + \int \int_\Sigma \left( \hat{G} \frac{\partial \hat{p}}{\partial y_i} - \hat{p} \frac{\partial \hat{G}}{\partial y_i} \right) n_i d^2y. \tag{2.66}
\]

Curle’s equation for the pressure field is obtained using the free field Green’s function, \(\hat{G}_o\), in the above expression:

\[
H\hat{p}(x, \omega) = \int \int V \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \hat{G}_o d^3y + \int \int_\Sigma \left( \hat{G}_o \frac{\partial \hat{p}}{\partial y_i} \hat{G}_o \frac{\partial \hat{G}_o}{\partial y_i} \right) n_i d^2y, \tag{2.67}
\]

and after some manipulations of the integrals and using the definition of \(T_{ij}\) it reaches:

\[
H\hat{p}(x, \omega) = \int \int V T_{ij} \frac{\partial^2 \hat{G}_o}{\partial y_i \partial y_j} d^3y + \int \int_\Sigma \hat{G}_o \frac{\partial (\hat{p} \hat{u}_i \hat{u}_j + \hat{p} \delta_{ij} - \hat{\sigma}_{ij})}{\partial y_j} n_i d^2y - \int \int_\Sigma (\hat{p} \hat{u}_i \hat{u}_j + \hat{p} \delta_{ij} - \hat{\sigma}_{ij}) \frac{\partial \hat{G}_o}{\partial y_j} n_i d^2y. \tag{2.68}
\]

When \(\Sigma\) is taken as the rigid body surface (\(u_i = 0\) and \(\partial \hat{p}/\partial n = 0\) on \(\Sigma\)), and if the viscous stresses are neglected (\(\hat{\sigma}_{ij} = 0\) on \(\Sigma\), for high Reynolds number flow [18]) the above equation becomes:

\[
H\hat{p}(x, \omega) = -\int \int V T_{ij} \frac{\partial^2 \hat{G}_o}{\partial y_i \partial y_j} d^3y - \int \int_\Sigma \hat{p} \frac{\partial \hat{G}_o}{\partial y_i} n_i d^2y, \tag{2.69}
\]

which is Curle’s integral solution for the pressure in the frequency domain. It is an expression for the pressure field in a quiescent medium in the presence of a rigid body. The extra term on the right side of the equation appears due to the presence of the body. It is directly associated with the unsteady forces of the wall on the fluid and has the form of a surface distribution of dipoles.

The presence of the body can be accounted for using a Green’s function tailored to the body surface instead of the free field Green’s function. A tailored Green’s function \(\hat{G}_t\) satisfies both the wave equation and the wall boundary condition:

\[
\frac{\partial \hat{G}_t}{\partial x_i} n_i = 0, \tag{2.70}
\]

and equation (2.69) becomes
$H\tilde{\rho}(\mathbf{x}, \omega) = -\iiint_V \hat{T}_{ij} \frac{\partial^2 \hat{G}_t}{\partial y_i \partial y_j} d^3y. \quad (2.71)$

Now let $\hat{G}_t = \hat{G}_o + \hat{G}_s$, where $\hat{G}_s$ is the scattering Green’s function. Introducing this expression in equation (1.13) it yields:

$$\iiint_{\Sigma} \hat{p} \frac{\partial \hat{G}_o}{\partial y_i} n_i d^2y = -\iiint_V \hat{T}_{ij} \frac{\partial^2 \hat{G}_s}{\partial y_i \partial y_j} d^3y. \quad (2.72)$$

The surface sources are equivalent to the volume sources associated with $\hat{G}_s$, the scattering Green’s function. The surface sources are not real sources of sound; the real sources of sound are within the flow \[31, 88\]. The acoustic field generated by the turbulence sources is diffracted/scattered by the bodies present in the flow. To increase the noise efficiency from quadrupole to dipole, the bodies must be acoustically compact. This mechanism has associated a more efficient conversion of hydrodynamic energy into acoustic energy than noise generation in absence of compact bodies. A method to calculate tailored Green’s functions of various (compact) bodies can be found in Howe \[18\].

Taking the noise generation mechanism into account, it is useful to define the high frequency range as the frequencies at which the large bodies within the flow are not compact. In this range the enhancement associated with the scattering mechanism will only happen at the roughness elements. For lower frequencies the scattering will also affect the large elements, and since the scattered field intensity increases with the element size, the scattering on the large elements will dominate the acoustic field. What makes the analysis of the effects of roughness very complicated is the fact that roughness elements can have a very significant effect on the turbulence structure around the large bodies, at all frequencies and length scales. For example, it has been shown that cable attached helicoidally on a circular cylinder can suppress the vortex shedding from it, thus diminishing drastically the turbulent energy associated with the large scales \[21, 22, 23\].
2.6.2 BEM integral equation

The conventional BEM method has been used to determine the Green’s function tailored to the bluff body, a smooth circular cylinder in this case. The method is based on solving numerically an integral equation, to determine the Green’s function in a particular observer location and a particular frequency, with sources located on the body surface.

The integral equation to be solved is now derived. The free field Green’s function $G_o$ of the Helmholtz equation is defined by:

$$-\frac{\omega^2}{c^2} G_o(x, y, \omega) - \nabla^2 G_o(x, y, \omega) = \delta(x - y), \tag{2.73}$$

with the Sommerfeld condition at infinity. The tailored Green’s function $G_t$ obeys the same equation:

$$-\frac{\omega^2}{c^2} G_t(x, y, \omega) - \nabla^2 G_t(x, y, \omega) = \delta(x - y), \tag{2.74}$$

but with the solid wall boundary condition:

$$\frac{\partial G_t(x, y, \omega)}{\partial y_i} n_i = 0, \text{ for } x \in \Sigma, \tag{2.75}$$

where $\Sigma$ is the body surface, and the equations hold in the volume $V$ outside $\Sigma$. Green’s theorem using $G_o$ and $G_t$ states:

$$\int\int\int_V \left( G_o(z, y, \omega) \nabla^2 G_t(x, z, \omega) - G_t(x, z, \omega) \nabla^2 G_o(z, y, \omega) \right) d^3z = -\int\int_{\Sigma} \left( G_o(z, y, \omega) \frac{\partial G_t(x, z, \omega)}{\partial z_i} - G_t(x, z, \omega) \frac{\partial G_o(z, y, \omega)}{\partial z_i} \right) n_i d^2z. \tag{2.76}$$

Using the previous relations the following equation is reached:

$$G_t(x, y, \omega) = G_o(x, y, \omega) + \int\int_{\Sigma} G_t(x, z, \omega) \frac{\partial G_o(z, y, \omega)}{\partial z_i} n_i d^2z, \tag{2.77}$$

which determines the Green’s function for an off-surface source located in $y$, given the free field Green’s function and the tailored Green’s function with on-surface sources. The
above equation, when the source located at $y$ is brought arbitrarily close to a surface point $z_p$, becomes:

$$G_t(x, z_p, \omega) \left(1 - \frac{\Omega(z_p)}{4\pi}\right) = G_o(x, z_p, \omega) + \iint_{\Sigma} G_t(x, z, \omega) \frac{\partial G_o(z, z_p, \omega)}{\partial z_i} n_i d^2z. \quad (2.78)$$

$\Omega(z_p)$ is the inner solid angle at $z_p$, which for a smooth surface is equal to $2\pi$. This is an integral equation whose only unknown is $G_t(x, z_p, \omega)$, and has to be solved numerically. Conventional BEM consists of solving the above equation using a discretisation of the surface integral and converting it to a system of linear equations, with as many equations as surface elements.
Chapter 3

Experimental Methods

In this chapter the facilities and experimental methods used will be presented.

3.1 Facilities

The aerodynamic tests and the near field acoustic tests have been performed in the 0.9 m × 0.6 m wind tunnel, and the far field acoustic measurements have been carried out in the anechoic chamber, at the University of Southampton.

3.1.1 0.9 m × 0.6 m wind tunnel

Phased microphone array measurements, PIV and Pitot tube measurements were conducted in a low speed wind tunnel at the University of Southampton. The tunnel is an open-loop, closed-section configuration, with a rectangular test section of dimensions 600 mm (height) × 900 mm (width). The maximum speed is 27 m/s (measured at the test section entrance). The incoming turbulence intensity is approximately 0.2% [97].

The arrangement of the circular cylinder in the closed-section wind tunnel is shown in figure [3.1a]. The cylinder is fixed vertically to two horizontal endplates with a streamlined leading edge. The endplates are fixed to the tunnel side walls with L-brackets, and there is a distance of approximately 5 cm between the endplates and the top and bottom tunnel walls. This way the endplates and the cylinder extents are outside of the BL’s growing on the tunnel walls. The arrangement corresponding to the flat wall tests are
shown in figure 3.1b. The plate span is equal to the width of the wind tunnel, and is fixed approximately 10 cm above the ground using two vertical streamlined struts. It has a streamlined leading edge, and includes a surface pressure taps system together with a trailing edge flap. The flap angle was fixed such that the streamwise pressure gradient was minimum.

3.1.2 Anechoic chamber

Far field acoustic measurements and on-surface pressure measurements were performed in an anechoic chamber equipped with an open jet facility at the University of Southampton (see figure 3.4). The chamber walls are covered with glass-fiber cored wedges, which extend 900 mm from the walls. The (empty) chamber provides free field conditions for frequencies larger than 80 Hz. To perform aeroacoustic tests, the nozzle of an open-jet wind tunnel can be located inside the chamber, the jet direction being horizontal. The cross-section of the nozzle is rectangular and has dimensions of 500 mm (height) × 350 mm (width). The maximum speed of the facility is 40 m/s, and the incoming turbulence level is 0.2% outside of the shear layers [96].

There are two sources of background noise in the chamber equipped with the open-jet. First of all there is a certain amount of noise that propagates from the fan that runs the air through the tunnel, and is mainly concentrated at low frequencies. Second of all, due to the need of removing the air supplied by the nozzle inside the chamber, a side door connecting to a non-anechoic room had to be open, allowing a certain amount of sound to reflect and enter the anechoic chamber again.

The arrangement of the circular cylinder in the anechoich chamber is shown in figure 3.4a. The cylinder is normal to the picture, fixed to two vertical endplates, which are themselves rigidly attached to the nozzle. The far field microphones are fixed to a vertical metallic arc, and located in the vertical midspan plane. Figure 3.4b shows the arrangement in the flat wall tests. A vertical flat plate is located at the nozzle midspan plane, with its streamlined leading edge close to the nozzle exit section. The plate is fixed rigidly to a metallic structure, which is fixed to the ground. The far field microphones were located in the model midspan plane (in this case it is horizontal). The location of the microphones are shown in figure 3.5.
3.2 Measurement techniques

The tests performed until now have consisted exclusively in measuring pressure signals using microphones. Therefore the properties of the microphones used will be described. To set the inflow velocity in the tunnel, a Pitot tube was located upstream of the model section.

3.2.1 Pitot tube and micromanometer

A Pitot tube was fixed upstream of the model to measure the incoming velocity, and set it to the desired value. It was positioned outside of the boundary layer developing on the tunnel walls. The total and static pressure inlets were connected to a digital Micromanometer FCO12, with a resolution of 0.1 mmH₂O. The sampling frequency was 100 Hz, and the measurements were averaged over 5 s to obtain the mean pressure and velocity. The data was acquired using an A/D converter and read by LabView software. The micromanometer consists of a pressure transducer with a linear relationship of electric voltage and pressure. Calibration provides the proportionality constant needed thereafter to determine the measured pressure. The micromanometer was calibrated once at the beginning of each set of experiments. In the case of the open-jet wind tunnel at the anechoic chamber, the Pitot inlet was located at the nozzle exit section, and the flow velocity was fixed in the presence of the smooth cylinder and the smooth flat plate.

The flow speed was determined from the equation:

\[ \Delta p = \frac{1}{2} \rho U_o^2, \]  

(3.1)

where \( \Delta p \) is the dynamic pressure (equal to the total pressure minus the static pressure). Using this equation, the uncertainty in the calculation of \( U_o \) is estimated from the contributions of the uncertainty in \( \Delta p \) and \( \rho \). The uncertainty in \( \Delta p \) is 0.1 mmH₂O, and the uncertainty in \( \rho \) is 0.005 Kg/m³ (calculated from the uncertainty in \( p_o \) and \( T_o \), using the perfect gas law). Finally, the uncertainty in \( U_o \) obtained is 0.04 m/s, at \( U_o = 20 \) m/s.
3.2.2 Far field microphones

The sound in the far field was measured using a set of free-field microphones fixed to a metallic arc. The microphones were Behringer ECM8000, omni-directional electret condenser microphones, with a frequency range of 15 Hz - 20 kHz. The signals were amplified using a DIGIMAX FS by Presonus, and acquired using National Instruments PXI-4472 data acquisition cards. The calibration of the microphones was performed by comparison with a 1/2” B&K standard microphone, using a white noise signal and taking the average value of the plateau present in the ratio of both spectra.

The power spectral density of the pressure signals was calculated using an averaged periodogram:

$$\text{PSD}(f_i) = \frac{2|\text{FFT}(p(t))|^2}{n \cdot F_s} \quad \text{for} \quad i = 1, \ldots, n. \quad (3.2)$$

where $p(t)$ is the measured pressure signal, $n = 8192$ is the number of samples per block, and $F_s = 48$ kHz is the sampling frequency. The frequency resolution is therefore $\Delta f = 5.9$ Hz, and the spectra were been averaged over 120 blocks. No windowing was applied in the calculation of the FFT. The spectra were plotted using narrow bands:

$$\text{SPL}(f_i) = 10\log_{10} \frac{\text{PSD}(f_i)}{p_{ref}^2}, \quad (3.3)$$

or 1/3-octave bands (each band centred at $f_b$):

$$\text{SPL}_{1/3}(f_b) = 10\log_{10} \frac{\sum_{i \in b} \text{PSD}(f_i) \Delta f_i}{p_{ref}^2} = 10\log_{10} \frac{\sum_{i \in b} \text{PSD}(f_i) \Delta f}{p_{ref}^2} \quad \text{for} \quad b = 1, \ldots, N_b, \quad (3.4)$$

where $p_{ref} = 2 \times 10^{-5}$ Pa. The overall levels (OASPL) were obtained by integrating the power spectral density within the desired frequency range:

$$\overline{p^2}_{f_1,f_2} = \int_{f_1}^{f_2} \text{PSD}(f) df \approx \sum_i \text{PSD}(f_i) \Delta f_i, \quad (3.5)$$

$$\text{OASPL}_{f_1,f_2} = 10\log_{10} \frac{\overline{p^2}_{f_1,f_2}}{p_{ref}^2}. \quad (3.6)$$
The velocity scaling was estimated in two ways. Firstly, the OASPL level at each free stream velocity was calculated from the PSD narrowband spectra, by integration in the desired frequency range. Then the scaling exponents associated with every pair of free stream velocities were determined:

\[ n = \frac{\text{OASPL}_2 - \text{OASPL}_1}{10 \log_{10}(U_o^2/U_o^1)}. \]  

(3.7)

Secondly, the 1/3-octave band spectra were normalised with various powers of the free stream velocity and the collapse between them was analysed at the desired frequency ranges. Assuming no error in the ratio of velocities \( (U_o^2 = 40 \text{ m/s and } U_o^1 = 27 \text{ m/s}, \) since the measurements of Smooth at the lowest velocity were not considered due to low signal-to-noise ratio), the error is \( \delta n \approx 0.6 \delta (\Delta \text{OASPL}). \) From long term repeatability tests, the errors of \( \delta (\Delta \text{OASPL}) \) were estimated in the low, medium and high frequency ranges, by adding the error at the two velocities. The errors obtained are \( \delta n \approx 0.4, \) \( \delta n \approx 0.7 \) and \( \delta n \approx 0.4, \) for the low, medium and high frequency ranges, respectively (low frequencies belong to the range \( 0.18 < St < 0.4, \) medium frequencies to \( 0.4 < St < 6, \) and high frequencies to \( 6 < St < 50). \)

The measurements were corrected for shear layer refraction and convective amplification. The shear layer correction was done using Amiet’s method [98] and assuming a zero-thickness shear layer. The method provides both the corrected pressure values and the corrected observer angles for the case in which there is no shear layer, i.e. in-flight conditions. Afterwards the convective amplification correction was applied:

\[ p(U_o = 0) = p(U_o) \left( 1 - \frac{U_o}{c} \cos(\pi - \theta) \right)^2, \]  

(3.8)

where \( p(U_o) \) is the measured pressure, and \( p(U_o = 0) \) is the pressure in absence of convective amplification. The above corrections are frequency-independent. Since the maximum Mach number in the tests was low \( (M = 0.12) \) the corrections are small.

The far field sound level was measured in the anechoic facility using a set of microphones fixed to a metallic arc. They were located in the positions listed in Table 3.1.
3.2.3 On-surface microphones

On-surface pressure signals were measured using microphones flush mounted with the wall (figure 3.6). A microphone consists of a pressure transducer which, as the micromanometer, is based on a linear relationship between the received pressure and the emitted electrical voltage. The on-surface microphones are ECM-10B type, electret condenser microphones. They have an approximately omni-directional response, a frequency range of 60 Hz - 13 kHz and a sensitivity of $-60 \pm 3$ dB. Their dimensions are a diameter of 6 mm and a thickness of 5.2 mm. The signal was amplified using an in-house built amplifier with adjustable gain, and it was acquired using National Instruments PXI-4472 data acquisition cards. The calibration of the microphones was performed by comparison with a 1/2” B&K standard microphone, in the same way as the far field microphones.

<table>
<thead>
<tr>
<th>Circular cylinder</th>
<th>$r$(m)</th>
<th>$\theta$(°)</th>
<th>Flat plate</th>
<th>$r$(m)</th>
<th>$\theta$(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1.69</td>
<td>86</td>
<td>M1</td>
<td>1.47</td>
<td>131</td>
</tr>
<tr>
<td>M2</td>
<td>2.13</td>
<td>67</td>
<td>M2</td>
<td>1.27</td>
<td>121</td>
</tr>
<tr>
<td>M3</td>
<td>1.66</td>
<td>45</td>
<td>M3</td>
<td>1.00</td>
<td>90</td>
</tr>
<tr>
<td>M4</td>
<td>1.85</td>
<td>39</td>
<td>M4</td>
<td>1.36</td>
<td>70</td>
</tr>
<tr>
<td>M5</td>
<td>2.22</td>
<td>28</td>
<td>M5</td>
<td>1.40</td>
<td>62</td>
</tr>
<tr>
<td>M6</td>
<td></td>
<td></td>
<td>M6</td>
<td>1.71</td>
<td>48</td>
</tr>
<tr>
<td>M7</td>
<td></td>
<td></td>
<td>M7</td>
<td>2.54</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 3.1: Location of the far field microphones for circular cylinder and flat plate tests.

<table>
<thead>
<tr>
<th>Circular cylinder</th>
<th>$\theta$(°)</th>
<th>$z/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>20</td>
<td>2.0</td>
</tr>
<tr>
<td>M2</td>
<td>60</td>
<td>2.0</td>
</tr>
<tr>
<td>M3</td>
<td>120</td>
<td>2.0</td>
</tr>
<tr>
<td>M4</td>
<td>30</td>
<td>0.4</td>
</tr>
<tr>
<td>M5</td>
<td>30</td>
<td>0.8</td>
</tr>
<tr>
<td>M6</td>
<td>30</td>
<td>1.2</td>
</tr>
<tr>
<td>M7</td>
<td>30</td>
<td>2.0</td>
</tr>
<tr>
<td>M8</td>
<td>30</td>
<td>2.4</td>
</tr>
<tr>
<td>M9</td>
<td>30</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table 3.2: Location of the on-surface microphones ($z$ measured from one of the end-plates).

The microphone surface was covered with a layer of foam, approximately 1 mm thick. The goal was to partly eliminate the hydrodynamic component of the pressure signal,
since it is much stronger than the acoustic one (the acoustic field fulfills \( k_o = \omega/c \), and the rest corresponds to the hydrodynamic flow field). It was also intended to prevent air going through the holes.

### 3.2.4 Microphone phased array

The microphones that formed the microphone phased array are of the same type as the on-surface ones described above, as well as the amplifier and acquisition system. It is composed of 49 microphones fixed on a square plate of dimensions 0.2 m × 0.2 m. They are located in the intersection of a multi-arm logarithmic spiral with a set of concentric circles. The array plate is mounted on the tunnel wall. All microphones used a sampling frequency of 48 kHz, a block size of 4096 was used to perform the FFTs of the signals and the spectra were averaged over 100 blocks. The frequency resolution was therefore 11.7 Hz. The beamforming algorithm is of delay-and-sum type, and implemented in an in-house code [83]. The algorithm is based in comparing the actual measured signals by each array microphone to the sound hypothetically received from a number of monopole sources distributed on a source plane. From the measured signals, the cross-power spectral densities corresponding to each pair of microphones is calculated. The source maps are obtained for individual frequencies specified by the user. On the other hand, a source plane is defined by the user, with a grid of points distributed uniformly on it. Assuming that each grid point emits sound as a monopole, the acoustic field it radiates at the specified frequency is determined exclusively by a complex pressure (complex number). The cross-spectral density corresponding to each pair of microphones of the array is then calculated, as a function of the source point complex pressure. A least-square fit from comparison of the cross-power spectral densities obtained from the measurement and the monopole sources is performed, and the optimum values of the monopole complex pressures are obtained, obtaining a source map. For each frequency specified by the user there is a source map. The code allows reducing the effect of the background noise by removing the main diagonal elements of the cross-power matrix (the auto power spectra of the microphones do not provide any directivity information). Since the array plate acts as a tunnel wall there is a boundary layer on it, and the received signals are highly contaminated by its turbulent pressure fluctuations. The method used to remove its contribution is subtracting the cross-powers corresponding to a measurement with the same conditions except with no
model. Therefore the measured cross-power densities used to calculate the source map are equal to the measured cross-power densities with the model minus the measured cross-powers without the model. Since the boundary layer fluctuations are not affected by the presence of the model they are greatly eliminated by the subtraction operation.

3.2.5 PIV

A PIV system (Quantel Twins BSL 200) was used in the low speed wind tunnel. The pulse gap between image pairs was set at 18 µs. The data was processed using a recursive Nyquist grid method, from 64 to 32 pixel spot dimensions. A total of 500 to 700 image pairs were averaged to obtain the mean and RMS velocity fields.

3.2.6 Oil flow visualisation

The oil consisted of a liquid suspension of titanium dioxide (TiO$_2$) in paraffin. The wind tunnel was run at the desired speed until the solvent had evaporated. The titanium dioxide particles remained on the surface following a set of streaklines.

3.3 Repeatability

The study consists mainly in comparing the rough cylinder noise with the smooth cylinder noise. The measurement setup was not changed during the tests of the various roughness configurations, and therefore the systematic error was constant for all the cases.

Short term repeatability tests were performed for all configurations at the higher speed. The maximum difference in level observed for the smooth cylinder at the highest speed was approximately 0.7 dB, from the 1/3-octave band spectra. Long term repeatability tests of the smooth cylinder far field measurements show that the difference in level of the 1/3-octave band spectra is smaller than approximately 1.2 dB at the higher speed and for all microphones, in the frequency range of interest ($0.18 < St < 50$). In the case of the flat plate, the long term repeatability tests showed a difference in level smaller than 0.9 dB at the higher speed, in the frequency range of interest ($800$ Hz $< f < 20000$ Hz). Regarding the OASPL levels, table 3.3 shows the maximum differences in level
from the long term repeatability tests. These values are higher than the uncertainty of
the microphones, and therefore are taken as the overall uncertainty of the results.

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>Circular cylinder</th>
<th>Flat plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.18 &lt; St &lt; 0.4$</td>
<td>1.4 dB</td>
<td>-</td>
</tr>
<tr>
<td>$0.4 &lt; St &lt; 6$</td>
<td>0.7 dB</td>
<td>-</td>
</tr>
<tr>
<td>$6 &lt; St &lt; 50$</td>
<td>1.7 dB</td>
<td>-</td>
</tr>
<tr>
<td>$800 \text{ Hz} &lt; f &lt; 20000 \text{ Hz}$</td>
<td>-</td>
<td>1.0 dB</td>
</tr>
</tbody>
</table>

Table 3.3: Maximum differences in far field noise level from long term repeatability
tests, at $U_o = 40 \text{ m/s}$.

To check the repeatability between the low speed wind tunnel and the anechoic facility,
on-surface microphone spectra were compared at $U_o = 27 \text{ m/s}$. The microphones were
placed at $\theta = 40^\circ$ and $\theta = 60^\circ$ for the open-jet wind tunnel and the closed-section
wind tunnel, respectively. The spectra for hemispherical roughness is shown in figure
3.7. The vortex shedding peak frequencies are close in both wind tunnels, as well as the
background levels. Since the microphones are not located in the same position, exact
matching is not expected.

The background noise levels corresponding to the empty test section configuration (but
with endplates) were measured as well, and compared with the smooth cylinder noise
levels. In the case of the cylinder the signal-to-noise ratio of the smooth case was too low
at the smallest speed, so this case was discarded to calculate the velocity scaling. In the
case of the flat plate, the smooth configuration radiated slightly higher noise levels than
the background. However, the difference in level between the rough and the smooth
plates was high enough for all speeds and roughness configurations.

The uncertainty of the source strength maps obtained from the phased microphone array
is difficult to estimate. Here the short term repeatability tests were used to determine
the uncertainty. The difference in the maximum source strength levels between two
consecutive measurements of the smooth cylinder at $U_o = 27 \text{ m/s}$, and at a frequency
of 1.9 kHz, was 0.6 dB. The maximum source levels have been considered because they
Correspond to the noise sources generated directly by the cylinder. Higher variations are
observed upstream of the cylinder, but there are no sources located in the scan plane,
so they correspond to background noise.
3.4 Roughness configurations

The roughness configurations used in the tests consist of distributed roughness and cable, together with a smooth cylinder which is the baseline case.

3.4.1 Smooth

Two configurations of smooth cylinder have been used. A first one without any tripping device, with a surface roughness which is $h/D < 0.002$, and a second one with two roughness strips attached at $\pm 50^\circ$ from the front stagnation line. The roughness strips consisted of carborundum particles with approximate size of $\approx 0.3$ mm (Grit 60), glued on double-sided adhesive tape. The width of the strips was approximately 10 mm which corresponds to an angle around the cylinder of $9^\circ$. Nakamura and Tomonari [36] obtained a transcritical flow for $Re = 1.3 \times 10^5$ using strips at this location, and covering an angle of $4^\circ$. In their case it was $h/D = 0.005$, while in the present tests it was $h/D = 0.002$. With the goal of assuring a proper transition it was decided to increase the area covered by the strips.

At the Reynolds numbers of the experiments the smooth configuration without transition strips lies within the critical regime in which there is no vortex shedding. The goal of the roughness strips is to cause the flow to be transcritical by causing the boundary layers to be turbulent before separation. This is a more representative situation of high Reynolds number flows.

3.4.2 Distributed roughness

The distributed rough surfaces have been manufactured using a three-dimensional printer. The material used is plastic and the roughness elements were built on a base plate of thickness 0.5 mm. The base plates were then glued to the surface of the flat plate and the circular cylinder. $Spheres4mm$ and $Cylinders4mm$ are shown in figure 1.7. For the circular cylinder the base plates are cylindrical, to adapt to the cylinder surface (figure 1.6). The way the rough surfaces are built is adding material layers one upon each other, resulting in the surface being rough on a finer scale. This scale is approximately 0.5 mm, which is much smaller than the roughness height (4 mm). Schultz and Flack [48] tested
the effect of secondary roughness of about 10% of the size of the main roughness, and observed no significant effects in the mean and RMS velocity profiles of the TBL. The effect of this finer roughness is neglected in the present work.

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>$h/D$</th>
<th>$\sigma$</th>
<th>$\zeta(^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SmoothNotTripped</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Distributed roughness**

<table>
<thead>
<tr>
<th>Roughness</th>
<th>$h/D$</th>
<th>$\sigma$</th>
<th>$\zeta(^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spheres4mm</td>
<td>0.031</td>
<td>0.42</td>
<td>-</td>
</tr>
<tr>
<td>Spheres4.5mm</td>
<td>0.035</td>
<td>0.53</td>
<td>-</td>
</tr>
<tr>
<td>Cylinders4mm</td>
<td>0.031</td>
<td>0.42</td>
<td>-</td>
</tr>
<tr>
<td>Cylinders4.5mm</td>
<td>0.035</td>
<td>0.53</td>
<td>-</td>
</tr>
</tbody>
</table>

**Cable**

<table>
<thead>
<tr>
<th>Roughness</th>
<th>$h/D$</th>
<th>$\sigma$</th>
<th>$\zeta(^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CablePerp6</td>
<td>0.047</td>
<td>0.08</td>
<td>79</td>
</tr>
<tr>
<td>CablePerp15</td>
<td>0.047</td>
<td>0.19</td>
<td>85</td>
</tr>
<tr>
<td>CablePerp40</td>
<td>0.047</td>
<td>0.50</td>
<td>88</td>
</tr>
</tbody>
</table>

Table 3.4: Roughness configurations.

### 3.4.3 Cable

In the case of the cable configurations the cable has been helicoidally wrapped around the cylinder tightly, to avoid it from moving during the tests. Three configurations were considered with different surface density and helicoidal angle, as seen in figure 1.10. The geometry is detailed in table 3.4.

### 3.4.4 Bar fetches

A set of additional configurations consisting of a circular cylinder together with a fetch of three circular metal bars located upstream, at the cylinder side, and downstream were tested (figure 1.12). The goal is to study the effect of the bar fetch present on the main strut of actual landing gears of the family Airbus320 (figure 1.11). The bars have a diameter of 10 mm, are parallel to the cylinder axis, and its extents are fixed to the endplates. There is a gap of $l_g = 0.5D_b$ between the bars and the cylinder surface, and the bars are separated an angle of $20^\circ$. Each fetch consists of three bars with a non-dimensional diameter $D_b/D = 0.08$, fixed at a height above the cylinder surface $y_b/D = 0.08$, and separated $20^\circ$ between them.
(a) Circular cylinder.

(b) Flat plate.

Figure 3.1: Circular cylinder and flat plate arrangements in the closed section wind tunnel.
Figure 3.2: Circular cylinder in the closed-section wind tunnel.

Figure 3.3: Trailing edge flap.
Figure 3.4: Circular cylinder and flat plate arrangement in the anechoic chamber.
Chapter 3. Experimental Methods

(a) Flat plate.  
(b) Circular cylinder.

Figure 3.5: Experimental setup in the acoustic measurements.

Figure 3.6: Detail of on-surface microphone in Cylinders4.5mm configuration.
Figure 3.7: On-surface pressure spectra of circular cylinder with hemispherical roughness (Spheres 4mm), from open-jet wind tunnel and closed-section wind tunnel, at $Re = 2.2 \times 10^5$.

Figure 3.8: Circular cylinder covered with Cylinders 4.5mm fixed between the end-plates, in the anechoic chamber.
Chapter 4

Numerical Methods

In this chapter the numerical methods used in the CFD simulations as well as the developed BEM algorithm will be presented.

4.1 Reynolds-Averaged Navier-Stokes

Within the RANS models the continuity and momentum equations are averaged in time in order to solve only for the mean fields $\bar{u}$ and $\bar{p}$:

\[
\begin{align*}
\bar{u}_i &= \bar{u}_i + u'_i, \quad \text{for } i = 1, 2, 3, \\
p &= \bar{p} + p'.
\end{align*}
\]

By definition $u'_i = p' = 0$. The equations of motion become:

\[
\begin{align*}
\frac{\partial \bar{u}_i}{\partial x_j} &= 0, \\
\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial u'_i u'_j}{\partial x_j \partial x_j} \quad \text{for } i = 1, 2, 3.
\end{align*}
\]
If the unsteady term is added to this equation we allow the flow to evolve in time. However, the time interval corresponding to the time derivative is much larger than the averaging time interval. The extended unsteady version of RANS is called URANS (Unsteady RANS). Within these formulations turbulence is modeled in all flow scales, as opposed to LES, which models only the small scales (which are assumed to be universal), and DNS, which solves all the flow scales. There are hybrid models which allow an increase in the accuracy with respect to RANS, still having less computational requirements than LES. The most commonly used in engineering applications of open flows is DES (Detached Eddy Simulation). DES is equivalent to LES far enough from the walls, where the relevant flow scales are large enough to be resolved using a relatively coarse grid. Close to the walls, in the turbulent boundary layers containing important small scales, DES is equivalent to RANS.

4.2 Two-dimensional simulations

The commercial code FLUENT has been used in the two-dimensional simulations. To solve the equations at a desired Reynolds number of $Re = 1.6 \times 10^5$ it has been necessary to use a turbulence model. This Reynolds number is low compared to a real landing gear application but the main goal of the CFD is the comparison with the experimental measurements, in which the desired Reynolds number cannot be reached. The URANS equations have been used, together with the Spalart-Allmaras model (S-A), which solves an additional equation for the eddy viscosity. The reason of choosing this particular model is that it reached the required convergence criteria, whereas the other considered model ($k\omega$-SST) did not. Since our particular incoming velocity is much smaller than the speed of sound the compressibility effects are neglectable to the hydrodynamic flow field and therefore we have assumed the fluid (air) to be incompressible.

4.2.1 Turbulence model

In the RANS model which was used in the two-dimensional simulations, the term $-\frac{\partial u_i u_j}{\partial x_j}$ is modelled in all scales of motion. Here we have used the S-A turbulence model, which is a one-equation linear eddy-viscosity model. It introduces the eddy viscosity $\mu_t$ and approximates the Reynolds stresses:
Table 4.1: Roughness configurations.

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>$h/D$</th>
<th>$\Delta \theta (^\circ)$</th>
<th>Two-dimensional ($Re$)</th>
<th>Three-dimensional ($Re$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>0</td>
<td>-</td>
<td>YES($1.7 \times 10^5$)</td>
<td>YES($1.7 \times 10^5, 10^6$)</td>
</tr>
<tr>
<td>Dense roughness</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spheres 4mm</td>
<td>0.031</td>
<td>10</td>
<td>YES($1.7 \times 10^5$)</td>
<td>NO</td>
</tr>
<tr>
<td>Cylinders 4mm</td>
<td>0.031</td>
<td>10</td>
<td>YES($1.7 \times 10^5$)</td>
<td>NO</td>
</tr>
<tr>
<td>Sparse roughness</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cable Par 8</td>
<td>0.047</td>
<td>45</td>
<td>YES($1.7 \times 10^5$)</td>
<td>YES($1.7 \times 10^5, 10^6$)</td>
</tr>
<tr>
<td>Cable Par 16</td>
<td>0.047</td>
<td>22.5</td>
<td>NO</td>
<td>YES($1.7 \times 10^5, 10^6$)</td>
</tr>
</tbody>
</table>

\[
\overline{u'_i u'_j} = -2 \frac{\mu_t}{\rho} S_{ij}, \tag{4.5}
\]

\[
S_{ij} = \frac{1}{2} \left[ \frac{\partial \mu_i}{x_j} + \frac{\partial \mu_j}{x_i} \right] - \frac{1}{3} \frac{\partial \mu_k}{x_k} \delta_{ij}, \quad \text{for } i, j = 1, 2, 3, \tag{4.6}
\]

where $S_{ij}$ is the mean strain rate. Another partial differential equation is solved for the eddy viscosity field $\mu_t$. This equation includes a set of empirical parameters which are fixed by comparison with experiments. Finally, the set of differential equations to be solved are the continuity and momentum equations, plus the eddy viscosity equation. Additional initial and boundary conditions for the eddy viscosity field need to be specified.

### 4.2.2 Discretisation

The differential equations are converted to a set of algebraic equations via discretisation in space and time. The unknowns are then the values of the velocity, pressure and eddy viscosity fields at a finite set of locations and time instants.

#### 4.2.2.1 Spatial discretisation

The spatial discretisation method used by FLUENT is the finite volume method (FVM). In this method space is divided in a set of control volumes which, in this case, coincide with the grid cells. The equations are then solved in its integral form within each volume element. In the integral form of the equations the terms in the equations are the fluxes of the transported magnitudes through the control volume faces, and the conservation
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of mass and momentum are exact in each volume. In FVM the values of the fields are calculated in the cell centroids. In the equations the values at the cell faces are required to determine the fluxes and they need to be estimated from the values in the centroids. This is done using a Taylor series expansion about the cell centroids. The order of the Taylor expansion determines the order of the scheme. In our case, in which a second order upwind approximation is used, the field is approximated as a linear function about the centroid of the upstream cell. In the first order scheme the value at the cell is equal as the value at the centroid.

It is also needed to estimate the field gradients at the faces. In our case the least-square method has been used, which assumes that the fields are linear between cell centroids. Therefore the gradients can be determined simply by dividing the difference of centroid values and the distance between centroids.

4.2.2.2 Temporal discretisation

After performing the spatial discretisation, the equations need to be discretised also in time. The time discretisation can be explicit, if the field values used to determine the next time step solution are the ones corresponding to the previous time step, or implicit, in which they correspond to the ones at the next time step. The implicit formulations has the advantage that is unconditionally stable with respect to time step size. Its main disadvantage is its high computational cost, since it needs an iterative process to solve the discretised equations. The order of the discretisation is the local accuracy obtained by the discretisation, and depends on the number of terms from the Taylor expansion series used. In the present work a second order implicit time discretisation has been used, and a timestep of $\Delta t = 10^{-5}$ s has been used in all simulations, which corresponds to a non-dimensional timestep of $\Delta t U_0/D = 1.6 \cdot 10^3$.

4.2.3 Solver

The system of algebraic equations obtained after discretising the governing equations is coupled and non-linear. It is solved in an iterative way, starting from the solution in the previous time step. The iteration continues until the residuals associated to velocity, pressure and eddy viscosity reach certain imposed limits. The iterative method
used is the so called pressure-based solver, in its segregated form. It consists in solving the momentum equations for the velocities in a sequential manner, using the pressure magnitudes corresponding to the previous time step, and then solving an equation for the corrected pressure, derived from the continuity equation and the momentum equation, and which ensures the mass conservation requirement. The segregated form refers to the fact that the momentum fluxes and the pressure are decoupled by solving firstly the momentum equations for the momentum fluxes using the pressures from the previous time step, and then solving a corrected equation for the pressure. This process is repeated in an iterative manner until the desired convergence is reached. The particular segregated scheme used has been SIMPLE. The PISO scheme caused the solution to diverge, which could be caused by the relatively high mesh skewness.

The number of iterations per time step was set to the minimum between 20 and the required number to achieve a reduction of the flow residuals of $10^{-3}$ in the smooth case, and $10^{-2}$ in the hemispheres cases. In the rough cases more iterations were required to achieve the same convergence criteria and in the hemispheres cases it was not possible to decrease the $10^{-2}$ threshold by reducing the time step or by increasing the number of iterations per time step.

As was remarked above, the Spalart-Allmaras model has been used instead of other more accurate models for detached flows because it was not possible to reach the required convergence criteria in the rough configurations. The $k\omega$-SST model reached only a reduction of the continuity residual of one order of magnitude, which was considered insufficient.

4.2.4 Initial conditions and boundary conditions

The starting flow field was taken as the solution of a steady RANS simulation. The number of iterations performed allowed the flow field to reach approximately periodic conditions according to the evolution of the residuals. In all cases 6000 iterations were sufficient.

The boundary conditions imposed in all configurations are the most common in aerodynamic problems. At the incoming flow face a velocity inlet condition of $u_1 = 20 \text{ m/s}$, $u_2 = u_3 = 0$ is used. At the outlet face zero-pressure outlet (gauge pressure) has
been applied. Symmetry conditions are used in the top and bottom faces, and no-slip and non-penetration condition is imposed on the cylinder wall. Finally, regarding the turbulent viscosity, an inlet and outlet value of $\mu_t = 5\mu$ is imposed. This turbulent boundary condition is recommended by Travin et al. [84] to ensure turbulent separation conditions, like the ones obtained by tripping the boundary layer to simulate flow at supercritical and post critical regimes.

### 4.2.5 Grids

The grids used in the spatial discretisation are shown in the following pictures. The grid geometry far from the body (outer grid) can be seen in figure 4.1. It is a T-type grid. The inlet is located at $5.5D$ from the cylinder axis, and the outlet at $11.8D$. The top and bottom faces are located at $5.5D$ from the axis. The outer grids are equal in all roughness configurations. The grid size transition between the inner mesh and the outer mesh is discontinuous in order to diminish the number of cells required. The skewness (equi-angle) has been limited to 0.6.

![Figure 4.1: Outer mesh geometry.](image-url)
A grid convergence study has been performed for the four configurations tested. The magnitudes taken into account in the study have been the mean drag and RMS lift coefficients. Three grids have been tested in each configuration, a coarse one, medium, and fine. The medium and fine grids have been observed to have associated values of $C_d$ and $\sqrt{C_l^2}$ closer than 2.5%. The medium grids have been used to obtain the flow fields. An additional test with an extended computational domain of the Smooth configuration has been performed to check the domain size sensitivity. The extended domain is 29% wider and 18% longer.

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>Coarse</th>
<th>Medium</th>
<th>Medium (extended domain)</th>
<th>Fine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>-</td>
<td>$C_d = 0.53$, $\sqrt{C_l^2} = 0.07$</td>
<td>$C_d = 0.52$, $\sqrt{C_l^2} = 0.07$</td>
<td>$C_d = 0.53$, $\sqrt{C_l^2} = 0.07$</td>
</tr>
<tr>
<td>Spheres4mm</td>
<td>$C_d = 1.14$, $\sqrt{C_l^2} = 0.71$</td>
<td>$C_d = 1.20$, $\sqrt{C_l^2} = 0.81$</td>
<td>-</td>
<td>$C_d = 1.18$, $\sqrt{C_l^2} = 0.83$</td>
</tr>
<tr>
<td>Cylinders4mm</td>
<td>$C_d = 0.74$, $\sqrt{C_l^2} = 0.21$</td>
<td>$C_d = 0.92$, $\sqrt{C_l^2} = 0.58$</td>
<td>-</td>
<td>$C_d = 0.91$, $\sqrt{C_l^2} = 0.58$</td>
</tr>
</tbody>
</table>

Table 4.2: $C_d$ and $\sqrt{C_l^2}$ of the various considered grids for each roughness configuration.

### 4.2.5.1 Smooth

The maximum wall $x_2^+$ reached is 0.6. The maximum and average skewness (equi-angle) are 0.28 and 0.06 respectively.

### 4.2.5.2 Spheres4mm

The maximum and average wall $x_2^+$ are 3.5 and 0.42, respectively. The maximum and average skewness (equi-angle) are 0.50 and 0.13 respectively. Figure 4.2 shows the mesh geometry in the vicinity of a roughness element. Since all the elements are equal the mesh was performed for a single element and afterwards copied to obtain the grid around all the cylinder. The geometry has been chosen as a compromise between simplicity and grid spacings required near the walls.
The maximum and average wall $x_2^+$ are 12 and 2.62, respectively. The maximum and average skewness (equi-angle) are 0.28 and 0.05 respectively. In this case the maximum wall $x_2^+$ has been allowed to go over 5 (beyond the viscous sublayer) because due to the sharp edges the flow is detached at the upper corners of the elements, so that there is no need of predicting the detachment point, as in the smooth and hemispheres cases. That may be the explanation why, while grid convergence in the other configurations required having a wall $x_2^+$ of order 1, in this case grid convergence was achieved with a higher wall $x_2^+$. Figure 4.3 shows the mesh geometry around the cylindrical elements.

4.2.5.4 CablePar8

The maximum wall $x_2^+$ is 0.65. The maximum and average skewness (equi-angle) are 0.57 and 0.01, respectively. The mesh surrounding the cables is shown in figure 4.4.
4.3 Three-dimensional simulations

Bluff body flows present important three-dimensional effects that are ignored by two-dimensional simulations. For example, in the case of cylindrical bodies vortex shedding decorrelates along the span, whereas two-dimensional flow assumes a perfect spanwise correlation. This causes the mean and fluctuating forces on the cylinder to be higher in two-dimensional flow, giving unrealistic results. This is why three-dimensional simulations are necessary in two-dimensional bluff-body geometries.

Three-dimensional simulations of smooth and rough circular cylinder flow were performed using the open source code OpenFOAM. The incompressible Navier-Stokes equations were solved since the desired Mach numbers are 0.06 and 0.2, and the additional difficulties inherent in the treatment of the acoustic boundary conditions and the acoustic propagation are avoided. The Reynolds numbers are $1.6 \times 10^5$ and $10^6$. 
In addition to Smooth, the configurations CablePar8 and CablePar16, with surface cables parallel to its axis, as well as FetchFront, FetchSide and FetchBack, with an off-surface bar fetch, have been tested computationally using three-dimensional simulations. Due to computational limitations it was not possible to perform three-dimensional simulations of dense roughness with large enough span length to resolve the shedding. Only the previously described two-dimensional simulations are available for those cases.

4.3.1 Turbulence model

Due to the high Reynolds number nature of the problem and the impossibility of performing pure Large Eddy Simulation (LES), Delayed Detached Eddy Simulation (DDES) was chosen to model turbulence [84, 85]. It is an extension of the original Detached Eddy Simulation (DES) model, which includes a correction to avoid an anomalous early separation for high aspect ratio wall cells [85]. DDES is in principle equivalent to LES away from the wall, and to RANS close to the wall. Therefore the detached flow structures are
resolved, whereas the TBLs are modelled with RANS. In this case the RANS turbulence
model used was S-A.

In the original DES model, the unsteady RANS equations together with an equation
for the transport of turbulent viscosity are solved. But the turbulent length is not the
wall distance everywhere, as in the pure unsteady RANS approach, but it changes to
the LES length scale far from the wall:

\[
l_t = \min(y_{wall}, C_{DES} \Delta),
\]

(4.7)

where \(C_{DES}\) is a model constant, approximately equal to 0.65, and \(\Delta\) is a characteristic
resolvable grid length defined as:

\[
\Delta = \sqrt[3]{\Delta x \Delta y \Delta z}.
\]

(4.8)

In DDES a continuous dependence of the turbulent length scale with the wall distance
is introduced:

\[
\tilde{l}_t = f(y_{wall}, C_{DES} \Delta).
\]

(4.9)

The details of the model can be found in the reference above.

4.3.2 Discretisation

As in the two-dimensional simulations the spatial discretisation used is FVM, and the
grid is fully structured, created using ’Gridgen’ software.

4.3.2.1 Spatial discretisation

The spatial derivatives are approximated by a second order central difference scheme
(called ’Gauss linear’ in OpenFOAM)
4.3.2.2 Temporal discretisation

The temporal discretisation scheme is second order implicit (called ‘backward’ in OpenFOAM). The timestep used in all cases is $\Delta t = 10^{-5}$ s, which corresponds to non-dimensional timesteps (normalised with the convection time across the cylinder) of $2 \times 10^{-3}$ and $3 \times 10^{-3}$ for Reynolds numbers of $1.6 \times 10^5$ and $10^6$, respectively. The timesteps normalised with the convection time across a single roughness element are $5 \times 10^{-2}$ and $10^{-1}$ at the considered Reynolds numbers.

4.3.3 Solver

The resulting discretised system of equations are solved iteratively using the PIMPLE algorithm (merged PISO and SIMPLE algorithms).

4.3.4 Initial conditions and boundary conditions

The initial velocity and pressure fields are uniform and equal to the outer boundary values (incoming velocity and ambient pressure), except on the walls. Therefore, from $t = 0$ s until the flow field has statistically converged there is a transient period in which the flow adapts to the boundary conditions. The force histories started to be sampled after the mean drag and the mean flow field had approximately converged.

The boundary conditions are the same as in the two-dimensional simulations, plus periodic boundary conditions in the spanwise direction. It must be remarked that also here a turbulent inflow was imposed, via a certain amount of turbulent viscosity, to simulate transition to turbulent flow prior to separation.

The cylinder span length used in the various configurations are shown in Table 8.1. The span length of the smooth cylinder and the cylinder with the bar fetches is $L = 2D$. The span length of the cylinder with on surface bars and rings is $L = 1D$, due to computational limitations. As stated by Travin et al. [84], in bluff body flows a strong correlation of the vortex shedding over the span is acceptable. In that work a span length of $L = 2D$ was used.
4.3.5 Grids

The maximum wall $x_2^+$ has been limited to 4 at the highest Reynolds number for all configurations, so that the first cell is contained within the viscous sublayer, as required by the RANS turbulence models. The complete mesh corresponding to Smooth, CablePar8 and CablePar16 is shown in figure 4.5a, and for FetchFront, FetchSide and FetchBack in figure 4.5b. In the following the grids surrounding roughness for the various configurations are presented, together with the maximum wall $x_2^+$ and mesh skewness.

4.3.5.1 Smooth

The maximum wall $x_2^+$ reached is 1.63. The maximum and average skewness (equi-angle) are 0.28 and 0.03 respectively. The spanwise cell size is $\Delta z/D = 0.057$.

4.3.5.2 Spheres4mm

The maximum and average wall $x_2^+$ are 3.50 and 0.42, respectively. The maximum and average skewness (equi-angle) are 0.50 and 0.13, respectively. Figure 4.6 shows the mesh geometry in the vicinity of a roughness element. Since all the elements were equal the mesh was performed for a single element and afterwards copied to obtain the grid around all the cylinder. The geometry has been chosen as a compromise between simplicity and grid spacings required near the walls.

4.3.5.3 CablePar8

The maximum and average wall $x_2^+$ are 3.95 and 0.50, respectively. The maximum and average skewness (equi-angle) are 0.57 and 0.03, respectively. The three-dimensional mesh was obtained by protruding the two-dimensional mesh of CablePar8. The spanwise cell size relative to the cylinder diameter is $\Delta z/D = 0.015$, and with respect to the cable diameter (roughness height) is $\Delta z/h = 0.32$. 
4.3.5.4 *CablePar16*

The maximum and average wall $x^+_2$ are 3.63 and 0.45, respectively. The maximum and average skewness (equi-angle) are 0.57 and 0.03, respectively. The spanwise cell size relative to the cylinder diameter is $\Delta z/D = 0.013$, and with respect to the cable diameter is $\Delta z/h = 0.28$. Figure 4.7 shows a three-dimensional view of the surface mesh.

4.3.5.5 *FetchFront*

The maximum and average wall $x^+_2$ are 1.09 and 0.35, respectively. The maximum and average skewness (equi-angle) are 0.55 and 0.02, respectively. The spanwise cell size relative to the cylinder diameter is $\Delta z/D = 0.015$, and with respect to the cable diameter is $\Delta z/h = 0.191$. The grid geometry surrounding the bar fetch is shown in figure 4.8.

4.3.5.6 *FetchSide*

The maximum and average wall $x^+_2$ are 1.40 and 0.49, respectively. The maximum and average skewness (equi-angle) are 0.55 and 0.02, respectively. The spanwise cell size relative to the cylinder diameter is $\Delta z/D = 0.015$, and with respect to the cable diameter is $\Delta z/h = 0.191$. The grid geometry surrounding the bar fetch is the same as in *FetchFront* and it is not shown.

4.3.5.7 *FetchBack*

The maximum and average wall $x^+_2$ are 3.87 and 0.51, respectively. The maximum and average skewness (equi-angle) are 0.55 and 0.02, respectively. The spanwise cell size relative to the cylinder diameter is $\Delta z/D = 0.015$, and with respect to the cable diameter is $\Delta z/h = 0.191$. The grid geometry surrounding the bar fetch is the same as in *FetchFront* and it is not shown.
(a) Outer mesh of Smooth, CablePar8 and CablePar16.

(b) Outer mesh of FetchFront, FetchSide and FetchBack.

Figure 4.5: Complete meshes of the three-dimensional simulations.
Figure 4.6: Three-dimensional mesh of Spheres4mm.

Figure 4.7: Mesh surrounding the cables in CablePar16.
Figure 4.8: Mesh surrounding the bar fetch of FetchFront.
Chapter 5

Distributed Roughness. Fully Covered Circular Cylinder and Flat Plate

5.1 Introduction

On the upstream face of the large components and if they are not immersed in the near wake of other large components, a TBL will grow until it separates, and some of the small components will be immersed in it. This situation is known to enhance the scattering of turbulent sound sources in the same way as any vortical flow interacting with a compact body \[18\], and the phenomenon is described by roughness noise models. There exist several models in the literature to calculate the radiated noise by rough flat walls immersed in a TBL, in terms of the roughness geometry and the turbulent pressure field of the TBL. The two models considered in the present work are the models of Howe \[55\] and Smol’yakov \[59\].

Bluff body noise is dominated by large scale shedding, and it peaks at low frequencies. Therefore the effect of roughness at low frequencies will be related to the changes in vortex shedding characteristics. The effect of surface roughness on a circular cylinder flow has been widely studied experimentally in the past. However no studies have been performed on the noise radiation of highly rough cylinders. Achenbach and Heinecke \[35\] measured coherent vortex shedding in the supercritical and postcritical regimes even
for the largest roughness \((h/D = 0.03)\), which had pyramidal shape. The measured Strouhal number of the primary vortex shedding peak was \(St \approx 0.22\) for the largest roughness. Achenbach \([33]\) measured the mean drag coefficient of cylinders covered with glued spheres with \(h/D = 0.017\) up to postcritical Reynolds numbers. The postcritical values of mean drag coefficient obtained were close to 1.2, i.e. the subcritical regime value. He observed as well that the drag coefficient kept increasing with increasing roughness size. The results were interpreted by Guven \textit{et al.} \([34]\) through the effect that roughness has on the TBLs. Due to roughness, the TBLs are thicker and have higher momentum deficit, which causes premature separation. The resulting lower pressure recovery causes the observed higher drag. The increase of drag coefficient with roughness size is also explained. The presence of coherent vortex shedding requires the separation lines to be straight along the span, i.e. the surface geometry should allow the TBLs to be straight \([21]\). If the roughness elements are large enough with respect to the cylinder diameter the TBLs will potentially not be well developed before separation. Therefore separation will not be uniform along the span, and coherent vortex shedding will be suppressed.

As a first approach a set of simple configurations were chosen. The bluff body was a circular cylinder, since it is a representative component of landing gears. The clearest example is the main strut. As surface roughness, a set of regular arrangements of hemispheres and circular cylinders protruding from the wall were considered. To directly compare the flat wall roughness noise and the bluff body roughness noise, a flat wall configuration was also tested, with the same roughness configurations. In this case the roughness noise models are directly applicable as they appear in the literature.

This chapter addresses the following issues:

1. Validation of the roughness noise models for a flat wall, using different roughness element shapes. This is relevant because small components present in landing gears have a number of different geometrical shapes, and the models should be robust.

2. The case of moderate values of TBL thickness to roughness height ratio at the upstream row of elements has been addressed, due to its relevance to landing gear applications. The predictive capability of the models in this situation, which they don’t contemplate, has been checked.
3. Study of the relevance of roughness noise in noise emitted by a bluff body. Applicability of the models to the case of a bluff body with relatively large roughness, and various roughness shapes.

4. Study of the effects of roughness at lower frequencies. At low and medium frequencies, bluff body noise is dominated by large scale shedding. The effect of roughness in this lower frequency range has been analysed.

The baseline configurations are a smooth circular cylinder and a smooth flat plate. Two transition strips at ±50° measured from the upstream stagnation line were attached on the circular cylinder, which consisted of carborundum particles of average size $2.7 \times 10^{-4} \text{m (Grit 60)}$ glued to a double-sided adhesive tape. The goal of the strips was to obtain turbulent separation, and therefore supercritical flow. A roughness strip was also attached to the flat plate in both the anechoic and the low speed wind tunnel tests. It was located immediately downstream of the leading edge, causing the TBL to be fully developed upstream of the rough surface. In the case of the circular cylinder oil flow visualisation tests were performed to validate the effect of the roughness strips. Figure 5.1 shows clearly a straight separation line at $\theta > 90^\circ$. The resulting surface oilflow pattern for the smooth cylinder without roughness strips is also shown. As can be seen in figure 5.2 the TBL thickness growth over a smooth plate is well predicted, indicating that the roughness strip causes transition close to the leading edge, and the streamwise pressure gradient was small.

5.2 Flat plate

Measurements of the TBL thickness at various streamwise locations were performed using a Pitot tube, and the results are shown in figure 5.2 together with the TBL models.

It is observed that the thickness is underpredicted in the upstream region of the rough surface, where the transition between the smooth surface to rough surface happens. The predictions are improved at downstream locations for both the hemispherical and the cylindrical roughness.

For frequencies lower than 60 Hz background noise is dominant. In the range $60 < f < 170$ Hz noise scales well with $U_\theta^7$ (figure 5.5), but since it is a fall-off region and the
spectra are plotted in fixed frequency bands, it is equivalent to a scaling with $U_o^8$, i.e. turbulence noise. For frequencies lower than approximately 800 Hz all spectra collapse with the smooth wall data. The calculated velocity scalings of all configurations in the relevant spectral regions are presented in table 5.1 and the spectra scaled with various powers of the velocity are shown in figure 5.5.

The measured and predicted far field noise spectra, at two observation angles and two flow speeds are shown in figure 5.3. A broadband peak is observed at all observation angles for the hemispherical elements, which scales well with $U_o^6$ (table 5.1), as is described by roughness noise models. The peaks collapse on a Strouhal number basis at $f h / U_o \approx 0.18$. The peak corresponds to roughness noise and is well predicted by Howe’s model (figure 5.3). The model predicts the peak to be at $f h / u_\tau \approx 3$, which is equivalent to the above relation since in typical rough wall TBLs, the friction velocity lays approximately in the range $0.05 < u_\tau / U_o < 0.07$ [56]. The two empirical constants of Smol’yakov’s model [59] have been fixed to best fit the measurements at $\theta = 121^\circ$, giving a good match with the experimental trends for this case ($K_R = 6.36 \times 10^{-19}$, and $St_\tau = 0.32$, i.e. $f h / U_o \approx 0.16$). The value of $St_\tau = 0.61$ determined by Smol’yakov
Chapter 5. Distributed Roughness. Fully Covered Cylinder and Flat Plate

causes the peak frequency to be too large (approximately 1.8 times larger than the measurements). However, at different observation angles, Smol’yakov’s model doesn’t predict the levels well. This shows that the lack of directivity information of the model is a serious drawback for its use in practical applications. Directivity needs to be added to the model.

In the case of cylindrical roughness, patterns are significantly different. Firstly, roughness noise peak levels are lower than those of hemispherical roughness, especially for downstream observers. The spectral levels deviate from hemispherical roughness before the maximum peak is reached and start decreasing with a fall-off rate close to that of the smooth wall spectra. The fact that at $\theta = 121^\circ$ the peak is close to the hemispherical roughness case up to higher frequencies suggests that the noise generation mechanism is only partially modified. As seen in table 5.5, the described peak scales approximately with $U_o^{5.5}$, instead of $U_o^{6}$.

At higher frequencies, cylindrical roughness configurations have significantly higher levels than hemispherical roughness, which in upstream observers appears as a peak centred at about 11 kHz, and is not accounted by the models. It is not Strouhal-based, but only weakly shifted to higher frequencies with increasing outer velocity, and it has been determined to scale approximately with $U_o^7$ (figure 5.5c and table 5.1).

Scattering on the right-angled edges of the roughness elements as described by Crighton and Leppington [88], which implies a scaling with $U_o^{5.3}$, cannot strictly apply here, since roughness elements are compact. However, this value is close to the measured scaling, $U_o^{5.5}$. Yang and Wang [69, 70, 71] performed LES simulations of a TBL flow over a roughness patch of hemispherical, cylindrical and cuboidal elements with $h/\delta \approx 0.1$ and $\sigma = 0.1$, and their results showed a monotonous increase of noise from $f h / U_o \approx 0.15$ for both the cylinders and cuboids, with respect to the hemispheres case. They observed that the turbulent surface pressure field was dominated by the flow structures in the vicinity of the front edges due to sharp edge separation. Also the pressure fluctuations were strongest in the leading row of roughness elements, and diminished downstream. The peaks are not detected in the reported simulation effort. Possible explanations for the observed peak in the present experiments are: a) it is due to interaction effects between neighbouring roughness elements, since the present roughness is more densely packed than the ones in the simulations ($\sigma = 0.42$ and $\sigma = 0.1$, respectively.), b) the difference
is related to the difference in roughness height to TBL thickness ratio \( h/\delta > 0.4 \) in the present experiments and \( h/\delta = 0.12 \) in the simulations, and c) the simulations (LES) did not resolve the flow structures responsible for the measured noise (the peak frequency is around 11 kHz, which is close to the reported maximum frequency). Despite the differences found between the present experiments and the simulations, it is reasonable to suspect that the peak measured in the present experiments with the sharp edge separation phenomenon was the same as that observed in the simulations.

The \( \Delta \text{SPL}_{1/3} \) plots obtained by subtracting the smooth wall noise levels from the rough wall levels are shown in figure 5.6. Roughness noise is more pronounced in the upstream and downstream directions, as was described by Liu and Dowling. The sharp-edge noise peak is most pronounced in the upstream direction and its level diminishes in the downstream direction.

The measured noise directivity patterns are shown in figure 5.7. The values correspond to overall sound pressure levels obtained by integration of PSD(\( \omega \)) between 1 kHz and 3 kHz (figure 5.7a) and between 3 kHz and 18 kHz (figure 5.7b). The roughness noise peak has a minimum at \( \theta = 90^\circ \), as is predicted by the model. The directivity pattern of Cylinders\( 4\text{mm} \) is similar to Spheres\( 4\text{mm} \), but the levels are somewhat lower. The directivity pattern of the high frequency noise peak present for the cylindrical roughness shows that higher noise levels are received by upstream observers.

Roughness noise models assume a fully developed rough wall TBL, which covers completely the roughness elements and interstitial flow is not relevant regarding noise generation. Roughness noise generation is higher for lower TBL thickness, which is predicted by Howe’s model. However, if the TBL is thin in the leading rows of roughness elements, there is a transition smooth-rough where the TBL adapts to roughness. In this region interstitial flow must be important regarding the turbulent stresses, and its relevance in the noise generation will be briefly assessed. The parameter that determines the strength of interstitial flow is the roughness height to TBL thickness ratio in the leading row of elements, \( h/\delta \). In the previous configurations it was \( h/\delta \approx 0.5 \), i.e. roughness elements are completely covered by the TBL. A configuration corresponding to a larger value of \( h/\delta \) in the leading row of elements was tested to assess the effect of interstitial flow and the possible miss-prediction of the roughness noise model.
In the additional configuration the distance from the leading edge of the plate to the leading row of roughness elements was reduced by half, resulting in a TBL thickness to roughness height ratio $h/\delta \approx 1$. Far field noise measurements of both configurations are shown in figure [5.4], together with Howe’s model predictions. The difference in measured peak level is about 1.4 dB for both hemispherical and cylindrical roughness, whereas the model predicts a level increase of only 0.7 dB. This difference indicates a possible miss-prediction. However, the spectral pattern is the same for both configurations and for both roughness shapes; there appear no additional features due to interstitial flow, which should be different than the model predictions. In any case, far field noise dependence on $h/\delta$ due to interstitial effects in the leading rows of elements is weak. This is true for both hemispherical and cylindrical elements.

## 5.3 Circular cylinder

### 5.3.1 Roughness noise

The far field noise spectra shown in figures [5.8a] and [5.8c] reveal a broadband peak in the distributed roughness configurations, which is especially prominent for hemispherical roughness, similarly to the flat plate cases. In the range dominated by the broadband peak, spectra scale well with $U_o^6$ (figure [5.9] and table [5.2]). It peaks at higher frequencies than the flat plate configuration, but it is Strouhal-based with a peak Strouhal number $St \approx 10$. However, for the circular cylinder there isn’t a unique value of outer velocity; the outer velocity changes along the TBL. In the flat plate configuration the roughness noise peak is located at $f h/U_o \approx 0.2$. If the frequency is normalised with the maximum outer velocity around the cylinder instead of the free stream velocity ($U_{max} \approx 1.7U_o$ based on PIV measurements), and the roughness height instead of the cylinder diameter, it yields:

$$\frac{f h}{U_{max}} = \frac{f D}{U_o} \cdot \frac{U_o}{U_{max}} \cdot \frac{h}{D} \approx 0.18$$  \hspace{1cm} (5.1)

i.e. the same value obtained for a flat plate. This suggests that the peak corresponds to roughness noise, and that it is mostly generated in the region of maximum outer velocity. The spectra of cylindrical roughness are also shown. They have lower peak levels than
hemispherical roughness, as in the flat wall case, but the peak scales with $U_o^{6}$, and not with $U_o^{5.5}$.

A set of phased microphone array measurements were performed for the distributed roughness configurations on the circular cylinder (see figures 5.10 and 5.11), to obtain information regarding the location of the noise sources. The source strength maps show that within the frequency range in which roughness noise is dominant the most powerful sources are located on the upstream face of the cylinder and the sides. Figure 5.10 shows the source strength maps at $f = 1.9$ kHz. The rough cylinders emit more noise at this frequency, and the sources are located on the cylinder surface, mainly around the separation lines. From the far field spectra it is seen that at this frequency roughness noise starts to dominate. $Spheres 4 mm$ and $Cylinders 4 mm$ have very similar source strength maps, as was expected from the far field spectra at this frequency. In figure 5.11 the maps at $f = 3$ kHz are presented. As in the previous case, noise levels of the rough cylinders are above the smooth case. At this frequency the strongest sources are located mainly on the upstream face of the cylinder, and not in the separation region. At this frequency the hemispherical roughness configuration radiates more intensely than the cylindrical roughness configuration, as observed in the far field spectra as well. It can also be observed that the highest contribution comes from the spanwise extents of the cylinder, due to the interaction of the TBLs growing on the endplates with the rough surface in the cylinder extents.

For frequencies higher than the peak described above, cylindrical roughness has higher spectral levels than hemispherical roughness, with the exception of $Spheres 4 mm$ at $\theta = 46^\circ$, which has similar levels as the cylindrical configurations. As in the flat plate case, this difference may be due to sharp edge separation. However, the increase in this case is much smaller than what is observed for the flat plate case. The sharp edge noise in the flat plate case mainly happens when the incoming thin smooth wall TBL impinges on the first rows of roughness elements, as was shown by Yang and Wang [70, 71]. In the circular cylinder this doesn’t occur because in the region of higher outer velocity the TBL has already adapted to the rough surface, and is thicker. Furthermore, in the flat plate case the peak scales approximately with $U_o^{7}$, while in the circular cylinder it scales with $U_o^{6}$. This implies that the noise sources are different in both cases.
Figure 5.12 shows the on-surface pressure spectra of the distributed roughness configurations. The on-surface pressure is dominated by the unsteady hydrodynamic field \(|k| \gg \omega/c\), not by the acoustic field \(|k| = \omega/c\). In this section the medium and high frequency ranges are discussed. The low frequency range is dominated by the vortex shedding peak(s).

At \(\theta = 20^\circ\) (M1) the spectral levels of the rough cylinders are of the order of 15 - 20 dB higher than Smooth, in the frequency range \(1 < St < 25\). This shows the wake of the rough cylinders have significantly higher turbulence intensity. Hemispherical and cylindrical roughness have close levels up to \(St = 25\). A broadband peak is observed, centred at \(St \approx 15\), higher than the roughness noise peak, and unlike the latter, hemispherical and cylindrical roughness have very close peak levels. The spectra of the microphone at \(\theta = 60^\circ\) (M2) shows differences with respect to the previous one. Smooth has a broadband hump centred at \(St = 2\), which is due to the proximity of the separation region and shear layer. The hump is present also for the rough cylinders, but is relatively weaker. Roughly, comparing figures 5.12b and 5.12d, the main difference between them is due to the broadband peak of Smooth. Early separation due to roughness causes the microphone at \(\theta = 60^\circ\) to be more distant to the separation region and the shear layer, and therefore the hump has lower levels. In figures 5.12c and 5.12d, slightly higher levels for the hemispherical roughness than for the cylindrical roughness are observed, similarly to the roughness noise peak.

The microphone at \(\theta = 120^\circ\) (M3) is located within the upstream attached flow, and the spectra are shown in figures 5.12e,f. The baseline configuration is SmoothNotTripped, because there is no available data of Smooth for this microphone. The spectra of the rough cylinders show a broadband hump associated with the TBL turbulent stresses. The peak level and frequency, as well as the fall-off levels, are significantly different between the various roughness types. For both hemispherical and cylindrical roughness the peak Strouhal number increases with roughness height, and the peak level decreases with roughness height. Also, differences between hemispherical and cylindrical roughness are observed. The peak Strouhal numbers of hemispherical roughness are larger than cylindrical roughness and their spectral levels are significantly higher in the range \(8 < St < 50\). These differences can be explained by the differences in TBL structure induced by different roughness shapes. Since the TBL structure constitutes roughness noise sound sources, it could also potentially cause the weaker roughness noise peak levels.
measured for cylindrical roughness. Scattering can be interpreted as a filter of turbulent 
stresses of a certain wavelength and frequency, and depends on the roughness geometry. 
The spectra of cylindrical roughness show lower levels than hemispherical roughness at 
$St \approx 10$, i.e. the roughness noise peak Strouhal number. Since cylindrical roughness 
generates Reynolds stresses equal or higher than hemispherical roughness, as observed 
from the RMS x-velocity fields obtained with PIV (figure 5.14), the lower values of on-
surface turbulent stresses suggests an outwards shift of the Reynolds stresses profile. 
This would cause a decrease of roughness noise generation efficiency, since the scattering 
mechanism, which increases the noise efficiency from quadrupole to dipole, becomes 
weaker with the distance from the turbulent sources to the roughness elements.

Blockage (frontal area of roughness elements per unit total frontal area) cannot account 
for this effect, since $Spheres_{4.5mm}$ and $Cylinders_{4mm}$ have close values of blockage 
(they differ 1%). Solidity (projected frontal roughness area per unit wall parallel area) 
is also the same for hemispherical and cylindrical roughness. The mean roughness height, 
deﬁned not by the mean height of the roughness element tops, but by the full integral of 
the rough surface, is 20% higher for cylindrical roughness than hemispherical roughness, 
and normalised with the TBL thickness could be an appropriate parameter to describe 
the outward shift of the turbulent stresses. It must be remarked that, since the TBL 
is potentially not fully developed, results from fully developed rough wall TBLs may 
be inappropriate in this case. Further understanding of rough wall TBLs and roughness 
parameterization is required.

The mean velocity fields obtained using PIV (figure 5.13) reveal that distributed rough-
ness induces thicker shear layers and a wider wake immediately after separation. The 
RMS velocity fields of the streamwise stresses (figure 5.14) show remarkably higher levels 
of turbulent ﬂuctuations in the shear layers and the near wake of the rough cylinders.

5.3.2 Low frequency noise

The far ﬁeld noise spectra measured at $\theta = 46^\circ$ (M3) are shown in ﬁgure 5.8a. At low 
frequencies, noise radiated by Smooth and the cylinders with distributed roughness is 
dominated by vortex shedding peak(s). The tripped smooth conﬁguration has a vortex 
shedding peak at $St \approx 0.26$, which is in agreement with previously reported measure-
ments [35] and indicates that the tripping is successful. The cylinders with distributed
roughness also have vortex shedding peaks, but at significantly smaller Strouhal numbers than the smooth case, at about $St \approx 0.20$. It was seen in past experiments that cylinders with roughness as large as $h/D = 0.03$ had strong coherent vortex shedding with values of Strouhal number and drag coefficient close to the subcritical regime values. The mean drag appeared to keep increasing with $h/D$ up to the highest tested roughness size. The present roughness configurations follow this trend. The explanation was due to the effect that roughness had on the TBLs. According to Guven et al. [34] roughness causes the TBLs to be thicker and with a higher momentum deficit, which implies an earlier separation than the smooth cylinder and a wider wake. This explains the decrease of Strouhal number. The PIV measurements of figures 5.13 and 5.14 are in agreement with the previous argument.

According to Zdravkovich [21] for coherent vortex shedding to be present it is required that there is homogeneous separation along the span. That appears to be the case for the present distributed roughness sizes.

The on-surface pressure spectra (figure 5.12 shows clearly the vortex shedding peaks. The fundamental peak Strouhal number is 0.2, the same value measured in the far field noise. At the downstream stagnation line, the first harmonic, corresponding to the alternating drag force, is more intense than the fundamental one. In the other locations the fundamental peak is dominant over the first and second harmonics. This indicates that the pressure fluctuations related with the oscillating lift are much stronger close to the separation region than close to the stagnation line, and the opposite happens for the pressure fluctuations associated with the oscillating drag, at twice the vortex shedding frequency.

Using the measurements of six on-surface microphones (M4-M9) located along the span, the spanwise correlation of the shedding was studied. The distance between microphones was $0.4D$, and the microphones closer to the cylinder endplates were far enough from them to avoid end effects. The two-point correlation coefficient of two time signals measured at locations $z_i$ and $z_j$ is:

$$\rho_{ij} = \frac{R_{ij}(0)}{\sqrt{R_{ii}(0)R_{jj}(0)}}$$  \hspace{1cm} (5.2)

where the cross-correlation function of two signals is defined as:
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\[ R_{ij}(\tau) = p(z_i, t)p(z_j, t + \tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} p(z_i, t)p(z_j, t + \tau)dt. \]  

(5.3)

The signal \( p_i(t) \) corresponds in this case to one of the microphones closer to the endplates, and \( p_j(t) \) to the signals obtained from the other microphones, in order of increasing distance to the first microphone. In this way the correlation coefficient is obtained for each of the six pairs of microphones. The results are shown in figure 5.15. The smooth cylinder exhibited an approximately constant decrease rate of correlation up to the maximum measured distance, \( \Delta z/D = 2.4 \), where \( \rho \approx 0.5 \), i.e. a significant correlation is still present. The smooth cylinder without roughness strips is also shown. The two-point correlation in this case falls abruptly to levels \( \rho < 0.1 \) for \( \Delta z/D > 0.7 \). In the subcritical regime a significant two-point correlation is present at least until \( \Delta z/D \approx 10 \). The present configuration, with an aspect ratio of 3.9, is insufficient to develop coherent vortex shedding due to the end effects. It is, however, enough to develop coherent vortex shedding in the supercritical regime, where the correlation length is shorter and the end effects are weaker. The oilflow results corresponding to the smooth cylinder configurations (figure 5.1) agree with the previous observations. The rough cylinders exhibit a stronger decrease of correlation than the smooth cylinder for distances up to approximately 0.5\( D \), but for larger distances the correlation diminishes slowly, resulting in a two-point correlation value similar to the smooth cylinder at \( \Delta z/D = 2.5 \). There are small differences between the various roughness elements, except for the larger hemispherical elements, which have a higher decrease of correlation for \( \Delta z/D > 1 \). It still presents, however, a correlation coefficient \( \rho \approx 0.3 \) at \( \Delta z/D = 2.5 \). These results explain why the vortex shedding peak noise levels are so high for the rough cylinders. The spanwise shedding coherence together with the early detachment of the TBL due to roughness causes a large increase of the radiated noise levels. It is remarkable that, despite the size of the roughness elements, the TBLs are homogeneous enough along the span to allow a uniform separation (observed previously by Zdravkovich for smaller roughness size \([22, 23]\)). It is expected that for larger roughness (larger value of \( h/D \)), the TBLs will be more heterogeneous causing a decrease of the spanwise shedding correlation length and of the noise peak levels. The state of the TBLs previous to separation is also important regarding the roughness noise generation mechanisms. If the TBL prior to separation are not developed enough, the noise sources will be significantly different from the ones on a fully developed TBL and so will be the radiated noise.
The far field and on-surface microphone signals were acquired simultaneously in order to analyse their coherence. The coherence is defined as:

$$\gamma_{ij}(f) = \frac{P_{ij}(f)^2}{P_{ii}(f)P_{jj}(f)},$$

(5.4)

where $P_{ij}$ is the cross-power spectrum of the signals $i$ and $j$, and $P_{ii}$ and $P_{jj}$ are the power spectral densities of signals $i$ and $j$. Only the results of one pair of microphones and one roughness configuration are shown (in addition to the smooth cylinder case), since all of them show the same features.

The coherence function of one of the on-surface microphones and one of the far field microphones is shown in figure 5.16. A clear peak close to the vortex shedding frequency is observed, for both the smooth and rough cylinders. The magnitude is slightly higher for the rough cylinder, and is broader. The rough cylinder also has weaker peaks corresponding to the harmonics of the fundamental shedding peak. No significant coherence is observed for frequencies other than vortex shedding.

The two cable configurations with lowest cable density (CablePerp6 and CablePerp15) show no vortex shedding peak, and spectral levels close to SmoothNotTripped in the low frequency range $St < 0.26$. The cable configuration with the highest surface density (CablePerp40) has a weak but discernible vortex shedding peak at $St \approx 0.2$, a value close to the one of cylinders with distributed roughness, as well as SmoothNotTripped (the peak for this configuration is seen in the on-surface pressure spectra of figure 8.22c).

It is known that dense circumferential grooves have the same effect on the mean drag and vortex shedding frequency as distributed roughness [22]. In the present case, results suggest that transition to the fully rough regime has not occurred. Higher surface cable density is needed to induce a fully rough TBL, and uniform along the span, before separation.

All vortex shedding peaks scale with $U_0^{5.5-6}$. For frequencies between $f_{vs}$ and $c/D$, corresponding to the spectral fall-off, all configurations show a scaling close to $U_0^6$ (except for CablePerp15, which is thought to be anomalous).
5.4 Summary

Roughness noise on flat walls is well described by Howe’s model in the case of hemispherical roughness. Cylindrical roughness elements emit weaker roughness noise than predicted by the model, and present remarkably higher levels at higher frequencies, which are not described by the model. It has been hypothesised that it is related with sharp edge separation effects. However, the dependence on $h/\delta$ appears to be relatively weak for both hemispherical and cylindrical roughness, which limits the relevance of interstitial flow in the leading rows of elements regarding noise radiation.

The effect of dense large roughness on the noise emitted by circular cylinders has been shown to be important over all the frequency range. At high frequencies, hemispherical roughness generates noise presumably well described by Howe’s model, since the peak frequency is well predicted assuming that the dominant noise sources are located in the maximum outer velocity region. To correctly use Howe’s model to determine the far field noise, a modification to account for the cylindrical geometry is needed. The cylinder with cylindrical roughness has, similarly to the flat wall case, a weaker roughness noise peak (as described by Howe’s model) compared to hemispherical roughness, and radiates higher noise levels at higher frequencies, as in the flat wall. However, this increase at higher frequencies is lower than for the flat wall case. As for the flat wall, the origin of the additional noise is hypothesized to be due to sharp edge separation.

Regarding the lower frequency range, noise levels are increased due to the presence of stronger vortex shedding peaks, and fall-off levels about 5 dB higher than the smooth cylinder. These higher levels happen despite a lower spanwise correlation length. The velocity scaling of the vortex shedding and spectral fall-off regions is similar to the smooth cylinder, i.e. noise scales with $U^6$. The vortex shedding peak Strouhal number is lower than for the smooth cylinder, and close to the subcritical regime value. This fact, together with the increase of the peak level is in agreement with earlier experimental studies with smaller roughness sizes and different roughness shapes. PIV has shown that distributed roughness causes significantly thicker shear layers after separation and a wider wake, in agreement with early separation and a lower Strouhal number.
Figure 5.2: Boundary layer thickness evolution measured in the closed section wind tunnel.

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>70 Hz $&lt; f &lt; 200$ Hz</th>
<th>$f \approx f_{rn}$</th>
<th>$f \gg f_{rn}$</th>
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<tr>
<td>Smooth</td>
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<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>Spheres4mm</td>
<td>7.6</td>
<td>5.9</td>
<td>6.1</td>
</tr>
<tr>
<td>Spheres4.5mm</td>
<td>7.5</td>
<td>6.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Cylinders4mm</td>
<td>7.5</td>
<td>5.5</td>
<td>7.2</td>
</tr>
<tr>
<td>Cylinders4.5mm</td>
<td>7.5</td>
<td>5.4</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Table 5.1: Velocity scaling exponents for all flat wall configurations, determined from the signals measured at M4 and M7. $f_{rn}$ is the roughness noise peak frequency.

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>$f \approx f_{vs}$</th>
<th>$f_{vs} \ll f \ll c/D$</th>
<th>$f \approx f_{rn}$</th>
<th>$f \gg c/D$</th>
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<td>-</td>
<td>5.1</td>
</tr>
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<td>Smooth</td>
<td>5.5</td>
<td>5.7</td>
<td>-</td>
<td>6.9</td>
</tr>
<tr>
<td>SmoothNotTripped</td>
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<td>5.7</td>
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<td>6.5</td>
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<tr>
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<td>6.3</td>
<td>6.1</td>
<td>6.3</td>
</tr>
<tr>
<td>Cylinders4mm</td>
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</tr>
<tr>
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<td>6.2</td>
<td>5.9</td>
<td>6.1</td>
</tr>
<tr>
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<td>CablePerp15</td>
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<td>5.4</td>
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<td>6.1</td>
</tr>
<tr>
<td>CablePerp40</td>
<td>5.8</td>
<td>6.1</td>
<td>-</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table 5.2: Inflow velocity scaling exponents for all configurations, determined from the signals measured at $\theta = 45^\circ$. $f_{vs}$ and $f_{rn}$ are the vortex shedding and the roughness noise peak frequencies, respectively.
Figure 5.3: Measurements and predictions of the far field noise 1/3-octave band spectra for the flat plate.
Figure 5.4: 1/3-octave band noise spectra of flat plate configurations at $U_o = 40$ m/s.
Figure 5.5: Noise spectra of rough flat plates normalised with $U_0^5$ (red), $U_0^6$ (black) and $U_0^7$ (blue), for $U_o = 20, 27, 34, 40$ m/s, measured at M1.
Figure 5.6: $\Delta$SPL spectra of the flat plate roughness noise.

Figure 5.7: Measured directivity patterns for the flat plate, obtained at $U_o = 40$ m/s.
Figure 5.8: Far field noise spectra of circular cylinders with distributed roughness, at $Re = 3.2 \times 10^5$ and measured at M3.
Figure 5.9: Velocity scaling of cylinders with distributed roughness. Scaling with $U_5^v$ (red), $U_6^v$ (black) and $U_7^v$ (blue), for $U_o = 20, 27, 34, 40$ m/s, measured at M3.
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Figure 5.10: Source strength maps at $f = 1.9$ kHz (flow from left to right).
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Figure 5.11: Source strength maps at $f = 3$ kHz (flow from left to right).
Figure 5.12: On-surface pressure spectra of cylinders with distributed roughness (angle measured from downstream stagnation line).
Figure 5.13: Mean x-velocity fields of the circular cylinders with distributed roughness, measured using PIV.

Figure 5.14: RMS x-velocity fields of the circular cylinder with distributed roughness, measured using PIV.
Figure 5.15: Cross-correlations of on-surface pressure signals measured along the span, at $Re = 2 \times 10^5$ (the measurements of Szepessy [40] in the subcritical regime are also included).

Figure 5.16: Coherence of far field and on-surface pressure signals.
Chapter 6

Distributed Roughness. Partially Covered Circular Cylinder

6.1 Introduction

In the previous chapter the aerodynamic and acoustic consequences of fully covering the circular cylinder with distributed roughness was studied. The far field noise spectra of smooth and fully rough cylinders present significant differences in all the frequency range. These differences were associated with the mechanisms by which roughness affects the BLs, causing early separation, and the attached flow at separation being uniform enough along the span to allow strong vortex shedding. Actually the vortex shedding peak level is remarkably increased by roughness. Also roughness itself becomes an efficient noise radiator through its interaction with the turbulent stresses within the TBLs.

In reality the large components in a landing gear need not be fully-covered uniformly by the dressings. The dressings may be concentrated upstream, or downstream, or at the sides. The effects of distributed roughness on the low and medium frequency ranges are to thicken the TBLs and cause an early detachment. But this mechanism will obviously have to be modified if the upstream face is not fully covered by roughness, e.g. if roughness is only present from a certain angle. In this case, depending on this angle, the TBLs in the separation region will be fully developed or will be still transitioning to the fully rough regime, affecting the separation location and the spanwise shedding correlation length.
In this chapter the effect of partially covering the cylinder surface with distributed roughness has been studied, with the goal of studying the transition from the smooth regime to the fully rough regime. Of course the transition depends on the particular partial coverings used. As a first approach only configurations maintaining the symmetry have been considered for simplicity.

### 6.2 Roughness partial coverings

![Figure 6.1: Partially roughened cylinders.](image)

Here only configurations uniform along the span and symmetric with respect to the incoming flow have been considered (see figure 6.1). In the first place progressive covering from upstream to downstream has been applied. This is shown in figure 6.1a. The angular increments of rough surface were 20°, applied simultaneously at both sides of the cylinder. The transition strips were used for \( \theta_u < 50° \). Secondly, progressive covering from downstream to upstream was applied, as sketched in figure 6.1b. The angular increments of rough surface were also 20°, and transition strips were used when \( \theta_d < 130° \).

The main distinctive features of the fully rough regime compared with the smooth regime are:

- Higher vortex shedding peak level and lower peak Strouhal number,
- Higher spectral fall-off levels,
• Similar spectral fall-off rate,

• Roughness noise.

These features are expected to gradually appear as the covered surface increases. However, the transition is in general dependent on the particular way the covering is applied. In this study only configurations maintaining spanwise symmetry have been considered.

The phenomena involved in the smooth-rough transition are complex. Firstly, the effect of upstream roughness in the separation location dictates the shedding peak level and Strouhal number. The questions that this work answers regarding the shedding peak and fall-off noise are: what are the minimum amounts of $\theta_u, \theta_d$ to reach fully rough shedding regime? How gradual is it (what ranges of $\theta_u, \theta_d$ does it involve)?

Secondly, the appearance of roughness noise is going to be analysed. The questions in this case are: what values of $\theta_u, \theta_d$ are required for roughness noise to be significant? How gradual is the transition (what ranges of $\theta_u, \theta_d$ does it involve)?

### 6.3 Partial roughness from upstream

Figure 6.2a,b shows the transition of far field noise spectra of cylinders partially covered with Spheres4.5mm, from $\theta_u = 0^\circ$ (Smooth) to $\theta_u = 150^\circ$ (fully rough regime, Spheres4.5mm). As in the previous chapter, it is $\Delta\text{SPL} = \text{SPL}_{\text{rough}} - \text{SPL}_{\text{smooth}}$. For $\theta_u = 30^\circ$ a global increase of 3 - 5 dB in $St < 1$ is observed. For $St > 1$ the difference in level with Smooth is smaller than about 2 dB. This indicates that roughness in $\theta < 30^\circ$ affects the TBLs until separation, and also that no significant roughness noise is generated for $\theta_u < 30^\circ$. It must be remarked that for $\theta_u = 30^\circ, 50^\circ$ roughness strips were attached at $\theta = 50^\circ$, to assure turbulent separation. For $\theta_u = 50^\circ$ the levels at $St < 3$ are within the Smooth level $\pm 2$ dB, indicating a similar effect of roughness to transition strips alone. Roughness noise appears at $St > 3$, with an increase of up to 6 dB at $St < 30$. For $\theta_u = 70^\circ$ the roughness noise levels further increase and a peak is noticeable. However at $St < 3$ the levels are close to $\theta_u = 30^\circ$ ($\pm 2$ dB). A sudden change to fully rough regime noise spectrum is observed in $\theta_u = 90^\circ$, reaching levels close to $\theta_u = 150^\circ$ (within $\pm 2$ dB). Finally, for $\theta_u = 110^\circ, 130^\circ$ the spectra match the
fully rough spectrum closely. So in the case of cylinders with upstream face covered, adding roughness on the downstream face doesn’t affect significantly the radiated noise.

Figure 6.3 shows measurements corresponding to Cylinders4mm. There is no available data corresponding to $\theta_u = 30^\circ$. For $\theta_u = 50^\circ$ noise levels are significantly higher at $St > 3$, which corresponds to roughness noise. They are though still 3 - 6 dB lower than the fully covered cylinder. At $St < 3$ the difference with Smooth is small, observing only a moderate decrease of the shedding peak level, but no change in peak Strouhal number. For $\theta_u = 70^\circ$ a clear change of the spectrum is observed at $St < 3$. The peak level is increased by 5 dB, but is still about 7 dB lower than the fully covered cylinder. The peak Strouhal number is shifted to $St \approx 0.2$, the value of the fully covered configuration. The fall-off levels are also increased by about 5 dB, and no noticeable change in the fall-off rate can be appreciated. The levels at $St > 3$ follow the trend of $\theta_u = 50^\circ$, but are 1 - 2 dB higher. Finally, for $\theta_u = 90^\circ, 110^\circ$ the spectra collapse to the fully covered cylinder within 1 - 2 dB in all the frequency range, indicating that the transition is complete, and as with hemispherical roughness, roughness located on the downstream face doesn’t change significantly the radiated noise.

The transition smooth-rough with the two considered roughness shapes is similar. It is noted that the fully covered cylinder spectra are reached for similar values of $\theta_u$ for both Spheres4.5mm and Cylinders4mm, in all the frequency range, so transition is independent of roughness geometry, at least for the geometries tested. Both the shedding peak and fall-off dominated range ($St < 3$) and the roughness noise dominated range ($St > 3$) reach fully covered cylinder regime for $70^\circ < \theta_u < 90^\circ$. For $\theta_u = 70^\circ$ hemispherical roughness merely contributes to roughness noise, and no significant change has happened in $St < 3$, whereas for this amount of rough surface cylindrical roughness has increased the spectral levels in all the frequency range. Particularly the shedding peak Strouhal number is already equal to the fully covered configurations.

The shedding peak Strouhal number as a function of $\theta_u$ is shown in figure 6.4. The difference between hemispherical and cylindrical roughness for the smooth case ($\theta_u = 0^\circ$) is due to the slightly different location of the transition strip. For both types of roughness the transition of the shedding regime happens in the range $70^\circ < \theta_u < 90^\circ$, but Spheres4.5mm presents a sharper change than Cylinders4mm. The reason of this difference is hypothesised to be related to the different behaviour of the TBL above
the two different roughness geometries. However this is hard to reconcile with the fact
that both present close values in the fully covered configuration. Figure 6.5a shows the
$\Delta$OASPL$\text{peak}$ values as a function of $\theta_u$. It is shown that the transition is concentrated
in $70^\circ < \theta_u < 90^\circ$. The difference is due to the higher peak levels of Smooth in the tests
corresponding to the cylindrical roughness, because of the slightly different location of
the transition strips, which changes the peak level by about 3 dB.

Regarding roughness noise, as seen in figure 6.5b for $\theta_u = 90^\circ$ the fully covered regime
is complete, but the transition appears less sharp than the shedding peak and fall-off. It
is clear that Cylinders$4mm$ generates higher roughness noise levels than Spheres$4.5mm$
at $\theta = 90^\circ$, but similar levels at $\theta = 45^\circ$. This agrees with the very high frequency noise
emitted by Cylinders$4mm$ and Cylinders$4.5mm$ on a flat wall, even if the directivity
pattern is changed.

The spanwise correlation was studied for upstream covered cylinders with Spheres$4.5mm$
and Cylinders$4.5mm$ (figure 6.6). For $\theta_u = 30^\circ$ no transition strips were applied. The
shedding decorrelates quickly along the span, as in SmoothNotTripped, especially for
Spheres$4.5mm$ ($\rho_{ij} < 0.3$ at $z/D = 0.4$). The far field noise measurements were per-
formed using transition strips and present spectra close to Smooth, typical of the super-
critical regime. There is no spanwise correlation data for $\theta_u = 30^\circ, 50^\circ$ with transition
strips. The low correlation coefficient of $\theta_u = 30^\circ$ at $z/D = 0.4$ suggests that rough-
ness doesn’t force turbulent separation at this low Reynolds number ($Re = 1.7 \times 10^5$),
especially for Spheres$4.5mm$. For $\theta_u \geq 50^\circ$ the correlation coefficient at $z/D = 0.4$
is similar to the values of higher $\theta_u$. For both types of roughness $\theta_u = 90^\circ$ shows
spanwise decorrelation rates close to the fully covered configurations. There are large
differences between the correlation coefficient trends for different $\theta_u$, especially in the
case of Cylinders$4mm$, but these differences are weakly translated to the far field noise
spectra. Furthermore, Spheres$4.5mm$, that presents more similar spanwise correlation
trends than Cylinders$4mm$, shows bigger differences between the noise spectral levels
than Cylinders. This eliminates the possibility of a direct link between the spanwise
shedding correlation and the noise levels.
Figure 6.2: Far field noise spectra of upstream partially covered cylinders with $Spheres4.5mm$, at $\theta = 97^\circ$.

6.4 Partial roughness from downstream

The far field noise spectra and $\Delta SPL_{1/3}$ spectra of the circular cylinders with partial coverings from downstream are shown in figures 6.7 and 6.8 $Spheres4.5mm$ covering
Figure 6.3: Far field noise spectra of upstream partially covered cylinders with Cylinders4mm, at \( \theta = 90^\circ \).

\( \theta_d = 50^\circ \) presents similar noise levels than Smooth except in \( St > 10 \), where its level starts to increase up to reaching similar levels than the fully covered configuration in \( St \leq 20 \). This suggests that for Spheres4.5mm, the radiated noise in this frequency
range is mainly generated by interaction of the near wake with the downstream roughness. However this causes a moderate noise level increase, of about 4 dB. For $\theta_d = 70^\circ$ the shedding peak level increases by about 7 dB with the same peak Strouhal number, and the fall-off levels also increase, but to levels still lower than the fully covered cylinder. The roughness noise peak appears to be still below the fall-off levels. All these features indicate an intermediate state between smooth and fully covered cylinders. The fact that the peak Strouhal number hasn’t changed would contradict the fact that roughness simply forces a slightly earlier separation, since that would imply a decrease of peak Strouhal number. For $\theta = 90^\circ$ the transition regarding the low and medium frequency noise is complete (the spectrum is close to the fully covered cylinder), but the roughness noise broadband peak level is still about 4 dB lower than the fully covered cylinder peak level. Therefore the transition regarding roughness noise is not complete. The TBL impinging the first rows of elements corresponds to a smooth wall and a transition to a rough wall TBL occurs. But in this case separation happens shortly after the impingement and it can be concluded that roughness noise generation within the transition smooth-rough region is not relevant. A similar conclusion was reached in the case of a rough flat wall for both hemispherical and cylindrical roughness. For $\theta_d = 110^\circ$ the roughness noise peak level is less than 1 dB lower than the fully covered peak level.
indicating an almost complete transition despite the close smooth-rough transition. For $\theta_d = 130^\circ$ slightly higher roughness noise peak levels (2 dB) than the fully covered case are observed. For $\theta_d = 150^\circ$ the peak levels diminish again close to the fully covered case. The higher roughness noise levels of $\theta_d \approx 130^\circ$ can be explained by the fact that the noise generation efficiency increases for thinner TBLs and larger outer velocity. Higher $\theta_d$ causes the TBL to be thicker in $60^\circ < \theta < 90^\circ$, where the outer velocity is maximum.
Cylinders4mm covering the cylinder from downstream presents a similar transition trend as Spheres4.5mm. $\theta_d = 70^\circ$ has a spectrum, as Spheres4.5mm, intermediate between the fully covered and smooth cylinders, with a shedding peak Strouhal number equal to Smooth. Roughness noise in high frequencies appears lower than the fall-off levels for this amount of downstream roughness. As with Spheres4.5mm, $\theta_d = 90^\circ$ shows a
complete transition to the fully covered regime regarding the spectral shedding peak and fall-off (Strouhal number lower than about 7). At $St > 7$ noise levels are 3 - 5 dB lower than the fully covered cylinder levels, indicating an incomplete transition. $\theta_d = 110^\circ$ and $\theta_d = 130^\circ$ present spectral levels 2 - 3 dB and 1 - 2 dB lower than the fully covered ones, respectively, at $St < 10$. In downstream observers this difference is present but is about 1 dB lower. The hemispherical roughness doesn’t present this level difference between $\theta_u = 110^\circ, 130^\circ, 150^\circ$ in the low and medium frequency range.

Figure 6.9 shows the change of vortex shedding peak Strouhal number as a function of $\theta_d$. The transition in this case is concentrated between $70^\circ < \theta_d < 90^\circ$ for both $Spheres4.5mm$ and $Cylinders4mm$. As was mentioned previously the difference is due to the roughness strips being located in slightly different locations in different tests.

The variation of $\Delta OASPL$ with the amount of downstream roughness is shown in figure 6.10. The significantly lower level of $\Delta OASPL_{peak}$ of cylindrical roughness for $\theta_d = 110^\circ$ is clear.

The mean and RMS x-velocity fields of downstream covered cylinders with $Spheres4.5mm$ are shown in figures 6.11 and 6.13 respectively. The mean and RMS flow fields of $\theta_d = 70^\circ$ are similar to Smooth, and the significant difference in far field noise levels (figure 6.7) cannot be associated with a significant change in the flow fields. In accordance with the far field noise spectra, a significant change is observed between $\theta_d = 70^\circ$ and $\theta_d = 90^\circ$. The shear layer thickness growth rate is higher for the latter, as well as the RMS x-velocity values in the shear layers and near wake, indicating an effect of roughness on the TBLs before and at separation. For $\theta_d = 90^\circ$, in which case the flow detaches shortly after impinging on the roughness elements, the shear layer after detachment is thinner than for $\theta_d = 110^\circ, 130^\circ$, but it has a faster growth rate and higher x-velocity fluctuations. For $\theta_d = 90^\circ$ at separation the ratio $h/\delta$ is the highest of all configurations and therefore the flow at separation is the most dependent on the roughness geometry. The RMS velocity fluctuations in the shear layers shortly after separation are higher than for $\theta_d = 110^\circ, 130^\circ$, which suggests that the detachment in a region with low $h/\delta$ causes higher turbulence intensity after separation and a higher mixing rate. However, these differences are not significantly translated to a far field noise changes at $St < 6$ (roughness noise levels are lower). Significant differences are not observed between $\theta_d = 110^\circ, 130^\circ, a50^\circ$, as in the far field noise spectra.
The cylinders covered with *Cylinders4mm* have mean and RMS x-velocity fields shown in figures 6.12 and 6.14, respectively, but only for three amounts of rough areas. This is merely due to time limitations. There are little differences between the mean fields of the three amounts of rough area. The RMS fields show a slight increase in the levels of x-velocity fluctuations for increasing values of $\theta_d$.

**Figure 6.7**: Far field noise spectra of downstream partially covered cylinders with *Spheres4.5mm*, at $\theta = 97^\circ$. 
Figure 6.8: Far field noise spectra of downstream partially covered cylinders with Cylinders4mm, at θ = 90°.

6.5 Summary

An experimental study of noise radiated by circular cylinders with various amounts of dense distributed roughness that maintain the spanwise symmetry has been performed,
with the goal of studying the transition of radiated noise from smooth to fully covered. The results can be used to increase the accuracy of the roughness noise model applied to landing gears.

Regarding the low and medium frequency range, upstream hemispherical roughness ($Spheres_{4.5mm}$) changes the noise spectra almost completely within $70^\circ < \theta_u < 90^\circ$. In contrast, for $\theta_u = 70^\circ$, cylindrical roughness ($Cylinders_{4mm}$) already presents a noise spectrum which has partly transitioned, which means that this type of roughness causes the regime transition to happen for lower $\theta_u$. Regarding the high frequency range dominated by roughness noise, transition appears more gradual between $30^\circ < \theta_u < 90^\circ$, for both types of roughness. In all cases and for all frequencies, transition is complete for $\theta_u = 90^\circ$. A direct link between the spanwise shedding decorrelation rates and the far field noise spectral levels hasn’t been observed, both changing independently through the transition.

When hemispherical roughness is added from downstream, transition starts for $\theta_d < 70^\circ$, and for $\theta_d = 90^\circ$ the transition is complete in frequencies dominated by shedding and fall-off, but not in the roughness noise range. For $\theta_d = 110^\circ$ the noise spectra are close to the fully covered cylinder spectra in all the frequency range. For $Cylinders_{4mm}$ the
transition also starts for $\theta_d < 70^\circ$, and for $\theta_d = 90^\circ$ the noise spectra are close to the fully covered one at $St < 7$. For $\theta_d = 110^\circ$ the low and medium frequency noise levels are about 4 dB lower than for the hemispherical roughness. This is thought to be due to the interaction between separation and smooth-rough transition. This noise level decrease is only of about 2 dB in the hemispherical roughness. Since both roughness elements have close values of blockage (frontal area of the roughness elements per unit total frontal
area), and similar values of roughness height and surface density, it is thought that the difference has to be associated with the different roughness element shapes. Cylindrical roughness may have a higher effective blockage, i.e. a higher $h_s$.

The effect of the present distributed roughness on separation has been seen to be relevant. For all tested configurations, there isn’t a significant decrease of low and medium frequency noise levels with respect to the smooth cylinder. A vortex shedding spectral peak with peak level higher than for Smooth is always present, for both upstream and
Figure 6.12: Mean x-velocity fields of downstream partially covered cylinders with \textit{Cylinders4mm}.

downstream roughness, indicating that the transitions smooth-rough and rough-smooth don’t prevent a uniform separation angle along the span.
Figure 6.13: RMS x-velocity fields of downstream partially covered cylinders with Spheres 4.5mm.
Figure 6.14: RMS x-velocity fields of downstream partially covered cylinders with $\text{Cylinders}_4$$\text{mm}$. 

$\theta_d = 110$ deg

$\theta_d = 130$ deg

$\theta_d = 150$ deg
Chapter 7

Distributed Roughness. Circular Cylinder Roughness Noise Model

7.1 Introduction

The analogy between small components attached to large components and roughness is exploited in order to develop a physical noise model for the small components. In chapters 5 and 6 the noise emitted by circular cylinders covered with large roughness elements ($h/D = 0.031, 0.035$) was investigated, and compared to flat wall roughness noise. The peak frequency was observed to be well predicted by flat wall models when the maximum outer velocity was used in the frequency scaling. It was deduced that the main modifications required to extend roughness noise models to bluff bodies were the scattering of sound around the cylinder, and the effects of a pressure gradient on the TBL.

The noise generation mechanism is assumed not to be significantly affected by the wall curvature, since the acoustic field perceives the wall as flat around each single roughness element in the limit $h/D \ll 1$. The scattering of the turbulent field on each roughness element takes place as if the surface was locally flat. Under this approximation the dipole strengths are the same in both situations. However, the wall curvature does affect the noise propagation once it has been generated. The flat wall roughness noise model of Howe [55] uses a Green’s function tailored to the roughness elements on a flat wall, together with Lighthill’s acoustic analogy, to calculate the far field noise. The noise
sources are contained in the TBL flow field above the roughness elements. In the present case the Green’s function used by Howe [55] is not, on its own, appropriate since the wall is not flat. Therefore, a new Green’s function is required. A tailored Green’s function obtained with BEM will be used to calculate the field around the cylinder created by the given dipole sources.

The advantages of BEM over other numerical techniques that resolve sound propagation with wall boundary conditions are that the Sommerfeld condition at infinity is automatically satisfied, there is an absence of dissipation and dispersion errors, there is no need to implement boundary conditions to avoid sound reflection, and it is straightforward to include far field observers. The main disadvantage is that for each observer location and frequency the solution must be computed, so many runs must be performed if a complete spectrum and directivity information are desired. BEM amounts to solving a linear system of equations, full and non-symmetric in general, with as many equations as surface boundary elements. Therefore, at high frequencies, which require fine boundary meshes to capture short wavelengths, the problem is computationally expensive.

There are various formulations of BEM [91, 92, 93, 94, 95]. In the present case the solution of the Helmholtz equation with wall boundary conditions and arbitrary sources, i.e. the Green’s function tailored to the bluff body geometry, is calculated. Since in our case the sound sources are known a priori from the TBL properties, it is convenient to implement the BEM solver so that it is independent of the sources. The source strength distribution can be specified a posteriori to determine the pressure signal, simply by multiplying the Green’s function by the source strengths.

Convection effects can be easily included in a straightforward manner in the BEM solver by substituting the free field Green’s function of the Helmholtz equation by the free field Green’s function of the convective Helmholtz equation [91]. Due to the inherent low Mach number nature of roughness noise, convection effects have not been accounted for here. However, the wind tunnel measurements used for comparison are corrected for convective amplification and shear layer refraction.
7.2 BEM

BEM was chosen to calculate the scattering around the bluff body and the propagation to the far field. The advantages of BEM over numerical techniques that resolve sound propagation with wall boundary conditions are well known: Sommerfeld condition at infinity automatically fulfilled, absence of dissipation and dispersion errors, no need to implement far field boundary conditions to avoid sound reflection, and straightforward inclusion of far field observers. The main disadvantage is that for each observer location and frequency the solution must be computed, so many runs must be performed if a complete spectrum and directivity information are desired. BEM amounts to solving a linear system of equations, full and non-symmetric in general, with as many equations as boundary elements. At high frequencies, where fine boundary meshes are required, the problem is computationally expensive.

The integral equation to be solved numerically in BEM is:

\[
G_t(x, z_{pi}, \omega) \left( 1 - \frac{\Omega(z_{pi})}{4\pi} \right) = G_o(x, z_{pi}, \omega) + \int\int_{\Sigma} G_t(x, z, \omega) \frac{\partial G_o(z, z_{pi}, \omega)}{\partial z_i} n_i d^2z, \tag{7.1}
\]

This is an integral equation whose only unknown is \( G_t(x, z_{pi}, \omega) \). The conventional BEM method, which has been used in the present work, consists of solving this equation using a discretisation of the surface integral and converting the above equation into a system of linear equations, with as many equations as surface elements. A piece-wise representation of the Green’s function is used on the surface, the surface integration being of second order. It must be remarked that the solution corresponds to the particular values taken for the observer location \( x \) and frequency \( \omega \), i.e. for every pair \((x, \omega)\) the linear system must be solved.

\[
G_t(x, z_{pi}, \omega) \left( 1 - \frac{\Omega(z_{pi})}{4\pi} \right) = G_o(x, z_{pi}, \omega) + \sum_{j=1, j\neq i}^{N_e} G_t(x, z_j, \omega) \frac{\partial G_o(z_j, z_{pi}, \omega)}{\partial z_i} \Delta S(z_j), \text{ for } i = 1, ..., N_e. \tag{7.2}
\]
The free field Green’s function and its wall normal derivatives are:

\[ G_o(x, z_{pi}, \omega) = \frac{e^{i\frac{\omega}{c}|x-z_{pi}|}}{4\pi |x - z_{pi}|}, \quad (7.3) \]

\[ \frac{\partial G_o(z_j, z_{pi}, \omega)}{\partial z_n} = \frac{e^{i\frac{\omega}{c}|z_j-z_{pi}|}}{4\pi |z_j - z_{pi}|^3} \left( i\frac{\omega}{c} |z_j - z_{pi}| - 1 \right) (z_j - z_{pi}) \cdot n_j. \quad (7.4) \]

Note that the free field Green’s function diverges when \( z_j \to z_{pi} \), but the integral exists. In the numerical integration the surface element corresponding to \( z_{pi} \) is excluded, eliminating the divergence. If the surface elements are small enough the contribution to the integral of that surface element is neglectable. The linear system matrix coefficients are:

\[ A_{ij} = \begin{cases} 
1/2 - \Delta S(z_j) \frac{\partial G_o(z_j, z_{pi}, \omega)}{\partial z_n}, & \text{if } i \neq j. \\
1/2, & \text{if } i = j. 
\end{cases} \quad (7.5) \]

Finally, the following linear system needs to be solved:

\[ A_{ij} G_t(x, z_{pi}, \omega) = G_o(x, z_{pi}, \omega). \quad (7.6) \]

The matrix \( A_{ij} \) is full and not symmetric in general. The number of operations to solve the system using one of the conventional methods (LU decomposition, Gauss elimination) grows with \( N^3 \), and so does the computing time. In the present case LU decomposition with partial pivoting has been used. The surface mesh has been parameterised using cylindrical coordinates, with uniform spacing.

### 7.2.1 Cylinder edges

The treatment of the cylinder edges has to be defined. It must be remarked that in this first stage model the effect of the edges is required to be small, i.e. we are interested in a nominally infinite circular cylinder. However, due to the, although long, finite length of the cylinder used, the edges can have a significant effect. Several geometries were considered initially: hemispherical, flat and empty (no edges). Flat edges cause the
linear system to be ill-conditioned and no sensible results are obtained in that case. The other two cases both provide similar results in the case that the observer distance is small compared with the cylinder length, $|x| \ll l_{max}$. The comparison between both cases is shown in figure 7.2 in which the grid used was $108 \times 36$. The differences are considered acceptable, especially at the highest frequency, where the roughness noise peak is located. Therefore, and due to its simplicity, the geometry with no edges was used and all results shown correspond to that case. The observer distance has been limited to $4D$ precisely to keep the ratio $|x|/l_{max}$ small. To compare with the experimental results, corresponding to larger observer distances, $1/|x|^2$ scaling has been applied to the power spectral density.

**Figure 7.1:** BEM surface grid of the cylinder with hemispherical extents.

**Figure 7.2:** Absolute value of $G_t$ for the cylinder with hemispherical edges and no extents. On-surface sources are located at an angle $\theta$ measured from the observer direction and the observer distance is $|x| = 4D$. 
7.2.2 Reduction of problem size using spanwise symmetry

The observer location \( x \) is located in the midspan plane of the cylinder, thus the problem being symmetric with respect to the midspan plane. A reduction of the problem size has been performed using this symmetry. The reduced problem has been obtained in the following way. The system can be expressed separating explicitly both sides:

\[
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\begin{pmatrix}
G_{t1} \\
G_{t2}
\end{pmatrix}
=
\begin{pmatrix}
G_{o1} \\
G_{o2}
\end{pmatrix}.
\] (7.7)

![Figure 7.3: BEM surface grid of the cylinder with no extents.](image)

Here the subindices don’t represent each surface element, but the surface elements at either side of the midspan plane. The matrix \( A_{11} \) accounts for the field in region 1 created by a source in region 1, \( A_{12} \) for the field in region 1 due to sources in region 2, etc. The symmetry means that the array \( G_{t2} \) is equal to a certain reordering of the elements of \( G_{t1} \): \( G_{t2} = \tilde{G}_{t1} \). The same happens with the free field Green’s function, \( G_{o2} = \tilde{G}_{o1} \). The particular reordering (represented by \( \tilde{\ldots} \)) depends on the order of the surface elements in equation (7.7). The first block equation is:

\[
A_{11}G_{t1} + A_{12}G_{t2} = G_{o1}.
\] (7.8)

Introducing the symmetry condition it becomes:

\[
A_{11}G_{t1} + A_{12}\tilde{G}_{t1} = \tilde{G}_{o1}.
\] (7.9)
Applying a reordering to the rows of $G_{t1}$ is equivalent to applying the same reordering to the columns of $A_{12}$: $A_{12} \tilde{G}_{t1} = \tilde{A}_{12} G_{t1}$, and the system can be expressed as:

$$A_{11} G_{t1} + \tilde{A}_{12} G_{t1} = (A_{11} + \tilde{A}_{12}) G_{t1} = \tilde{G}_{o1}, \quad (7.10)$$

which is a system of $N_{e}/2$ equations. Note that the tilde in $\tilde{A}_{12}$ represents the reordering to the columns, and not the rows, of $A_{12}$.

### 7.2.3 Grid size

The frequency resolution depends on the mesh spacing. The roughness noise peak frequency measured in the experiments of Alomar et al. [44] was $f_{rn} \approx 0.18 U_{max}/h = 2.7$ kHz, which corresponds to a wavelength of 0.13 m. It has been required to have at least ten points per wavelength for a correct resolution, so a grid spacing smaller than 0.013 m. For a cylinder with a length of 2.2 m and a diameter of 0.127 m, it corresponds to 176 (spanwise) $\times$ 32 (azimuth) grid points. Finer meshes than this have been used in all cases. The results of a mesh sensitivity study are shown in figures 7.4 and 7.5, where a coarse, a medium and a fine grid are compared. It is observed that the medium and the fine meshes provide similar results at the peak frequency. The medium grid has been used in the roughness noise predictions. Evidently the required grid density depends on the frequency that has to be calculated, as well as on the body dimensions. Figure 7.6 shows the calculated Green’s function at various frequencies, with the coarse grid $220 \times 40$. For frequencies up to 2 kHz the solutions are smooth in most of the angles. However at the highest frequency it is not.

### 7.2.4 Cylinder length

The effect of the cylinder length is shown in figure 7.7. At an observer distance of $|x| = 4D$, the two smaller cylinder lengths present significant variation, but the two highest are similar, indicating a certain convergence. A length $L = 23.6D$ was chosen.
7.2.5 Location of roughness noise sources

It is known that the accuracy of the calculated Green’s function decreases for sources in the vicinity of the surface, i.e. for distances source-wall of the order of the cell size. This is the present case, and this problem has been confronted in the following way. The first step of the BEM solver calculates the (exact) Green’s functions for on-surface sources. It was observed that the levels of the Green’s function obtained with on-surface sources are close to twice the levels for sources located close to the wall (figure 7.8, sources at $z = 0.07D$). This is justified theoretically by taking the limit in which the source-wall distance goes to zero. In that case this approximation is exact (analytically). In our case there are enough dipole sources when they are located exactly on the cell centres. Therefore, the final Green’s function has been taken as twice the value of the Green’s function with on-surface sources located on the cell centres. This approximation allows to diminish the computation time, since the second step of the method, which calculates the Green’s function for off-surface sources, is eliminated.
7.3 Extension of roughness noise model to bluff bodies

7.3.1 TBL

The TBL is described in Howe’s roughness noise model by the wall pressure wavenumber-frequency spectrum along the TBL, for which there exist semi-empirical models in the literature, in the case of smooth wall ZPG TBLs. Under Townsend’s similarity hypothesis [47] the turbulent structure of ZPG TBLs over smooth and rough walls at high Reynolds numbers are equivalent. Recently there has been evidence in favour of it [48, 49], and the evidence against it found in other studies for certain types of roughness was attributed to their large values of $h_s/h$ causing $h_s \approx \delta$ ($h_s$ is the equivalent roughness height). The surface pressure wavenumber-frequency spectrum scales with the friction velocity $u_r$, the displacement thickness $\delta^*$, the outer velocity $U_o$ and the convection velocity $U_c$ [45]. Their streamwise evolution is well modelled for fully developed ZPG TBLs. Liu and Dowling [45] and Alomar et al. [44] used smooth wall models of the surface pressure wavenumber-frequency spectrum in the calculation of roughness
noise radiated by ZPG TBLs with dense distributed roughness, and good agreement was found with the experimental measurements.

Muck et al. [80] tested the effect of mild convexity on a smooth wall TBL, and observed that the curvature attenuated the Reynolds stresses, without changing significantly the eddy structure. Schloemer [81] observed that, for smooth walls, a moderate FPG caused a decrease from $\delta^*/\delta \approx 1/7$ to $\delta^*/\delta \approx 1/13$, and an increase from $U_c/U_o \approx 0.65$ to $U_c/U_o \approx 0.8$. Cal et al. [82] tested rough wall TBLs under a FPG. It was observed that the effect of roughness on the Reynolds stresses ($\langle u_1^2 \rangle$ and $\langle u_2^2 \rangle$) was absorbed when they are scaled with $u_\tau$, independently for ZPG and FPG. But this scaling didn’t absorb the FPG. Presently, the correct scaling of the mean and turbulent fields to account for the FPG is not known. This invalidates Townsend’s similarity hypothesis for FPG TBLs.

He also observed that while roughness increased the value of $\delta^*/\delta$ in absence of FPG, the FPG caused it to decrease. Finally, a FPG increased significantly the skin friction coefficient of rough walls. These studies only considered single point measurements, and not the evolution along the TBL.
Two-dimensional simulations (figure 8.10a) as well as experimental evidence [33] reveal that there is a FPG until $\theta \approx 75^\circ$, and then an APG until separation, which happens shortly after.

In addition to the rough wall ZPG TBL models and the experimental results of Achenbach [32, 33], the outer velocity $U_o(\theta)$, TBL thickness $\delta(\theta)$ and friction coefficient $c_f(\theta)$ along the TBLs were obtained from two-dimensional and three-dimensional simulations of a smooth cylinder ($L = 2D$), and a rough cylinder with Spheres4mm ($L = 0.26D$). Due to the short span length of the rough cylinder the results are incorrect downstream of the separation point since the wake cannot be resolved. Only the upstream attached flow was used. Computational limitations due to the need of resolving the flow around the roughness elements are the cause of the short cylinder span.

Finally, the extent of the TBL has to be fixed in the model, since it detaches of the bluff body at a certain angle which may depend on the roughness configuration and the Reynolds number. TBLs over smooth circular cylinders detach later than on rough
FIGURE 7.8: $|G_t|$ for on-surface and off-surface sources, a cylinder length $L = 23.6D$, the grid $360 \times 56$, (13 points per wavelength at $f = 3200$ Hz) and observer and sources located as in figure 7.2.

cylinders due to the thickening of the TBL and its higher momentum deficit in the wall normal direction [34].
7.3.2 Coupling of roughness noise model and the tailored Green’s function

The surface dipole strength distribution $\Lambda(\omega)$ (equation (2.46)) has been coupled with the computed tailored Green’s function. The Green’s functions corresponding to two sources separated a small distance have been determined, and therefore the acoustic field created by this pair of monopoles, with the same strength and opposite phase is:
\[ p_{\text{dip}}(x, \omega) = p_1(x, \omega) + p_2(x, \omega) = a(\omega)(G_t(x, y_1, \omega) - G_t(x, y_2, \omega)) \]  
(7.11)

The power spectral density of \( p_{\text{dip}}(x, \omega) \) is:

\[ P_{\text{dip}}(x, \omega) = \Lambda(\omega)|G_t(x, y_1, \omega) - G_t(x, y_2, \omega)|^2 \]  
(7.12)

since the Green’s function is a deterministic function. The function \( \Lambda(\omega) \) is the source strength spectrum, which was determined previously. The final acoustic field emitted by a rough surface that contains \( N = \sigma / \pi h^2 \) roughness elements (dipoles) per unit area is the sum of the sound emitted by all area elements:

\[ P_R(x, \omega) = \sum_{i=1}^{N} \Delta S_i N \Lambda_i(\omega)|G_t(x, y_{1i}, \omega) - G_t(x, y_{2i}, \omega)|^2 \]  
(7.13)

The far field noise is independent of the distance between the monopoles in each dipole, \( l_{\text{dip}} \), since \( \Lambda(\omega) \propto l_{\text{dip}}^{-2} \), but \( |G_t(x, y_{1i}, \omega) - G_t(x, y_{2i}, \omega)|^2 \propto l_{\text{dip}}^2 \), so \( P_R(x, \omega) \) is independent of \( l_{\text{dip}} \). The only requirement is that \( l_{\text{dip}} \ll \lambda \). It has been taken \( l_{\text{dip}} = 0.11D \). The difference of the dipole field obtained with two different values of \( l_{\text{dip}} \) is shown in figure 7.13 and it is small.
7.3.3 Sensitivity analysis of the TBL parameters

It was found that noise levels depend strongly on the parameters and functions defining the TBL from the upstream stagnation line until separation, as well as on the separation angle. These parameters and functions are: the displacement thickness to TBL thickness ratio $\delta^*/\delta$, the convection velocity to outer velocity ratio $U_c/U_o$, the friction coefficient along the TBL $c_f(\theta)$, and the separation angle $\theta_{sep}$ (it is assumed that no roughness noise is radiated in $\theta > \theta_{sep}$). A study of the sensitivity of the noise spectrum to changes of these parameters has been performed.

The functions $c_f(\theta)$ that have been used were obtained from two-dimensional $S-A$ simulations of dense surface bars with $h/D = 0.031$ at $Re = 1.7 \times 10^5$, and the measurements of Achenbach [33] on a rough cylinder with $h/D = 0.0045$, at $Re = 3 \times 10^6$. Extrapolating these trends to other Reynolds numbers and other roughness sizes is required but it is a source of inaccuracy. The effect of changing the absolute levels of $c_f(\theta)$ (obtained from two-dimensional simulations) on the noise spectrum at two observation angles is shown in figure 7.14. A 20% increase in $c_f(\theta)$ induces an increase of about 2 dB/Hz.
across all the spectral peak and all observers. To reach the measured noise levels in M2, M3, M4 and M5 the friction coefficient must be multiplied by about 0.5.

The sensitivity on the separation angle is weak when the skin friction corresponding to rough cylinders is used (from S-A simulations and the experimental results of Achenbach [33], figure 7.10), as shown in figure 7.15. This is due to the low values of the friction coefficient for $\theta > 75^\circ$.

The sensitivity on the ratio $\delta^*/\delta$ is shown in figure 7.16. A 50% increase in $\delta^*/\delta$ causes a 3 dB/Hz increase of the far field PSD, as well as a small decrease of the peak frequency.

Finally the sensitivity of the noise PSD on the ratio $U_c/U_o$ is shown in figure 7.17. A 20% increase of $U_c/U_o$ causes an increase of approximately 1 dB in the peak level and a slight increase of the peak frequency.

Finally it should be noted that there might be significant variations of the previous parameters along the TBL, and to capture that variation might be necessary to calculate the noise levels with the desired accuracy.

### 7.4 Model predictions and performance

The parameter values that best fit the experimental trends are $c_f(\theta) = 0.6 \times c_f(\theta)_{\text{roughcyl,}(S-A)}$, $\delta^*/\delta = 1/6$, $U_c/U_o = 0.75$ and $\theta_{\text{sep}} \approx 77^\circ$. With them the predicted spectra at different observation angles are shown in figure 7.18. The peak levels are well predicted at $\theta = 21^\circ, 39^\circ, 61^\circ$ (measured from downstream). At the downstream observers $\theta = 83^\circ, 94^\circ$ the peak levels are overpredicted by about 5 dB/Hz. The peak frequency is well predicted except at $\theta = 61^\circ$, where it is overpredicted, and at $\theta = 21^\circ$ is slightly underpredicted.

The directivity pattern of the peak level is shown in figure 7.19. For comparison the predictions using the flat plate model directly extended to the cylinder is plotted. It has been obtained by considering the flat wall tangent to the circular cylinder at the surface element that radiates noise. It is clearly observed the overprediction of the peak level at the over-head observers. Changing the value of the TBL parameters can diminish the difference in level at over-head observers, but it also increases the difference at downstream observers.
The cause of the miss-prediction is hypothesized to be mainly due to the failure of the wall pressure wavenumber-frequency spectrum of ZPG smooth TBLs. As observed by Cal et al. [82] the existence of a FPG changes significantly the Reynolds stresses levels through all the TBL. In addition, for $\theta > 75^\circ$ appears an APG, which can change the TBL structure even more, and that region generates a significant amount of acoustic energy because the outer velocity is high.

\[ f \text{(Hz)} \]
\[ \text{SPL (dB/Hz)} \]

(a) $\theta = 45^\circ$.

(b) $\theta = 90^\circ$.

**Figure 7.14:** Effect of multiplying $c_f(\theta)$ of a rough cylinder by a constant on the noise spectrum at $Re = 3.2 \times 10^5$. 
Figure 7.15: Effect of changing $\theta_{sep}$ on the noise spectrum at $Re = 3.2 \times 10^5$.

### 7.5 Summary

A bluff body roughness noise prediction model has been developed and implemented, for the particular case of a circular cylinder covered with distributed dense roughness of height $h/D = 0.031$ and surface density $\sigma \approx 0.5$. The model is an extension of the flat wall roughness noise model of Howe to the case of bluff bodies. BEM is used to
Chapter 7. Distributed Roughness. Circular Cylinder Roughness Noise Model

Figure 7.16: Effect of changing $\delta^*/\delta$ on the noise spectrum at $Re = 3.2 \times 10^5$.

calculate the Green’s function tailored to the cylinder, which accounts for the scattering around the bluff body and the propagation to the far field. The roughness noise peak frequencies are well predicted, and the peak levels are well predicted except for over-head observers, where the predicted levels are about 5 dB/Hz higher than the measurements. The reasons for this failure are thought to be a deficient modelling of the TBLs, which are described by a ZPG (smooth wall) model of the wall pressure wavenumber-frequency
Figure 7.17: Effect of changing $U_c/U_o$ on the noise spectrum at $Re = 3.2 \times 10^5$.

spectrum. The attached TBLs are potentially not well developed and have a strong FPG followed by an APG and separation, which can significantly affect the turbulent structure. To improve the model predictions the TBLs need to be described with more accuracy.
Figure 7.18: Predicted and measured SPL spectra at various observation angles, for Spheres 4.5mm. Predictions with $c_f = 0.6c_{f,ROUGHCYL(S-A)}$, $U_c = 0.75U_o$ and $\delta^* = \delta/6$.

Figure 7.19: Far field noise directivity at $Re = 3.2 \times 10^5$, compared with the flat wall model and the measurements.
Figure 7.20: Far field noise directivity at $Re = 3.2 \times 10^5$, compared with the flat wall model.
Chapter 8

Two-Dimensional Roughness.
Fully Covered Circular Cylinder

8.1 Introduction

It is not possible to simulate circular cylinder flow resolving dense distributed roughness with a large enough aspect ratio, due to computational limitations. Therefore in the three-dimensional case other configurations, also relevant to real applications, have been considered.

- Dense bars of hemispherical and rectangular section on the cylinder surface (figure 1.8) have been studied with two-dimensional simulations (URANS, S-A). The bar height and distance between bars are the same as the roughness height and the distance between roughness elements of the distributed roughness (experimental) configurations.

- Two sparse bars of circular section on the surface of a circular cylinder (figure 1.9a,b) have been studied using three-dimensional (DDES) and two-dimensional simulations (URANS, $k\omega$-SST, S-A). Also sparse circular section bars on the surface of a flat plate in cross flow have been tested experimentally in the anechoic chamber (figure 1.9c).
Three configurations, each consisting of three off-surface bars of circular section (figure 1.12) have been studied using three-dimensional (DDES) and two-dimensional simulations ($k\omega$-SST).

The configurations of bar fetches are a simple model of a landing gear main strut, which has attached an off-surface bar fetch, as is shown in figure 1.11. The difference in noise radiation between three locations of the bar fetch (upstream, side and downstream) have been studied.

The two-dimensional simulations are necessary in the dense configurations due to computational limitations ($x_2^+ < 4$). In the sparse configurations and the bar fetches they are only complementary to the three-dimensional simulations, and are useful to assess the relevance of three-dimensional effects, as well as between the various turbulence models.

Aerodynamic information has been extracted and analysed for its implication on the far field noise. The aerodynamic data consist of force histories and spectra, on-surface mean and RMS pressures, and mean and RMS velocity fields. The far field noise spectra have been estimated using the compact source assumption, so they are only strictly valid in the range of large wavelengths compared to the cylinder diameter $\lambda >> D$, i.e. $St << 1/M = 5$ at $M = 0.2$.

The configurations tested are Smooth, Spheres4mm, Spheres4.5mm, Cylinders4mm, CablePar8, FetchFront, FetchSide and FetchBack in the two-dimensional simulations, and Smooth, Spheres4mm, CablePar8, CablePar16, FetchFront, FetchSide and FetchBack in the three-dimensional simulations. Additionally the configurations CablePerp15 and CablePerp40 of surface helicoidal cable were tested experimentally. The inflow velocities were 20 m/s and 70 m/s, and the Reynolds numbers based on the smooth cylinder diameter were $1.7 \times 10^5$ and $10^6$. The maximum flow velocity corresponds to a Mach number of 0.21. Using the following relation the order of magnitude of the density changes can be estimated:

$$\frac{\Delta \rho}{\rho} \approx \frac{1}{2} M^2 = 0.022.$$ (8.1)
These density changes are considered small enough for the compressibility effects to be neglectable.

The results are summarised in table 8.1. The recirculation length is the distance from the cylinder axis to the location of zero mean streamwise velocity.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Approach</th>
<th>Re</th>
<th>L/D</th>
<th>St</th>
<th>$C_d$</th>
<th>$\sqrt{C_l^2}$</th>
<th>$L_r/D$</th>
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<td>Smooth</td>
<td>Exp.</td>
<td>$3.2 \times 10^5$</td>
<td>2.8</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>0.20</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>0.20</td>
<td>-</td>
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<td>2.8</td>
<td>0.20</td>
<td>-</td>
<td>-</td>
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<td>0.20</td>
<td>-</td>
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<tr>
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<td>-</td>
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<td>CablePerp40</td>
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<td>0.20</td>
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<td>-</td>
<td>-</td>
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<tr>
<td>FetchBack</td>
<td>Exp.</td>
<td>$3.2 \times 10^5$</td>
<td>2.8</td>
<td>0.26</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>1.05</td>
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<td>0.89</td>
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<td>-</td>
<td>0.27</td>
<td>0.69</td>
<td>0.24</td>
<td>1.51</td>
</tr>
</tbody>
</table>

Table 8.1: Summary of experimental and numerical results.

The power spectral densities of the drag and lift forces ($D_F$ and $L_F$, respectively) have been calculated using:
\[
\text{PSD}(C_d) = \log_{10} \frac{D_F(\omega)}{\left(1/2\rho_o U_o^2 LD\right)^2},
\]
(8.2)

\[
\text{PSD}(C_l) = \log_{10} \frac{L_F(\omega)}{\left(1/2\rho_o U_o^2 LD\right)^2},
\]
(8.3)

where

\[
D_F(\omega) = \frac{2|\text{FFT}(D_F(t))|^2}{n \cdot F_s},
\]
(8.4)

\[
L_F(\omega) = \frac{2|\text{FFT}(L_F(t))|^2}{n \cdot F_s}.
\]
(8.5)

The far field noise spectra have been determined analogously to the experimental data, but the length of the signals is necessarily shorter. The sampling frequency was \(10^5\) Hz. The number of samples per block is 8192 and at least 4 blocks were averaged to obtain all spectra. The difference between averaging the noise spectrum over 4 blocks and 120 blocks was calculated from the experimental data. The resulting spectra showed a maximum difference of 1.5 dB/Hz in all the frequency range. The initial transient was not included to calculate the spectra, mean flow field and mean forces. In all cases the resulting spectra were weakly sensitive to the addition of one more block.

### 8.2 Smooth

The three-dimensional and two-dimensional simulations present values of drag and lift coefficients, and shedding peak Strouhal number similar to previous studies \[30, 84, 86\]. The drag coefficient and shedding peak Strouhal number are typical of supercritical flows. From the skin friction curve shown in figure 8.10 obtained from two-dimensional simulations (S-A), separation occurs where \(c_f = 0\), i.e. at \(\theta \approx 100^\circ\). This value corresponds to supercritical flow.

The \(Q\)-isosurfaces \((Q = 10)\) are shown in figure 8.1 for \(Re = 1.7 \times 10^5\). Resolved eddies of various sizes along the span are observed.
Chapter 8. Two-Dimensional Roughness. Fully Covered Circular Cylinder

The difference in level of the forces obtained from two-dimensional (kω-SST) and three-dimensional simulations (figure 8.2) is attributable to the decrease of shedding coherence in the three-dimensional case. Between the two turbulence models used in the two-dimensional simulations, S-A provides significantly lower force levels than kω-SST, due to its highly dissipative nature. The fact that S-A has similar levels of both drag and lift coefficients than three-dimensional DDES suggests that its dissipation compensates for the loss of shedding coherence of the real three-dimensional case to get the correct force values per unit of span.

Figure 8.8a (top) shows the mean streamwise velocity field of the smooth cylinder from PIV measurements (roughness in the PIV tests are not cables, but distributed roughness). Comparing the results of the smooth cylinder from PIV and three-dimensional simulations a similarity is observed (figure 8.8a (top) and 8.8b (top)). PIV doesn’t capture the separation point (due to wall reflections), but the angle and position of the shear layer after detachment agrees well with the simulations. The shear layers spreading rates are also close. The RMS streamwise velocity fields obtained from PIV and three-dimensional simulations are also remarkably similar (figure 8.9a (top) and 8.9b (top)).

Figure 8.3 compares the three-dimensional and two-dimensional simulations. Big differences are observed, especially for kω-SST. This model presents a faster mixing rate (spreading) of the shear layers which results in a significantly shorter recirculation
length than three-dimensional DDES and two-dimensional S-A (table 8.1). The last two agree better with each other (regarding the mean velocity). However the shedding peak Strouhal number of $k\omega$-SST is closer to the three-dimensional case than S-A (table 8.1). The RMS streamwise velocity plots are shown in figure 8.3b. The levels of the different turbulence models agree with the force levels, i.e. higher mean drag and lift fluctuations are associated with higher velocity fluctuations in the near wake.

**Figure 8.2:** $C_l$ of Smooth for three-dimensional (DDES) and two-dimensional (S-A, $k\omega$-SST) simulations.

**Figure 8.3:** Mean and RMS streamwise velocity fields of the smooth cylinders at $Re = 1.7 \times 10^5$. 
In the case that the cylinder is acoustically compact, i.e. \( \lambda \gg D \), the far field noise can be calculated from the overall force history on the cylinder. Takaishi et al. \cite{91} compared the far field noise of a smooth circular cylinder using the full Curle’s solution with the solution for an acoustically compact cylinder and found agreement for acoustic wavelengths as low as \( \lambda/D \approx 2.3 \), which corresponds to a Strouhal number:

\[
St = \frac{fD}{U_o} = \frac{D}{\lambda} \frac{c}{U_o} \approx \frac{1}{2.3M} = 2.2, \text{ for } M = 0.2. \tag{8.6}
\]

The far field noise of an acoustically compact body is \cite{18}:

\[
p_{\text{compact}}(x,t) = \frac{x_i}{4\pi c|x|^2} \frac{dF_i(t - |x|/c)}{dt}, \tag{8.7}
\]

where \( F_i(t) \) is the instantaneous force acting on the cylinder.

Figure 8.4 shows the far field noise spectra of a smooth cylinder of length \( L = 0.448 \) m and diameter \( D = 0.222 \) m from experiments and three-dimensional simulations (assuming the cylinder is compact). The spectra obtained at different speeds and different cylinder sizes were obtained assuming the following scaling law for compact sources of surface area \( S \):

\[
\overline{p^2} \propto \frac{U_o^6 S}{r^2} \approx \frac{U_o^6 L D}{r^2}. \tag{8.8}
\]

Spectra from both experiments and simulations have a vortex shedding peak. In the simulations it has a Strouhal number of 0.35, whereas in the experiments it is 0.25. The value obtained in the simulations is similar to the ones obtained by Travin et al. \cite{84} (DES) and Squires et al. \cite{86} (DDES). The difference between the experiments and simulations is thought to be due to the difference in Reynolds number as well as the effects that the strips introduce. There is a difference of about 7 dB in level of the peaks, and the fall-off rate is higher for the simulations, causing the difference to increase up to 15 dB at \( St \approx 5 \), where the compact cylinder model ceases to be valid. This remarkable difference in levels is caused by the high dissipation of the low order numerical schemes, as well as the turbulence model.
8.3 Surface parallel cables

The configurations considered in this section are the dense parallel cables Spheres4mm, Spheres4.5mm, Cylinders4mm, which have been studied using two-dimensional simulations (S-A), and the sparse parallel cable configurations CablePar8 and CablePar16, studied with both two-dimensional ($k\omega$-SST and S-A) and three-dimensional simulations (DDES).

The $Q$-isosurfaces ($Q = 10$) of the cylinder with sparse cables are shown in figure 8.5. Resolved eddies of various sizes along the span are observed in both configurations, both on the upstream face and in the near wake.

It must be remarked that the dense cable configurations correspond to ‘D-type’ roughness [50], which means that the high blockage of one cable to the immediately downstream cable causes the presence of highly isolated recirculation regions between them, and the dependence of the TBL mean flow on the roughness size is different than in ‘K-type’ roughness, i.e. three-dimensional distributed roughness, such as sandpaper. ‘D-type’ roughness presents, in general, lower friction coefficient and lower TBL thickness.

The force histories on the rough cylinders are shown in figure 8.6. The three-dimensional simulations of the sparse cable configurations present significantly higher levels of $C_d$ and $\sqrt{C_d'^2}$ than the smooth configuration. The three-dimensional force histories appear highly irregular (non-periodic), as expected. This large irregularity is lacking in the
patterns of two-dimensional simulations. The forces corresponding to CablePar8 are the largest between the three-dimensional simulations, followed by CablePar16 and Smooth. The two-dimensional simulations of CablePar8 present highly regular patterns, like the smooth case, but with harmonics at higher frequencies.

There is a significant difference between the patterns and levels of $k\omega$-SST and S-A. In this case the latter presents larger $C_d$ and $C_l$, as opposed to the smooth configuration. It must be remarked that the pressure and velocity fields obtained with S-A are not symmetric with respect to the symmetry plane, giving a non-zero mean lift. This asymmetry is not observed in $k\omega$-SST or in the three-dimensional simulations. The origin of the asymmetry is unknown.

The dense cable two-dimensional configurations Spheres4mm and Cylinders4mm, approached only with S-A, present remarkably larger mean drag and lift fluctuations than the smooth configuration. Its patterns are highly regular, and no high frequency content is discernible, as opposed to the patterns of CablePar8 obtained with S-A, which have discernible higher frequency harmonics.

The power spectra of the drag and lift coefficients are shown in figure 8.7. In the $C_l$ spectra of CablePar8 and CablePar16 shedding peaks are discernible at $St \approx 0.25$, lower than Smooth ($St \approx 0.35$), and the peaks are also broader. This, together with the larger $C_d$ and $\sqrt{C_l^2}$, suggests early separation caused by the cables. It appears that lower cable
surface densities cause larger forces. The reason could be related to the fact that the attached flow adapts better to denser distributions of obstacles, and $\theta_{\text{sep}}$ is larger. The lower values of $St$ are in agreement with early separation and a wider wake. The fall-off rates are similar to Smooth. The $C_d$ spectra of CablePar8 and CablePar16 present significantly higher broadband levels than Smooth.

The dense cables (figure 8.8b) cause an increase of the shear layer spreading rate and a decrease of the recirculation length. The cylindrical and rectangular section cables (figure 8.8b, centre and bottom) show similar patterns, but the latter has slightly thinner TBL’s at separation, which detach later, causing a larger pressure recovery (figure 8.10) and a lower mean drag. Both configurations show a lower pressure recovery than the smooth cylinder. The TBL separation location of the rough cylinders is difficult to determine from the skin friction trends (figure 8.10c). The skin friction of the rough cylinders presents clear patterns that repeat on each cable. The peaks are located close to the top of the cables and have values significantly larger than the smooth case. At the sides of the cables and the region between them the skin friction is significantly lower, since the flow is detached in these regions. As in all TBLs above dense roughness, the highest mean velocity gradients (and shear stresses) are located above the cables. The wall viscous shear stress ($\tau_\omega = \mu \partial U / \partial n$) is shown in figure 8.11. Only the values at the top of the roughness elements are significant, where the outer flow is attached and the skin friction is mainly due to viscous stress (as on a smooth wall TBL).

Figure 8.12 shows the mean streamwise velocity profiles in the wake. The cylinders with surface parallel cables have recovered a greater amount of mean velocity than the smooth case, especially the sparser configuration, and their wake is wider. The increase of wake width of the rough cylinders is higher than the corresponding to the higher diameter caused by the presence of the cables. The two-dimensional S-A simulations of dense surface bars captures, at least qualitatively, the effect of distributed roughness observed in previous experiments. This is shown in figures 8.8b, 8.9b and 8.10b,c. Roughness causes the boundary layer at separation to be thicker, and so are the separated shear layers, which have a higher momentum transfer normal to them. This phenomenon was observed to happen not only for distributed roughness, but also for parallel surface bars and surface rings. Therefore the mixing length is shorter. The velocity fluctuations in Spheres4mm are larger than in Cylinders4mm. This may be explained by the fact that the flow above the rectangular cables is more isolated from the recirculating flow.
between the cables, and the mixing caused by roughness is lower. The configuration
with rectangular cables is closer to a wall with axial grooves than the hemispherical
cable configuration.

The far field noise spectra are shown in figure 8.16 at two observation angles. The force
spectra show the same patterns, as expected. At $Re = 10^6$ there is a difference of about
10 dB between the shedding peak levels of the rough cylinders and the smooth one, and
the lower peak Strouhal numbers are appreciated, especially of the sparser configuration.
The highest differences appear for larger frequencies. The rough cylinders present larger
noise levels than the smooth cylinder in the range $1 < St < 20$. At $St \approx 5$, the limit
of validity of the compact cylinder assumption, the level difference reaches $\approx 40$ dB at
$\theta = 45^\circ$, and $\approx 30$ dB at $\theta = 90^\circ$. At $Re = 1.7 \times 10^5$ the vortex shedding peaks of the
smooth and rough cylinders are broader, and the peak Strouhal numbers of the rough
cylinders are noticeably higher. Also, at both $\theta = 45^\circ$ and $\theta = 90^\circ$ there is a peak for
CablePar16 configuration at $St \approx 4$, only present at the lower Reynolds number. Scaling
the frequency with the cable diameter and the maximum outer velocity instead of the
cylinder diameter the peak Strouhal number is $St_c = fD_c/U_{max} \approx 0.12$. CablePar8
presents a peak in $\theta = 45^\circ$, at $St \approx 6$ also only at the lower Reynolds number.

Directivity trends are presented in figure 8.17 corresponding to the shedding peak
Strouhal number band (figure 8.17a) and a mid-frequency band (figure 8.17b). The
directivity patterns in the shedding peak range have the maximum and the minimum
levels in over-head and upstream/downstream observers, respectively, in all three con-
figurations and both Reynolds numbers. The difference between the maximum and
minimum levels changes between the configurations though. The smooth cylinder at
the higher Reynolds number presents the maximum level difference, of about 22 dB.
The sparser cable configuration, CablePar8, has the lowest level difference, 12 dB, at
the lower Reynolds number. In the higher frequency range $0.6 < St < 5$ the patterns
of the rough cylinders are different than in the shedding peak range. CablePar16 and
CablePar8 present the maximum level in upstream/downstream observers and the min-
imum in over-head observers, whereas Smooth presents the opposite pattern (same as in
the peak frequency range). This implies that for the rough cylinders, in this frequency
range the noise radiation is dominated by the drag dipole, rather than the lift dipole.
The lift dipole dominates all the frequency range in Smooth. The trends for the lower
and higher Reynolds numbers are similar, the levels at $Re = 1.7 \times 10^5$ being higher than at $Re = 10^6$. 
(a) $C_d$.

(b) $C_l$.

(c) $C_l$ of Smooth and CablePar8 for three-dimensional (DDES) and two-dimensional ($k\omega$-SST, S-A) simulations.

**Figure 8.6:** Force coefficient histories of surface bar configurations.
Figure 8.7: PSD of the drag and lift coefficients of cylinders with parallel cables at two Reynolds numbers.
Figure 8.8: Mean streamwise velocity fields of various roughness configurations.
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Figure 8.9: RMS streamwise velocity fields of various roughness configurations.

(a) Experimental, $Re = 1.9 \times 10^5$.

(b) Simulations, $Re = 1.7 \times 10^5$.

(c) Simulations, $Re = 10^6$.

(d) Simulations, $Re = 10^6$ (top), $Re = 1.7 \times 10^5$ (centre and bottom).
Figure 8.10: Surface pressure and skin friction of various configurations.
The mean velocity profiles obtained in the simulations were used to obtain the TBL thickness and outer velocity evolution in the attached flow region. The main goal was to use them in the circular cylinder roughness noise model, instead of the flat wall models.

Figures 8.13 and 8.14 show the TBL thickness growth for various configurations. Since the outer velocity is not uniform, in the cases where the velocity profile has a maximum the outer limit of the TBL has been taken at that height. The rough flat wall model is plotted for comparison. Since the flat wall friction coefficient model is independent of the Reynolds number, so is the flat wall thickness growth rate. The dense distributed roughness presents the largest thickness (figure 8.13a). CablePar8 and CablePar16 present very similar thickness evolutions for $45^\circ < \theta < 90^\circ$, at the highest Reynolds number. However, at the low Reynolds number the difference between CablePar8 and CablePar16 increases up to 25% at $90^\circ$ (figure 8.13b).

The dense parallel bars configurations have significantly lower TBL thickness growth rates than the distributed roughness, but similar to CablePar16. This is observed in flat wall parallel bars configurations as well, and is associated with the sheltering of consecutive bars in dense arrangements (‘D-roughness’, [50]).
The two-dimensional and three-dimensional simulations of \textit{CablePar8} are compared in figure 8.14b. The $k\omega$-SST model underpredicts the TBL thickness of the three-dimensional DDES simulations by up to a 20% at $\theta = 90^\circ$. The S-A model presents a different pattern in the range $70^\circ < \theta < 90^\circ$, which suggests that it is not capable of predicting even qualitatively the flow field.

The outer velocity plots are shown in figure 8.15. The potential flow solution has been plotted for comparison. All simulations present lower values than the potential solution. All parallel bars configurations, dense and sparse, have similar outer velocity patterns from $\theta = 55^\circ$, within 10%. These are in agreement with the PIV measurements of the smooth cylinders and cylinders with distributed roughness, which present a maximum outer velocity of about $1.7U_o$.

In figure 8.15b the patterns from two-dimensional and three-dimensional simulations, for \textit{CablePar8} at the lowest Reynolds number. The differences between both the three-dimensional and two-dimensional simulations, and the $k\omega$-SST and S-A turbulent models for the two-dimensional case, are small, up to about 13% in the range $60^\circ < \theta < 90^\circ$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{mean_streamwise_velocity_profiles.png}
\caption{Mean streamwise velocity profiles in the wake of the cylinders with parallel cables at $x/D = 6$, from three-dimensional simulations (DDES), at $Re = 10^6$.}
\end{figure}
The configuration of surface bars in cross flow on a flat plate was tested to explore the noise sources and compare it with the parallel cable configurations on the cylinder.
Figure 8.14: TBL thickness for various configurations, normalised to the cylinder diameter, and to a roughness height $h/D = 0.031$. 

(CablePar8 and CablePar16). Measured noise spectra at three observation angles are presented in figure 8.18. A Strouhal-based peak is observed at $St_b = f D_b/U_o \approx 0.13$ ($D_b = 0.01$ m). This value is close to the one obtained in CablePar16, and they are thought to be caused by the same flow mechanism. Since a circular cylinder on a flat wall in cross-flow (with $D_b/\delta \approx 1$) doesn’t induce vortex shedding [89], but it induces
transition to turbulence in its wake, the noise sources are thought to be associated with the interaction of turbulent eddies with downstream cables/bars. This is similar to the case of roughness noise from distributed roughness, with two main differences. Firstly, the scattering of the turbulent stresses can differ due to the change in roughness geometry. And secondly, the structure of the turbulent field above the cables/bars and the roughness elements might be significantly different. Sparse bars/cables are known
to induce TBLs with different structure than dense roughness for fully developed TBLs \cite{90}. For developing TBLs (low $D_b/\delta$ and $h/\delta$) the differences are expected to be even larger.

The broadband noise levels are higher than Smooth especially in upstream observers.
The directivity plots are shown in figure 8.19, and the OASPL levels are obtained integrating the spectra in the band 0.09 < Stb < 4. The maximum levels are perceived by upstream and downstream observers, and upstream observers perceive higher levels than downstream observers. This pattern is similar to the one of Spheres4mm and Cylinders4mm (distributed roughness) for the roughness noise peak, but in that case
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it is centred at \( f h / U_o \approx 0.18 \). The noise scales approximately with \( U_o^6 \) (table 8.2). A scaling with the sixth power of velocity has a sound physical interpretation as dipole noise due to the scattering of compact sources on the bars. The sources can be associated with shedding, as well as with turbulent stresses present in the TBL above the bars (roughness noise).

<table>
<thead>
<tr>
<th>CONFIGURATION</th>
<th>70 Hz &lt; f &lt; 200 Hz</th>
<th>f \approx f_{rn}</th>
<th>f \gg f_{rn}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>7.5</td>
<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>Spheres4.5mm</td>
<td>7.5</td>
<td>6.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Bars10mm</td>
<td>7.4</td>
<td>6.2</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Table 8.2: Velocity scaling exponents for Bars10mm, determined from the signals measured at M4 and M7. (Smooth and Spheres4.5mm shown for comparison).

\[ \text{SPL}^{1/3} + 20 \log_{10} r \ (\text{dB}) \]

\[ \theta = 83^\circ \text{ - Smooth} \]
\[ \theta = 41^\circ \text{ - Smooth} \]
\[ \theta = 124^\circ \text{ - Bars10mm} \]
\[ \theta = 83^\circ \text{ - Bars10mm} \]
\[ \theta = 41^\circ \text{ - Bars10mm} \]

Figure 8.18: Far field noise spectra of Bars10mm at various observation angles, at \( U_o = 40 \text{ m/s} \).

8.5 Helicoidal surface cable

The configurations considered in this section are the helicoidal cable configurations tested experimentally in the anechoic chamber CablePerp6, CablePerp15 and CablePerp40.

The noise spectra of the cable configurations are shown in figure 8.20. At all Reynolds numbers and for all surface cable densities, the far field noise spectra collapse between them and with the smooth cylinders in the range \( 1.5 < St < 30 \), to within \( \pm 1 \text{ dB} \) (figures 8.20a,b). This shows that, even for surface cable density as high as \( \sigma = 0.5 \),
approximately streamwise cables don’t affect noise generation, either by roughness noise or by a change of the fall-off spectral levels.

It is known that, for a fully developed ZPG TBL over a flat wall with streamwise riblets, the diagonal Reynolds stresses (in wall units) normal to the riblets, and parallel to the wall, are only slightly higher than for a smooth wall, and the streamwise Reynolds stresses are lower (riblets induce a decrease of the skin friction compared to a smooth wall) [99, 100]. Therefore, even if the cable, which can be considered as (sparse) riblets, causes transition before separation, the generation of roughness noise due to interaction of turbulent stresses with the cable, is low. The presence of a FPG is thought not to change this picture significantly.

At lower frequencies, the far field spectra have a broadband peak. It is located around $St \approx 0.7$, although, due to its weakness relative to the background levels, it is not clear whether it is Strouhal-based or not. The peak seems to scale approximately with $U_o^6$ for all configurations. The far field noise measurements of Hutcheson and Brooks [38] on cylinders with large cable wrapped helicoidally showed a broadband peak at $St \approx 0.7$, Figure 8.19: Far field noise directivity of Bars10mm within the range $0.09 < St_b < 4$, at $U_o = 40$ m/s.
Hutcheson and Brooks observed that, when the peak frequency was scaled with the cable diameter and the incoming velocity, a Strouhal number \( St \approx 0.2 \) was obtained, typical of a vortex shedding mechanism. However, it is known that the presence of the wall prevents vortex shedding \[89\]. Further experiments are required to clarify the origin of this peak.

**Figure 8.20**: Far field noise spectra of the circular cylinders with helicoidal cable, at \( Re = 3.2 \times 10^5 \) and measured at M3.
CONFIGURATION | $f \approx f_{vs}$ | $f_{vs} \ll f \ll c/D$ | $f \approx f_{rn}$ | $f \gg c/D$
---|---|---|---|---
Endplates | 3.2 | 4.1 | - | 5.1
Smooth | 5.5 | 5.7 | - | 6.9
SmoothNotTripped | 5.1 | 5.7 | - | 6.5
CablePerp6 | 4.5 | 6.1 | - | 6.5
CablePerp15 | 4.6 | 5.4 | - | 6.1
CablePerp40 | 5.8 | 6.1 | - | 6.3

Table 8.3: Inflow velocity scaling exponents for smooth and helicoidal cable configurations, determined from the signals measured at $\theta = 45^\circ$. $f_{vs}$ and $f_{rn}$ are the vortex shedding and the roughness noise peak frequencies, respectively.

Figure 8.22 shows the on-surface pressure spectra of the smooth cylinders and the cylinders with helicoidal cable. The spectra of Spheres4mm have been included for comparison. The spectra of the cable configurations are more similar to SmoothNotTripped than Smooth for all microphones. However, the differences with SmoothNotTripped are still significant, especially at $\theta = 60^\circ$ (figures 8.22c,d). For this microphone there are large differences between the various cable densities, and they have significantly higher levels than distributed roughness and Smooth. Regarding the upstream attached flow, the spectra measured at $\theta = 120^\circ$ show the cable configurations have significantly higher levels than SmoothNotTripped, but still much lower than distributed roughness (figures 8.22e,f). Cables induce unsteady flow structures on the upstream face, responsible for the higher levels of on-surface pressure fluctuations than SmoothNotTripped. These are, though, still much weaker than the pressure fluctuations of distributed roughness.

It is not clear whether cables induce transition to turbulence prior to separation. However, if there is transition, cables induce spanwise non-uniformities of the flow prior and in the surroundings of separation, and uniform TBLs, which would allow a higher vortex shedding coherency, are prevented. Only for the highest cable density (CablePerp40), a vortex shedding peak is apparent in the far field noise spectrum, at a Strouhal number of approximately 0.2, the same as the one of SmoothNotTripped, typical of the subcritical regime. Further investigation is required to interpret the observations.

8.6 Bar fetches

The configurations considered here are FetchFront, FetchSide and FetchBack, for all of which far field noise measurements and simulations have been performed. As with the
parallel bars and ring cases the aerodynamic flow field will be presented first, and after the measured and predicted far field noise will be presented and discussed.

The $Q$-isosurfaces ($Q = 10$) are shown in figure 8.23, for $Re = 10^6$. Resolved eddies of various sizes along the span are observed. In *FrontFetch* eddies shed from the frontal fetch can be appreciated. In *FetchSide*, resolved eddies are present downstream from the middle fetch bar. The upstream bar has a Reynolds number of $Re \approx 7.5 \times 10^4$ (based on $U_o$), and therefore has laminar separation. This is a source of inaccuracy because it is assumed that the attached flow is turbulent (transition is forced artificially the incoming turbulent viscosity).
Figure 8.21: Velocity scaling of cylinders with helicoidal cable. Scaling with $U_0^5$ (red), $U_0^6$ (black) and $U_0^7$ (blue), for $U_0 = 27, 34, 40$ m/s, measured at M3.
Figure 8.22: On-surface pressure spectra of cylinder with helicoidal cable at $Re = 2.2 \times 10^5$ (angle measured from downstream stagnation line).
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Figure 8.23: \( Q \)-isosurfaces \((Q = 10)\) of cylinder with bar fetches at \( Re = 10^6 \).
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Figure 8.24: Force coefficient histories of the bar fetches configurations.

The force history patterns of the cylinder with bar fetches from three-dimensional simulations at $Re = 10^6$ are shown in figure 8.24. A clear difference is observed between the drag force history of FrontSide and the rest (figure 8.24a). The mean value is significantly higher and it has a larger content of higher frequencies. The differences in fluctuating lift are less evident, but a higher frequency content of FetchSide can be appreciated (figure 8.24b). Figure 8.24c shows a comparison between the two-dimensional
Figure 8.25: Drag coefficient histories from two-dimensional and three-dimensional simulations.

and three-dimensional simulations. The two-dimensional simulation patterns are highly regular (close to periodic), especially for FetchFront and FetchBack. The FetchSide pattern presents also clear periodicity, but with larger content of higher frequencies.

The force spectra (figure 8.27) show that there are significant differences between the two different Reynolds numbers. At $Re = 1.7 \times 10^5$ the vortex shedding peaks are broader than at $Re = 10^6$, and the relative levels of the various configurations are different. At the low Reynolds number FetchFront presents a significantly larger peak level than the rest. At the highest Reynolds number FetchFront and FetchBack have similar shedding characteristics, with close peak levels. FetchSide doesn’t present a vortex shedding peak at all. Smooth and FetchBack spectra have a straight fall-off for $St > 1$, but FetchFront and FetchSide have larger high-frequency levels. The former presents a single peak at $St \approx 3.7$. The latter presents three peaks at $St = 1.5, 3, 6$.

In agreement with the force spectra, the mean streamwise velocity fields of Smooth, FetchFront and FetchBack (figure 8.28) are similar, with close recirculation lengths. The only appreciable difference is that FetchBack presents lower recirculating flow velocity values. The velocity field of FetchSide is different than the former ones. The bars break the symmetry, and the recirculation length becomes about 30% larger than the other configurations.

Figure 8.31 shows the noise spectra of the various bar fetches configurations from experiments, for a Strouhal basis (scaled with the bar diameter, not the cylinder diameter).
Roughness strips were used in all cases to ensure turbulent separation. The spectra of \textit{Smooth} and \textit{FetchBack} collapse in all the frequency range. All configurations present a vortex shedding peak, with close peak level. The peak Strouhal number of \textit{FetchFront} is close to \textit{Smooth} (and \textit{FetchBack}), but \textit{FetchSide}'s peak Strouhal number is slightly lower, at $St \approx 0.20$. 
At higher frequencies the additional noise of \textit{FetchFront} and \textit{FetchSide} is remarkable. Both have a peak at $St \approx 0.23$, which is assumed to be directly associated with the bars, and both have higher broadband levels than \textit{Smooth}, up to $St_b \approx 3$. The frequency scaling of the peaks of \textit{FetchFront} has been studied (figure 8.32). The lower frequency peak doesn’t scale with the free stream velocity (magenta arrows), whereas the higher frequency peak does, but its Strouhal number is $\approx 0.4$, so for it to be caused by direct shedding from the bars a velocity approximately $1.5U_o$ must be used (black arrows). This is plausible since the bars are located in the region where the flow is accelerating.
The lower frequency peak is observed to change its spectral shape, and it moves to lower Strouhal numbers and higher frequencies (figure 8.32c and figure 8.32d) with increasing velocity. This implies a Reynolds number dependence. The noise spectra scaled with various powers of the incoming velocity are shown in figure 8.26. The peaks scale approximately with $U_o^7$. For increasing frequencies, the spectrum of FetchFront changes gradually to a scaling with $U_o^6$.

The noise spectra from the simulations at $Re = 10^6$ present a single peak of similar level than the experiments (figure 8.33a,b), at a Strouhal number between the two experimental peaks ($St_b \approx 0.28$). At $Re = 1.7 \times 10^5$ the simulations present the same peak but it has higher level, and a slightly lower Strouhal number ($St_b \approx 0.25$). Also a harmonic of this peak is present at $St_b \approx 0.5$. From the mean velocity fields and the Q-isosurfaces it is clear that the lateral bars exhibit vortex shedding, and the peak Strouhal numbers are compatible with that. Therefore, the differences between the experiments and the simulations suggest that the simulations capture the vortex shedding from the individual bars, but not the interactions between the bars. It is also possible that the compactness assumption is not valid anymore to describe the peaks at $St \approx 4$, since it is strictly valid only for $St << 5$. In this case a full (off-surface) FW-H expansion would improve the results. In the present work it hasn’t been performed. The peak of FetchSide scales approximately with $U_o^6$. For higher frequencies, noise scales with $U_o^7$ (figure 8.26).

The comparison with the simulations is shown in figure 8.34. The peak at $St_b = 0.12$ present in the simulations at $Re = 10^6$ doesn’t appear in the experimental trends or in the simulations at $Re = 1.7 \times 10^5$. The second peak appears in all three cases. Both simulations overpredict it by about 8 dB at $\theta = 90^\circ$, and at $\theta = 45^\circ$ the high Reynolds number simulation overpredicts the peak level by about 5 dB, and the low Reynolds one underpredicts it by about 3 dB. Also the peak Strouhal number from the low Reynolds number simulations is slightly lower. As with FetchFront the far field noise from the simulations at $Re = 1.7 \times 10^5$ is closer to the experimental levels than the high Reynolds number simulations. The latter underpredict the noise levels in all the frequency range. From the on-surface RMS pressures shown in figure 8.29 it is observed that the largest pressure fluctuations correspond to the middle bar, followed by the upstream bar. Therefore most of the noise generation is due to the interaction of the wake from the upstream bar with the middle bar.
The directivity plots are shown in figure 8.36 for the shedding peak frequency range (figure 8.36a) and the higher frequency range (figure 8.36b). All configurations are dominated by the lift dipole in the peak frequency range, but for FetchSide the difference in levels between over-head and upstream/downstream observers is about 4 dB. This is much lower than the other configurations. This agrees with the weak shedding peak that appears in the spectra of this configuration.

In the higher frequency range all configurations are also dominated by the lift dipole, except for FetchSide, which has maximum levels in the upstream and downstream observers, and minimum levels close to the over-head observers. Therefore the radiated noise of FetchSide for $St > 0.6$ is dominated by the drag dipole. These trends exist at both Reynolds numbers considered.
Figure 8.27: PSD of the lift and drag coefficients of the bar fetches configurations at two Reynolds numbers.
Figure 8.28: Mean x-velocity fields of Smooth and the bar fetches configurations, from three-dimensional simulations (DDES) at $Re = 10^6$. 

(a) Smooth.  
(b) FetchFront.  
(c) FetchSide.  
(d) FetchBack.
Figure 8.29: RMS surface pressure coefficient from two-dimensional simulations, at $Re = 1.7 \times 10^5$.

Figure 8.30: Mean streamwise velocity profiles in the wake of the cylinders with bar fetches measured at $x/D = 6$, from three-dimensional simulations (DDES), at $Re = 10^6$. 
Figure 8.31: Far field noise spectra of the cylinders with bar fetches at $r = 1.97 \text{ m}$ and $\theta = 97^\circ$ ($D_b$ is the bar diameter).
Figure 8.32: Far field noise spectra of FetchFront scaled with $U_o^6$ and in a Strouhal basis ($D_b$ is the fetch bars diameter).
Figure 8.33: Far field noise spectra of Smooth and Fetchfront, at $r = 16D = 3.56$ m.

8.7 Summary

A set of circular cylinders covered with surface cables of various sections and surface densities have been studied by both numerical and experimental means, to assess the effect that they have on the radiated noise.
Both the dense and sparse bars configurations cause an increase in the vortex shedding peak level and a decrease in the peak Strouhal number. These effects are qualitatively the same as the ones observed in cylinders with distributed roughness.

Regarding the two-dimensional simulations of dense roughness (Spheres4mm, Spheres4.5mm and Cylinders4mm), the comparison of the computed mean and RMS velocity fields with the PIV measurements of the cylinders with distributed roughness shows similar
changes with respect to the smooth cylinder; the shear layers are thicker, the wake is wider and the wake velocity fluctuations are larger. As happens for distributed roughness the TBLs are thicker and detach earlier, which causes a wider wake and a lower peak Strouhal number. Accordingly, the mean surface pressure plots reveal a lower pressure recovery for the rough cylinders, as well as larger surface pressure fluctuations. The effect of rectangular section bars is weaker than hemispherical section bars, due to the
more effective isolation of the recirculation flow present between the bars and the fast flow above the elements, causing the TBLs to be thinner, more similar to a smooth wall TBL, which detaches later. This difference is not observed in the case of distributed roughness between the cylindrical and hemispherical roughness elements.

The effects of sparse cable are qualitatively the same as dense cable regarding the mean
and RMS velocity fields. However the mean and RMS surface pressure fields reveal clear differences with the dense configurations. The sparse cables don’t allow a uniform TBL to develop above them before separation and these configurations are closer to flow over obstacles than a TBL. They present an even lower separation angle than denser bar configurations. This increases not only the noise levels due to vortex shedding, in the low frequency range, but also at higher frequencies due to the interaction of turbulent stresses with the cables. Since surface cables don’t induce vortex shedding the noise peak that appear in the flat wall surface bars configuration (Bars10mm) is hypothesized to be associated with the interaction of the bars with eddies shed from upstream bars or with the scattering of noise sources above the bars.

At the higher Reynolds number the simulations estimate the OASPL increments presented in tables 8.4 and 8.5.

The cylinders with on-surface helicoidal cable, close to ring configurations (cable aligned with the mean flow) were studied only experimentally and show that these roughness configurations have little effect on the far field noise in the high frequency range, up to the point that the spectra collapse with the smooth case for $St > 2$. This indicates that cables aligned with the mean flow act weakly as noise sources, as opposed to cables perpendicular to the mean flow and distributed roughness. It is known that dense configurations of rings cause the flow to be supercritical, i.e. the BL’s are turbulent before detachment, so the lack of significant roughness noise is not due to the weakness of turbulent stresses in the BL’s.

The configurations with bar fetches show significant effects of the bars on the aerodynamics and radiated noise with respect to the smooth cylinder. Little differences are observed in the vortex shedding peaks from the acoustic measurements in the experiments. The higher Reynolds number simulations show a strong weakening of the vortex shedding peak in FetchSide, which is not observed in the lower Reynolds number experiments and simulations.

FetchBack far field noise is remarkably close to Smooth in all the frequency range, and for both the experiments and the simulations. It can be concluded that the bars located in the near wake have no significant effect on the flow structure and the radiated noise. The most important differences are observed in FetchSide and FetchFront at higher frequencies. In both configurations the high frequency noise levels are increased
significantly, especially in FetchSide. Both configurations present a peak in the range \(0.23 < St_b < 0.35\) which appears to be Reynolds number dependent, and which scales with \(U_o^7\).

Measurements of FetchFront present a second peak at \(St_b \approx 0.4\), which is Strouhal-based. The simulations spectra present a single peak at \(St \approx 0.25\) \((Re = 1.7 \times 10^5)\) and \(St \approx 0.3\) \((Re = 10^6)\), and higher harmonics at twice these Strouhal numbers, which are presumably due to individual shedding from the bars. The experiments suggest a complex flow Reynolds number dependence related with the bars and their interaction, but the simulations do not capture that dependence. The higher Reynolds number simulations estimate an increase of OASPL with respect to Smooth from 4 dB (upstream/-downstream) to 12 dB (over-head) in the range \(0.05 < St_b < 1.6\). For \(St_b < 0.05\) the additional is lower than 2 dB for all observers.

FetchSide measurements present a peak at \(St_b \approx 0.23\), Strouhal-based, which is captured, but overpredicted, by the simulations at both Reynolds numbers. The surface pressure fluctuations reveal that the centre bar suffers the larger pressure fluctuations, and therefore is mostly responsible for the peak. The higher Reynolds number simulations present a peak at \(St_b \approx 0.12\), too low to be associated with shedding from the bars. It doesn’t appear in the lower Reynolds number cases, and its origin is unknown. The higher Reynolds number simulations estimate an increase of OASPL with respect to Smooth from 9 dB (upstream/downstream) to 29 dB (over-head) in the range \(0.05 < St_b < 1.6\). Whereas in the low frequency range \(St_b < 0.05\) the noise is reduced due to the weakening of the vortex shedding peak. The reduction reaches up to 17 dB for over-head observers.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>(\Delta\text{OASPL (dB) at } \theta = 90^\circ)</th>
<th>(\Delta\text{OASPL (dB) at } \theta = 0^\circ, 180^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CablePar8</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>CablePar16</td>
<td>11</td>
<td>16</td>
</tr>
<tr>
<td>FetchFront</td>
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<td>0</td>
</tr>
<tr>
<td>FetchSide</td>
<td>-17</td>
<td>2</td>
</tr>
<tr>
<td>FetchBack</td>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>

Table 8.4: \(\Delta\text{OASPL}\) of various configurations in the range \(0.17 < St < 0.6\), from the three-dimensional simulations (DDES) at \(Re = 10^6\).
<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\Delta$OASPL (dB) at $\theta = 90^\circ$</th>
<th>$\Delta$OASPL (dB) at $\theta = 0^\circ, 180^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CablePar8</td>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>CablePar16</td>
<td>16</td>
<td>26</td>
</tr>
<tr>
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<tr>
<td>FetchSide</td>
<td>9</td>
<td>29</td>
</tr>
<tr>
<td>FetchBack</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Table 8.5: $\Delta$OASPL of various configurations in the range $0.6 < St < 20$, from the three-dimensional simulations (DDES) at $Re = 10^6$. 
Chapter 9

Conclusions

9.1 Fully covered circular cylinder with dense distributed roughness

With the goal of modelling the small components of landing gear noise an experimental investigation of circular cylinders fully covered with various configurations of dense distributed roughness \( h/D \approx 0.03 \) and \( \sigma \approx 0.5 \) was performed. The same roughness configurations were tested on a flat wall to validate the roughness noise models.

9.1.1 Vortex shedding noise

- The vortex shedding peak level increases by 14 dB and the peak Strouhal number diminishes to 0.2.

- The shear layers are thicker and the velocity fluctuations in them are higher, which is due to the effect of roughness on the TBLs.

- A wider wake is observed, and together with the lower peak Strouhal number (close to the value of subcritical flow), indicates early detachment, which was already observed in previous works with smaller roughness. It was explained by the rough wall TBLs having a higher momentum deficit, which causes less resistance to APG and a corresponding earlier detachment.
• The spanwise correlation length of the rough cylinders is lower. They present a high decorrelation rate up to a distance of half the cylinder diameter, but the two-point correlation coefficient is close to the smooth cylinder at distances of about 2.5 cylinder diameters. It is remarkable that despite this fact the noise peak level is much higher than the smooth cylinder.

• The spectral fall-off levels of the rough cylinders increase by about 5 dB and present a similar fall-off rate than the smooth case.

9.1.2 Roughness noise

9.1.2.1 Flat wall

• Flat wall ZPG TBL roughness noise is well predicted by Howe’s model for hemispherical roughness. Roughness noise levels are weakly dependent on $h/\delta$ in the first row of elements, despite in the transition smooth-rough the dominant turbulent stresses are presumably not well described by the smooth wall pressure wavenumber-frequency spectrum model.

• Cylindrical-type roughness presents slightly lower levels of the peak described by the model, but has remarkably higher broadband levels at higher frequencies, not present for the hemispherical roughness and not described by the models. It is hypothesized that it’s caused by sharp edge separation effects. This noise mechanism is also weakly dependent on $h/\delta$ at the first row of elements.

• The lack of directivity information provided by Smol’yakov’s roughness noise model causes a miss-prediction of noise levels at observation angles that were not considered in the calibration of the model.

9.1.2.2 Circular cylinder

• Regarding the rough circular cylinders with hemispherical roughness, a broadband peak is observed in the far field noise spectra which is associated with roughness noise. The peak frequency is well predicted by Howe’s roughness noise model.

• Cylindrical-type roughness has lower levels of the peak described by the model, but presents higher levels at higher frequencies, as in the flat wall configuration.
In this case, though, the difference is lower than for the flat wall. The higher levels at high frequencies have been also associated with sharp edge separation effects.

9.2 Partially covered circular cylinder with distributed roughness

An experimental study of noise radiated by circular cylinders with various amounts of dense distributed roughness that maintain the spanwise symmetry has been performed, with the goal of studying the transition of radiated noise from smooth to fully covered. The results can be used to increase the accuracy of the roughness noise model applied to landing gears.

9.2.1 Roughness from upstream to downstream

- Regarding low and medium frequencies, upstream hemispherical roughness ($Spheres_{4.5mm}$) changes the noise spectra almost completely within $70^\circ < \theta_u < 90^\circ$.
- For $\theta_u = 70^\circ$, cylindrical roughness ($Cylinders_{4mm}$) already presents a noise spectrum which has partly transitioned, which means that this type of roughness causes the regime transition to happen for lower $\theta_u$.
- Regarding the high frequency range dominated by roughness noise the transition appears more gradual between $30^\circ < \theta_u < 90^\circ$, for both types of roughness. In all cases and for all frequencies, transition is complete for $\theta_u = 90^\circ$.
- A direct link between the spanwise shedding decorrelation rates and the far field noise spectral levels is not observed, both changing independently through the transition.

9.2.2 Roughness from downstream to upstream

- In the case of hemispherical roughness, transition starts for $\theta_d < 70^\circ$, and for $\theta_d = 90^\circ$ the transition is complete for frequencies dominated by vortex shedding and spectral fall-off, but not in the roughness noise range.
• For $\theta_d = 110^\circ$ noise spectra of cylinders with hemispherical and cylindrical roughness are close to the fully covered cylinder spectra in all the frequency range.

• Transition of cylinders with cylindrical roughness also starts for $\theta_d < 70$, and for $\theta_d = 90^\circ$ the noise spectra are close to the fully covered configuration at frequencies $St < 7$.

• For $\theta_d = 110^\circ$ the low and medium frequency noise levels of the cylinder with cylindrical roughness are about 5 dB lower than the fully covered cylinder. This is thought to be due to the interaction between separation and smooth-rough transition. This noise level decrease is only of about 2 dB in the hemispherical roughness. Since both roughness elements have close values of blockage, roughness height and surface density, the difference should be associated with the different roughness element shapes.

• For all tested configurations, there isn’t a significant decrease of low- and mid-frequency noise levels with respect to the smooth cylinder. The vortex shedding spectral peak with peak and level higher than for Smooth is always present, for both upstream and downstream roughness.

9.3 Bluff body roughness noise model

A bluff body roughness noise prediction model has been developed for the particular case of a circular cylinder covered with distributed dense roughness, and has been validated against experimental measurements, with hemispherical roughness of height $h/D = 0.031$, and surface density $\sigma \approx 0.5$.

• The model is an extension of the flat wall roughness noise model of Howe [55] to the case of bluff bodies.

• BEM is used to calculate the Green’s function tailored to the cylinder, which accounts for the scattering around the bluff body and the propagation to the far field.

• Within the roughness noise model, roughness is equivalent to a set of incoherent dipole sources, whose strength is fixed by the local TBL properties. These surface
dipoles are then coupled with the tailored Green’s function to calculate the far field noise spectrum.

- Roughness noise peak frequencies are well predicted, and the peak levels are well predicted except for over-head observers, where the predicted levels are about 5 dB/Hz higher than the measurements.

- This failure is thought to be due to a deficient modelling of the TBLs, which are described by a ZPG (smooth wall) model of the surface pressure wavenumber-frequency spectrum. The attached TBLs are potentially not well developed and have a strong FPG followed by an APG and separation, which can significantly affect the turbulent structure. To improve the model the TBLs need to be described with more accuracy.

### 9.4 Two-dimensional roughness

A set of configurations consisting of circular cylinders covered with surface cables of various sections, surface densities and orientations have been studied, mainly by numerical means, to assess the effect that they have on the radiated noise. The noise of a configuration consisting of sparse surface parallel bars on a flat wall was measured to compare with the cylinder case. In addition, three configurations consisting of a fetch of three circular off-surface bars located upstream, downstream and at the side of the circular cylinder have been tested experimentally and computationally.

#### 9.4.1 Parallel surface cable

- Both the dense and sparse cable configurations cause an increase of the vortex shedding noise peak level and a decrease of the peak Strouhal number. These effects are qualitatively the same as the ones observed in cylinders with distributed roughness.

- Regarding dense roughness, the comparison of the computed mean and RMS velocity fields with the PIV measurements of the cylinders with distributed roughness shows similar changes with respect to the smooth cylinder: the shear layers are thicker and the wake velocity fluctuations are larger. The lift force spectrum peaks
at a (slightly) lower Strouhal number, and the pressure recovery is lower, in agreement with the higher mean drag coefficient. The value of peak Strouhal number is between the smooth cylinder and the one measured for three-dimensional distributed roughness, which can be explained by the differences between ‘K-type’ and ‘D-type’ roughness.

- The effect of rectangular section cables is slightly smaller than of hemispherical section cables. This can be explained by a higher ‘d-type’ roughness effect (despite they have the same value of solidity) of blockage between the consecutive surface cables.

- The effects of sparse cable are qualitatively the same as dense cable regarding the mean and RMS velocity fields: shear layers with a faster thickening rate and higher velocity fluctuations in the shear layers and the wake.

- The mean and RMS surface pressure fields reveal clear differences with the dense configurations. Sparse cables don’t allow a uniform TBL to develop above them before separation, and the pressure recovery is lower than in the dense cable configurations, resulting in early separation, higher forces on the cylinder, and therefore, higher shedding peak levels.

- Forces and noise levels are higher also at higher frequencies. A noise peak was identified in both the sparse cable configurations and the surface bars on a flat wall, at the same Strouhal number (too low to be vortex shedding from the bars). It is hypothesized to be due to the interaction of eddies shed by upstream cables with downstream ones.

It must be remarked that the noise spectra calculated from the simulations are strictly valid only for low frequencies ($St \leq 2$), since it is assumed the cylinder is compact. The levels are underpredicted for higher frequencies, but the qualitative features are likely to be correct.

### 9.4.2 Helicoidal surface cable

- The cylinders with surface helicoidal cable (cable nearly aligned with the mean flow) present little differences on the far field noise at high frequencies with respect
to the smooth cylinder: the spectra collapse with the smooth case for $St > 1.8$. This indicates that cables aligned with the mean flow act as weak noise sources, as opposed to cables perpendicular to the mean flow and distributed roughness.

### 9.4.3 Bar fetches

- Little differences are observed in the vortex shedding peaks from the measured noise spectra. The higher Reynolds number simulations show a strong weakening of the vortex shedding peak in for the cylinder with the side bar fetch, which is not observed in the lower Reynolds number experiments and simulations. This can be a real Reynolds number effect.

- The bar fetch located downstream has no significant effect on the flow structure and the radiated noise in all the frequency range.

- The noise spectra corresponding the upstream fetch and side fetch present a peak in the range $0.23 < St_b < 0.35$ which appears to be Reynolds number dependent, and which scales with $U_o^7$. This suggests a Reynolds number dependence related with the bars and their interaction, which is not captured by the simulations.

- The measured noise spectra corresponding to the upstream bar fetch present a second peak at $St_b \approx 0.4$, which is Strouhal-based.

- The higher Reynolds number simulations of the upstream bar fetch estimate an increase of OASPL with respect to the smooth cylinder from 4 dB (upstream/-downstream) to 12 dB (over-head) in the range $0.05 < St_b < 1.6$. For $St_b < 0.05$ the increase is lower than 2 dB for all observers.

- The measurements corresponding to the side bar fetch present a Strouhal-based peak at $St_b \approx 0.23$, which is captured, but overpredicted, by the simulations at the two Reynolds numbers. The surface pressure fluctuations reveal that the centre bar suffers the larger pressure fluctuations, and therefore is mostly responsible for the peak.

- The higher Reynolds number simulations of the side bar fetch configuration present a peak at $St_b \approx 0.12$, too low to be associated with shedding from the bars. It doesn’t appear in the lower Reynolds number cases, and its origin is unknown.
• The higher Reynolds number simulations of the side bar fetch estimate an increase of OASPL with respect to the smooth cylinder from 9 dB (upstream/downstream) to 29 dB (over-head) in the range $0.05 < St_b < 1.6$. Whereas in the low frequency range $St_b < 0.05$ the noise is reduced due to the weakening of the vortex shedding peak. The reduction reaches up to 17 dB for over-head observers.

9.5 Future work

9.5.1 Roughness noise model for two-dimensional roughness

As shown by Volino et al. [90], TBLs over two-dimensional sparse surface bars in cross-flow present differences with respect to dense distributed roughness. The normalised Reynolds stress profiles of the former don’t collapse with the latter, and the roughness sublayer appears significantly thicker. For denser configurations, though, Townsend’s hypothesis has clear experimental support. If the friction velocity and the TBL thickness are known the noise sources can be estimated, and to calculate the far field noise using Howe’s approach only the Green’s function tailored to the surface bars, instead of the distributed hemispherical elements, is required. The compact Green’s function tailored to two-dimensional circular surface bars (figure 9.1) is [18]:

\[
G(x, y, t, \tau) = \frac{\delta(t - \tau - |x - Y|/c)}{2\pi |x - Y|},
\]  

(9.1)

where

\[
Y_2 = y_2.
\]

\[
Y_3 = y_3.
\]

\[
Y_1 = y_1 + \sum_m \frac{\mu H h^2 (y_1 - x_{m1})}{2 |y - x_m|^2}.
\]

(9.2)

The sum is over all roughness elements in the unit area. The far field noise spectrum can be derived using the above Green’s function instead of (2.17) and (2.18), and may lead to better predictions for the case of (dense) two-dimensional roughness elements, such as CablePar16.
The roughness noise model for two-dimensional roughness elements oriented in the streamwise direction can be equally derived, however this configuration was tested experimentally (they were actually oriented only approximately in the streamwise direction) and no significant roughness noise was observed.

9.5.2 TBL on bluff bodies

It was shown that the description of the TBL is fundamental to accurately determine roughness noise. The structure of TBLs over rough walls with PGs have been rarely studied in the past, especially the presence of roughness and a FPG simultaneously, and therefore there are no physical models of the TBL properties as a function of the roughness geometry and the pressure gradient.

9.5.3 Larger roughness

The studied roughness sizes (with respect to the cylinder diameter) permit a uniform TBL to develop prior to separation, and a uniform spanwise separation. Despite the decrease of spanwise correlation length with respect to the smooth configuration, the vortex shedding noise peaks are significantly higher than the background broadband noise. It has been suggested that for higher roughness sizes, TBLs at separation won’t be uniform enough due to a lack of streamwise distance to develop, and vortex shedding will be suppressed, or at least weakened. In this situation, what will be the noise levels at low and medium frequencies?

Regarding roughness noise, in the described situation the validity of the model is a priori highly questionable. Since the TBL is not well developed, the dominant turbulent
stresses associated with interstitial flow must be relevant, and therefore geometry dependent. How well do the model predictions hold in this case? Maybe Smol’yakov’s model, which accounts for the individual roughness elements shedding, can describe better noise generation.

The experimental study of larger roughness sizes is necessary to assess these equations, and to improve landing gear noise prediction.

9.5.4 Application to landing gear noise

In order to apply the developed bluff body roughness noise model to actual landing gear noise further work is needed.

9.5.4.1 Extension to other bluff bodies

Firstly, for bluff bodies that are not cylindrical (the current model deals with circular cylinders, but the mesh for cylinders with a different cross-section is relatively straightforward) new surface meshes need to be defined in the BEM algorithm. The difficulty of this task increases for bluff bodies with complicated geometries, and if multiple body interactions are to be considered. For complicated geometries it is appropriate to use meshing software. In a second stage a coupling with the BEM algorithm has to be added.

9.5.4.2 Modelling of small components

Secondly, the modelling of roughness has to be adapted to apply to the elements in actual landing gears. The current model describes roughness as a distribution of hemispherical elements of height $h$, surface density $\sigma$ and a certain location within the TBL (the roughness elements can be identified with individual sources or with a continuous source distribution). The variability of the size and surface distribution of the landing gear small components makes it difficult, and possibly unworthy, to describe all the surface elements as if they were roughness elements in the model. The performed experiments with cylindrical roughness elements present certain differences with the hemispherical case, which implies that hemispherical roughness is probably inadequate to describe
other element geometries. This difference was hypothesized to be due to an upward shifting of the TBL due to the higher blockage of the cylindrical roughness, the dominant turbulent sources being at a larger distances from the roughness elements. This shows that the TBL definition as it stands in the model is strictly limited to hemispherical-type roughness. This geometry dependence implies that an accurate description of variable geometries cannot be provided by the model. A certain degree of empiricism will have to be introduced to deal with variable roughness geometries.

Here it is proposed to perform an intermediate step prior to the final application to real configurations. It consists of testing a single bluff body, such as a circular cylinder, covered with realistic configurations of small components, such as hoses, rings, etc. The roughness noise model can then be calibrated regarding the definition of the geometry inputs (roughness size, surface density, ...) as well as several parameters regarding the flow structure (separation location, spanwise dependence, ...). Once the model is calibrated it must be used to predict the radiated noise with additional configurations, to assess the predictive capabilities.

9.5.4.3 Low frequency noise

The effect of the small components in the low- and mid-frequency range will also be better understood after performing the previously described tests. The present results show that uniformly distributed roughness along the span enhances the vortex shedding noise peak as well as the fall-off range. But it is likely that roughness configurations not uniform along the span will cause a decrease of spanwise shedding coherence and therefore a decrease of the peak level and fall-off levels. This needs to be further studied to accurately parameterise the peak level, peak Strouhal number and fall-off levels as a function of the small components geometry and distribution.
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