Accurate near-body predictions at intermediate Reynolds numbers

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Maritime field adopting new design criterion and applying novel concepts
Marine field (biology, paleontology, and sport) eager to utilize technology
Hydrodynamic object recognition using pressure sensing

- Blind cave fish use pressure in efficient propulsion, navigation, and predator avoidance

- Applications in autonomous vehicles, harbor safeguarding, energy extraction...

- \( \text{Re} = 2K-100K \)
Predictions of near-field pressure and separation at low angles of attack

Induced change in pressure field passing circular cylinder
Can induce separation, but not in this case!

Fernandez et al *MTS* 2011
Direct Forcing generates nonphysical separation and surface pressures
Jump in velocity gradient across boundary causes numerical pressure instability

Griffith & Peskin *JCP* 2005
Muldoon & Acharya *IJNM* 2008
Guy & Hartenstine *JCP* 2010

Problem is exacerbated at higher Reynolds numbers
Obviously, it’s time to throw in the towel

- Switch to cut-cell method
  - Sharp interface
Obviously, it’s time to throw in the towel

• Switch to cut-cell method
  – Sharp interface
  – Much more complex
  – Stability problems which require lots of numerical trickery to avoid

The smoothing wasn’t the problem
Let’s try reformulating the system...
Coupled two-domain problem:
Fluid system with irregular boundary data

\[ F : \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \nabla p - \nu \nabla^2 \mathbf{u} = 0 \quad \Omega_f \]

\[ F : \mathbf{u} - \mathbf{u}_0 + \int_{t'=0}^{t} (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \nabla p - \nu \nabla^2 \mathbf{u} \, dt' = 0 \]

\[ \mathbf{u} = \mathbf{f}(\mathbf{u}) \]

\[ B : \mathbf{u} - \mathbf{V} = 0 \]

\[ \mathbf{u} = \mathbf{b}(\mathbf{u}) \text{ boundary} \quad \Omega_b \]
Coupled two-domain problem:
Fluid system with irregular boundary data

Want one *continuous* GEQ over the complete domain

\[ \vec{u} = \vec{f}(\vec{u}) \]

\[ \vec{u} = \vec{b}(\vec{u}) \text{ boundary} \]
Finite kernel convolution of the GEQs unifies the equation domain

\[
\begin{align*}
\vec{f}_\epsilon(\vec{u}, \vec{x}, t) &= \int_{\Omega_f} \vec{f}(\vec{u}, \vec{x}_f, t) K_\epsilon(\vec{x}, \vec{x}_f) d\vec{x}_f \\
\vec{b}_\epsilon(\vec{u}, \vec{x}, t) &= \int_{\Omega_b} \vec{b}(\vec{u}, \vec{x}_b, t) K_\epsilon(\vec{x}, \vec{x}_b) d\vec{x}_b \\
\vec{u}_\epsilon &= \vec{f}_\epsilon + \vec{b}_\epsilon, \quad \forall \vec{x} \in \Omega
\end{align*}
\]
Second-order Taylor expansion simplifies the integrated governing equations

\[ b_\epsilon(\vec{u}, \vec{x}) = \int_{\Omega_b} b(\vec{u}, \vec{x}_b) K_\epsilon(\vec{x}, \vec{x}_b) \, d\vec{x}_b \]

\[ \approx \int_{\Omega_b} \left( b(\vec{u}, \vec{x}) + \vec{\nabla} b(\vec{u}, \vec{x}) \cdot (\vec{x}_b - \vec{x}) \right) K_\epsilon(\vec{x}, \vec{x}_b) \, d\vec{x}_b \]

\[ \approx b(\vec{u}, \vec{x}) \int_{\Omega_b} K_\epsilon(\vec{x}, \vec{x}_b) \, d\vec{x}_b + \frac{\partial b}{\partial n}(\vec{u}, \vec{x}) \int_{\Omega_b} (\vec{x}_b - \vec{x}) \cdot \hat{n} K_\epsilon(\vec{x}, \vec{x}_b) \, d\vec{x}_b \]

\[ \approx b(\vec{u}, \vec{x}) \mu_0^{\epsilon,B} + \frac{\partial b}{\partial n}(\vec{u}, \vec{x}) \mu_1^{\epsilon,B} \]

Boundary Data Immersion Method

*Maertens & Weymouth, JCP (in review)*
Kernel moments are smooth functions of distance to the fluid/body boundary

Never need to explicitly integrate over domains

\[ \mu_0^\epsilon \equiv \mu_0^{\epsilon, F} = 1 - \mu_0^{\epsilon, B} \]
Second-order BDIM governing equation

\[ \vec{u}_\varepsilon = \mu_0^e \vec{f} + (1 - \mu_0^e) \vec{b} + \mu_1^e \frac{\partial}{\partial n} \left( \vec{f} - \vec{b} \right) \]

- First two terms are “mixed” fluid/body system
- Last term is a correction for discontinuity
- Simple algebraic adjustment to uncoupled fluid system \( \vec{u} = \vec{f} \)
- Enables (near trivial) Cartesian-grid method to make excellent predictions
Direct Forcing is first-order and neglects the pressure weighting

\[ \vec{f} = \vec{R} - \partial \vec{P} \]

\[ \vec{u}_\epsilon = \vec{f} + \left( 1 - \mu_0^\epsilon - \mu_1^\epsilon \frac{\partial}{\partial n} \right) \left( \vec{b} - \vec{R} + \partial \vec{P} \right) \]

\[ \vec{\nabla} \cdot \mu_0^\epsilon \partial \vec{P} = \vec{\nabla} \cdot \left[ \vec{R} + \left( 1 - \mu_0^\epsilon - \mu_1^\epsilon \frac{\partial}{\partial n} \right) (\vec{V} - \vec{R}) \right] \]
Inclusion of $\mu_0$ in Poisson equation stabilizes pressure in nonlinear cases

- Enforces correct pressure boundary condition
- Improves pressure conditioning
Inclusion of $\mu_0$ in Poisson equation stabilizes pressure in nonlinear cases

- Giant cavity remains stable and gives correct pinch-off
- Simple adjustment ensures robust solution

Bergmann et al *PRL* 2006
Second-order term controls shear gradient discontinuity and removes bias

- 1D unsteady channel flow at Re=1000
- Both 1\textsuperscript{st} order methods have similar error
- 2\textsuperscript{nd} order is improved throughout
Second-order enables near field predictions on separated foil test case

- SD7003 at 4° AOA, Re=10K
  - Low curvature foil with fine trailing edge
  - Very sensitive to IB treatment
Second-order enables near field predictions on separated foil test case

- Sharp SD7003 at 4° AOA
- Enables analytic treatment of sharp trailing edge
Second-order enables near field predictions on separated foil test case

- $O(1)$ method diverges due to pressure instability
- BDIM $O(2)$ matches body-fitted predictions
Second-order at higher Reynolds number

- At $\text{Re} \sim 25K$ the flow becomes three-dimensional at the trailing edge
Heaving and pitching foil at Re=100K

Leading edge vortex remains attached

Tuncer & Platzer 2000
Heaving and pitching foil at Re=100K

- O(2) predicts smooth lift
- Reduces mean drag error from 20% to 5%
Fish model open water pressure

- 3D axisymmetric foil at Re=6K

Windsor et al. *JEB* 2003
Direct Forcing generates nonphysical separation and surface pressures
Second-order method predicts correct near-body pressure and velocity
Current Status

• BDIM: Analytic convolution GEQ with simple numerical implementations
• First-order BDIM applies accurate nonlinear pressure BCs
• Second-order BDIM controls gradient discontinuity at intermediate $Re$

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Future Directions

• Speed-up through optimizing $\vec{u} = \vec{f}$
• High $Re$ by incorporating BL data
• Continue fun and challenging applications

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