Accurate near-body predictions at intermediate Reynolds numbers

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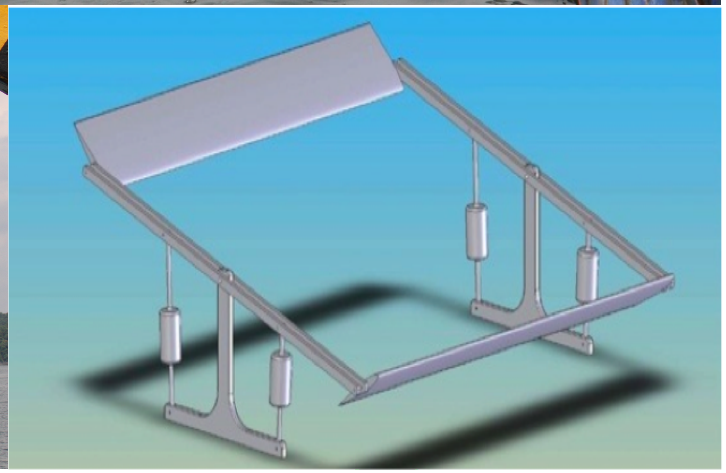
Maritime field adopting new design criterion and applying novel concepts



Marine field (biology, paleontology, and sport) eager to utilize technology

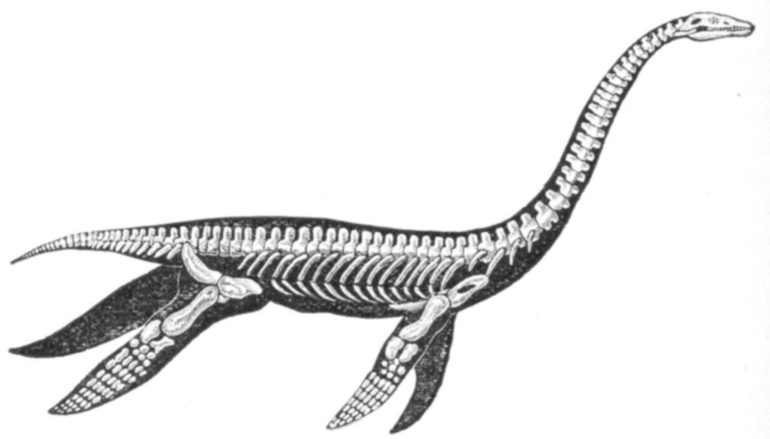
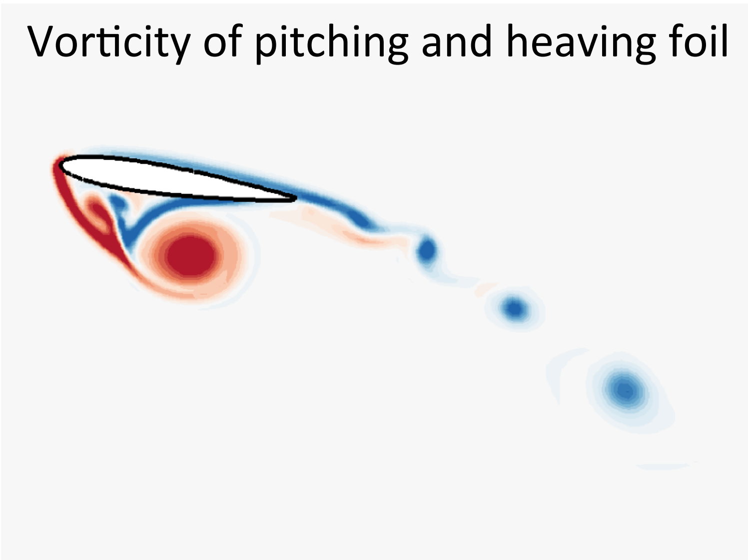


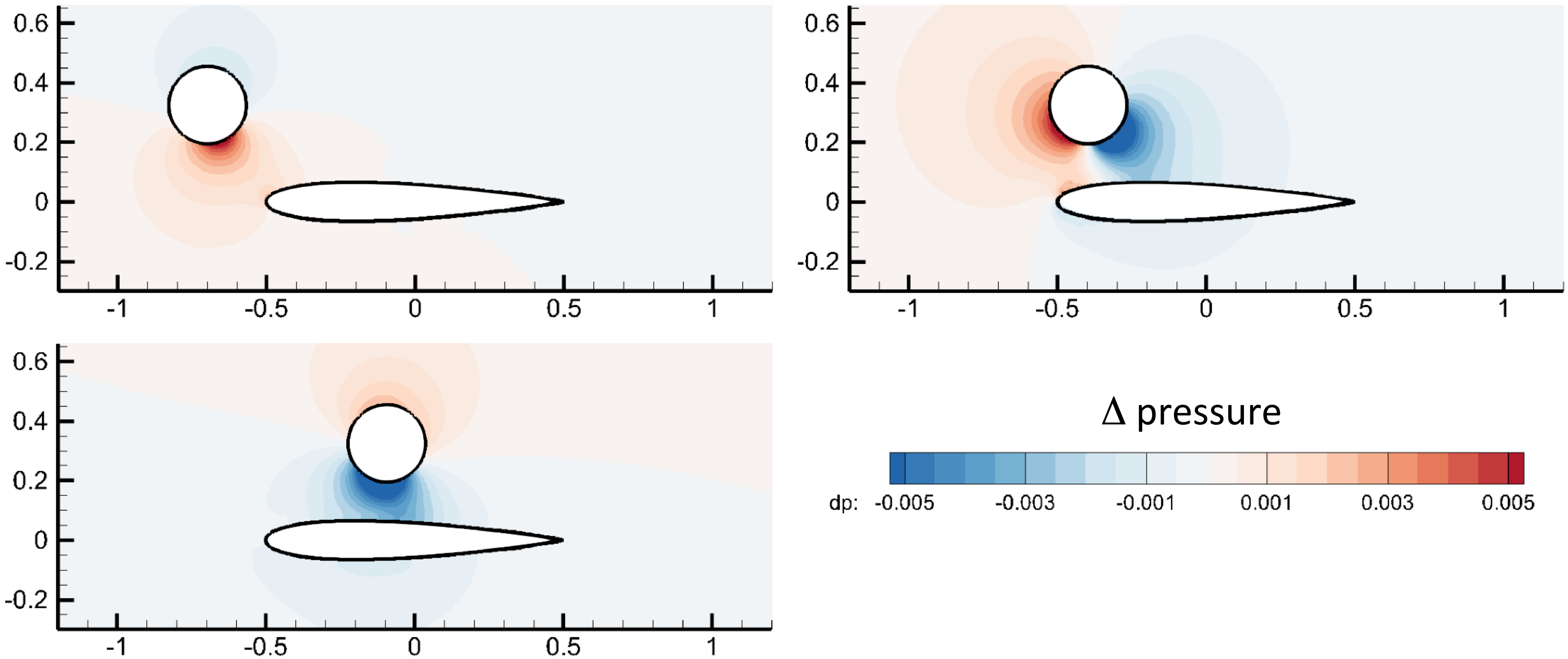
Fig. 177.—*Plesiosaurus dolichodeirus*, restored. Lias.

Hydrodynamic object recognition using pressure sensing



- Blind cave fish use pressure in efficient propulsion, navigation, and predator avoidance
- Applications in autonomous vehicles, harbor safeguarding, energy extraction…
- Re = 2K-100K

Predictions of near-field pressure and separation at low angles of attack

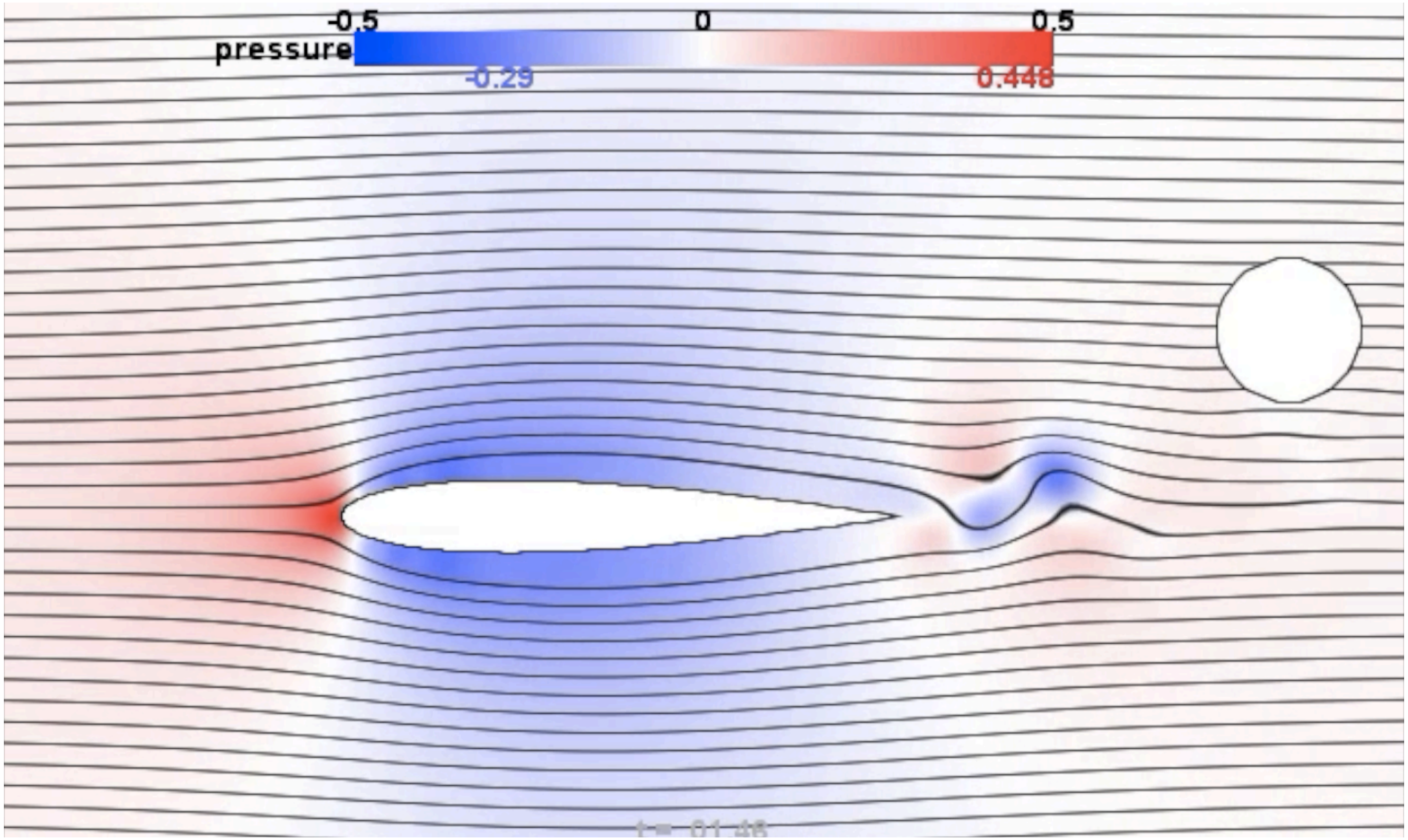


Induced change in pressure field passing circular cylinder

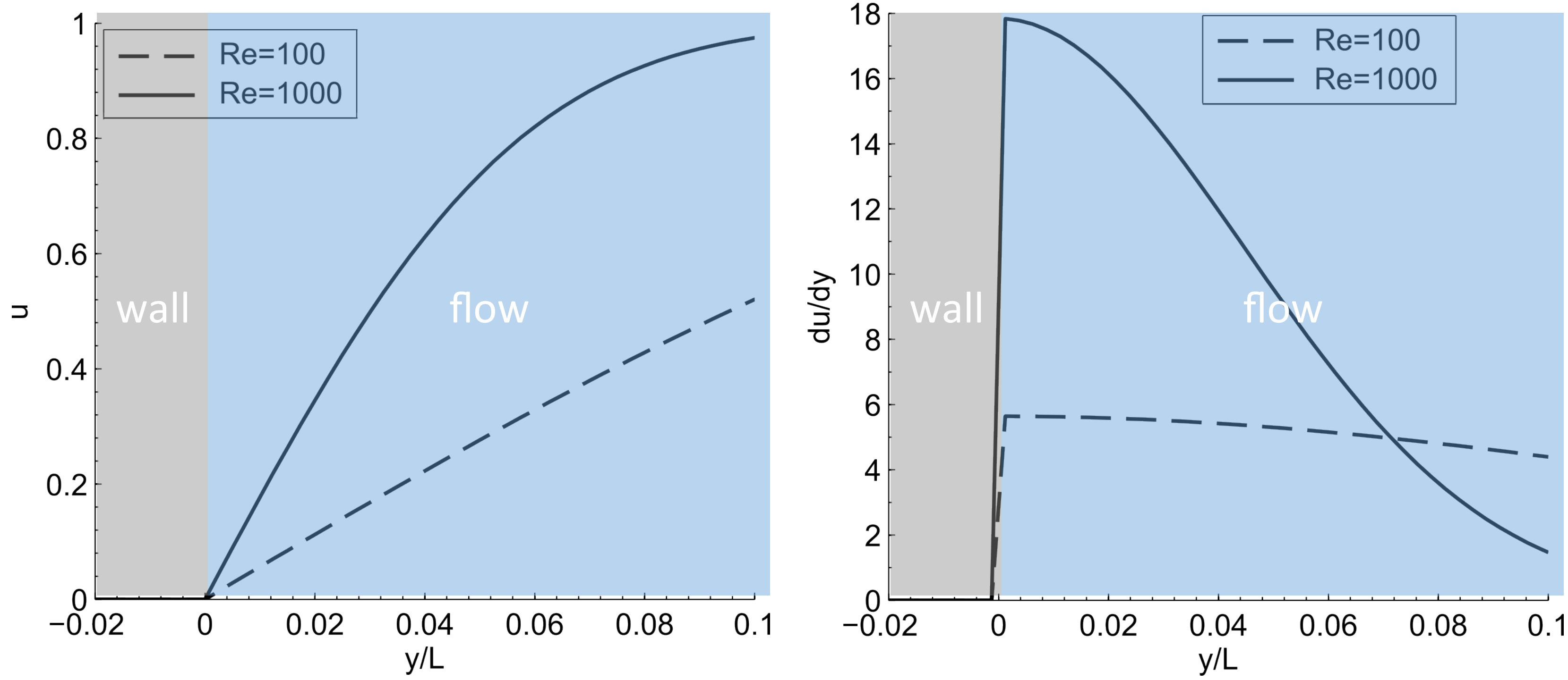
Can induce separation, but not in this case!

Fernandez et al *MTS* 2011

Direct Forcing generates nonphysical separation and surface pressures



Jump in velocity gradient across boundary causes numerical pressure instability



Griffith & Peskin *JCP* 2005
Muldoon & Acharya *IJNM* 2008
Guy & Hartenstine *JCP* 2010

Problem is exacerbated at higher Reynolds numbers

Obviously, it's time to throw in the towel

- Switch to cut-cell method
  - Sharp interface

~~Obviously, it's time to throw in the towel~~

- Switch to cut-cell method
  - Sharp interface
  - Much more complex
  - Stability problems which require lots of numerical trickery to avoid

The smoothing wasn't the problem
Let's try reformulating the system...

Coupled two-domain problem:
Fluid system with irregular boundary data

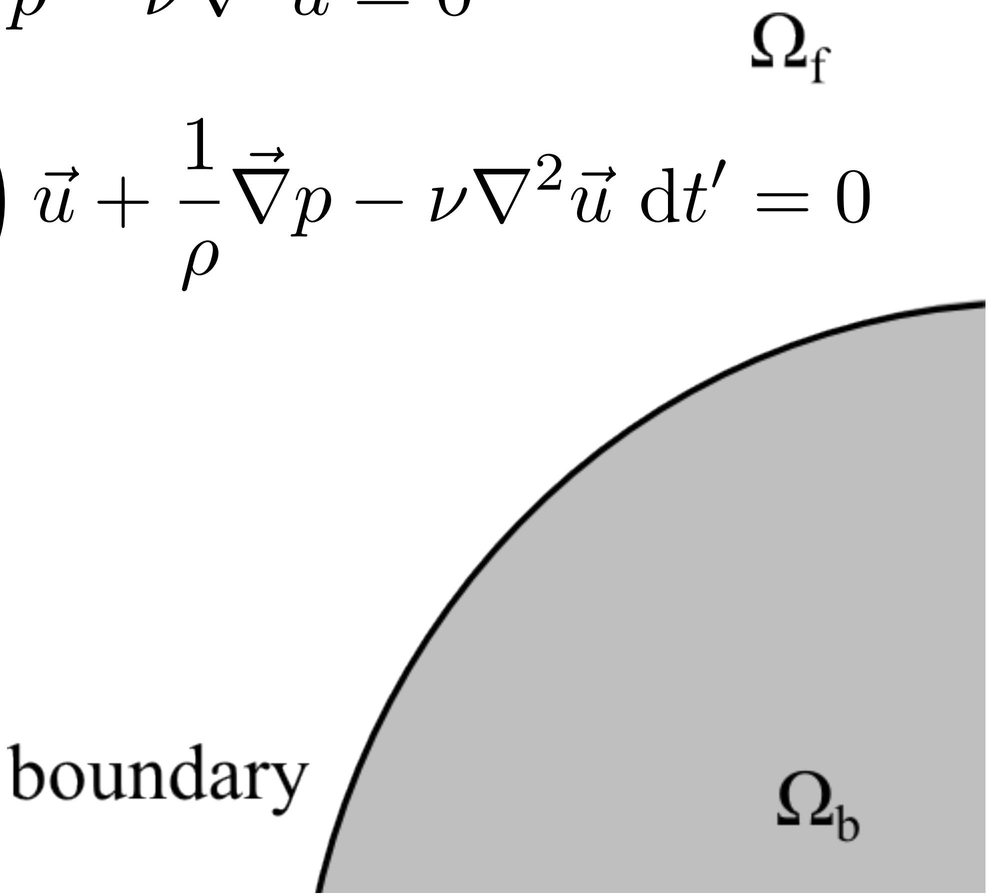
$$F : \frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right) \vec{u} + \frac{1}{\rho} \vec{\nabla} p - \nu \nabla^2 \vec{u} = 0$$

$$F : \vec{u} - \vec{u}_0 + \int_{t'=0}^{t} \left(\vec{u} \cdot \vec{\nabla}\right) \vec{u} + \frac{1}{\rho} \vec{\nabla} p - \nu \nabla^2 \vec{u} \; \mathrm{d}t' = 0$$

$$\vec{u} = \vec{f}(\vec{u})$$

$$B : \vec{u} - \vec{V} = 0$$

$$\vec{u} = \vec{b}(\vec{u})$$

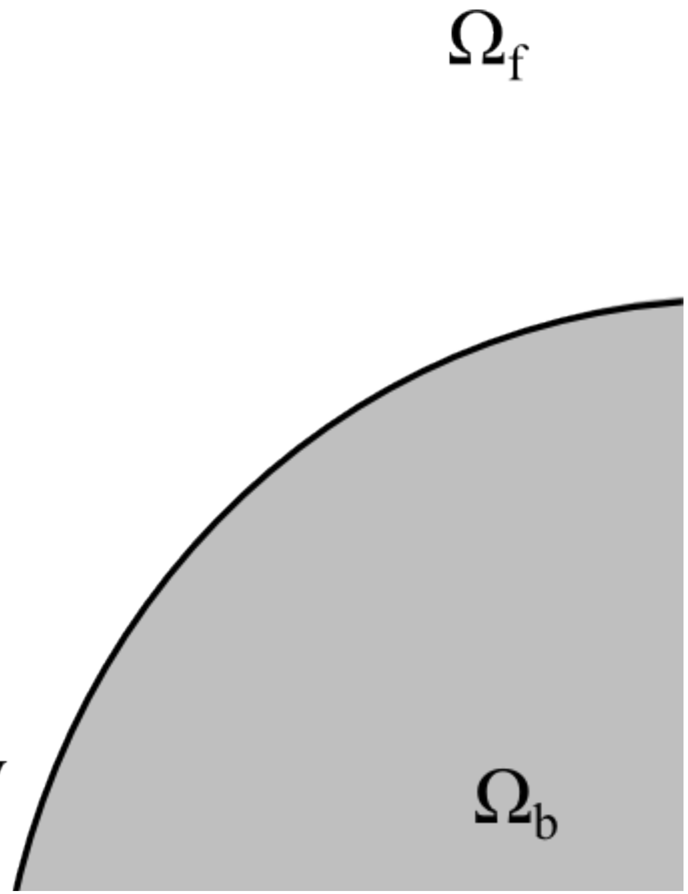


Coupled two-domain problem:
Fluid system with irregular boundary data

Want one *continuous* GEQ over the complete domain

$$\vec{u} = \vec{f}(\vec{u})$$

$$\vec{u} = \vec{b}(\vec{u})$$ boundary



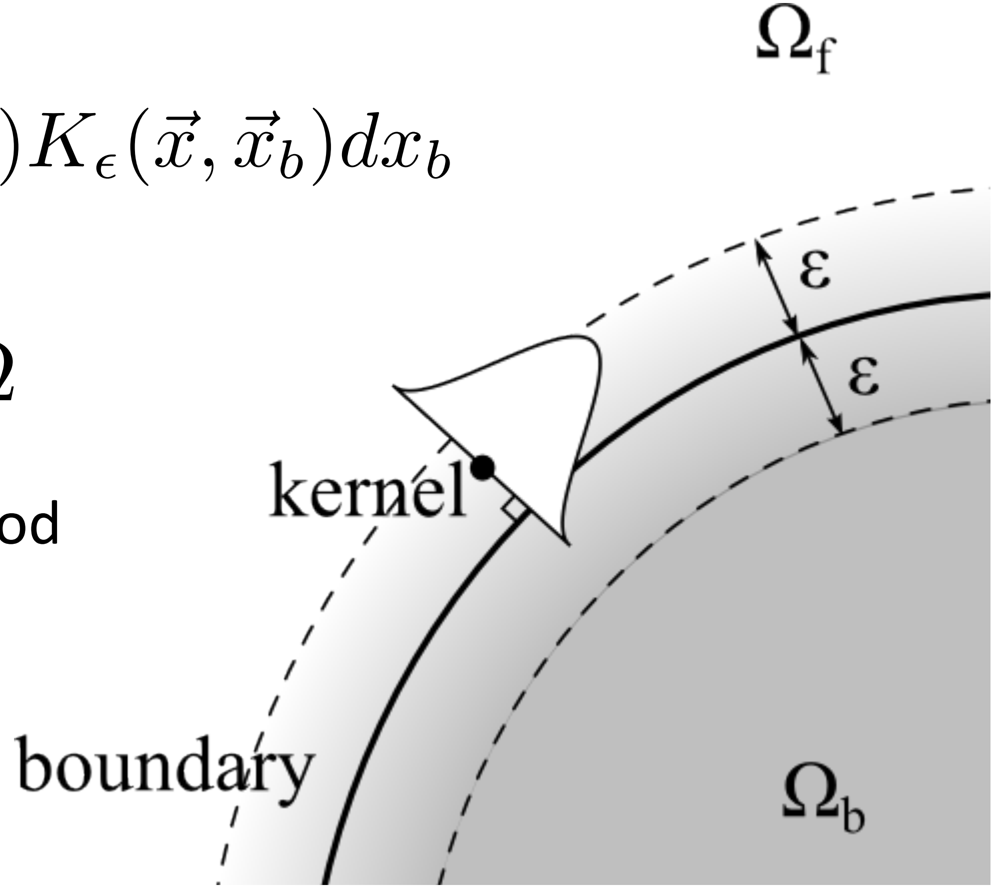
Finite kernel convolution of the GEQs unifies the equation domain

$$\vec{f}_\epsilon(\vec{u},\vec{x},t) = \int_{\Omega_f} \vec{f}(\vec{u},\vec{x}_f,t) K_\epsilon(\vec{x},\vec{x}_f) dx_f$$

$$\vec{b}_\epsilon(\vec{u},\vec{x},t) = \int_{\Omega_b} \vec{b}(\vec{u},\vec{x}_b,t) K_\epsilon(\vec{x},\vec{x}_b) dx_b$$

$$\vec{u}_\epsilon = \vec{f}_\epsilon + \vec{b}_\epsilon, \quad \forall\ \vec{x} \in \Omega$$

Boundary Data Immersion Method
Weymouth & Yue, JCP 2011



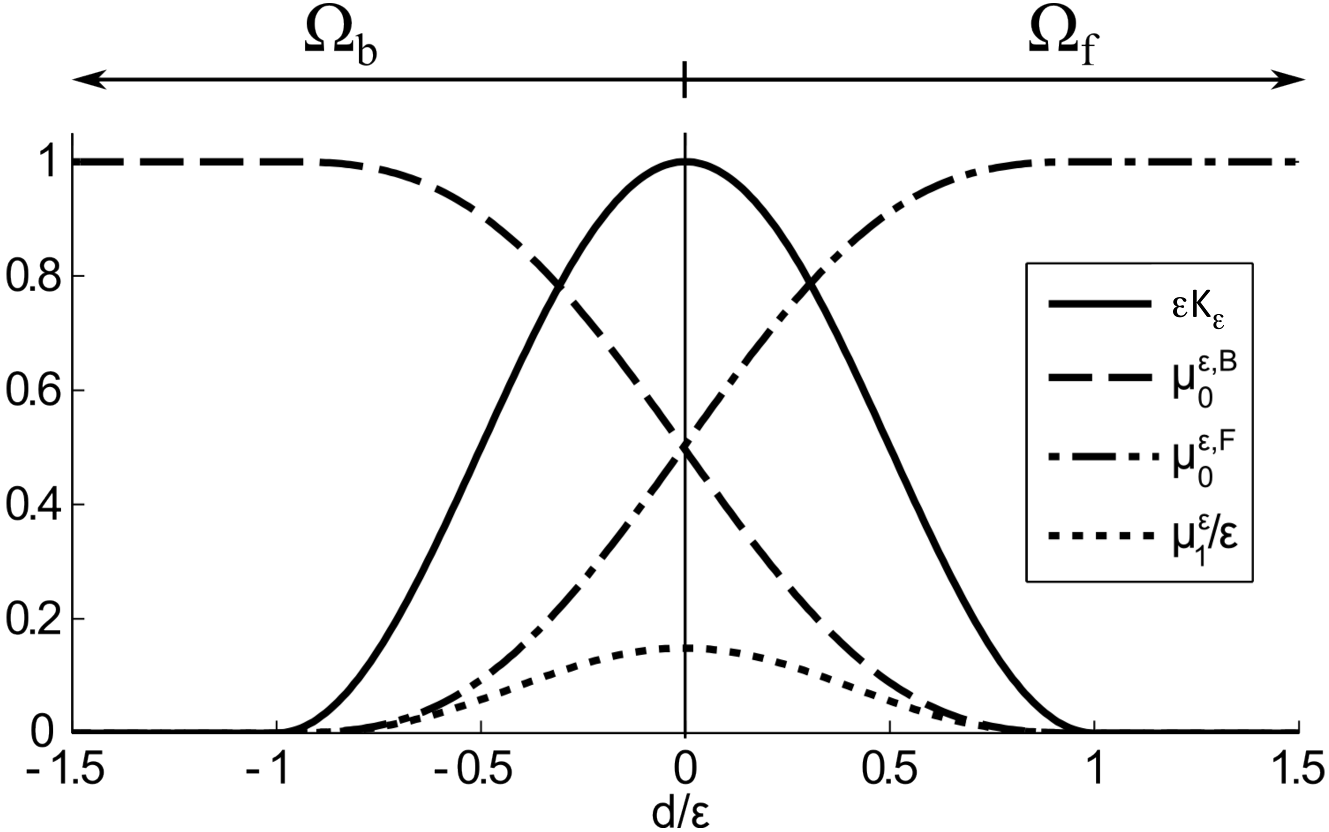
Second-order Taylor expansion simplifies the integrated governing equations

$$
\begin{aligned}
b_\epsilon(\vec{u},\vec{x}) &= \int_{\Omega_b} b(\vec{u},\vec{x}_b) K_\epsilon(\vec{x},\vec{x}_b)\ \mathrm{d}\vec{x}_b \\
&\approx \int_{\Omega_b} \Big(b(\vec{u},\vec{x}) + \vec{\nabla} b(\vec{u},\vec{x}) \cdot (\vec{x}_b - \vec{x}) \Big) K_\epsilon(\vec{x},\vec{x}_b)\ \mathrm{d}\vec{x}_b \\
&\approx b(\vec{u},\vec{x}) \boxed{\int_{\Omega_b} K_\epsilon(\vec{x},\vec{x}_b)\ \mathrm{d}\vec{x}_b} + \frac{\partial b}{\partial n}(\vec{u},\vec{x}) \boxed{\int_{\Omega_b} (\vec{x}_b - \vec{x}) \cdot \hat{n}\ K_\epsilon(\vec{x},\vec{x}_b)\ \mathrm{d}\vec{x}_b} \\
&\quad \cancel{+ \frac{\partial b}{\partial \tau}(\vec{u},\vec{x}) \int_{\Omega_b} (\vec{x}_b - \vec{x}) \cdot \hat{\tau}\ K_\epsilon(\vec{x},\vec{x}_b)\ \mathrm{d}\vec{x}_b} \\
&\approx b(\vec{u},\vec{x}) \boxed{\mu_0^{\epsilon,B}} + \frac{\partial b}{\partial n}(\vec{u},\vec{x}) \boxed{\mu_1^{\epsilon,B}}
\end{aligned}
$$

Boundary Data Immersion Method

Maertens & Weymouth, JCP (in review)

Kernel moments are smooth functions of distance to the fluid/body boundary



Never need to explicitly integrate over domains

$$\mu_0^\epsilon \equiv \mu_0^{\epsilon,F} = 1 - \mu_0^{\epsilon,B}$$

Second-order BDIM governing equation

$$\vec{u}_\epsilon = \underbrace{\mu_0^\epsilon \vec{f} + (1 - \mu_0^\epsilon)\vec{b}} + \underbrace{\mu_1^\epsilon \frac{\partial}{\partial n}\left(\vec{f} - \vec{b}\right)}$$

- First two terms are “mixed” fluid/body system
- Last term is a correction for discontinuity
- Simple algebraic adjustment to uncoupled fluid system $\vec{u} = \vec{f}$
- Enables (near trivial) Cartesian-grid method to make excellent predictions

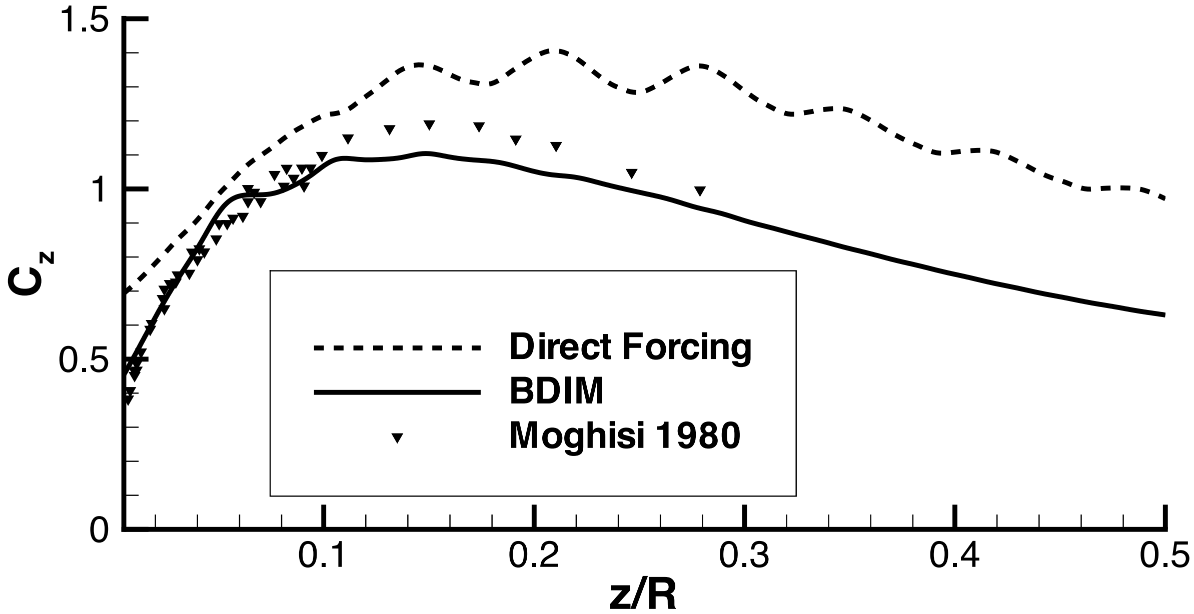
Direct Forcing is first-order and neglects the pressure weighting

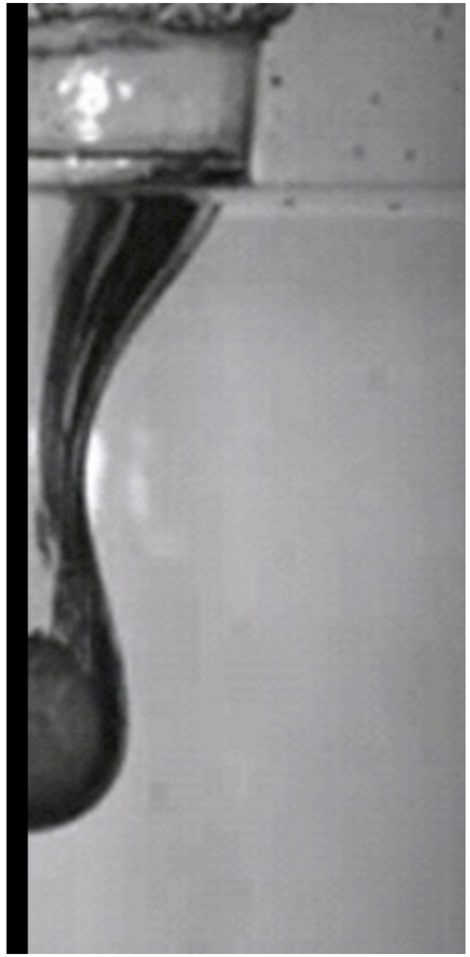
$$\vec{f} = \vec{R} - \partial\vec{P}$$

$$\vec{u}_\epsilon = \vec{f} + \left(1 - \mu_0^\epsilon - \cancel{\mu_1^\epsilon \frac{\partial}{\partial n}}\right)\left(\vec{b} - \vec{R} + \cancel{\partial\vec{P}}\right)$$

$$\vec{\nabla} \cdot \overset{1}{\cancel{\mu_0^\epsilon}}\,\partial\vec{P} = \vec{\nabla} \cdot \left[\vec{R} + \left(1 - \mu_0^\epsilon - \cancel{\mu_1^\epsilon \frac{\partial}{\partial n}}\right)(\vec{V} - \vec{R})\right]$$

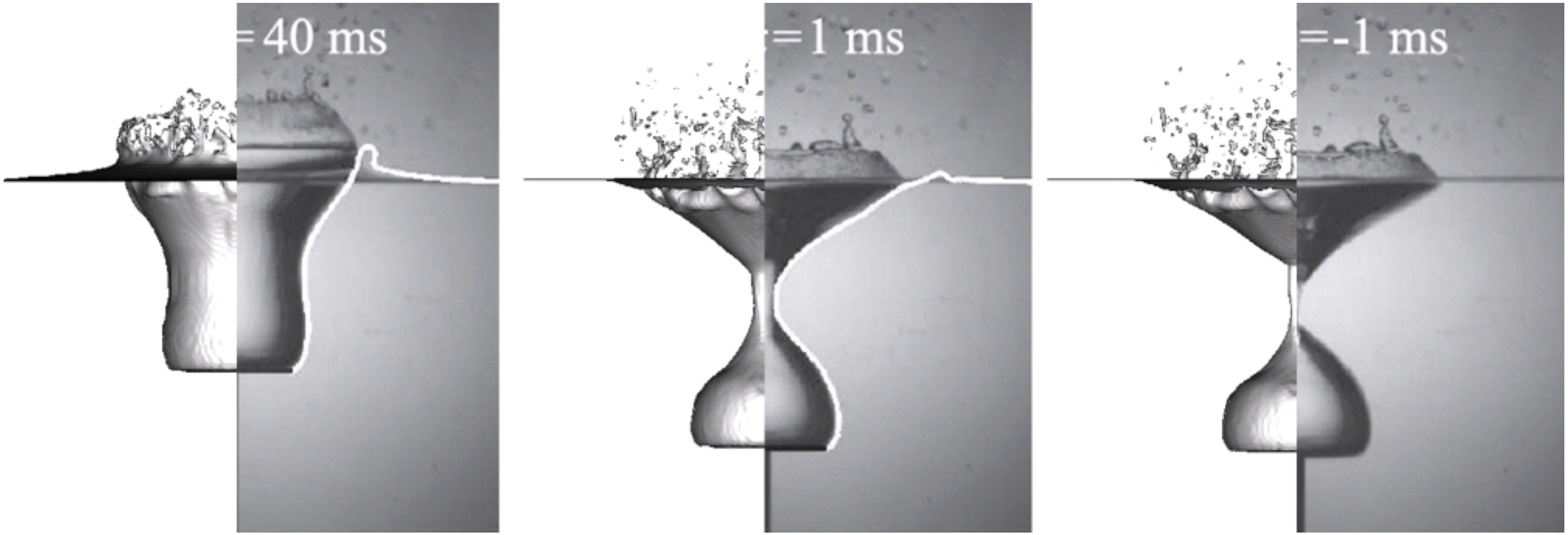
Inclusion of μ_0 in Poisson equation stabilizes pressure in nonlinear cases





- Enforces correct pressure boundary condition
- Improves pressure conditioning

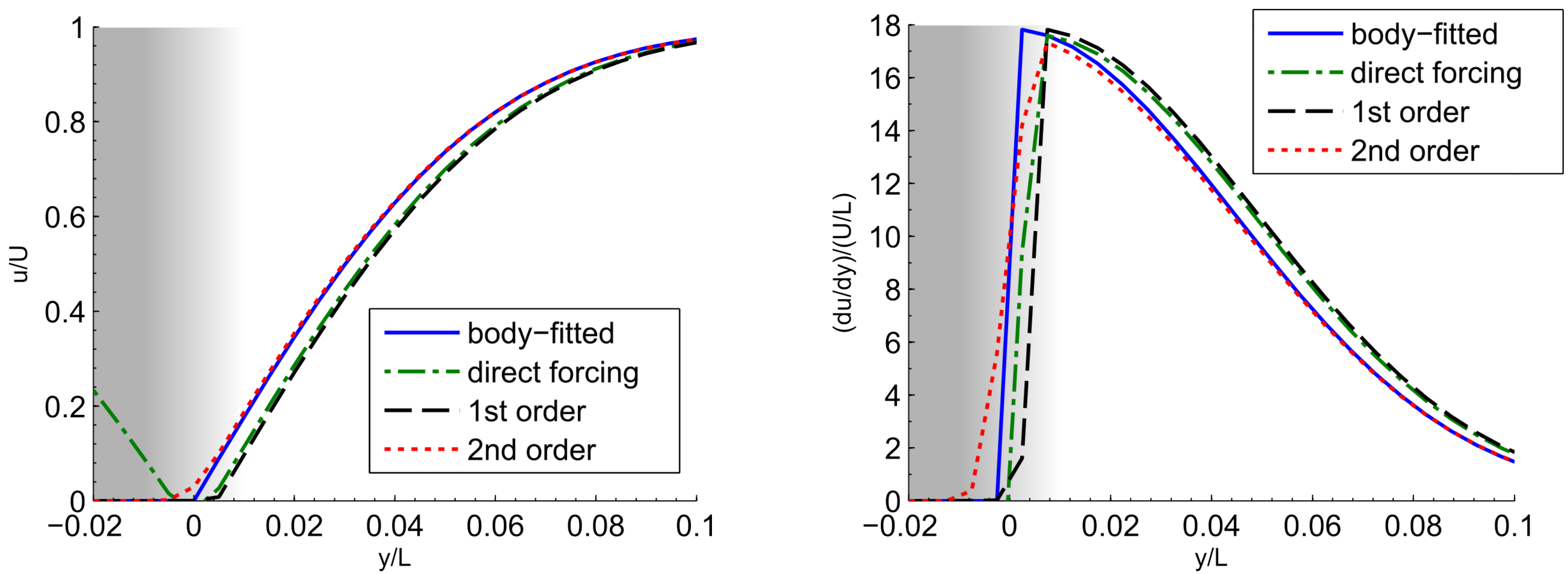
Inclusion of μ_0 in Poisson equation stabilizes pressure in nonlinear cases



Bergmann et al *PRL* 2006

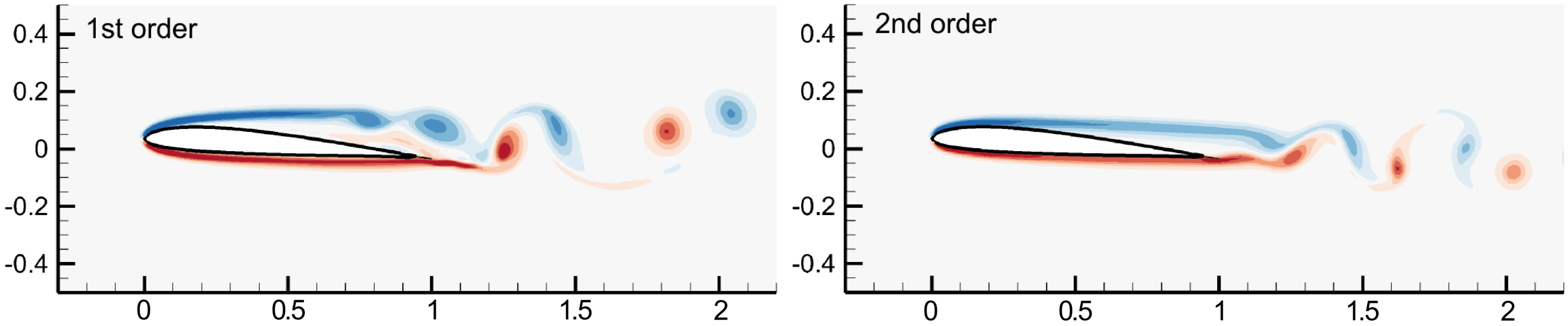
- Giant cavity remains stable and gives correct pinch-off
- Simple adjustment ensures robust solution

Second-order term controls shear gradient discontinuity and removes bias



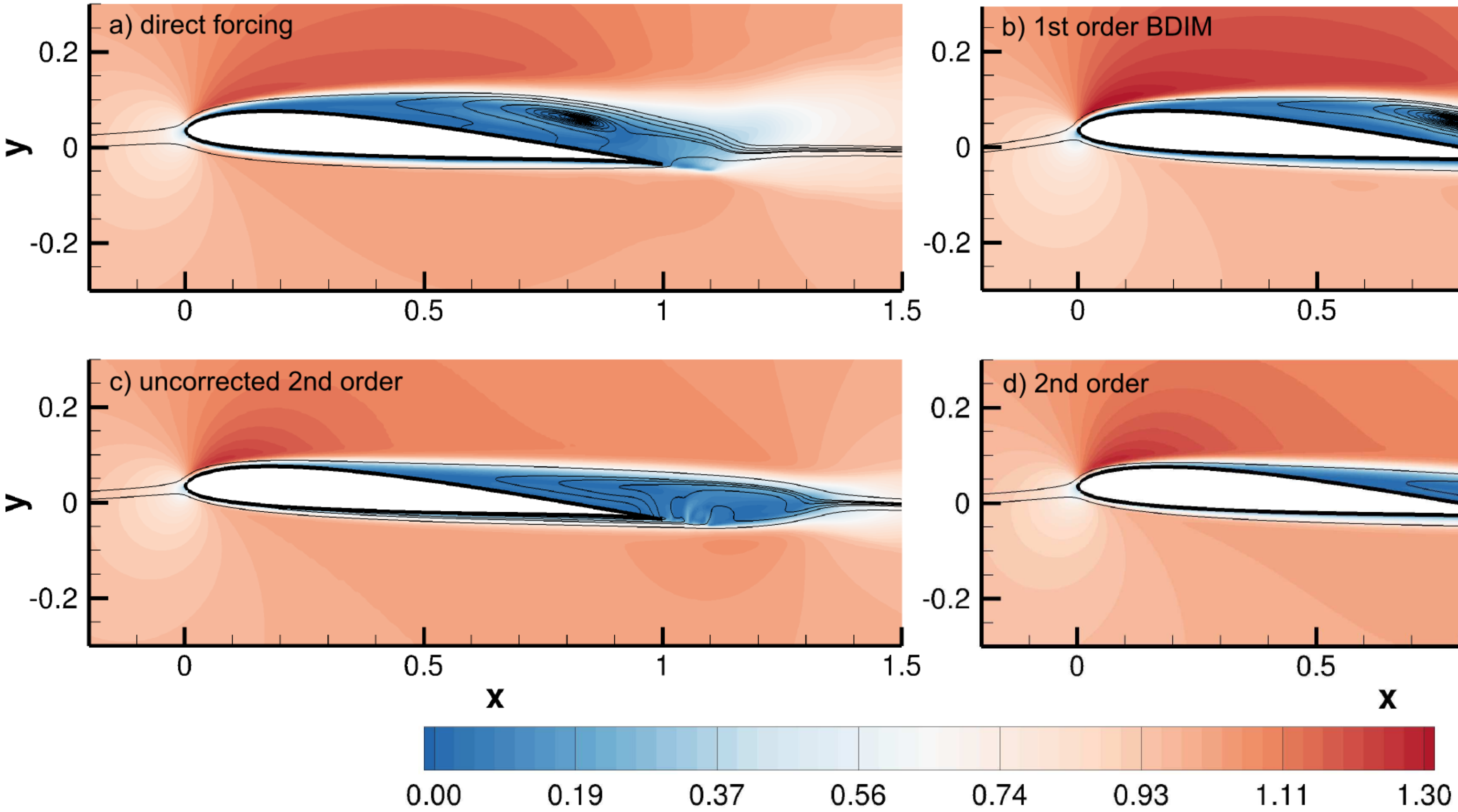
- 1D unsteady channel flow at Re=1000
- Both 1st order methods have similar error
- 2nd order is improved throughout

Second-order enables near field predictions on separated foil test case



- SD7003 at 4^o AOA, Re=10K
  - Low curvature foil with fine trailing edge
  - Very sensitive to IB treatment

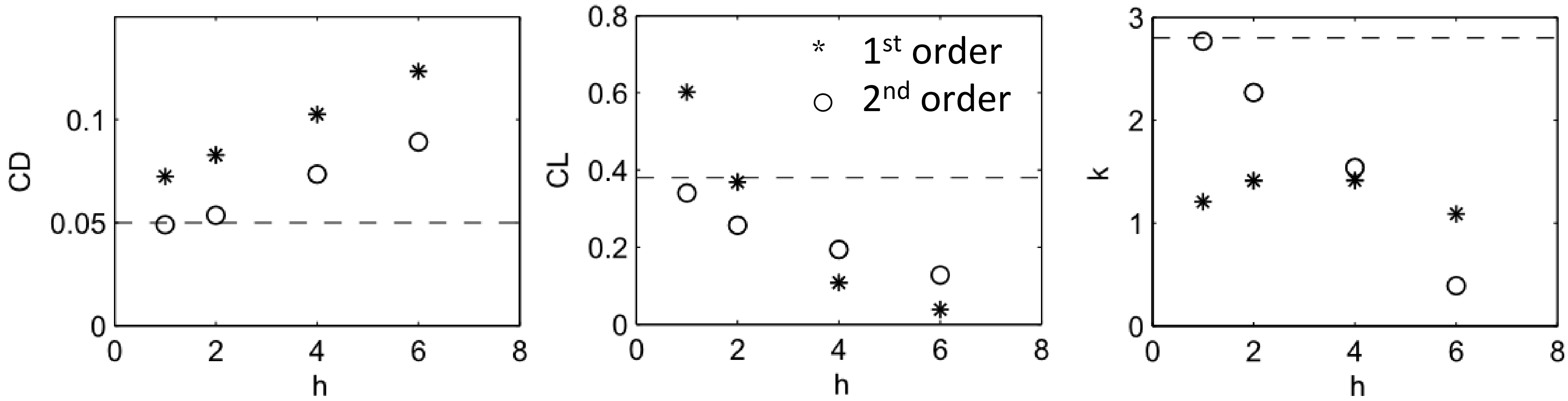
Second-order enables near field predictions on separated foil test case



Time-averaged velocity magnitude

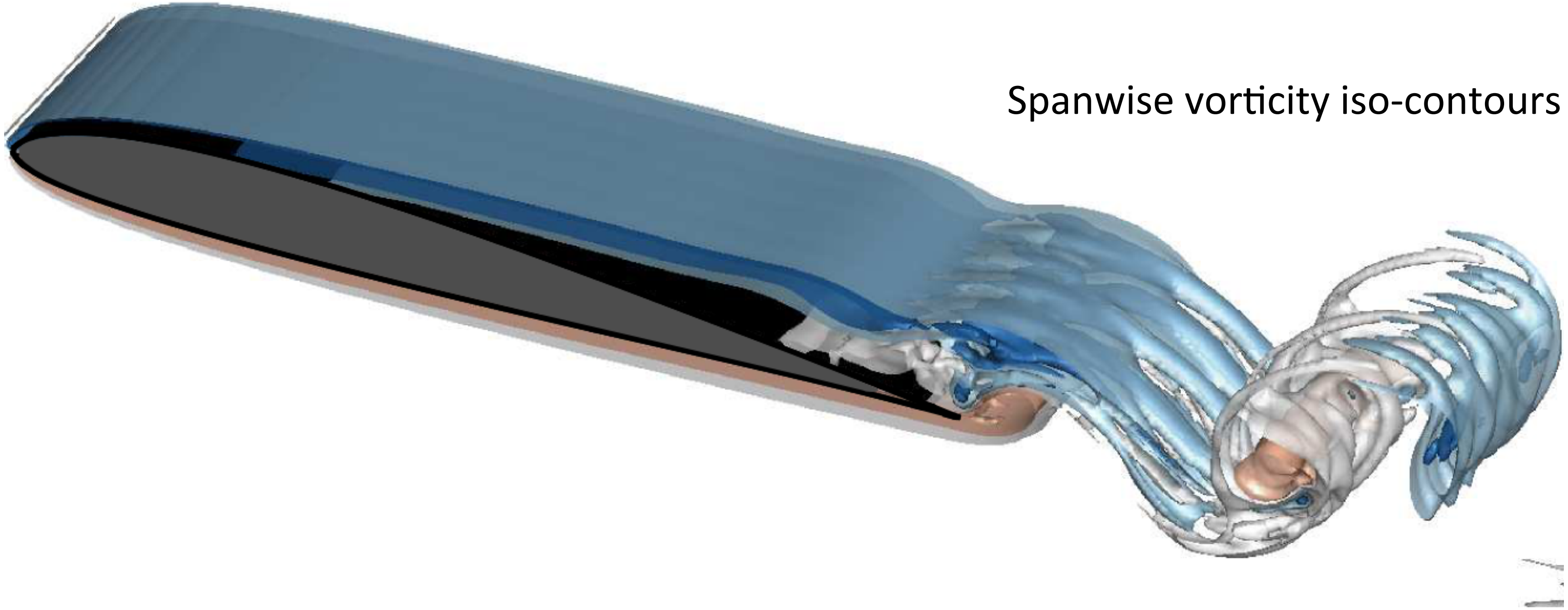
- Sharp SD7003 at 4° AOA
- Enables analytic treatment of sharp trailing edge

Second-order enables near field predictions on separated foil test case



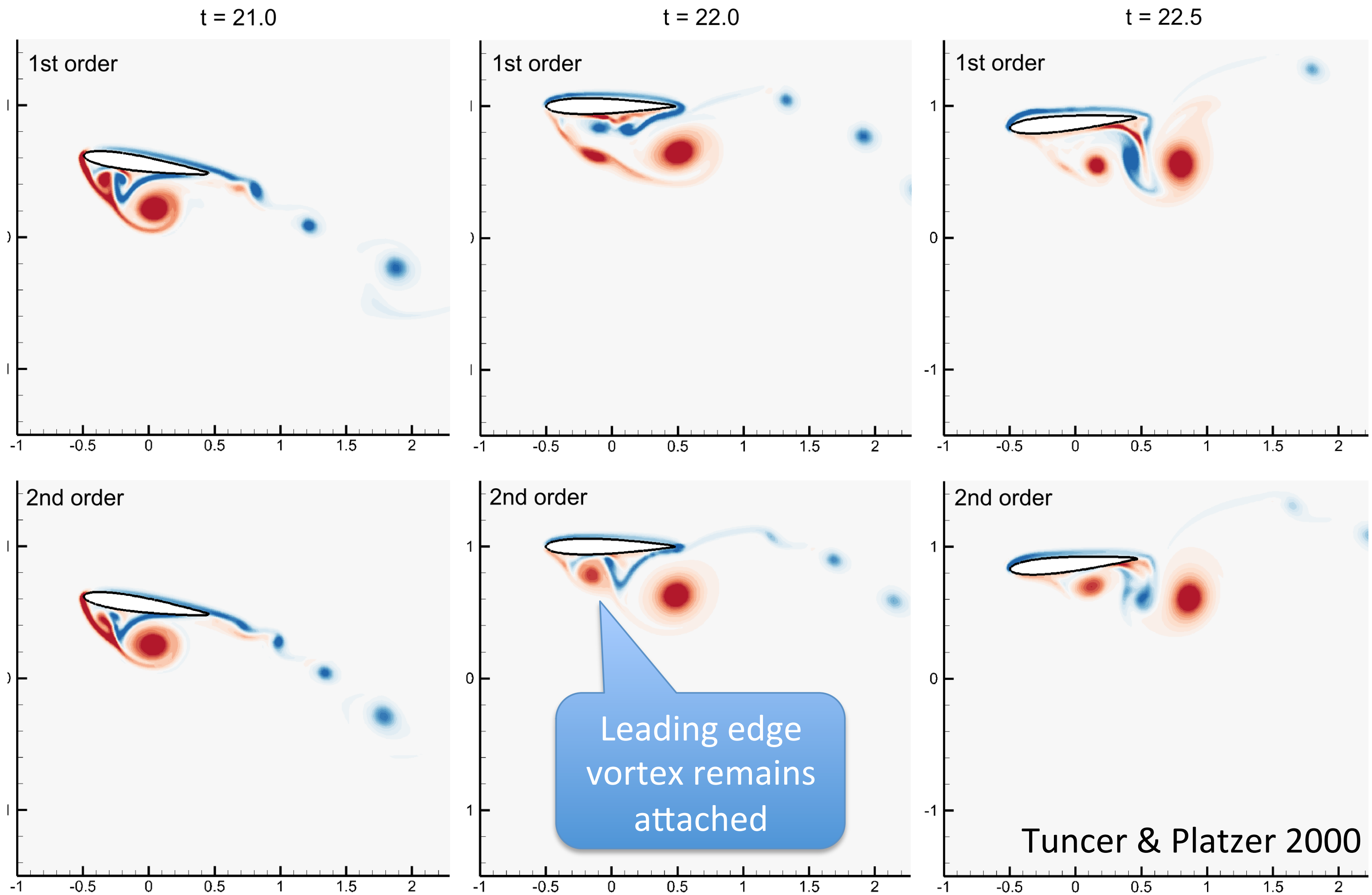
- O(1) method diverges due to pressure instability
- BDIM O(2) matches body-fitted predictions

Second-order at higher Reynolds number

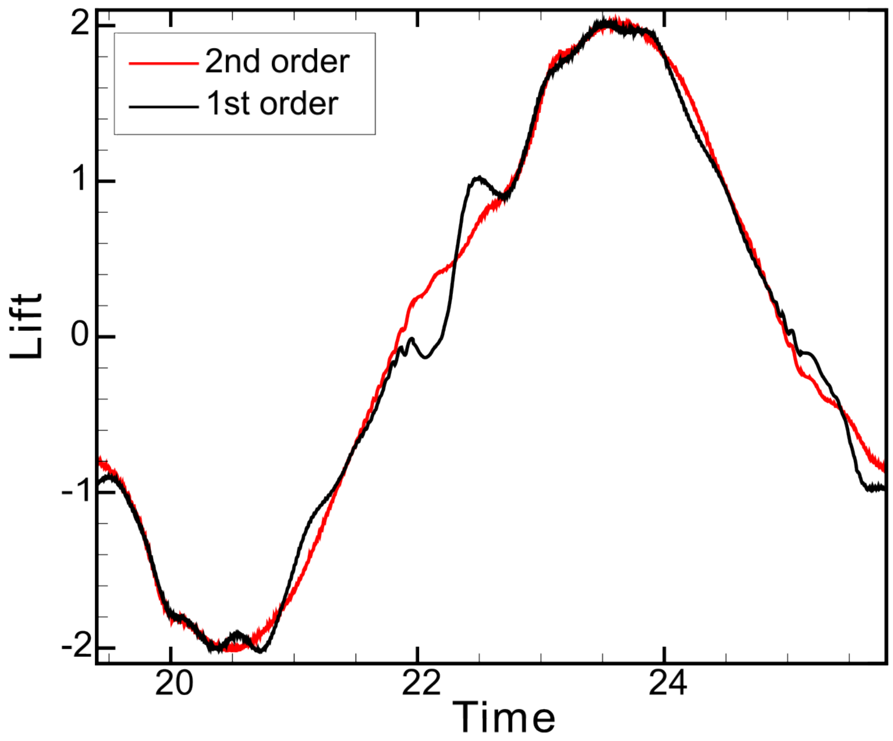


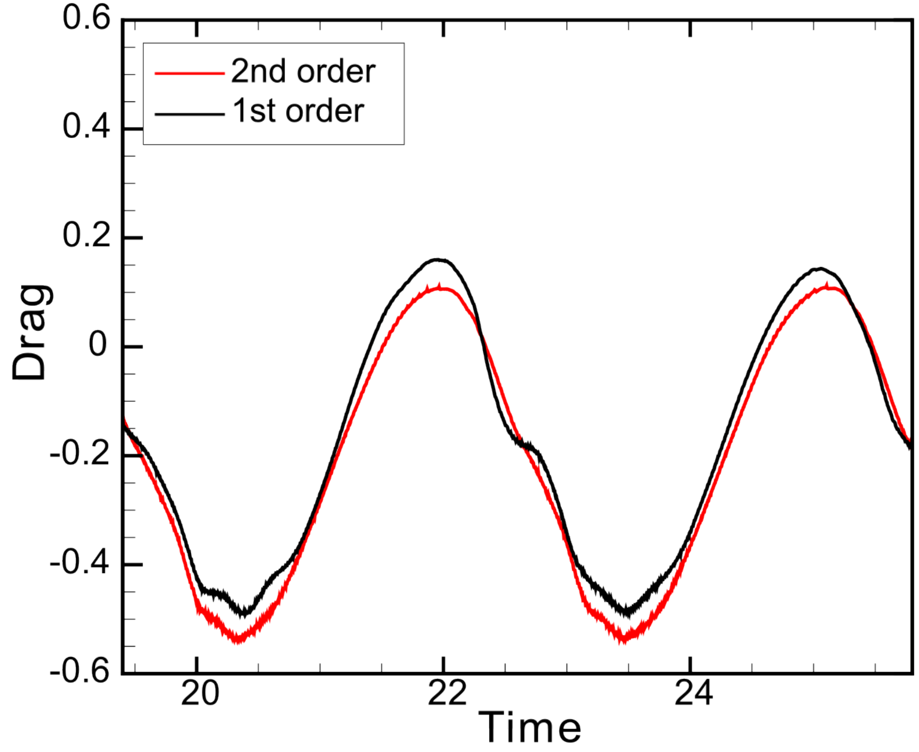
- At Re ~ 25K the flow becomes three-dimensional at the trailing edge

Heaving and pitching foil at Re=100K



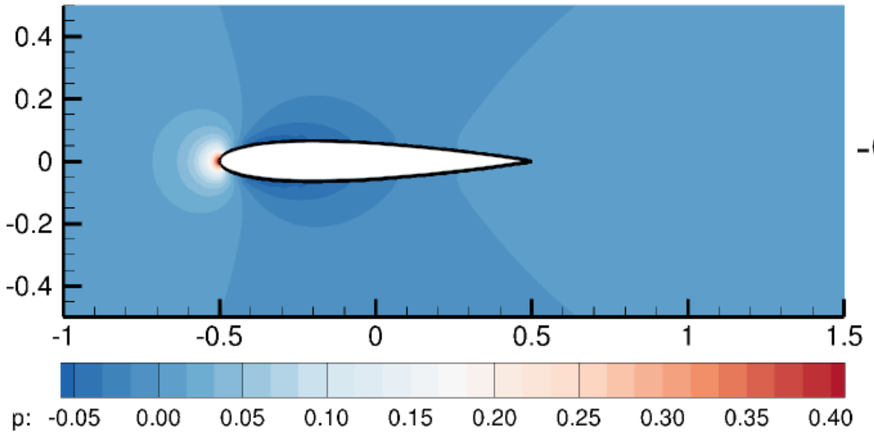
Heaving and pitching foil at Re=100K

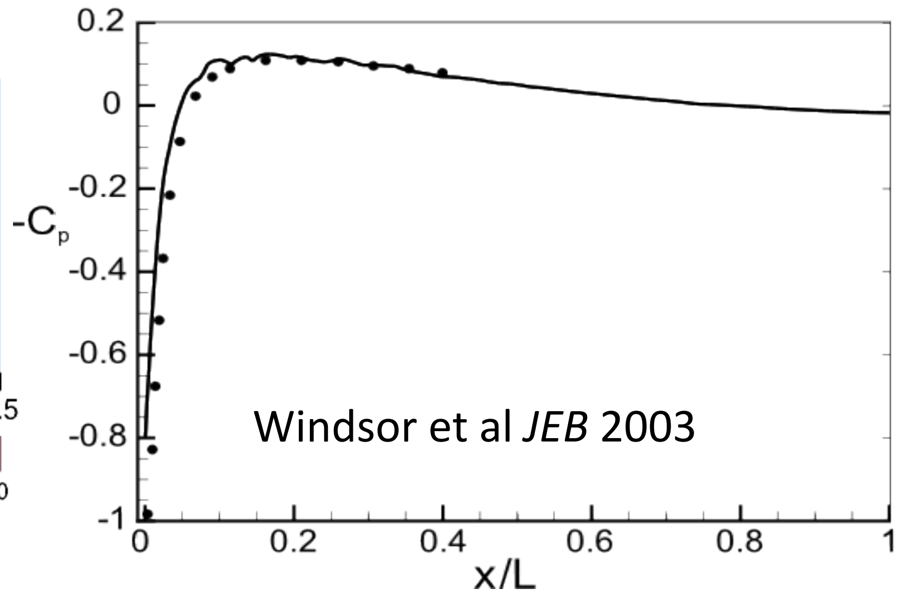




- O(2) predicts smooth lift
- Reduces mean drag error from 20% to 5%

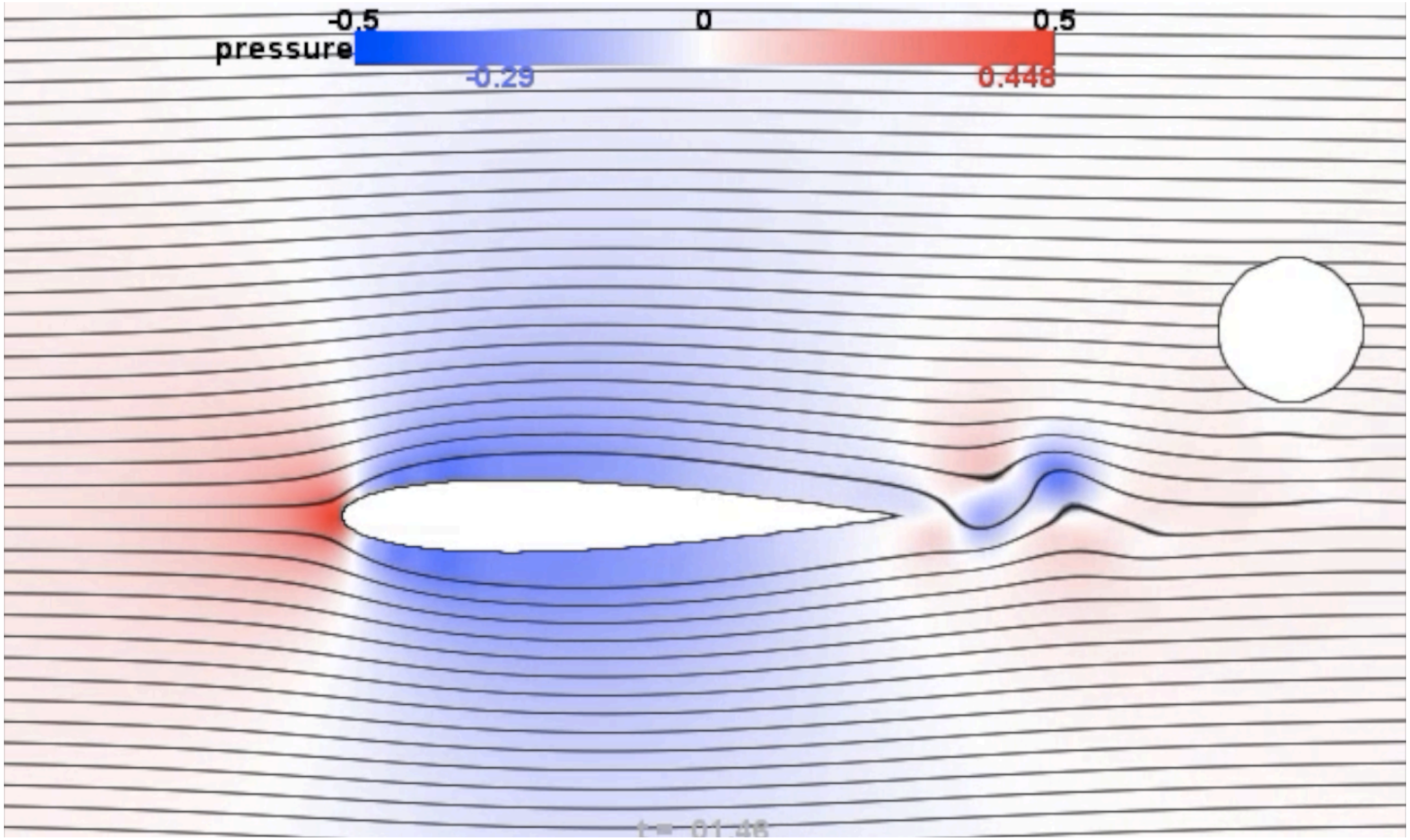
Fish model open water pressure



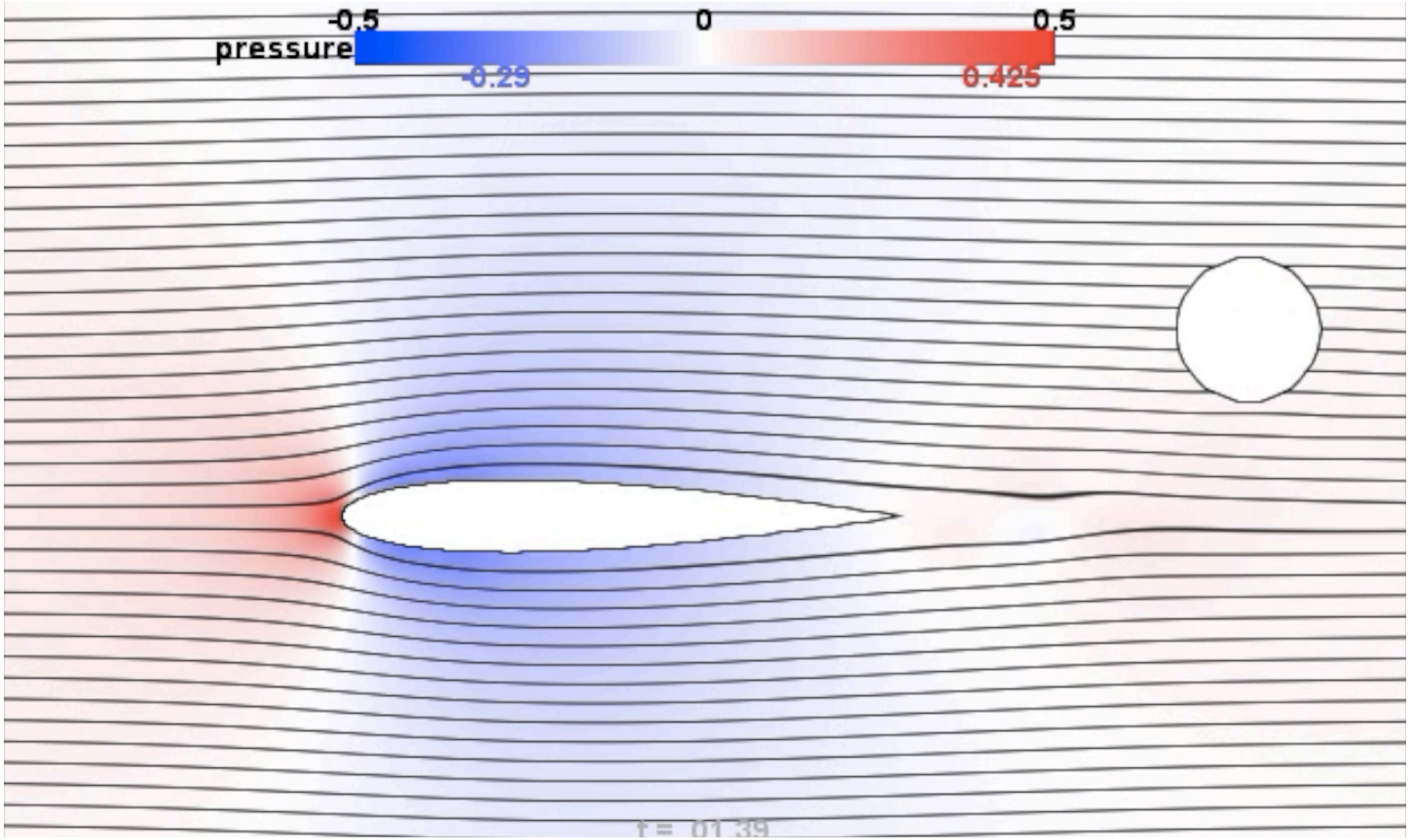


- 3D axisymmetric foil at Re=6K

Direct Forcing generates nonphysical separation and surface pressures



Second-order method predicts correct near-body pressure and velocity



Current Status

- BDIM: Analytic convolution GEQ with simple numerical implementations
- First-order BDIM applies accurate nonlinear pressure BCs
- Second-order BDIM controls gradient discontinuity at intermediate *Re*

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Thanks to:
Audrey Maertens, Michael Triantafyllou, Dick Yue @ MIT

Future Directions

- Speed-up through optimizing $\vec{u} = \vec{f}$
- High *Re* by incorporating BL data
- Continue fun and challenging applications

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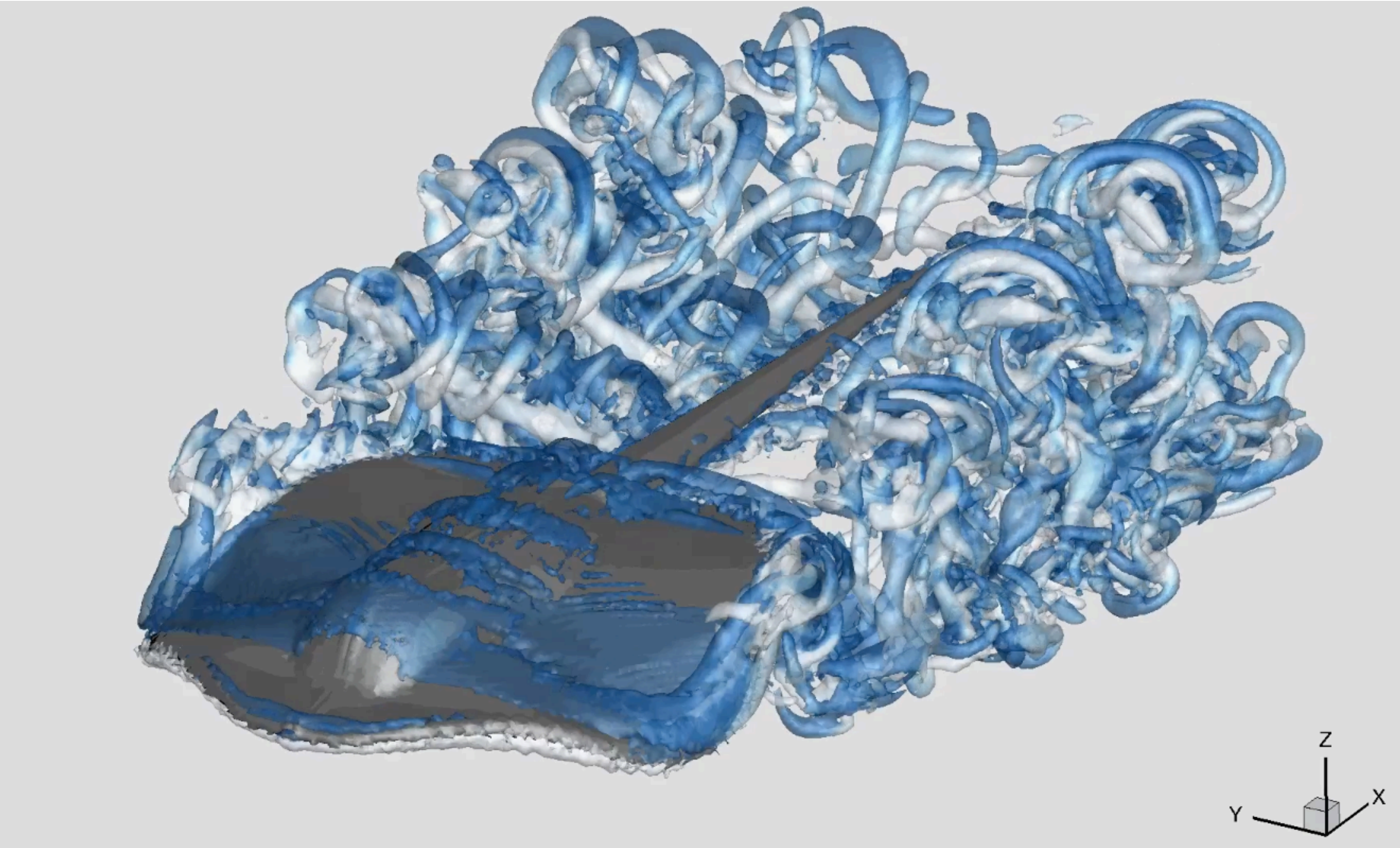

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