

## ORIGINAL ARTICLE

## Nonlinear dielectric optomechanical metamaterials

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**We introduce a dielectric photonic metamaterial presenting a giant nonlinear optical response driven by resonant optomechanical forces. Being inherently free of Joule losses, it exhibits optical bistability at intensity levels of less than  $0.2 \text{ mW } \mu\text{m}^{-2}$  and, furthermore, manifests nonlinear asymmetric transmission with a forward : backward optical extinction ratio of more than 30 dB.**

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## INTRODUCTION

Optical forces are extremely important in mesoscopic systems: they are exploited in all forms of optical tweezing, manipulation and binding.<sup>1–5</sup> The dynamic back-action caused by optical forces has been proposed for optomechanical laser cooling and amplification and underpins the emerging field of ‘cavity optomechanics’<sup>6</sup> where enormous progress has been made in recent years.<sup>7–15</sup> Optical forces can also be harnessed for actuation of nanophotonic devices.<sup>16–19</sup> Indeed, the convergence of nanophotonics and nanomechanics through optical forces presents enormous potential for all-optical operation of nanomechanical systems, reconfigurable and ultra-widely tunable nanophotonic devices, and novel nonlinear and self-adaptive photonic functionalities.<sup>20–26</sup>

Here we introduce the concept of optomechanical metamaterials as a new paradigm for achieving strong optical nonlinearity, optical bistability<sup>27</sup> and asymmetric transmission.<sup>28–31</sup> Metamaterials are artificial media with unusual and useful electromagnetic properties achieved through subwavelength structuring.<sup>32</sup> They provide a unique platform for manipulating electromagnetic fields, and thereby optical forces,<sup>33,34</sup> on the nanoscale. Numerical analyses reveal that optomechanical forces, acting within and among the constituent cells of a dielectric (silicon/silicon nitride) metamaterial, provide a strong nonlinear optical response mechanism (i.e., one through which light may change the optical properties of the medium) delivering high contrast, near-infrared asymmetric transmission and optical bistability at intensity levels of only a few hundred  $\mu\text{W } \mu\text{m}^{-2}$ .

## MATERIALS AND METHODS

Within the framework of classical electrodynamics, the components of the total time-averaged force  $F$  acting on an object illuminated with light can be calculated using a surface integral:<sup>35</sup>

$$\langle F_i \rangle = \oint_S \langle T_{ij} \rangle n_j dS \quad (1)$$

where  $S$  is a bounding surface around the object and  $T_{ij}$  is the time-averaged Maxwell stress tensor:

$$\langle T_{ij} \rangle = \frac{1}{2} \text{Re} \left[ \epsilon \epsilon_0 \left( E_i E_j^* - \frac{1}{2} \delta_{ij} |E|^2 \right) + \mu \mu_0 \left( H_i H_j^* - \frac{1}{2} \delta_{ij} |H|^2 \right) \right] \quad (2)$$

The stress tensor integral Equation (1) encompasses both radiation pressure, which arises through transfer of momentum between photons and any object on which they impinge, and the gradient force, which is associated with strong intensity variations in the local field around an object.<sup>34</sup>

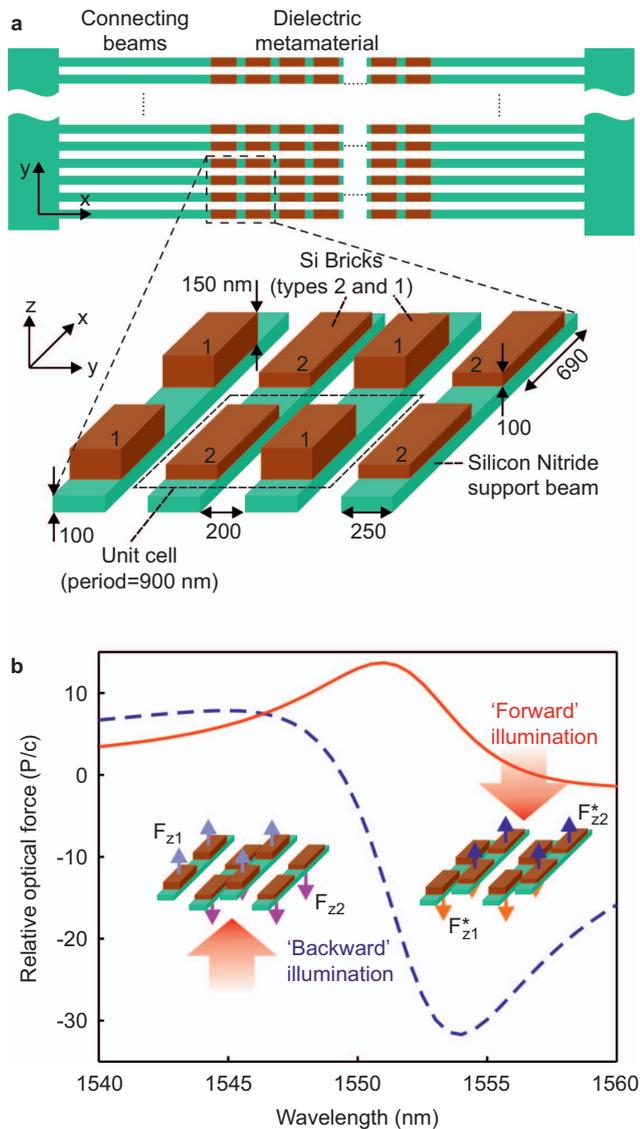
Figure 1a shows an artistic impression and dimensions of the optomechanical metamaterial under consideration: it comprises an array of 250 nm wide, 690 nm long silicon ‘nano-bars’ with thicknesses alternating in the  $y$  direction between 100 and 150 nm; these are supported on parallel 100 nm thick, 250 nm wide silicon nitride strips running parallel to  $x$  and separated from each other (in  $y$ ) by 200 nm. A unit cell of the metamaterial, with  $x$  and  $y$  dimensions of 900 nm, thus comprises a pair of dissimilar silicon nano-bars on parallel, independently mobile silicon nitride beams. It has recently been shown experimentally<sup>36</sup> that geometrically asymmetric silicon structures such as this support strong near-infrared magnetic resonances akin to the familiar ‘trapped mode’ of metallic asymmetric split ring designs.<sup>37</sup> Here we show computationally that in the vicinity of such a resonance (detailed further in the Supplementary Materials) strong optical forces are generated, which act to change the spatial arrangement of nano-bars within each cell and thereby the optical properties of the array:

- Electric  $E$  and magnetic  $H$  field distributions for the metamaterial are obtained from fully three-dimensional finite-element Maxwell solver simulations (in COMSOL Multiphysics). By modeling a single-unit cell with periodic boundary conditions in the  $x$  and  $y$  directions, these calculations assume an infinite planar array (which amounts in practical terms only to an assumption that one’s probe beam is smaller than the array);

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**Figure 1** Asymmetric optomechanical forces in a dielectric photonic metamaterial. (a) Artistic impression and dimensional details of the parallel silicon nitride beam, silicon nano-bar metamaterial configuration studied. (b) Spectral dispersion of the relative optical force  $F_{\text{opt}}^{\text{cell}}$  on the two nanobar elements of a single metamaterial unit cell under normally incident  $x$ -polarized illumination for both forward ( $-z$ ) and backward ( $+z$ ) directions of light propagation ( $F_{\text{opt}}^{\text{cell}} = F_{z2} - F_{z1}$ , where  $F_{z1}$  and  $F_{z2}$  are the optical forces on the thick and thin bars, respectively; Optical force is presented in units of  $P/c$ , where  $P$  is the incident power per unit cell and  $c$  is the speed of light in vacuum).

- Through the Maxwell stress tensor integral (Equation (1)), these  $E$  and  $H$  fields provide for the evaluation of optical force (on the nano-bar elements of a unit cell) as a function of relative displacement for selected wavelengths and/or incident power levels (as shown in Figure 2a).
- By translating these forces to a purely mechanical model of the silicon nitride beam structure (again in COMSOL), one may identify the equilibrium displacement position(s) at which the total optical force on each beam (for given illumination conditions) is balanced by the elastic restoring force. This mechanical model necessarily assumes a metamaterial array of finite extent in the  $x$  direction (12 Si nano-bars on a 10.8  $\mu\text{m}$  long, 100 nm thick

silicon nitride support), with the nitride beams extending, at a reduced thickness of 50 nm, to fixed mounting points 15  $\mu\text{m}$  from the boundaries of the nano-bar array (edge effects relating to the finite number of silicon nano-bars on a beam are ignored because the optical force on each nano-bar is primarily a function of its electromagnetic interaction with dissimilar near-neighbors in the  $y$  direction.) By modeling a single pair of neighboring beams (one supporting 100 nm thick silicon nano-bars, the other 150 nm bars) the model effectively assumes an infinite array in the  $y$  direction—periodic boundary conditions being implied by the absence of any mechanical coupling between beams.

- The optical transmission coefficients for identified equilibrium states can then be obtained using the original model for the electromagnetic properties of the metamaterial array (the mechanical model shows that, within the range of applied force considered, less than 5% of the total out-of-plane beam deflection occurs across the central 100 nm thick section supporting the nano-bar array. As such, it validates the assumption of zero array curvature implied by the electromagnetic model's periodic boundary conditions).

Silicon and silicon nitride are assumed to be lossless with refractive indices of 3.5 and 2.0 respectively in the near-infrared range under consideration.<sup>38</sup> Further detail of modeling procedures may be found in the Supplementary Information.

All in-plane ('horizontal'; perpendicular to  $z$ ) optical forces generated within the metamaterial structure are canceled and only out-of-plane ('vertical'; parallel to  $z$ ) forces act on the dielectric beams. These drive each beam to move up or down until the optical forces are balanced by elastic restoring forces. While the electromagnetic components of the modeling procedure assume an infinite planar metamaterial array, optomechanical properties are presented below for a representative square domain of  $12 \times 12$  unit cells subject to uniform (plane wave) illumination at a total incident power level  $P_0$ .

Figure 1b shows the relative magnitude of optical forces  $F_{\text{opt}}^{\text{cell}}$  acting on the pair of adjacent thick (150 nm) and thin (100 nm) nano-bars within a unit cell for light impinging on the metamaterial in the forward and backward directions—defined respectively as being incident on the silicon bar and supporting silicon nitride beam sides of the structure. A significant difference is found between these two configurations. The total force of radiation pressure on a unit cell is  $\pm 2RP/c$  (where  $R$  is the reflection coefficient,  $P$  is the incident power per unit cell and  $c$  is the speed of light in vacuum), depending on the direction of incident light propagation. Forces on the individual nano-bar elements (see Supplementary Information) can be much larger in magnitude and may act against the direction of light propagation. Resonantly enhanced gradient forces are therefore the primary driver of the differential movement between neighboring beams, which gives rise to the asymmetric, nonlinear and bistable optical responses described below.

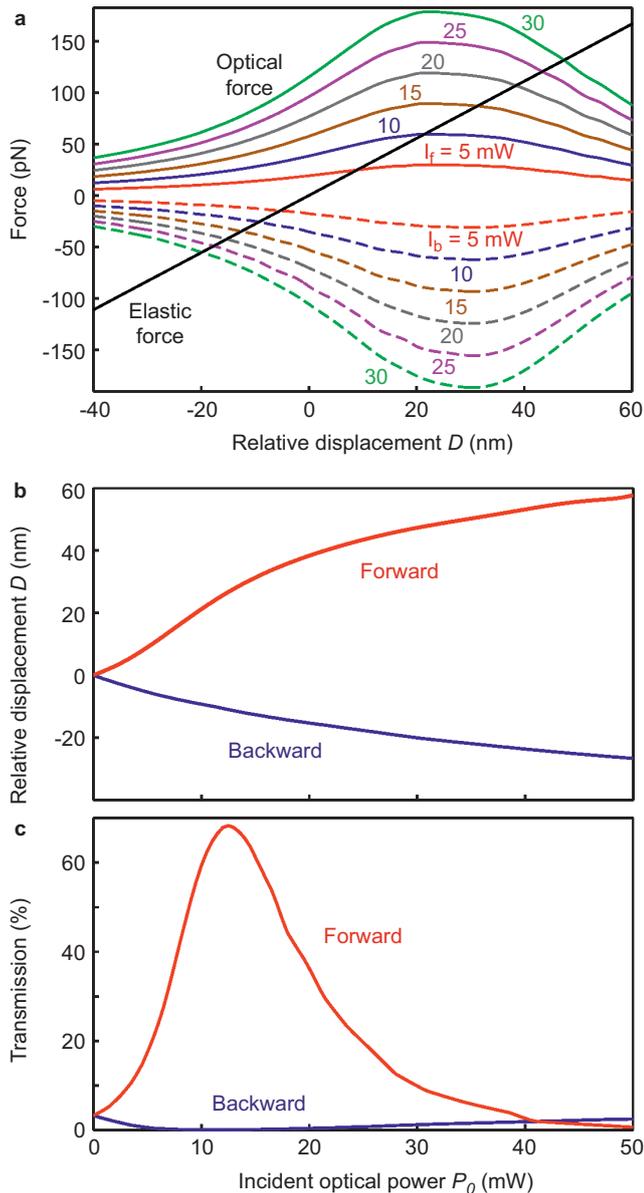
## RESULTS AND DISCUSSION

### Nonlinearity and asymmetric transmission

By providing a mechanism whereby light can induce changes in the spatial arrangement of a metamaterial's constituent parts, and thereby in its optical properties, the optomechanical effect described here acts as the foundation of a strong optical nonlinearity. Figure 2a shows the relative optical force acting on two neighboring beams  $F_{\text{opt}}^{\text{beam}}$  ( $= NF_{\text{opt}}^{\text{cell}}$  where  $N=12$  is the number of silicon nano-bars on each beam within the mechanical model) as a function of mutual displacement  $D$  in the vertical ( $z$ ) direction for a selection of incident light power levels at a wavelength  $\lambda=1551$  nm. To achieve stable equilibrium, not only

should the optical force be equal to the elastic restoring force  $F_{el}$ , but they should also satisfy the conditions that  $F_{opt}^{beam} > F_{el}|_{D-\Delta D}$  and  $F_{opt}^{beam} < F_{el}|_{D+\Delta D}$ . If these last conditions are not met, the equilibrium will be unstable and any perturbation would cause  $D$  to increase or decrease rapidly towards a stable balance point.

In the example shown, stable equilibrium is achieved for forward incidence of light with a power of 5 mW at a relative displacement of 9 nm (the point at which the black elastic force line intercepts the solid red 5 mW optical force line). For backward incidence, the direction of relative movement is inverted and equilibrium is achieved for the same



**Figure 2** Nonlinear optical response and asymmetric transmission. (a) Dependence of the relative optical force  $F_{opt}^{beam}$  on the mutual out-of-plane displacement  $D$  of neighboring silicon nitride beams supporting thick and thin silicon nano-bars under 1551 nm forward (solid lines) and backward (dashed lines) illumination at a range of incident power levels  $P_0$  (as labeled). The straight black line corresponds to the opposing elastic force. (b, c) Dependencies of (b) relative nano-beam displacement and (c) metamaterial optical transmission on total incident power at a wavelength of 1551 nm ( $D = D_2 - D_1$  where  $D_1$  and  $D_2$  are the absolute displacements of beams supporting thick and thin Si nano-bars respectively).

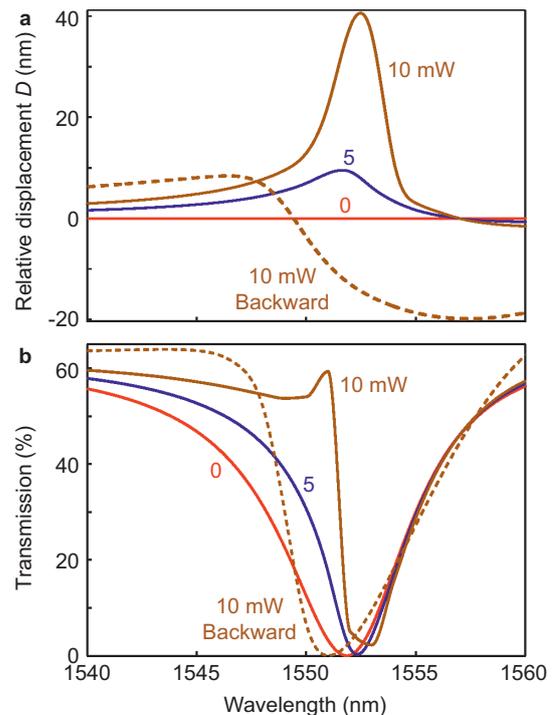
power level at a mutual separation of  $-5$  nm (the intersection of black and dashed red lines).

Figure 2b shows the dependence of relative beam displacement on optical power  $P_0$  at a wavelength of 1551 nm for forward and backward light propagation directions. As a result of this movement, the dielectric metamaterial changes its optical properties as shown in Figure 2c. For forward propagation, transmission increases from a zero-illumination level of 3.2% to 68.3% at a power of 12.5 mW and then drops back to  $\sim 0.6\%$  at 50 mW. The backward incidence optical transmission is less than 3.2% across the entirety of the 0–50 mW power range and at  $P_0 = 12.5$  mW is only 0.034%, giving a forward:backward extinction ratio  $>30$  dB.

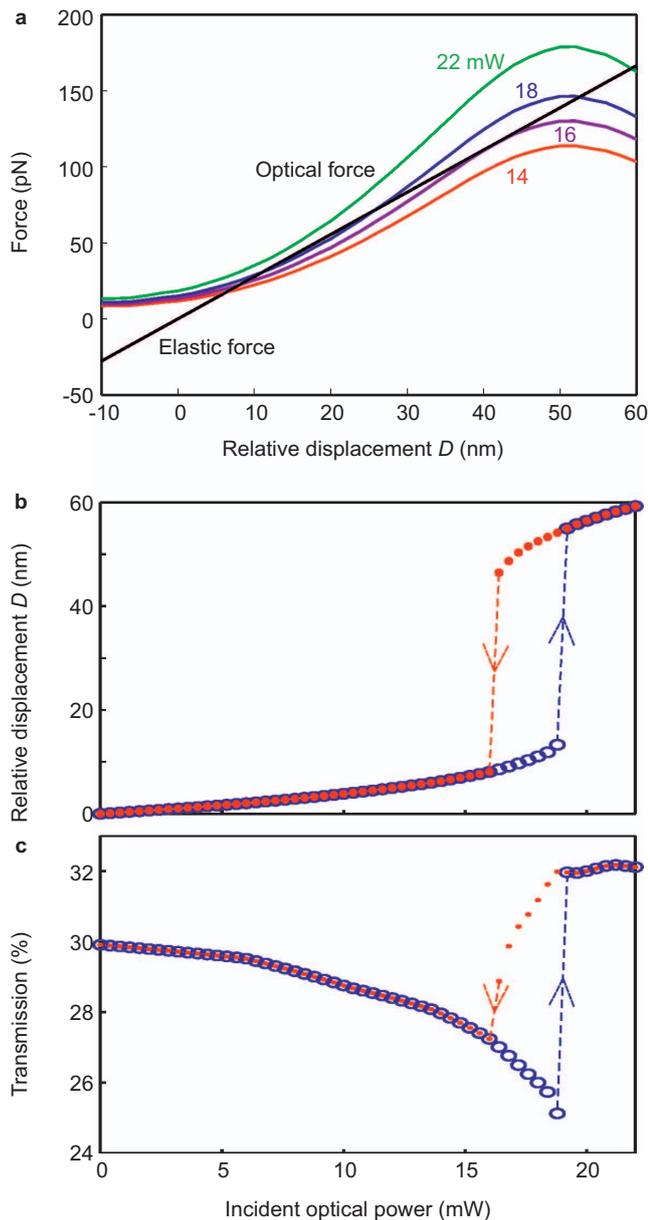
Figure 3a shows the spectral dispersion of relative beam displacement under forward illumination at a selection of optical power levels and Figure 3b shows the corresponding transmission resonances, which become increasingly narrow and asymmetric as the optical power increases from 0 to 10 mW. This figure further illustrates the property of optomechanically-induced asymmetric transmission based on differing displacement responses for forward and backward directions of light propagation: at an optical power of only 10 mW, one observes an asymmetric transmission band centered at 1551 nm wherein the forward:backward extinction exceeds 20 dB.

### Bistability

The dielectric nano-bar metamaterial also exhibits optical bistability based on its nano-mechanical response:<sup>27,39</sup> Figure 4a presents the relative optical force (for forward incidence at a wavelength of  $\lambda = 1555$  nm) and the corresponding elastic restoring force acting on two neighboring beams as a function of relative displacement. At both low and high incident powers, the system has only one equilibrium



**Figure 3** Optomechanical nonlinearity and asymmetric transmission resonances. (a) Spectral dispersion of the steady-state relative nano-beam displacement and (b) corresponding optical transmission for forward incidence of light (solid lines) at a selection of total incident power levels (as labeled) and backward incidence at 10 mW (dashed lines).



**Figure 4** Optomechanical bistability. (a) Dependence of the relative optical force  $F_{\text{opt}}^{\text{beam}}$  and elastic restoring force  $F_{\text{el}}$  on the mutual out-of-plane separation  $D$  of neighboring silicon nitride nano-beams under 1555 nm forward illumination at a selection of incident power levels  $P_0$  (as labeled). (b, c) Corresponding bistable incident power dependencies of (b) mutual displacement and (c) optical transmission.

position (the single points of intersection between  $F_{\text{el}}$  and  $F_{\text{opt}}^{\text{beam}}$  lines at  $D=6.3$  nm for  $P_0=14$  mW and at 59 nm for 22 mW). But at intermediate levels, here between about 16 and 19 mW, the optical and elastic force lines intersect at more than one point and the system becomes bistable, as illustrated by the relative displacement and corresponding optical transmission versus power curves in Figure 4b and 4c.

### Switching dynamics

In such a highly nonlinear system, the switching dynamics are complex and depend strongly on initial conditions and control input dynamics. The above study is concerned with steady-state displacements of the nanostructure (i.e., assumes constant illumination at any given incident power level) and with the associated changes in optical

properties, but an indicative estimate of switching time can be obtained by numerically solving the nonlinear equations of motion for individual beams. This analysis yields a characteristic transition time, from the zero-illumination equilibrium position to maximum deflection, of order  $5 \mu\text{s}$  (assuming instantaneous application of a force equivalent to that generated at  $P_0=10$  mW for forward illumination at a wavelength of 1551 nm, ignoring the damping of any subsequent oscillation).

### CONCLUSIONS

In summary, we introduce a new type of dielectric metamaterial, inherently free of Joule losses, which exhibits strong optomechanical nonlinearity, asymmetric transmission and optical bistability at optical intensities of less than  $0.2 \text{ mW } \mu\text{m}^{-2}$ .

While changes in the transmission spectrum of a metamaterial such as considered here may also result from the Kerr nonlinearity of the silicon bars, this nonlinear response mechanism is around five orders of magnitude weaker than the optomechanical nonlinearity.

With regard to the practical realization of optomechanical metamaterials, resonance quality factors will be affected slightly by the small but non-zero optical absorption coefficients of the constituent media, but more so by manufacturing imperfections. However, as the first experimental studies of dielectric metamaterials have shown,<sup>36</sup> neither of these factors is an obstacle to the achievement of near-infrared resonances at least as sharp as is possible in metallic metamaterials. Optical absorption (including two-photon absorption) and variations in ambient temperature may lead to thermomechanical changes in the structure. But while these would affect every beam equally, the nonlinear, asymmetric and bistable behaviors of interest are derived from differential movements of neighboring beams. As such, their visibility would not be compromised. The viscosity of the ambient medium, though not relevant to the steady state optical properties of such a system, will clearly be a determining factor in the limits of dynamic switching performance.

This metamaterial concept for nanoscale photonic functionality may exploit unique technological and manufacturing opportunities provided by semiconductor membrane technology and, in being driven by forces generated among its constituent parts (as opposed to external actuators), holds notable advantages over more conventional M/NEMS structures, particularly in relation to size scaling for different operational wavelength bands.

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