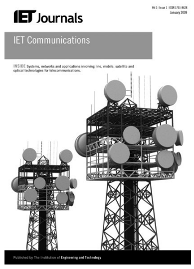
Published in IET Communications
Received on 1st March 2013
Revised on 6th August 2013
Accepted on 17th September 2013
doi: 10.1049/iet-com.2013.0173



ISSN 1751-8628

Channel estimation relying on the minimum bit-error-ratio criterion for BPSK and QPSK signals

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Abstract: The authors consider the channel estimation problem in the context of a linear equaliser designed for a frequency selective channel, which relies on the minimum bit-error-ratio (MBER) optimisation framework. Previous literature has shown that the MBER-based signal detection may outperform its minimum-mean-square-error (MMSE) counterpart in the bit-error-ratio performance sense. In this study, they develop a framework for channel estimation by first discretising the parameter space and then posing it as a detection problem. Explicitly, the MBER cost function (CF) is derived and its performance Q3 studied, when transmitting BPSK and QPSK signals. It is demonstrated that the MBER based CF aided scheme is capable of outperforming existing MMSE, least square-based solutions.

1 Introduction

The problem of estimating the channel coefficients of a frequency selective channel in a communication system has been extensively addressed in the literature. Various methods have been proposed based on numerous criteria, namely the maximum-likelihood [1], the linear minimum-mean-square-error [1–3], least square (LS) [4, 5], expectation maximisation [6] and so on. Recent channel estimation methods including [7–9] use these criteria for channel estimation. In this work, we propose a new channel estimation framework based on the minimum bit-error-ratio (MBER)-based criterion [10–12].

We consider a linear equalizer-based channel estimation problem in the presence of additive white Gaussian noise (AWGN) for a frequency selective channel. Although, we consider a single antenna system for the proposed method in the current work, it can be extended to multiple antenna system also.

It was demonstrated in [10] that the optimal MBER linear equaliser can be designed by directly minimising the bit-error-ratio (BER) and that in the presence of a non-Gaussian equaliser output distribution it is capable of outperforming the MMSE-based system [11–16]. The concept of detection-aided channel estimation relying on discretising channel parameter space was proposed for a fixed time-invariant channel and for BPSK signals in [17]. Our new contribution is that we further develop this to a dispersive Rayleigh fading channel conveying both BPSK and QPSK signal sets and characterise the achievable performance of the proposed method.

We commence by first posing the channel estimation problem as a detection problem, where the parameter space is discretised into sufficiently fine-grained intervals (or bins) and we devise a technique of detecting the correct parameter interval containing the true parameter. More specifically, we invoke the MBER criterion as our cost function (CF), which is then optimised by finding the tap-weights of the MBER-equaliser. We will demonstrate that – under certain circumstances – the proposed MBER channel estimate results in a BER, which is better than that of the channel estimator relying on the MMSE CF across a range of bit-energy-to-noise (E_b/N_0) ratio values. Although, this framework is analogous to the multi-level hypothesis testing philosophy [18], to the best of our knowledge, this problem formulation framework has not been used before.

The primary contributions of this work beyond those presented in [17] are as follows:

1. MBER-based channel estimation is conceived for a complex-valued channel conveying both channel-coded QPSK data and pilot symbols.
2. Closed-form expressions are derived for the MBER equaliser's coefficient-optimisation CF for transmission over circularly symmetric zero-mean Gaussian channels.
3. The complexity analysis of the proposed method is presented. The analysis is approximate in nature.

Notation: $\mathbb{E}(x)$ denotes the expectation operation of a random variable. The variables, such as x, $\boldsymbol{x}$, $\boldsymbol{X}$ represent a scalar, a vector and a matrix, respectively, whereas x^{R} and x^{I} represent the real and imaginary parts of any complex number x. For a vector $\boldsymbol{s}$, $[\boldsymbol{s}]_i$ represents the ith element of $\boldsymbol{s}$. Furthermore, $P_r\{x\}$ denotes a general probability term. $\mathrm{sgn}(x) = +1$, if $x > 0$, $\mathrm{sgn}(x) = -1$, if $x < 0$. For any positive integer n, we have $n! \triangleq n(n-1)\ldots \cdot 2 \cdot 1$ and $\mathring{n} \triangleq (n-1)(n-3)..3.1$.

2 Background

2.1 Data model

We consider a linear discrete-time frequency-selective channel contaminated by AWGN, whose output at time instant k is

$$r_k = \sum_{m=0}^{M-1} h_m x_{k-m} + v_k \qquad (1)$$

where x_k is an input drawn from any discrete signal constellation set, $\boldsymbol{h} = \left[h_0 h_1 \ldots h_{M-1}\right]^{\mathrm{T}} \in \mathbb{C}^{M\times 1}$ is the complex-valued channel impulse response (CIR) having a length of M symbol-duration with h_m being the m^{th} CIR coefficient and v_k is the AWGN noise component with zero mean and a power spectral density of σ_v^2.

Let us assume that an L-symbol equaliser, $\boldsymbol{c} \in \mathbb{C}^{L\times 1}$, is used for detecting the input symbol x_k. The output of the linear equaliser is given by $y_k = \boldsymbol{c}^{\mathrm{H}}\boldsymbol{r}_k$, where the channel's output vector is $\boldsymbol{r}_k = \left[r_k r_{k-1} \cdots r_{k-(L-1)}\right]^{\mathrm{T}} \in \mathbb{C}^{L\times 1}$. More explicitly, we have [11]

$$\boldsymbol{r}_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{v}_k \qquad (2)$$

where $\boldsymbol{H} \in \mathbb{C}^{L\times(L+M-1)}$ is the channel matrix defined as

$$\boldsymbol{H} = \begin{bmatrix} h_0 & h_1 & . & h_{M-1} & . & 0 \\ 0 & h_0 & h_1 & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & h_0. & . & h_{M-1} \end{bmatrix} \qquad (3)$$

and the input symbol vector is $\boldsymbol{x}_k = \left[x_k \cdots x_{k-(L+M-2)}\right]^{\mathrm{T}} \in \mathbb{C}^{(L+M-1)\times 1}$, whereas the noise vector is $\boldsymbol{v}_k = \left[v_k \cdots v_{k-(L-1)}\right]^{\mathrm{T}} \in \mathbb{C}^{L\times 1}$.

2.2 Overview of MMSE and MBER-based symbol detection

We continue with a brief description of the MMSE and MBER symbol detection frameworks, where the optimum equaliser coefficients are found using both the MMSE and the MBER CFs.

MMSE method: The optimum equaliser solution based on the MMSE criterion for the zero-mean data symbol x_k is obtained by Kay [1]

$$\boldsymbol{c}^{\mathrm{MMSE}} = \boldsymbol{e}_1 \boldsymbol{R}_x \mathbf{H}^{\mathrm{H}}\left(\boldsymbol{H}\boldsymbol{R}_x\boldsymbol{H}^{\mathrm{H}} + \sigma_v^2 \boldsymbol{I}\right)^{-1} \qquad (4)$$

where $\boldsymbol{e}_1 = [100\ldots 0] \in \mathbb{R}^{(L+M-1)\times 1}$ and $\boldsymbol{R}_x \in \mathbb{R}^{(L+M-1)\times(L+M-1)}$ and represents the covariance matrix of data vector $\boldsymbol{x}_k$. Symbol detection is carried out at the output of the equaliser using $(\boldsymbol{c}^{\mathrm{MMSE}})^{\mathrm{H}}\boldsymbol{r}_k$.

MBER method: In this context, the optimum MBER equaliser coefficients are designed by minimising the BER. If each symbol value x_k is equiprobable and it is drawn from a BPSK signal set, the estimated value of x_k at the decision device's output becomes

$$\hat{x}_k = \mathrm{sgn}(y_k) \qquad (5)$$

The error probability, P_{e}, for transmission over a dispersive non-fading channel is evaluated as [11]

$$\begin{aligned} P_{\mathrm{e}} &= \Pr\left(\hat{x}_k \neq x_k\right) \\ &= \mathbb{E}\left[Q\left(\frac{\boldsymbol{c}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{x}_k\, x_k}{\|\boldsymbol{c}\|\sigma_v}\right)\right] \end{aligned} \qquad (6)$$

where $Q(x)$ is the Gaussian Q-function, as defined in [1].

The expectation in (6) is computed over the $N = 2^{M+L-1}$ equally likely $\boldsymbol{x}_k$ vectors. If $\boldsymbol{x}_{k,n}$ is the nth realisation of $\boldsymbol{x}_k$, then we define $\boldsymbol{s}_n \triangleq \boldsymbol{H}\boldsymbol{x}_{k,n}\, x_{k,n}$. From (6), the error probability is simplified to

$$P_{\mathrm{e}} = \frac{1}{N}\sum_{n=1}^{N} Q\left(\frac{\boldsymbol{c}^{\mathrm{T}}\boldsymbol{s}_n}{\|\boldsymbol{c}\|\sigma_v}\right) \qquad (7)$$

The MBER solution is then formulated as

$$\boldsymbol{c}^{\mathrm{MBER}} = \boldsymbol{c}\arg\left[\min\left[P_{\mathrm{e}}\right]\right] \qquad (8)$$

where the optimum solution may be found using diverse optimisation techniques, as exemplified by the adaptive MBER (AMBER) solution [11]. The symbol detection is carried out at the output of the equaliser using $(\boldsymbol{c}^{\mathrm{MBER}})^{\mathrm{H}}\boldsymbol{r}_k$.

3 Channel estimation within the detection framework

Let us now reformulate the above-mentioned detection framework in the context of our CIR-parameter estimation problem. Consider the equivalent form of (2) as

$$\boldsymbol{r}_k = \boldsymbol{X}_k\boldsymbol{h} + \boldsymbol{v}_k \qquad (9)$$

where the matrix $\boldsymbol{X}_k \in \mathbb{C}^{L\times M}$ is defined as

$$\boldsymbol{X}_k = \begin{bmatrix} x_k & x_{k-1} & . & x_{k-(M-1)} \\ x_{k-1} & x_{k-2} & . & x_{k-M} \\ . & . & . & . \\ x_{k-(L-1)} & . & . & x_{k-(L+M-2)} \end{bmatrix} \qquad (10)$$

Equation (9) may be interpreted as the model of a virtual communication system associated with the 'channel model' $\boldsymbol{X}_k$ and the 'data input vector' $\boldsymbol{h}$. In reality, $\boldsymbol{X}_k$ contains the known training sequence and $\boldsymbol{h}$ is the CIR to be estimated. From the point of parameter estimation, an estimation technique estimates the parameter within a given uncertainty interval that contains the true value. As an example, with h being a parameter and $\hat{h}$ being its estimation, we can say $\hat{h}$ lies within some ϵ-neighbourhood $(\epsilon > 0)$ of the parameter [1]. The entire continuous channel parameter space is visualised as the union of an infinite number of contiguous ϵ-neighbourhood spaces as shown in Fig. 1. The mth entry of the hypothetical data vector $\boldsymbol{h}$ is drawn from discrete values of the discretised CIR model. Consider the intervals encapsulating the true CIR estimate, which contiguously cover the entire discretised CIR space. If an estimate lies within its own native interval, it is assumed to be a 'correct decision'. This framework is well suited for discrete channel detection (estimation) using the MBER CF.

doi: 10.1049/iet-com.2013.0173

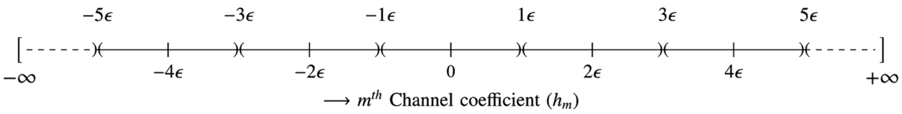

Fig. 1 *Discrete interval representation of range of values a channel parameter estimate can take with associated* ϵ-neighbourhood

4 Proposed MBER-based channel estimation

Let us now define the proposed methodology conceived for both real-valued and complex-valued channels. Assuming that the noise statistics will not change with time for the estimation duration, we drop the symbol index k.

4.1 CF for real-valued channel taps

To estimate $\boldsymbol{h}$ from (9), a detection mechanism similar to the previously described data-detection model is proposed. Let us consider estimation of the mth tap, h_m, $m = 0, 1, \ldots, M-1$. We define an L-tap equaliser with coefficients $\boldsymbol{w}_m = \left[w_{0,m}, w_{1,m}, w_{2,m}, \ldots, w_{L-1,m}\right]^{\mathrm{T}} \in \mathbb{C}^{L\times 1}$ for $m = 0, 1, \ldots, M-1$. We choose a different equaliser symbol $\boldsymbol{w}_m$ for channel estimation to differentiate it from $\boldsymbol{c}$, which is used for symbol ($\boldsymbol{x}$) detection after the channel estimation step. The output of the mth L-tap equaliser $\boldsymbol{w}_m$ is defined as

$$\hat{h}_m = \boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{r} \tag{11}$$

The probability of a correct decision, when detecting h_m, $m = 0, 1, \ldots, M-1$ belonging to the interval containing the true value h_m, is denoted by $P_{c,m}$ and given by

$$\begin{aligned} P_{c,m} &= P_r\left\{-\epsilon \le \hat{h}_m - h_m \le \epsilon\right\} \\ &= P_r\left\{-\epsilon \le \left(\boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{X}\boldsymbol{h} + \boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{v} - h_m\right) \le \epsilon\right\} \\ &= \mathbb{E}_{\boldsymbol{h}\left\{P_r\left\{-\epsilon \le \left(\boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{X}\boldsymbol{h} + \boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{v} - h_m\right) \le \epsilon | \boldsymbol{h}\right\}\right\}} \end{aligned} \tag{12}$$

We define

$$\mu_m \triangleq \boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{X}\boldsymbol{h} - h_m \triangleq \sum_{i=0}^{M-1} b_{i,m}\, h_i \tag{13}$$

where $b_{i,m} \triangleq \left[\boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{X}\right]_i$ for $i \ne m$ and $b_{m,m} \triangleq \left[\boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{X}\right]_m - 1$. Also, $u_m \triangleq \boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{v}$ which is a zero-mean Gaussian random variable with a variance of $\|\boldsymbol{w}_m\|^2\sigma_v^2$ and having a probability density function (pdf), $p(u_m)$. We consider the term

$$\begin{aligned} &P_r\left\{-\epsilon < \left(\boldsymbol{w}_m^T X\boldsymbol{h} + \boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{v} - h_m\right) < \epsilon | \boldsymbol{h}\right\} \\ &= P_r\left(-\epsilon < \left(\mu_m + \boldsymbol{w}_m^{\mathrm{T}}\boldsymbol{v}\right) < \epsilon | \boldsymbol{h}\right) \\ &= P_r\left(-\epsilon - \mu_m < u_m < \epsilon - \mu_m | \boldsymbol{h}\right) \end{aligned} \tag{14}$$

The probability of error associated with detecting h_m is formulated as

$$\begin{aligned} P_{\mathrm{e},m} &= 1 - P_{c,m} \\ &= 1 - \mathbb{E}_{\boldsymbol{h}}\left[\int_{-(\epsilon+\mu_m)}^{(\epsilon-\mu_m)} p(u_m)\, \mathrm{d}u_m\right] \\ &= \mathbb{E}_{\boldsymbol{h}}\left[Q\left(\frac{\epsilon+\mu_m}{\|\boldsymbol{w}_m\|\sigma_v}\right)\right] + \mathbb{E}_{\boldsymbol{h}}\left[Q\left(\frac{\epsilon-\mu_m}{\|\boldsymbol{w}_m\|\sigma_v}\right)\right] \\ &= \int_{h_0}\cdots\int_{h_{M-1}}\left[Q\left(\frac{\epsilon+\mu_m}{\|\boldsymbol{w}_m\|\sigma_v}\right) + Q\left(\frac{\epsilon-\mu_m}{\|\boldsymbol{w}_m\|\sigma_v}\right)\right]P(\boldsymbol{h})\, \mathrm{d}\boldsymbol{h} \end{aligned} \tag{15}$$

where $P(\boldsymbol{h})$ is the joint distribution of all the M CIR coefficients. Every h_m is a random variable having a variance of $\sigma_{h,m}^2$, independent of h_i for $i \ne m$. $P_{j_i}^h$, (with $i = 1, 2, \ldots, M$), an optimum solution for $\boldsymbol{w}_m$ is obtained by minimising (15), which can be written as [17]

$$\boldsymbol{w}_m^{\mathrm{MBER}} = \arg_{\boldsymbol{w}_m}\left[\min\left[P_{\mathrm{e},m}\right]\right] \tag{16}$$

4.2 CF for complex-valued channel taps

For complex-valued CIR coefficients, the probability of error function is an extension of the real-valued case. A complex-valued CIR can be interpreted as a pair of independent, parallel real-valued channels. The probability of erroneous decision for the complex-valued CIR coefficient h_m, for $m = 0, 1, \ldots, M-1$, can thus be calculated as [16]

$$P_{\mathrm{e},m} = P_{\mathrm{e},m}^{\mathrm{R}} \times P_{\mathrm{e},m}^{\mathrm{I}} \tag{17}$$

where $P_{e,m}^{\mathrm{R}}$ and $P_{e,m}^{\mathrm{I}}$ are the probabilities of erroneous detection (estimation) of the real and imaginary components of h_m, respectively. We define

$$\mu_m^{\mathrm{R}} \triangleq \Re\left\{\boldsymbol{w}_m^{\mathrm{H}}\boldsymbol{X}\boldsymbol{h} - h_m\right\} = \sum_{i=0}^{2M-1} b_{i,m}^{\mathrm{R}} h_i^{\mathrm{R,I}} \tag{18}$$

$$\mu_m^{\mathrm{I}} \triangleq \Im\left\{\boldsymbol{w}_m^{\mathrm{H}}\boldsymbol{X}\boldsymbol{h} - h_m\right\} = \sum_{i=0}^{2M-1} b_{i,m}^{\mathrm{I}} h_i^{\mathrm{R,I}} \tag{19}$$

where $2M$ length stacked variables are defined as $\boldsymbol{a}_1 \triangleq \left[\Re\{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}\} - \Im\{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}\}\right] \in \mathbb{R}^{2M\times 1}$, $\boldsymbol{a}_2 \triangleq \left[\Im\{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}\}\Re\{\boldsymbol{w}^{\mathrm{H}}\boldsymbol{X}\}\right]$ and $\boldsymbol{h}^{\mathrm{R,I}} \triangleq \left[\Re\{\boldsymbol{h}\}\Im\{\boldsymbol{h}\}\right] \in \mathbb{R}^{2M\times 1}$. We define $b_{i,m}^{\mathrm{R}} \triangleq [\boldsymbol{a}_1]_i$ for $i \ne m$ and $b_{i,m}^{\mathrm{R}} \triangleq [\boldsymbol{a}_1]_m - 1$ for $i = m$. Similarly $b_{i,m}^{\mathrm{I}} \triangleq [\boldsymbol{a}_2]_i$, for $i \ne M+m+1$ and $b_{i,m}^{\mathrm{I}} \triangleq [\boldsymbol{a}_2]_{M+m+1} - 1$ for $i = m$. The probabilities of error, $P_{\mathrm{e},m}^{\mathrm{R}}$, $P_{\mathrm{e},m}^{\mathrm{I}}$, can be obtained from (5) as

$$P_{\mathrm{e},m}^{\mathrm{R}} = \mathbb{E}_{\boldsymbol{h}}\left[Q\left(\frac{\epsilon+\mu_m^{\mathrm{R}}}{\|\boldsymbol{w}_m\|\sigma_v}\right)\right] + \mathbb{E}_{\boldsymbol{h}}\left[Q\left(\frac{\epsilon-\mu_m^{\mathrm{R}}}{\|\boldsymbol{w}_m\|\sigma_v}\right)\right] \tag{20}$$

$$P_{\mathrm{e},m}^{\mathrm{I}} = \mathbb{E}_{\boldsymbol{h}}\left[Q\left(\frac{\epsilon+\mu_m^{\mathrm{I}}}{\|\boldsymbol{w}_m\|\sigma_v}\right)\right] + \mathbb{E}_{\boldsymbol{h}}\left[Q\left(\frac{\epsilon-\mu_m^{\mathrm{I}}}{\|\boldsymbol{w}_m\|\sigma_v}\right)\right] \tag{21}$$

4.3 Closed-form expression of $P_{e,m}$ for zero-mean Gaussian channels

In order to obtain a closed-form expression for (15), (20), (21) using multinomial expansion of series, we define $K_m \triangleq \left(1/\left(\|w_m\|\sigma_v\right)\right)$ and recall $n = (n-1)(n-3)\ldots 3.1$ for any positive integer n.

4.3.1 Real-valued channel scenario

Proposition 1: The closed-form expression for (15) when the channel is a zero-mean Gaussian is given by

$$P_{\mathrm{e},m} = 1 - \sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n K_m^{2n+1}}{n!} \times \left[\sum_{k_0+k_1+\cdots k_M=2n+1}\frac{(2n)!\epsilon^{k_M}}{k_0!k_1!\ldots k_M!}\prod_{0\le i\le(M-1)}(b_{i,m}\sigma_{h,i})^{k_i}\overset{\circ}{k}_i\right] \left(k_M\ \text{odd}, k_0, k_1, \ldots, k_{M-1}\ \text{even}\right) \tag{22}$$

Proof: Let us consider the real-valued channel scenario, using the Taylor series expansion of e^{-x^2}, the Q-function can be expanded as

$$Q(x) = \frac{1}{2} - \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty}\frac{(-1)^{(n)}x^{2n+1}}{n!(2n+1)} \tag{23}$$

The argument of the Q-function in (15) is $K_m\left(\epsilon + \sum_{i=0}^{M-1} b_{i,m}h_i\right)$. Thus in (23), we have to evaluate $\left(\epsilon + \sum_{i=0}^{M-1} b_{i,m}h_i\right)^{2n+1}$ using multinomial expansion of series. The series can be expressed as [19] (see (24))

$$\left(\epsilon + \sum_{i=0}^{M-1} b_{i,m}h_i\right)^{2n+1} = \sum_{k_0+k_1+\cdots k_M=2n+1}\frac{(2n+1)!\epsilon^{k_M}}{k_0!k_1!\ldots k_M!}\prod_{0\le i\le(M-1)}(b_{i,m}h_i)^{k_i} \tag{24}$$

All the tap coefficients are assumed to be zero-mean, independent Gaussian random variables. We take expectation of (24) with respect to the Gaussian random vector $\boldsymbol{h}$. Thus, $\mathbb{E}_{\boldsymbol{h}}\{h_i^{k_i}\}$, for $i = 0, 1, \ldots, M-1$, can be expressed as

$$\begin{aligned}\mathbb{E}_{\boldsymbol{h}}\{h_i^{k_i}\} &= (\sigma_{h,i})^{k_i}k_i, \left(k_i\ \text{even}, 0 \le i \le (M-1)\right) \\ &= 0, \left(k_i\ \text{odd}, 0 \le i \le (M-1)\right)\end{aligned} \tag{25}$$

since it is evident that if any of $k_0, k_1, \ldots, k_{M-1}$ is odd in (24), then the whole term would be zero after taking the expectation. Since $(2n+1)$ is an odd number for any $n \ge 0$, k_M must also be an odd number. This ensures that only the specific terms having all even numbered $k_0, k_1, \ldots, k_{M-1}$ and odd numbered k_M would have a non-zero value. After the expectation operations

$$\mathbb{E}_{\boldsymbol{h}}\left[\left(\epsilon - \sum_{i=0}^{M-1} b_{i,m}h_i\right)^{2n+1}\right] = \mathbb{E}_{\boldsymbol{h}}\left[\left(\epsilon + \sum_{i=0}^{M-1} b_{i,m}h_i\right)^{2n+1}\right] \tag{26}$$

This is because, all the negative coefficients will be positive as each $k_0, k_1, \ldots, k_{M-1}$ are even numbers. Using (23)–(26) in (15), we obtain (22). □

4.3.2 Complex-valued channel scenario:

Similar closed-form expressions are derived for the complex-valued channel scenario using Proposition 1 and given as follows

$$P_{\mathrm{e},m}^{\mathrm{R}} = 1 - \sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n K_m^{2n+1}}{n!} \times \left[\sum_{k_0+k_1+\cdots k_{2M}=2n+1}\frac{(2n)!\epsilon^{k_{2M}}}{k_0!k_1!\ldots k_{2M}!}\prod_{0\le i\le(2M-1)}\left(b_{i,m}^{\mathrm{R}}\sigma_{h,i}\right)^{k_i}k_i\right] \left(k_{2M}\ \text{odd}, k_0, k_1, \ldots, k_{2M-1}\ \text{even}\right) \tag{27}$$

$$P_{\mathrm{e},m}^{\mathrm{I}} = 1 - \sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n K_m^{2n+1}}{n!} \times \left[\sum_{k_0+k_1+\cdots k_{2M}=2n+1}\frac{(2n)!\epsilon^{k_{2M}}}{k_0!k_1!\ldots k_{2M}!}\prod_{0\le i\le(2M-1)}\left(b_{i,m}^{\mathrm{I}}\sigma_{h,i}\right)^{k_i}\overset{\circ}{k}_i\right] \left(k_{2M}\ \text{odd}, k_0, k_1, \ldots, k_{2M-1}\ \text{even}\right) \tag{28}$$

leading to $P_{\mathrm{e},m}$ in (17).

4.4 Channel estimation algorithm

To obtain the channel estimation algorithm using the MBER method, we need to obtain the optimal w_m, $m = 0, 1, \ldots, M-1$ using the CF given by (15) and (17), using existing optimisation tools. For the special case of Gaussian channels, we can use the closed-form expressions of the CF. For practical purposes, we consider a finite number of terms, N_a, to approximate the infinite-term summation in the CF. It is clear that the approximation is better if N_a is large. The corresponding algorithmic steps are summarised in Algorithm 1 (see Fig. 2).

4.5 Complexity analysis

We present an approximate complexity analysis using the closed-form approximation of the CF. We obtain the approximate computational cost with the real case as the complex one is a simple extension. We express the cost in terms of the number of 'operations', which refers to the combination of addition, subtraction, multiplication, division and other simple arithmetic operations. The major computation is involved in evaluating the CF for each value of the equaliser vector in the optimisation routine and e note that the calculation of all the we consider terms, such as K_m^{2n+1}, $\left(b_{i,m}^{R}\right)^{2n+1}$ which are dependent on equaliser. Other terms like $k_i!$, ϵ^{k_M} and k_i can be pre-computed for a given N_a, so we ignore them for the iterative computation of the CF.

doi: 10.1049/iet-com.2013.0173

Algorithm 1

Given : N_a, ϵ, σ_v^2, L, M, $\boldsymbol{r}$, $\sigma_{h,m}^2$ for $m = 0, 1, \ldots, M-1$.
for $m = 0$ **to** $M-1$ **do**
  Use an optimisation tool to optimise (22) or (27) (28) to obtain $\boldsymbol{w}_m^{MBER}$, with $\boldsymbol{w}_m^{MMSE}$ (MMSE solution of the equaliser) as the initial value.
  $\hat{h}_m = (\boldsymbol{w}_m^{MBER})^H \boldsymbol{r}$.
end for

Fig. 2 *Channel estimation using MBER method*

1. For each value of w, $b_{i,m}$ requires $\simeq 2L - 1$ operations (inner product of two $L \times 1$ vectors requires L multiplications and $L - 1$ additions). Next, $b_{i,m}\sigma_{h,i}$ needs 1 operation, so $(b_{i,m}\sigma_{h,i})^{k_i}$ needs $2L(k_i - 1)$ operations as $x^n = x.x.x.x \ldots x$ (n times) requires $n - 1$ operations. Now, $\left(b_{i,m}\sigma_i^h\right)^{k_i}\mathring{k}_i$ incurs $2L(k_i - 1) + 1$ operations.
2. $\prod_{0 \le i \le (M-1)} (b_{i,m}\sigma_i^h)^{k_i}\mathring{k}_i$ incurs $\simeq 2L\sum_{i=0}^{M-1}(k_i - 1) + 2M + 1$ operations. Considering the factorials $k_i!$, $2n!$ and ϵ^{k_M}, we further obtain the cost as $2L\sum_{i=0}^{M-1}(k_i - 1) + 3M + 1$. This value is for a particular choice of various k_0, k_1, k_2, …, k_M. Thus, considering the inner summation part of (22), we obtain $\sum_{k_0+k_1+\cdots+k_M=2n+1}\left[2L\sum_{i=0}^{M-1}(k_i - 1) + 3M + 1\right]$ operations.
3. We now calculate the approximate cost of $(K_m)^{(2n+1)}$. Each K_m takes $\simeq (2L + 1)$ operations as $\|w\|$ takes $2L - 1$ operations and then multiplication by σ_v and then a division take the cost to $2L + 1$. Thus, $(K_m)^{(2n+1)}/n!$ needs $\simeq 2L + 2n + 2$ operations.

We now consider the cost for all values of n which varies from 0 to N_a. The complete cost would be (see (29))

For the complex case, M, L need to be replaced by $2M$, $2L$ in (29), respectively, and the total cost will be two times the cost of $P_{e,m}^{R}$ because of $P_{e,m}^{I}$ component. The practical value of N_a, for a squared error of 10^{-16} between the approximated value of $Q(x)$ and its true value, is found to be close to 400. For 10^{-8}, it is 40 and $N_a = 20$ gives 10^{-6}. With $N_a = 100$ as the golden case, we choose $N_a = 20$ for a $M = 2$ tap real channel. We see the computational cost reduction of almost 90% with negligible performance drop compared to the golden case. For, MMSE case, we obtain the computational comparison by observing the machine time. We observed that almost 15 times more computations are required (with $M = 2$ real channel case) including the complete search of equalisers with $N_a = 20$ for the MBER method compared to the MMSE method.

5 Simulation results

We study the attainable BER performance of the proposed MBER method and compare it to that of the MMSE method as a function of both the E_b/N_0 and of the equaliser length as well as of the channel discretisation parameter ϵ. We considered convolutionally coded BPSK and QPSK symbol sets transmitted over an uncorrelated dispersive Rayleigh fading channel. Furthermore, we considered a CIR length of $M = 3$ associated with CIR-tap variances of 0, −4 and −8 dB in conjunction with an equaliser length of $L = 3$ and $\epsilon = 0.001$. We have chosen $N_a = 40$. The E_b/N_0 (dB) is defined as $10\log_{10}\left(E_b E\|\boldsymbol{h}\|^2/2\sigma_v^2\right)$, where E_b is defined as the average input bit power. $E\|\boldsymbol{h}\|^2$ is computed as per the tap variances. For a given E_b/N_0 value, σ_v^2 is obtained using the above expression for generating appropriate noise realisations.

At the receiver, a training sequence length of 18 pilot-symbols is used for estimating the CIR. The estimated CIR, using Algorithm 1 (see Fig. 2) with Nelder-Mead [20] based optimisation tool, is obtained. This CIR estimate is then used for detecting the signals, whereas employing both the MMSE and the MBER-based equalisers. The equaliser length L is the same for the estimation and detection stages. The AMBER algorithm of [11] is used for MBER symbol detection. For the coded system, we employed a 1/2-rate convolution code (CC) using the hard-decision Viterbi decoding algorithm, for which the octal representation of the generator polynomials is (117_8, 115_8). For benchmarking purposes, we consider channel estimation relying on both the classic MMSE and LS methods [4].

Experiment 1: We consider the bit-stream to be encoded using a convolutional coder and BPSK symbols are transmitted over a dispersive uncorrelated Gaussian fading real-valued channel. We use a 3-tap CIR and an equaliser length of 3.

Remarks 1: Fig. 3 plots the BER against E_b/N_0 performance. It is observed that in the E_b/N_0 range between 3 to 12 dB, the MBER CIR estimation performs better than MMSE estimation. At BER = 2×10^{-3}, the MBER estimation combined with MBER detection requires $E_b/N_0 = 8$ dB, whereas the MMSE estimation combined with MBER detection requires 9 dB, yielding an MBER-gain of 1 dB. At low (< 3 dB) and high (> 12 dB) E_b/N_0 the BER values recorded for the MMSE and MBER estimation methods are close to each other. Similarly, with the aid of the MMSE detection method, at BER 3×10^{-3}, the MBER technique requires 9 dB of E_b/N_0, whereas the LS method necessitates 10.4 dB of E_b/N_0. Thus the MBER channel estimation technique achieves an E_b/N_0 gain of about 1.4 dB compared to the LS method.

Experiment 2: In this experiment, we consider the dispersive uncorrelated complex-valued Rayleigh channel for transmitting convolutionally coded QPSK symbols. Again, we use a 3-tap CIR associated with an equaliser length of 3.

$$C \simeq \sum_{n=0}^{N_a}\left[\sum_{k_0+k_1+\cdots+k_M=2n+1}\left[2L\sum_{i=0}^{M-1}(k_i - 1) + 3M + 1\right] + 2L + 2n + 2\right] \quad (29)$$

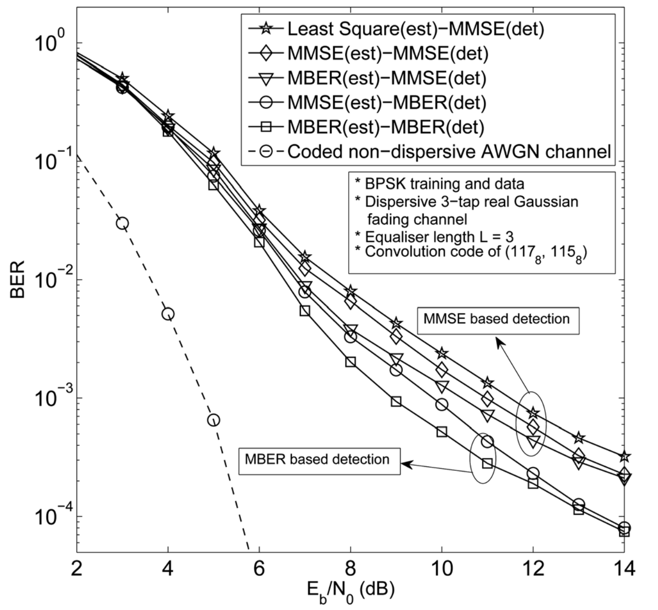

Fig. 3 *BER performance of MBER, MMSE, LS estimation methods with BPSK training and data detection over the dispersive real-valued fading channel with convolution coding*

Channel length $M = 3$ and equaliser length $L = 3$
'est' refers to estimation and 'det' refers to detection

Remarks 2: Fig. 4 plots the BER against E_b/N_0 performance. For the coded system, at $\text{BER} = 6 \times 10^{-3}$, the MBER estimation combined with MBER detection requires $E_b/N_0 = 9$ dB, whereas the MMSE estimation combined with MBER detection requires 10.0 dB, yielding a gain of 1 dB. Similarly, with the aid of the MMSE detection method, at BER 7×10^{-3}, the MBER method needs 10 dB of E_b/N_0, whereas the LS method requires 11.3 dB of E_b/N_0. Thus the MBER method of channel estimation gives an E_b/N_0 gain of about 1.3 dB. At relatively low E_b/N_0 values, the CC degrades the attainable performance because of precipitating the detection errors, which is a widely recognised phenomenon. As seen in Fig. 3, in the E_b/N_0 range of 4–14 dB, the MBER channel estimator performs better than the MMSE estimation technique. The MBER solution is shown to perform better in Fig. 4, when the detector's output signal is likely to be non-Gaussian [10, 11]. At the receiver, the equaliser output is dominated by the Gaussian noise at low E_b/N_0 values, hence, it is expected that the MMSE and MBER estimation would perform in a similar manner. At higher E_b/N_0, the equaliser output tends to become Gaussian [21] with the increase of the equaliser length. The equaliser output may be non-Gaussian for a certain range of E_b/N_0 and this is where the MBER method gives an advantage over the MMSE method.

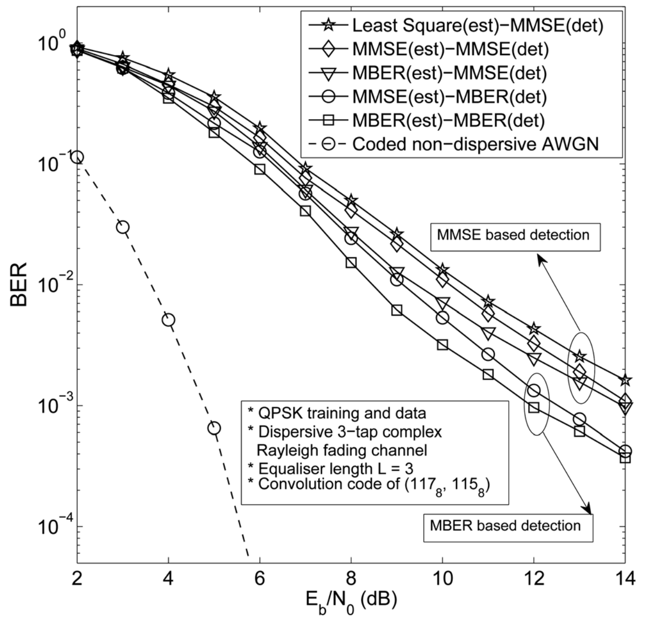

Fig. 4 *BER performance of MBER, MMSE, LS estimation with QPSK training and data detection over dispersive uncorrelated Rayleigh fading complex-valued channel with convolution coding*

Channel length $M = 3$ and equaliser length $L = 3$

Experiment 3: The effect of the equaliser length on the performance of a coded system is studied. Again, we consider the dispersive complex-valued uncorrelated Rayleigh fading channel using convolutionally coded QPSK symbols. We use a 3-tap CIR in conjunction with various equaliser lengths and record the E_b/N_0 value corresponding to the BER of 10^{-2} for different equaliser lengths.

Remarks 3: Fig. 5 shows that for $L = 3$, the E_b/N_0 required by the MMSE method is about 1.0 dB higher than that of the MBER method and 1.6 dB higher than that of perfect channel estimation. For $L = 9$, the difference becomes negligible. Note that the MMSE performance approaches the MBER performance upon increasing the equaliser length, again because the equaliser's output is tending to be Gaussian and this is a well-known phenomenon in the context of MBER [11]. For higher equaliser lengths, both methods perform close to the perfect channel estimation scenario.

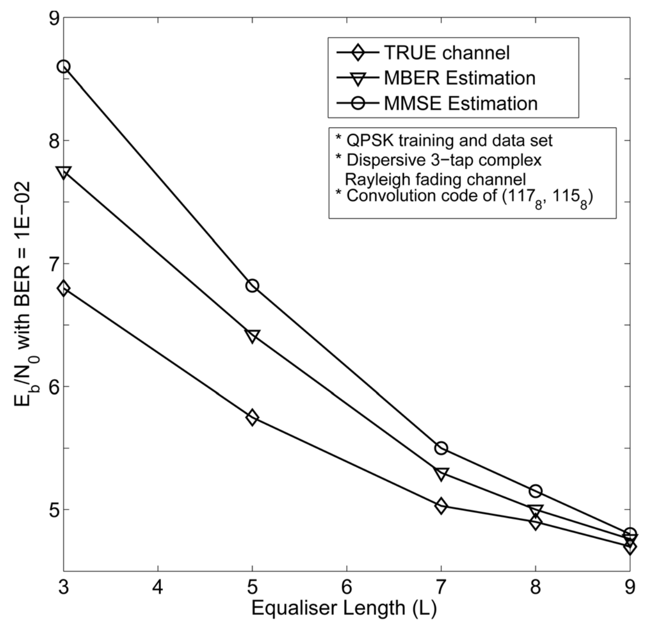

Fig. 5 *E_b/N_0 performances for MBER and MMSE methods are plotted with respect to different equaliser lengths fixing the BER = 1×10^{-2} over a dispersive complex-valued uncorrelated Rayleigh fading channel, when using convolutional coding*

The performance of the ideal channel is also given as reference. Channel length $M = 3$

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doi: 10.1049/iet-com.2013.0173

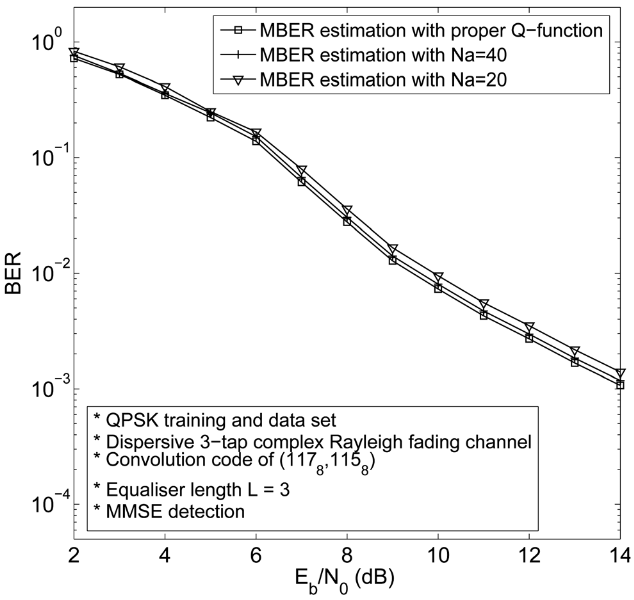

Fig. 6 *Performance comparisons of MBER estimation for different values of N_a over a dispersive uncorrelated Rayleigh fading complex-valued channel with convolution coding*

Channel length $M = 3$ and equaliser length $L = 3$

Experiment 4: We study Q-function approximation technique based on the Taylor series for different N_a values. MBER channel estimation is invoked for transmission over the dispersive complex-valued uncorrelated Rayleigh fading channel. We use a 3-tap CIR with equaliser length 3. The detection method is based on the MMSE criterion.

Remarks 4: Fig. 6 compares the BER performance for various N_a values. The Taylor series-based approximation closely approaches the accurate Q-function result, as the value of N_a increases. For $N_a = 40$, the results are quite close to the actual value of the function.

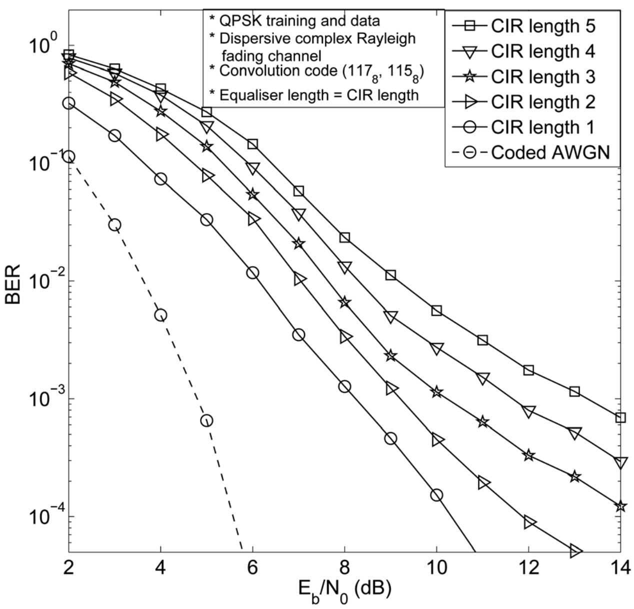

Fig. 7 *Performance comparison of MBER channel estimation with QPSK training and data detection for different CIR lengths of a dispersive uncorrelated Rayleigh fading complex-valued channel with convolution coding*

Equaliser length is the same as the CIR length

Experiment 5: Fig. 7 studies the BER performance of MBER-based channel estimation for various CIR lengths. The channel is a dispersive complex-valued uncorrelated Rayleigh fading medium conveying convolutionally coded QPSK symbols. Here, the CIR lengths of 2, 3, 4, 5 are considered in conjunction with equaliser lengths of 2, 3, 4, 5, respectively. The consecutive tap variances start from 0 dB and decay by 4 dB with each CIR length increment. The BER performance of the corresponding coded AWGN system is also included for comparison.

Finally, the sensitivity of the MBER-based channel estimation to ϵ was also studied. We observed that the performance of the MBER-based channel estimation becomes worse than that of the MMSE-based estimation for larger ϵ values ($\epsilon > 0.01$).

6 Conclusions

The channel estimation problem was posed as a detection problem and a new channel estimation method was conceived for convolutionally coded BPSK and QPSK signals relying on the MBER framework. The CF of this channel estimation method was derived and a closed-form expression was presented for Gaussian channels. The performance of the proposed method was studied by simulations. The MBER-based channel estimation was shown to improve the BER as compared with that obtained using the MMSE estimator for a moderate range of E_b/N_0 values. The MBER-based estimation was shown to be sensitive to the channel estimation equaliser length. The performance improvement was achieved at an increased computational complexity.

7 Acknowledgment

The financial support of the DST, India and of the EPSRC, UK under the auspices of the India-UK Advanced Technology Centre (IUATC) is gratefully acknowledged.

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doi: 10.1049/iet-com.2013.0173

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