Upper Limb Electrical Stimulation using Input-Output Linearization and Iterative Learning Control

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Abstract—A control scheme is developed for multi-joint upper limb reference tracking using functional electrical stimulation (FES). In accordance with the needs of stroke rehabilitation, FES is applied to a reduced set of muscles in the arm and shoulder, with support against gravity provided by a passive exoskeletal mechanism. The approach fuses input-output linearization with iterative learning control (ILC), one of the few techniques to have been applied in clinical treatment trials with patients. This powerful hybrid control structure hence extends performance and scope of clinically proven technology for widespread application in rehabilitation robotic and FES domains. In addition to simplifying tracking and convergence properties of the stimulated joints, the framework enables conditions for the stability of unstimulated joints to be derived for the first time. Experimental results confirm tracking performance of the stimulated joints, together with unstimulated joint stability.

I. INTRODUCTION

Annually 15 million people suffer a stroke worldwide and 5 million are left permanently disabled. Conventional therapy to improve upper limb function following stroke is not effective, and only 5% of survivors with severe paralysis regain upper limb function [1]. In recent years there has been growing evidence supporting the effectiveness of rehabilitation robots [2] and functional electrical stimulation (FES) [3], to reduce impairment post-stroke. Both technologies enable a person with limited physical ability to practice tasks, and the resulting sensory feedback is associated with cortical changes that can bring about recovery of functional movement. In particular, there is substantial clinical evidence [4], [5], [6], [3] indicating that increased functional recovery is closely related to the accuracy with which FES assists the subject’s own voluntary completion of a task. This finding also has theoretical support from neurophysiology [7], [8] and motor learning research [9].

Model-based FES control is key in providing the required accuracy, but few such approaches have transferred into clinical practice [10], despite a wide variety of FES upper limb control techniques having been applied in simulation or laboratory conditions. This is due to difficulties in obtaining an accurate model since the identification time available is restricted by the onset of fatigue and the time constraints of the patient, carer, physiotherapist and/or engineer. Time-varying physiological effects also mean that models must be re-identified at the start of each treatment session. A further feature of the rehabilitation problem is that FES is applied only to a subset of weak or paralysed muscles, and hence mechanisms are required to ensure stability of the resulting underactuated system. It is also beneficial that control parameters may be tuned in a transparent manner so that performance can be maintained despite changes in underlying dynamics.

To address these needs, this paper employs input-output linearization, an approach that embeds decoupling and so provides a route to both simplified control design (the dynamics around each joint becoming independent), as well as providing a framework for analysis of unactuated joint stability. This is the first time this approach has been used in upper limb FES, with the only prior application in this field to a single degree of freedom (DOF) knee model [11]. This set-up contained no unactuated DOFs, identification was not considered and no experimental results were given. Input-output linearization also benefits from existing robustness analysis for both parametric [12], [13] and unstructured uncertainty cases [14].

Upper limb FES control approaches capable of embedding precision tracking include neural networks, which have recently been combined with feedback controllers to enable nonlinear components in the model to be approximated while providing guaranteed stability properties, giving rise to asymptotic convergence [15], [16]. Iterative learning control (ILC) is another leading approach, and has exploited the repetitive nature of the rehabilitation process, where patients attempt the same task multiple times in order to promote re-learning. ILC sequentially improves accuracy by using data from previous attempts to adjust the FES supplied during the next execution of the task, and has been successfully used by several groups to assist movement in the lower limb [17], [18], [19], [20].

This paper focuses on the upper limb, and combines input-output linearization with a general linear ILC form that has been employed in three sets of clinical trials [21] during which the tracking accuracy provided by ILC translated into statistically significant results across a range of outcome measures [22], [23]. The ILC structure has minimal parameters that can be tuned to compromise performance and robustness in an intuitive manner, including stipulation of convergence and trial-to-trial control effort. When combined with input-output linearization, these properties can be set for each joint independently, and hence generalise to arbitrary DOF. The system used in this paper is shown in Fig. 1, and applies FES to two muscles in the arm and shoulder to assist patient’s tracking of a virtual reality tracking task. The arm is supported by a passive mechanism to provide a safe, productive environment for training across a wide spectrum of patient ability.

This paper is arranged as follows: Section II develops a combined upper limb and passive robotic arm model of broad relevance, with corresponding identification procedure given in Section III. The input-output linearization and ILC strategy is developed in Section IV with experimental evaluation in Section V. Conclusions are set out in Section VI.

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II. COMBINED FES & ROBOTIC UPPER LIMB PLATFORM

A. Mechanical Support

The clinically trialled system shown in Fig. 1 combines FES and robotic therapy, using a popular form of commercial arm support (ArmeoSpring, Hocoma AG) to provide adjustable force against gravity via two springs. Each joint is aligned in either the horizontal or vertical plane, as shown in Fig. 2a), with measured joint variables \( \Theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T \). The patient’s arm is rigidly strapped to the exoskeleton support.

B. Muscle Selection and Modeling

Spasticity (velocity-dependent stiffness) in stroke typically produces resistance to arm extension due to overactivity of biceps, wrist and finger flexors, and loss of activity of triceps, anterior deltoid, wrist and finger extensors [24]. Triceps and anterior deltoid are hence selected for stimulation to align with the clinical need to increase muscle tone and restore motor control of weakened muscles. The relationship between muscle stimulation and subsequent movement is well explored, and sophisticated muscle models exist with multiple attachment points across more than one joint, and movement over complex sliding surfaces [25]. However, simplification opens up routes for both parameter identification and controller derivation that have not yet been possible for more complex models [10]. Hence it is assumed that applying FES to the triceps produces a moment about an axis orthogonal to both the forearm and upper arm, and FES to the anterior deltoid produces a moment about an axis that is fixed with respect to the shoulder. The corresponding joint variables (\( \phi_3 \) and \( \phi_5 \)) are shown in Fig. 2b), with additional axes chosen to account for the remaining DOF. A dynamic model of the support is given by

\[
B_a(\Theta) \ddot{\Theta} + C_a(\Theta, \dot{\Theta}) \dot{\Theta} + F_a(\Theta, \dot{\Theta}) + G_a(\Theta) + K_a(\Theta) = 0
\]

where \( B_a(\cdot) \) and \( C_a(\cdot) \) are 5×5 inertial and Coriolis matrices, and \( F_a(\cdot) \) and \( G_a(\cdot) \) are friction and gravitational vectors. Moments produced through gravity compensation provided by each spring yield the form \( K_a(\cdot) = [0, 0, k_3(\theta_3), 0, k_5(\theta_5)]^T \).

Similarly, a dynamic model of the human arm is given by

\[
B_h(\Phi) \ddot{\Phi} + C_h(\Phi, \dot{\Phi}) \dot{\Phi} + F_h(\Phi, \dot{\Phi}) + G_h(\Phi) = \tau(u, \Phi, \dot{\Phi})
\]

in which \( \tau(\cdot) \) comprises moments produced through application of FES, and \( \Phi = [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5]^T \) contains anthropomorphic joint angles. As discussed in [26], the most prevalent form of muscle representation is the Hill-type model

\[
t_i(u_i(t), \phi_i, \dot{\phi}_i) = h_i(u_i(t)) \cdot F_{m,i}(\phi_i, \dot{\phi}_i), \quad i \in \{2, 5\}
\]

Here \( u_2(t) \) and \( u_5(t) \) are the FES control signals that comprise the pulselwidth of the stimulation signal applied to the anterior deltoid and triceps respectively (see Fig. 1). The term \( h_i(u_i(t)) \) is a Hammerstein structure with static non-linearity, \( h_{180,1}(u_i) \), representing the isometric recruitment curve, cascaded with linear activation dynamics, \( h_{\text{LAB},i}(t) \). The multiplicative effect of joint angle and joint angular velocity on the muscle torque is modeled by \( F_{m,i}(\phi_i, \dot{\phi}_i) \), and the state-space system realising \( h_{\text{LAB},i}(t) \) has state, input and output matrices \( A_{m,i}, B_{m,i}, C_{m,i} \) respectively. Writing \( u = [0, u_2, 0, u_5]^T \) yields

\[
\tau(u, \Phi, \dot{\Phi}) = [0, \tau_2(u_2, \phi_2, \dot{\phi}_2), 0, 0, \tau_5(u_5, \phi_5, \dot{\phi}_5)]^T
\]

Within suitable joint ranges there exists a bijective transformation \( \Phi = k(\Theta) \) which enables the Lagrangian equation in one variable to be expressed in terms of the other to produce

\[
B(\Phi) \ddot{\Phi} + C(\Phi, \dot{\Phi}) \dot{\Phi} + F(\Phi, \dot{\Phi}) + G(\Phi) + K(\Phi) = \tau(u, \Phi, \dot{\Phi}) - J_h^T(\Phi) h
\]

where \( J_h(\Phi) \) is the system Jacobian and \( h \) the externally applied force and torque. This combined model has terms

\[
B(\Phi) = B_h(\Phi) + k_1(\Phi)^T B_a(k(\Phi)) k_1(\Phi), \\
C(\Phi, \dot{\Phi}) = C_h(k(\Phi), \dot{\Phi}) + k_1(\Phi)^T C_a(k(\Phi), k_1(\Phi)) k_1(\Phi), \\
F(\Phi, \dot{\Phi}) = F_h(\Phi, \dot{\Phi}) + k_1(\Phi)^T F_a(k(\Phi), k_1(\Phi)) + k_1(\Phi)^T B_a(k(\Phi)) k_2(\Phi), \\
G(\Phi) = G_h(\Phi) + k_1(\Phi)^T G_a(k(\Phi)), \\
K(\Phi) = k_1(\Phi)^T K_a(k(\Phi))
\]

with \( k_1(\Phi) = \frac{d}{d\Phi} k(\Phi) \) and \( k_2(\Phi, \dot{\Phi}) = \frac{d}{dt} \left( \frac{d}{d\Phi} k(\Phi) \right) \).

III. MODEL IDENTIFICATION

Procedures are now developed to identify parameters in (3). The set of joint angle indices actuated by FES is denoted \( I_C = \{2, 5\} \), and the remaining set is denoted \( I_U = \{1, 3, 4\} \).
A. Muscle Axis Identification

Lengths $l_u$ and $l_f$ are first measured, and then axes about which muscles produce torque are identified. To orientate the $\phi_2$ axis to correspond with the stimulated anterior deltoid, two additional rotations, with variables $\alpha$ and $\beta$, are introduced as shown in Fig. 3. After initial rotation of the base frame by $\phi_1$, it is rotated about the z-axis by $\alpha$ and about the x-axis by $\beta$. Appendix A describes the identification procedure used.

B. Passive Parameter Identification

With no applied FES, system (3) simplifies to

$$B(\Phi) \dot{\Phi} + C(\Phi, \dot{\Phi}) \dot{\Phi} + F(\Phi, \dot{\Phi}) + G(\Phi) + K(\Phi) = -J_h^T(\dot{\Phi}) h$$

and can be written in a form which is linear in parameters. First introduce a matrix $Y_B$ containing kinematic data, and a vector $\pi_B$ containing a minimal parameter set, such that

$$Y_B (\Phi(t), \dot{\Phi}(t)) \pi_B = B(\Phi(t)) \dot{\Phi}(t) + C(\Phi(t), \dot{\Phi}(t)) \dot{\Phi}(t) + F(\Phi(t), \dot{\Phi}(t)) + G(\Phi(t)) + K(\Phi(t))$$

Similarly represent $F(\Phi, \dot{\Phi})$, $G(\Phi)$, and $K(\Phi)$ using piecewise linear functions by introducing matrix $Y_F$ containing kinematic data, and vector $\pi_F$ containing a minimal parameter set, such that

$$Y_F (\Phi(t), \dot{\Phi}(t)) \pi_F = F(\Phi(t), \dot{\Phi}(t))$$

Using these (4) is written as

$$\begin{bmatrix} Y_B(t), Y_F(t) \end{bmatrix} \begin{bmatrix} \pi_B \pi_F \end{bmatrix}^T = -J_h^T(\dot{\Phi}(t)) h(t)$$

A 6-axis force/torque sensor is attached to the underside of the extreme link of the mechanical support to provide externally applied force and torque vector $h$. A handle attached to the sensor is used to kinematically excite the system, during which the kinematic variables $Y(t)$ and forces $\tau(t)$ are recorded at times $t_j$, $j = 1, \ldots, N$. From these assemble the matrices

$$\bar{Y} = [Y(t_1)^T \ldots Y(t_N)^T]^T, \quad \bar{\tau} = [\tau(t_1)^T \ldots \tau(t_N)^T]^T$$

The least squares solution for the parameter vector is $\pi = \bar{Y}^T \bar{\tau}$ where $A^T = (A^T A)^{-1}A^T$ is the pseudoinverse of $A$.

To affect a compromise between accuracy and repeatability, the form taken for the $i$th row of $F(\Phi, \dot{\Phi})$ is

$$F_{m,i}(\phi_i) + F_{s,i}(\phi_i), \quad i \in I_C \cup I_U$$

This is accurate provided effects such as spasticity in bi-articular elbow/shoulder muscles, which introduce biomechanical coupling between joints [27], are sufficiently mild [28].

C. Muscle Identification

Hammerstein structures $h_i(u_i, t), i \in I_C$ appearing in (1) are identified by fixing the sensor handle and applying FES inputs, $u_i(t)$, to each muscle. The vector $\bar{\tau}(t)$ is recorded and the torque generated about the $i$th joint axis is extracted using

$$\tau_i(u_i(t), \phi_i, \dot{\phi}_i) = Y_i(t) \pi - \bar{\tau}_i, \quad i \in I_C$$

where $\bar{\tau}$ is provided by the previous tests. Here $Y_i(t)$ corresponds to static operating conditions $\Phi = \bar{\Phi}, \dot{\Phi}, \ddot{\Phi} = 0$, and taking without loss of generality $F_{m,i}(\phi_i, 0) = 0, F_{s,i}(\phi_i, 0) = h_i(u_i, t)$

$$\tau_i(u_i(t), \phi_i, 0) = h_i(u_i(t)) \cdot F_{m,i}(\phi_i, 0) = h_i(u_i(t))$$

(9)

Algorithms developed for stroke patients in [26] are applied to data sets $\{u_i, \tau_i(t)\}$ to identify the Hammerstein structures $h_i(u_i, t), i \in I_C$. These comprise static nonlinearity $h_{IRC,i}(\cdot)$ and linear activation dynamics $h_{L,I,i}(\cdot)$. The latter is then expressed using state-space matrices $M_{A,i}, M_{B,i}, M_{C,i}$.

D. Multiplicative Muscle Function Identification

To identify $F_{m,i}(\phi_i, \dot{\phi}_i), i \in I_C$, kinematic excitation is again applied and $Y(t)$ and $\bar{\tau}(t)$ recorded at samples $t_j, j = 1, \ldots, N$. However now FES sequences $u_i(t)$ are applied and using the Hammerstein models previously identified, the isometric muscle torque is calculated using $h_i(u_i(t), t)$, so that

$$F_{m,i}(\phi_i, \dot{\phi}_i) = \tau^*_i(t) - \tau_i(t), \quad i \in I_C$$

(10)

Here $\tau^*(t) = Y(t) \bar{\tau}$ is the passive torque, with $\bar{\tau}$ provided by previous tests. $F_{m,i}(\cdot)$ is now represented as $Y_{F_{m}}(t) \pi_{F_{m}}$, with an optimal parameter set $\pi_{F_{m}} = Y^T \bar{\tau}$, where

$$\begin{bmatrix} Y_{F_{m}}(t_1) \ldots Y_{F_{m}}(t_N) \end{bmatrix}^T, \quad \begin{bmatrix} \tau'_i(t_1) \ldots \tau'_i(t_N) \end{bmatrix}^T$$

$$\begin{bmatrix} h_i(u_i(t_1), t_1) \ldots h_i(u_i(t_N), t_N) \end{bmatrix}^T$$

IV. FES CONTROL STRATEGY

The clinical objective is to control FES inputs $u_2(t)$ and $u_3(t)$ so that $\phi_2(t)$ and $\phi_3(t)$ track references $\phi_2(t)$ and $\phi_3(t)$ respectively, with the remaining joint angles stable [29], [21].

A. Input-output Linearization

The ILC approaches so far used clinically involve canceling each muscle non-linearity, $h_{IRC,i}(\cdot)$, and assuming each muscle actuates only the corresponding joint before designing separate feedback and single input, single output (SISO) ILC loops. To improve performance and enable extension to a far broader range of support mechanisms and choice of stimulated muscles, a combined input-output linearization and ILC scheme is now developed. Conditions will also be derived for stability of the unactuated joint angles.

The linear actuation dynamics are typically assumed to be second order [26] and hence can be written in state-space form

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -d_{i,1} & -d_{i,2} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_i(t), \quad i \in I_C$$

$$\tau_i(u_i) = A_{m,i} x_1 + B_{m,i} u_i, \quad i \in I_C$$

(11)
so that \( \mathcal{L} \{ h_{\text{IRC},i}(t) \} = \frac{n_{i,1}+n_{i,2}}{x^T + d_{i,1}x + d_{i,2}}. \) Using \( x = [\Phi^T, \Phi^T, x_2^T, x_5^T]^T \) express the controlled dynamics of (3) as

\[
\dot{x} = f(x) + g(x) \begin{bmatrix} h_{\text{IRC},2}(u_2) \\ h_{\text{IRC},5}(u_5) \end{bmatrix}
\]

where \( \phi_2^T = \begin{bmatrix} \phi_2 \\ \phi_5 \end{bmatrix} \) and

\[
f(x) = \begin{bmatrix} p_2(\Phi, \Phi) + (B(\Phi)^{-1})_{2,2} F_{m,2}(\phi_2, \phi_2)(C_{m,2} x_2) \\ p_3(\Phi, \Phi) \\ p_4(\Phi, \Phi) \end{bmatrix} \begin{bmatrix} \Phi \\ \Phi \end{bmatrix} + (B(\Phi)^{-1})_{5,5} F_{m,5}(\phi_5, \phi_5)(C_{m,5} x_5) - \begin{bmatrix} A_{m,2} x_2 \\ A_{m,5} x_5 \end{bmatrix}
\]

\[
g(x) = \begin{bmatrix} g_1(x)^T \\ g_2(x)^T \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

Using \( h(x) = [h_1(x), h_2(x)]^T = [\phi_2 \phi_5]^T. \) From [30], for an \( m \times m \) system the input-output linearizer is

\[
\begin{bmatrix} h_{\text{IRC},2}(u_2) \\ h_{\text{IRC},5}(u_5) \end{bmatrix} = (v(x) - \mu(x) \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix})
\]

which with control input \( v = [v_1, v_2]^T. \) The components of \( \mu, \nu \) are

\[
\mu_i(x) = L_i^T h_i(x), \quad \nu_i(x) = L_g_i L_i^{T-1} h_i(x)
\]

respectively, with \( i, j = 1, \ldots, m, \) and \( k_i \) the relative degree of output \( i. \) The Lie derivatives of \( h_i(x) \) are defined by

\[
L_i h_i(x) = \frac{\delta h_i}{\delta x} f(x), \quad L_g_i h_i(x) = \frac{\delta h_i}{\delta x} g_i(x)
\]

and \( L_i^T h_i(x) \) and \( L_g_i L_i^{T-1} h_i(x) \) are respectively given by

\[
L_i^T(L_i^{T-1} h_i(x)) \quad \text{and} \quad L_g_i (L_i^{T-1} h_i(x))
\]

Relative degree \( k_i \) satisfies \( L_g_i L_i^{k_i-1} h_i(x) \neq 0, \) and \( L_g_i L_i h_i(x) = 0 \) for \( n = 1, 2, \ldots, (k_i-2). \) Hence (13) becomes

\[
\begin{bmatrix} h_{\text{IRC},2}(u_2) \\ h_{\text{IRC},5}(u_5) \end{bmatrix} = \begin{bmatrix} L_g_i L_i^{k_i-1} h_i(x) \\ L_g_i L_i^{k_i-2} h_i(x) \end{bmatrix} \frac{1}{L_g_i L_i^{k_i} h_i(x)} - \begin{bmatrix} L_g_i L_i^{k_i-1} h_i(x) \\ L_g_i L_i^{k_i-2} h_i(x) \end{bmatrix} \frac{1}{L_g_i L_i^{k_i} h_i(x)}
\]

with \( k_1 = 3 \) if \( n_{2,1} = 0, \) and \( k_5 = 4 \) otherwise, and \( k_2 = 3 \) if \( n_{5,1} = 0, \) and \( k_2 = 4 \) otherwise. Applied to (12) this yields

\[
u_2 = \begin{cases} h_{\text{IRC},2} \left( \frac{\delta}{\delta x} \left( \frac{\delta}{\delta x} \left( \frac{\delta}{\delta x} f(x) \right) f(x) \right) - v_2 \right) & \text{if } n_{2,1} = 0 \\ h_{\text{IRC},2} \left( \frac{\delta}{\delta x} f(x) - v_2 \right) & \text{otherwise} \end{cases}
\]

\[
u_5 = \begin{cases} h_{\text{IRC},5} \left( \frac{\delta}{\delta x} f(x) - v_5 \right) & \text{if } n_{5,1} = 0 \\ h_{\text{IRC},5} \left( \frac{\delta}{\delta x} f(x) \right) & \text{otherwise} \end{cases}
\]

The case \( n_{i,1} \neq 0, i \in \mathcal{C}, \) is now used, however the same analysis applies to all cases. This yields the decoupled signals

\[
\phi_2^{(4)} = v_1, \quad \phi_5^{(4)} = v_2 \quad \text{where} \quad \phi_i^{(k)} = \frac{\delta^k}{\delta t^k} \phi_i
\]

### B. Optimal Tracking Controller

System (17) is next stabilized by a pre-compensator to achieve baseline tracking and disturbance rejection. A linear quadratic tracking controller has been selected due to its well-established robust performance, and is realised by state feedback \( v = [\hat{\phi}_2^{(4)}, \hat{\phi}_5^{(4)}]^T - K \xi, \) to yield error dynamics

\[
\begin{bmatrix} e^{(1)} \\ e^{(2)} \\ e^{(3)} \\ e^{(4)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -A_0 & -A_1 & -A_2 & -A_3 \end{bmatrix} \begin{bmatrix} \phi_2^{(4)} \\ \phi_5^{(4)} \end{bmatrix}
\]

where \( e = \Phi_{\text{IRC}} - \Phi_{\text{IC}}, \) and \( K = [A_0, A_1, A_2, A_3]. \) These are stabilized by choosing \( K \) to minimize the cost

\[
J(v) = \int_0^\infty (\xi^T Q \xi + v^T R v) \, dt
\]

which weights error and input energy norms, subject to

\[
\xi + [0 \ 0 \ 0 \ I]^T \left(v - \begin{bmatrix} \phi_2^{(4)} \\ \phi_5^{(4)} \end{bmatrix}\right)
\]

An observer is used to provide estimates of \( \xi \) by minimizing error covariance, as shown in Fig. 4. Note that (19) can be solved over a finite range \([0, T], T < \infty \) resulting in a time-varying \( K. \) This has no effect on the analysis of Section IV-C, which relies only on stability of the error dynamics (18).

### C. Stability of Unactuated Joints

Feedback tracking has been implemented for the controlled joints, but to guarantee stability of the remaining joints first express components of \( C(\Phi, \Phi) \) in standard form as

\[
c_{i,j} = \sum_{k=1}^n \frac{1}{2} \frac{\partial b_{i,j}}{\partial \phi_k} \frac{\partial b_{i,j}}{\partial \phi_k} + \frac{\partial b_{i,j}}{\partial \phi_k} \frac{\partial b_{i,j}}{\partial \phi_l}
\]

where \( b_{i,j} \) are components of \( B(\Phi). \) Partition uncontrolled and controlled joint angles as \( \Phi_U = [\phi_{\mathcal{U}(1)}, \ldots, \phi_{\mathcal{U}(N_U)}]^T \) and \( \Phi_C = [\phi_{\mathcal{C}(1)}, \ldots, \phi_{\mathcal{C}(N_C)}]^T \) respectively, where \( N_U \) and \( N_C \) are the number of elements in each. Then using \( \eta_1 = \Phi_U, \) the system (12) and controller (13) yield the system

\[
\xi = A \xi
\]

\[
\eta = \omega (\xi, \eta, t)
\]

\[
e = \eta - N \xi
\]

where \( \eta = [\eta_1^T, \eta_2^T]^T, \) and \( \omega (\xi, \eta, t) =

\[
\begin{bmatrix} I \\ 0 \end{bmatrix}
\]

The case \( n_{i,1} \neq 0, i \in \mathcal{C}, \) is now used, however the same analysis applies to all cases. This yields the decoupled signals

\[
\phi_2^{(4)} = v_1, \quad \phi_5^{(4)} = v_2 \quad \text{where} \quad \phi_i^{(k)} = \frac{\delta^k}{\delta t^k} \phi_i
\]
and likewise \( B_U(\Phi) \) and \( B_{UC}(\Phi) \) have elements
\[
B_{U,i,j} = b_{U,i}(i,\xi_U(j)), \quad B_{UC,i,j} = b_{I_U,i}(i,\xi_C(j)) \tag{25}
\]
\( F_U(\Phi_U, \dot{\Phi}_U) \) has elements \( F_{U,i} \), which from (7) have form
\[
F_{U(i)}(\Phi_U(i), \dot{\Phi}_U(i)) = F_{x,i}(\Phi_U(i)) \tag{26}
\]
From (22) and (23) the surface \( \xi = 0 \) defines an integral manifold for the system given by
\[
\dot{\eta} = \omega(0, \eta, t) \tag{27}
\]
Since \( A \) is stable this system is globally attractive and defines the zero dynamics [30] relative to the output \( e \).

**Theorem 1.** Suppose that \( \omega(0, \eta^*_t, t) = 0 \) for \( t \geq 0 \), i.e. \((0, \eta^*_t)\) is an equilibrium of the full system (22) - (24), and \( \eta^*_t \) is an equilibrium of the zero dynamics (27), and that \( A \) is stable. Then \((0, \eta^*_t)\) of the full system (22) - (24) is locally stable if \( \eta^*_t \) is locally stable for the zero dynamics (27).

**Proof.** This uses the Center Manifold Theorem, see [30]. □

Stability of the complete system (12) is hence assured if both the actuated and unactuated subsystems are independently stable. The former is guaranteed by the linearizing controller, and the following theorem gives conditions for the latter.

**Theorem 2.** A sufficient condition for the zero dynamics to be stable is that the function \( F_v(\cdot) \) is passive, that is
\[
F_{v,i}(\Phi) \dot{\phi}_i \geq 0, \quad i \in \mathcal{I}_U \tag{28}
\]
and the function \( F_{v,i}(\cdot) \) satisfies the sector bound
\[
F_{v,i}(\Phi) \begin{cases} 
\geq F_{v,i}(\Phi) \dot{\phi}_i \quad \text{if } \dot{\phi} > 0, \\
< F_{v,i}(\Phi) \dot{\phi}_i \quad \text{otherwise.}
\end{cases} \tag{29}
\]
where \( F_{v,i} = \sum_{i \neq j} \sum_{k=1}^{N_C} c_{I_U(i),I_C(k),I_U(j)} \hat{\Phi}_{I_C(k)}^{(1)} \), \( i, j \in \mathcal{I}_U \).

**Proof.** The unactuated system dynamics are given by
\[
B_U(\Phi) \dot{\Phi}_U + C_U(\Phi, \dot{\Phi}) \dot{\Phi}_U + C_{UC}(\Phi, \dot{\Phi}) \dot{\Phi}_C + F_U(\Phi, \dot{\Phi}) \tag{30}
\]
and
\[
B_{UC}(\Phi) \dot{\Phi}_C = 0 \tag{31}
\]
The term \( C_{UC} \) can be partitioned as \( C_{UC}(\Phi, \dot{\Phi}) = C_{UC}^C(\Phi, \dot{\Phi}_C) + C_{UC}(\Phi, \dot{\Phi}) \), with respective elements
\[
C_{UC,i} = \sum_{k=1}^{N_C} c_{I_C(i),I_C(k),I_C(k)} \dot{\Phi}_{I_C(k)}^{(1)} \\
C_{UC,i} = \sum_{k=1}^{N_C} c_{I_C(i),I_C(k),I_C(k)} \dot{\Phi}_{I_C(k)}^{(1)}
\]
Now \( C_{UC}(\Phi, \dot{\Phi}_C) \Phi_C \) can be written as \( C_{\Phi}(\Phi, \Phi_C) \Phi_U \) with
\[
C_{\Phi,i} = \sum_{k=1}^{N_C} c_{I_U(i),I_C(k),I_C(k)} \dot{\Phi}_{I_C(k)}^{(1)}
\]
This enables (31) to be rewritten using substitutions \( C_{UC} \leftrightarrow C_{UC} \) and \( C_{\Phi} \leftrightarrow C_{\Phi} \). Here \( C_{\Phi} = C_{\Phi} \) to give
\[
B_U(\Phi) \dot{\Phi}_U + C_U(\Phi, \dot{\Phi}) \dot{\Phi}_U + C_{UC}(\Phi, \dot{\Phi}_C) \dot{\Phi}_C + F_U(\Phi, \dot{\Phi}) + B_{UC}(\Phi) \dot{\Phi}_C = 0 \tag{32}
\]
When \( \xi = 0 \) the zero dynamics correspond to
\[
B_U(\eta_1) \dot{\eta}_2 + C_U(\eta_1, \eta_2) \dot{\eta}_2 + C_{UC}(\eta_1) \dot{\Phi}_C^{(1)} = 0 \tag{33}
\]
This equates to \( \dot{\eta}_2 = -h(\eta_1, \eta_2) - g(\eta_1) \) where
\[
h(\eta_1, \eta_2) = B_U(\eta_1)^{-1}(C_U(\eta_1, \eta_2) + F_U(\eta_1) + F_c(\eta_2)) \tag{34}
\]
and
\[
g(\eta_1) = B_U(\eta_1)^{-1}(C_{UC}(\eta_1) \dot{\Phi}_C^{(1)} + B_{UC}(\eta_1) \dot{\Phi}_C^{(2)}) \tag{35}
\]
The equilibrium point satisfies \( h(\eta_1^*, 0) + g(\eta_1) = 0 \) and the system can be interpreted as conservative system \( \dot{\eta_2} + g(\eta_1) = 0 \) acted on by external force \(-h(\eta_1, \eta_2)\) where \( \eta_1 = \eta_1 - \eta_1^* \). Accordingly, introduce potential energy function \( V(\eta_1, \eta_2) = \frac{1}{2} B_U(\eta_1)^T \dot{\eta}_2 + \int_0^{\eta_1} F_u(\sigma) \delta \sigma + \int_0^{\eta_2} C_U(\eta_1, \eta_2, \eta_2 - \eta_2^* F_v(\eta_2) \eta_2 \leq \frac{1}{2} B_U(\eta_1)^T \delta \sigma \tag{36}
\]
for stability. This is equivalent to the requirement
\[
\psi_i(\frac{B_U(\eta_1)^T}{2} - C_U(\eta_1, \eta_2) - C_{\Phi}^T(\eta_1, \eta_2) - F_v(\eta_2)) < 0 \tag{37}
\]
As \( \frac{1}{2} B_U(\eta_1)^T - C_U(\eta_1, \eta_2) \) is skew-symmetric, a sufficient condition is that \( C_{\Phi}(\eta_1, \eta_2) + F_v \) is diagonally dominant with positive diagonal entries, which is satisfied by (29). □

Condition (29) can always be met by adding viscous damping to the unactuated joints. Alternatively, (30) shows bounds on \( F_v(\cdot) \) scale with \( \Phi_C^{(1)} \) and hence (29) is relaxed if \( \Phi_C^{(1)} \) is small. The above result holds for any subset of uncontrolled joints, however the amount of damping required for stability is dictated by the degree of axis coupling. With no interaction \( \psi_{v,vi} = 0 \), reducing to the requirement that \( \psi_{v,vi} \) are passive. Theorem 2 relies on stability of the actuated joints, which clearly is subject to model uncertainty. Few robustness results exist for input-output linearization, however [14] provides conditions for stability in terms of unstructured uncertainties using the Gap Metric, as well as an overview of results for structured uncertainties. These hence can be applied to provide bounds on admissible model mismatch such that the analysis in this paper holds in the presence of model uncertainty.

**D. Error Estimation**

Control inputs (15), (16) require the state systems and while \( \Phi, \Phi \) are read directly, muscle states \( x, x_5 \) must be estimated using an observer around the arm system. Each channel is designed for system (11) with input \( h_{B_U, i}(\xi_i) \) and output
\[
F_{m,i}(\phi_i, \dot{\phi}_i) \Phi_i^{-1} (B(\Phi_i)^{-1})_{i,i} \dot{\phi}_i - p_i(\Phi_i, \dot{\Phi}_i), \quad i \in \mathcal{I}_C
\]
These provide the estimates \( \hat{x}_i \) shown in Fig. 4.
E. Linear model-based ILC

ILC is applicable to systems which repeatedly track a fixed reference signal over a finite time interval, termed a trial. After each trial the system is reset to the same initial conditions, and previous trial data are used to modify the control input in order to reduce the error in the subsequent trial. ILC is often applied in combination with a feedback controller to ensure disturbance rejection and baseline tracking performance. There are many available update procedures, the most common form of which is shown in Fig. 4 [31]. Here $\Phi_k(t)$ denotes the plant output on trial $k \in \mathbb{N}$ over finite duration $t \in [0,T]$ and likewise $e_k(t)$ denotes tracking error $\Phi_{zc}(t) - \Phi_{zc,k}(t)$. The ILC update $f_k(t)$ is calculated off-line between trials, using learning filter $L(s)$ and robustness filter $Q(s)$. The ILC objective is to update $f_k(t)$ so that the error converges to zero, that is $\lim_{k \to \infty} \|e_k\| = 0$. This is achieved by the ILC update

$$f_{k+1}(s) = Q(s) (f_k(s) + L(s)e_k(s))$$

(32)

with reset condition $\Phi_{zc,k}(0) = \Phi_{zc}(0) \forall k$. From (18), (20), (24), the relation between $v_k$ and $\Phi_k$ is

$$G(s) = C(sI - A)^{-1}B = \operatorname{diag} \{G_1(s), G_2(s)\}$$

(33)

where $G_i(s) = \frac{-1}{a_{i1}s^4 + a_{i2}s^3 + a_{i3}s^2 + a_{i4}s + a_{i5}}$. Assuming forms $Q(s) = \operatorname{diag} \{Q_1(s), Q_2(s)\}$ and $L(s) = \operatorname{diag} \{L_1(s), L_2(s)\}$ this yields independent error evolutions $e_{i,k+1}(s) = Q_i(s) (1 - G_i(s)L_i(s)) e_{i,k}(s)$, $i = 1, 2$. Hence from [31] a sufficient condition for $\lim_{k \to \infty} \|e_{i,k}\| = 0$ is

$$|Q_i(j\omega) (1 - G_i(j\omega)L_i(j\omega))| < 1, \quad \forall \omega$$

(34)

and ILC simplifies to designing two separate SISO controllers. In the current application the voluntary effort of the patient can also be treated as an iteration-invariant disturbance. However, in practice all ILC systems are subject to iteration varying disturbance and modelling uncertainty. The effects of model uncertainty results in the input-output linearising action producing a system that does not equate to $G(s)$, but whose stability can be studied using [14] and references therein. This mismatch then propagates through to the ILC action, but can be addressed through robust ILC design which can tolerate this uncertainty (see [31] for robustness properties of the form (32), including suitable design of robustness filter $Q(s)$).

V. EXPERIMENTAL RESULTS

Following ethical approval (Faculty of Health Sciences ETHICS-2010-30), tests were conducted with three unimpaired subjects and representative results are given in this section. Each subject was seated in the ArmeoSpring, which was adjusted to their arm dimensions. The level of support was modified so their arm was raised 5cm above their lap. Each subject’s workspace was established and nine reference trajectories then calculated for $\phi_2$ and $\phi_5$. These corresponded to lifting and extending the upper arm and forearm in three different directions (centre, off-centre and far) and with three different lengths (proximal, middle and distal). These are shown in Fig. 5, and the duration of each trajectory was set between 5 and 10 seconds. Surface electrodes were placed on the anterior deltoid and triceps muscles to elicit maximum appropriate movement. Each FES channel comprised a sequence of electrical pulses with a fixed frequency of 40Hz and a fixed amplitude of 5v. The pulsewidth was the controlled variable (0 - 300ms), denoted $v_2$ or $v_5$ for anterior deltoid and triceps respectively. This form is shown in Fig. 1. Each signal is then amplified by a commercial stimulator, to obtain a fixed pulse amplitude of between 0 and 120mA, this being set at the beginning of each test session to produce a comfortable contraction [21]. The $\phi_2$ axis was identified by slowly increasing $v_2$ to lift the arm, and then decreasing it back to zero, the procedure of Appendix A then yielding parameters $\alpha$ and $\beta$, which were inserted into the augmented model of Section III-A. Next the procedures of Section III were applied to yield the parameter vector $\pi$ appearing in (6). The functional forms (7) relating to joint 1 are shown in Fig. 6 and the remaining parameters are given in [32]. These, and the forms identified for joints 3 and 4, all satisfy the conditions given in Theorem 2 for stability of the unactuated joints. In clinical trials, graphical software displayed the reference...
were not shown the trajectory. After each trial their arm was repositioned at the start position.

The control scheme of Section IV was implemented with weights $Q = I$ and $R = 0.01I$ in tracking cost (19) found to achieve a satisfactory balance between tracking accuracy and oscillatory behavior across all subjects. Representative tracking results for joints $\phi_2$ and $\phi_5$ are shown in Fig. 7, together with the FES control signals $u_2$ and $u_5$. Corresponding error norm results are shown in Fig. 8. Trial $k = 1$ corresponds to the linearizing control action alone. Gradient ILC was selected due to previous use in clinical trials with the planar system [21], [22]. This corresponds to (32) with $L_i(s) = \gamma G_i^2(s)$, where $(\cdot)^*$ is the adjoint operator, and $Q_i(s) = 1$ with $\gamma$ a scalar gain. This satisfies (34) if

$$0 < \gamma < \frac{2}{|G_2(j\omega)|^2}, \quad \forall \omega$$

and accordingly a gain of $\gamma = 0.8$ was selected to achieve a satisfactory compromise between convergence rate and robustness of the ILC update (32). The results confirm error convergence to low levels in a small number of trials, and an input signal which is comfortable for each subject.

Tests were conducted with 10 trials of three trajectories (faro-distal, off-centre-distal and centre-distal) for each subject, and summary data are shown in Table I. To assess stability of the unactuated joints, their movement from their initial position, $\phi_{i,k} - \phi_{1,k}(0)$, $i \in I_U$ is also quantified. For comparison, the table also shows norm results using Newton method based ILC, a nonlinear scheme that does not involve linearising the system and has produced accurate tracking in previous tests [21]. It can be seen that the lack of decoupling action in the Newton method based ILC scheme causes ILC transients which lead to greater oscillation in the unactuated joint axes. Since the Newton based method is designed without consideration of the unactuated joint dynamics, these also degrade the tracking performance of the actuated joints, leading to greater norms $\|\epsilon_{i,k}\|_2$, $i \in I_C$. The results hence confirm the efficacy of the separate linearization and ILC actions, and confirm that a high level of tracking accuracy is possible. Although a comprehensive robustness analysis of Newton method based ILC does not exist in the literature, it is clear from these results that neglecting unactuated dynamics, combined with a lack of feedback action, reduces its robust performance.

VI. CONCLUSION

A combined input-output linearization and ILC approach has been developed, building on control structures that have a proven track record in upper limb stroke rehabilitation. The proposed control scheme significantly extends the scope of current ILC approaches in this area, and has a form that is suitable for general application across a broad range of support structures and stimulated muscle sets. The framework is a foundation for future research using ILC in FES-based rehabilitation, an area which is increasing in terms of the muscles which are stimulated, and the functionality of the movements controlled. Future work will include analysis of iteration-varying model uncertainties within this structure.

APPENDIX

From Fig. 2, stimulation of the anterior deltid muscle acts about an axis fixed with respect to the trunk, corresponding to $\phi_2$. Its orientation, and that of $\phi_3$, are found by considering their dynamics in response to FES signal $u_2(t)$, given by

$$B_{A1}(\Phi) \ddot{\Phi} + C_{A1}(\Phi, \ddot{\Phi}) \Phi + F_A(\Phi, \dot{\Phi}) + G_A(\Phi) + K_A(\Phi) = \begin{bmatrix} \tau_2(u_2, \phi_2, \phi'_2) \\ 0 \end{bmatrix}$$

where matrices $B_{A1}(\Phi)$ and $C_{A1}(\Phi)$ have elements $B_{A1,i,j} = b_{I_A(i,j)}$, $C_{A1,i,j} = c_{I_A(i,j)}$, where $I_A = \{2, 3\}$ and $F_A(\Phi, \dot{\Phi})$, $G_A(\Phi)$ and $K_A(\Phi)$ have elements $F_{A1}(\Phi, \dot{\Phi}) = F_{I_A(1)}(\Phi, \dot{\Phi})$, $G_{A1}(\Phi) = G_{I_A(1)}(\Phi)$, $K_{A1}(\Phi) = K_{I_A(1)}(\Phi)$.
If the frictional form (7) is taken, the spring term is again assumed to approximately cancel the gravity term, and \( u_2(t) \) and an initial position \( \Phi \) are chosen such that

\[
\begin{align*}
\dot{b}_{i,j} &\ll \dot{b}_{j,i}, \quad c_{i,j} \ll c_{j,i}, \quad i \notin \mathcal{I}_A, \quad j \in \mathcal{I}_A, \quad (38)
\end{align*}
\]

is satisfied throughout the movement, then joints \( \phi_j, \ i \notin \mathcal{I}_A \) remain approximately stationary, and (36) simplifies to

\[
B_A(\Phi) \dot{\Phi} + C_A(\Phi) \dot{\Phi} \Phi + F_A(\Phi, \Phi, \Phi) = \begin{bmatrix} \tau_2(\phi_2, \phi_2, \phi_2) \\ 0 \end{bmatrix}
\]

where the submatrices \( B_A(\Phi) \) and \( C_A(\Phi) \) have elements

\[
B_{A,i,j} = b_{\mathcal{I}_A(i)\mathcal{I}_A(j)}, \quad C_{A,i,j} = c_{\mathcal{I}_A(i)\mathcal{I}_A(j)}
\]

and the substitution \( \dot{\Phi} = \Phi \dot{\Phi} \), \( \Phi_i = \dot{\Phi}_i \), \( i \notin \mathcal{I}_A \) has been used.

Terms \( B_A(\cdot) \) and \( C_A(\cdot) \) are diagonal due to orthogonality between axes \( \phi_2 \) and \( \phi_3 \), hence \( u_2(t) \) only produces motion about the \( \phi_2 \) axis. Let \( p(1) \cdots p(N) \) contain the points the elbow passes through when stimulation \( u_2(t) \) is applied, where \( p(i) = [p_x(i), p_y(i), p_z(i)]^T \) is the base frame location at sample \( i \). Define the vector normal to the plane as \( [a, b, c]^T \).

The orthogonal distance \( \lambda \) from the plane to the \( i \)-th point is

\[
\lambda(i) = \frac{ap_x(i) + bp_y(i) + pc_z(i)}{a^2 + b^2 + 1}
\]

so, given \( N \) points, the \( \phi_2 \) axis can be identified by solving

\[
\min_{a,b} \sum_{i=1}^{N} \lambda(i)^2 = \sum_{i=1}^{N} \left( \frac{(ap_x(i) + bp_y(i) + pc_z(i))^2}{a^2 + b^2 + 1} \right)
\]

via non-linear regression. From this, values \( a \) and \( b \) are found through inversion of the kinematic transforms shown in Fig. 3. To supply an approximate starting point for (41), locate two points approximately \( \pi/2 \) rad apart, by solving

\[
\min_{i,j} \frac{\arccos\left( \frac{p(i) \cdot p(j)}{|p(i)||p(j)|} \right)}{\pi/2}
\]

then use \( p(i) \times p(j) = x_s = [x_s(1), x_s(2), x_s(3)]^T \) to give \( a = x_s(1)/x_s(3) \) and \( b = x_s(2)/x_s(3) \) for use in (41).

REFERENCES