

Research Article

Application of the G'/G Expansion Method in Ultrashort Pulses in Nonlinear Optical Fibers

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With the increasing input power in optical fibers, the dispersion problem is becoming a severe restriction on wavelength division multiplexing (WDM). With the aid of solitons, in which the shape and speed can remain constant during propagation, it is expected that the transmission of nonlinear ultrashort pulses in optical fibers can effectively control the dispersion. The propagation of a nonlinear ultrashort laser pulse in an optical fiber, which fits the high-order nonlinear Schrödinger equation (NLSE), has been solved using the G'/G expansion method. Group velocity dispersion, self-phase modulation, the fourth-order dispersion, and the fifth-order nonlinearity of the high-order NLSE were taken into consideration. A series of solutions has been obtained such as the solitary wave solutions of kink, inverse kink, the tangent trigonometric function, and the cotangent trigonometric function. The results have shown that the G'/G expansion method is an effective way to obtain the exact solutions for the high-order NLSE, and it provides a theoretical basis for the transmission of ultrashort pulses in nonlinear optical fibers.

1. Introduction

It is understood that a soliton is excited by a nonlinear field, and its energy is relatively concentrated in a small area. The elastic scattering phenomenon occurs during the interaction between two solitons. The energy is nondispersed, so that the shape and speed can remain unchanged during its propagation. The history of the studies of solitons can be traced back to 1834, when James Scott Russell, a British scientist, accidentally observed that a bulge of water in Edinburgh-Glasgow Canal was propagating undistorted over several kilometers [1]. In 1973, Hasegawa and Tappert [2] first proposed the idea of applying “optical solitons” to photonic communication. After rigorous theoretical and mathematical deduction, he predicted that both bright and dark soliton pulses were present in an optical fiber. He also proved that any nondestructive optical pulse could travel as stable as a soliton during its transmission in an optical fiber. As an intrinsically nonlinear phenomenon, a soliton is the product of the optical fiber dispersion and the nonlinear interaction. It obeys the NLSE and is controlled by the optical fiber dispersion and the nonlinear effects of self-phase modulation. In 1980, bright solitons were observed for the first time in an optical fiber by

Mollenau [3] and his colleagues during their experiments at the Bell Laboratory of the United States. In 1987, Emplit observed dark solitons in a fiber by using an amplitude technique and a phase filtering technique. In mathematics, the progress of the soliton theory has been achieved mainly by finding a large number of soliton solutions for nonlinear partial differential equations, so as to gradually establish a systematic theory of solitons. The special properties of a soliton, the related mathematical description of its rich structure, and the deep physical origins have profoundly expanded the applications of soliton theory and methods. The study of the exact solutions for the NLSEs has become an important research and development direction. The G'/G expansion method [4], one of the methods for obtaining exact solutions for NLSE, has attracted much attention recently [4–6]. In particular, it plays an important role in deriving the propagation wave solution of NLSE. Therefore, a good understanding of the G'/G expansion method is of great significance to the transmission application of ultrashort pulses in nonlinear optical fibers. This paper presents the method in detail with examples for obtaining multiple soliton solutions.

2. G'/G Expansion Method

The nonlinear equation with two variables x and t could be written as

$$P\left(u(x, t), \frac{\partial u}{\partial x}, \frac{\partial u}{\partial t}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial x \partial t}, \frac{\partial^2 u}{\partial t^2}, \dots\right) = 0. \quad (1)$$

The first step of the G'/G expansion method is to assume a variable $\xi = x - vt$; thus, $u = u(x, t) = u(\xi)$; it can then be expressed as a G'/G power polynomial:

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \alpha_{m-1} \left(\frac{G'}{G}\right)^{m-1} + \dots + \alpha_0, \quad (2)$$

in which $G = G(\xi)$ satisfies the second-order linear ordinary differential equation:

$$G'' + \lambda G' + \mu G = 0, \quad (3)$$

where $\alpha_m, \alpha_{m-1}, \dots, \alpha_0, \lambda, \mu$ in (2) and (3) are undetermined parameters, in which, $\alpha_m \neq 0$, and the general solution of (3) is expressed as

$$G(\xi) = Ae^{(-\lambda + \sqrt{\lambda^2 - 4\mu})\xi/2} + Be^{(-\lambda - \sqrt{\lambda^2 - 4\mu})\xi/2}. \quad (4)$$

The derivative of $G(\xi)$ can be derived from (4):

$$\begin{aligned} G' &= \left(-\frac{\lambda}{2}\right) e^{(-\lambda/2)\xi} \\ &\times \left[(A+B) \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right. \\ &\quad \left. + (A-B) \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right] \\ &+ e^{(-\lambda/2)\xi} \left[(A+B) \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right. \\ &\quad \left. + (A-B) \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right] \\ &\times \frac{\sqrt{\lambda^2 - 4\mu}}{2}. \end{aligned} \quad (5)$$

The ratio of $G(\xi)$ and $G'(\xi)$ is expressed as [6]

$$\begin{aligned} \frac{G'}{G} &= -\frac{\lambda}{2} \\ &+ \frac{\sqrt{\lambda^2 - 4\mu}}{2} \\ &\times \frac{(A+B) \sinh\left(\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\right)\xi\right) + (A-B) \cosh\left(\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\right)\xi\right)}{(A+B) \cosh\left(\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\right)\xi\right) + (A-B) \sinh\left(\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\right)\xi\right)}. \end{aligned} \quad (6)$$

3. Applying the G'/G Expansion Method to Solve NLSE for Ultrashort Pulses in Optical Fibers

Many problems in the study of natural science can be related to nonlinear evolution equations, and the optical soliton is one of them. It is always problematic to find soliton solutions for nonlinear evolution equations by physicists and mathematicians. With the development of modern computers, many methods have been developed, such as the homogeneous balance method and the hyperbolic function method [7–9]. This research uses the G'/G expansion method to solve the high-order NLSE.

The generalized NLSE is given by

$$\begin{aligned} \sum_{l=0}^{\infty} i^l \alpha_l \frac{\partial^l}{\partial x^l} \psi + \sum_{m=0}^{\infty} \frac{i^m \beta_m}{m!} \frac{\partial^m}{\partial t^m} \psi \\ + \sum_{n=0}^{\infty} \gamma_{2n} |\psi|^{2n} \psi = 0. \end{aligned} \quad (7)$$

Considering the case when $l = 1, m = 2, 3, 4$, and $n = 1, 2$, the corresponding coefficients are nonzero and the rest of the coefficients are zero. Assuming the coefficient $\alpha_1 = 1$ and allowing the variable x to be replaced by z and the function ψ to be replaced by q , the high-order NLSE (with items included, namely, the group velocity dispersion, the self-phase modulation effect, the fourth-order dispersion, and the fifth-order nonlinear term) can then be expressed as

$$\begin{aligned} i \frac{\partial q}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 q}{\partial t^2} + \gamma_2 |q|^2 q \\ = i \frac{\beta_3}{6} \frac{\partial^3 q}{\partial t^3} + \frac{\beta_4}{24} \frac{\partial^4 q}{\partial t^4} - \gamma_4 |q|^4 q, \end{aligned} \quad (8)$$

in which β_2, β_3 , and β_4 are the second-, third-, and fourth-order dispersion coefficients and γ_2 and γ_4 are the third- and fifth-order nonlinear coefficients. The second-order dispersion coefficient β_2 is also known as the group velocity dispersion coefficient. With increasing optical power in a fiber, it is necessary to take the third-order nonlinear term into account, which represents the phase equilibrium between the group velocity dispersion and the nonlinear effect. This will allow the maintaining of unchanged properties for soliton pulses in an optical fiber, which is necessary for long distance transmission, and is also beneficial to WDM.

Other methods, such as the extended tanh-function method, the subsidiary ordinary differential method, and the extended hyperbolic auxiliary equation method, have been used in order to find the exact solutions for (8) [10–12]. It has been found that there is no solution for (8) when $\beta_3 - \beta_4 \nu \neq 0$. In order to obtain the exact solution, $\nu = \beta_3/\beta_4$ is assumed and (8) is then converted to a real equation after its real part and imaginary part are separated. The G'/G expansion method is then used for solving the equation.

3.1. *Reduction Travelling Wave.* Assume the form of the exact solution is

$$q(z, t) = u(\xi) e^{i(Lz - vt)}, \quad \xi = kz - \omega t. \quad (9)$$

The parameters of k, L, ω, v are undetermined parameters. A series of expressions can be obtained based on (9). The key equation (10) can be obtained after the real part and the imaginary part of (8) are separated:

$$\begin{aligned} & \beta_4 \omega^4 u'''' + \left(6\beta_3 \omega^2 \frac{\beta_3}{\beta_4} + 12\beta_2 \omega^2 \right) u'' \\ & + \left(-12\beta_2 \frac{\beta_3^2}{\beta_4^2} + 24L - 3 \frac{\beta_3^4}{\beta_4^3} \right) u \\ & - 24\gamma_2 u^3 - 24\gamma_4 u^5 = 0. \end{aligned} \quad (10)$$

3.2. *The Exact Solution Obtained Using the G'/G Expansion Method.* The G'/G expansion method was proposed for the exact solutions by Wang et al. [4]. The method is effective to solve the exact solutions for the nonlinear partial differential equation.

The form of the exact solution for (10) is

$$u(\xi) = \sum_{i=0}^n \alpha_i \left(\frac{G'}{G} \right)^i, \quad (11)$$

where $G = G(\xi)$ and it satisfies (3).

The result of $n = 1$ can be obtained by balancing the highest derivative u'''' and the highest derivative with high nonlinearity u^5 in (10). The general expression for (10) can be expressed as

$$u(\xi) = \alpha_1 \left(\frac{G'}{G} \right) + \alpha_0. \quad (12)$$

Based on (12) and (10), and assuming the coefficients of $(G'/G)^i, i = 0, 1, 2, 3, 4, 5$ are zero, a set of six algebraic equations can be obtained:

$$\alpha_1 (\beta_4 \omega^4 - \gamma_4 \alpha_1^4) = 0, \quad (13)$$

$$\alpha_1 (\beta_4 \omega^4 \lambda - 2\gamma_4 \alpha_1^3 \alpha_0) = 0, \quad (14)$$

$$\begin{aligned} & \alpha_1 \left[\beta_4 \omega^4 (25\lambda^2 + 20\mu) + \left(6\beta_3 \omega^2 \frac{\beta_3}{\beta_4} + 12\beta_2 \omega^2 \right) \right. \\ & \left. - 12\gamma_2 \alpha_1^2 - 120\gamma_4 \alpha_1^2 \alpha_0^2 \right] = 0, \end{aligned} \quad (15)$$

$$\begin{aligned} & \alpha_1 \left[\beta_4 \omega^4 (5\lambda^3 + 20\lambda\mu) + \left(6\beta_3 \omega^2 \frac{\beta_3}{\beta_4} + 12\beta_2 \omega^2 \right) \lambda \right. \\ & \left. - 24\gamma_2 \alpha_1 \alpha_0 - 80\gamma_4 \alpha_1 \alpha_0^3 \right] = 0, \end{aligned} \quad (16)$$

$$\begin{aligned} & \alpha_1 \left[\beta_4 \omega^4 (\lambda^4 + 22\lambda^2 \mu + 16\mu^2) \right. \\ & \left. + \left(6\beta_3 \omega^2 \frac{\beta_3}{\beta_4} + 12\beta_2 \omega^2 \right) (\lambda^2 + 2\mu) \right. \end{aligned}$$

$$\begin{aligned} & \left. - \left(12\beta_2 \frac{\beta_3^2}{\beta_4^2} - 24L + 3 \frac{\beta_3^4}{\beta_4^3} \right) - 72\gamma_2 \alpha_0^2 \right. \\ & \left. - 120\gamma_4 \alpha_0^4 \right] = 0, \end{aligned} \quad (17)$$

$$\begin{aligned} & \beta_4 \omega^4 (\mu \lambda^3 + 8\lambda \mu^2) \alpha_1 + \left(6\beta_3 \omega^2 \frac{\beta_3}{\beta_4} + 12\beta_2 \omega^2 \right) \lambda \mu \alpha_1 \\ & + \left(-12\beta_2 \frac{\beta_3^2}{\beta_4^2} + 24L - 3 \frac{\beta_3^4}{\beta_4^3} \right) \alpha_0 \\ & - 24\gamma_2 \alpha_0^3 - 24\gamma_4 \alpha_0^5 = 0. \end{aligned} \quad (18)$$

The parameter α_1 can be obtained from (13):

$$\alpha_1 = \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega. \quad (19)$$

It was mistakenly written as $\alpha_1 = \sqrt[5]{(\beta_4/\gamma_4)} \omega$ in [6].

3.2.1. *The First Type Exact Solution.* From (14) when $\lambda = 0$, the result is $\alpha_0 = 0$:

$$\begin{aligned} \mu &= \frac{3}{10\beta_4 \omega^3} \left[2\gamma_2 \sqrt{\frac{\beta_4}{\gamma_4}} - \left(\frac{\beta_3^2}{\beta_4} + 2\beta_2 \right) \omega \right], \\ L &= \left[2\gamma_2 \sqrt{\frac{\beta_4}{\gamma_4}} - \left(\frac{\beta_3^2}{\beta_4} + 2\beta_2 \right) \omega \right] \\ & \times \left[\frac{-3\gamma_2}{25\beta_4 \omega^2} \sqrt{\frac{\beta_4}{\gamma_4}} + \frac{3}{50\beta_4 \omega} \frac{\beta_3^2}{\beta_4} + \frac{3\beta_2}{25\beta_4 \omega} \right. \\ & \left. - \frac{3}{20\omega} \left(\frac{\beta_3}{\beta_4} \right)^2 - \frac{3\beta_2}{10\beta_4 \omega} \right] \\ & + \frac{1}{2} \beta_2 \frac{\beta_3^2}{\beta_4^2} + \frac{1}{8} \frac{\beta_3^4}{\beta_4^3}. \end{aligned} \quad (20)$$

The exact solution of (21) based on (6) is

$$\begin{aligned} q_1 &= \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \sqrt{-\mu} \\ & \frac{(A+B) \sinh((\sqrt{-4\mu}/2)\xi) + (A-B) \cosh((\sqrt{-4\mu}/2)\xi)}{(A+B) \cosh((\sqrt{-4\mu}/2)\xi) + (A-B) \sinh((\sqrt{-4\mu}/2)\xi)} \\ & \times e^{i(Lz - vt)}. \end{aligned} \quad (21)$$

Consider the following.

(a) When $\mu < 0$ and $A = B = 1$, the kink wave solution is

$$q_{11} = \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \sqrt{-\mu} \tanh\left(\frac{\sqrt{-4\mu}}{2} \xi\right) e^{i(Lz - vt)}. \quad (22)$$

(b) When $\mu < 0$ and $A = -B = 1$, the inverse kink wave solution is

$$q_{12} = \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \sqrt{-\mu} \coth \left(\frac{\sqrt{-4\mu}}{2} \xi \right) e^{i(Lz-vt)}. \quad (23)$$

(c) When $\mu \geq 0$ and $A = B = 1$, the solitary wave solution of the tangent trigonometric function is

$$q_{13} = -\omega \sqrt[4]{\frac{\mu\beta_4}{\gamma_4}} \tan(\sqrt{\mu}\xi) e^{i(Lz-vt)}. \quad (24)$$

(d) When $\mu \geq 0$ and $A = -B = 1$, the solitary wave solution of the cotangent trigonometric function is

$$q_{14} = \sqrt[4]{\frac{\mu\beta_4}{\gamma_4}} \omega \cot(\sqrt{\mu}\xi) e^{i(Lz-vt)}. \quad (25)$$

3.2.2. The Second Type Exact Solution. From (14) when $\lambda \neq 0$, the result is $\alpha_1 = \sqrt[4]{(\beta_4/\gamma_4)}\omega$, $\alpha_0 = (\beta_4\omega^4\lambda/2\gamma_4\alpha_1^3) = (\lambda/2)\sqrt[4]{(\beta_4/\gamma_4)}\omega = (\lambda/2)\alpha_1$,

$$\mu = -\frac{3}{10} \left(\frac{\beta_3}{\beta_4\omega} \right)^2 - \frac{3}{5} \frac{\beta_2}{(\beta_4\omega)^2} + \frac{3\gamma_2}{5\omega^2} \sqrt{\frac{1}{\beta_4\gamma_4}} + \frac{\lambda^2}{4}; \quad (26)$$

$$\begin{aligned} L = & \beta_2 \frac{\beta_3^2}{2\beta_4^2} + \frac{\beta_3^4}{8\beta_4^3} + \gamma_2 \frac{3\lambda^2}{4} \omega^2 \sqrt{\frac{\beta_4}{\gamma_4}} + 5 \frac{\lambda^4}{16} \beta_4 - \frac{\beta_4\omega^4\lambda^4}{24} \\ & - \frac{1}{4} \beta_3 \omega^2 \frac{\beta_3}{\beta_4} \lambda^2 - \frac{1}{2} \beta_2 \omega^2 \lambda^2 \\ & - \left\{ \frac{11}{12} \beta_4 \omega^4 \lambda^2 + \frac{1}{2} \beta_3 \omega^2 \frac{\beta_3}{\beta_4} + \beta_2 \omega^2 \right\} \mu \\ & - \frac{2}{3} \beta_4 \omega^4 \mu^2. \end{aligned} \quad (27)$$

The exact solution of (26) based on (6) is

$$\begin{aligned} q_2 = & \left[\sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \right. \\ & \times \left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \right. \\ & \cdot \left((A+B) \sinh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\ & \left. \left. \left. + (A-B) \cosh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right) \right) \right] \end{aligned}$$

$$\begin{aligned} & \times \left((A+B) \cosh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right. \\ & \left. \left. \left. + (A-B) \sinh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right) \right)^{-1} \right) \end{aligned}$$

$$\left. + \frac{\lambda}{2} \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \right] e^{i(Lz-vt)}. \quad (28)$$

The item of $-(\lambda/2)$ in [6] was lost and the item of α_0 was not compensated.

Consider the following.

(a) When $\lambda^2 - 4\mu \geq 0$ and $A = B = 1$, the kink wave solution is

$$q_{21} = \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \frac{\sqrt{\lambda^2 - 4\mu}}{2} \tanh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) e^{i(Lz-vt)}. \quad (29)$$

(b) When $\lambda^2 - 4\mu \geq 0$ and $A = -B = 1$, the inverse kink wave solution is

$$q_{22} = \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \frac{\sqrt{\lambda^2 - 4\mu}}{2} \coth \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) e^{i(Lz-vt)}. \quad (30)$$

(c) When $\lambda^2 - 4\mu < 0$ and $A = B = 1$, the solitary wave solution of the tangent trigonometric function is

$$q_{23} = -\omega \sqrt[4]{\frac{\beta_4}{\gamma_4}} \frac{\sqrt{4\mu - \lambda^2}}{2} \tan \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) e^{i(Lz-vt)}. \quad (31)$$

(d) When $\lambda^2 - 4\mu < 0$ and $A = -B = 1$, the solitary wave solution of the cotangent trigonometric function is

$$q_{24} = \sqrt[4]{\frac{\beta_4}{\gamma_4}} \omega \frac{\sqrt{4\mu - \lambda^2}}{2} \cot \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) e^{i(Lz-vt)}. \quad (32)$$

The application of the feature equations can be found in [13–17].

4. Conclusions

A G'/G expansion method was applied to obtain the exact solutions for NLSE which is the governing equation for ultrashort pulse transmission in a nonlinear fiber. The items of the group velocity dispersion, self-phase modulation effect, the fourth-order dispersion, and the fifth-order nonlinearity

were considered. Two kinds of the kink wave solution, inverse kink wave solutions, the solitary wave solution of the tangent trigonometric function, and the solitary wave solution of the cotangent trigonometric function were found by using the G'/G expansion method.

These results have important significance in optical fiber communication and its applications.

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