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CSPC Communications, Signal Processing and Control

Quantum-Aided Solutions in Wireless Systems

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Near-Capacity Code Design for Entanglement-Assisted Classical Communication over Quantum Depolarizing Channels [1]

Entanglement-assisted classical capacity sets the ultimate capacity limit on the reliable transmission of classical information over a noisy quantum channel, when an unlimited amount of noiseless entanglement is shared between the sender and the receiver.

Classical-quantum-classical transmission model, whereby classical information is transmitted over a quantum channel with the aid of the Superdense (SD) coding protocol [3].

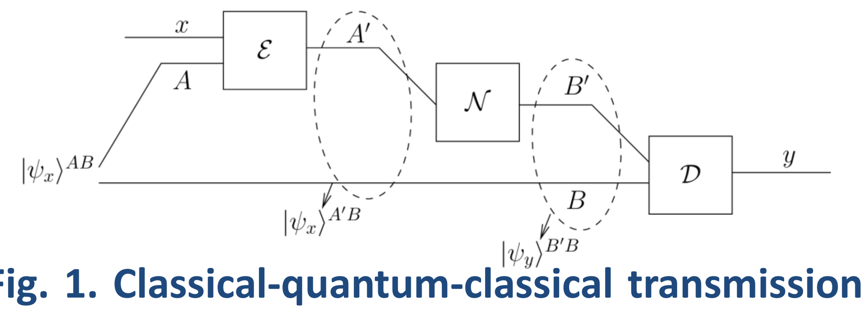

Fig. 1. Classical-quantum-classical transmission model employing 2-qubit SD [1].

Symbol-by-symbol Bell-basis measurement at the receiver reduces the overall transmission model of Fig. 1 to a classical discrete-memoryless channel, whose entanglement-assisted classical capacity is quantified as follows [4]:

$$C_{2sd} = 2 + (1-p)\log_2(1-p) + p\log_2(p/3)\ \text{cbits/use}$$

For an N-particle system with N-qubit SD, the capacity may be generalized.

$$C_{Nsd} = \frac{N + \sum_{m=0}^{M-1} P\left(y|x=x^{(m)}\right)\log_2 P\left(y|x=x^{(m)}\right)}{N-1}$$

We exploit the efficient near-capacity classical code designs of [5] for the sake of approaching the entanglement-assisted classical capacity of the depolarizing channel. The proposed system, named IRCC-URC-SD, incorporates classical Irregular Convolutional Code (IRCC) and Unity Rate Code (URC), and a soft-decision aided Superdense Code (SD). We have intrinsically amalgamated our scheme with 2-qubit as well as 3-qubit superdense codes and benchmarked it against the associated bit-based capacity.

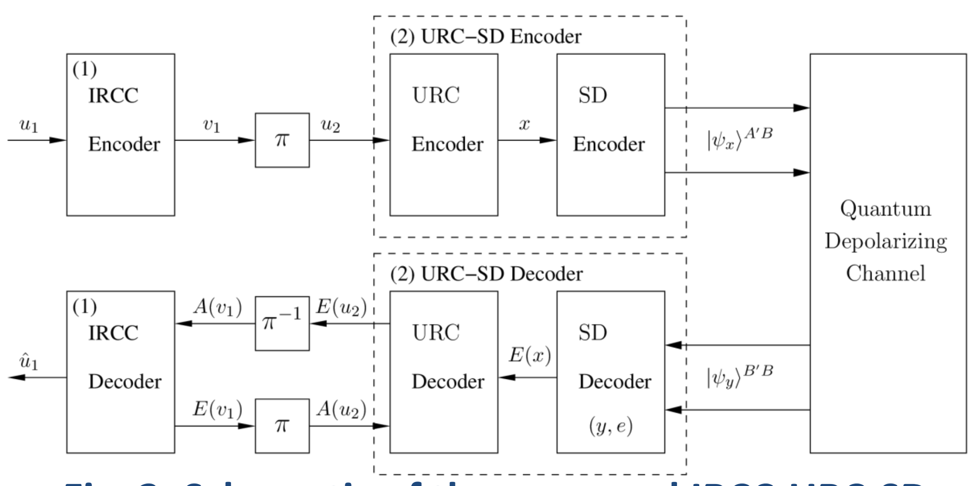

Fig. 2. Schematic of the proposed IRCC-URC-SD classical-quantum communication system [1].

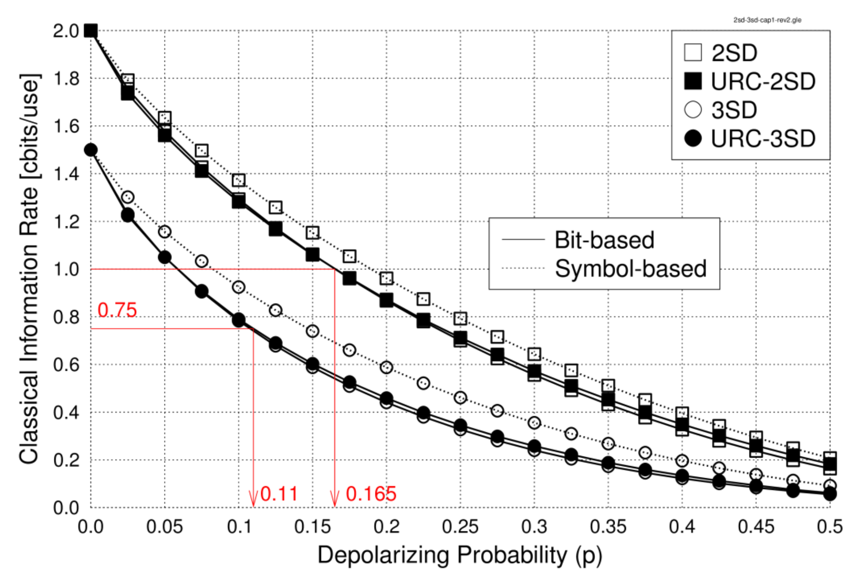

Fig. 3. Classical information rate (cbits/use) versus quantum depolarizing probability for 2-qubit and 3-qubit SD with and without URC [1].

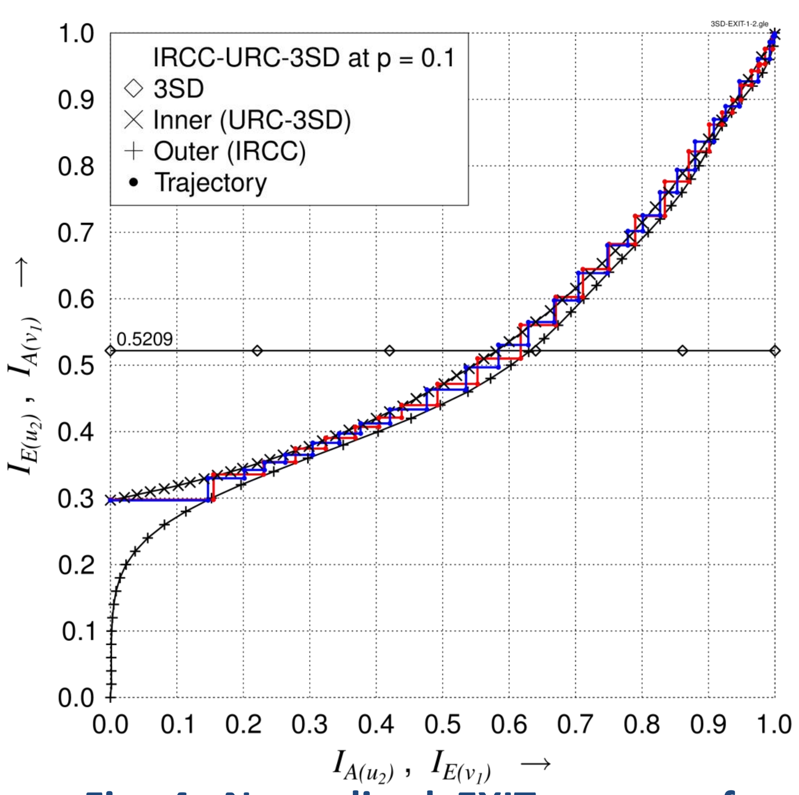

Fig. 4. Normalized EXIT curves of the IRCC-URC-2SD at a depolarizing probability of 0.15 [1].

The area under the inner EXIT curve is approximately equal to the attainable channel capacity, while the area under the outer EXIT curve is equivalent to (1 − R), where R is its coding rate. Therefore, our near-capacity design aims for creating a narrow, but marginally open tunnel between the inner and outer EXIT curves at the highest possible channel depolarizing probability.

We have benchmarked the performance of our system against the classical Turbo Code (TC) and the achievable capacity.

	TC-SD	IRCC-URC-SD
2SD	1.9 dB	0.6 dB
3SD	2.5 dB	0.75 dB

Table 1. Deviation of TC-SD & IRCC-URC-SD from the capacity at a BER of 10^{-4} [1].

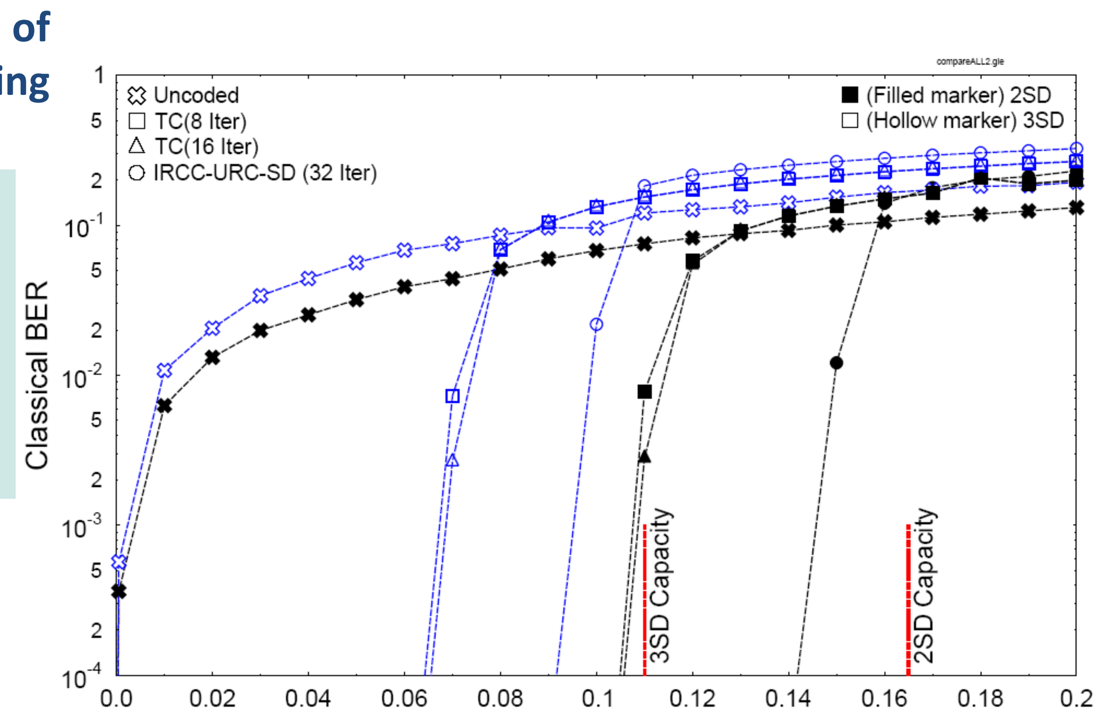

Fig. 5. Comparison of IRCC-URC-SD with TC-SD [1].

Quantum Search Algorithms, Quantum Wireless, and a Low-Complexity Maximum Likelihood Iterative Quantum Multi-User Detector Design [2]

In a Code Division Multiple Access (CDMA) system [6] such as the one illustrated in Fig. 1, the classical optimal Maximum Likelihood (ML) MUD that accepts soft inputs and provides soft outputs is the one that computes the bit Log-Likelihood Ratios (LLR) of every bit of every symbol of each user:

$$L_{m,po}\left(b_{k,l}^{(m)}\right) = \log \frac{\sum_{\mathbf{x}\in\chi(k,l,m,0)} P(\mathbf{x})f(\mathbf{x})/f(\mathbf{x}_{\max})}{\sum_{\mathbf{x}\in\chi(k,l,m,1)} P(\mathbf{x})f(\mathbf{x})/f(\mathbf{x}_{\max})} \qquad f(\mathbf{x}) = P(\mathbf{r}|\mathbf{x}) = \exp\left(-||\mathbf{r}-\mathbf{H}\mathbf{x}||^2/2\sigma^2\right)$$

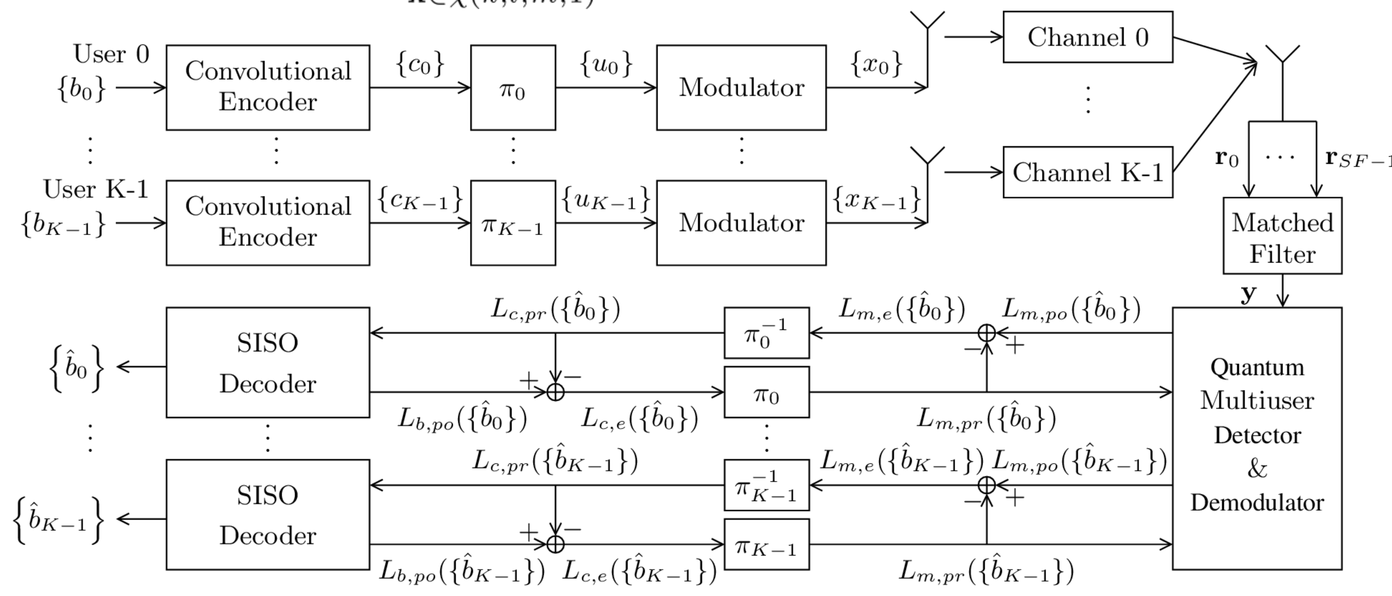

Fig. 6. CDMA system supporting *K* users employing Bit Interleaved Coded Modulation with Iterative Decoding, *M*-ary modulation and Quantum MUD [2].

The proposed Quantum MUD achieves same performance as the ML MUD while being less complex.

Grover's Quantum Search Algorithm [7]: Given a database with *N* entries and a known value *δ*, Grover's QSA may find the *unique solution x* so that *f(x) = δ*, with ~100% probability after *O(√N)* Cost Function Evaluations in the Quantum Domain.

Durr - Hoyer Algorithm [8]:
The DHA succeeds in finding the index *x* that minimizes the Cost Function *f(x)* with ~100% probability after *4.5√N* or *22.5√N* Cost Function Evaluations in the Quantum Domain in the best-case and worst-case scenarios, respectively.

Quantum Weighted Sum Algorithm [2]:
The QWSA estimates the weighted sum of a function $f: \{0, 1, \ldots, 2^n-1\} \rightarrow [0, 1]$, while the precision of the resultant sum *α* and the complexity of the QWSA depend on the number of qubits *l* in the Quantum Control Register (QCR). The QWSA's circuit is depicted in Fig. 8 and it is based on the Quantum Mean Algorithm [9].

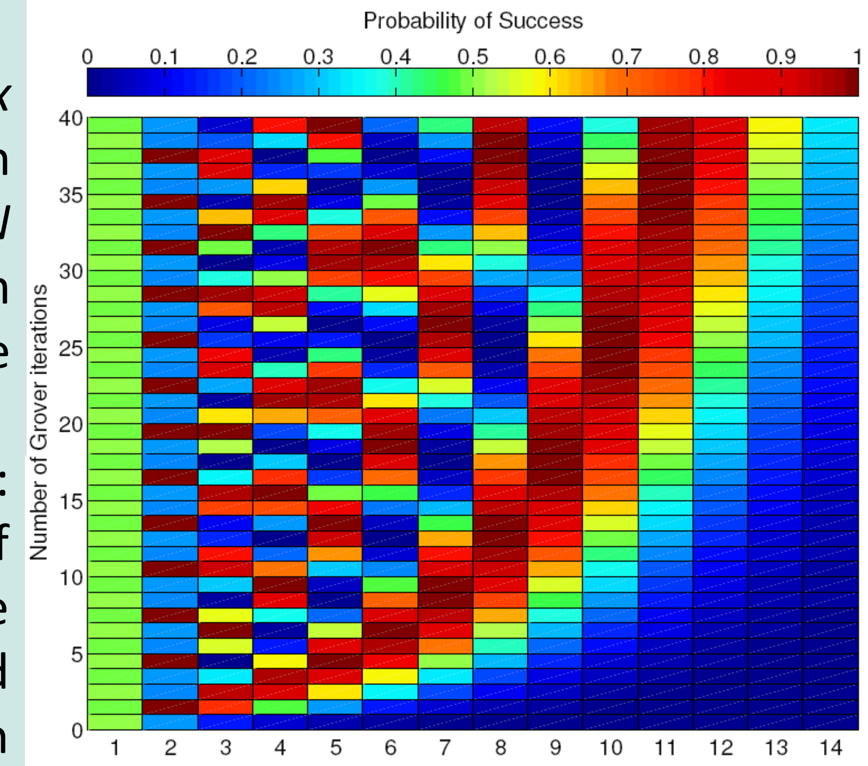

Fig. 7. Grover's QSA's probability of success with respect to the QCFEs and $N = 2^n$ [2].

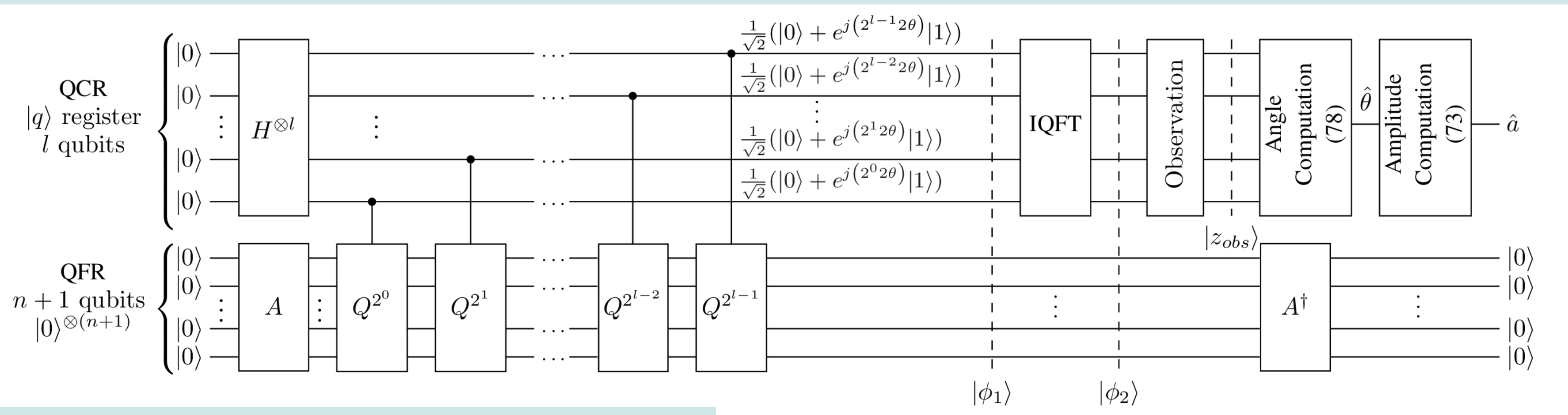

Fig. 8. Quantum Circuit of the QWSA [2].

Figure 9 depicts the Bit Error Ratio (BER) performance of a CDMA system with 4 users, Gold codes with 7 chips, Bit Interleaved Coded Modulation with Iterative Detection (BICM-ID) employing Quadrature Phase Shift Keying (QPSK) modulation and a Turbo Code relying on Convolutional Codes with rate ½, 8 trellis states and 5 inner iterations. The Maximum Likelihood (ML) MUD and the DHA-aided QWSA QMUD for various number of qubits *l* in the QCR have been employed. As the number of qubits *l* increases, the BER of the DHA-aided QWSA QMUD approaches that of the optimal classical ML MUD and it becomes essentially the same when multiple MUD-decoder outer iterations have been performed. The complexity of the QWSA QMUD is smaller than that of the ML MUD in systems supporting more than 20 users.

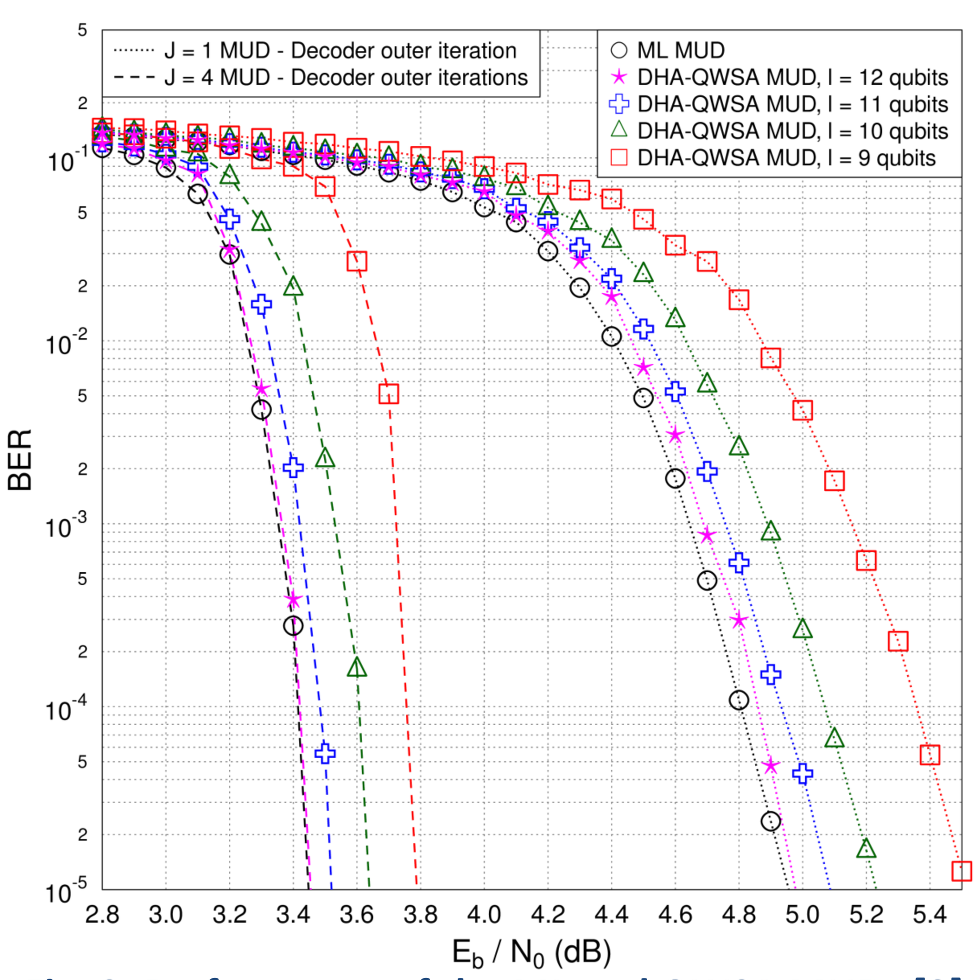

Fig. 9. Performance of the ML and QWSA MUDs [2].

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