' THE STRUCTURE OF THE NUCLEI OF MASS 37 AND 38 '

(A shell model calculation)

Being a

THESIS

submitted for the degree of

DOCTOR OF PHILOSOPHY

in the

FACULTY OF SCIENCE

of the

UNIVERSITY OF SOUTHAMPTON

by

R. V. TURLEY, B.Sc.

Mathematics Department, Southampton University,

' Viviamo in questo mondo per imparare sempre industriosamente, e per mezzo dei raggionamenti di illuminarsi l' un l' altro e d' affatigarsi di portar via sempre avanti le scienze e le belle arti.'

Wolfgang Amadeus Mozart

from a letter to the Padre Martini at Bologna, dated Salzburg 1776.

(' We live in this world in order always to learn industriously, and to enlighten each other by means of discussion, and to strive vigorously to promote the progress of science and the fine arts.'

Translation by Lady Wallace, 1865)

CONTENTS

****

Introduction

This thesis describes an attempt to investigate the structure of the nuclei of mass 37 and 38. The problem, which has not been tackled in detail before, is an interesting one in that it concerns a region about which not too much is yet known experimentally and where the methods of calculation described here may begin to break down. Indeed the results obtained will indicate the applicability of an approach that has met with much success elsewhere, (see, later, reference to the work of Elliott and Flowers).

The nuclear problem is a many-body problem and as such cannot be solved exactly. The situation is further complicated by the fact that, unlike the atomic case, the precise nature of the inter-particle forces is not known. Hence most of the attempts to describe nuclear structure have involved the setting up of a model to represent the nucleus, deduction of properties associated with the model and comparison with experimental nuclear data.

The idea of models in theoretical physics is not new. In less sophisticated form they became the foundations of that great edifice nineteenth century ' classical physics ' thereby causing its downfall - for the conception of a model has now undergone subtle and far-reaching development. It would be fair to say that the classical physicist did not consider himself to have understood a phenomenon until he had devised some mechanical model for its theoretical interpretation. As most of the experiments up to that time had been concerned with phenomena which could be more or

less satisfactorily ' explained ' along these lines, it is not difficult to understand the attitude in the 1890's that there was little else for coming generations of physicists to do than measure the next decimal place. This illusion was shattered at the turn of the century by the discovery of radioactivity and Plancks novel derivation of the black-body radiation law, neither of which admitted interpretation via a mechanical model.

In the first decade of the present century theoretical physics was flung into confusion as one experiment after another exposed flaws in the previously well-established ideas. It is perhaps a mistake to discard a philosophy until there is something better with which to replace it! A turning point was reached however with the advent of Bohrs atomic model and De Broglies hypothesis, the full significance of neither being realised at the time. The structure of theoretical physics has now been strengthened by the reintroduction of models, not this time mechanical, but mathematical. Thus if models are to be successful, their limitations should be accepted and distinction made between natural phenomena and the predicted behaviour of artificial systems.

In order that progress shall be made in any branch of science, it is necessary that both experimental and theoretical work are done in conjunction with each other. This is particularly true for problems in nuclear structure where the theoretical basis is not well understood. In fact the most successful approaches begin with a broad theoretical outline based on the experimental systematics of nuclei,

followed by an attempt to fill in detail from nucleus to nucleus with constant reference to the measured properties. It will be seen later in this thesis just how much the work has depended on the availability of nuclear data.

The shell model

The shell model has been used exclusively as the basis of the calculations described here. It is one of the earliest nuclear models, being the natural extension of its highly successful atomic counterpart.

The shell model has been most useful in accounting for the systematics of nuclei in a general way. It has been able to deal adequately with the structural details of nuclei with mass lying between 6 and 20. Elsewhere detailed calculations are not always easy to perform, but generally speaking the model is not expected to give such good results.

Other nuclear models have been proposed and some have been successful where the shell model failed. However as there exists no single or unified model valid for all nuclei, the choice of a model for a particular calculation is mainly governed by the region in which the nucleus is found. In the present instance the nuclei concerned occur just before a double closed shell at Ca^{40} and it was felt that in this region the individual particle shell model with intermediate coupling would prove most satisfactory as a first choice.

A description of the contents of this thesis

Chapter 1 contains an account of the Hamiltonian constructed for the calculations. The classification of nuclear states is explained and details of the type of interaction used are

given. Also included is a section on the evaluation of central force matrix elements and one on the choice of an oscillator well parameter.

Chapter 2 is concerned with the calculated energy levels for the mass 38 nuclei. It describes how the interaction parameters were varied in order to find an ' optimum ' spectrum and discusses the final values obtained in the light of those deduced by other workers.

Chapter 3 describes the evaluation of β -decay log ft values from the calculated mass 38 wavefunctions. The results are used to assist verification of the interaction parameters deduced in the previous chapter.

Chapter 4 consists of an attempt to account for the forbidden β -decay of Cl^{38}, which could not be done using the simple approximation to the Cl wavefunction successfully employed by Goldstein and Talmi. It is shown how the Cl wavefunction may be expanded using perturbation theory and how this detailed wavefunction is essential to an understanding of the decay. Suggestions for the resolution of this paradox are also included.

Chapter 5 considers the energy levels calculated for the mass 37 nuclei. The evaluation of magnetic and quadrupole moments in Cl^{37} is also discussed.

Chapter 6 deals with the determination of β -decay log ft values from the calculated mass 37 wavefunctions. A forbidden decay, similar to that described in Chapter 4, is satisfactorily accounted for in the same way.

Chapter 7 reviews the conclusions to be drawn concerning various aspects of the problem. Suggestions for further work on the mass 37, 38 nuclei are given together with those for possible future application of the Chapter 4 techniques.

Previous theoretical work on the mass 37, 38 problem

Almost all the previous work on the mass 37, 38 problem appears to consist of passing references to one or other of the nuclei in papers dealing with systematics. A few of the more interesting references are given below.

(1) Goldhammer: Phys. Rev. 101; 1375. (1956)
This paper refers to work on Cl^{37}. A ground state with spin $^3/_2$ is predicted followed by a low-lying excited state with spin $^1/_2$. A correction to the Schmidt value of the magnetic moment is suggested which is about half as large again as the experimental deviation. Mention is made of a derived quadrupole moment rather larger than the experimental one. The approach used here was to consider the coupling of nuclear orbitals through the medium of surface waves, treated by an intermediate coupling procedure.

(2) Grayson and Nordheim: Phys. Rev. 102; 1093. (1956)
This paper deals with the systematics of certain β - decay transition probabilities. It shows how the ratio of theoretical ft value to observed ft value can be reduced when going from the single particle model to one using simple jj-configurations.

Allowed decay	$\frac{(s.p.)ft}{(obs.)ft}$	$\frac{(jj-)ft}{(obs.)ft}$	$\frac{ft\ obtained\ here}{observed\ ft}$
$A^{37} \rightarrow Cl^{37}$	15	4	2·5
$K^{38} \rightarrow A^{38}$	11	4·5	1·5

(3) Kurath: Phys. Rev. 91; 1430. (1953)
Mention is made of the ground state spin of Cl^{38}, the value 2 being favoured. A similar conclusion is reached by Hitchcock: Phil. Mag. 45; 385.

(4) Tauber and Wu: Phys. Rev. 93; 295. (1954)
Here an intermediate coupling calculation gives a ground state spin 0 followed by a first excited state spin 2 for A^{38}. Incorrect levels for K^{38} are inferred but not discussed.

(5) Thieberger and Talmi: Phys. Rev. 102; 923. (1956)
Using pure jj-coupled wavefunctions the splitting between ground and first excited states in A^{38} and some similar even-even nuclei is deduced for various forces. The level of agreement obtained in A^{38} is not as good as that for the other nuclei, being only 80% of the experimental value.

(6) Pandya and Shah: Nuc. Phys. 24; 326. (1961)
A central two-body interaction is deduced here which fits the energy levels (of simple jj-configurations) observed in K^{38}, Cl^{36}. Spin and parity are assigned to the lowest three levels in K^{38} and the spectrum is successfully related to that of Cl^{36}, enabling the interaction parameters to be estimated. In conclusion, however, it is shown that by this means the very small splittings of the $s^1/_2$ -particle doublets in P^{30}, P^{32} cannot be explained.

This brief account of some previous investigations closes the introduction.

Chapter 1

The Structure of the Hamiltonian

The basic process involved in finding the eigenvalues and eigenvectors of a quantum mechanical system is the solution of Schrödingers equation

$$H \Psi = E \Psi \qquad 1.1$$

In applying this to the problem of nuclear structure H is the Hamiltonian, Ψ the eigenfunctions or wave-functioms, E the eigenvalues or energy levels of a system which acts as model for the nucleus.
The wavefunctions Ψ may be expanded in terms of a complete set of orthogonal functions, such as the states formed from harmonic oscillator wavefunctions, ψ_n

$$\Psi = \sum_{n=1}^{\infty} a_n \psi_n$$

Schrödingers equation may then be written in matrix form, $[H]$ being the matrix Hamiltonian whose rows and columns are labelled by the ψ_n and whose ij^{th} element is $(\psi i | H | \psi j)$

The solution of 1.1 is then equivalent to diagonalising $[H]$. This cannot be done exactly because $[H]$ is an infinite matrix, but an approximate solution is found by only considering a finite number of terms in the expansion of Ψ; ie. the lowest configuration of oscillator well states, which are the most likely to contribute to the nuclear wavefunction for low energies. The finite $[H]$ so obtained may be diagonalised either by hand or, as in this instance, using a digital computer.

The structure of the Hamiltonian depends on the details of the model used.

Classification of the nuclear states

The initial assumption made is that Ca^{40} has doubly closed shells in neutrons and protons, from this it is supposed that low lying even-parity levels in nuclei of mass 39, 38, 37, are due to one, two, three holes in the 1d, 2s oscillator shells. It is further assumed that the classification of states is independent of whether they are formed by particles, or holes.

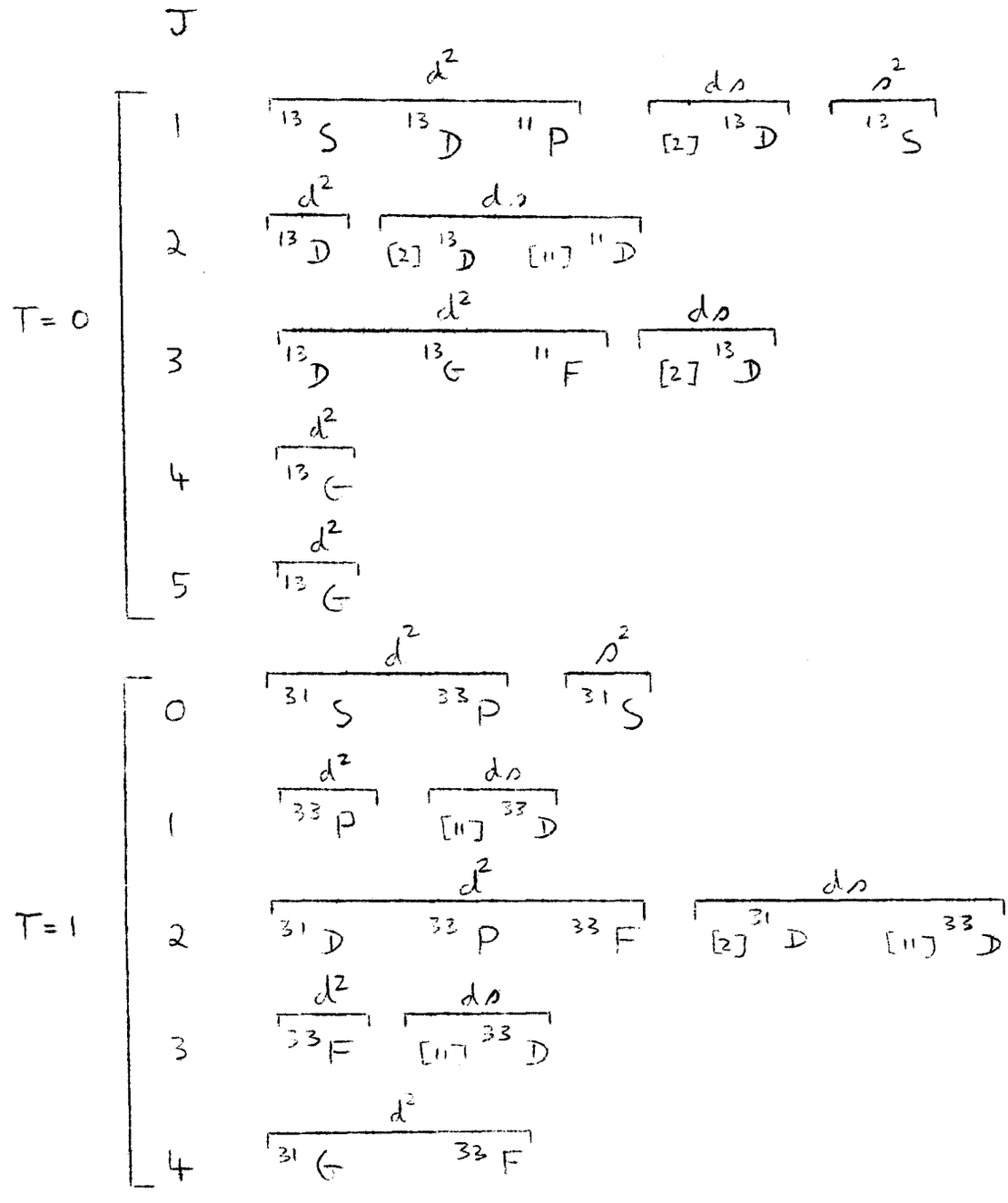
The next step is the enumeration of possible nuclear states formed by two or three holes in the 1d, 2s shell, such states being constructed with due regard to symmetry in the usual way. These are the 'pure' configurations in terms of which the final wavefunctions will be expressed. In the present case they have been constructed using the LS-coupling scheme for convenience, although the final wavefunctions will be intermediate between this and the jj-extreme.

For example, in mass 38 the possible configurations of two holes are d^2, ds and s^2, the LS-coupled states being given in the table below. The wavefunctions involve the products of charge-spin and orbital components, each with appropriate symmetry to produce a finally anti-symmetric wavefunction.

Columns 1, 2 represent the T, S values of the two hole combination, column 3 the symmetry of the orbital component and the last three columns list the L values of the final state.

T	S	L	(2T+1)(2S+1)	d^2	d_s	s^2
0	1	S	13	SDG	[2]D	S
1	0	S	31			
1	1	a	33	PF	[11]D	-
0	0	a	11			

These lead to the following T, J states



These states label the rows and columns of $[H]$ and as J, T are good quantum numbers of the system, $[H]$ may be partitioned and the submatrices dealt with separately.

The J, T states for mass 37, which are partially classified group-theoretically, can be found in appendix IV .

Equivalence of particle systems and hole systems

It has already been mentioned that there is no difference in the classification of states formed by particles and those formed by holes.
In most nuclear structure calculations it is both easier and more convenient to work in terms of particles rather than holes. The question arises 'how may a result derived for a system of particles be made applicable to a system of holes ?'

The type of result envisaged is obtained using the matrix elements of tensor operators. A rule given by Racah (Theory of Complex Spectra II. Phys. Rev. 62; 438. 1942) states that for a single body operator the transition from particles to holes only involves a change of sign given by $(-)^{1 + k_1 + k_2 + k_3}$

where the tensor operator in question has component tensors of rank k_1 in isotopic spin space, k_2 in intrinsic spin space, k_3 in orbital space.
For example matrix elements of the spin-orbit force (operator $\bar{s}.\bar{l}$) change sign when particles are replaced by holes.

Racah (1942) has also shown that the matrix elements of a two-body scalar operator (eg. central force) are unaltered when holes are substituted for particles, provided only differences in energy are considered.

Terms involved in the Hamiltonian

The Hamiltonian is taken to consist of three terms:

(1) a two-body central force,

(2) a spin-orbit force,

(3) a term involving the d-s single hole level separation.

These terms will be discussed in more detail below. Tensor forces have not been included as there is no conclusive justification for their inclusion in the many body problem, (Elliott and Flowers. The structure of the nuclei of mass 18 and 19. Proc. Roy. Soc. 229; 536. 1955), the spin-orbit term giving rise to similar effects.

From this it will be seen that the calculation has been carried out in Intermediate coupling, a coupling scheme lying between the extremes of jj-coupling (obtained with spin-orbit force and no central force) and LS-coupling (obtained with central force and no spin-orbit force).

The Hamiltonian matrix is given by

$$[H] = \zeta \left[\begin{matrix}\text{spin-orbit matrix}\\ \text{fof holes}\end{matrix}\right] + \frac{Vc}{50} \left[\begin{matrix}\text{central force matrix}\\ \text{for holes}\end{matrix}\right] + \left[\begin{matrix}\text{d-s difference matrix}\\ \text{for holes}\end{matrix}\right]$$

$$= -\zeta \left[\begin{matrix}\text{spin-orbit matrix}\\ \text{for particles}\end{matrix}\right] + \frac{Vc}{50} \left[\begin{matrix}\text{central force matrix}\\ \text{for particles}\end{matrix}\right] + \left[\begin{matrix}\text{d-s difference matrix}\\ \text{for particles}\end{matrix}\right]$$

where ζ is the strength of the spin-orbit force,

Vc is the strength of the central force.

This can be rewritten

$$H = -\left\{ \left\{ [\text{spin-orbit force}] + x\,[\text{central force}] - \frac{1}{\ell}[\text{d-s difference}] \right\} \right. \qquad 1.2$$

where $x = \frac{-Vc}{50\ell}$ is the so-called intermediate coupling parameter.

The calculations were performed for several values of x. The value of ℓ may be determined from the single hole levels in Ca^{39} and a value of Vc chosen to give the most satisfactory agreement with experimental data. This value of Vc should not of course be inconsistent with that found by other workers.

The Central force

The central interaction between two particles outside a closed shell may be written

$$V_{12} = V(r_{12}) \left[a_0 + a_\sigma (\bar{\sigma}_1 . \bar{\sigma}_2) + a_\tau (\bar{\tau}_1 . \bar{\tau}_2) + a_{\sigma\tau} (\bar{\sigma}_1 . \bar{\sigma}_2)(\bar{\tau}_1 . \bar{\tau}_2) \right] Vc$$

where r_{12} is the distance between the nucleons

$\bar{\sigma} = 2\bar{s}$, s being the intrinsic spin of a nucleon

$\bar{\tau} = 2\bar{t}$, t being the isotopic spin of a nucleon

This may be expanded in terms of the permutation operators

P^r_{12}, P^σ_{12}, P^τ_{12} as

$$V_{12} = V(r_{12}) \left[W + M\,P^r_{12} - H\,P^\tau_{12} + B\,P^\sigma_{12} \right] Vc$$

$$= V(r_{12}) \left[W + M(-)^{1+S+T} + H(-)^T + B(-)^{1+S} \right] Vc \qquad 1.3$$

for two-body matrix elements, where S, T are for the two-body states.

Throughout the mass 38 problem the exchange mixture parameters

$$A^{13} = W + M + H + B$$

$$A^{31} = W + M - H - B$$

$$A^{33} = W - M - H + B$$

$$A^{11} = W - M + H - B$$

will be used.

An indication of the techniques involved in calculating the central force matrix elements will be given later in this chapter.

The spin-orbit force.

The spin-orbit force is introduced to account for the experimentally observed splitting of certain levels.

Eg. In Ca^{39} the 1d single hole level is split into two,

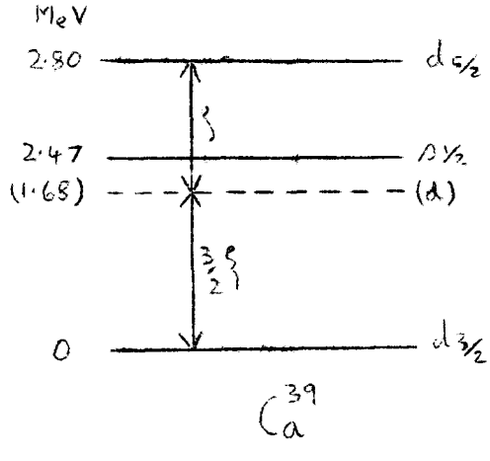
$$d^3/2 \quad \text{and} \quad d^5/2$$

Its value is

$$\langle \zeta\,(\bar{s}.\bar{l})\rangle = -\tfrac{1}{2}\zeta \quad \text{for} \quad d^5/2$$

$$= +\tfrac{3}{2}\zeta \quad \text{for} \quad d^3/2$$

The strength ζ may be calculated from the mass 39 experimental levels given below (Middleton. Private communication) based on the reaction Ca^{40} (He^3, α) Ca^{39}. It must be said at this point that the interpretation of these experimental levels is by no means certain. In particular the $d^5/2$ single hole level may well be somewhat higher.



The experimental results are difficult to analyse, furthermore Ca^{40} may not be a good closed shell. The effects of changing these levels will be considered throughout the calculations.

The value of $\{$ deduced from this data is $-1{\cdot}12$ MeV.

The matrix elements of the spin-orbit force for mass 37 and 38 have been taken from Elliott and Flowers' previous work on mass 18 and 19 (Private Communication).

The d-s difference.

This term arises from the interaction of particles outside closed shells with the particles inside. It is assumed to depend only on orbital angular momentum and is independent of the spin-orbit force.

The energy of particles in the unsplit d state relative to the 2s state is, from the Ca^{39} data,

$$-2{\cdot}47 + 1{\cdot}68 = -0{\cdot}79 \text{ MeV}$$

Thus the d-s difference term has diagonal matrix elements

$$\left.\begin{array}{r} 0 \text{ in } s^2,\ s^3 \\ -0{\cdot}79 \text{ in } ds,\ ds^2 \\ -1{\cdot}58 \text{ in } d^2,\ d^2s \\ -2{\cdot}37 \text{ in } d^3 \end{array}\right]$$

Changing the Ca^{39} single hole levels must also affect these terms, however this is judged to be of small importance relative to its other effects.

Calculation of central force matrix elements

An expression for the two-body central force has already been given (equation 1.3)

Writing the charge-spin part of the two-body states as

$$(2T+1)(2S+1)\ \square$$

the two-body states are

$^{13}\square$, $^{31}\square$: antisymmetric in charge-spin, requiring symmetric $[2]$ orbital functions for final antisymmetric state.

$^{11}\square$, $^{33}\square$: symmetric in charge-spin, requiring antisymmetric $[11]$ orbital functions for final antisymmetric state.

where the matrix elements of the $\square$'s are simply the A's defined on page 13 .

Writing the appropriate orbital wavefunctions for inequivalent particles (the symmetry of equivalent particle states being determined by the final L value) as

$$\Psi(\ell\ell'[2]LM) = \sqrt{\tfrac{1}{2}}\left(\psi(\ell_1\ell_2'LM) + \psi(\ell_2\ell_1'LM)\right)$$
$$\Psi(\ell\ell'[11]LM) = \sqrt{\tfrac{1}{2}}\left(\psi(\ell_1\ell_2'LM) - \psi(\ell_2\ell_1'LM)\right)$$

the orbital matrix elements are

$$\left.\begin{aligned}(\ell\ell'[2]LM|V_{12}|\tilde{\ell}\tilde{\ell}'[2]L'M') &= (\ell_1\ell_2'LM|V_{12}|\tilde{\ell}_1\tilde{\ell}_2'L'M') \\ &\quad + (-)^{\tilde{\ell}+\tilde{\ell}'-L'}(\ell_1\ell_2'LM|V_{12}|\tilde{\ell}_1'\tilde{\ell}_2L'M') \\ (\ell\ell'[11]LM|V_{12}|\tilde{\ell}\tilde{\ell}'[11]L'M') &= (\ell_1\ell_2'LM|V_{12}|\tilde{\ell}_1\tilde{\ell}_2'L'M') \\ &\quad + (-)^{1+\tilde{\ell}+\tilde{\ell}'-L'}(\ell_1\ell_2'LM|V_{12}|\tilde{\ell}_1'\tilde{\ell}_2L'M')\end{aligned}\right\}\ 1.4$$

Expanding $$V(r_{12}) = \sum_{k=0}^{\infty} P_k(\cos\omega_{12})\, V_k(r_1, r_2) \qquad 1.5$$

where $P_k(\cos\omega_{12})$ is a Legendre polynomial and ω_{12} is the angle between $\bar{r}_1$ and $\bar{r}_2$, gives rise to matrix elements:

$$\left((n\ell)_1 (n'\ell')_2 LM \,|\, V(r_{12}) \,|\, (\tilde{n}\tilde{\ell})_1 (\tilde{n}'\tilde{\ell}')_2 LM\right)$$
$$= \sum_{k=0}^{\infty} \left(\ell_1 \ell_2' LM \,|\, P_k(\cos\omega_{12}) \,|\, \tilde{\ell}_1 \tilde{\ell}_2' LM\right) F^k(n\ell, n'\ell', \tilde{n}\tilde{\ell}, \tilde{n}'\tilde{\ell}') \qquad 1.6$$

where F^k are radial integrals of the form

$$F^k = \int_0^\infty \int_0^\infty u_{n\ell}(r_1)\, u_{n'\ell'}(r_2)\, u_{\tilde{n}\tilde{\ell}}(r_1)\, u_{\tilde{n}'\tilde{\ell}'}(r_2)\, V_k(r_1, r_2)\, r_1^2 r_2^2\, dr_1\, dr_2$$

the $u_{n\ell}$ being radial parts of the wavefunction normalised by

$$\int_0^\infty u_{n\ell}^2(r)\, r^2\, dr = 1$$

From the spherical harmonic addition theorem

$$P_k(\cos\omega_{12}) = \frac{4\pi}{2k+1} \left[Y^k_{(1)}, Y^k_{(2)}\right]$$

the matrix elements of such operators being known.

In fact

$$\left(\ell_1 \ell_2' LM \,|\, P_k(\cos\omega_{12}) \,|\, \tilde{\ell}_1 \tilde{\ell}_2' LM\right)$$
$$= \frac{(-)^{\frac{1}{2}(\ell+\ell'+\tilde{\ell}+\tilde{\ell}')+L}}{2} \left[(2\ell+1)(2\ell'+1)(2\tilde{\ell}+1)(2\tilde{\ell}'+1)\right] C^{\ell\,\tilde{\ell}\,k}\, C^{\ell'\,\tilde{\ell}'\,k}$$
$$\times W(\ell\ell'\tilde{\ell}\tilde{\ell}' : Lk) \qquad 1.7$$

where $C^{\ell\tilde{\ell}k}$ are tabulated as the Shortley-Fried coefficients

(Shortley and Fried. Phys. Rev. 54; 739. 1938)

The possible values of k are restricted by the Racah function.

The following radial integrals occur in the types of matrix element given below

$(s^2 \,\|\, \|\, s^2)$	$F^0(2s^2)$				
$(d^2 \,\|\, \|\, d^2)$	$F^0(1d^2)$	$F^2(1d^2)$		$F^4(1d^2)$	
$(ds \,\|\, \|\, ds)$	$F^0(1d\,2s)$	$G^2(1d2s)$			

$(d^2 | \ | \ ds)$ $\quad M^2\ (1d^2,\ 1d2s)$

$(d^2 | \ | \ s^2)$ $\quad G^2\ (1d,2s)$

where the F^k have already been defined and

$$G^k = \int_0^\infty\int_0^\infty u_{n\ell}(r_1)\, u_{n'\ell'}(r_2)\, u_{n\ell}(r_2)\, u_{n'\ell'}(r_1)\, V_k(r_1,r_2)\, r_1^2\, r_2^2\, dr_1 dr_2$$

$$M^k = \int_0^\infty\int_0^\infty u^2_{n\ell}(r_1)\, u_{n\ell}(r_2)\, u_{n'\ell'}(r_2)\, V_k(r_1,r_2)\, r_1^2\, r_2^2\, dr_1 dr_2$$

In order to evaluate the radial integrals two things must be decided,

(a) the shape of the nuclear potential well, which determines the $u_{n\ell}(r)$

(b) the shape of the two-body force, which determines $V_k(r_1,r_2)$

Throughout most of these calculations the nuclear potential well has been represented by an infinite harmonic oscillator shape, following many other workers and in particular Elliott and Flowers (1955).

$$V(r) = \tfrac{1}{2}\left(\frac{\hbar^2}{M b^4}\right) r$$

where M is the nucleon mass

b is a constant related to the size of the nucleus whose value will be dealt with later.

This gives rise to

$$u_{n\ell}(r/b) = (r/b)^\ell\, e^{-r^2/2b^2}\, f'_{n\ell}(r/b)$$

where the $f'_{n\ell}$ include a factor to normalise $u_{n\ell}$ as indicated above and may be found in 'The nuclear shell model' Elliott and Lane (Hand. der. Phys. 39; 241. 1957)

Concerning the two-body interaction, in the first instance a Yukawa potential

$$V(r) = \frac{e^{-r/a}}{r/a}$$

based on simple meson field theory was chosen, following Elliott and Flowers (1955), with $a = 1{\cdot}37 \times 10^{-13}$ cm

Later on it was decided to vary a, the range of the force, and to facilitate this a Gaussian two-body potential

$$V(r) = e^{-r^2/a^2}$$

was employed.

Radial intergrals with Yukawa potential.

Elliott and Flowers (private communication) have derived expressions for the d,s shell radial integrals with Yukawa potential involving the Hh functions. These are listed in appendix I, a typical example being given below

$$F^0(2s^2) = \frac{1}{\sqrt{2\pi}} \frac{e^{1/2\alpha}\alpha^{1/2}}{96} \{123\,Hh_1 - 632\,Hh_3 + 6160\,Hh_5 - 33600\,Hh_7 + 120960\,Hh_9\}$$

where $\alpha^{1/2} = a/r$ and $Hh_n(r/a) = \int_{r/a}^{\infty} Hh_{n-1}(t)\,dt$; $Hh_0(r/a) = \int_{r/a}^{\infty} e^{-\frac{1}{2}t^2}\,dt$

The Hh functions are tabulated in 'Report of the British Association for the Advancement of Science' (Glasgow) 1928.

Radial integrals with Gaussian potential.

In changing from Yukawa to Gauss potential it is necessary to replace the Hh functions by their Gaussian equivalent.

From Jahn's 'Tabulation of the radial integrals by the Talmi method' (Private communication) it is seen that

$$e^{\frac{1}{2}x} x^{\frac{1}{2}} H_{2n-1}(x^{-\frac{1}{2}})$$

must be replaced by

$$\frac{(2n-1)!!}{(2n-1)!} \beta_0^{2n+1} \sqrt{\frac{\pi}{2}}$$

where $\beta_0 = \sqrt{\frac{\alpha}{x+2}}$

Three particle central force matrix elements.

So far only the simple two particle central force elements, such as occur explicitly in mass 38, have been considered.

In mass 37 the matrix elements will be given by

$$3\left(\bar{\Psi}'_{123}(TS'L') \left| V_{12} \right| \bar{\Psi}_{123}(TSL)\right)$$

Using the concept of fractional parentage coefficients (c.f.p.) to expand

$$\bar{\Psi}_{123}(TSL) = \sum_{\bar{\Psi}}' a_{\bar{\Psi}}^{\bar{\Psi}} \left\{ \bar{\Psi}_{12} \phi_3 \right\}^{TSL}$$

where $\{\ \}^{TSL}$ denotes vector coupling in the T, S, L spaces and the $a_{\bar{\Psi}}^{\bar{\Psi}}$ are the coefficients of fractional parentage, the three particle matrix elements may be expressed in terms of the two particle ones

$$\left(\bar{\Psi}'_{123}(TS'L') \left| V_{12} \right| \bar{\Psi}_{123}(TSL)\right) = \sum_{\bar{\Psi}'}' \sum_{\bar{\Psi}}' a_{\bar{\Psi}'}^{\bar{\Psi}'} a_{\bar{\Psi}}^{\bar{\Psi}} \left(\left\{ \bar{\Psi}'_{12} \phi'_3 \right\}^{TS'L'} \left| V_{12} \right| \left\{ \bar{\Psi}_{12} \phi \right\}^{TSL} \right)$$

Using the fact that V_{12} only operates in the coordinate space of particles 1, 2 it can be shown that

$$\left(\left\{ \bar{\Psi}'_{12} \phi'_3 \right\}^{TS'L'} \left| V_{12} \right| \left\{ \bar{\Psi}_{12} \phi_3 \right\}^{TSL} \right)$$

reduces to two-body terms of the form

$$\left(\bar{\Psi}'_{12}(\bar{t}\bar{s}'\bar{l}') \left| V_{12} \right| \bar{\Psi}_{12}(\bar{t}\bar{s}\bar{l})\right) \int \phi_3^* \phi_3 \, d\tau$$

where the integral must be unity from the normalisation of the wavefunctions.

The a^{Ψ}_{Ψ} have been tabulated and are given together with the central force elements for d^2, d^3 configurations by Jahn 'Theoretical studies in Nuclear Structure II' (Proc. Roy. Soc. 205; 192. 1951). The remaining elements have been computed from expressions used by Elliott and Flowers (Private communication).

A list of some central force matrix element expressions is given in Appendix II.

Evaluation of the oscillator well parameter b.

A value for b may be determined by evaluating $\langle r^2 \rangle$ for the nucleus in two different ways and equating the results.

(1) For the quantum-mechanical system $\langle r^2 \rangle$ is defined by

$$\langle r^2 \rangle = \frac{1}{A} \sum_i \left(u_{n\ell}(i) \left| r_i^2 \right| u_{n\ell}(i) \right)$$

where A is the mass number of the nucleus

r_i is the distance of particle i from the centre

$u_{n\ell}(i)$ is the wavefunction of particle i .

Thus $$\langle r^2 \rangle = \frac{1}{A} \sum_i \int u_{n\ell}^2(i)\, r_i^4\, dr_i = \frac{1}{A} \sum_i I_{n\ell}(i)$$

Expressions for $u_{n\ell}$ are taken from Elliott and Lane (1957) and the integrals $I_{n\ell}$ for nuclei with $A \leq 40$ are

$$I_{1s} = {}^3/_2\ b^2$$

$$I_{1p} = {}^5/_2\ b^2$$

$$I_{2s} = {}^7/_2\ b^2$$

$$I_{1d} = {}^7/_2\ b^2$$

ds

Hence for mass 38 all shells are closed with the

exception of two holes in the d, s and so

$$\langle r^2 \rangle_{38} = \frac{1}{38}\left\{4 I_{1s} + 12 I_{1k} + 22 I_{ds}\right\}$$

$$= \frac{113}{38} \quad b^2$$

and

$$\langle r^2 \rangle_{37} = \frac{219}{74} \quad b^2$$

(2) Now $\langle r^2 \rangle$ is calculated using a somewhat crude technique.

Suppose the nucleus to be represented by a uniform spherical distribution of matter with definite radius R.

Then $\langle r^2 \rangle$ is given by

$$r^2 = \frac{\iiint_0^R r^2 \cdot r^2 dr \sin\theta \, d\theta \, d\phi}{\iiint_0^R r^2 dr \sin\theta d\theta \, d\phi} = \frac{\int_0^R r^4 dr}{\int_0^R r^2 dr} = \frac{3}{5} \times R^2$$

This approach seems no more arbitrary than that of Swiatecki (Proc. Roy. Soc. 205; 238. 1951) who defines b by associating R with the point at which the probability density $u^2(r/b)$ falls to a quarter of its maximum value.

Of course the nucleus has no definite radius, but Moszkowski in his 'General Survey of Nuclear Models' (Hand. der. Phys. 39; 411. 1957) quotes the following empirical formula for R

$$R = 1{\cdot}3\, A^{\frac{1}{3}} \times 10^{-13}\text{cm}$$

Thus $\langle r^2 \rangle = 1{\cdot}014\, A^{\frac{2}{3}} \times 10^{-26}\text{cm}$ and equating the expressions for $\langle r^2 \rangle$ yields

$$b_{38} = 1{\cdot}963 \times 10^{-13}\text{cm}$$

$$b_{37} = 1{\cdot}950 \times 10^{-13}\text{cm}$$

The value of b obtained this way for the mass 19 nuclei is

$$b_{19} = 1{\cdot}672 \times 10^{-13}\text{cm}$$

which compares with $1{\cdot}64 \times 10^{-13}$cm used by Elliott and Flowers (1954).

Finally it must be remarked that it is the ratio a/b which occurs in the evaluation of central force matrix elements, although b occurs explicitly in the expressions for forbidden β -decay half lives and quadrupole moments. Later in these calculations a/b will be treated as a free parameter.

Note on the diagonalisation of [H].

The matrix [H] was formed by the addition of three matrices after the fashion of 1.2 for several values of the intermediate coupling parameter x . [H] was subsequently diagonalised using EROS, a routine for symmetric matrices developed by Howell (Southampton University Ph.D Thesis 1960).
All computations were carried out on the Southampton University 'Pegasus' computer.

Chapter 2

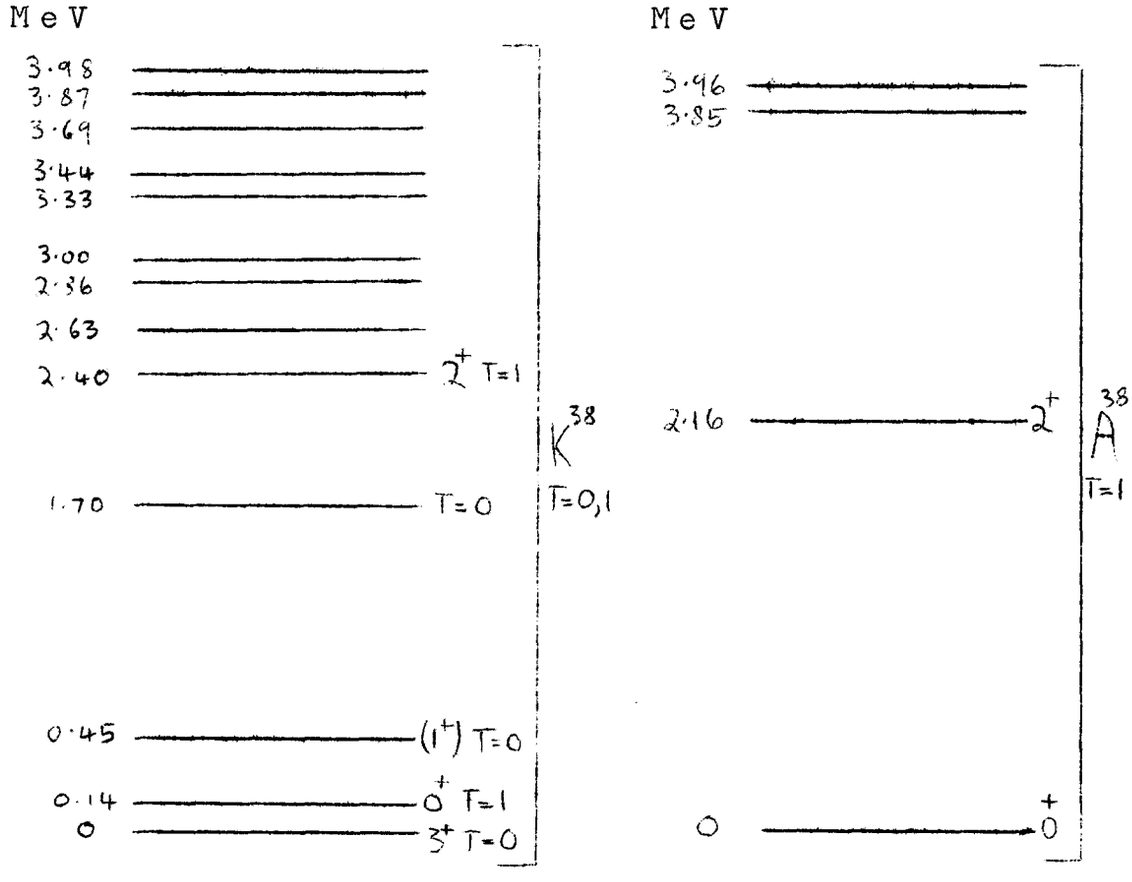
The Mass 38 Spectrum

The methods outlined in chapter 1 enable the energy levels with T = 0 or 1 to be calculated for the mass 38 nuclei. The available experimental data summarised in the diagram below has been taken from the following sources,

(1) Endt and Braams, Rev. Mod. Phys. 29; 683. 1957

(2) Jänecke (University of Michigan), Private communication

(3) Taylor (Rice University, Texas), Private communication



The assignment of spin 1 to the 0·45 MeV T = 0 state in K^{38} is not absolutely certain.

The levels above 2·4 MeV are given by Taylor, those at 2·4 and 1·7 MeV in K^{38} by Taylor and Jänecke, the others being common to all three sources.

The bulk of the data given on the last page, has only been recently determined. Most of the work described in this chapter was done at a time when only the lowest three levels were known in K^{38} and the lowest two in A^{38}. This explains why so much attention has been paid to the lowest lying levels obtained from the calculation.

It is to be hoped that still more experiments will be performed in this region, which is a difficult one as many of the substances involved are gaseous, so that the new levels may be confirmed and spin, parity assignments made. The results of the energy level calculations will now be given and compared with the data on the last page.

Yukawa interaction ($a = 1{\cdot}37 \times 10^{-13}$cm)

Initially it was decided to use the interaction which Elliott and Flowers (1955) found to give such good agreement at the beginning of the ds shell.

An exchange mixture

$$A^{13} = -1$$

$$A^{31} = -0{\cdot}7$$

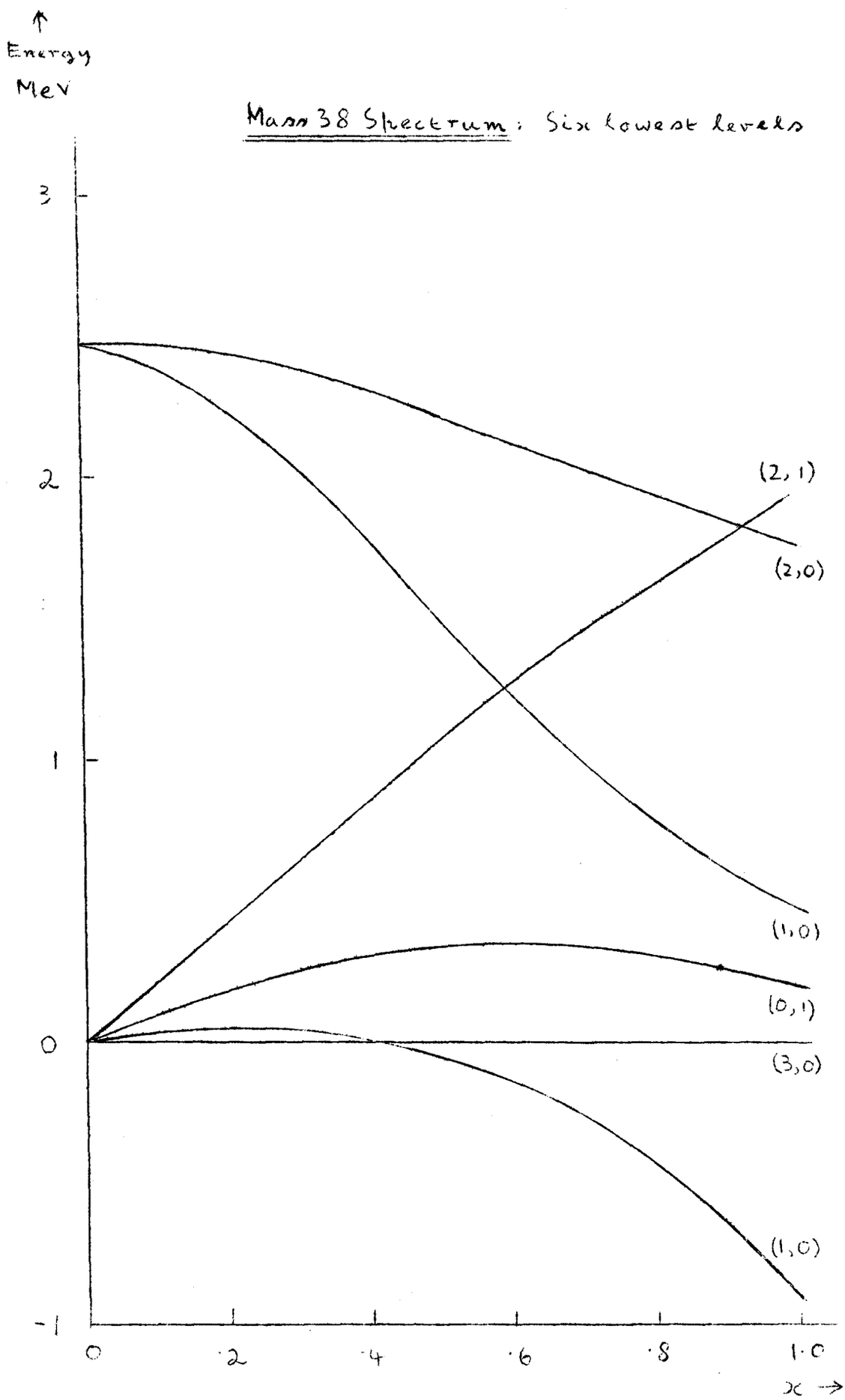
$$A^{33} = +0{\cdot}26$$

$$A^{11} = -0{\cdot}5$$

was chosen following that employed in O^{16} by Elliott and Flowers (Proc. Roy. Soc. 242; 57. 1957). The radial integrals were evaluated using a Yukawa two-body interaction ($a = 1{\cdot}37 \times 10^{-13}$cm) and the oscillator nuclear potential well ($b = 1{\cdot}96 \times 10^{-13}$cm).

In mass 18 it was found that Vc should take a value of

FIG. 1.


Energy
MeV
Mass 38 Spectrum: Six lowest levels
3
2
1
0
-1
(2, 1)
(2,0)
(1,0)
(0,1)
(3,0)
(1,0)
0
·2
·4
·6
·8
1·0
x →

around 40 MeV, which would correspond to $x = 0{\cdot}7$ in this case.

The spectrum obtained is given in fig: 1 for varying values of x. These spectra have all been plotted relative to the $J = 3$, $T = 0$ (3, 0) level, which is the experimental ground state of K^{38}.

The following points are apparent,

(1) The first (1,0) level, which should be above the (0,1) level is much too low and falls down sharply as x increases.

(2) The (0,1) level is approximately correct, but to gain a 2 MeV difference between this and the (2,1) level, a large value of x ($\sim 1{\cdot}2$) would be needed, which is inconsistent with (1) above.

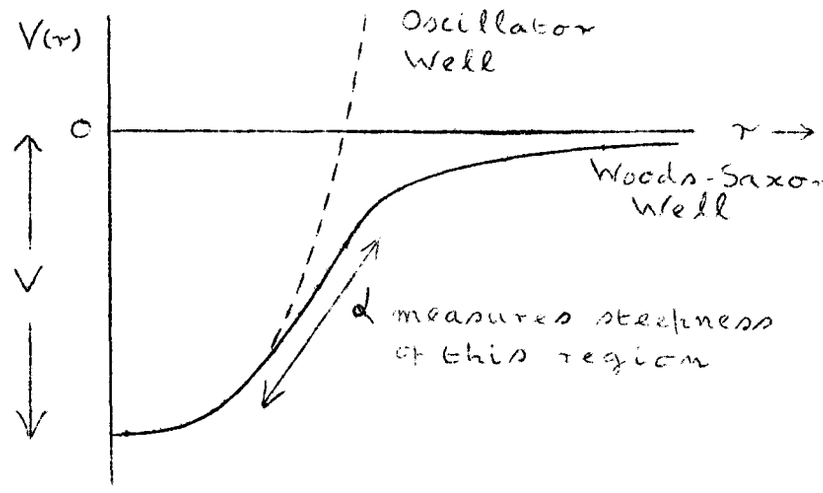
(3) A second (1,0) and a (2,0) level, which have not yet been identified, may occur among the low lying levels in K^{38}.

Woods-Saxon Integrals

In chapter 1 it was shown how the radial integrals depend on the nuclear potential well. One objection to the use of an oscillator potential in representing the nuclear well is that it does not approach zero as r becomes large. A better approximation to the nuclear well is the Woods-Saxon potential

$$V(r) = \frac{-V}{1 + \exp[\alpha(r - a)]}$$

where a is the 'nuclear radius' and α determines the thickness of the surface region.



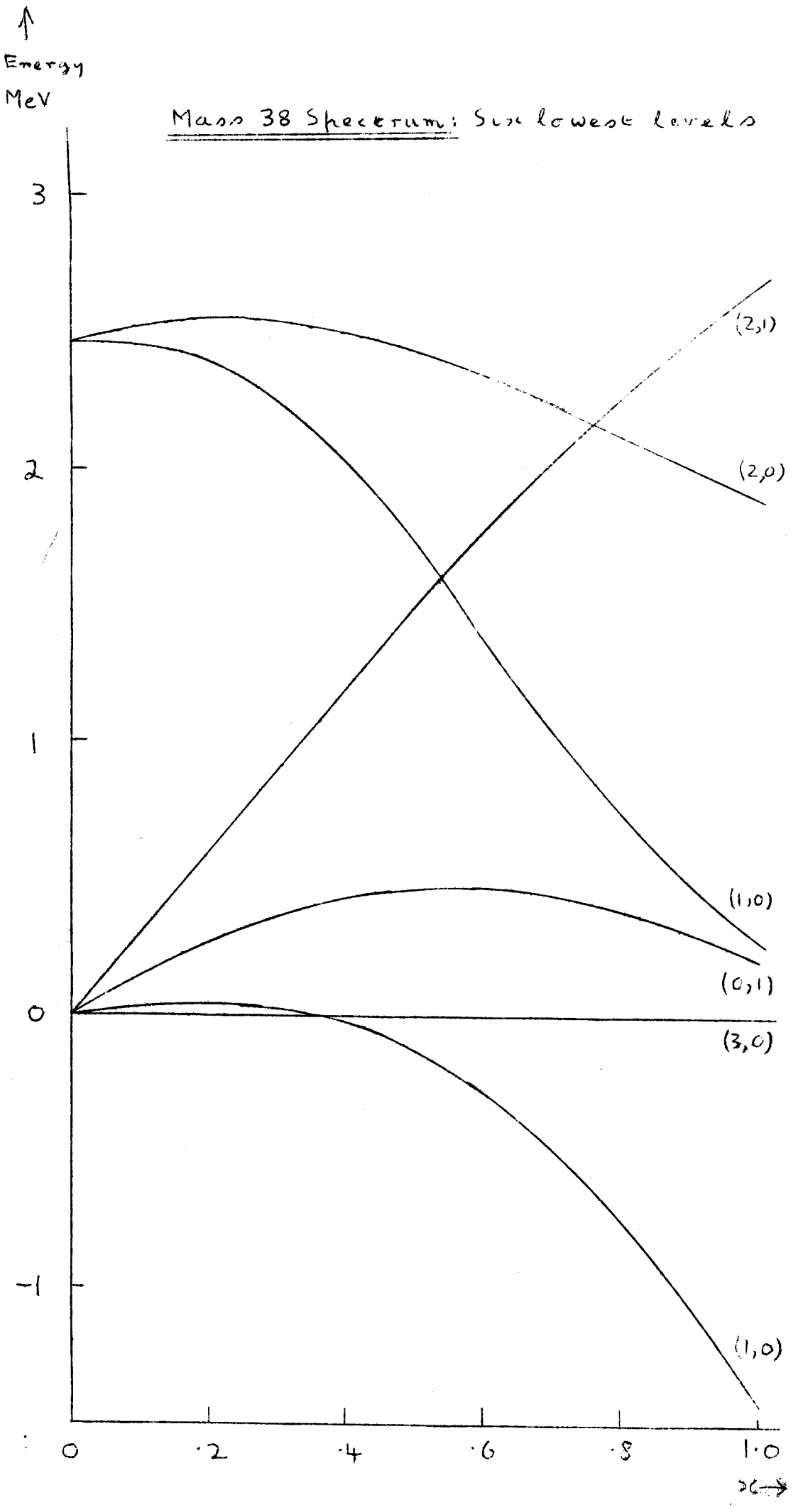
The radial integrals have been evaluated for a Woods-Saxon well by Wilmore (Manchester University, Private communication) with α = 1·16 (Ross Mark and Lawson Phys. Rev. 102; 1613. (1956)) and V = 80 MeV.
These integrals were used in the present calculation retaining the Yukawa interaction (a = 1·37 x 10^{-13}cm). The spectrum so obtained was plotted, but is not shown here as there is no noticeable deviation from the fig: 1 curves.

This indicates that for these energies the oscillator wavefunctions are quite adequate to describe the nuclear potential well and they have been used in all the other calculations here.

Variation of the $d^5/2$ single hole level

It was shown in chapter 1 how the matrix elements of the spin-orbit force depended on the mass 39 single hole levels. Mention was made of the fact that these levels are not well known, there being a good chance that the $d^5/2$ level might be higher than 2·80 MeV, (which is certainly a lower limit for this level).

FIG. 2


Energy
MeV
Mass 38 Spectrum: Six lowest levels
3
2
1
0
-1
(2,1)
(2,0)
(1,0)
(0,1)
(3,0)
(1,0)
0
·2
·4
·6
·8
1·0
x

The calculation giving rise to the fig: 1 spectra was repeated, assuming that the $d_{5/2}$ level is at 4 MeV, the results obtained being given in fig: 2.

At first sight the splitting between the (0,1) and (2,1) levels has increased favourably, but the (1,0) has been pushed further below the (3,0) level. Also ζ has altered (to 1·6) and so a given value of x now corresponds to a larger value of Vc. As far as the lowest levels are concerned nothing can be gained from this effect alone. However the spacing between the higher levels can be increased favourably by this change.

Initial attempts to improve the mass 38 spectrum

In this type of nuclear structure problem there are a great many quantities which can be varied. It was decided to retain the Yukawa two-body interaction and seek improvement by variation of exchange mixture.

As a preliminary to doing this the wavefunctions for intermediate values of x were examined and found to be reasonably close to jj-coupling (see Appendix III).
It is not difficult to derive expressions for the slopes of the curves E(x) at the origin (x = 0 corresponding to pure jj-coupling) and from such expressions it is possible to argue whether the (1,0) curve for example can be kept sufficiently above the (3,0), (by ensuring that its slope relative to that of the latter is as large as possible).

The slopes of the energy spectrum curves at x = 0 are given by

$$\left(\frac{dE}{dx}\right)_{x=0} = -\zeta\left(\psi(jj)_J \,|\, 50\, H_c \,|\, \psi(jj)_J\right)$$

where Hc is the central force part of the Hamiltonian given by V_{12} / Vc (see equation 1.3) and $\psi (jj)_J$ are the pure jj-coupled wavefunctions responsible for the curve with spin J.

Denoting the slope of (3,0) by E_3'

(1,0) by E_1'

(0,1) by E_0'

(2,1) by E_2'

the following relative slopes are of interest; $E_1' - E_3'$, $E_2' - E_0'$ and $E_1' - E_0'$. Using the Yukawa interaction the following expressions are obtained (in units of $-\xi$)

$$E_1' - E_3' = -0{\cdot}49\ A^{13} + 0{\cdot}49\ A^{11}$$

$$E_2' - E_0' = -1{\cdot}47\ A^{31} - 0{\cdot}32\ A^{33}$$

$$E_1' - E_0' = 1{\cdot}60\ A^{13} + 0{\cdot}65\ A^{11} - 2{\cdot}06\ A^{31} - 1{\cdot}08\ A^{33}$$

For an improved low level spectrum the following conditions would have to be satisfied.

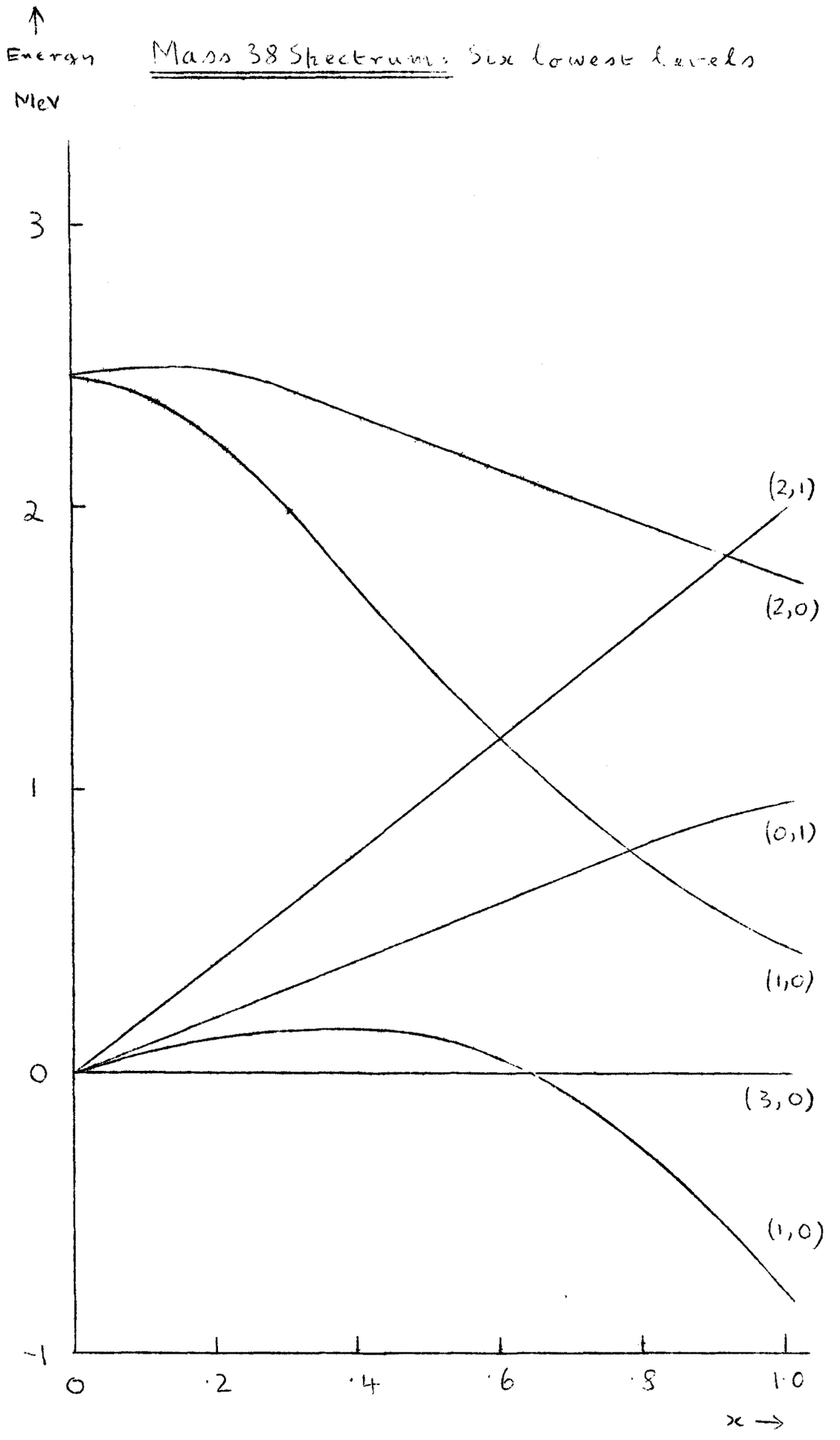
(a) $E_1' - E_3' > 0$

(b) $E_2' - E_0' > 0$ and as large as possible

(c) $E_1' - E_0' > 0$

The table below illustrates the result of substituting some well known exchange mixtures into the formulae above.

FIG. 3


Energy
MeV
Mass 38 Spectrum: Six lowest levels
3
2
1
0
-1
0
·2
·4
·6
·8
1·0
x →
(2,1)
(2,0)
(0,1)
(1,0)
(3,0)
(1,0)

Exchange used by:-

Values	Elliott and Flowers	Rosenfeld	Kurath	Soper
A^{13}	- 1	- 1	- 1	- 1
A^{31}	- 0·7	- 0·6	- 0·6	- 0·46
A^{33}	+ 0·26	+ 0·33	+ 0·6	- 0·15
A^{11}	- 0·5	+ 1·8	+ 1·0	+ 0·4

Type of exchange:-

Slope differences	Elliott and Flowers	Rosenfeld	Kurath	Soper
$E_1' - E_3'$	+ 0·24	+ 1·37	+ 0·98	+ 0·69
$E_2' - E_0'$	+ 0·95	+ 0·78	+ 0·69	+ 0·72
$E_1' - E_0'$	- 0·76	+ 0·45	- 0·36	- 0·23

It will be seen that none of the exchange mixtures listed here would appear to give all round improvement, with the possible exception of Rosenfelds, which is not thought to be completely satisfactory from Elliott and Flowers (1957) O^{16} work. Also this exchange would not help the splitting between the (0,1) and (2,1) levels.

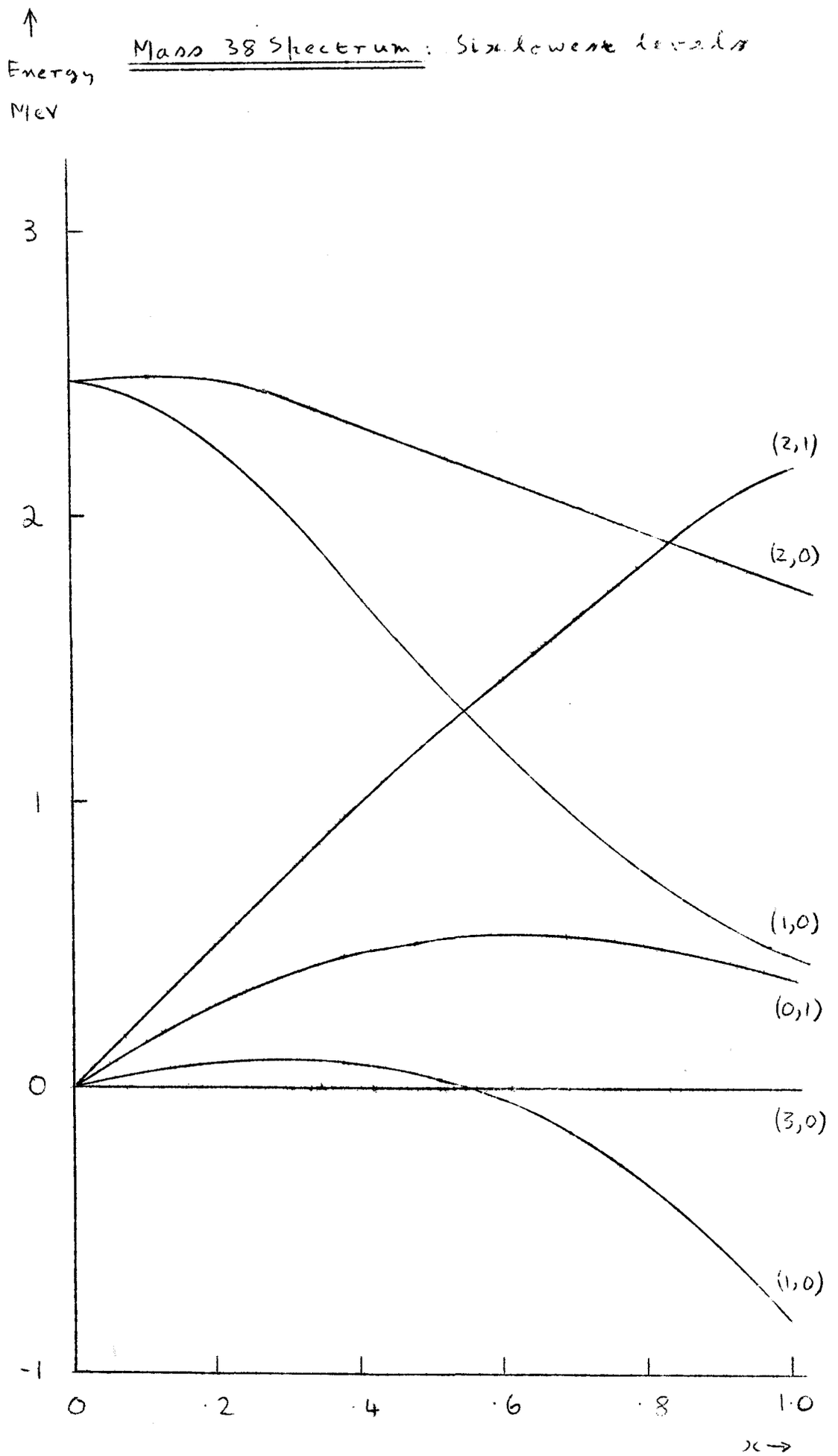
In fact two calculations were carried out using different exchanges, the results being given in figs: 3, 4.

In the fig: 3 curves Sopers' exchange was used.
Points to be noted are

(1) The position of the first (1,0) level has improved, although it is still below the (0,1) level and is too low.

(2) The (0,1) level is now too high for a realistic value of x and consequently good splitting between the (0,1) and (2,1) levels cannot be obtained.

FIG. 4


Energy
MeV
Mass 38 Spectrum : Six lowest levels
3
2
1
0
-1
(2,1)
(2,0)
(1,0)
(0,1)
(3,0)
(1,0)
0
·2
·4
·6
·8
1·0
x →

In the fig: 4 curves the exchange used was

$$A^{13} = -1$$
$$A^{31} = -0{\cdot}7$$
$$A^{33} = +0{\cdot}8$$
$$A^{11} = 0$$

a modified version of that used by Elliott and Flowers, chosen to preserve $A^{33} - A^{11} \sim 0{\cdot}8$ from O^{16}.

Most of the remarks made about the fig: 3 curves also apply to this case, although (0,1) and (2,1) level splitting is a little better.

From these results it was concluded that variation of exchange mixture alone could not improve the mass 38 spectrum. This was born out by the β -decay results to be discussed in chapter 3.

Attempt to simulate a variation in the range of the two-body force.

The expressions derived for $E_1' - E_3'$ etc. above depend on the exchange mixture and the two-body interaction. As variation of the former yielded no improvement, it was decided to vary the latter.

An attempt to simulate the effect of varying the range a of the two-body force was made by introducing a quadrupole force of strength χ defined by

$$V(r) = \chi \sum_{i<j} r_i^2 \; r_j^2 \; P_2 \; (\cos \omega_{ij})$$

and replacing V(r) by V(r) + V'(r), thus affecting the value of certain radial integrals.

There is little justification for this new force. Its physical interpretation is vague; all that can be said is that it resembles the interaction between two particles outside a closed shell and the vibrations of the closed shell core.

New expressions for $E_1' - E_3'$ etc. were worked out, various exchanges considered, and a rough estimate of the value of χ made. Detailed calculations were then performed following the usual techniques.

It has not been thought worthwhile to include a summary of the results obtained, some of which seem rather peculiar. The quadrupole force did help to increase the splitting between the (0,1) and (2,1) levels but this was completely offset by a considerable deterioration in the position of the lowest (1,0) level.

This approach was then abandoned in favour of a straightforward range variation.

Variation of the range of the two-body force

The effects of changing the two-body interaction are felt in the evaluation of the radial integrals. It was decided that as the range might have to be changed several times, this would become easier if the Yukawa potential were replaced by its Gaussian equivalent. This had the additional advantage of demonstrating the effects of changing the shape of the two-body potential.

It was shown in chapter 1 how this step was carried out.

Gauss potential, $a = 1{\cdot}80 \times 10^{-13}$ cm.

This short range potential is similar to the initial

FIG. 5

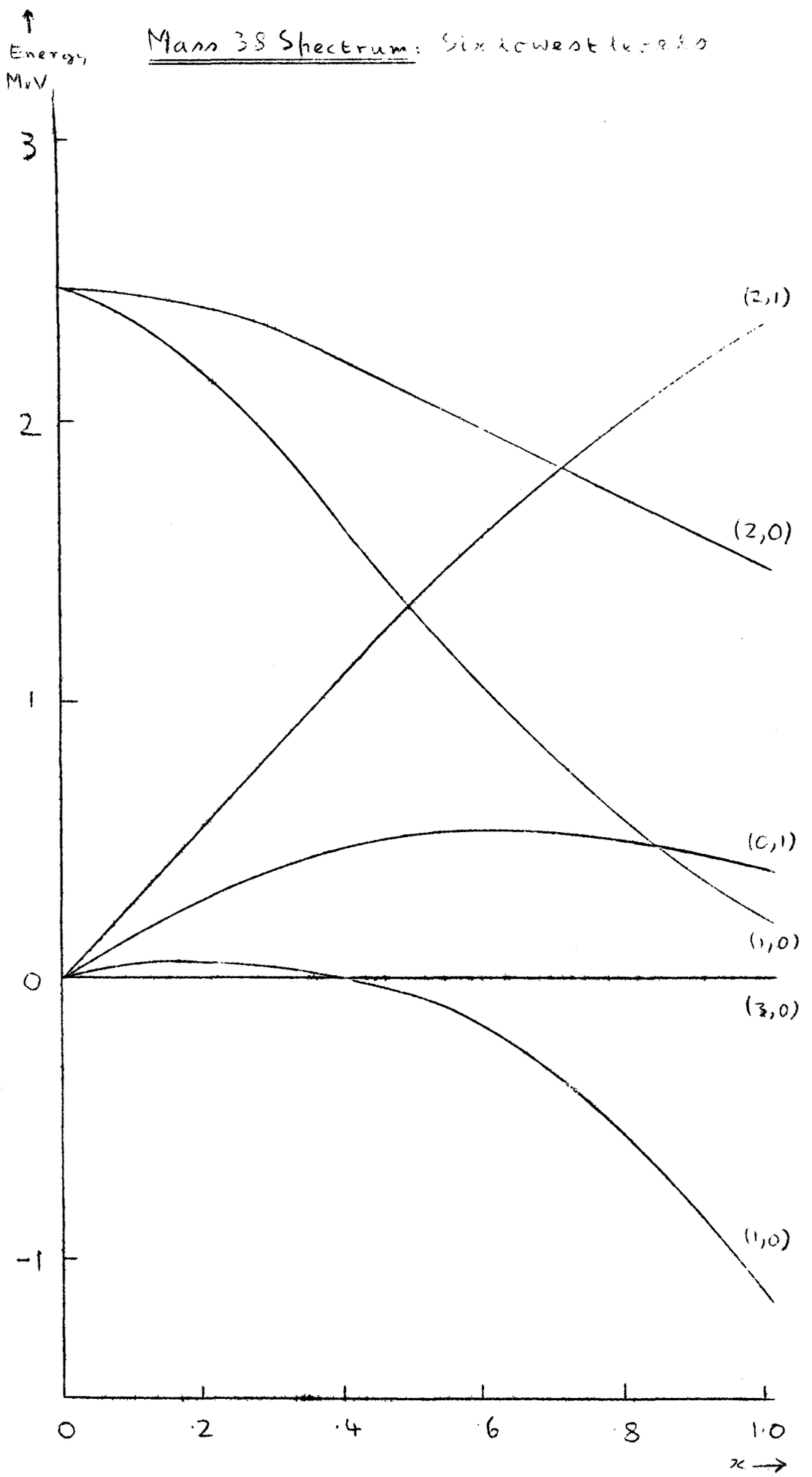
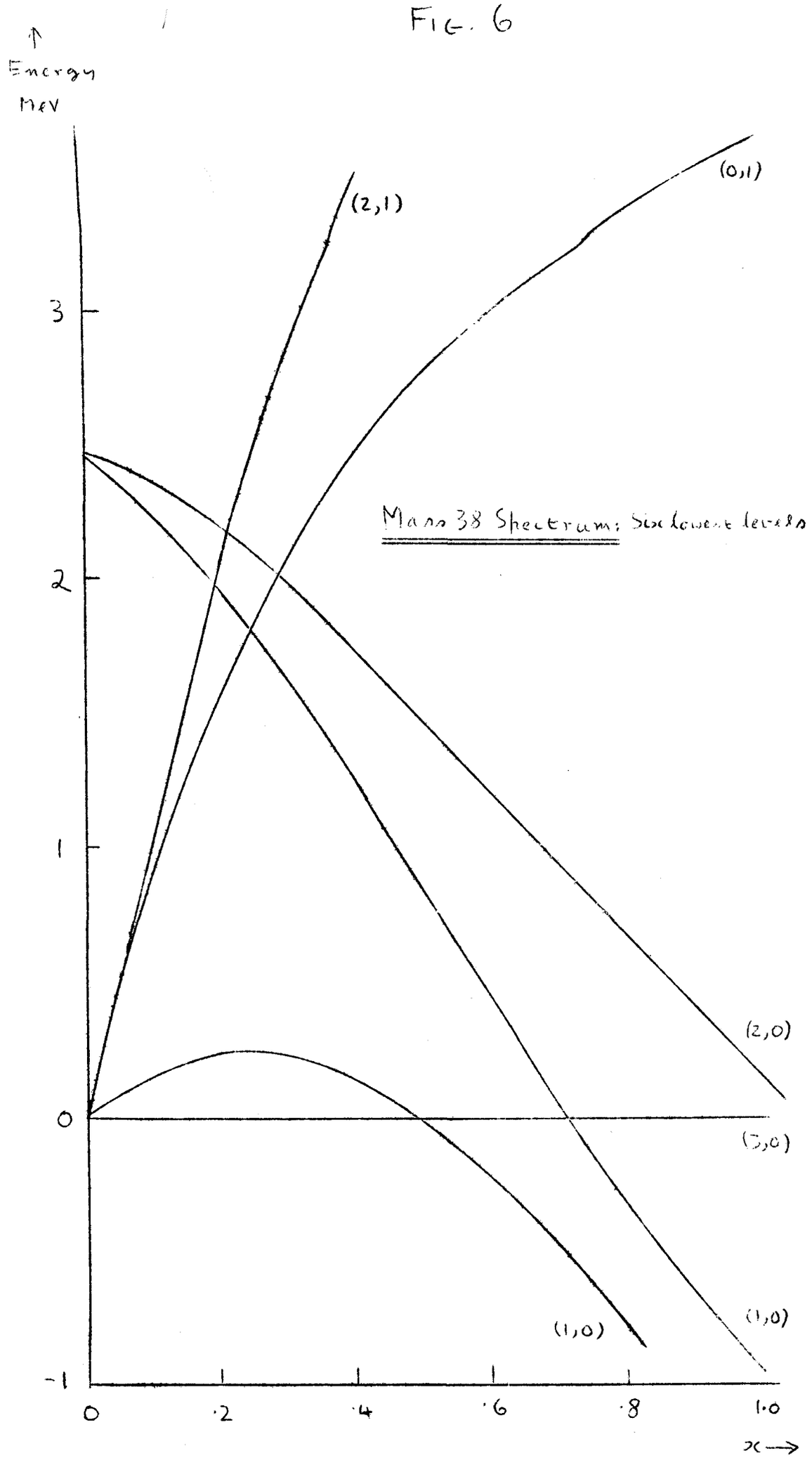

Energy, M.V
Mass 38 Spectrum: Six lowest levels
3
2
1
0
-1
(2,1)
(2,0)
(0,1)
(1,0)
(3,0)
(1,0)
0
·2
·4
·6
·8
1·0
x

FIG. 6



Mass 38 Spectrum: Six lowest levels

Yukawa calculation with a = 1·37 x 10^{-13}cm. The Elliott and Flowers exchange was again used.

The spectra obtained are illustrated in fig: 5. It will be seen that there is little or no improvement.

(1) The first (1,0) level is much too low and again falls sharply as x increases.

(2) The (0,1) level is much too high and consequently cannot easily be split 2 MeV from the (2,1) level.

(3) A second (1,0) and a (2,0) level should be found in the low lying levels of K^{38}.

The range of the potential was then lengthened.

Gauss potential, a = 3·40 x 10^{-13}cm.

This calculation was first performed for the Elliott and Flowers exchange, the results obtained appearing in fig: 6.

Significant changes have occurred, but no improvement.

(1) The first (1,0) level is now a little higher.

(2) The (0,1) level is ridiculously high and cannot be split 2 MeV from the (2,1) level.

(3) The second (1,0) and (2,0) levels are again low.

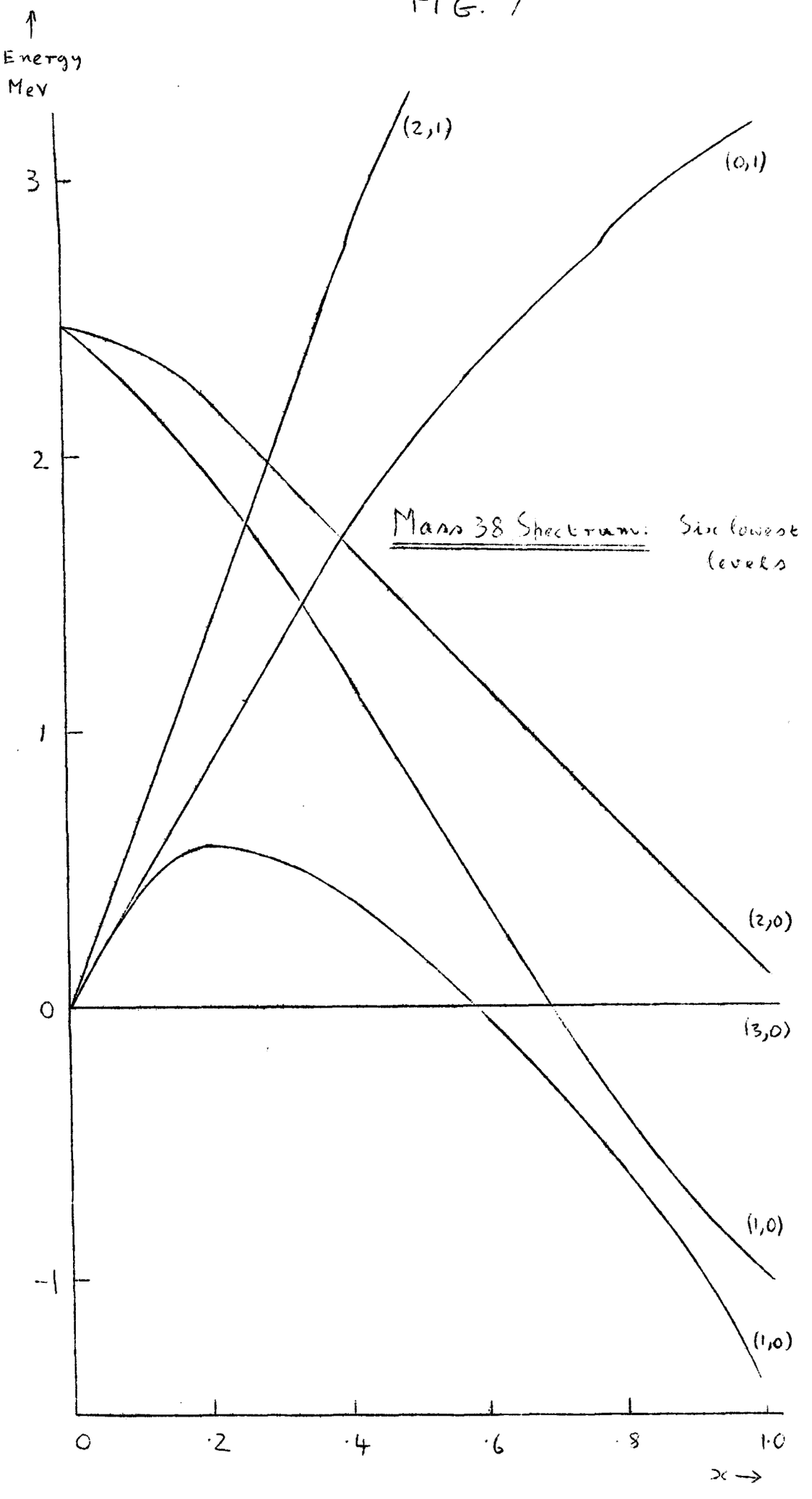
Having observed the effect of a range variation it was decided to see whether altering the exchange could cause an improvement. Expressions were worked out for the slopes of the curves at x = o as indicated before.

$$E_1' - E_3' = -2{\cdot}84\ A^{13} + 2{\cdot}84\ A^{11}$$

$$E_2' - E_o' = -2{\cdot}74\ A^{31} - 0{\cdot}45\ A^{33}$$

$$E_1' - E_o' = 6{\cdot}90\ A^{13} + 4{\cdot}30\ A^{11} - 5{\cdot}63\ A^{31} - 7{\cdot}17\ A^{33}$$

FIG. 7



Mass 38 Spectrum: Six lowest levels

From these it becomes clear that one way of reducing the slope of the (0,1) curve was to change the sign of A^{33}. This also slightly assists with the splitting between the (0,1) and (2,1) levels. Changing the sign of A^{11} also helps. From other considerations it is not advisable to alter A^{13} or A^{31} and so it was decided to try the exchange

$$A^{13} = -1$$

$$A^{31} = -0{\cdot}7$$

$$A^{33} = -0{\cdot}26$$

$$A^{11} = +0{\cdot}5$$

which is similar to that favoured by Soper (and will hereafter be referred to as the Soper-like exchange).

The curve slope data is summarised below

Exchange:-	Elliott and Flowers	Soper-like
Values		
$E_1' - E_3'$	+ 1·42	+ 4·26
$E_2' - E_0'$	+ 1·80	+ 2·03
$E_1' - E_0'$	- 6·97	+ 1·05

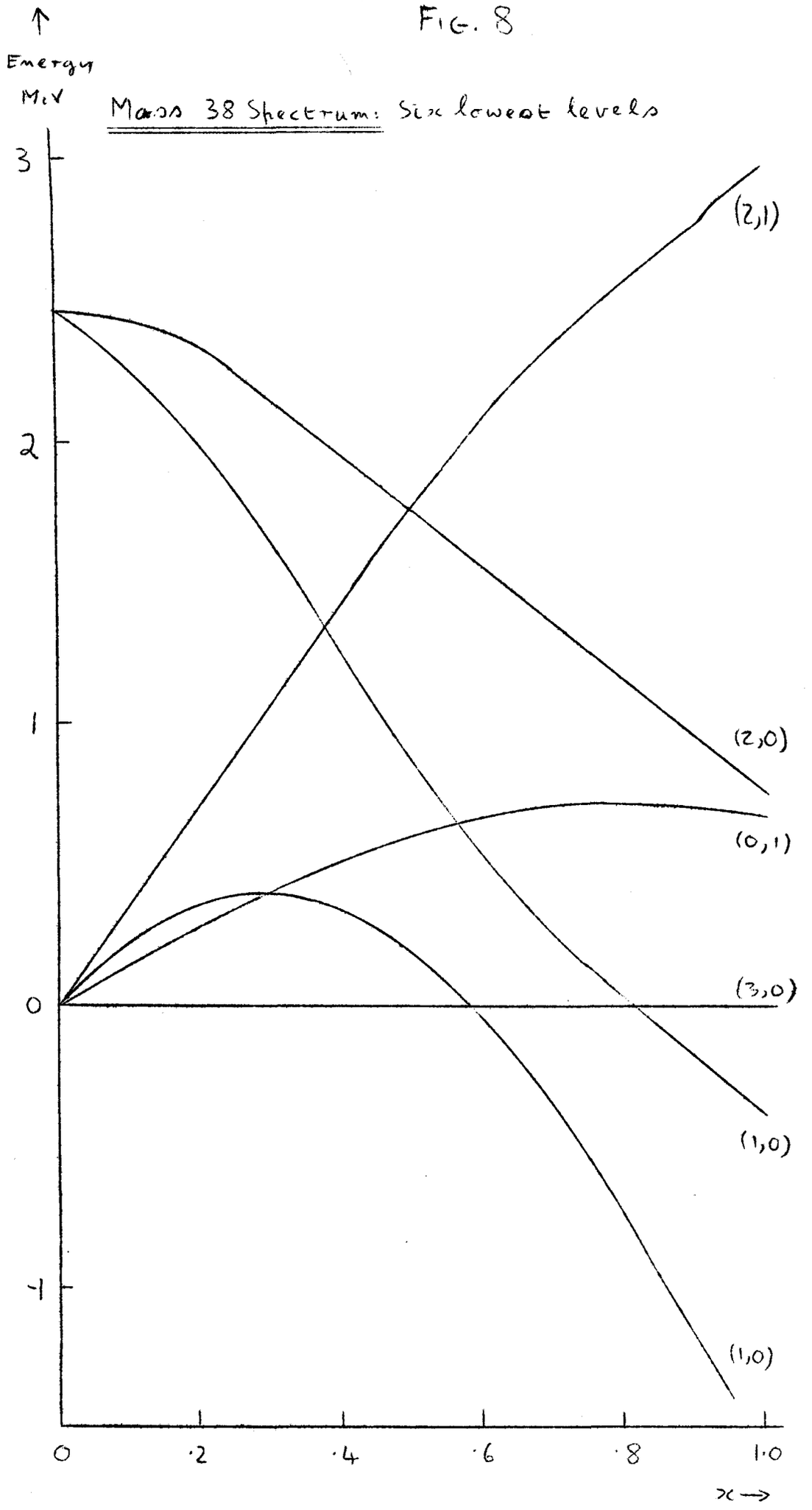
The calculation was repeated using the new exchange and the result obtained is illustrated in fig: 7.

(1) The position of the first (1,0) level has improved and is now roughly correct for $0{\cdot}1 < x < 0{\cdot}5$

(2) The (0,1) level is still much too high and the splitting between this level and the (2,1) has only improved slightly.

(3) The second (1,0) and (2,0) levels are again low lying.

FIG. 8


Energy
MeV
Mass 38 Spectrum: Six lowest levels
3
2
1
0
-1
(2,1)
(2,0)
(0,1)
(3,0)
(1,0)
(1,0)
0
·2
·4
·6
·8
1·0
x →

It is known that reducing the range of the force improves the position of the (0,1) level. It was decided to try a mid range potential to see whether the (0,1) level could be lowered and the lowest (1,0) level kept sufficiently high.

Gauss potential, a = 2·31 x 10^{-13}cm.

The calculation was performed using the Soper-like exchange, the results being given in fig: 8. These were the best obtained for the low lying levels in mass 38, the wavefunctions associated with them also being the most satisfactory as will be seen in the next chapter.

(1) The position of the first (1,0) level is still improved but it is still too low for a realistic value of x.

(2) The (0,1) level is again too high (but at least for $x < 0{\cdot}3$ it is above the (1,0) level for the first time). If this level could be further lowered and the (2,1) raised, the splitting between them would be correct.

(3) The second (1,0) and (2,0) levels should be low lying in K^{38}.

The β -decay calculations in chapter 3 indicated that a value of x around 0·5 should be taken, corresponding to Vc ~ 30 MeV. The K^{38} spectrum so obtained is compared with experimental data below.

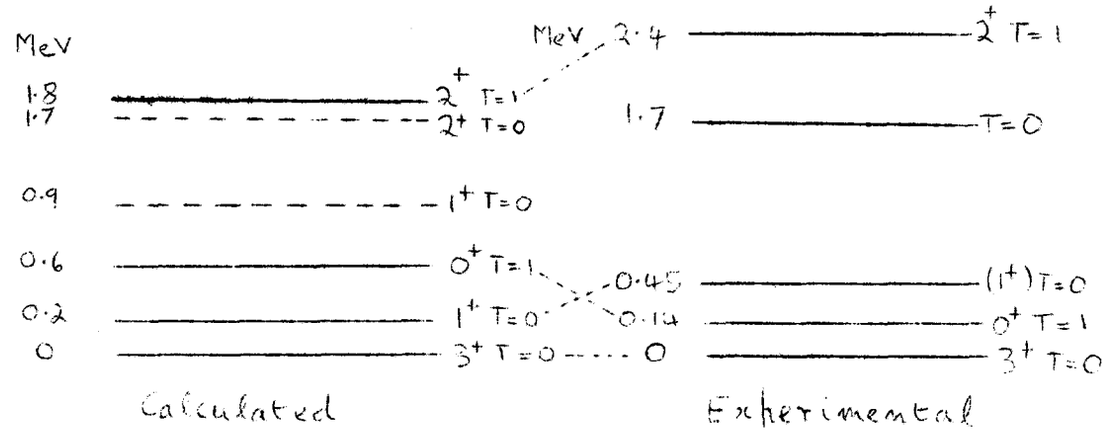
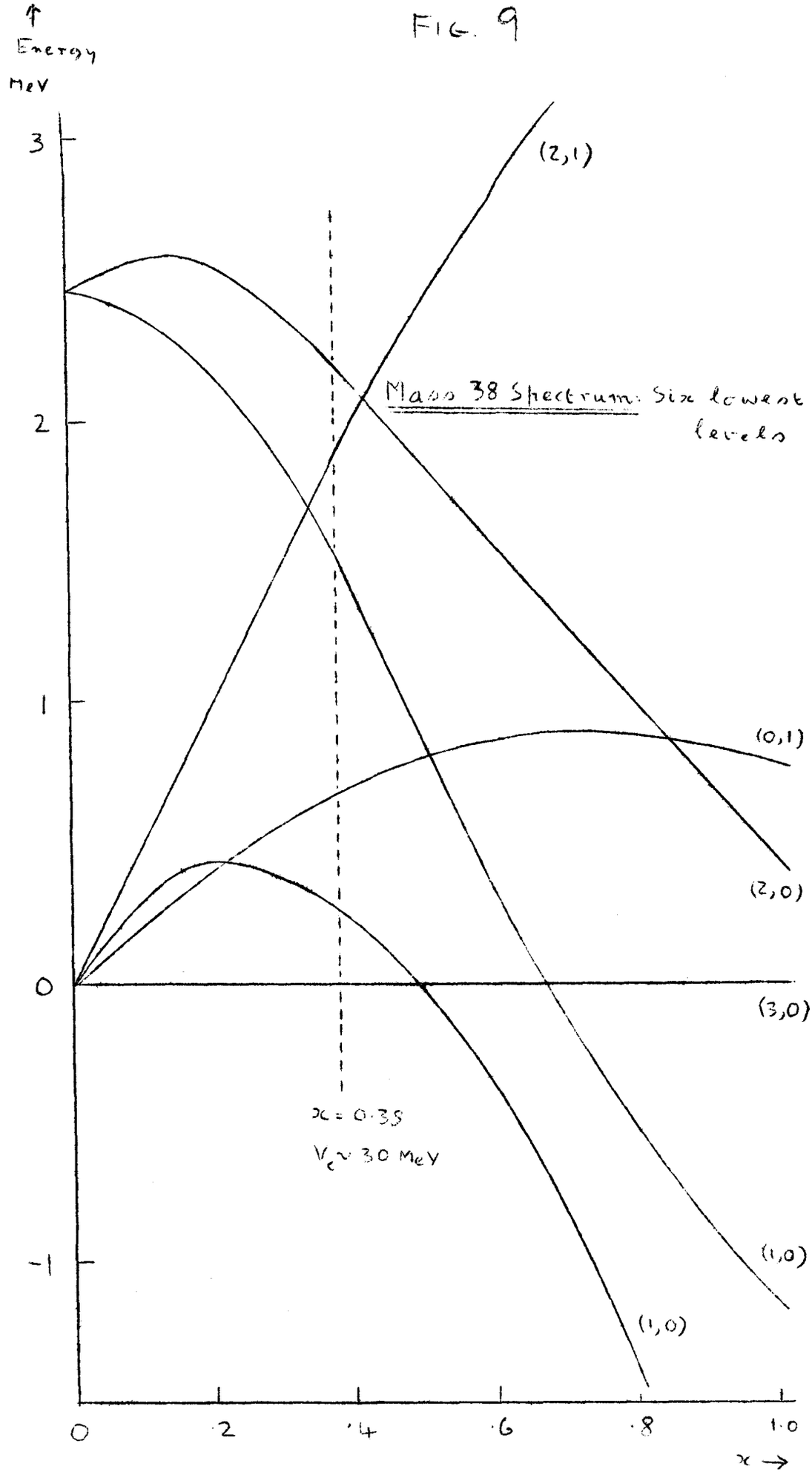


FIG. 9


Energy
MeV
3
2
1
0
-1
(2,1)
Mass 38 Spectrum: Six lowest levels
(0,1)
(2,0)
(3,0)
(1,0)
(1,0)
x = 0·35
V_c ~ 30 MeV
0
·2
·4
·6
·8
1·0
x →

The arrival of new experimental data caused attention to be turned towards some of the higher levels.
The evidence was strongly in favour of there being only one level between $(1^+)T = 0$ at 0·45 Me V and $2^+T = 1$ at 2·4 MeV, whereas all the calculations so far have advocated two (for realistic values of x).

The position of higher levels in this kind of treatment must depend quite strongly on the single hole levels which represent the pure jj-coupled states at x = o.
It was said earlier that the $d^5/2$ single hole level at 2·80 Me V could only be taken as a lower limit. If this level were to be raised, it should lead to the raising of the higher calculated levels leaving the lower ones relatively unchanged.

The calculations were repeated using the Gauss mid range force, the Soper-like exchange, and a $d^5/2$ single hole level shifted from 2·80 to 4 Me V. The results for the lowest levels are shown in fig: 9.

(1) The position of the first (1,0) level is acceptable for $0{\cdot}1 < x < 0{\cdot}4$.

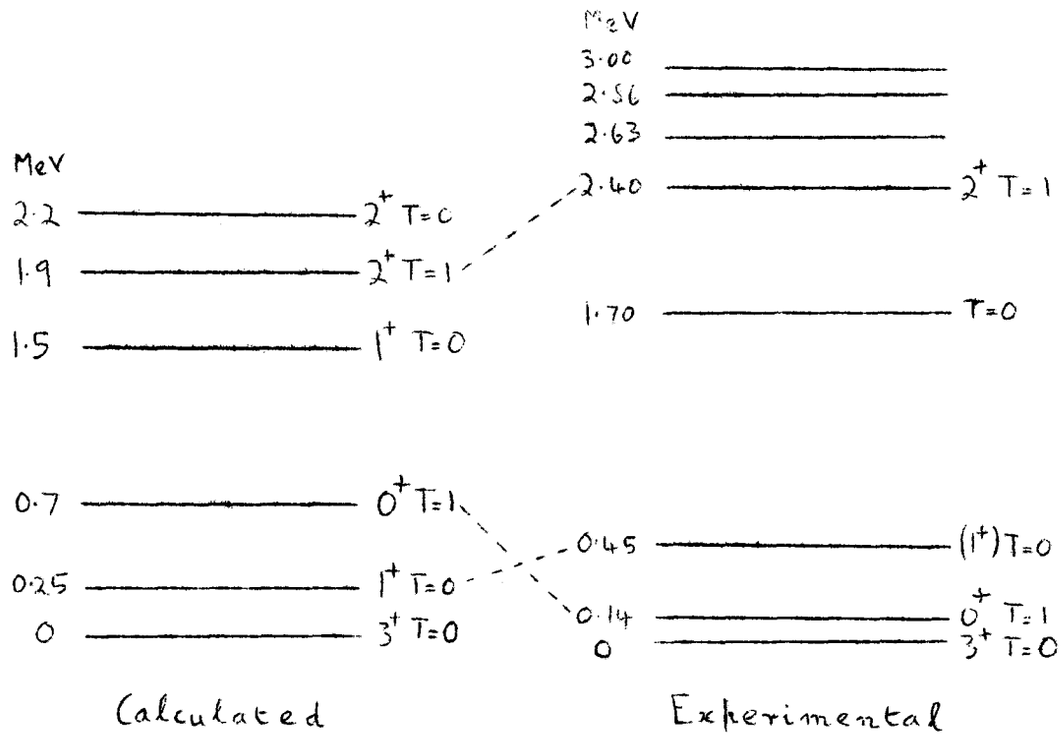
(2) The (0,1) level is again too high and is only below the (1,0) for $x < 0\ 2$. In order to obtain good splitting between the (0,1) and (2,1) levels a high value of x ($\sim 0{\cdot}7$) would be necessary, at which both of these levels are too high by 1 Me V.

(3) If there is to be only one level between the $(1^+)T = 0$ and the $2^+T = 1$ then $0{\cdot}34 < x < 0{\cdot}42$.

Taking all these factors into account together with the β -decay data of chapter 3, the best overall agreement is

still obtained with Vc = 30 MeV which now corresponds to x = 0·38 (since ξ has changed to −1·6 MeV).

The K^{38} spectrum with Vc = 30 MeV is compared with the experimental data below for the lowest levels.



The following remarks can be made,

(1) The T = 0 level observed at 1·70 MeV is probably the 1^+ T = 0 calculated to be at 1·5 MeV.

(2) The next even parity level above 2^+T = 1 at 2·40 MeV is most likely to have spin 2 with T = 0.

The A^{38} spectrum with Vc = 30 MeV is compared with the experimental data below.

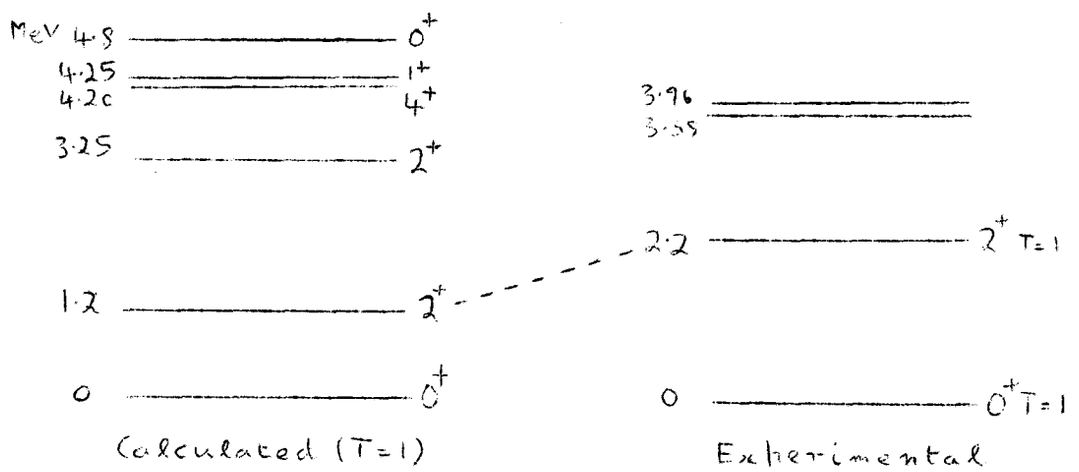
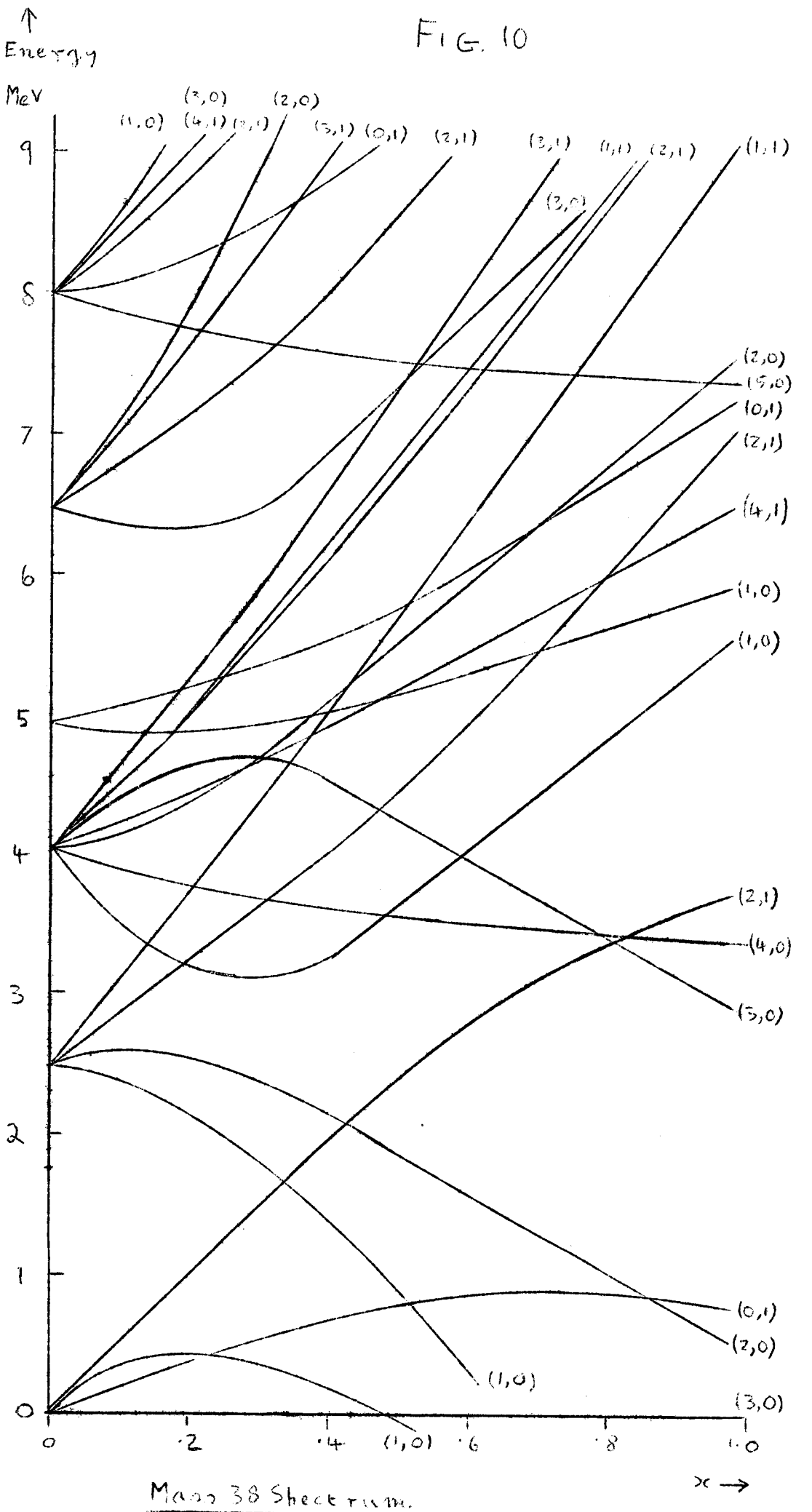


FIG. 10



Mass 38 Spectrum.

The lowest calculated $2^+T = 1$ level is out by 1 Me V and further comment is impossible until more experimental data is available.

Fig: 10 illustrates the complete level scheme obtained from this last calculation.

The final conclusions to be drawn from these results will be left until chapter 7.

Justification of the longer range force

The mid range Gauss force used here has longer range than the conventional p shell forces and indeed longer range than that required by Elliott and Flowers (1955) at the beginning of the ds shell.

Some workers at the beginning of the pf shell have also found it necessary to employ these longer range forces.

The assumption of a long range force was first made by Kurath (Phys. Rev. 80; 98. 1950). Levinson and Ford (Phys. Rev. 100; 13. 1955) had to employ such a force when fitting a spectrum to Ca^{42}. French and Raz (Phys. Rev. 104; 1411. 1956) also used a longer range force (though not as long as Levinson and Ford's) when investigating the spectra of Ca isotopes.

If all these results are valid, there is no reason why a force of longer range should not be required just below Ca^{40}.

Analysing the results quoted in French and Raz (1956)

(1) Levinson and Ford used a very long range Gaussian force ($a = 3{\cdot}4 \times 10^{-13}$cm) and in order to fit their spectrum an overall Vc $\sim$ 20 Me V was required.

Potential and Range	Nucleus	L/K	K/Vc	Vc (MeV)
Yukawa $a = 1.37 \times 10^{-13}$ cm	Li^6	6.21	0.031	32
	N^{14}	5.62	0.024	42
Gauss $a = 1.80 \times 10^{-13}$ cm	Li^6	8.47	0.023	44
	N^{14}	6.78	0.022	45
Gauss $a = 3.40 \times 10^{-13}$ cm	Li^6	46.6	0.011	93
	N^{14}	28.1	0.015	68
Gauss $a = 2.31 \times 10^{-13}$ cm	Li^6	14.7	0.020	49
	N^{14}	10.5	0.022	45

(2) French and Raz used a Gaussian force of range $a = 2{\cdot}7 \times 10^{-13}$cm. To fit the Ca^{43} spectrum they required an overall Vc $\sim$ 25 M e V. (Nb it proved easier to fit the Ca^{43} spectrum than the Ca^{42}).

They also used an exchange mixture having

$$A^{31}/_{A}{}^{33} = 2{\cdot}3$$

which is similar to the value 2·7 used here.

Using a Gaussian force of range $a = 2{\cdot}31 \times 10^{-13}$cm and overall Vc $\sim$ 30 M e V would not be inconsistent with the above.

Some calculations were made to investigate the effects these longer range forces would have had in the p shell.

The quantities $^{L}/_{K}$ and $^{K}/_{Vc}$ were evaluated. Kurath (Intermediate coupling in the 1p shell. Phys. Rev. 101; 216. 1956) found empirically that $^{L}/_{K}$ should lie between 6 and 7 whilst K $\sim$ 1 M e V.

The results obtained are summarised in the table opposite. Vc was estimated by putting K = 1 Me V.

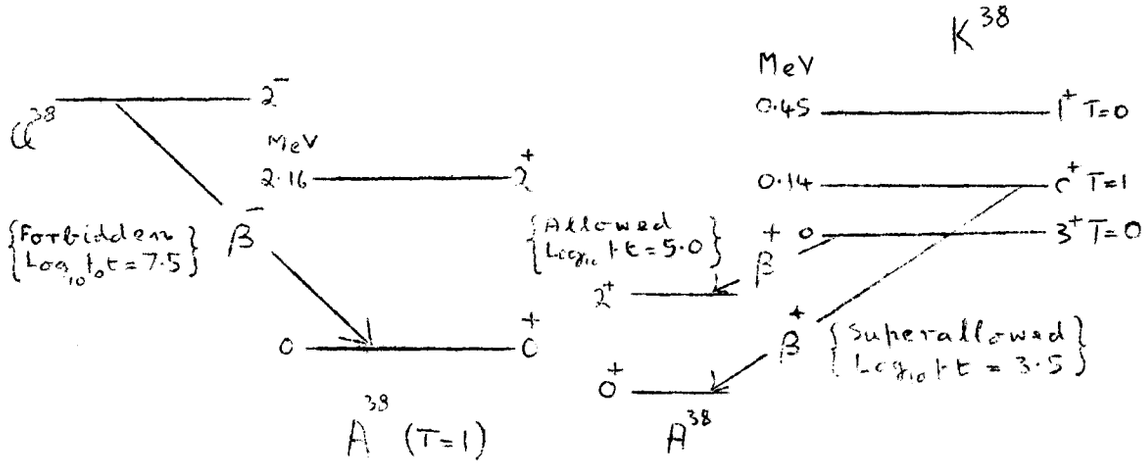
It will be seen from this that the Gauss mid range force is not in complete conflict with the p shell results although it could not be used there satisfactorily.

Chapter 3

The Mass 38 Wavefunctions

When the Hamiltonian matrix [H] described in chapter 1 is diagonalised, a set of eigenfunctions are obtained which are the approximate solutions of Schrödingers equation. These functions represent the nuclear wavefunction and may be used to calculate such things as transition probability for electromagnetic radiations, β-decay ft values, magnetic and quadrupole moments.

The mass 38 wavefunctions have been tested via the calculation of several β-decay ft values. The available experimental data, taken from Endt and Braams (1957) is summarised in the diagram below.



In addition to this there is the decay $Ca^{38} \xrightarrow{\beta^+} K^{38}$ which will be considered later.

The composition of some mass 38 wavefunctions is described in Appendix III

The allowed β-decay transitions

In light nuclei most β-decay transitions are of the

allowed type for which the ft value is given by (Elliott and Lane (1957))

$$ft = \frac{B}{(1-x)\ F^2 + x\ G^2} \qquad 3.1$$

where the square of the Fermi matrix element is

$$F^2 = \tfrac{1}{4}\left(\psi' \left| \sum_{\gamma}' \left(\tau_x(\gamma) \pm i\tau_y(\gamma)\right) \right| \psi\right)^2 \qquad 3.2$$

and the Gamon-Teller term

$$G^2 = \frac{1}{4} \sum_{M'}' \left(\psi' \left| \sum_{\gamma}' \left(\tau_x(\gamma) \pm i\tau_y(\gamma)\right)\sigma(\gamma) \right| \psi\right)^2 \qquad 3.3$$

the 'dashed' symbols representing final states and the $\pm$ signs referring to $\beta^{\pm}$ emission

B and x are constants given by

$$B = 2783 \pm 70$$

$$x = 0{\cdot}560 \pm 0{\cdot}012$$

The Fermi term

Rewriting the Fermi term operator in terms of $T_{\pm 1}$

where $T_{\pm 1} = \frac{1}{2} \sum_{\gamma}' \tau_{\pm 1}(\gamma)$

and $\tau_{\pm 1} = \mp \frac{1}{\sqrt{2}} \tau_{\pm} = \mp \frac{1}{\sqrt{2}} \left(\tau_x \pm i\tau_y\right)$

are defined such that τ_{+1} is the operator which changes a proton into a neutron and τ_{-1} is the operator which changes a neutron into a proton, and using the Wigner-Eckart theorem, it can easily be shown that

$$F^2 = (T \pm M')(T \mp M' + 1)\ \delta(TT')\ \delta(M\ M' \pm 1) \qquad 3.4$$

where T is the nuclear isotopic spin and M' the M_T value for the final nucleus. It will be seen that this term cannot contribute if isotopic spin changes.

For example consider the super allowed decay

$$K^{38}\ (0^+,\ T=1;\ M_T=0) \xrightarrow{\beta^+} A^{38}\ (0^+,\ T=1;\ M_T=1)$$

then $F^2 = (1 + 1)\ (1 - 1 + 1) = 2$

The Gamow-Teller term does not contribute in this case because the vector operator concerned cannot couple two $J = 0$ states.

Thus from 3.1

$$ft = \frac{2783}{(1-0\cdot56)\ 2} \qquad \text{whence } \log_{10} ft = 3\cdot5$$

This example gives no fresh information because the decay concerned was used in establishing the constants of the ft formula.

The Gamow-Teller term

Writing the operator in terms of $\tau_{\pm1}$,

$$G^2 = \tfrac{1}{2} \sum_{M'_J} \left(\psi'(T' M'_T J' M'_J) \left| \sum_j \tau_{\pm1}(j) \sigma'_q(j) \right| \psi(T M_T J M_J) \right)^2$$

Applying the Wigner-Eckart theorem to remove M dependence

$$G^2 = \tfrac{1}{2} \frac{1}{(2T'+1)(2J'+1)} \begin{pmatrix} T' M'_T \\ C \\ T M_T\ 1\pm1 \end{pmatrix}^2 \left\{ \sum_{M'_J(q)} \begin{pmatrix} J' M'_J \\ C \\ J M_J\ q \end{pmatrix}^2 \right\}$$

$$\times \left(\psi'(T'J') \left\| \sum_j \tau(j) \sigma(j) \right\| \psi(TJ) \right)^2$$

where $C^{JM}_{j_1 m_1 j_2 m_2}$ are the Wigner coefficients.
This is an amplitude matrix in both T and J spaces.

Using the latent symmetries of the Wigner coefficients and their orthogonalities it is not difficult to show

$$\sum_{M'_J(a)} \left(C^{J'M'_J}_{J M_J a} \right)^2 = \frac{2J'+1}{2J+1}$$

Hence

$$G^2 = A \left(\psi'(T'J') \Big\| \sum_i' \tau(i)\sigma(i) \Big\| \psi(TJ) \right)^2 \qquad 3.5$$

where

$$A = \tfrac{1}{2} \frac{1}{(2T'+1)(2J+1)} \left(C^{T'M'_T}_{TM_T 1\pm 1} \right)^2$$

Consider

$$N = \left(\psi'(T'S'(\ell'\tilde{\ell}')[f']L'J' \Big\| \sum_i' \tau(i)\sigma(i) \Big\| \psi(TS(\ell\tilde{\ell})[f]LJ) \right)$$

where $(l\,\tilde{l})[f]L$ denotes a two-body orbital angular momentum state where l, $\tilde{l}$ of the two particles have been coupled to form a state of symmetry $[f]$.

As the operator in N does not operate in the L space but only in the T, S spaces

$$N = \delta\left((\ell'\tilde{\ell}')[f']L', (\ell\tilde{\ell})[f]L\right) \left[(2J'+1)(2J+1)\right]^{\frac{1}{2}} W(LJS'1;SJ') \times \left(T'S' \Big\| \sum_i' \tau(i)\sigma(i) \Big\| TS\right) \qquad 3.6$$

using standard expressions of the Racah algebra, to be found forvexample in Elliott and Lane (1957).

The Kronecker δ shows that the operator cannot change the L value of the initial state, its symmetry or its two particle configuration.

For antisymmetric two particle states

$$\left(T'S' \left\| \sum_j \tau(j)\sigma(j) \right\| TS\right) = 2\left(T'S' \left\| \tau(2)\sigma(2) \right\| TS\right)$$

$$= 2\left(T' \left\| \tau(2) \right\| T\right)\left(S' \left\| \sigma(2) \right\| S\right)$$

since τ only operates in the T space and σ in the S space.

These terms may be evaluated using the fact that the operator does not act on the first particle

$$\left(T' \left\| \tau(2) \right\| T\right) = \left[(2T'+1)(2T+1)\right]^{1/2} W(\tfrac{1}{2}T\tfrac{1}{2}1;\tfrac{1}{2}T')\left(\tfrac{1}{2} \left\| \tau \right\| \tfrac{1}{2}\right)$$

and similarly for the other.

Also

$$\left(\tfrac{1}{2} \left\| \tau \right\| \tfrac{1}{2}\right) = \left(\tfrac{1}{2} \left\| \sigma \right\| \tfrac{1}{2}\right) = \sqrt{6}$$

Hence

$$N = \delta\left((\ell'\tilde{\ell}')[1']L', (\ell\tilde{\ell})[1]L\right)\left[(2J'+1)(2J+1)(2T'+1)(2T+1)(2S'+1)(2S+1)\right]^{1/2}$$

$$\times 12(-)^{S+T+L+J+S'+1} \begin{Bmatrix} L & J & S \\ 1 & S' & J' \end{Bmatrix} \begin{Bmatrix} \tfrac{1}{2} & T & \tfrac{1}{2} \\ 1 & \tfrac{1}{2} & T' \end{Bmatrix} \begin{Bmatrix} \tfrac{1}{2} & S & \tfrac{1}{2} \\ 1 & \tfrac{1}{2} & S' \end{Bmatrix} \qquad 3.7$$

In solving Schrödingers equation the nuclear wave-functions were expanded in terms of certain LS-coupled states

$$\Psi = \sum_i a_i \psi_i$$

$$\Psi' = \sum_j b_j \psi_j'$$

Hence from 3.5

$$G^2 = A\left(\sum_j b_j \psi_j' \left\| \mathcal{O} \right\| \sum_i a_i \psi_i\right)^2$$

$$= A\left(\sum_i \sum_j a_i b_j \left(\psi_j' \left\| \mathcal{O} \right\| \psi_i\right)\right)^2 \qquad 3.8$$

where $\mathcal{O}$ represents the appropriate operator, and

$(\psi'_\gamma \| \mathcal{O} \| \psi_i)$ is just N evaluated between states ψ'_γ, ψ_i which are known.

The evaluation of G^2 from 3.8 is equivalent to forming the matrix representation of $\mathcal{O}$ with rows labelled by the ψ'_γ and columns by the ψ_i, pre- and post-multiplying by the appropriate eigenvectors obtained from the calculation, squaring the result and multiplying by A.

ie. $$G^2 = \left\{ [b_1\ b_2\ \ldots] \left[\begin{array}{c|c} \psi' & \mathcal{O} \\ \hline & \psi \end{array} \right] \begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix} \right\}^2 \times A \qquad 3.9$$

<u>The allowed decay $K^{38} \xrightarrow{\beta^+} A^{38}$</u>

The details of the decay are

$$K^{38}\ (3^+,\ T = 0) \xrightarrow{\beta^+} A^{38}\ (2^+,\ T = 1)$$

The matrix $[\mathcal{O}]$ is labelled by the states (J = 2, T = 1) and (J = 3, T = 0) as in the diagram.

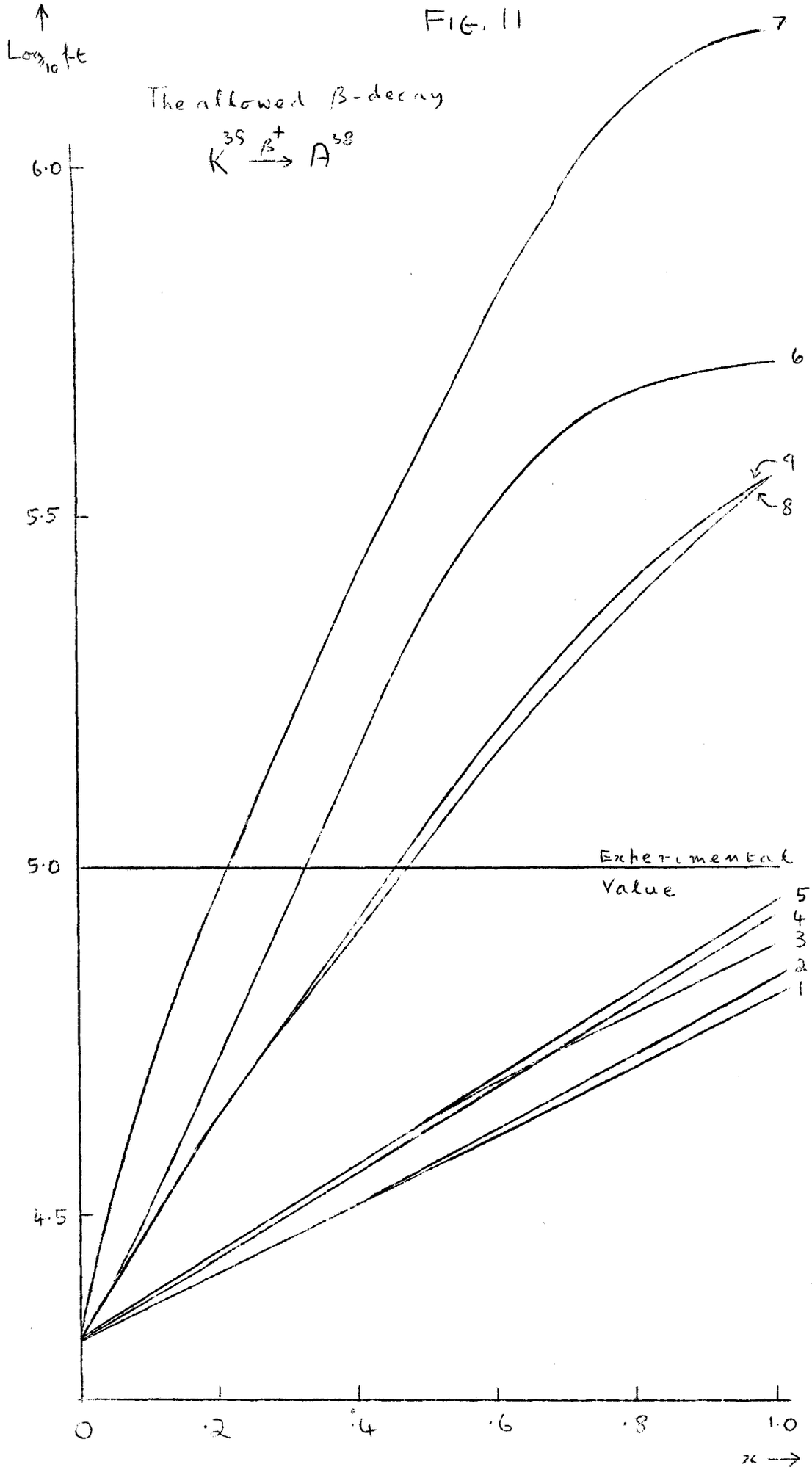
K^{38} / A^{38}	d^2			ds
	^{13}D	^{13}G	^{11}F	$^{13}[2]D$
d^2 ^{31}D	$-6\sqrt{\frac{7}{3}}$	0	0	
^{33}P	0	0	0	0
^{33}F	0	0	$6\sqrt{\frac{5}{3}}$	
ds $^{31}[2]D$		0		$-6\sqrt{\frac{7}{3}}$
$^{33}[10]D$				0

The zeros are caused by

$$\delta((\ell'\tilde{\ell}')[1]L', (\ell\tilde{\ell})[1]L)$$

in 3.7 from which the non-zero matrix elements are also computed.

FIG. 11


Log10 ft
The allowed β-decay
K38 β+ → A38
6·0
5·5
5·0
4·5
Experimental
Value
7
6
9
8
5
4
3
2
1
0
·2
·4
·6
·8
1·0
x →

M_r is determined from the relation

$$M_T = \langle T_z \rangle = \tfrac{1}{2}(N - Z) \qquad 3.10$$

for a nucleus with N neutrons and Z protons.

Eg. for K^{38} which has $N = Z = 19$; $M_T = 0$

for A^{38} which has $N = 20, Z = 18$; $M_T = 1$

In this example A, evaluated from 3.5, has the value $^1/_{42}$.

Thus G^2 can be found when the calculated eigenvectors are substituted in 3.9 and $\log_{10}$ ft obtained from 3.1 where the Fermi term does not contribute, there being a change of isotopic spin.

The matrix elements of $[\mathcal{O}]$ were checked by calculating this β-decay using pure jj-coupled wavefunctions directly, which corresponds of course to $x = 0$ in the eigenvector calculation.

The results obtained are given for several exchanges and ranges in Fig: 11; a key to these curves is given below.

Curve no.	Potential	a x 10^{13} cm	$d_{5/2}$ position	Exchange A^{13}	A^{31}	A^{33}	A^{11}
1	Yukawa	1·37	2·8 MeV	-1	-0·7	+0·26	-0·5
2	"		4	"	"	"	"
3	"		2·8	"	-0·46	-0·15	+0·4
4	"		"	"	-0·7	+0·8	0
5	Gauss	1·80	"	"	"	+0·26	-0·5
6	"	3·40	"	"	"	"	"
7	"	3·40	"	"	"	-0·26	+0·5
8	"	2·31	"	"	"	"	"
9	"	2·31	4	"	"	"	"

The experimental value for $\log_{10}$ ft is 5·0. The results will be discussed on the basis that the calculated $\log_{10}$ ft is acceptable if it lies between 4·8 and 5·2.

(a) Curves 1 to 5.

These curves were obtained using short range potentials and various exchange mixtures. One variation of $d_{5/2}$ single hole level is included. For an acceptable $\log_{10}$ ft x must be restricted as follows

Curves 1, 2 $\quad x > 0·9$

Curves 3, 4, 5 $\quad x > 0·8$

Reference to the spectra associated with these curves (Chapter 2) shows that with these values of x no kind of agreement can be obtained with the experimental levels, due mainly to the positions of the two lowest (1, 0) levels.

(b) Curves 6 and 7.

These curves correspond to the long range Gauss potential, with a variation of exchange. For an acceptable $\log_{10}$ ft x is restricted as follows

Curves 6 $\quad 0·24 < x < 0·42$

Curves 7 $\quad 0·14 < x < 0·3$

Reference to the spectra associated with these curves (Chapter 2) shows that if x is so chosen, the spectra will not agree with experiment due mainly to the position of levels (0, 1) and (2, 1) in addition to the totally inadequate splitting between them.

(c) Curves 8 and 9.

These curves represent the Gauss mid range force with a change of $d_{5/2}$ single hole level. It will be

observed that this calculation is particularly insensitive to the latter effect. For an acceptable $\log_{10}$ ft x would be restricted by

$$0{\cdot}32 < x < 0{\cdot}6$$

Reference to the appropriate calculated spectra (Chapter 2) shows that x lying within this range will produce a spectrum that may reasonably be compared with experiment.

In particular Vc = 30 M e V which gave the best spectrum corresponds to x = 0·5 in curve 8 and x = 0·38 in curve 9, both of which lie within the range above.

The very faint indication that increasing the $d5_{/2}$ single hole level leads to a decreased value of x for the same level of agreement with experimental β -decay data should be noted; this result would seem to apply with regard to energy levels (Chapter 2).

Conclusions to be drawn are

(i) the calculation is sensitive to range of potential and exchange, it being necessary to employ the longer range to obtain agreement with experiment for β -decay and energy levels,

(ii) this calculation is insensitive to the $d5_{/2}$ single hole level position.

<u>The forbidden β -decay $Cl^{38} \xrightarrow{\beta^-} A^{38}$</u>

The β -decay Cl^{38} $(2^-) \xrightarrow{\beta^-} A^{38}$ (0^+) is of the unique first forbidden type, involving a change of parity and spin change 2.

Expressions may be derived for the $f_0 t$ value of such a decay, one of which is given below (see Blatt and Weisskopf 'Theoretical Nuclear Physics' or Bohr and Mottelson Dan. Mat. Fys. Medd. 27; 16. 1953)

$$\frac{1}{f_0 t} = \left(\frac{2\pi}{9}\right)\left(\frac{\kappa}{B}\right)\left\{\frac{f_1(Z,E_0)}{f_0(Z,E_0)}\right\}\sum_{M}^{1}\left|\left(\psi'\left|\sum_{n}^{'} r^{\frac{1}{2}}\gamma_5(\sigma'\times Y')^2_m\right|\psi\right)\right|^2 \qquad 3.11$$

where x, B were given at the beginning of the chapter

Z is the number of protons in the nucleus E_0 is the total energy available for the decay (units mc^2)

$$\left\{\frac{f_1}{f_0}\right\} = 24\left[a(z)\left(E_0^2-1\right) + b(z)\left(E_0-1\right)\right]$$

is given by

Davidson (Phys. Rev. 82; 50. 1951) who also plots $a(z)$, $b(z)$ as functions of Z.

Now

$$\sum_{M(M_J')}\left|\left(T'M_T'\,J'M_J'\left|\sum_{n} r^{\frac{1}{2}}\gamma_5(\sigma'\times Y')^2_M\right|T M_T\, J M_J\right)\right|^2$$

can be reduced using the Wigner-Eckart theorem and the orthogonality of Wigner coefficients to

$$\frac{1}{2J+1}\left(T'M_T'\,J'\left\|\sum_{n}^{'} r^{\frac{1}{2}}\gamma_5(\sigma'\times Y')^2\right\|T\,M\,J\right)^2 \qquad 3.12$$

an amplitude matrix in the J space. This expression can be evaluated using suitable wavefunctions.

The technique followed here will be to substitute crude wavefunctions for Cl^{38}, A^{38} on both sides of 3.12 and add detail later.

The lowest energy odd parity configuration of Cl (J = 2) is

$$\left\{ \begin{array}{l} \left\{ (p_{1/2})^2 (h_{1/2})^2 (h_{3/2})^4 (d_{5/2})^6 (2p_{1/2})^2 (d_{3/2})^4 f_{7/2} \right\}^{\text{neutron}} \\ \left\{ (p_{1/2})^2 (h_{1/2})^2 (h_{3/2})^4 (d_{5/2})^6 (2p_{1/2})^2 d_{3/2} \right\}^{\text{proton}} \end{array} \right\}$$

which would appear from the work of Goldstein and Talmi (Phys. Rev. 102; 589. 1956) to be a good approximation to the real wavefunction.

If this is to decay as a first forbidden transition to A^{38} (0^+), the only part of the argon ground state wavefunction that can contribute is

$$\left\{ \begin{array}{l} \left\{ (p_{1/2})^2 (h_{1/2})^2 (h_{3/2})^4 (d_{5/2})^6 (2p_{1/2})^2 (d_{3/2})^4 \right\}^{\text{neutron}} \\ \left\{ (p_{1/2})^2 (h_{1/2})^2 (h_{3/2})^4 (d_{5/2})^6 (2p_{1/2})^2 (d_{3/2})^2 \right\}^{\text{proton}} \end{array} \right\}$$

Thus the nuclear states may be represented by the following simple wavefunctions in jj-coupling

$$Cl^{38} = \frac{1}{\sqrt{2}} \left\{ f_{7/2}(1) \chi_+(1) d_{3/2}(2) \chi_-(2) - f_{7/2}(2) \chi_+(2) d_{3/2}(1) \chi_-(1) \right\}_{J=2} = \psi(12)$$

$$A^{38} = \left\{ d_{3/2}(1) \chi_-(1) \, d_{3/2}(2) \chi_-(2) \right\}_{J=0} = \psi'(12)$$

where χ_+, χ_- represent wavefunctions for neutron, proton.

Defining

$$N = \left(\psi' \left\| \sum_{12} \tau \tfrac{1}{2} \tau_- (\sigma' \times Y')^2 \right\| \psi \right)$$

$$= 2 \left(\psi'(12) \left\| \left(\tau \tfrac{1}{2} \tau_- (\sigma' \times Y')^2 \right)_{(1)} \right\| \psi(12) \right) \qquad 3.13$$

where $\psi'(12)$, $\psi(12)$ are just as given above, and using the properties of τ_- : $\tau_- \chi_+ = 2 \chi_-$

$$\tau_- \chi_- = 0$$

reduces this to

$$N = 2 \left(\left\{ d_{3/2}(1) d_{3/2}(2) \right\}_0 \left\| \left(\tau (\sigma' \times Y')^2 \right)_{(1)} \right\| \frac{1}{\sqrt{2}} \left\{ f_{7/2}(1) d_{3/2}(2) \right\}_2 \right)$$

Since the operator only acts in the space of particle 1,

$$N = \frac{1}{\sqrt{2}} \left(d_{3/2} \| r \, (\sigma' \times Y')^2 \| f_{7/2} \right)$$

$$= \frac{-4}{\sqrt{105}} \left(\tfrac{1}{2} \| \sigma \| \tfrac{1}{2} \right) \left(Y^{\ell'=2} \| Y' \| Y^{\ell=3} \right) \int_0^\infty u_{\substack{n'=2 \\ \ell'=2}} r \, u_{\substack{n=3 \\ \ell=3}} r^2 \, dr$$

from further properties of the operator.

Evaluating this expression

$$\left(\tfrac{1}{2} \| \sigma \| \tfrac{1}{2} \right) = \sqrt{6}$$

$$(Y^2 \| Y^1 \| Y^3) = -\frac{1}{4} \sqrt{\frac{35}{11}}$$

$$\int_0^\infty u_{\frac{3}{2}} \, u_{\frac{3}{3}} \, r^3 dr = \sqrt{\frac{7}{2}} \quad b$$

whence $N = \sqrt{\frac{7}{11}}\, b$, where b is the oscillator well parameter.

The value of 3.12 is therefore $\frac{7}{5\pi} \, b^2$

For this decay $E_0 = 4{\cdot}9$ MeV (Endt and Braams (1957))

$= 9{\cdot}68 \; (mc^2)$

for Z = 18 $\left.\begin{aligned} a(z) &= 0{\cdot}052 \\ -b(z) &= 0{\cdot}054 \end{aligned}\right\}$ Davidson (1957)

and $b = 1{\cdot}963 \times 10^{-13}$ cm (Chapter 1)

$= 5{\cdot}103 \times 10^{-3}$ cm ($\hbar/mc$)

Substituting these numbers into equation 3.11 yields

$$\log_{10} f_0 t = 6{\cdot}77$$

<u>The detailed A^{38} wavefunction</u>

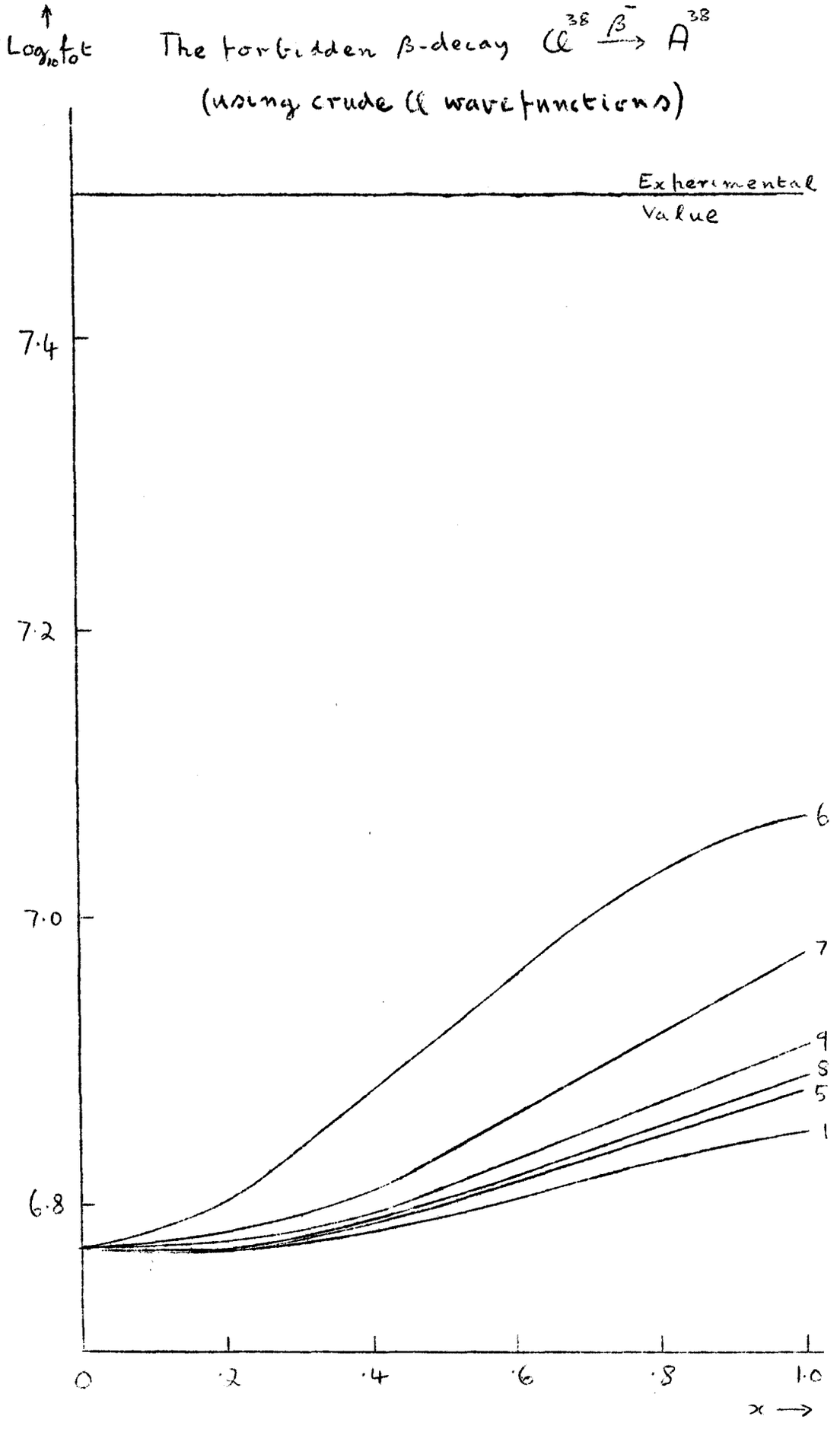
The log ft above was evaluated using a $(d3/2)_0^2$ configuration for the ground state of A^{38}.

The J = 0, T = 1 ground state wavefunction calculated for A^{38} consists of a linear combination of jj-coupled states

$$\psi' (A^{38}) = \alpha \, (d_{5/2})_0^2 + \beta \, (d_{3/2})_0^2 + \gamma \, (s_{1/2})_0^2 \qquad \underline{3.14}$$

where $\alpha^2 + \beta^2 + \gamma^2 = 1$

FIG. 12


Log10 ft
The forbidden β-decay Cl38 β⁻ → A38
(using crude Cl wavefunctions)
Experimental Value
7·4
7·2
7·0
6·8
6
7
9
8
5
1
0
·2
·4
·6
·8
1·0
x →

Since only the $(d_{3/2})^2_0$ configuration contributes to the decay it will be necessary to include a factor ($\beta < 1$ is a function of x) in 3.13, the initial expression for N carrying this factor through, the final expression becomes

$$\log_{10} f_o t = 6{\cdot}77 - 2 \log_{10} \beta (x) \qquad 3.15$$

The values of β (x) are determined by transforming the LS-coupled A^{38} wavefunctions into their jj-coupled equivalent.

Fig: 12 illustrates the results obtained for several ranges and exchanges; the curve numbering here is as in Fig: 11.

It will be seen that none of these curves predict the experimental figure. This result is disappointing in view of the previous success of the Gauss mid range force (curves 8 and 9).

One reason for this non-agreement might be that too crude an approximation has been made in the Cl^{38} wavefunctions. It was decided to repeat the calculation using perturbation theory to form more realistic wavefunctions by the mixing in of higher configurations. The next chapter (4) will be devoted to this approach and the conclusions to be drawn therefrom.

Another possibility to account for the bad agreement is that the oscillator well approximation to the nuclear potential is too crude. It seemed hardly worthwhile to replace this by the Woods-Saxon well, which in chapter 2 led to no significant change in the energy levels.

However the more drastic change of substituting an infinite square well potential was investigated.

The effect of this is to replace $N = \sqrt{\frac{7}{11}}\, b$ in the original calculation by

$$N = \frac{2 \cdot 9360}{11^{3/2}} \quad R$$

where R is the 'nuclear radius' obtained from the empirical formula quoted in Chapter 1.

Expression 3.15 is thus replaced by

$$\log_{10} f_o t = 6 \cdot 98 - 2 \log_{10} \beta(x)$$

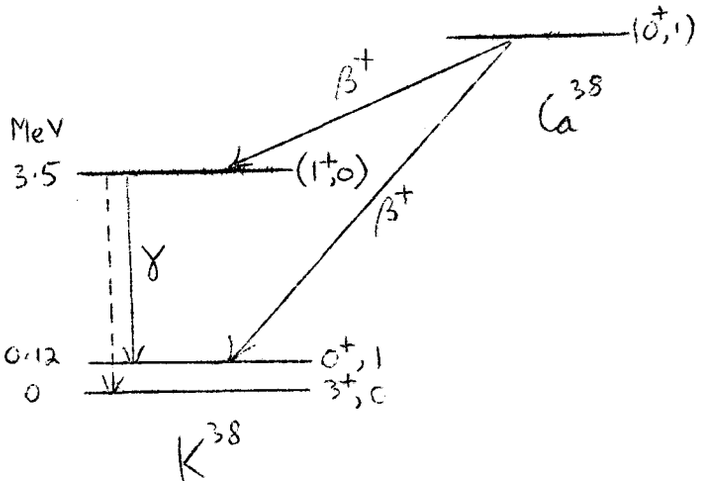
and even including the expansion of the A^{38} ground state it would not be possible to obtain good agreement with the experimental result.

β -decay of Ca^{38}

Cline and Chagnon (Phys. Rev. 108; 1495. 1957) have reported the decay of an unstable calcium isotope formed from the reaction

$$Ca^{40} (\gamma, 2n) Ca^{38}$$

The experimental data for this decay is given in the diagram below.



On the basis of the shell model Ca^{38} is a T = 1 nucleus which should have a 0^+ ground state (cf. A^{38})

Two β^+ decays were observed consistent with $\log_{10} ft = 3{\cdot}5$. One of these was associated with the superallowed transition

$$Ca^{38}\ (0^+,\ 1) \xrightarrow{\beta^+} K^{38}\ (0^+,\ 1)$$

The other apparently gave rise to a γ-ray of energy 3·5 Me V, and was interpreted as a decay from the Ca^{38} ground state to a 1^+, T = 0 level at 3·5 Me V in K^{38}.

A theoretical investigation was made as to whether this interpretation is consistent with energy levels and wavefunctions obtained in the present calculation. An alternative interpretation of the second decay, suggested by Cline and Chagnon (1957) was

$$Ca^{38}\ (0^+,\ 1) \xrightarrow{\beta^+} K^{38}\ (0^+,\ 1) \text{ at } 3{\cdot}5 \text{ Me V},$$

which would be inconsistent with these calculations as they do not predict a $(0^+,\ 1)$ level for K^{38} in this region.

A theoretical point also arises here. If there were no Coulomb force the decay would go exclusively via the transition to the lowest $(0^+,\ 1)$ level in K^{38}. The existence of a Coulomb force will admittedly mix the $(0^+,\ 1)$ states, but hardly to such an extent as to account for the second decay observed by Cline and Chagnon.

It was decided to treat the Ca^{38} ground state as $(0^+,\ 1)$ and calculate the log ft values for β^+-decay to all the predicted $(1^+,\ 0)$ levels.

The theory of such allowed transitions has already been described earlier in the chapter. The decay under consideration involves a change of isotopic spin, so only the Gamow-Teller term can contribute.

FIG. 13 (1)

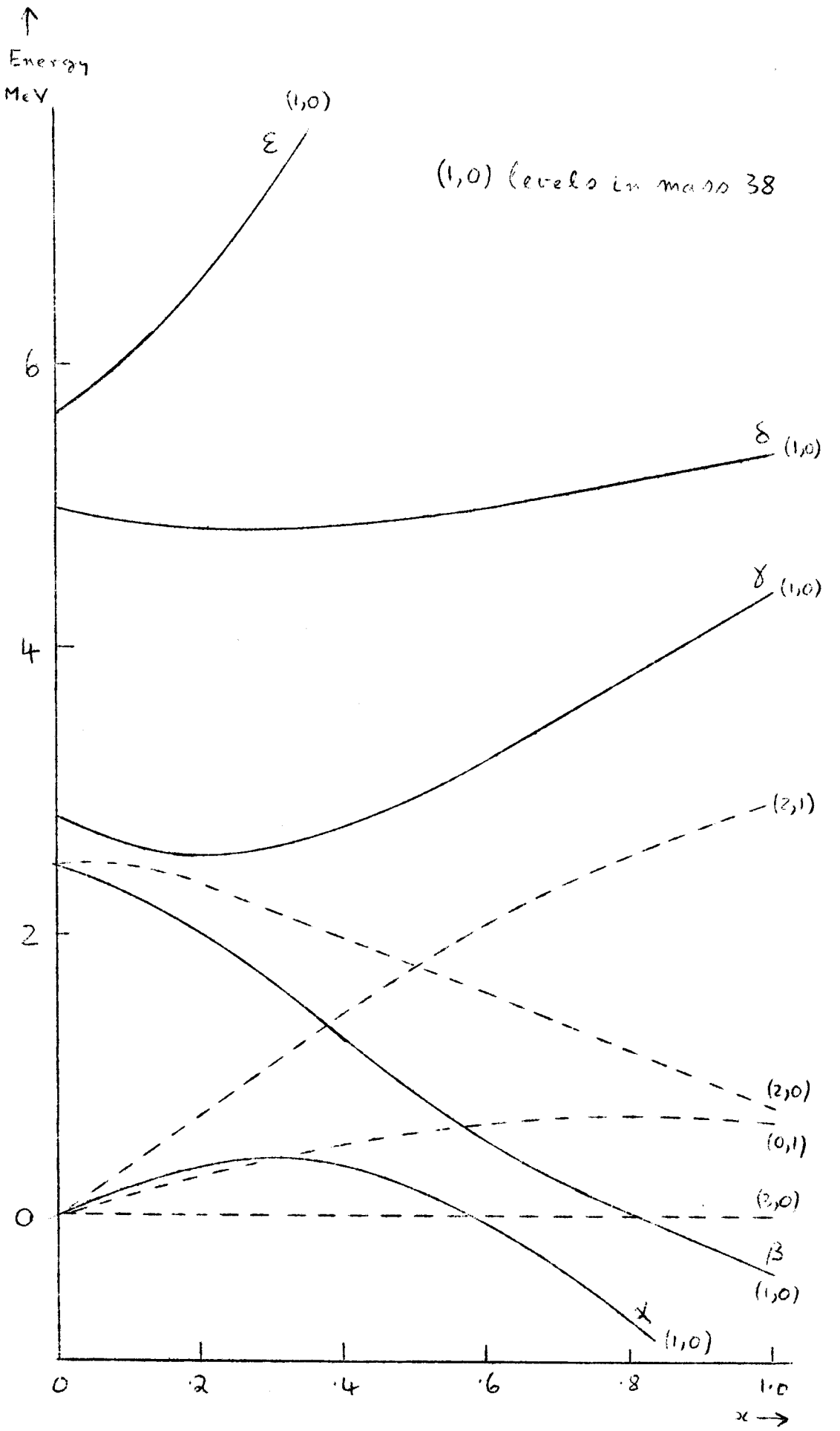
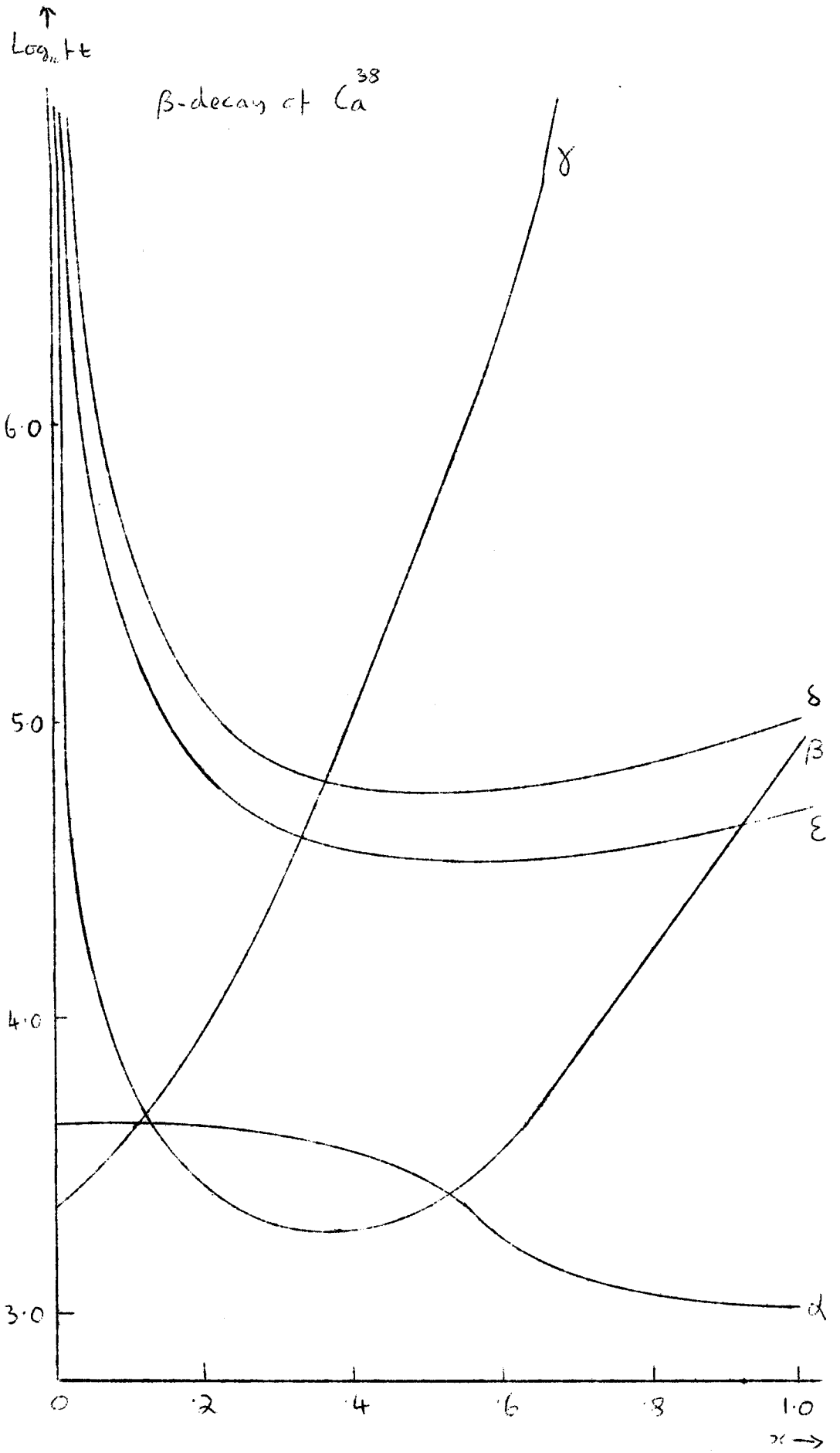
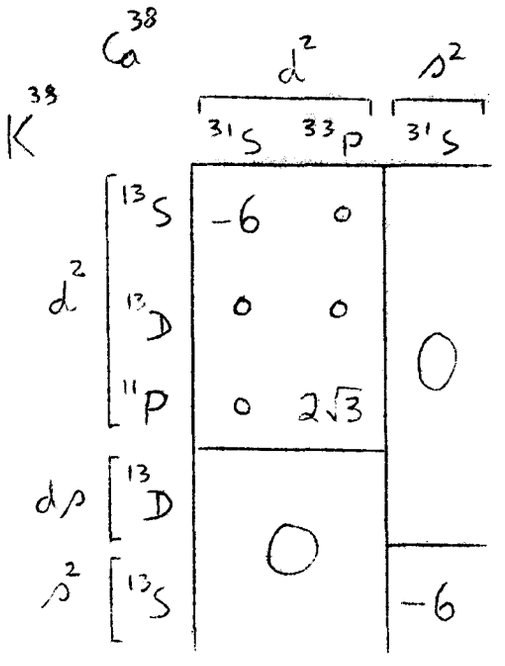

Energy
MeV
(1,0)
ε
(1,0) levels in mass 38
6
δ
(1,0)
γ
(1,0)
4
(2,1)
2
(2,0)
(0,1)
(2,0)
0
β
(1,0)
α
(1,0)
0
·2
·4
·6
·8
1·0
x →

FIG. 13 (2)



An expression for G^2 was given in 3.9, the appropriate matrix $[\emptyset]$ being illustrated below; the matrix elements were calculated as before.



In this case the value of A (3.5) is $1/6$.

The calculations were performed for the mid range Gauss curves with the $d5/2$ single hole level at both 2·80 and 4 M e V.

Fig: 13 (i) shows the position of the (1, 0) levels with a $d5/2$ hole level at 2·80 M e V, Fig: 13 (ii) shows the calculated log ft value for a decay to each of these levels.

The best spectrum was obtained here with $x \sim 0{\cdot}5$. In this case there is a $(1^+, 0)$ level at 3M e V with a log ft of 5·6. This does not agree particularly well with the experimental interpretation.

There are also two lower $(1^+, 0)$ levels with log ft $\sim 3{\cdot}4$ which were not seen by Cline and Chagnon (1957).

Since it is the higher levels that are involved in this decay, the effect of raising the $d5/2$ single hole level should lead to improvement as in Chapter 2.

FIG. 14(1)

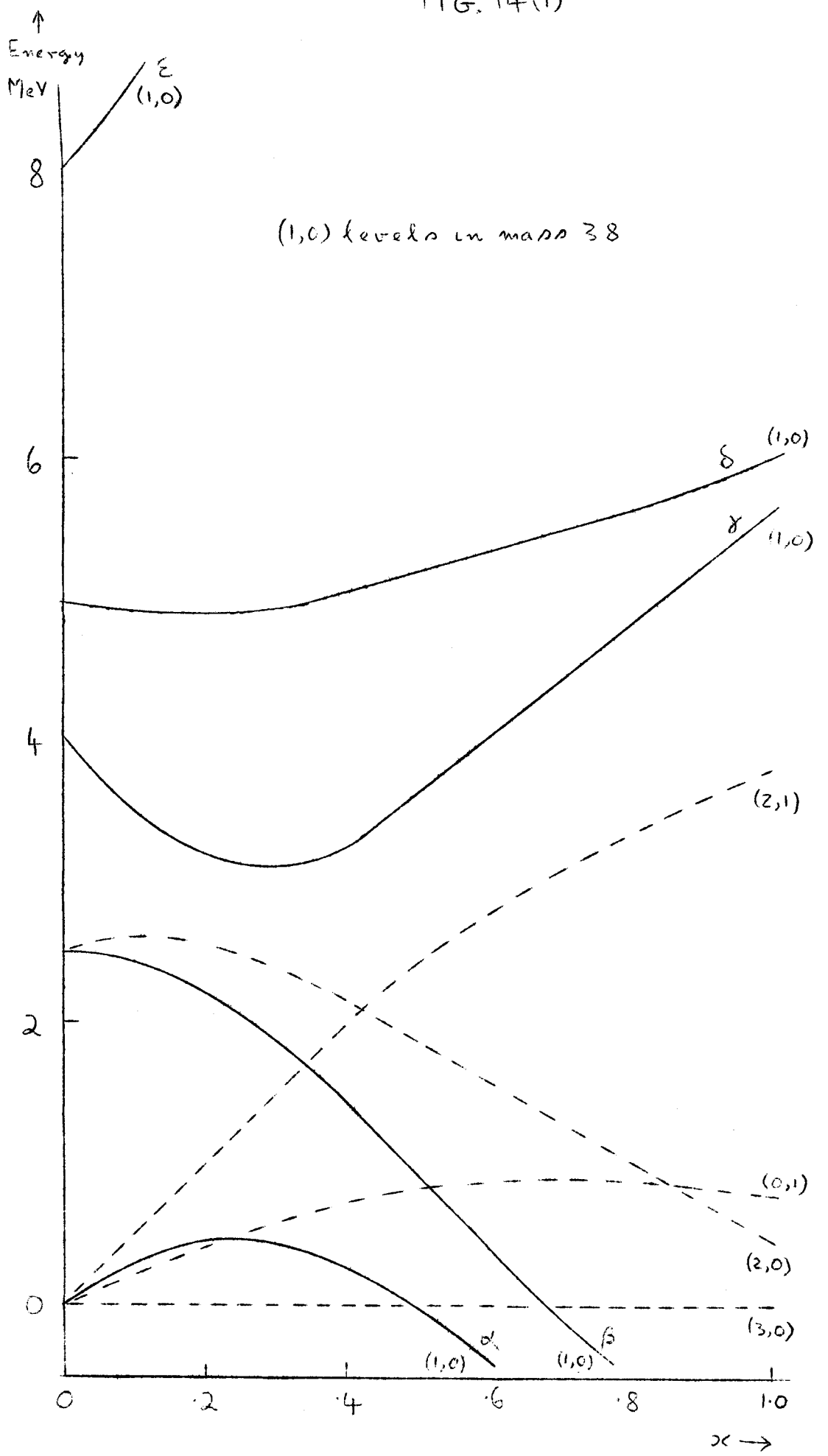

Energy
MeV
ε
(1,0)
8
(1,0) levels in mass 38
6
δ
(1,0)
γ
(1,0)
4
(2,1)
2
(0,1)
(2,0)
0
(3,0)
α
(1,0)
β
(1,0)
0
·2
·4
·6
·8
1·0
x →

FIG. 14 (2)

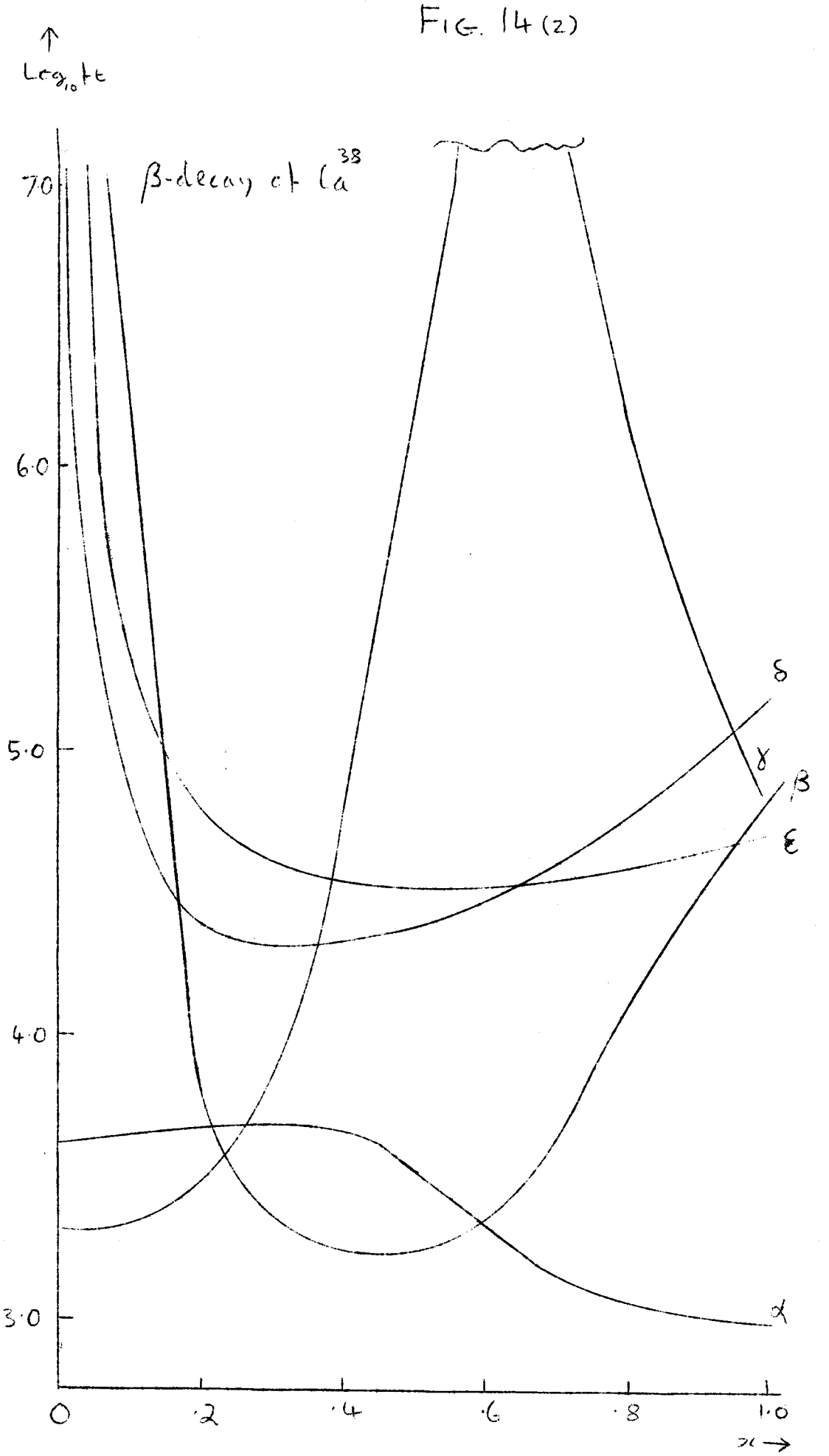


Fig: 14 (i) shows the position of the (1, 0) levels with a $d5_{/2}$ single hole level at 4MeV; Fig: 14 (ii) shows the calculated log ft value for a decay to each of these levels.

In this case for a good spectrum x should take a value 0·38. This would predict a $(1^+, 0)$ level at 3·2 MeV and a log ft of 4·4, both improvements on the Fig: 13 results from the point of view of verifying the experimental interpretation.

The two lower $(1^+, 0)$ levels now have log ft values 3·7 and 3·3 associated with them.

The conclusions to be drawn are

(1) According to the calculations a$(0^+, 1)$ state of Ca^{38} could β-decay to a a$(1^+, 0)$ level K^{38} at 3·5 MeV with a log ft value ~ 4.

(2) If it does so then this is an argument in favour of using the Gauss mid range force with a $d5_{/2}$ single hole level raised above 2·80 MeV.

(3) β-decays to the provisionally labelled $(1^+, 0)$ levels at 1·7 and 0·45 MeV in K^{38} should be observed with log ft $\sim 3·5$.

Chapter 4

The Forbidden β -Decay $Cl^{38}(2^-) \xrightarrow{\beta^-} A^{38}(0^+)$

The last chapter showed that it was not possible to explain the forbidden β -decay $Cl^{38}(2^-) \xrightarrow{\beta^-} A^{38}(0^+)$, log $f_o t = 7{\cdot}5$, with any of the detailed A^{38} wavefunctions, using a crude jj-coupled wavefunction for Cl^{38}.

The object of this chapter is to recalculate the log $f_o t$ value using more detailed wavefunctions for Cl^{38} obtained by first order perturbation theory.

Basic first order perturbation theory, (non-degenerate levels).

It is assumed that a wave equation differing from the true wave equation by the omission of small terms can be solved.

Let

$$(H - E)\psi = 0 \qquad 4.1$$

represent the true wave equation, with eigenvalue solutions E^k and eigenfunctions ψ^k.

Assume H can be written

$$H = H_o + H_1 \qquad 4.2$$

where H_1 is the perturbation correction and

$$(H_o - E_o)\psi_o = 0 \qquad 4.3$$

can be solved.

Let the solutions of 4.3 be the ψ_o^k which form a complete orthogonal set of functions normalised by

$$\int \psi_o^i \, \psi_o^j \, d\tau = \delta(ij) \qquad 4.4$$

The effect of the perturbation is to be small by hypothesis and so expanding

$$\begin{aligned} \psi^k &= \psi_0^k + \psi_1^k \\ E^k &= E_0^k + E_1^k \end{aligned} \qquad 4.5$$

where ψ_1^k, E_1^k are perturbation corrections, substitution in 4.1 yields

$$(H_0 - E_0^k)\psi_0^k + (H_0 - E_0^k)\psi_1^k + (H_1 - E_1^k)\psi_0^k + (H_1 - E_1^k)\psi_1^k = 0$$

The first term in this expression is zero from 4.3 and the last is of second order which may be neglected.

Hence $$(H_0 - E_0^k)\psi_1^k + (H_1 - E_1^k)\psi_0^k = 0 \qquad 4.6$$

Expanding ψ_1^k in terms of the ψ_0^k

$$\psi_1^k = \sum_\ell{}' a_\ell \psi_0^\ell \qquad 4.7$$

whence

$$H_0 \psi_1^k = \sum_\ell{}' a_\ell H_0 \psi_0^\ell = \sum_\ell{}' a_\ell E_0^\ell \psi_0^\ell$$

and

$$E_0^k \psi_1^k = \sum_\ell{}' a_\ell E_0^k \psi_0^\ell$$

4.6 may now be written

$$\sum_\ell{}' a_\ell (E_0^\ell - E_0^k)\psi_0^\ell = (E_1^k - H_1)\psi_0^k \qquad 4.8$$

Multiplying both sides of this equation by $\psi_0^{j\,*}$ and integrating over all space yields

$$\sum_\ell{}' a_\ell (E_0^\ell - E_0^k)\,\delta(j\ell) = E_1^k \delta(jk) - (\psi_0^j | H_1 | \psi_0^k)$$

or

$$a_j = \frac{-(\psi_0^j | H_1 | \psi_0^k)}{E_0^j - E_0^k} \quad \text{if } j \neq k \qquad 4.9$$

a_k is generally chosen to normalise the resultant ψ and in first order theory its value is zero.

From 4.2

$$H_1 = H - H_0$$

thus 4.9 can be rewritten as

$$a_j = -\frac{(\psi_0^j | H - H_0 | \psi_0^k)}{E_0^j - E_0^k} = -\frac{(\psi_0^j | H | \psi_0^k)}{E_0^j - E_0^k} + \frac{(\psi_0^j | H_0 | \psi_0^k)}{E_0^j - E_0^k}$$

or

$$a_j = -\frac{(\psi_0^j | H | \psi_0^k)}{E_0^j - E_0^k} \qquad 4.10$$

from 4.3 and 4.4 remembering that 4.9 holds only provided $j \neq k$.

Expansion of the nuclear wavefunction

Consider the transition from an initial nuclear state ϕ to a final nuclear state ψ. Suppose that these states may be expanded in terms of complete sets of states ϕ_j, ψ_i but that the transition is mainly goverened by the leading states ϕ_0, ψ_0

ie.
$$\phi = \sqrt{1 - \sum_j \eta_j^2}\,\phi_0 + \sum_j \eta_j \phi_j$$
$$\psi = \sqrt{1 - \sum_i \varepsilon_i^2}\,\psi_0 + \sum_i \varepsilon_i \psi_i \qquad 4.11$$

where the square root factors have been included to give normalisation.

The terms $\sum_j \eta_j \phi_j$, $\sum_i \varepsilon_i \psi_i$ can be considered as perturbations after the fashion of 4.7 and the perturbation coefficients will be given by 4.10

$$\eta_j = -\frac{(\phi_j | H | \phi_0)}{E_j - E_0} \qquad 4.12$$

etc.

	ϕ_0		ϕ_1		ϕ_2		ϕ_3		ϕ_4		ϕ_5		ϕ_6		ϕ_7		ϕ_8				ψ_0		ψ_1		ψ_2	
	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h	n	h			n	h	n	h	n	h
$f_{5/2}$							1										1									
$h_{1/2}$					1										1											
$h_{3/2}$			1						1				1													
$f_{7/2}$	1										1															
$d_{3/2}$		1		1		1		1		2		2		2		2		2				2				
$p_{1/2}$										1														0		
$d_{5/2}$												5		5		5		5								4
$h_{1/2}$																										
$h_{3/2}$																										
$p_{1/2}$																										

ϕ_j are odd parity states with $J=2$

ψ_i " even " " " $J=0$

Diagonal lines denote closed shells.

FIG. 15

Once the ϕ_j have been chosen, the η_j can be evaluated if the $E_j - E_0$ are known. The latter quantities are difficult to find theoretically, but to a first approximation they may be determined from single particle energy levels.

Applying this to the forbidden decay $Cl^{38} \longrightarrow A^{38}$ let ϕ, ψ be the Cl, A wavefunctions and ϕ_0, ψ_0 the crude approximations used in Chapter 3 (page 50). Denoting the decay operator by β, the nuclear matrix element is obtained from

$$(\psi(A^{38}) \| \beta \| \phi(Cl^{38}))$$
$$= \sqrt{1-\Sigma_i \eta_i^2}\sqrt{1-\Sigma_i \varepsilon_i^2}\,(\psi_0 \| \beta \| \phi_0) + \sum_i{}' \varepsilon_i \sqrt{1-\Sigma_i \eta_i^2}\,(\psi_i \| \beta \| \phi_0)$$
$$+ \sum_j{}' \eta_j \sqrt{1-\Sigma_i \varepsilon_i^2}\,(\psi_0 \| \beta \| \phi_j) + \sum_i{}' \sum_j{}' \varepsilon_i \eta_j (\psi_i \| \beta \| \phi_j) \qquad 4.13$$

using 4.11 .

The possible ϕ_j and ψ_i which can contribute are listed in fig: 15. All the states with one oscillator quantum of excitation ($\hbar\omega$) have been considered.

(i) The last term in 4.13 is of second order and so may be neglected.

(ii) The second term in 4.13 is the coupling of ψ_i with ϕ_0 via the β operator. This is a single body operator and since ψ_i (i = 1, 2) has $(d^{\hbar}_{3/2})^4$ and ϕ_0 has only $(d^{\hbar}_{3/2})^1$, the operator cannot couple these states.

ie. $(\psi_i \| \beta \| \phi_0) = 0$

(iii) Consider the term $(\psi_0 \| \beta \| \phi_j)$. States similar to ϕ_4 in which the extra neutron is in an f orbit cannot contribute because the operator would have to change f^n into s^p and it can only change angular momentum by one unit. The state similar to ϕ_4 with the excess neutron in the $p_{1/2}$ state is not considered because it has no final J value of 2.

From (i), (ii) above 4.13 can be written

$$(\psi(A^{38}) \| \beta \| \phi(Cl^{38}))$$
$$= \sqrt{1-\Sigma_1 \varepsilon^2}\left[\sqrt{1-\Sigma_1 \eta^2}\,(\psi_0 \| \beta \| \phi_0) + \sum_j{}' \eta_j (\psi_0 \| \beta \| \phi_j)\right] \qquad 4.14$$

Now $\sqrt{1-\Sigma_1 \varepsilon^2}$ represents the coefficient of the leading state in the jj-expansion of the A^{38} wavefunction, which is effectively

$$(d^n_{3/2})^2_0$$

and has coefficient $\beta(x)$ defined in 3.14

Thus

$$(\psi(A^{38}) \| \beta \| \phi(Cl^{38}))$$
$$= \beta(x)\left[\sqrt{1-\Sigma_1 \eta^2}\,(\psi_0 \| \beta \| \phi_0) + \sum_j{}' \eta_j (\psi_0 \| \beta \| \phi_j)\right] \qquad 4.15$$

The term $(\psi_0 \| \beta \| \phi_0)$ is just that which was considered in the last chapter. It remains to find the nuclear matrix elements for the excitation coupling terms and to determine the η_j by perturbation theory.

The terms $(\psi_0 \| \beta \| \phi_\gamma)$

Throughout this section reference will be made to the corresponding section in Chapter 3 (pages 48 to 51) where $(\psi_0 \| \beta \| \phi_0)$ was calculated.

In evaluating these nuclear matrix elements the ϕ_j can be divided into two groups, (i) j = 1 to 4
(ii) j = 5 to 8

The first group of ϕ_j

The Cl^{38} wavefunction ϕ_j may be written as

$$\frac{1}{\sqrt{2}} \left\{ l_{1_{j_1}}(1)\, \chi_+(1)\, l_{2_{j_2}}(2)\, \chi_-(2) - l_{1_{j_1}}(2)\, \chi_+(2)\, l_{2_{j_2}}(1)\, \chi_-(1) \right\}_{J=2}$$

where $l_{1_{j_1}} = h_{3/2}, h_{1/2}$, or $f_{5/2}$

$l_{2_{j_2}} = d_{3/2}$ or $s_{1/2}$

Nb. In ϕ_4 the $(d^{n}_{3/2})^2$ must be coupled to spin zero as the β operator will not affect these particles and they are coupled to spin zero in ψ_0

The A^{38} wavefunction ψ_0 may be written

$$\left\{ l_j(1)\, X_-(1)\, l_j(2)\, X_-(2) \right\}_{J=0}$$

where $l_j = d_{3/2}$ or $s_{1/2}$

Then as in 3.13

$$N(\phi_j) = (\psi_0 \| \sum_{12}' \tau_{1/2} Y_- (\sigma' \times Y')^2 \| \phi_\gamma)$$

$$= 2 (\psi_0 \| (\tau_{1/2} Y_- (\sigma' \times Y')^2)(1) \| \phi_1)$$

Using the detailed wavefunctions and the properties of τ this may be reduced to

$$N(\phi_j) = 2\left(\{l_j^{(1)} l_j^{(2)}\}_0 \,\|\, (r(\sigma' \times Y')^2)^{(1)} \,\|\, \tfrac{1}{\sqrt{2}}\{l_1 j_1^{(1)} l_2 j_2^{(2)}\}_2\right)$$

Since the operator only acts in the space of particle 1

$$N(\phi_j) = \delta(l_j, l_2 j_2)\sqrt{2}\,\frac{(-)^{3j+j_1}}{[2j+1]^{1/2}}\left(l_j \| r(\sigma' \times Y')^2 \| l_1 j_1\right) \qquad 4.16$$

Consider

$$(l_j \| r(\sigma' \times Y')^2 \| l_1 j_1) = ((\tfrac{1}{2}l)_j \| r(\sigma' \times Y')^2 \| (\tfrac{1}{2}l_1)_{j_1}) \qquad 4.17$$

$$= \sqrt{5}\,[(2j+1)(2j_1+1)]^{1/2} \begin{Bmatrix} \tfrac{1}{2} & l & \lambda \\ \tfrac{1}{2} & l_1 & j_1 \\ \infty & 1 & 2 \end{Bmatrix} (\tfrac{1}{2}\|\sigma\|\tfrac{1}{2})(l\|Y'\|l_1)R(ll_1)$$

where $$R(ll_1) = \int_0^\infty u_{nl}\, r\, u_{n_1 l_1}\, r^2\, dr$$

using the Racah algebra.

These quantities can all be evaluated

$$(\tfrac{1}{2}\|\sigma\|\tfrac{1}{2}) = \sqrt{6}$$

$$(Y^l \| Y' \| Y^{l_1}) = \left[\frac{3(2l_1+1)}{4\pi}\right]^{1/2} \begin{matrix} l\,\tfrac{1}{2} \\ C \\ l_1\,\tfrac{1}{2}\,0 \end{matrix}$$

$$R(21) = b \qquad R(23) = \sqrt{\tfrac{7}{2}}\,b \qquad R(01) = \sqrt{\tfrac{5}{2}}\,b$$

using oscillator well wavefunctions.

Thus $N(\phi_j)$ can be found from 4.16 and 4.17 where the l_j etc. are given by

State	l_j	$l_1\ j_1$	$l_2\ j_2$ = l_j from 4.16
ϕ_1	$d^3/_2$	$p^3/_2$	
ϕ_2	$d^3/_2$	$p^1/_2$	
ϕ_3	$d^3/_2$	$f^5/_2$	
ϕ_4	$s^1/_2$	$p^3/_2$	

For example

$$N(\phi_1) = \sqrt{2}\ \frac{(-)^{9/2+3/2}}{\left[2.3/2+1\right]^{1/2}} \sqrt{5}\left[(2.3/2+1)(2\,3/2+1)\right]^{1/2} \begin{Bmatrix} 1/2 & 2 & 3/2 \\ 1/2 & 1 & 3/2 \\ 1 & 1 & 2 \end{Bmatrix}$$

$$\times \sqrt{6}\ \left[\frac{3(2.1+1)}{4\pi}\right]^{1/2} \overset{2\,1/2}{C}_{1\,1/2\,1\,0}\ b$$

$$= -1/4\sqrt{\frac{3}{\pi}}\ b$$

Similarly

$$N(\phi_2) = \frac{1}{8} \times \sqrt{\frac{3}{2\pi}} \times b$$

$$N(\phi_3) = -\frac{1}{4} \times \sqrt{\frac{21}{2\pi}} \times b$$

$$N(\phi_4) = \frac{5}{4} \times \sqrt{\frac{3}{\pi}} \times b$$

<u>The second group of ϕ_j</u>

The Cl^{38} wavefunction ϕ_j may now be written as an anti-symmetric version of

$$\left\{ l_j\ X_+\ (d5/2)^5_{5/2}\ (X_-)^5 \right\}_2$$

where l_j = $f^7/_2$, $p^3/_2$, $p^1/_2$, or $f^5/_2$

Suppose it is desired to form an antisymmetric state from wavefunctions

$$\psi\ (\ 1,\ \ldots\ldots,m)$$
$$\psi^1(\ m+1,\ \ldots\ldots,m+n\ )$$

This is done by writing

$$\sqrt{\frac{m!\ n!}{(m+n)\ !}}\ \sum_{P}(-)^{h}P\psi\ (1,\ldots\ldots,m)\psi^1(m+1,\ldots,m+n) \qquad \underline{4.18}$$

where P is the permutation operator of parity p and includes only those permutations which preserve natural order in each group.

Thus the antisymmetric wavefunction for Cl^{38} may be written

$$\frac{1}{\sqrt{6}}\left\{\begin{array}{l} l_y X_+(1)\, d_{5/2} X_-(23456) - l_y X_+(2)\, d_{5/2} X_-(13456) \\ + l_y X_+(3)\, d_{5/2} X_-(12456) - l_y X_+(4)\, d_{5/2} X_-(12356) \\ + l_y X_+(5)\, d_{5/2} X(12346) - l_y X_+(6)\, d_{5/2} X_-(12345) \end{array}\right\}_{J=2}$$

The corresponding A^{38} wavefunction ψ_0 is now written

$$\left\{d_{5/2}\, X_-(12345)\, d_{5/2}\, X_-(6)\right\}_{J=0}$$

Then

$$N(\phi j) = \left(\psi_0 \left\| \sum_n r^{1/2} Y_- (\sigma' \times Y')^2 \right\| \phi_y\right)$$
$$= 6\left(\psi_0 \left\| \left(r^{1/2} Y_- (\sigma' \times Y')^2\right)(6) \right\| \phi_y\right)$$

Substituting the detailed wavefunctions, using the fact that the operator only acts on particle 6, removing the τ dependence, yields

$$N(\phi j) = 6\left(\left\{\left(d_{5/2}\right)^5_{5/2}\, d_{5/2}(6)\right\}_0 \left\|\left(r(\sigma' \times Y')^2\right)(6)\right\| \frac{-1}{\sqrt{6}} \left\{\ell_j(6)\left(d_{5/2}\right)^5_{5/2}\right\}_2\right)$$

which is evaluated just as in the last section giving the following expressions to be compared with 4.16 and 4.17

$$N(\phi j) = (-)^{j+3/2}\left(d_{5/2} \| r(\sigma' \times Y')^2 \| \ell_j\right) \qquad 4.19$$

$$\left(d_{5/2} \| r(\sigma' \times Y')^2 \| \ell_j\right) = \sqrt{30}\,[2j+1]^{1/2} \begin{Bmatrix} 1/2 & 2 & 5/2 \\ 1/2 & \ell & j \\ 1 & 1 & 2 \end{Bmatrix} \times \left(\tfrac{1}{2}\|\sigma\|\tfrac{1}{2}\right)(2\|Y'\|\ell)\, R(2\ell) \qquad 4.20$$

where all these quantities have been discussed before.

The $N(\phi j)$ can be evaluated where the lj are given by

State	lj	$N(\phi j)$
ϕ_5	$f_{7/2}$	$\frac{3}{2}\sqrt{\frac{7}{2\pi}} \times b$
ϕ_6	$p_{3/2}$	$-\frac{3}{4}\sqrt{\frac{7}{2\pi}} \times b$
ϕ_7	$p_{1/2}$	$\frac{3}{2}\sqrt{\frac{1}{\pi}} \times b$
ϕ_8	$f_{5/2}$	$-\sqrt{\frac{7}{3\pi}} \times b$

Evaluation of the η_j

An expression for the η_j has been given in 4.12. In order to evaluate this the matrix elements $(\phi_j \mid H \mid \phi_o)$ are required. These could be found by expressing the ϕ in either jj- or LS-coupling. Here it was decided to

employ the latter approach.

Writing $$H = H_{\text{spin-orbit}} + H_{\text{central}}$$

then since LS-coupling is to be used H can be replaced by H_{central} as $H_{\text{spin-orbit}}$ cannot couple different configurations.

The numerical evaluation of two-body central force matrix elements has been simplified using the 'Tables of Transformation Brackets' published by Brody and Moshinsky (Monografias del Instituto de Fisica Mexico 1960.

The working out of $(\phi_j | H | \phi_0)$ is a lengthy process of which only a bare outline is possible here. The ϕ_j are expressed as jj-coupled wavefunctions. They have to be transformed into their LS-coupled equivalent and reduced to two-body form. The calculation of $(\phi_1 | H | \phi_0)$ will be followed through in a little detail, although results for the rest must just be quoted.

Evaluation of $(\phi_1 | H | \phi_0)$

ϕ_1, ϕ_0 have been written (Chapter 3, page 50 , Chapter 4, page 62)

$$\phi_1 = \left\{ \Phi_0 \left(h^n_{3/2} \, d^h_{3/2} \right)_2 \right\}_2$$

$$\phi_0 = \left\{ \Phi_0 \left(h^n_{3/2} \, d^h_{3/2} \right)_2 \right\}_2$$

where both these functions are taken to be properly anti-symmetrised and Φ_0 represents a group of closed shells common to each wavefunction.

The first step is to reduce $(\phi_1 | H | \phi_0)$ to two-body form and this is done as follows.

Consider

$$(\underline{\Phi}\,\psi' | H | \underline{\Phi}\,\psi)$$

where in this case $\psi' = (h^n_{3/2}\, d^h_{3/2})$; $\psi = (f^n_{7/2}\, d^h_{3/2})$

Assuming $\underline{\Phi}$, ψ', ψ are individually antisymmetric the final states may be antisymmetrised by (see 4.18)

$$\sqrt{\frac{(n-2)!\,2!}{n!}} \sum_P (-)^h P\, \underline{\Phi}(1,\ldots,n-2)\, \psi'(n-1,n)$$

and similarly for the $\underline{\Phi}\,\psi$ state.

The matrix element is therefore

$$\frac{(n-2)!\,2!}{n!} \left(\sum_P (-)^h P \underline{\Phi}(1,..,n-2)\, \psi'(n-1,n) \middle| \sum_{i<j} H_{ij} \middle| \sum_P (-)^h P\, \underline{\Phi}(1,..,n-2)\, \psi(n-1,n) \right)$$

$$= \frac{2}{n(n-1)} \left(\sum_P (-)^h P\, \underline{\Phi}(\,)\, \psi'(n-1,n) \middle| \frac{n(n-1)}{2} H_{(n-1)n} \middle| \sum_P (-)^h P \underline{\Phi}(\,)\, \psi(n-1,n) \right)$$

Expanding the wavefunction on L.H.S. gives

$\underline{\Phi}(1,\ldots,n-2)\ \psi'(n-1,n)$ + [terms in which one particle in the last pair is in $\underline{\Phi}$, the other in ψ'] + [terms in which both of the last pair are $\underline{\Phi}$].

and similarly for R.H.S.

The operator $H_{(n-1)n}$ acts only on the last pair of particles.

(i) If both of these are in $\underline{\Phi}$ on the left, consider the states of particles not involved. One of them is in a p

state on the left and not on the right. Hence by orthogonality all terms of this sort vanish.

(ii) If one of the involved particles is in Φ and the other in ψ' on the left, there are two possibilities:

(a) the one in ψ' is in the d state. In this case an uninvolved particle will be in the p state similar to (i) above and these terms vanish as before.

(b) the one in ψ' is in the p state. Therefore the d state is occupied by an uninvolved particle; there are $n-3$ uninvolved particles in Φ on the left, so there must be $n-3$ in Φ on the right, with one in the d state, leaving one involved particle in Φ and the other in the f state.

$$\text{L.H.S.}\left\{\Phi\begin{pmatrix}n-3 & \text{uninvolved}\\ 1 & \text{involved}\end{pmatrix}_0 \psi'\left(p_{3/2}(\text{involved})\; d_{3/2}(\text{uninvolved})\right)_2\right\}_2$$

$$\text{R.H.S.}\left\{\Phi\begin{pmatrix}n-3 & \text{uninvolved}\\ 1 & \text{involved}\end{pmatrix}_0 \psi\left(f_{7/2}(\text{involved})\; d_{3/2}(\text{uninvolved})\right)_2\right\}_2$$

Changing the order of coupling of angular momenta this can be written (apart from constants)

$$\text{L.H.S.}\left\{\left(\Phi_0\, p_{3/2}(\text{involved})\right)_{3/2}\quad d_{3/2}(\text{uninvolved})\right\}_2$$

$$\text{R.H.S.}\left\{\left(\Phi_0\, f_{7/2}(\text{involved})\right)_{7/2}\quad d_{3/2}(\text{uninvolved})\right\}_2$$

These are of the form $\{ \underline{\Psi}'_{3/2} d_{3/2}(\text{uninvolved}) \}_2$

$\{ \underline{\Psi}_{1/2} d_{3/2}(\text{uninvolved}) \}_2$

and the matrix element becomes

$$\left(\{ \underline{\Psi}'_{3/2} d_{3/2} \}_2 \mid H_{(n-1)n} \mid \{ \underline{\Psi}_{1/2} d_{3/2} \}_2 \right)$$

and using the fact that $H_{(n-1)n}$ does not operate on $d_{3/2}$, this may be extracted to leave a term

$$\left(\underline{\Psi}'_{3/2} \mid H_{(n-1)n} \mid \underline{\Psi}_{1/2} \right)$$

which is zero, because H is a scalar and cannot couple different values of J.

(iii) The only other possibility is that both involved particles are in ψ' and ψ, leaving uninvolved ones in $\underline{\phi}$.

This argument, which is of a type used frequently in this work, has reduced $(\phi_1 | H | \phi_0)$ to two-body form

$$(\phi_1 | H | \phi_0) = \left((j^n_{3/2} \alpha^h_{3/2})_2 \mid H_{12} \mid (j^n_{1/2} \alpha^h_{3/2})_2 \right) \qquad \underline{4.21}$$

where the two-body states are taken to be antisymmetric.

Now the transition to LS-coupling must be made.

Two particles with intrinsic spin $\frac{1}{2}$ can couple to S=0, 1

Since J=2

(a) S=0, L must be 2, gives LS-state ^1D

(b) S=1, L=1, 2, or 3, gives LS-states ^3P, ^3D, or ^3F.

Rewriting 4.21 as

$$(\phi_1 | H | \phi_0) =$$

$$\left(\frac{1}{\sqrt{2}}\left\{h^n_{3/2}(1)\, d^h_{3/2}(2) - h^n_{3/2}(2)\, d^h_{3/2}(1)\right\}_2 \left|H_{12}\right| \frac{1}{\sqrt{2}}\left\{f^n_{7/2}(1)\, d^h_{3/2}(2) - f^n_{7/2}(2)\, d^h_{3/2}(1)\right\}_2\right) \qquad 4.22$$

Then

$$\left(h^n_{3/2}(1)\, d^h_{3/2}(2)\right)_2 = \left(h_{3/2}(1)\, d_{3/2}(2)\right)_2 X_+(1)\, X_-(2)$$

$$= \left(h_{3/2}(1)\, d_{3/2}(2)\right)_2 \sum_T C^{TO}_{\frac{1}{2}\frac{1}{2}\frac{1}{2}-\frac{1}{2}} \psi(12TO)$$

etc. , and since T takes only the values 0, 1 the sums over T may be easily performed.

Furthermore

$$\left(h_{3/2}(1)\, d_{3/2}(2)\right)_2 =$$

$$\sum_{S,L} \left[(2\tfrac{3}{2}+1)(2\tfrac{3}{2}+1)(2S+1)(2L+1)\right]^{1/2} \begin{Bmatrix} \frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{2} & 2 & \frac{3}{2} \\ S & L & 2 \end{Bmatrix} \left(\phi^S(12)\, \phi^L\left(h(1)\, d(2)\right)\right)_{J=2}$$

etc. Using the table below and substituting espressions of the type shown above into 4.22 yields 4.23

Value of T	Value of S	Symmetry of L
0	0	[11]
0	1	[2]
1	0	[2]
1	1	[11]

$$(\phi_1 | H | \phi_0) = \tfrac{1}{2} \times$$

$$\left[\left(\alpha^{\,11}_{L=2}[11]\, D_2 + \sum_L \beta_L^{\,13}[2]\, L_2 \,\middle|\, H_{12} \,\middle|\, \alpha'^{\,11}_{L=2}[11]\, D_2 + \sum_L \beta_L'^{\,13}[2]\, L_2 \right) + \left(\alpha^{\,31}_{L=2}[2]\, D_2 + \sum_L \beta_L^{\,33}[11]\, L_2 \,\middle|\, H_{12} \,\middle|\, \alpha'^{\,31}_{L=2}[2]\, D_2 + \sum_L \beta_L'^{\,33}[11]\, L_2 \right) \right] \qquad 4.23$$

where

$$\alpha_L = \left[(2.\tfrac{3}{2}+1)(2.\tfrac{3}{2}+1)(2.0+1)(2L+1)\right]^{1/2} \begin{Bmatrix} \frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{2} & 2 & \frac{3}{2} \\ 0 & L & 2 \end{Bmatrix}$$

$$\alpha'_L = \left[(2.\tfrac{3}{2}+1)(2.\tfrac{3}{2}+1)(2.0+1)(2L+1)\right]^{1/2} \begin{Bmatrix} \frac{1}{2} & 3 & \frac{3}{2} \\ \frac{1}{2} & 2 & \frac{3}{2} \\ 0 & L & 2 \end{Bmatrix}$$

$$\beta_L = \left[(2\tfrac{3}{2}+1)(2.\tfrac{3}{2}+1)(2.1+1)(2L+1)\right]^{1/2} \begin{Bmatrix} \frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{2} & 2 & \frac{3}{2} \\ 1 & L & 2 \end{Bmatrix}$$

$$\beta'_L = \left[(2.\tfrac{7}{2}+1)(2.\tfrac{3}{2}+1)(2.1+1)(2L+1)\right]^{1/2} \begin{Bmatrix} \frac{1}{2} & 3 & \frac{7}{2} \\ \frac{1}{2} & 2 & \frac{3}{2} \\ 1 & L & 2 \end{Bmatrix}$$

and L = 1, 2, or 3.

In matrix language this may be rewritten as $\frac{1}{2}$ x sum of four following terms:

(1) $$[\alpha_2] \times \overbrace{hd}^{4d} \left\{ {}^{11}_{[11]} D_2 \left[{}^{11}E_2 \right] \times [\alpha'_2] \right\}$$

(2) $$\left[\beta_1 \beta_2 \beta_3\right] \times \overbrace{hd}^{4d} \left\{ \begin{matrix} {}^{13}_{[2]} P_2 \\ {}^{13}_{[2]} D_2 \\ {}^{13}_{[2]} F_2 \end{matrix} \right. \overbrace{\begin{bmatrix} {}^{13}E_1 & 0 & \\ & {}^{13}E_2 & \\ 0 & & {}^{13}E_3 \end{bmatrix}}^{4d} \times \begin{bmatrix} \beta'_1 \\ \beta'_2 \\ \beta'_3 \end{bmatrix} \Bigg\} \; T=0$$

(3) $$[\alpha_2] \times \overbrace{hd}^{4d} \left\{ {}^{31}_{[2]} D_2 \left[{}^{31}E_2 \right] \times [\alpha'_2] \right\}$$

(4) $$\left[\beta_1 \beta_2 \beta_3\right] \times \overbrace{hd}^{4d} \left\{ \begin{matrix} {}^{33}_{[11]} P_2 \\ {}^{33}_{[11]} D_2 \\ {}^{33}_{[11]} F_2 \end{matrix} \right. \overbrace{\begin{bmatrix} {}^{33}E_1 & 0 & \\ & {}^{33}E_2 & \\ 0 & & {}^{33}E_3 \end{bmatrix}}^{4d} \times \begin{bmatrix} \beta'_1 \\ \beta'_2 \\ \beta'_3 \end{bmatrix} \Bigg\} \; T=1$$

where $(2T + 1)(2S + 1)\, E_L$ (pd:fd) are the two-body central force matrix elements.

It is the E's which are evaluated using Brody and Moshinsky tables.

Consider a typical E,

$$^{11}E_2(hd:hd) = \left(hd\{^{11}_{[11]}D_2 \mid H_{central} \mid hd\{^{11}_{[11]}D_2\right)$$

$$= \left\{(hd\,2M \mid V_{12} \mid hd\,2M) + (-)^{1+3+2-2}(hd\,2M \mid V_{12} \mid dh\,2M)\right\}A^{11}$$

from 1.4

Brody and Moshinsky label the oscillator well levels by

$$^1p\ 0d;\ 0f\ 0d$$

and the matrix elements by

$$(n_1l_1,\ n_2l_2,\ \lambda\mu \mid V(r) \mid n_1'l_1',\ n_2'l_2',\ \lambda\mu) \qquad \underline{4.24}$$

and so $^{11}E_2$ (pd:fd) is given in terms of

$$(11,\ 02,\ 2\mu \mid V(r) \mid 03,\ 02,\ 2\mu)$$

and $\quad (11,\ 02,\ 2\mu \mid V(r) \mid 02,\ 03,\ 2\mu)$

<u>General evaluation of 4.24</u>

(i) Find $\rho = 2n_1 + l_1 + 2n_2 + l_2$ and note the value of λ, both of which should be the same for each side.

(ii) Use table III (p. 129) to find possible values of p. This table gives the values of nl NL and p for a particular λ and ρ.

A table is then drawn up as below,

nl NL	$\langle 1 \rangle\langle 1 \rangle$	0	1	2	3
abcd					
sum	α	A	B	C	D

Table III of Brody and Moshinsky will indicate the positions of numbers to be entered in the table above whose rows are labelled by nl NL and whose columns are labelled by p.

(iii) The second column is filled with products of transformation brackets $\langle 1 \rangle$. These are given in Table I of Brody and Moshinsky,

$$1st \quad \langle 1 \rangle = f\,(l_1 l_2 \lambda\,;\ nl\ NL\ ;\ \rho)_{n_1,\, n_2}$$

$$2nd \quad \langle 1 \rangle = f\,(l_1' l_2' \lambda\,;\ nl\ NL\ ;\ \rho)_{n_1',\, n_2'}$$

Nb. It may be necessary to use the relation

$$\langle 1 \rangle = f(l_1 l_2\ \lambda\ ;\ nl\ NL; \rho)_{n_1,\, n_2} = (-)^{l-\lambda} f(l_2 l_1 \lambda\,;\ nl\ NL; \rho)_{n_2 n_1}$$

(iv) The element of the i^{th} row and p^{th} column of the table above is given by the product of the number in the corresponding row of the second column and a factor which depends on row and column.

This factor is given in Table II of Brody and Moshinsky and is listed as B (nl, nl, p) .

(v) The final value of the matrix element is

$$A\,I_0 + B\,I_1 + C\,I_2 + D\,I_3 + \ldots .$$

where A, B etc. are the vertical sums of the columns labelled by p and I_p are radial integrals.

There are two checks

(a) Sum of elements in second column = α = 0

(b) Horizontal sum A + B + C + D + ... = 0

Example (11, 02, 2 μ | V(r) | 03, 02, 2 μ)

$\rho = 5$, $\lambda = 2$ both sides

nl NL	⟨1⟩ ⟨1⟩	1	2	3	4
0 1 1 2	0·18708287	0·18708287			
0 2 0 3	-0·07483314		-0·07483314		
0 2 1 1	-0·11224972		-0·11224972		
0 3 0 2	-0·07483314			-0·07483314	
1 1 0 2	-0·11224972	-0·28062430	0·56124860	-0·39287402	
1 2 0 1	0·18708288		0·65479008	-1·30958016	0·84187296
	0	-0·09354143	1·02895582	-1·77728732	0·84187296

Hence

$$(\,|V(r)|\,) = -0.09354143\,I_1 + 1.02895582\,I_2$$

$$-1.77728732\,I_3 + 0.84187296\,I_4$$

It is using these techniques that enable a value for $(\phi_1 | H | \phi_0)$ to be found.

An expression for the I_p's

Brody and Moshinsky, following Talmi, define

$$I_p = \frac{2}{\Gamma(p + 3/2)} \int_0^\infty r^{2p} e^{-r^2} \tilde{V}(r)\, r^2\, dr$$

where $r = \frac{1}{\sqrt{2}} (r_i - r_j) = \frac{1}{\sqrt{2}} r_{ij}$ in units of b

and $\tilde{V}(r) = V(r_{ij}) = V e^{-r_{ij}^2/a^2}$ in c.g.s. units

Inserting these expressions into the definition and performing an integration finally gives

$$I_p = \beta_o^{2p+3} \quad \text{in units of V}$$

where $\beta_o = \sqrt{\frac{\alpha}{\alpha + 2}}$ and $\alpha = a^2/b^2$

The values of $(\phi_j | H | \phi_o)$

It is not possible to examine the calculation of the $(\phi_j | H | \phi_o)$ in further detail here, The final numerical values, obtained using a Gauss force of range

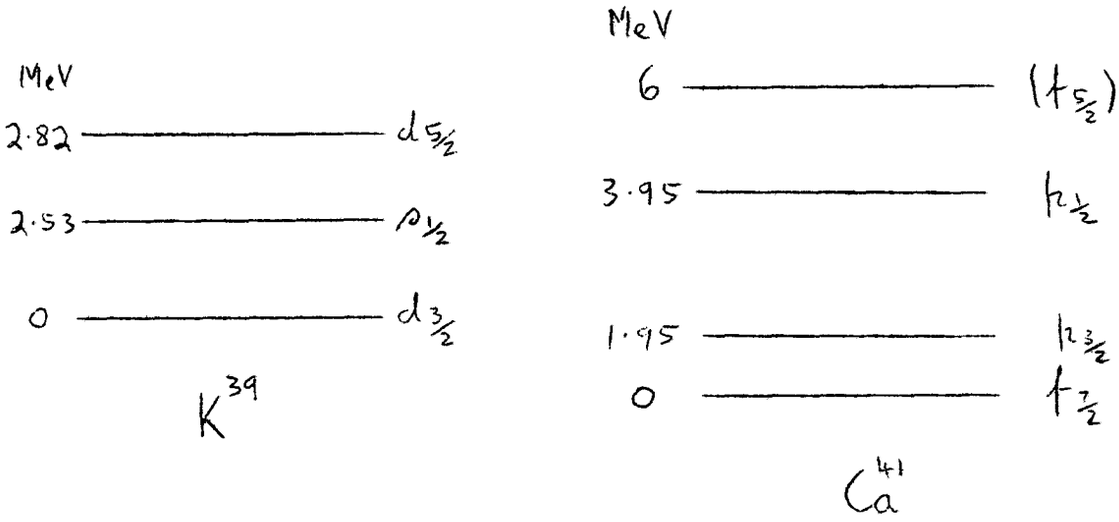
$$a = 2{\cdot}31 \times 10^{-13} \text{cm}$$

will just be quoted.

j	$(\phi_j \mid H \mid \phi_o)$ units of Vc	j	$(\phi_j \mid H \mid \phi_o)$ units of Vc
1	− 0·0080 8722	5	+ 0·0250 8079
2	− 0·0008 4641	6	+ 0·0058 1648
3	− 0·0011 1384	7	− 0·0072 8279
4	+ 0·0222 0319	8	− 0·0200 0774

Evaluation of the η_j

The expression for the η_j (4.12) involves the $E_j - E_o$ which are given approximately by differences in single particle energy levels.



The position of the $f5/2$ level in Ca^{41} is not well known, however it is said not to fall below 6 MeV. It will be seen later that the calculation is not very sensitive to the position of this level.

These single particle levels are taken from Endt and Braams (1957).

Later the effect of raising the $d5/2$ level in mass 39 will be considered.

The table below lists the $(E_j - E_o)$ and the η_j

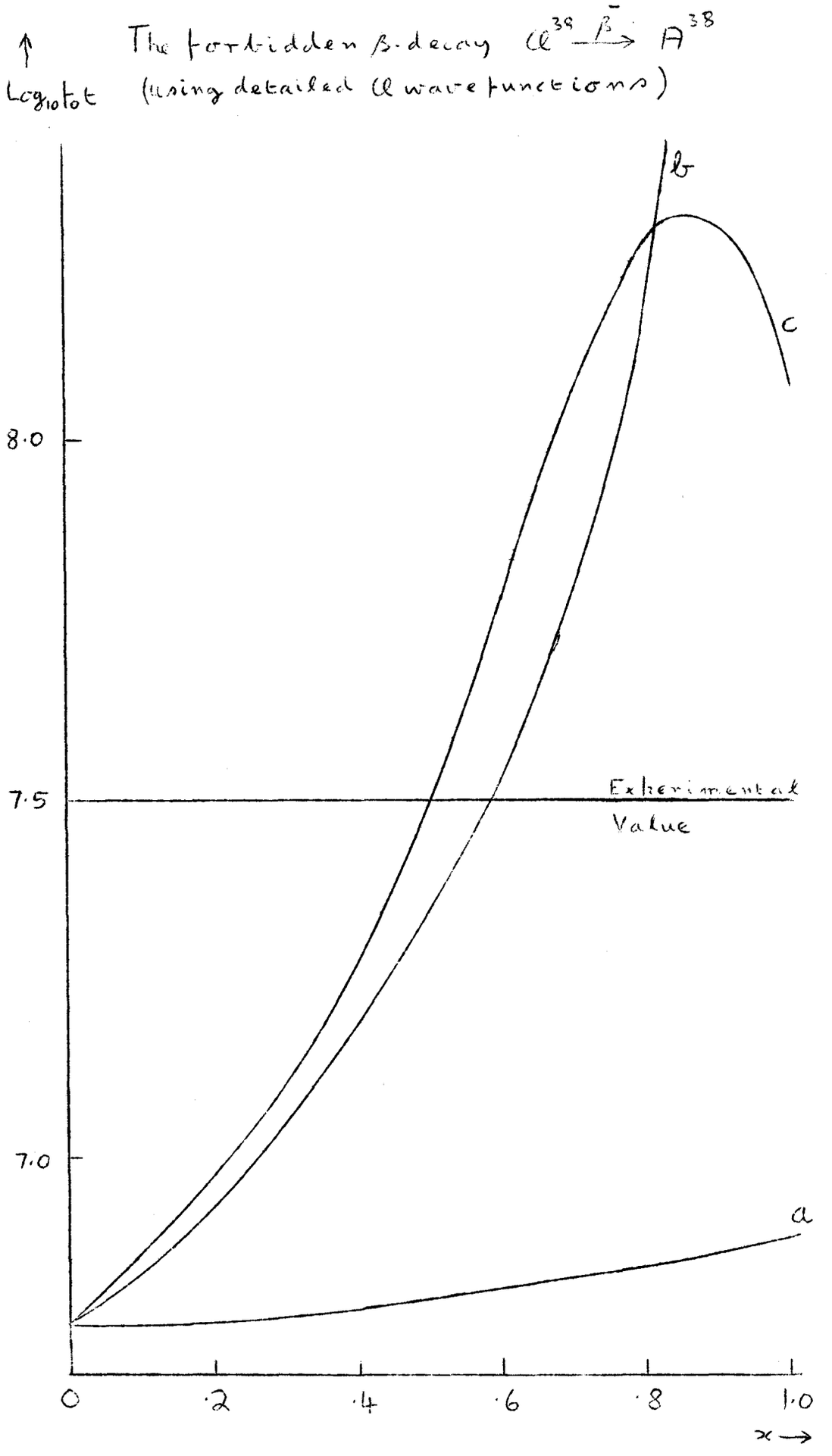
j	$E_j - E_o$ (MeV)	η_j (units of Vc)
1	1·95	+ 0·0041 4729
2	3·95	+ 0·0002 1428
3	6·00	+ 0·0001 8564
4	4·48	− 0·0049 5607
5	2·82	− 0·0088 9390
6	4·77	− 0·0012 1939
7	6·77	+ 0·0010 7574
8	8·82	+ 0·0022 6845

In 4.15 the products $\eta_j (\psi_o \| \beta \| \phi_j) = \eta_j N(\phi_j)$ are required.

j	$\eta_j N(\phi_j)$			
1	− 0·0017 9583	x	$b/\sqrt{11}$	x Vc
2	+ 0·0000 3280	"		"
3	− 0·0001 5038	"		"
4	− 0·0107 3021	"		"
5	− 0·0249 5844	"		"
6	+ 0·0017 1095	"		"
7	+ 0·0016 1361	"		"
8	− 0·0034 6511	"		"

It will be noticed that the largest contributions come from states ϕ_4 and ϕ_5. The $f5_{/2}$ level in mass 41 is only involved in η_3 and η_8 ; the contributions from these terms are small.

FIG. 16


The forbidden β-decay $Cl^{39} \xrightarrow{\beta^-} A^{38}$
(using detailed Cl wave functions)
↑
Log10 ft
8·0
7·5
7·0
Experimental
Value
a
b
c
0
·2
·4
·6
·8
1·0
x →

Using the result in Chapter 3 for $(\psi_o \| \beta \| \phi_o)$, 4.15 can now be written

$$(\psi(A^{38}) \| \beta \| \phi(Cl^{38}))$$

$$= \beta(x) \left[\sqrt{1-0\cdot0001\ 2873\ Vc^2}\ \sqrt{7-0\cdot0377\ 4261\ Vc} \right] \frac{b}{\sqrt{H}} \qquad \underline{4.25}$$

This is a function of x (since $x = Vc/-50\xi$ and $\xi = -1\cdot12$) which can now be evaluated.

$\mathrm{Log}_{10}\ f_o t$ is found from 3.11 using 3.12 where

$$\frac{1}{2J+1} \left(\psi'_{J'} \| \sum_n r^{\frac{1}{2}} \tau_- (\sigma' \times Y')^2 \| \psi_J \right)^2$$

must now be replaced by

$$\frac{1}{5} \left(\psi(A^{38})_o \| \beta \| \phi(Cl^{38})_2 \right)^2$$

given in 4.25 .

The result obtained is illustrated in Fig: 16. The effect of raising the $d5/2$ level in mass 39 to 4 Me V is also shown. This is calculated in just the same way, the only differences being in the values of some $E_j - E_o$ and $\beta(x)$.

Curve a represents the crude calculation using a Gauss mid range force with the $d5/2$ single hole level at 2·80 Me V,

Curve b represents the perturbation correction for the same force and single hole level position,

Curve c represents the perturbation correction for the same force with the $d5_{/2}$ single hole level raised to 4 MeV.

It will be seen that there is great improvement, the correction being of the right order of magnitude and sign. The best spectrum was obtained using the parameters of curve c with Vc $\sim$ 30 MeV and this corresponds to $\log_{10} f_o t = 7{\cdot}3$.

Once again there is an indication that to obtain the same level of agreement with experiment a smaller value of x should be chosen when the $d5_{/2}$ single hole level is raised.

In a perturbation calculation of this kind not too much emphasis should be laid on the final numerical figure, it is the sign and order of magnitude of the correction that are important.

Some conclusions

The result obtained here helps to confirm the previous success of the Gauss mid range force.

In order to obtain some kind of agreement with the experimental $\log_{10} f_o t$ it has been necessary to replace the crude jj-coupled approximation to the Cl^{38} wavefunction used in Chapter 3.

At first sight this would appear to be at variance with the work of Goldstein and Talmi (1956) who related the spectra of K^{40} and Cl^{38} very convincingly, using pure jj-configurations, without calculating or specifying the nuclear forces, apart from the assumption of a two-body force. Herein perhaps lies the explanation of the paradox.

The basic problem is always to solve

$$H\psi = E\psi$$

where H is the Hamiltonian and ψ its wavefunctions. It is usual to specify an H which is to some extent arbitrary and approximate to ψ in order to solve the equation.

Suppose now that there are a 'model' set of wavefunctions ϕ and that some operator P exists that transforms the 'model' wavefunctions into real ones

$$\text{ie.} \quad \psi = P\phi$$

then

$$H P\phi = E P\phi = P E \phi$$

Now assume that P posesses an inverse P^{-1}, so that

$$P^{-1} H P \phi = P^{-1} P E \phi = E \phi$$

and this new equation is similar to Schrödingers equation, but relates 'model' wavefunctions ϕ with a 'model' Hamiltonian $P^{-1} H P$.

It would then be possible to deduce a good spectrum from 'model' wavefunctions, provided a suitable 'model' Hamiltonian were used.

Let the nuclear matrix element in β -decay be represented by

$$(\psi' \| \beta \| \psi)$$

where ψ' is the final state wavefunction

ψ is the initial state wavefunction

β is the decay operator.

This decay problem could be solved equally well using 'model' wavefunctions, provided the decay operator were suitably transformed into a 'model' operator, ie. the nuclear matrix element would be given by

$$(\phi' \| P^{-1} \beta P \| \phi)$$

where ϕ' is the final state 'model' wavefunction

ϕ is the initial state 'model' wavefunction

$P^{-1} \beta P$ is the 'model' decay operator.

However only the 'real' β -decay operator is known and the problem is not solved in this way.

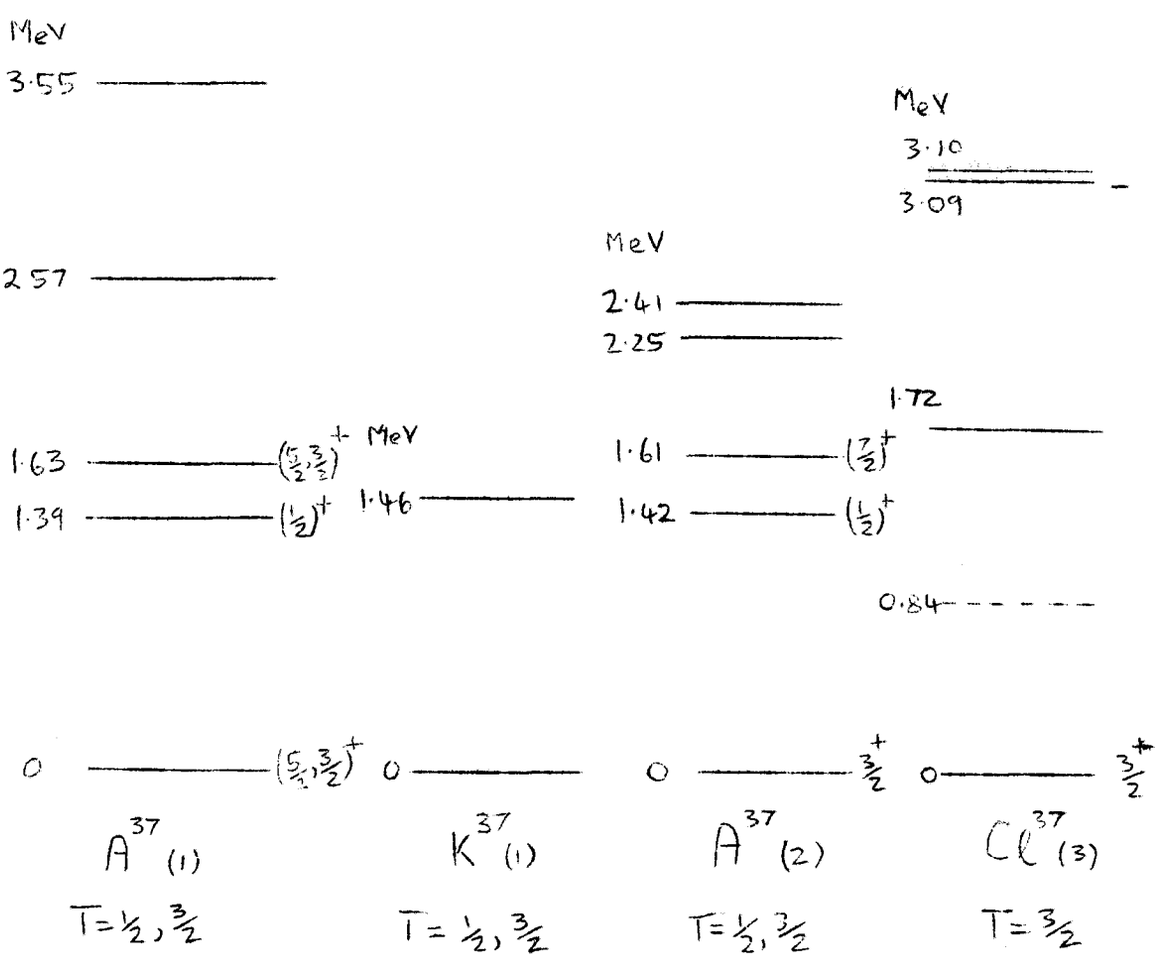
As Goldstein and Talmi made no assumptions about their Hamiltonian (apart from its two-body nature), they could be considered to have used a 'model' operator, which with 'model' wavefunctions might give a perfectly good spectrum. Here however the 'model' wavefunctions are of no use as only the real operator is available.

This may be why it is necessary to approximate more closely to the Cl^{38} wavefunctions in order to explain the forbidden β -decay than did Goldstein and Talmi in order to relate the spectra of two nuclei.

Chapter 5

The Spectrum and Electromagnetic Moments of Mass 37

The methods outlined in Chapter 1 enable the energy levels with $T = {}^1/2, {}^3/2$ to be calculated for the mass 37 nuclei. It is to be regretted that there is so little experimental data available for these levels. The diagram below summarises all that is so far known.



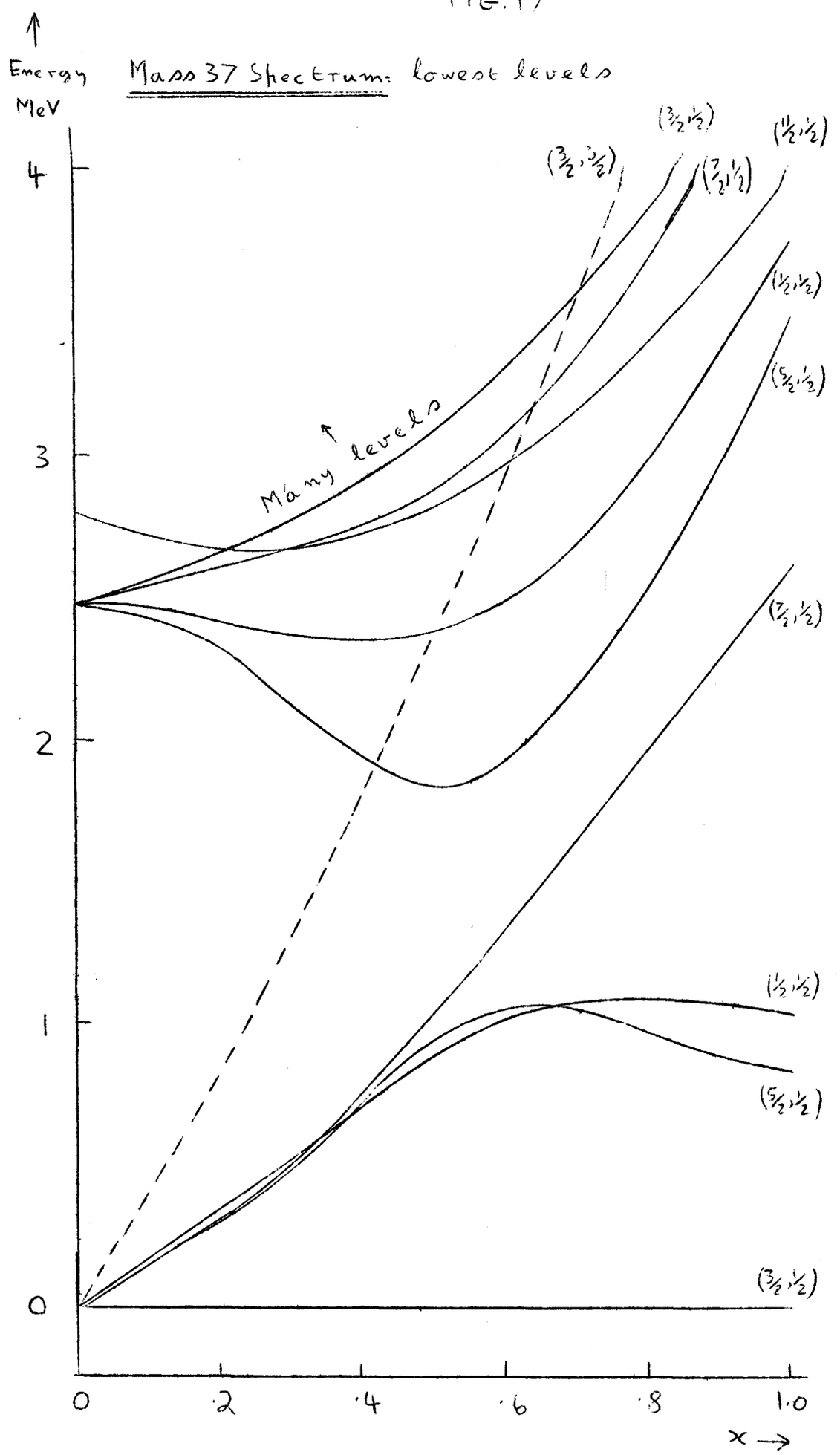
The sources of this data are as follows

(1) Yamomoto and Steigert: Phys. Rev. 121; 600 (1961)

(2) Ferguson and McCallum: Private communication.

(3) Endt and Braams (1957)

FIG. 17


Energy MeV
Mass 37 Spectrum: lowest levels
4
3
2
1
0
0
·2
·4
·6
·8
1.0
x →
(3/2, 3/2)
(3/2, 1/2)
(7/2, 1/2)
(11/2, 1/2)
(1/2, 1/2)
(5/2, 1/2)
(7/2, 1/2)
(1/2, 1/2)
(5/2, 1/2)
(3/2, 1/2)
Many levels

The lowest lying levels in A^{37}, K^{37} would be expected to have $T = {}^{1}/_{2}$

It will be seen that there is a disagreement between sources (1) and (2) in the assignment of spin to the lowest A^{37} levels. The shell model predicts a $^{3}/_{2}^{+}$ ground state for this nucleus, a fact used by Ferguson and McCallum in their assignment of spin to the first two excited levels.

Yamomoto and Steigert note that their first excited level in K^{37} was probably a doublet, not resolvable by this particular experiment.

In Cl^{37} there is possibly a level at 0·84 MeV. The level at 3·09 MeV is believed to have odd parity as there is an allowed β-decay from S^{37} $(^{7}/_{2}^{-})$ to this level.

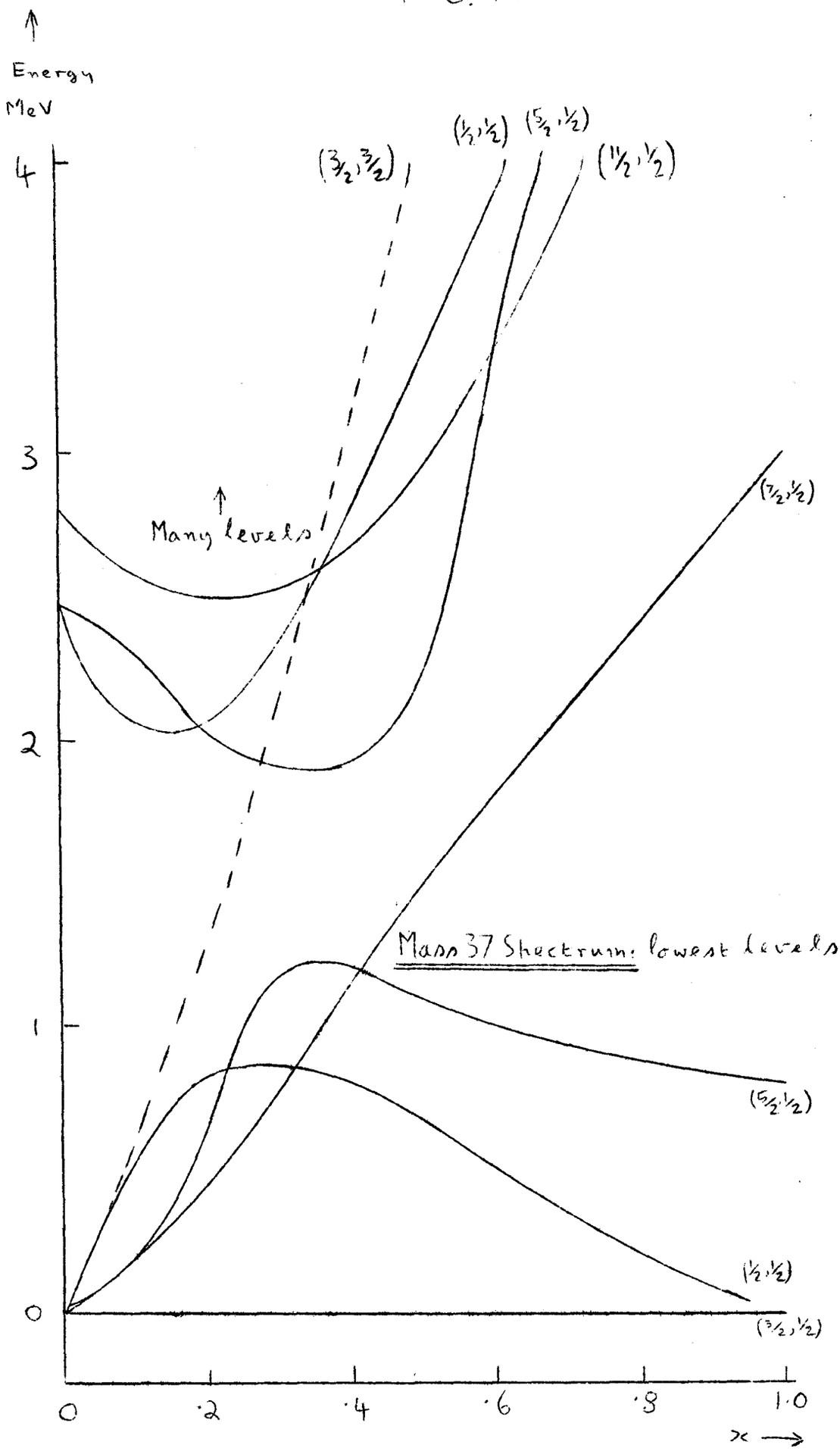
The problem of calculating energy levels and wave-functions in mass 37 is very much greater than that in mass 38, since three holes in the ds shell form more states than two and larger matrices have to be diagonalised.

Furthermore the mass 37 calculations were made at a time when there was no definite evidence in favour of a $d5_{/2}$ single hole level higher than 2·80 MeV.

Consequently only two full calculations have so far been made, both of which use the minimum value for the $d5_{/2}$ single hole level.

Fig: 17 illustrates the low lying levels obtained with a Yukawa interaction ($a = 1{\cdot}37 \times 10^{-13}$cm) with the Elliott and Flowers (O^{16}) exchange mixture; Fig: 18 illustrates the spectrum obtained with a Gauss mid range

FIG. 18


Energy
MeV
4
3
2
1
0
(3/2, 3/2)
(1/2, 1/2)
(5/2, 1/2)
(11/2, 1/2)
(7/2, 1/2)
Many levels
Mass 37 Spectrum: lowest levels
(5/2, 1/2)
(1/2, 1/2)
(3/2, 1/2)
0
·2
·4
·6
·8
1·0
x

interaction ($a = 2{\cdot}31 \times 10^{-13}$cm) using the Soper-like exchange.

The spectra for K^{37} and A^{37}

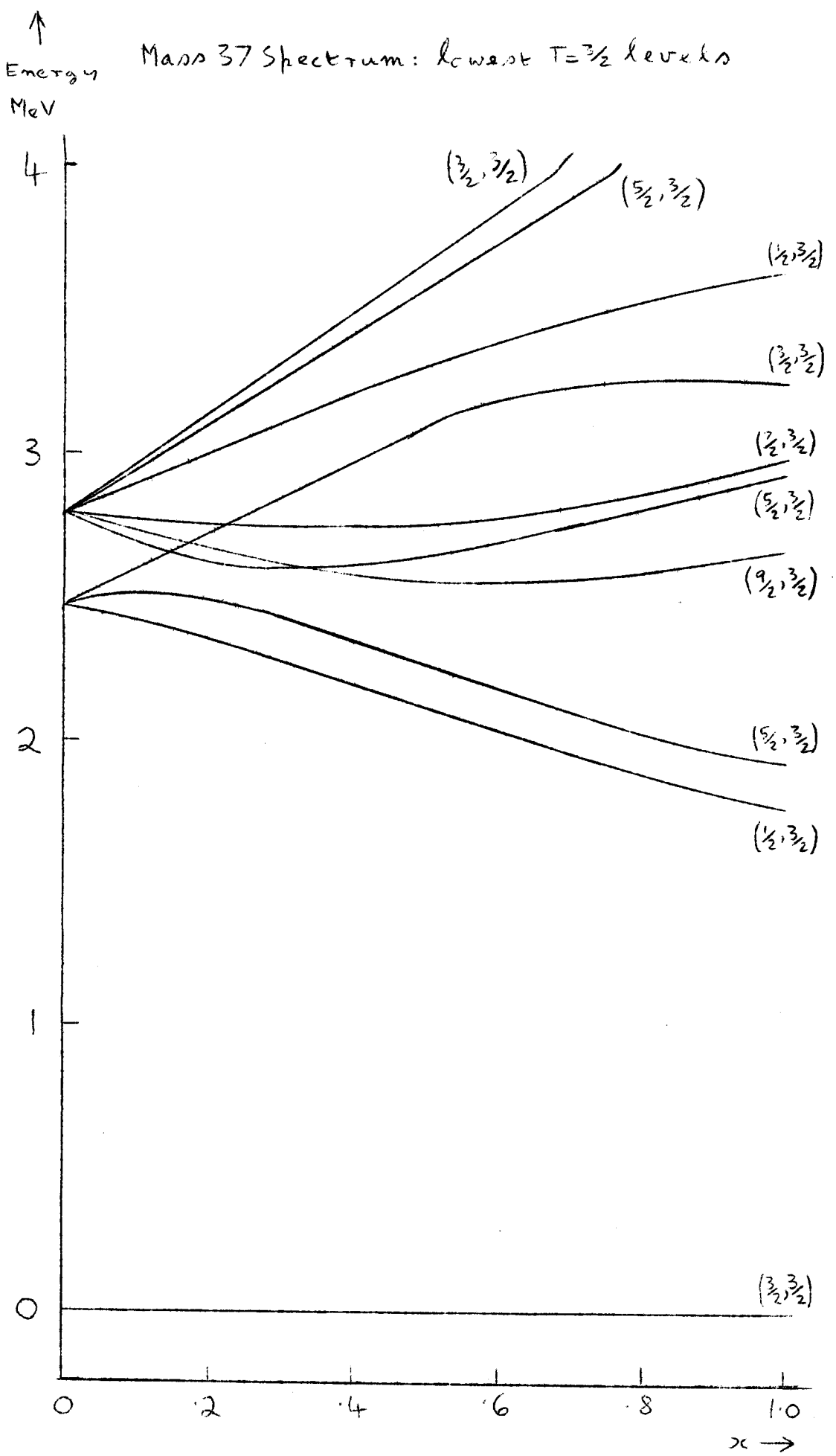
(1) Both calculations predict $T = {}^1/_2$ low lying levels in K^{37}, A^{37} with ground state ${}^3/_2^+$

(2) The earlier work of Elliott and Flowers (1955) who used the interaction of Fig: 17 would suggest that Vc takes a value $\sim$40 MeV corresponding to $x = 0{\cdot}7$. In mass 37 this leads to two excited states with spins ${}^5/_2$, ${}^1/_2$ very close to each other at about 1·1 MeV, followed by a spin ${}^7/_2$ state at 1·7 MeV.

This does not agree with the Ferguson and McCallum spin assignment, but favours that of Yamomoto and Steigert. In the latter case the ${}^1/_2$, ${}^5/_2$ levels are both too low and too close. The ${}^7/_2$ level would be difficult to fit into this scheme.

(3) In the case of Fig: 18, the conclusion drawn from the mass 38 calculations (Chapters 2 and 3) were that Vc should take a value $\sim$30 MeV ($x = 0{\cdot}5$) for this kind of interaction. In mass 37 a first excited state with spin ${}^1/_2$ at 0·7 MeV would be predicted, followed by a ${}^5/_2$ state at 1·1 MeV and a ${}^7/_2$ state at 1·5 MeV. This result again favours the spin assignment of Yamomoto and Steigert; the calculated levels are all much too low, but there is now a 0·4 MeV separation between the first and second excited states (experimental value $\sim$0·2 MeV) which could only have been approximated to in the Fig: 17

FIG. 19


Mass 37 Spectrum: lowest T=3/2 levels
Energy
MeV
4
3
2
1
0
(3/2, 3/2)
(5/2, 3/2)
(1/2, 3/2)
(3/2, 3/2)
(7/2, 3/2)
(5/2, 3/2)
(9/2, 3/2)
(5/2, 3/2)
(1/2, 3/2)
(3/2, 3/2)
0
·2
·4
·6
·8
1·0
x →

spectrum (keeping the first excited state as $^{1}/_{2}^{+}$) for too small a value of $x(\sim 0{\cdot}5)$

(4) It is to be expected from the mass 38 work that an improved spectrum should result from the Fig: 18 interaction when the $d5/_2$ single hole level is raised.

Without detailed calculation it is difficult to say precisely what the effect of this will be.

However some tentative suggestions are

(a) the overall level density should decrease

(b) the $^{1}/_{2}$, $^{5}/_{2}$, $^{7}/_{2}$ levels should be raised, the $^{7}/_{2}$ probably being least affected

At the present time it is not possible to say which interaction is most acceptable in fitting the A^{37}, K^{37} spectrum. These preliminary results favour the level order given by Yamomoto and Steigert. The raising of the $d5/_2$ single hole level would increase the energy of the excited levels, which is certainly desirable.

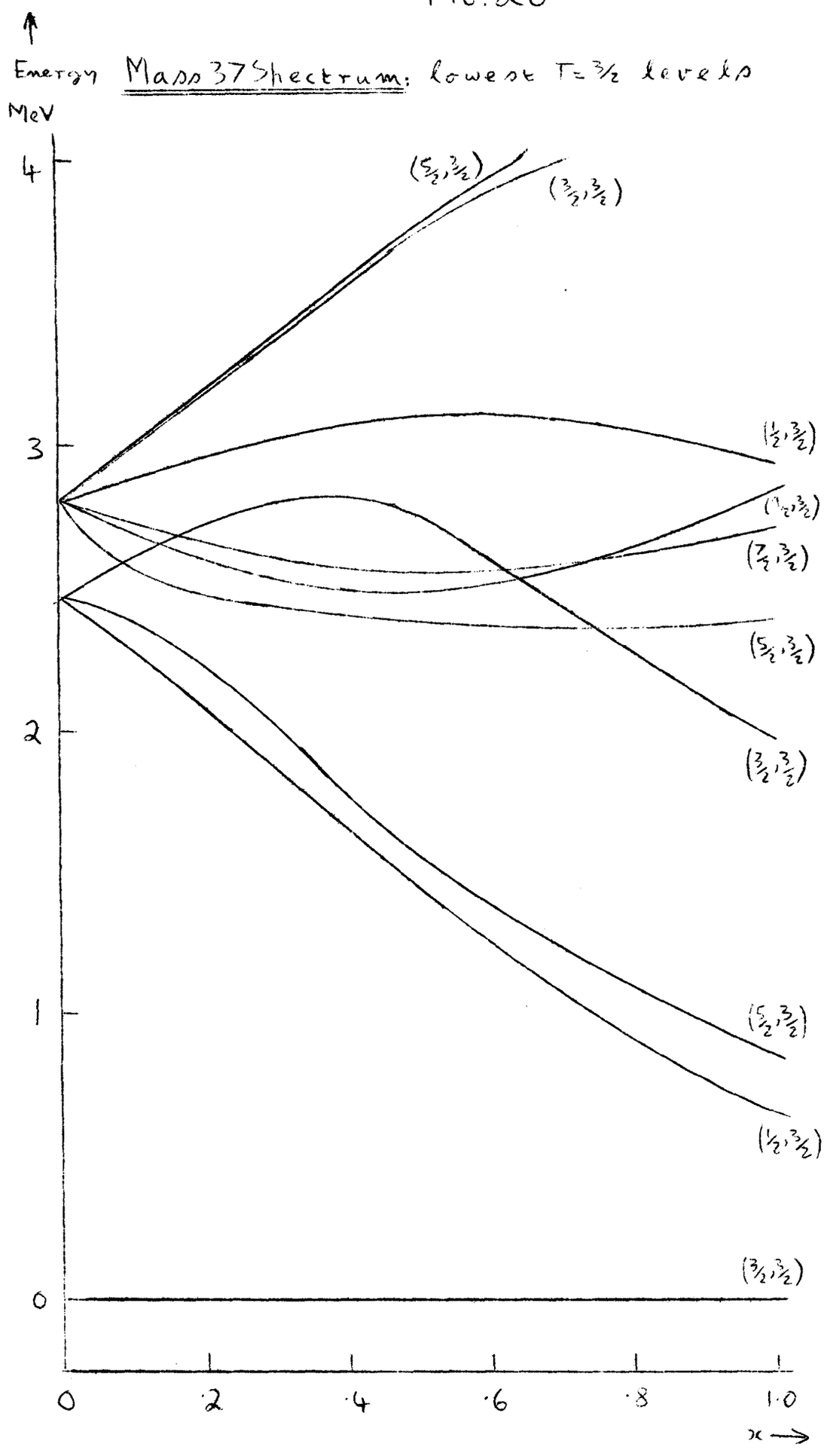
<u>The spectrum for Cl^{37}</u>

Figs: 19, 20 reproduce the $T = {}^{3}/_{2}$ levels obtained by calculation applicable to Cl^{37}. Fig: 19 is for the Yukawa and Fig: 20 is for the mid range Gauss interaction.

(1) Both interactions predict a $^{3}/_{2}^{+}$ ground state

(2) Both interactions predict first and second even parity excited states of spin $^{1}/_{2}$, $^{5}/_{2}$ respectively separated by about 0·1 Me V. For the values of x discussed above, the Yukawa interaction predicts

FIG. 20


Energy
Mass 37 Spectrum: lowest T=3/2 levels
MeV
4
3
2
1
0
(5/2, 3/2)
(3/2, 3/2)
(1/2, 3/2)
(1/2, 3/2)
(7/2, 3/2)
(5/2, 3/2)
(3/2, 3/2)
(5/2, 3/2)
(1/2, 3/2)
(3/2, 3/2)
0
·2
·4
·6
·8
1·0
x

these excited levels at $\sim$ 2 MeV and the Gauss interaction at $\sim$1·5 MeV.

(3) The effect of raising the $d_{5/2}$ single hole level in this case would probably lead to a raising of the $^5/_2$ level relative to the $^1/_2$, which would itself be raised. It is not easy to predict an even parity level at 0·84 MeV. If such a level is to be found in Cl^{37}, on the basis of this calculation it must surely be of odd parity.

It seems possible that the excited state observed at 1·72 MeV corresponds to the first even parity excited state with spin $^1/_2$.

More experimental data is badly needed for the mass 37 nuclei before the validity of these calculations can be fully discussed.

Magnetic moment of Cl^{37}

Blin-Stoyle (Theories of Nuclear Moments. O.U.P. 1957) quotes the magnetic moment of Cl^{37} as 0·7 nuclear magneton, the single particle model prediction being 0·1 nuclear magneton.

The magnetic moment μ is defined by Blin-Stoyle (1957) as

$$\mu = (J\ M = J \mid \mu_0' \mid J\ M = J) \qquad 5.1$$

where $$\mu_0' = \frac{e\hbar}{2Mc} \left\{ \sum_{k=1}^{A} g_L(k) L(k) + \sum_{k=1}^{A} g_s(k) S(k) \right\}$$

and L, S are vector operators

$e\hbar/2Mc$ is a constant known as the nuclear magneton (n.m.)

the g's are constants given by

	proton	neutron
g_L	1	0
g_S	5·587	− 3·826

Using the Wigner-Eckart theorem 5.1 may be reduced to

$$\mu = \sqrt{\frac{J}{(J+1)(2J+1)}} \qquad (J \parallel \mu' \parallel J) \qquad 5.2$$

From the values of g_L for neutron and proton, the first term in the expression for μ'_0 may be written

$$\sum_{k=1}^{A} g_L(k)\, L(k) = \sum_{p} L(p) \quad \text{a sum over protons only}$$

$$= \sum_{i} {}^1/_2 \,(1 - \tau_0(i))\, L(i)$$

$$= {}^1/_2 \, L - {}^1/_2 \sum_{i} \tau_0(i)\, L(i) \qquad 5.3$$

since ${}^1/_2\,(1 - \tau_0(i)) = 0$ for neutrons, 1 for protons.

Similarly the second term in μ'_0 may be written

$$\sum_{k=1}^{A} g_s(k)\, S(k) = \tfrac{1}{2}\left(g_s(p) + g_s(n)\right) S + \tfrac{1}{2}\left(g_s(n) - g_s(p)\right) \sum_{i} \tau_0(i)\, S(i) \qquad 5.4$$

Thus in general the operator μ'_0 can be expanded in terms of the four operators

$$L \qquad S \qquad \sum_{i} \tau_0(i)\, L(i) \qquad \sum_{i} \tau_0(i)\, S(i)$$

Cl^{37} is a three proton hole system. It will be convenient first to develop expressions for μ applicable to the three proton system and then exchange particles for holes using Racah's rule (Chapter 1).

Expanding μ'_o using 5.3, 5.4 and the fact that $\gamma_o = -1$ for protons

$$\mu'_o = \frac{e\hbar}{2Mc} \left\{ L + g_s (p)\, S \right\} \qquad 5.5$$

Replacing L by J − S the amplitude matrix of 5.2 becomes

$$(J \| \mu' \| J) = \frac{e\hbar}{2Mc} \left(J \| J + (g_s (p) - 1)\, S \| J \right)$$

$$= \frac{e\hbar}{2Mc} (J \| J \| J) + (g_s (p) - 1)(J \| S \| J) \qquad 5.6$$

These terms are easily evaluated using the Wigner-Eckart theorem and Racah algebra.

In fact

$$(J \| J \| J) = \sqrt{J(J+1)(2J+1)}$$

$$(J \| S \| J) = (-)^{L+J+S+1} (2J+1) \begin{Bmatrix} L & J & S \\ 1 & S & J \end{Bmatrix} \sqrt{S(S+1)(2S+1)}$$

whence from 5.2 and 5.6

$$\mu = \frac{e\hbar}{2Mc} \left[J + (g_s (p) - 1) \sqrt{\frac{J(2J+1)}{J+1}}\, (-)^{L+J+S+1} \begin{Bmatrix} L & S & J \\ 1 & S & J \end{Bmatrix} \sqrt{S(S+1)(2S+1)} \right] \qquad 5.7$$

The calculated mass 37 states were expanded in terms of a set of LS-coupled states

$$\underline{\Psi} = \sum_i a_i \psi_i$$

and so 5.1 should be written

$$\mu = \left(\sum_i a_i \psi_i \,|\, \mu'_0 \,|\, \sum_j b_j \psi_j \right)$$

$$= \sum_i \sum_j a_i b_j \left(\psi_i \,|\, \mu'_0 \,|\, \psi_j \right)$$

$$= \begin{bmatrix} a_1 & a_2 & \cdots \end{bmatrix} \begin{bmatrix} \text{Matrix} \\ \text{of} \\ \mu'_0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \end{bmatrix} \qquad 5.8$$

in matrix language (compare with 3.9)

The elements of the matrix $[\mu'_0]$ may be calculated using 5.7 , and the matrix must be diagonal as the only terms which can contribute have the same state on each side of the operator in 5.1

Cl^{37} has J = 3/2, T = 3/2 and the form of $[\mu'_0]$ is given below.

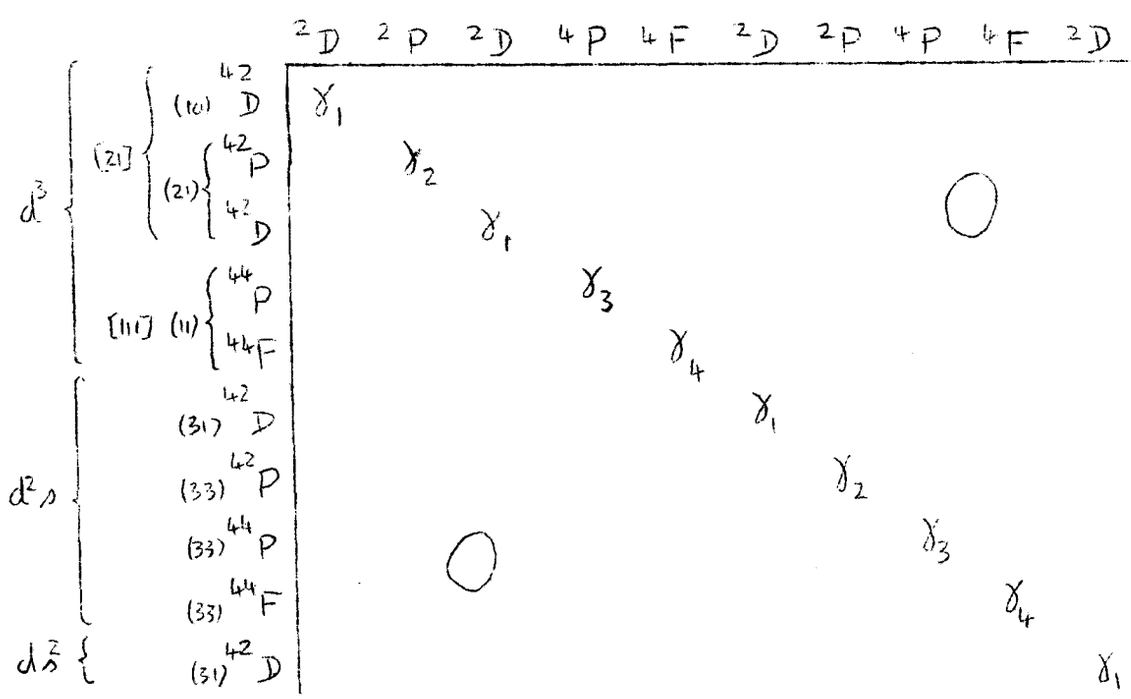
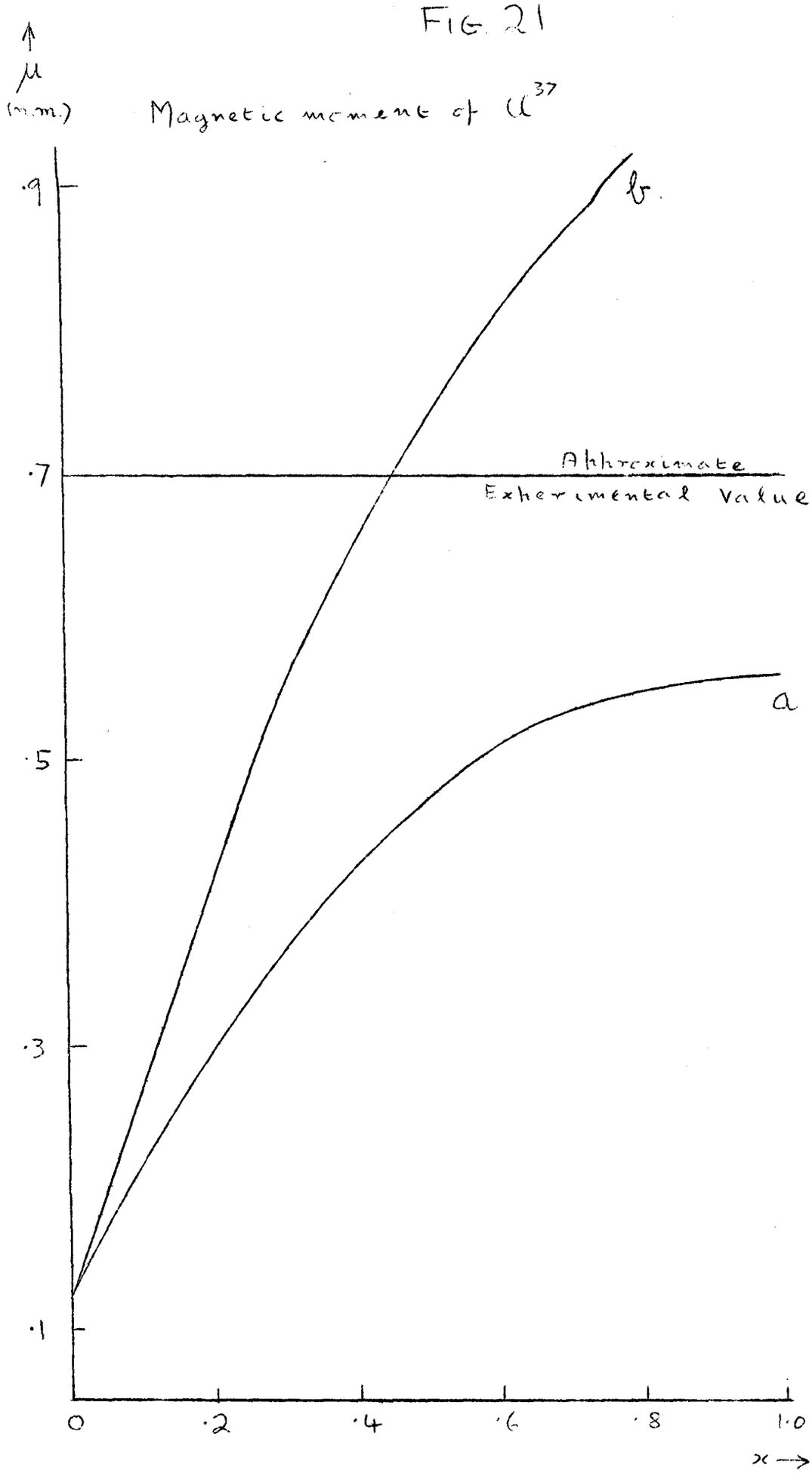


FIG. 21


μ
(n.m.)
Magnetic moment of U^{237}
·9
·7
·5
·3
·1
b
Approximate
Experimental Value
a
0
·2
·4
·6
·8
1·0
x →

Since 5.7 depends on L, S only (J being the same for all these states) there are only four distinct elements γ .

For example

$$\gamma_1 = \frac{et}{2Mc}\left[\frac{3}{2} + (5.587-1)\sqrt{\frac{\frac{3}{2}\cdot 4}{5/2}}\,(-)^{2+\frac{3}{2}+\frac{1}{2}+1}\begin{Bmatrix} 2 & \frac{3}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{3}{2} \end{Bmatrix}\sqrt{\frac{1}{2}\cdot\frac{3}{2}\cdot 2}\,\right]$$

$$= 0.1239\ \frac{et}{2Mc}$$

Similarly

$$\gamma_2 = 3.7935\ e\hbar/2Mc$$

$$\gamma_3 = 6.5457\ e\hbar/2Mc$$

$$\gamma_4 = -2.6283\ e\hbar/2Mc$$

Changing from protons to proton holes has no effect in this case as Racah's rule predicts a sign change

$$(-)^{k+1}$$

where k is the rank of the tensor operator and the only terms involved have the operator L^1 or S^1 .

Expression 5.8 was evaluated using the calculated $J = {}^3/2$, $T = {}^3/2$ wavefunctions and the results obtained are shown in Fig: 21.

Curve a represents the Yukawa two-body force

$(a = 1.37 \times 10^{-13}\text{cm})$

Curve b represents the Gauss two-body force

$(a = 2.31 \times 10^{-13}\text{cm})$

both with $d5/2$ single hole level at 2·80 MeV.

In each case it will be seen that intermediate coupling can increase the single particle value by the correct order of magnitude. Assuming that an acceptable calculated μ lies between 0·5 and 0·9 n.m., then x is restricted as follows:

Curve a: $x > 0.58$ Curve b: $0.26 < x < 0.76$

These values of x are quite consistent with all the work described here and it would not be sensible to distinguish between the two curves in this instance.

As the $(^3/_2,^3/_2)$ Cl^{37} ground state is of the lowest configuration in the jj-coupling limit it is thought that raising the $d5_{/2}$ single hole level would make very little difference to these results.

The final conclusion is that the single particle value for the magnetic moment of Cl^{37} can be greatly improved by intermediate coupling, but that the calculation is not unduly sensitive to the various parameters.

Quadrupole moment of Cl^{37}

Blin-Stoyle (1957) quotes the quadrupole moment of Cl^{37} as $-0{\cdot}06 \times e \times 10^{-24} cm^2$ with a single particle value of $-0{\cdot}04 \times e \times 10^{-24} cm^2$.

The quadrupole moment Q is defined (see for example Elliott and Lane (1957) by

$$Q = \sqrt{\frac{16\pi}{5}} \quad (J \; M{=}J \mid Q_o^2 \mid J \; M{=}J) \qquad 5.9$$

where $Q_o^2 = e \sum_p Y_o^2 (p) \; r^2 (p)$, a sum over protons only.

Using the Wigner-Eckart theorem 5.9 can be reduced to

$$Q = \sqrt{\frac{16\pi}{5}} \sqrt{\frac{J(2J-1)}{(J+1)(2J+1)(2J+3)}} \quad (J \| Q^2 \| J) \qquad 5.10$$

Racah algebra enables the amplitude matrix of 5.10 to be written

$$((S'L')J \| Q^2 \| (SL)J)$$
$$= \delta(S'S)(2J+1)(-)^{S+L+J} \begin{Bmatrix} S & J & L' \\ 2 & L & J \end{Bmatrix} (L' \| Q^2 \| L) \qquad 5.11$$

In order to evaluate $(L' \| Q^2 \| L)$ it is necessary to use fractional parentage coefficients and expand the three-body states L', L in terms of two-body states.

$$\Psi'_{123}(L') = \sum_{\bar{\Psi}'} a^{\bar{\Psi}'}_{\bar{\Psi}'} \left\{ \bar{\Psi}'_{12}(\bar{L}') \, \phi'_3(\ell') \right\}^{L'}$$

$$\Psi_{123}(L) = \sum_{\bar{\Psi}} a^{\bar{\Psi}}_{\bar{\Psi}} \left\{ \bar{\Psi}_{12}(\bar{L}) \, \phi_3(\ell) \right\}^{L} \qquad 5.12$$

Also for a three proton system (the change to holes will be made later) $Q^2_0 = 3e\, Y^2_0(3)\, r^2(3)$

Inserting this expression and 5.12 into $(L' \| Q^2 \| L)$ and using the fact that the operator only acts on particle 3,

$$(L' \| Q^2 \| L) = 3e \sum_{\bar{\Psi}'} \sum_{\bar{\Psi}} a^{\bar{\Psi}'}_{\bar{\Psi}'} \, a^{\bar{\Psi}}_{\bar{\Psi}} \, \delta\left(\bar{\Psi}'(\bar{L}'), \bar{\Psi}(\bar{L})\right)$$

$$\times \left[(2L'+1)(2L+1)\right]^{\frac{1}{2}} (-)^{\bar{L}+L'+\ell} \begin{Bmatrix} \bar{L} & L' & \ell' \\ 2 & \ell & L \end{Bmatrix} (\ell' \| Y^2_{r^2} \| \ell) \qquad 5.13$$

Now $(\ell' \| Y^2_{r^2} \| \ell) = (\ell' \| Y^2 \| \ell) \int_0^\infty u_{n'\ell'}(r)\, r^2\, u_{n\ell}(r)\, r^2\, dr$

and terms like this have already been dealt with in Chapters 1 and 4.

The only contributions are from

$$(d \| Y^2 \, r^2 \| d) = -\frac{5}{2}\sqrt{\frac{7}{2}}\,\frac{b^2}{\sqrt{\pi}}$$

$$(d \| Y^2 \, r^2 \| s) = 5\sqrt{\frac{1}{2}}\,\frac{b^2}{\sqrt{\pi}} \qquad 5.14$$

since Y^2 cannot couple two $l = 0$ states.

Combining 5.10, 5.11, 5.13,

$$Q = \sqrt{\frac{16\pi}{5}}\sqrt{\frac{J(2J-1)(2J+1)}{(J+1)(2J+3)}}\ \delta(SS')\,(-)^{S+L+L'+J}\begin{Bmatrix} S & J & L \\ 2 & L & J \end{Bmatrix}\left[(2L+1)(2L+1)\right]^{1/2}$$

$$\times 3e \sum_{\bar{\Psi}} a^{\bar{\Psi}'}_{\bar{\Psi}}\, a^{\bar{\Psi}}_{\bar{\Psi}}\, \delta(\bar{L}'\bar{L})\,(-)^{\bar{L}+\ell}\begin{Bmatrix} \bar{L} & L' & \ell' \\ 2 & \ell & L \end{Bmatrix}(\ell' \| Y^2_{r^2} \| \ell) \qquad 5.15$$

which can be evaluated using 5.14 .

The calculated wavefunctions are expressed in terms of a set of LS-coupled states

$$\Psi = \sum_i' a_i \psi_i$$

thus 5.9 should be written

$$Q = \sqrt{\frac{16\pi}{5}}\ \left(\sum_i' a_i\psi_i \,|\, Q_0^2 \,|\, \sum_j' b_j \psi_j\right)$$

$$= \sum_i \sum_j' a_i b_j \sqrt{\frac{16\pi}{5}}\ (\psi_i \,|\, Q_0^2 \,|\, \psi_j)$$

$$= [a_1\ a_2\ \cdots]\begin{bmatrix} \text{Matrix} \\ \text{of} \\ \sqrt{\frac{16\pi}{5}}\ Q_0^2 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ \vdots \end{bmatrix} \qquad 5.16$$

in matrix language.

The elements of the matrix $\left[\sqrt{\frac{16\pi}{5}}\ Q_0^2\right]$ may be evaluated directly from 5.15.

This time in changing from particles to holes there is a change of sign, since Racah's rule (Chapter 1) predicts

$$(-)^{1+k}$$

and the operator is a tensor of rank 2.

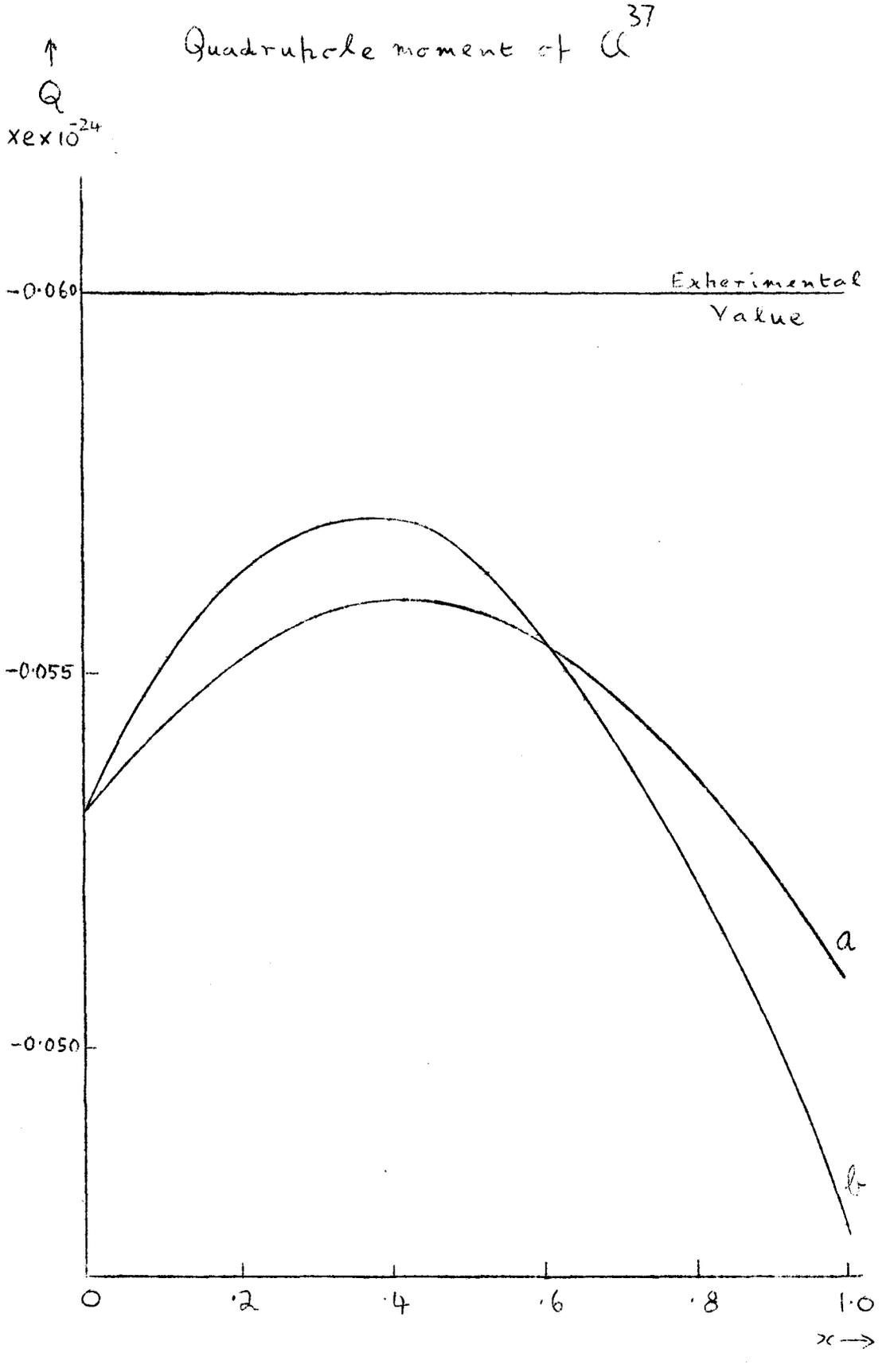
The matrix $\left[\sqrt{\frac{16\pi}{5}}\ Q_0^2\right]$ for Cl^{37} ($J=3/2$, $T=3/2$) is given below. It is symmetric because expression 5.15 is symmetric in S, S' ; L, L' ; $\bar{L}', \bar{L}$; and l is always even.

			^2D	^2P	^2D	^4P	^4F	^2D	^2P	^4P	^4F	^2D
d^3	[21]	(10) ^{42}D	1	2	3			17	20			
		(21) ^{42}P	2	4	5	0		18	21	0		0
		^{42}D	3	5	6			19	22			
	[111]	(11) ^{44}P		0		7	8		0	25	27	
		^{44}F				8	9			26	28	
d^2s		(31) ^{42}D	17	18	19			10	11			23
		(33) ^{42}P	20	21	22	0		11	12	0		24
		(33) ^{44}P		0		25	26		0	13	14	0
		(33) ^{44}F				27	28			14	15	
dp^2		(31) ^{42}D		⬭				23	24	0		16

The zero's here are due to the δ's in 5.15.

Element	Value		Element	Value		Element	Value	
1	$7/10$	eb^2	10	$-3/5$	eb^2	20	$3/5$	eb^2
2	$\frac{3}{5}\sqrt{\frac{7}{2}}$	"	11	0	"	21	$-\sqrt{14}/5$	"
3	$3\sqrt{21}/10$	"	12	$-7/5$	"	22	$\sqrt{21}/5$	"
4	$-2/5$	"	13	$28/25$	"	23	$-4\sqrt{7}/5$	"
5	$-\frac{1}{5}\sqrt{3/2}$	"	14	$4\sqrt{21}/25$	"	24	0	"
6	$3/10$	"	15	$12/25$	"	25	$16\sqrt{2}/25$	"
7	$-28/25$	"	16	$7/5$	"	26	$-2\sqrt{42}/25$	"
8	$-4\sqrt{21}/25$	"	17	$\sqrt{7}/5$	"	27	$-2\sqrt{42}/25$	"
9	$-12/25$	"	18	$6\sqrt{3}/5$	"	28	$24\sqrt{2}/25$	"
			19	$-3\sqrt{3}/5$	"			

FIG. 22


Quadrupole moment of Cl^{37}
Q
$xe \times 10^{-24}$
-0·060
Experimental Value
-0·055
-0·050
a
b
0
·2
·4
·6
·8
1·0
x →

These non zero matrix elements have been worked out directly from 5.15 and 5.14 taking the fractional parentage coefficients from the sources mentioned in Chapter 1.

b was evaluated in Chapter 1,

$$b_{37} = 1{\cdot}950 \times 10^{-13}\text{cm}.$$

Expression 5.16 was evaluated using the calculated wave-functions and the results obtained are shown in Fig: 22.

Curve a is for the Yukawa interaction ($a = 1{\cdot}37 \times 10^{-13}$cm)

Curve b is for the Gauss interaction ($a = 2{\cdot}31 \times 10^{-13}$cm)

Once again there is little information to be obtained from the result. If an acceptable calculated value of lies between $-0{\cdot}050 \times e \times 10^{-24}\text{cm}^2$ and $-0{\cdot}070 \times e \times 10^{-24}\text{cm}^2$ then both curves are satisfactory for $x < 0{\cdot}9$.

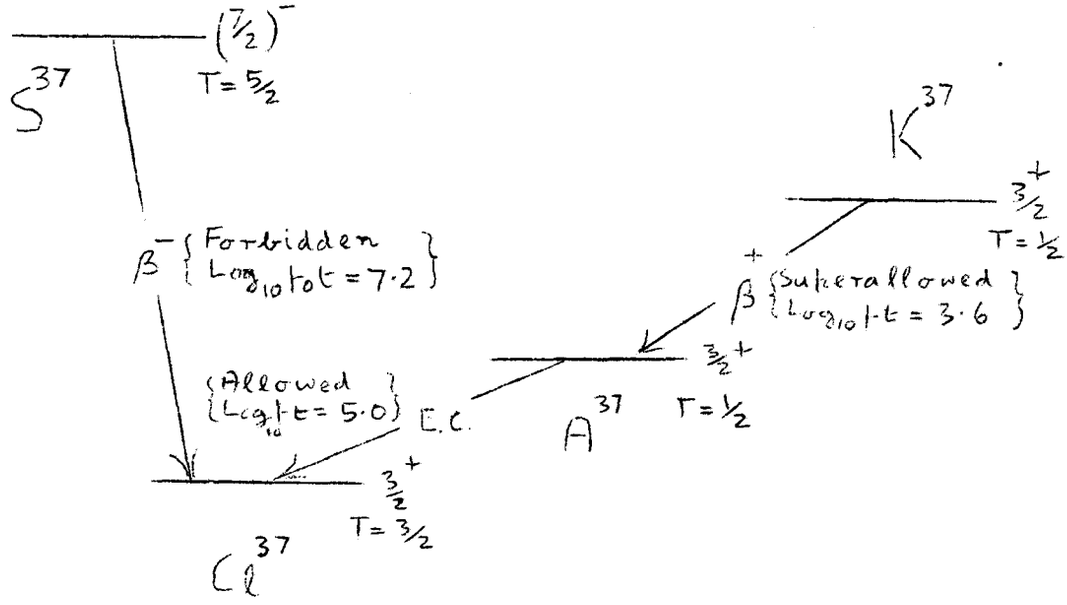
The effect of intermediate coupling is to slightly shift the single particle value ($-0{\cdot}053 \times e \times 10^{-24}\text{cm}^2$, in this case) nearer the experimental figure. The difference between the single particle value obtained here (and checked by a pure jj-coupling calculation) and that quoted by Blin-Stoyle (1957) is probably due to different ways of estimating b.

The conclusion to be drawn from this calculation is that the quadrupole moment of Cl^{37} is fairly well described on the single particle model. The effect of intermediate coupling is a slight improvement, but the calculation is very insensitive. The agreement between the experimental quadrupole moment and that predicted by the shell model is an indication that the Cl^{37} nucleus is not particularly deformed and so should be better understood on the basis of the shell model rather than that of a rotational model.

Chapter 6

β-Decay in Mass 37

Some of the nuclear wavefunctions obtained from the mass 37 calculation have already been used to estimate electromagnetic moments, in the last chapter.
Others will be used in this chapter to calculate log ft values for various β-decays.
The available experimental information is summarised in the diagram below.



The composition of some mass 37 wavefunctions is described in appendix IV.

The mirror transition $K^{37} \xrightarrow{\beta+} A^{37}$

The details of this decay are

$$K^{37}\ (3/2^+,\ T = 1/2) \xrightarrow{\beta+} A^{37}\ (3/2^+,\ T = 1/2)$$

where K^{37} and A^{37} are mirror nuclei.

The theory of allowed β -decay has already been discussed in some detail; see the first part of Chapter 3. As the isotopic spin is the same for both nuclei, there will be contributions from both Fermi and Gamow-Teller terms (3.1).

The Fermi term is easily calculated from 3.4, putting $T = {}^1/2$ and $M' = {}^1/2$ (for A^{37}) ,

$$F^2 = ({}^1/2 + {}^1/2)({}^1/2 - {}^1/2 + 1) = 1 \qquad 6.1$$

for this β^+ decay.

The Gamow-Teller term

A general expression for the Gamow-Teller term has been given in 3.5 .

Defining

$$N = (\psi'(T' J') \| \sum_j \tau(j)\sigma(j) \| \psi(T J)) \qquad 6.2$$

$$= (T' S' (l_1' l_2' l_3')[f'](\gamma')L' J' \| \sum_j \tau(j)\sigma(j) \| T S(l_1 l_2 l_3)[f](\gamma)LJ)$$

where $(l_1 l_2 l_3)[f](\gamma)$ L denotes a three-body orbital angular momentum state with $l_1 l_2 l_3$ of the three particles coupled to form a state labelled by symmetry $[f]$ and any other quantum numbers (γ).
Nb. In this case 1, 2, and 3 are not particle numbers.

Using the fact that the operator in 6.2 acts only in the T, S spaces and not in the L space,

$$N = \delta\Big((l_1' l_2' l_3')\,[f']\,(\gamma')\,L',\ (l_1 l_2 l_3)\,[f]\,(\gamma)\,L\Big) \qquad 6.3$$

$$\times \Big[(2J'+1)(2J+1)\Big]^{1/2}\, W(LJS'1{:}SJ')\ (T'S' \| \sum_j \tau(j)\sigma(j) \| TS)$$

from standard expressions of the Racah algebra.

For three particle states

$$(T'S' \| \sum_j \tau(j)\sigma(j) \| T\,S) = 3(T'S' \| \tau(3)\sigma(3) \| T\,S)$$

and expanding these states in terms of two particle states coupled to a single particle

$$\underline{\Psi}'_{123}(T'S') = \sum_{\bar{\Psi}'} a^{\bar{\Psi}'}_{\Psi'} \left\{ \bar{\psi}'_{12}(\bar{T}'\bar{S}')\,\phi'_3(t's') \right\}^{T'S'}$$

$$\underline{\Psi}_{123}(TS) = \sum_{\bar{\Psi}} a^{\bar{\Psi}}_{\Psi} \left\{ \bar{\psi}_{12}(\bar{T}\bar{S})\,\phi_3(ts) \right\}^{TS} \qquad 6.4$$

where the a's are coefficients of fractional parentage, the above amplitude matrix can be written

$$3 \sum_{\bar{\Psi}'} \sum_{\bar{\Psi}} a^{\bar{\Psi}'}_{\Psi'} a^{\bar{\Psi}}_{\Psi} \left(\left\{ \bar{\psi}'_{12}\phi'_3 \right\}^{T'S'} \| \tau(3)\sigma(3) \| \left\{ \bar{\psi}_{12}\phi_3 \right\}^{TS} \right)$$

$$= 3 \sum_{\bar{\Psi}'} \sum_{\bar{\Psi}} a^{\bar{\Psi}'}_{\Psi'} a^{\bar{\Psi}}_{\Psi} \left(\left\{ \bar{\psi}'_{12}\phi'_3 \right\}^{T'} \| \tau(3) \| \left\{ \bar{\psi}_{12}\phi_3 \right\}^{T} \right) \left(\left\{ \bar{\psi}'_{12}\phi'_3 \right\}^{S'} \| \sigma(3) \| \left\{ \bar{\psi}_{12}\phi_3 \right\}^{S} \right)$$

using an abbreviated notation for some of the symbols in 6.4 .

Since the operators involved only act on the last particle, with the help of Racah algebra this expression can also be written as

$$3 \sum_{\bar{\Psi}'} \sum_{\bar{\Psi}} a^{\bar{\Psi}'}_{\bar{\Psi}'} a^{\bar{\Psi}}_{\bar{\Psi}} \delta\left(\bar{\psi}'_{12}(\bar{T}')\,\bar{\psi}_{12}(\bar{T})\right) \delta\left(\bar{\psi}'_{12}(\bar{S}')\,\bar{\psi}_{12}(\bar{S})\right)$$

$$\times \left[(2T'+1)(2T+1)(2S'+1)(2S+1)\right]^{1/2} W(\bar{T}\,T'\,t\,1 : t'\,T)$$

$$\times W(\bar{S}\,S'\,s\,1 : s'\,S)\,(t' \| \tau \| t)\,(s' \| \sigma \| s)$$

which reduces to a single sum over $\bar{\Psi}$ as the δ's imply

$$\bar{\psi}'(\bar{T}'\bar{S}') = \bar{\psi}(\bar{T}\bar{S})$$

For single-body states

$$t' = t = 1/2$$
$$s' = s = 1/2$$

and so 6.2 can finally be written

$$N = 18\,\delta\left((l'_1 l'_2 l'_3)[\lambda'](\gamma')L', (l_1 l_2 l_3)[\lambda](\gamma)L\right)$$

$$\times (-)^{T'+2S'+L+J} \begin{Bmatrix} L & J & S \\ 1 & S' & J' \end{Bmatrix} \qquad 6.5$$

$$\times \left[(2J'+1)(2J+1)(2T'+1)(2T+1)(2S'+1)(2S+1)\right]^{1/2}$$

$$\times \sum_{\bar{\Psi}} a^{\bar{\Psi}'}_{\bar{\Psi}} a^{\bar{\Psi}}_{\bar{\Psi}} \delta\left(\bar{\psi}'_{12}(\bar{T}'\bar{S}')\,\bar{\psi}_{12}(\bar{T}\bar{S})\right) (-)^{\bar{T}+\bar{S}} \begin{Bmatrix} \bar{T} & T' & 1/2 \\ 1/2 & T & 1 \end{Bmatrix} \begin{Bmatrix} \bar{S} & S' & 1/2 \\ 1/2 & S & 1 \end{Bmatrix}$$

and $G^2 = A\,N^2$, see equation 3.5.

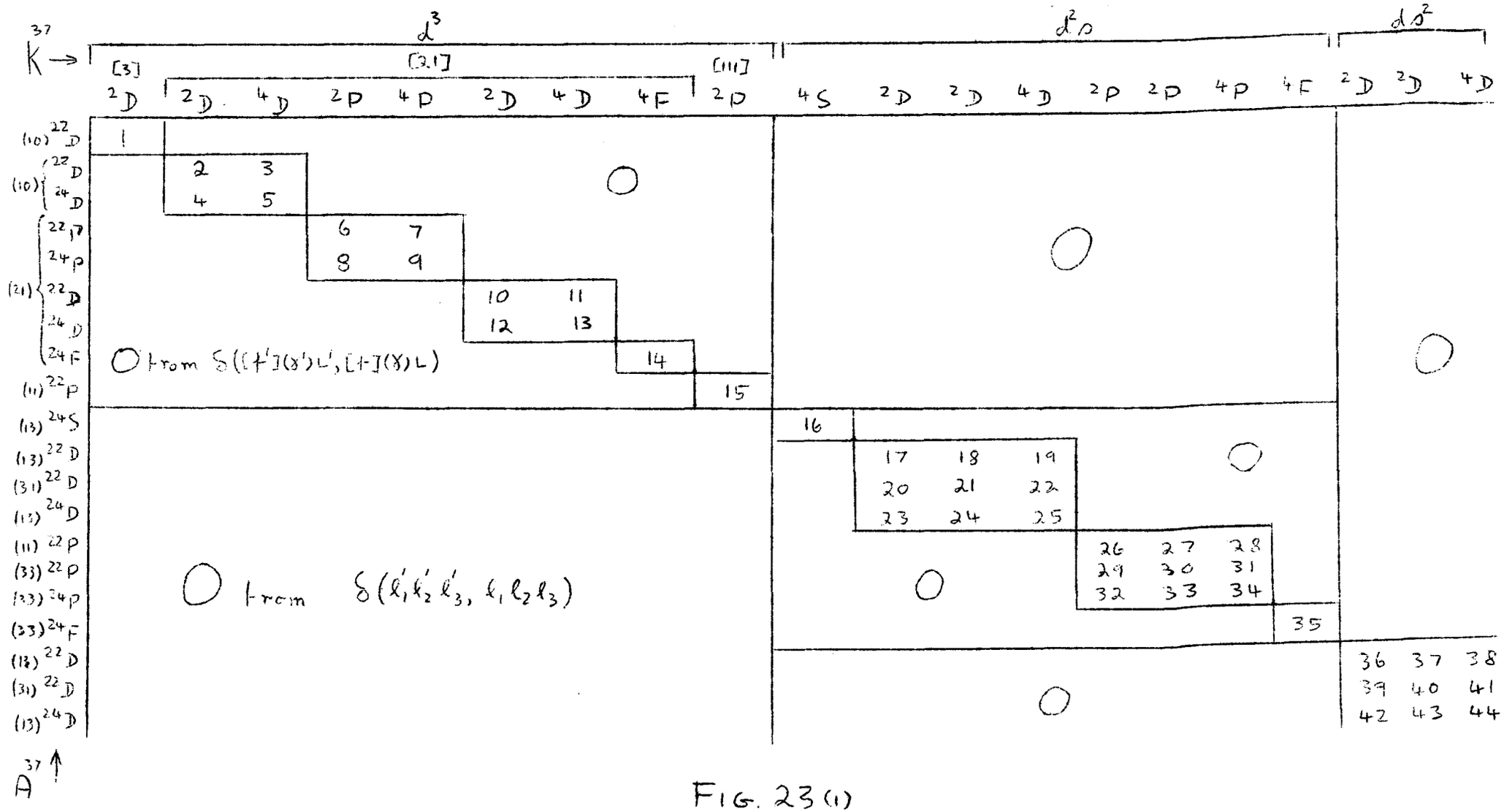

K → 37
A 37 ↑
d^3
$d^2 p$
$d s^2$
[3] [21] [111]
^2D ^2D ^4D ^2P ^4P ^2D ^4D ^4F ^2P ^4S ^2D ^2D ^4D ^2P ^2P ^4P ^4F ^2D ^2D ^4D
(10) ^{22}D
(10) ^{22}D ^{24}D
(21) ^{22}P ^{24}P ^{22}D ^{24}D ^{24}F
(11) ^{22}P
(13) ^{24}S
(13) ^{22}D
(31) ^{22}D
(13) ^{24}D
(11) ^{22}P
(33) ^{22}P
(33) ^{24}P
(33) ^{24}F
(13) ^{22}D
(31) ^{22}D
(13) ^{24}D
1
2 3
4 5
6 7
8 9
10 11
12 13
14
15
16
17 18 19
20 21 22
23 24 25
26 27 28
29 30 31
32 33 34
35
36 37 38
39 40 41
42 43 44
○ from $\delta([l']([s']L', [l](sL)$
○ from $\delta(l_1' l_2' l_3', l_1 l_2 l_3)$

FIG. 23(1)

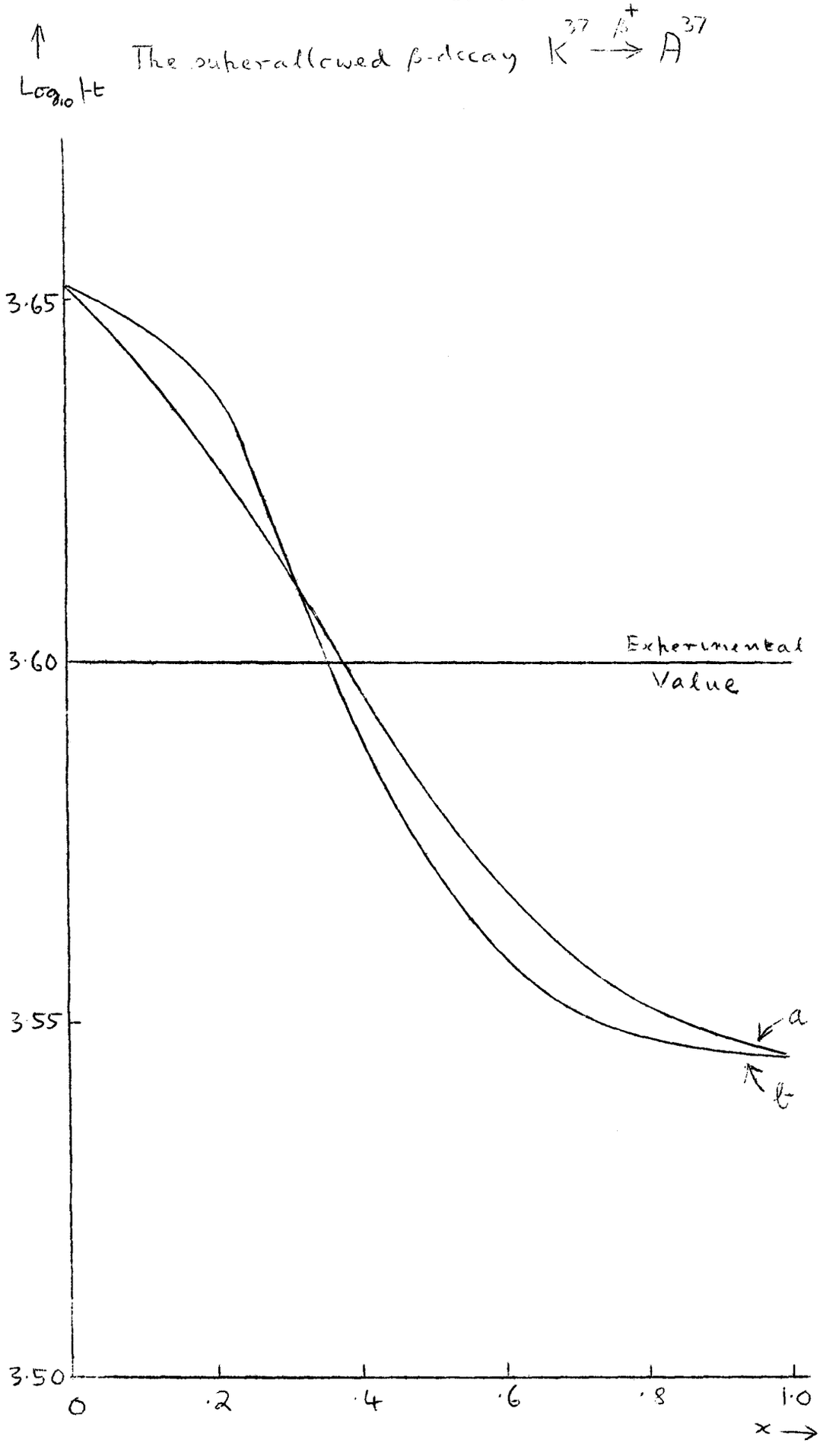
Value of matrix elements in fig. 23(1)

Element number	Value	Element number	Value
1	$+12/\sqrt{10}$	23	$+8\sqrt{2}/\sqrt{5}$
2	$-4/\sqrt{10}$	24	$+8\sqrt{2}/\sqrt{5}$
3	$+16/\sqrt{5}$	25	$+2\sqrt{2}/\sqrt{5}$
4	$+16/\sqrt{5}$	26	$+2\sqrt{10}$
5	$+4/\sqrt{10}$	27	$+4\sqrt{10}/3$
6	$+2\sqrt{10}/3$	28	$+8\sqrt{2}/3$
7	$+16/3$	29	$+4\sqrt{10}/3$
8	$+16/3$	30	$+2\sqrt{10}$
9	$+22\sqrt{2}/3\sqrt{5}$	31	$-8\sqrt{2}/3$
10	$-4/\sqrt{10}$	32	$+8\sqrt{2}/3$
11	$+16/\sqrt{5}$	33	$-8\sqrt{2}/3$
12	$+16/\sqrt{5}$	34	$+22\sqrt{2}/3\sqrt{5}$
13	$+4/\sqrt{10}$	35	$-6\sqrt{2}/\sqrt{5}$
14	$-6\sqrt{2}/\sqrt{5}$	36	$+4/\sqrt{10}$
15	$+10\sqrt{10}/3$	37	$-8/\sqrt{10}$
16	$+2\sqrt{10}$	38	$-8\sqrt{2}/\sqrt{5}$
17	$+4/\sqrt{10}$	39	$-8/\sqrt{10}$
18	$-8/\sqrt{10}$	40	$+4/\sqrt{10}$
19	$+8\sqrt{2}/\sqrt{5}$	41	$-8\sqrt{2}/\sqrt{5}$
20	$-8/\sqrt{10}$	42	$-8\sqrt{2}/\sqrt{5}$
21	$+4/\sqrt{10}$	43	$-8\sqrt{2}/\sqrt{5}$
22	$+8\sqrt{2}/\sqrt{5}$	44	$+2\sqrt{2}/\sqrt{5}$

FIG. 23 (2)

FIG. 24

The super-allowed β-decay $K^{37} \xrightarrow{\beta^+} A^{37}$



However the calculated mass 37 wavefunctions are expressed as a linear combination of certain LS-coupled states,

$$\psi = \sum_i a_i \psi_i$$

$$\psi' = \sum_j b_j \psi'_j$$

and so for the purpose of this calculation

$$G^2 = A\ N^2$$

$$= A \left(\sum_j b_j \psi'_j \| \mathcal{O} \| \sum_i a_i \psi_i \right)^2$$

$$= A \left(\sum_i \sum_j a_i b_j \left(\psi'_j \| \mathcal{O} \| \psi_i \right) \right)^2$$

where $\mathcal{O}$ represents the decay operator (see 6.2) and $\left(\psi'_j \| \mathcal{O} \| \psi_i \right)$ is just N evaluated between the known states ψ'_j, ψ_i using 6.5 .

G^2 can be evaluated by matrix algebra just as before, see 3.9 .

The elements of matrix $[\mathcal{O}]$ with rows and columns labelled by the LS-coupled states having $J = {}^3/_2$, $T = {}^1/_2$ are calculated directly from 6.5. Some sources of fractional parentage coefficients are mentioned in the first chapter.

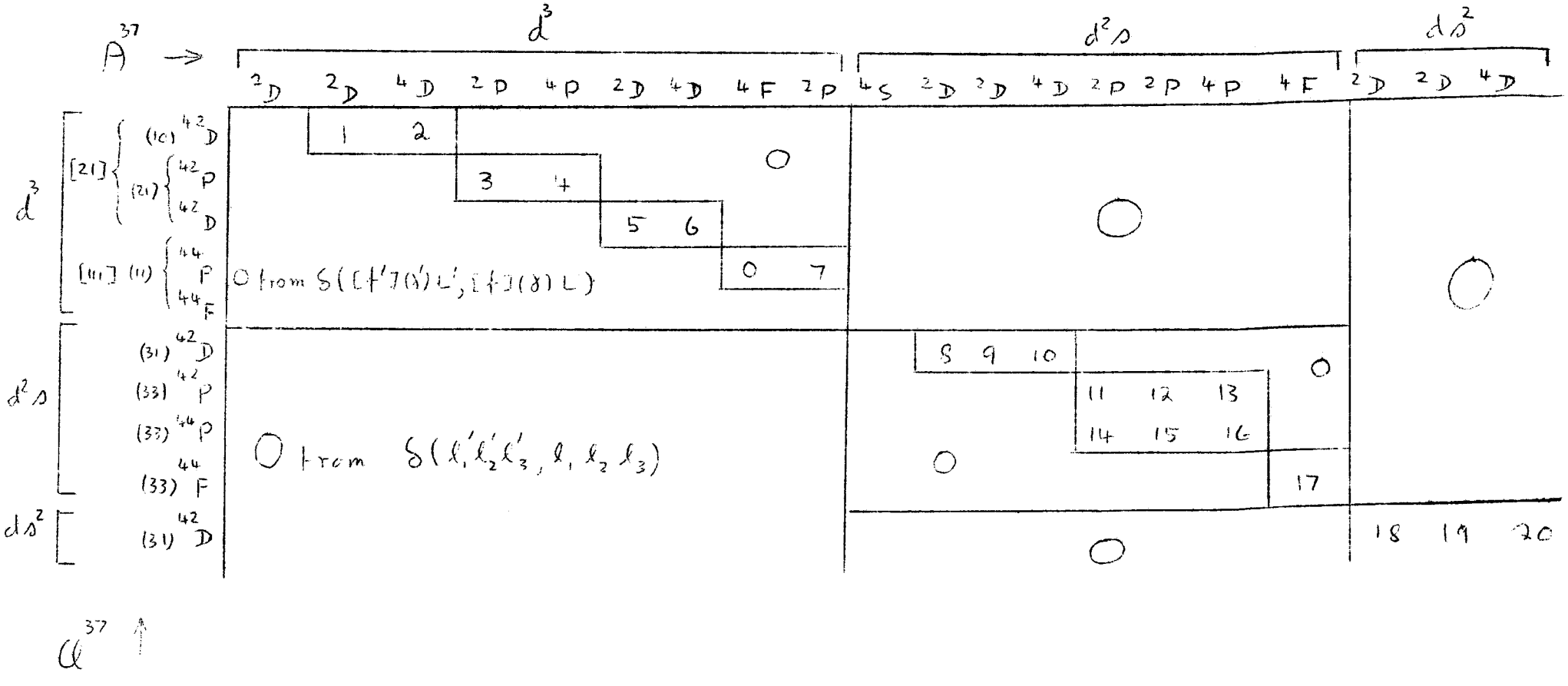
The matrix $[\mathcal{O}]$ and its elements are given for convenience in Fig: 23 (1) and (2).

In this decay A takes the value ${}^1/_{24}$ (see 3.5).

G^2 was evaluated as usual and $\log_{10}$ f t calculated from 3.1 using 6.1, the results obtained being illustrated in Fig: 24.

Curve a represents the Yukawa and curve b the mid range Gauss wavefunction.

If an acceptable calculated $\log_{10}$ ft lies between 3·5 and 3·7, then both interactions are satisfactory for $0 < x < 1$.



The detailed labelling of A^{37} states is given in fig. 23(1)

FIG. 25 (1)

Value of matrix elements in fig 25 (1)

Element number	Value
1	$+ 8\sqrt{2}/\sqrt{5}$
2	$- 16/\sqrt{5}$
3	$- 8\sqrt{10}/3$
4	$- 16/3$
5	$+ 8\sqrt{2}/\sqrt{5}$
6	$- 16/\sqrt{5}$
7	$- 16\sqrt{2}/3$
8	$+ 8/\sqrt{5}$
9	$+ 8/\sqrt{5}$
10	$+ 16/\sqrt{5}$
11	$- 8\sqrt{5}/3$
12	$+ 8\sqrt{5}/3$
13	$- 16/3$
14	$- 16/3$
15	$- 16/3$
16	0
17	0
18	$- 8/\sqrt{5}$
19	$- 8/\sqrt{5}$
20	$- 16/\sqrt{5}$

FIG. 25 (2)

Thus the calculation is very insensitive. The Gamow-Teller term is of the same order as the Fermi term, but does not vary much as the parameters change.

It can be said that this mirror transition is well enough described by the single particle model; the very slight effect of intermediate coupling being an improvement, although the calculation is too insensitive for deductions about the interaction parameters to be made.

The allowed decay $A^{37} \xrightarrow{E.C.} Cl^{37}$

The details of this decay are

$$A^{37}\ (3/2^+,\ T = 1/2) \xrightarrow{E.C.} Cl^{37}\ (3/2^+,\ T = 3/2)$$

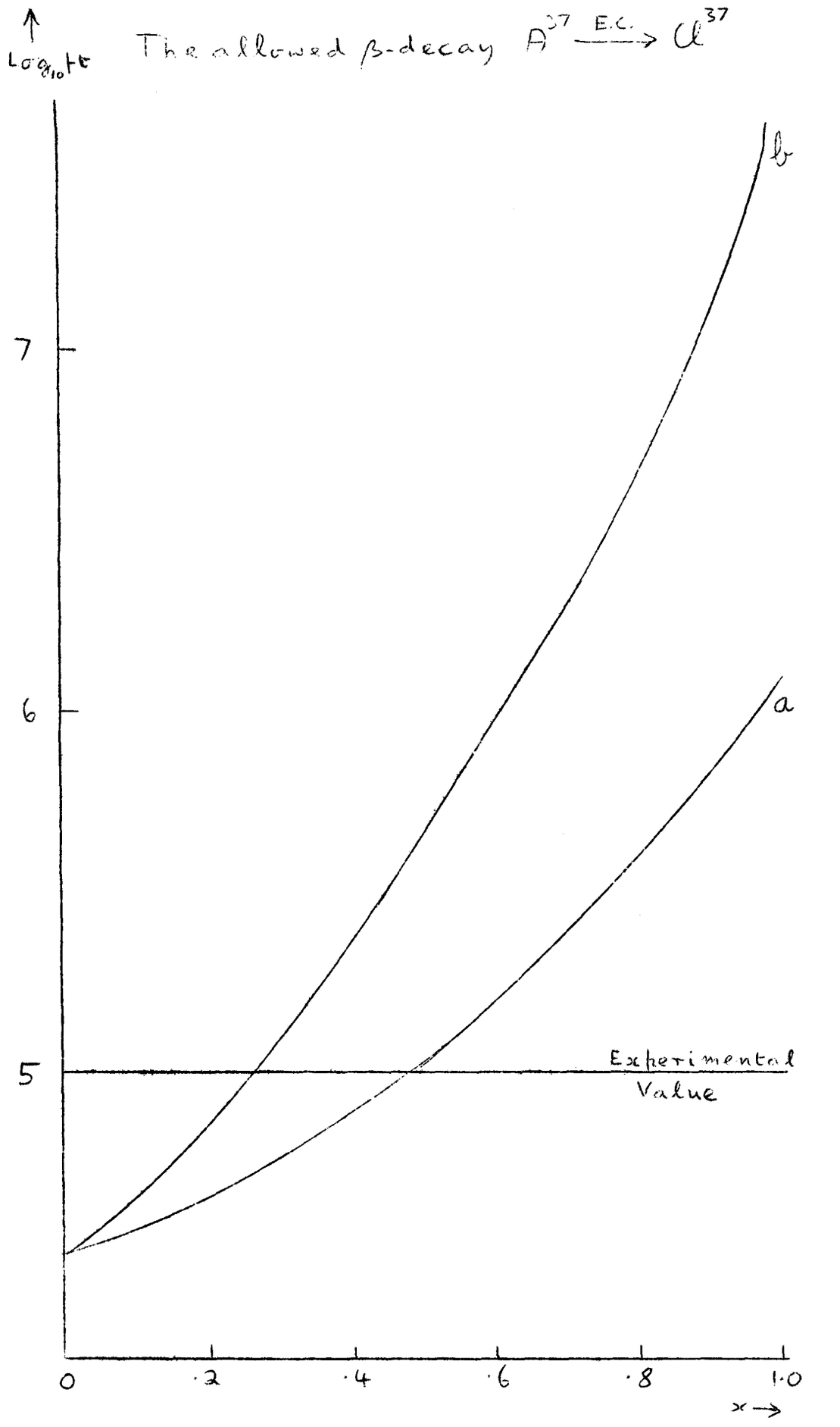
The E.C. (electron capture) process can be treated in the same way as a β-decay, since it involves the capture of an orbital electron by the nucleus and the conversion of a proton into a neutron, equivalent to a β^+-decay.

In this particular example only the Gamow-Teller term will contribute on account of the isotopic spin change.

The theory developed in the last section can also be applied here. Expression 6.5 may be used to evaluate elements of the matrix $[\mathcal{O}]$ and G^2 found from 3.9 as before. The matrix $[\mathcal{O}]$ together with its elements is given in Fig: 25 (1),(2). In this decay A takes the value $1/32$ (see 3.5).

The results obtained are illustrated in Fig: 26, where curve a represents the Yukawa and curve b the mid range Gauss interaction.

FIG. 26


Log10 ft
The allowed β-decay A^{37} E.C. → Cl^{37}
b
a
7
6
5
Experimental Value
0
·2
·4
·6
·8
1·0
x →

If an acceptable calculated $\log_{10} ft$ lies between 4·8 and 5·2, then x is restricted by

Curve a: $0{\cdot}34 < x < 0{\cdot}6$

Curve b: $0{\cdot}18 < x < 0{\cdot}35$

These values of x are rather smaller than those hitherto associated with these interactions, see Chapters 2 to 6.

The result should be slightly improved by raising the $d_{5/2}$ single hole level, which usually leads to smaller values of x being acceptable.
It will be seen that this calculation is more sensitive than the previous one, although it is still not sufficiently so to enable distinctions to be made between the interaction parameters.

<u>The forbidden decay $S^{37} \xrightarrow{\beta^-} Cl^{37}$</u>

The details of this decay are

$$S^{37}\ (\ (7/2)^-,\ T = 5/2\ ) \xrightarrow{\beta^-} Cl^{37}\ (\ 3/2^+,\ T = 3/2\ )$$

which is of the unique first forbidden kind, just like that discussed in Chapters 3 and 4.
It was decided first to evaluate $\log_{10} f_o t$ using a crude approximation to the S^{37} wavefunction as for the forbidden β-decay in Chapter 3.

<u>Calculation using crude S^{37} wavefunction</u>

The lowest energy odd parity configuration of S^{37} with $J = 7/2$ is

$$\left\{(p_{1/2})^2 (h_{11/2})^2 (h_{3/2})^4 (d_{5/2})^6 (2p_{1/2})^2 (d_{3/2})^4 f_{7/2}\right\}^{\text{neutron}}$$
$$\left\{(p_{1/2})^2 (h_{11/2})^2 (h_{3/2})^4 (d_{5/2})^6 (2p_{1/2})^2\right\}^{\text{proton}}$$

If this is to decay as a first forbidden transition to Cl^{37} $(^{3}/_{2}^{+})$ the only part of the chlorine ground state wavefunction that can contribute is

$$\left\{(p_{1/2})^2\,(h_{1/2})^2\,(h_{3/2})^4\,(d_{5/2})^6\,(2p_{1/2})^2\,(d_{3/2})^4\right\}_{\text{neutron}}$$
$$\left\{(p_{1/2})^2\,(h_{1/2})^2\,(h_{3/2})^4\,(d_{5/2})^6\,(2p_{1/2})^2\,d_{3/2}\right\}_{\text{proton}}$$

and so the nuclear states may be represented by the following simple wavefunctions

$$S^{37}: \quad f_{7/2}\,(1)\;X_{+}\,(1) \quad = \quad \psi$$

$$Cl^{37}: \quad d_{3/2}\,(1)\;X_{-}\,(1) \quad = \quad \psi'$$

These wavefunctions may be inserted in 3.12 and $f_o t$ found from 3.11.

Defining

$$N = \left(d_{3/2}(1)\,X_{-}(1)\,\|\,\left(T\,\tfrac{1}{2}\,\tau_{-}(\sigma'\times Y')^2\right)(1)\,\|\,f_{7/2}(1)\,X_{+}(1)\right)$$

as in 3.13 and using the properties of τ_-

$$N = \left(d_{3/2}(1)\,\|\,\left(T(\sigma'\times Y')^2\right)(1)\,\|\,f_{7/2}(1)\right)$$

which is closely related to the quantity evaluated in Chapter 3. This time

$$N = \sqrt{\frac{14}{11}}\;b$$

The value of 3.12 is now $^{7}/_{4\pi}\,b^2$

For this decay

$$E_o = 4{\cdot}7\ \text{MeV} = 9{\cdot}29\ (mc^2)$$

$$b = 5{\cdot}069 \times 10^{-3}\ (\hbar/mc)$$

the functions $a(z)$, $b(z)$ being approximately the same as in the Chapter 3 case.

Proceeding as before yields

$$\log_{10} f_o t = 6{\cdot}72$$

in the jj-coupling limit, to be compared with an experimental figure of 7·2.

So far no use has been made of the detailed Cl^{37} wavefunctions obtained from the mass 37 calculation. Writing an equation analagous to 3.15

$$\log_{10} f_o t = 6{\cdot}72 - 2\log_{10} \beta(x) \qquad 6.6$$

where $\beta(x)$ is now defined as the coefficient of that part of the Cl^{37} wavefunction which contributes to the decay, ie. $(d_{3/2}^{p})^{-3}$

Now the calculated wavefunctions are expressed in terms of a set of LS-coupled states and $\beta(x)$ is the coefficient of a jj-coupled state. The transformation matrix between LS- and jj-coupling is not readily available in this case and so use has been made of a slight trick in order to find β.

The following discussion has been restricted to d^3 configurations which are the only ones involved.

The calculated wavefunctions are expressed as a linear combination of the following LS-coupled states with $J = 3/2$, $T = 3/2$

$$\underbrace{(10)\,{}^{42}D \quad (21)\,{}^{42}P \quad (21)\,{}^{42}D}_{[21]} \qquad \underbrace{{}^{44}P \quad {}^{44}F}_{[111]\,(11)}$$

The corresponding set of jj-coupled states are formed from

$$\left(\tfrac{3}{2}\right)^3_{3/2} \quad \left(\tfrac{3}{2}\right)^2_2\left(\tfrac{5}{2}\right) \quad \left(\tfrac{3}{2}\right)\left(\tfrac{5}{2}\right)^2_0 \quad \left(\tfrac{3}{2}\right)\left(\tfrac{5}{2}\right)^2_2 \quad \left(\tfrac{5}{2}\right)^3_{3/2}$$

In general there is a matrix transformation between eigenvectors labelled by the LS-coupled states, and those labelled by the jj-coupled states,

$$\begin{matrix}(3/2)^3\\(3/2)^2(5/2)\\(3/2)(5/2)^2_0\\(3/2)(5/2)^2_2\\(5/2)^3\end{matrix}\begin{bmatrix}\beta(x)\\\cdot\\\cdot\\\cdot\\\cdot\end{bmatrix} = \begin{bmatrix}t_1 & t_2 & t_3 & t_4 & t_5\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\end{bmatrix}\begin{bmatrix}\rho_1(x)\\\rho_2(x)\\\rho_3(x)\\\rho_4(x)\\\rho_5(x)\end{bmatrix}\begin{matrix}(10)\,{}^{42}D\\(21)\,{}^{42}P\\(21)\,{}^{42}D\\{}^{44}P\\{}^{44}F\end{matrix}\begin{matrix}\}[21]\\\\\\\}[111](11)\end{matrix}$$

where $\beta(x)$ is the appropriate coefficient of the $(3/2)^3$ part of the wavefunction, t_1 to t_5 are the elements of the transformation matrix which enable β to be calculated; s_1 to s_5 are elements of an eigenvector expressed in terms of LS-coupled states which represents the Cl^{37} ground state.

If the t's were known, β could be determined for any LS-labelled eigenvector.

However in the jj-limit at $x = 0$ it is known that the jj-coupled wavefunction corresponding to the lowest eigenvalue is pure $(3/2)^3_{3/2}$ and since the mass 37 calculation was performed for $x = 0$, the d^3 part of an eigenvector labelled by LS-coupled states corresponding to the lowest eigenvalue is available.

Hence in the jj-limit the matrix expression above can be written

$$\begin{matrix}(3/2)^3\\(3/2)^2(5/2)\\(3/2)(5/2)^2_0\\(3/2)(5/2)^2_2\\(5/2)^3\end{matrix}\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}t_1 & t_2 & t_3 & t_4 & t_5\\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot & \cdot & \cdot\end{bmatrix}\begin{bmatrix}\rho_1(0)\\\rho_2(0)\\\rho_3(0)\\\rho_4(0)\\\rho_5(0)\end{bmatrix}\begin{matrix}(10)\,{}^{42}D\\(21)\,{}^{42}P\\(21)\,{}^{42}D\\{}^{44}P\\{}^{44}F\end{matrix}\begin{matrix}\}[21]\\\\\\\}[111](11)\end{matrix}$$

where from the normalisation of the wavefunctions

$$s_i(0) = t_i \;;\quad i = 1, \ldots 5.$$

Thus the t's are just the coefficients of the d^3 part of the eigenvector corresponding to the lowest eigenvalue obtained by diagonalising Hamiltonian matrix $[H]$ (described in Chapter 1, 1.2) for $x = 0$.

$\beta(x)$ is given by

$$\beta(x) = \sum_i t_i \rho_i(x) \qquad 6.7$$

where $s_i(x)$ are for the eigenvector corresponding to the lowest $J = {}^3/_2$, $T = {}^3/_2$ eigenvalue, representing the Cl^{37} ground state.

The correction in 6.6 proves to be small and will be illustrated for the Gauss mid range force in Fig: 28, curve a.

Calculation using corrected S^{37} wavefunction

It was seen in the last section that the forbidden β -decay of S^{37} could not be satisfactorily accounted for using a crude approximation for the S^{37} initial state, even when the Cl^{37} final state was considered in detail.

It was decided to try and improve the S^{37} wavefunction using the perturbation theory techniques outlined in Chapter 4.
As the calculation is very similar to that described before, very little detail has been included here.

The nuclear wavefunctions are expanded just as in 4.11 where ϕ now represents the S^{37} wavefunctions and ψ the Cl^{37} wavefunction.

	ϕ_0		ϕ_1		ϕ_2		ϕ_3		ϕ_4		ϕ_5				ψ_0	
	n	h	n	h	n	h	n	h	n	h	n	h			n	h
$f_{5/2}$											1					
$h_{1/2}$									1							
$h_{3/2}$			1				1									
$f_{7/2}$	1				1											
$d_{3/2}$				1		1		1		1		1				1
$p_{1/2}$				1												
$d_{5/2}$						5		5		5		5				
$h_{1/2}$																
$h_{3/2}$																
$p_{1/2}$																

ϕ_j are odd harity states with $J = 3/2$
ψ_0 is an even harity state with $J = 3/2$

Diagonal lines denote closed shells

FIG. 27

An equation analagous with 4.15 can be written

$$\left(\psi(Cl^{37}) \| \beta \| \phi(S^{37})\right)$$

$$= \sqrt{1-\Sigma_i \eta^2}\sqrt{1-\Sigma_i \varepsilon^2}\,(\psi_0 \| \beta \| \phi_0) + \sum_i \varepsilon_i \sqrt{1-\Sigma_i \eta^2}\,(\psi_i \| \beta \| \phi_0) \qquad 6.8$$

$$+ \sum_\gamma \eta_\gamma \sqrt{1-\Sigma_i \varepsilon^2}\,(\psi_0 \| \beta \| \phi_\gamma) + \sum_i \sum_\gamma \varepsilon_i \eta_\gamma (\psi_i \| \beta \| \phi_\gamma)$$

The possible ϕ_γ and ψ_i which can contribute are listed in Fig: 27. Once again all those states with one oscillator quantum ($\hbar\omega$) of excitation have been considered.

(1) The last term in 6.8 is of second order and so may be neglected.

(2) The second term in 6.8 is the coupling of ψ_i with ϕ_0 via the β operator. As ψ_i is an excited state above ψ_0 , it must have more than one proton in the $d3/2$ orbit and ϕ_0 has no protons in this orbit. The operator can only change one neutron into a proton, hence coupling between ψ_i and ϕ_0 via the β operator is impossible, so this term must be zero.

(3) Other states similar to the ϕ_γ listed in Fig: 27 will not contribute, for one of the following reasons:

(a) ϕ_γ must have $J = 7/2$

(b) The operator can only change angular momentum by one unit.

Finally 6.8 can be reduced to a form resembling 4.15

$$\left(\psi(Cl^{37}) \| \beta \| \phi(S^{37})\right)$$

$$= \beta(x)\left[\sqrt{1-\Sigma_i \eta^2}\,(\psi_0 \| \beta \| \phi_0) + \sum_\gamma \eta_\gamma (\psi_0 \| \beta \| \phi_\gamma)\right] \qquad 6.9$$

where ($\psi_0 \| \beta \| \phi_0$) was that term considered in the last section and $\beta(x)$ is given by 6.7.

The wavefunction ϕ_1 is written as an antisymmetric version (see 4.18) of

$$( )_0 \left\{ \{ h^n_{3/2} s^h_{1/2} \}_J d^h_{3/2} \right\}_{1/2}$$

The wavefunctions ϕ_2 to ϕ_5 are written as antisymmetric versions of

$$( )_0 \left\{ \{ l_j^n (d^h_{5/2})^5_{5/2} \}_J d^h_{3/2} \right\}_{1/2} \quad \text{where } l_j = h_{1/2}, h_{3/2}, h_{1/2}, h_{5/2}$$

whilst the corresponding ψ_0 are antisymmetric versions of

$$( )_0 \left\{ (s^h_{1/2})^2_0 \, d^h_{3/2} \right\}_{3/2} = ( )_0 \left\{ (d^h_{5/2})^6_0 \, d^h_{3/2} \right\}_{3/2}$$

(that these last expressions are equal can be seen by re-coupling the particle groups and taking into account the permutations)

In the above $( )_0$ denotes a group of closed shells. The angular momentum J in the ϕ_j wavefunctions must be equal to 2, a fact which appears during the calculation, as a tensor operator of rank 2 couples the group of particles with spin J to a group of particles with spin zero.

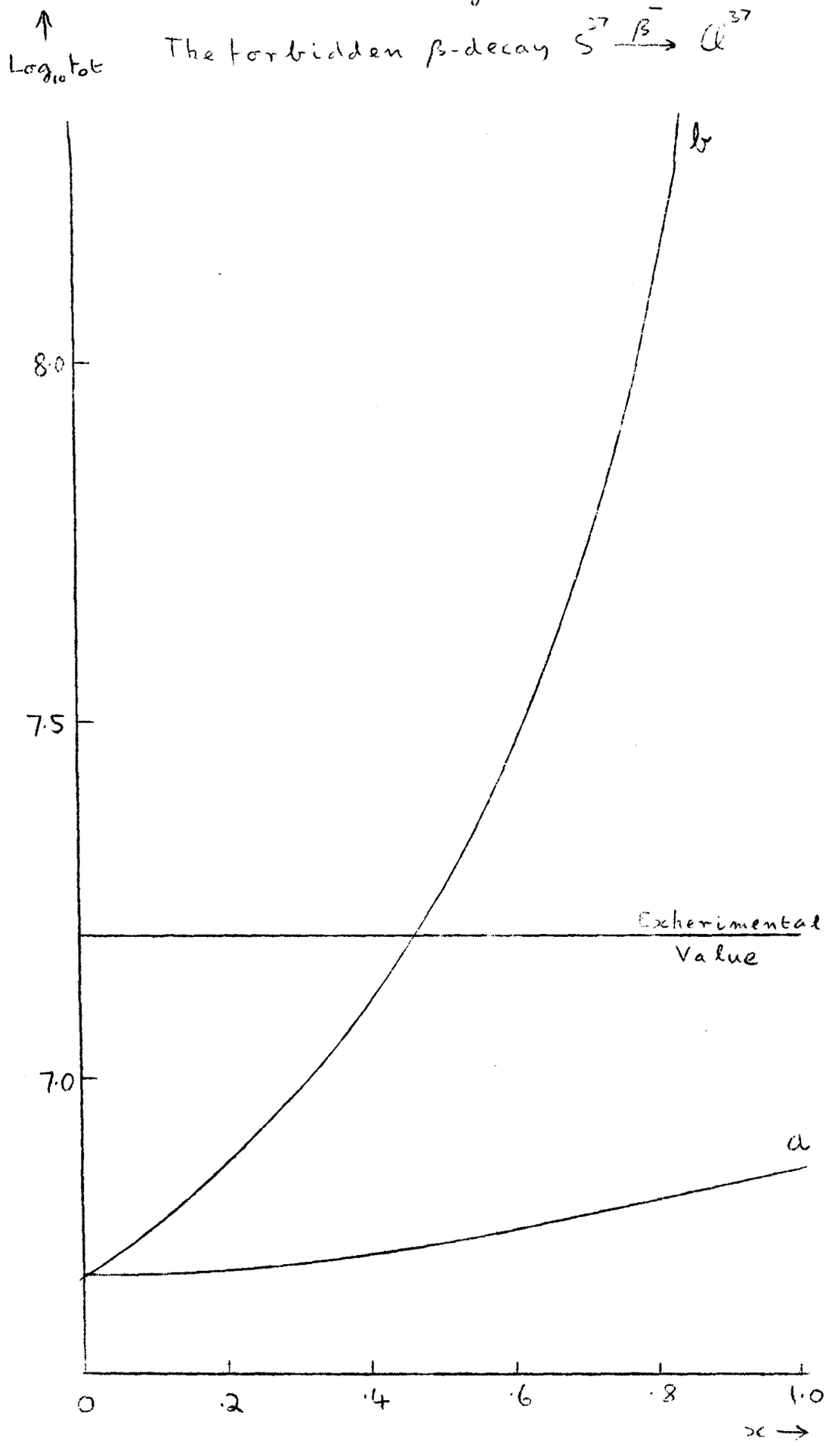
Defining

$$N(\phi_j) = \left(\psi_0 \| \sum_n^1 \tau^n_{1/2} r_- (\sigma^1 \times Y^1)^2 \| \phi_j \right)$$

just as before, the β-decay matrix elements are worked out as indicated in Chapter 4 using the wavefunctions described above.

j	$N(\phi_j)$	
1	$\sqrt{\frac{15}{2}}$	x $b/\sqrt{\pi}$
2	$3\sqrt{\frac{2}{5}}$	"
3	$-3/2\sqrt{\frac{7}{5}}$	"
4	$3\sqrt{\frac{2}{5}}$	"
5	$-2\sqrt{\frac{14}{15}}$	"

Fig. 28


The forbidden β-decay $S^{37} \xrightarrow{\beta^-} Cl^{37}$
$\uparrow$ $Log_{10} ft$
b
8·0
7·5
Experimental Value
7·0
a
0
·2
·4
·6
·8
1·0
$\alpha \rightarrow$

The η_j in 6.9 are given by 4.12 and are evaluated as before, the numerical values for the Gauss mid range force using the single particle energy differences of Chapter 4 being quoted below.

j	η_j	$\eta_j N(\phi_j)$
1	−0·0055 4105 x Vc	−0·0151 7479 x $b/\sqrt{\pi}$ x Vc
2	−0·0099 4368 "	−0·0352 9656 "
3	−0·0013 6331 "	+0·0024 1963 "
4	+0·0012 0272 "	+0·0022 8200 "
5	+0·0025 3620 "	−0·0049 0040 "

Equation 6.9 can now be written (c.f. 4.25)

$$\left(\psi((\ell^{37}) \| \beta \| \phi(S^{37})\right)$$

$$= \beta(x)\left[\sqrt{1-0.0001\,3932\,V_c^2}\,\sqrt{14} - 0.0506\,7012\,V_c\right] b/\sqrt{\pi} \qquad \underline{6.10}$$

$\mathrm{Log}_{10}\, f_o t$ can be found from 3.11 using 3.12 which now takes the value

$$1/8\ \ (\psi(Cl^{37}) \| \beta \| \phi\ (S^{37}))$$

and the result is shown in Fig: 28.

Curve a represents the calculation using a crude S^{37} wavefunction whilst curve b represents that using an improved wavefunction.

Once again the correction is of the right order of magnitude and sign.

If an acceptable calculated $\log_{10} f_o t$ lies between 7·0 and 7·4, then x should be restricted by

$$\text{curve b:}\quad 0{\cdot}32 < x < 0{\cdot}58$$

which is in agreement with other estimates of x for this interaction in both mass 37 and 38.

The effect of raising the $d5_{/2}$ single hole level will be felt mostly in the evaluation of perturbation coefficients and from the mass 38 result this should lead to the same level of agreement with experiment for a smaller value of x.

It is to be noticed that in this kind of calculation the jj-limit approximation to the forbidden β-decay $\log_{10} f_o t$ value is improved mainly by taking a more detailed wave-function for the odd parity state of the parent nucleus. The details of the daughter nucleus even parity wave-function are only of secondary importance

Chapter 7

Conclusions and Matters Arising

In this final chapter it is proposed to summarise the conclusions reached at each stage of the work, review the problem as a whole and lastly indicate extensions of the calculations; an outline of possible future investigation of topics arising from this thesis will be included.

The mass 38 calculation

The calculation of energy levels and wavefunctions for the mass 38 nuclei has proved to be a sensitive one. It was necessary to employ variations in exchange mixture, range of two-body interaction and single hole energy level in order to obtain even a fair approximation to the experimental spectra, (Chapter 2) . The wavefunctions were also sensitive to these parameters, exemplified by the calculation of β -decay log ft values, (Chapter 3) .

Concerning the energy levels:

(1) There has been great difficulty in keeping the lowest (1,0) level sufficiently high. This level tends to dip below the (3,0) ground state as the intermediate coupling parameter is increased. In the most satisfactory spectrum so far obtained the lowest (1,0) level has been kept above the (3,0) for a value of x consistent with the β -decay results, but is still too low.

(2) There has been difficulty in obtaining satisfactory splitting between the (0,1) and (2,1) levels, both of

which originate from the lowest configuration in jj-coupling. Even in the optimum spectrum this is still too small, the necessary correction being a raising of the (2,1) level and a lowering of the (0,1), which would also improve the position of the latter level relative to the lowest (1,0).

The bulk of these energy level calculations were performed at a time when the available experimental data was restricted to the four lowest levels in K^{38}, (excluding that at 1·7 MeV), and the two lowest in A^{38}. It is the variation of range and exchange parameters to which these levels are greatly sensitive. When some higher energy levels became known the calculated results were improved by variation of the $d5/2$ single hole level, which was uncertain from the beginning. Luckily the two effects are practically independent and the information concerning higher levels did not lead to reconsideration of range and exchange, which had been determined by fitting the lowest known levels.

There are three β -decays in mass 38 which have been of interest:

(1) The allowed decay $K^{38} \longrightarrow A^{38}$, which proved sensitive to range and exchange parameters, but insensitive to the single hole level position.

(2) The forbidden decay $Cl^{38} \longrightarrow A^{38}$, which proved almost completely insensitive to the A^{38} wavefunction. This decay could only be satisfactorily explained by taking a detailed wavefunction for Cl^{38} in apparent contradiction with the sucessful work of Goldstein and Talmi

who used a very simple approximation. A suggested resolution of this paradox was given at the end of chapter 4.

(3) The allowed decay of Ca^{38}, which is sensitive to the position of the $d5/_2$ single hole level. This decay will be mentioned again later. The result obtained in chapter 3 tended to confirm the experiment as far as it went, however there is a suggestion that the observations were incomplete.

In order to obtain a fair approximation to the low energy spectrum and explain the β decays, the following parameters were required:

(a) A Gauss mid range two-body interaction,

$a = 2{\cdot}31 \times 10^{-13}$ cm.

(b) A Soper-like exchange mixture

$$A^{13} = -1$$
$$A^{31} = -0{\cdot}7$$
$$A^{33} = -0{\cdot}26$$
$$A^{11} = +0{\cdot}5$$

(c) A $d5/_2$ single hole level above the 2·80 MeV minimum suggested by Middleton (Chapter 1) and provisionally taken at 4 MeV.

(d) An overall two-body force strength $V_c \sim 30$ MeV.

The range of the force used here is longer than that required by Elliott and Flowers (1955) at the beginning of the shell. This might appear inconsistent were it not for the fact that the nuclear mass has doubled and other workers (notably French and Raz, Levinson and Ford) require an even longer range force to fit the spectra of

Ca^{42}, Ca^{43} at the beginning of the pf shell (Chapter 2). These longer range forces could not be of use in the p shell, but do seem to be needed in the Ca^{40} region.

The mass 37 calculation

Only two detailed calculations were performed for the mass 37 nuclei, both utilising the minimum $d5/2$ single hole level position. One of these used the range and exchange parameters which proved most satisfactory in mass 38, the other had those found best for the mass 18, 19 problem by Elliott and Flowers (1955).

The calculations have proved on the whole rather insensitive to the various parameters:

(1) As far as energy levels are concerned the different interactions do predict different levels, but there is insufficient data available for serious comparison to be made. Also the effect of raising the single hole level has yet to be investigated.

(2) The single particle magnetic moment of Cl^{37} can be improved by intermediate coupling, but once again the calculation is insensitive to the parameters. The quadrupole moment of Cl^{37} is already close to the single particle value (indicating an undeformed nucleus). The slight effect of a very insensitive intermediate coupling calculation is to bring single particle and experimental values even closer.

(3) The calculation of several mass 37 β-decays is described in Chapter 6. One of these is a forbidden transition very similar to that in mass 38. The others again prove insensitive to the interaction

parameters and can equally well be explained with either set.

The forbidden β -decays

These calculations stand apart from the rest of the work described here. They were made because an interaction which proved satisfactory in describing other β-decays in mass 38 failed badly with $Cl^{38} \longrightarrow A^{38}$ when only a simple approximation was used for the Cl^{38} wavefunction.

When the odd parity (Cl^{38}, S^{37}) nuclear wavefunctions were improved using perturbation theory, the correction to the $\log_{10} f_0 t$ was of the right order of magnitude and sign, the final result proving almost independent of the detailed even parity wavefunction corresponding to the daughter nucleus (A^{38}, Cl^{37}).

Review of the mass 37, 38 nuclear structure problem

The work described in this thesis represents the first detailed investigation of the structure of the mass 37, 38 nuclei. Such an investigation has not been made before owing to lack of experimental information concerning the single hole energies in mass 39.

The calculations have been carried out using standard mathematical methods and techniques.
There is nothing startling or revolutionary about the results obtained; they indicate that this kind of shell model calculation can be expected to give a fairly reasonable account of nuclei at the end of the ds shell.

The agreement is not as good as that obtained by

Elliott and Flowers (1955) for mass 18, 19. In particular the energy levels have not been fitted so easily. There is doubt concerning the $d5/_2$ single hole level, indeed the difficulties may stem from Ca^{40} not being a good 'closed shell' nucleus. There are signs that this kind of shell model begins to break down at Ca^{40}; the absence of clearly defined single particle levels in mass 39, the apparent need to vary the range of the two-body interaction from nucleus to nucleus in going from the beginning of the ds shell to the beginning of the pf, both tend to support this idea.

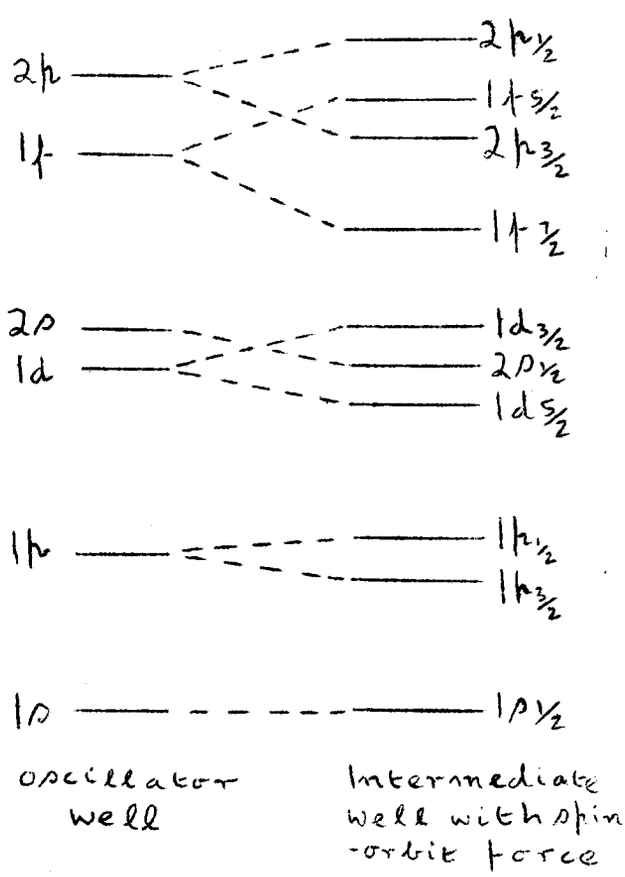
Elliott and Flowers (1955) find that the value of V_c required for the best spectrum is higher by about 5 MeV than that needed to fit the β-decay data. Conclusions of this kind may not be formed here, the agreement between calculation and experiment being insufficiently precise.

French and Raz (1956) have been quoted as saying that it was easier to fit the Ca^{43} spectrum than the Ca^{42}, (Chapter 2). Whether, when more experimental data becomes available, the three hole spectrum will be easier to fit than the two hole, remains to be seen; however there is less difficulty in fitting the β-decays in mass 37 than in mass 38 by virtue of the calculation being much less sensitive to the interaction parameters, (a fact also true for the Cl^{37} electromagnetic moments).

When thinking of possible ways to improve the calculation it is necessary to bear in mind the limitations of the model. It may be feasible to correct the lowest mass 38 levels by a small variation of range or

exchange mixture designed to improve the splitting between the (0,1), (2,1) levels without adversely affecting the others. Whether, if this could be done, the interaction would prove satisfactory in mass 37 is not obvious, but in any case more data is needed for this nucleus.

Otherwise an alternative approach at the end of a shell would be to consider the effect of higher configurations formed when particles are excited into the next shell. This is felt to be more realistic than a consideration of core particle excitations, which may lead to a deformed nucleus and are perhaps more likely at the beginning of a shell. It is difficult to justify these remarks and the following simple argument is not all-embracing.



The diagram illustrates the level scheme associated with a well shape intermediate between square and oscillator taking into account spin-orbit splitting. (Elliott and Lane (1957))

In the oscillator well the energy gaps between different shells are roughly equal, but this is not so for the more realistic well.

(a) Consider a nucleus at the beginning of the ds shell which in its lowest energy state has some holes in the $d5/_2$ orbit. Suppose it is desired to form a low energy configuration with one quantum of excitation.

The simplest possibilities are:

(1) The excitation of a $d5/_2$ particle into the lowest level of the next shell, ie. $f7/_2$.

(2) The excitation of a $p1/_2$ core particle into the ds shell lowest level, ie. $d5/_2$.

Of these less energy is required to form (2) and in this instance core excitations would probably be considered first when taking into account higher configurations.

(b) Consider a nucleus at the end of the ds shell which in its lowest energy state has some holes in the $d3/_2$ orbit. If it is desired to form a low energy configuration with one quantum of excitation the simplest possibilities are:

(1) The excitation of a $d3/_2$ particle into the lowest level of the next shell, ie. $f7/_2$.

(2) The excitation of a $p1/_2$ core particle into the lowest available ds shell level, ie. $d3/_2$.

Of these less energy is required to form (1) and so at the end of the shell it would probably be more realistic to consider this kind of excitation when taking into account higher configurations.

The result is unaltered if, in order to retain the parity of the original state, either

(1) the excitation of two particles by one quantum, or

(2) the excitation of one particle by two quanta to form higher configurations is considered.

These arguments seem quite consistent with the experimental fact that rotational spectra (associated with deformed nuclei and hence core excitations) are observed more towards the beginning of a shell than the end.

It is not then expected that a rotational model would improve the mass 37, 38 results; indeed judging from the quadrupole moment of Cl^{37} this nucleus does not appear in the least deformed (Chapter 5) .

Further work on the mass 37, 38 problem

Possibly the most urgent extension of the calculations described here is the evaluation of energy levels and wavefunctions for mass 37 using a higher value than 2·80 MeV for the $d5/2$ single hole level. It will be interesting to see precisely how this affects the spectrum and quantities deduced from the wavefunctions.

No attention has yet been paid to the calculation of electromagnetic moments in mass 38. This would be a quite straightforward and, if any experimental data is available, worthwhile exercise which might provide further information concerning the interaction parameters.

The possibilities of the Ca^{38} β -decay have not yet been exhausted. In its present state the theory suggests that decay to at least three $(1^+,0)$ levels in K^{38} should be observed and decay to only one has been recorded. It would not be difficult to extend the calculation and predict the branching ratio of the decay to each level, thereby estimating the lifetime of this Ca^{38} state.

In mass 37 there is an allowed β -decay

$$S^{37}\left((\tfrac{7}{2})^-\ T=\tfrac{5}{2}\right)\rightarrow Cl^{37}\left((\tfrac{5}{2},\tfrac{7}{2},\tfrac{9}{2})^-\ T=\tfrac{3}{2}\right)$$

with $\log_{10}$ ft = 4·2. A description of the odd parity S^{37} ground state was given in Chapter 6. The final nuclear state is probably the lowest lying odd parity state in Cl^{37} and is likely to consist of the configurations

$$\left\{(p_{\frac{1}{2}})^2\ (h_{\frac{1}{2}})^2\ (h_{\frac{3}{2}})^4\ (d_{\frac{5}{2}})^6\ (2p_{\frac{1}{2}})^2\ (d_{\frac{3}{2}})^4\right\}_{\text{neutron}}$$
$$\left\{(p_{\frac{1}{2}})^2\ (h_{\frac{1}{2}})^2\ (h_{\frac{3}{2}})^4\ (d_{\frac{5}{2}})^6\ (2p_{\frac{1}{2}})^2\ f_{\frac{7}{2}}\right\}_{\text{proton}}$$

and

$$\left\{(p_{\frac{1}{2}})^2\ (h_{\frac{1}{2}})^2\ (h_{\frac{3}{2}})^4\ (d_{\frac{5}{2}})^6\ (2p_{\frac{1}{2}})^2\ (d_{\frac{3}{2}})^3\ f_{\frac{7}{2}}\right\}_{\text{neutron}}$$
$$\left\{(p_{\frac{1}{2}})^2\ (h_{\frac{1}{2}})^2\ (h_{\frac{3}{2}})^4\ (d_{\frac{5}{2}})^6\ (2p_{\frac{1}{2}})^2\ d_{\frac{3}{2}}\right\}_{\text{proton}}$$

both of which are states of 'mixed' T. However it should be possible to find some combination of these which has $T = {}^3/_2$ and the decay might then be investigated.

The magnetic octupole moment of Cl^{37} has been measured according to Schwarz (Phys. Rev. 105; 180. 1957) The value quoted is $-0{\cdot}0146 \times 10^{-24}$ cm^2 and that predicted by the single particle model is $-0{\cdot}008 \times 10^{-24}$ cm^2 which is down by a factor 2. The predicted value is said to be small on account of cancellation between spin and orbital contributions. The magnetic octupole moments of Cl^{35}, Cl^{37} are somewhat unique in that they fall outside the Schmidt-like lines. It would be possible to calculate the octupole moment using the mass 37 wavefunctions and this might be more sensitive to the interaction parameters than were the magnetic and quadrupole moments.

Further application of the forbidden β -decay techniques

The main criterion for sucessful application of the Chapter 4 techniques is the existence of relatively simple models to describe the ϕ_f, ψ_i states.

Finally it must be emphasised that the work described here is only a first attempt at detailed investigation of ' the structure of the nuclei of mass 37 and 38.' As more experimental data is published the inadequacy must surely become obvious. Nevertheless it is by such painful processes as these that man gropes his way towards an understanding " de rerum natura."

Appendix I

Some expressions for the ds shell radial integrals

Each of these expressions should be further multiplied by a factor $\frac{1}{\sqrt{2\pi}}\, e^{1/2\alpha}\, \alpha^{1/2}$

See text (Chapter 1, page 18) for explanation of the symbols.

$$F^0(2s^2) = \frac{1}{96}\left[123\,Hh_1 - 632\,Hh_3 + 6160\,Hh_5 - 33600\,Hh_7 + 120960\,Hh_9\right]$$

$$F^0(d^2) = \frac{1}{120}\left[63\,Hh_1 + 56\,Hh_3 + 464\,Hh_5 + 1344\,Hh_7 + 24192\,Hh_9\right]$$

$$F^2(d^2) = \frac{5}{120}\left[63\,Hh_1 - 112\,Hh_3 - 112\,Hh_5 - 2688\,Hh_7 + 24192\,Hh_9\right]$$

$$F^4(d^2) = \frac{189}{40}\left[Hh_1 - 8Hh_3 + 48\,Hh_5 - 192\,Hh_7 + 384\,Hh_9\right]$$

$$F^0(d,2s) = \frac{1}{48}\left[15\,Hh_1 + 80\,Hh_3 + 272\,Hh_5 - 2688\,Hh_7 + 24192\,Hh_9\right]$$

$$G^2(d,2s) = \frac{5}{48}\left[15\,Hh_1 - 88\,Hh_3 + 848\,Hh_5 - 6720\,Hh_7 + 24192\,Hh_9\right]$$

$$M^2(d^2,d2s) = \frac{7\sqrt{10}}{48}\left[3Hh_1 - 4\,Hh_3 + 32\,Hh_5 - 672\,Hh_7 + 3456\,Hh_9\right]$$

Appendix II

Some Central Force Matrix Elements

s^2 matrix elements

$$\left.\begin{matrix}(^{13}S \mid {}^{13}S) \\ (^{31}S \mid {}^{31}S)\end{matrix}\right\} = \left\{\begin{matrix}A^{13} \\ A^{31}\end{matrix}\right\} F^0(2s^2)$$

ds matrix elements

$$\left.\begin{matrix}(^{13}D \mid {}^{13}D) \\ (^{31}D \mid {}^{31}D)\end{matrix}\right\} = \left\{\begin{matrix}A^{13} \\ A^{31}\end{matrix}\right\} \left(F^0(d,2s) + G^2(d,2s) \right)$$

$$\left.\begin{matrix}(^{33}D \mid {}^{33}D) \\ (^{11}D \mid {}^{11}D)\end{matrix}\right\} = \left\{\begin{matrix}A^{33} \\ A^{11}\end{matrix}\right\} \left(F^0(d,2s) - G^2(d,2s) \right)$$

Two-body coupling term matrix elements

$$(d^2 \mid s^2) \qquad \left.\begin{matrix}{}^{13}S \\ {}^{31}S\end{matrix}\right\} = \left\{\begin{matrix}A^{13} \\ A^{31}\end{matrix}\right\} \frac{G^2(d,2s)}{\sqrt{5}}$$

$$(d^2 \mid ds) \qquad \left.\begin{matrix}{}^{13}D \\ {}^{31}D\end{matrix}\right\} = \left\{\begin{matrix}A^{13} \\ A^{31}\end{matrix}\right\} - \frac{2M^2(d^2,d2s)}{\sqrt{35}}$$

It has not been found possible to include the three-body matrix element expressions; there are a great many of them and it hardly seems worthwhile devoting several pages to quantities which were not derived in the course of the other work described here.

Appendix III

Some mass 38 wavefunctions

These wavefunctions correspond to the calculation using the Gauss mid range force with $d5/2$ single hole level at 4 MeV and x = 0 4 (Vc 32 MeV)

(a) In IS-coupling

		(3,0)	(1,0)	(0,1)	(1,0)	(2,1)	(2,0)	(J,T)
d^2	^{13}S	·	+0·39	·	+0·77	·	·	
	^{31}S	·	·	+0·76	·	·	·	
	^{11}P	·	−0·30	·	−0·14	·	·	
	^{33}P	·	·	+0·63	·	−0·25	·	
	^{13}D	−0·07	−0·67	·	+0·19	·	−0·53	
	^{31}D	·	·	·	·	+0·63	·	
	^{11}F	+0·21	·	·	·	·	·	
	^{33}F	·	·	·	·	+0·69	·	
	^{13}G	+0·97	·	·	·	·	·	
	^{31}G	·	·	·	·	·	·	
$d_{\frac{5}{2}}$	[2] ^{13}D	+0·02	+0·54	·	−0·48	·	+0·82	
	^{31}D	·	·	·	·	−0·23	·	
	[1] ^{11}D	·	·	·	·	·	+0·22	
	^{33}D	·	·	·	·	−0·10	·	
p^2	^{13}S	·	+0·12	·	+0·34	·	·	
	^{31}S	·	·	+0·15	·	·	·	

(b) In jj-coupling

	(3, 0)
$(d_{5/2})^2_3$	−0.07
$(d_{5/2}\, d_{3/2})$	+0.19
$(d_{3/2})^2$	+0.98
$(d_{5/2}\, s_{1/2})$	+0.02

	(1,0)	(1,0)
$(d_{5/2})^2_1$	+0.25	+0.23
$(d_{5/2}\, d_{3/2})$	+0.04	+0.76
$(d_{3/2})^2$	−0.80	−0.14
$(d_{3/2}\, s_{1/2})$	−0.54	+0.48
$(s_{1/2})^2$	+0.12	+0.34

	(0,1)
$(d_{5/2})^2_0$	+0.99
$(d_{3/2})^2$	0
$(s_{1/2})^2$	+0.15

	(2,1)
$(d_{5/2})^2_2$	+0.09
$(d_{5/2}\, d_{3/2})$	−0.14
$(d_{3/2})^2$	+0.95
$(d_{5/2}\, s_{1/2})$	−0.12
$(d_{3/2}\, s_{1/2})$	−0.22

	(2,0)
$(d_{5/2}\, d_{3/2})_2$	−0.53
$(d_{5/2}\, s_{1/2})$	−0.34
$(d_{3/2}\, s_{1/2})$	+0.77

Appendix 4

Some mass 37 wavefunctions

These LS-coupled wavefunctions correspond to the lowest levels obtained using the Yukawa force with $x = 0.6$ ($V_c \sim 34$ MeV)

		$(\frac{3}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{5}{2},\frac{1}{2})$	$(\frac{7}{2},\frac{1}{2})$
d^3 [3] (10)	^2D	+0.75	·	+0.66	·
(30)	^2S	·	−0.48	·	·
	^2F	·	·	+0.28	−0.13
	^2G	·	·	·	−0.57
[21] (10)	^2D	+0.35	·	−0.22	·
	^4D	+0.22	+0.30	+0.33	−0.03
(21)	^2P	+0.11	−0.42	·	·
	^4P	+0.05	+0.27	+0.01	·
	^2D	+0.18	·	+0.10	·
	^4D	+0.19	−0.20	+0.09	−0.05
	^2F	·	·	+0.36	−0.17
	^4F	−0.33	·	+0.02	−0.15
	^2G	·	·	·	−0.39
	^4G	·	·	−0.32	−0.06
	^4H	·	·	·	+0.61
[111] (11)	^2P	−0.11	−0.23	·	·
	^2F	·	·	−0.17	−0.10
				·	·
d^2p	(13) ^2S	·	−0.30	·	·
	^4S	+0.01	·	·	·
	(31) ^2S	·	+0.37	·	·
	(13) ^2D	+0.11	·	+0.08	·
	^4D	+0.03	−0.09	0.00	+0.01
	(31) ^2D	−0.11	·	−0.09	·
	(13) ^2G	·	·	·	−0.20
	^4G	·	·	−0.03	−0.06
	(31) ^2G	·	·	·	+0.11
	(11) ^2P	+0.04	−0.15	·	·
	(33) ^2P	−0.03	+0.15	·	·
	^4P	+0.01	+0.20	0.00	·
	(11) ^2F	·	·	+0.02	−0.04
	(33) ^2F	·	·	−0.04	+0.02
	^4F	−0.07	·	−0.02	−0.02
ds^2	(13) ^2D	+0.08	·	+0.14	·
	^4D	+0.04	+0.03	+0.06	0.00
	(31) ^2D	−0.12	·	−0.06	·
p^3 [3]	^2S	·	+0.11	·	·

References

Blatt and Weisskopf: 'Theoretical Nuclear Physics'

Blin-Stoyle: 'Theories of Nuclear Moments' (1957)

Bohr and Mottelson: Dan. Mat. Fys. Medd. 27; 16. (1953)

Brody and Moshinsky: 'Tables of Transformation Brackets' (1960)

Cline and Chagnon: Phys. Rev. 108; 1495. (1957)

Elliott and Flowers: Proc. Roy. Soc. 229; 536. (1955)

Elliott and Flowers: Proc. Roy. Soc. 242; 57. (1957)

Elliott and Lane: Hand. der. Phys. 39; 241. (1957)

Endt and Braams: Rev. Mod. Phys. 29; 683. (1957)

French and Raz: Phys. Rev. 104; 1411. (1956)

Goldhammer: Phys. Rev. 101; 1375. (1956)

Goldstein and Talmi: Phys. Rev. 102; 589. (1956)

Grayso and Nordheim: Phys. Rev. 102; 1093. (1956)

Howell: Southampton University Ph.D Thesis (1960)

Jahn: Proc. Roy. Soc. 205; 192. (1957)

Kurath: Phys. Rev. 80; 98. (1950)

Kurath: Phys. Rev. 91; 1430. (1953)

Kurath: Phys. Rev. 101; 216. (1956)

Levinson and Ford: Phys. Rev. 100; 13. (1955)

Moszkowski: Hand. der. Phys. 39; 411. (1957)

Pandya and Shah: Nuc. Phys. 24; 326. (1961)

Racah: Phys. Rev. 62; 438. (1942)

Ross, Mark and Lawson: Phys. Rev. 102; 1613. (1956)

Schwarz: Phys. Rev. 105; 180. (1957)

Shortley and Fried: Phys. Rev. 54; 739. (1938)

Swiatecki: Proc. Roy. Soc. 205; 238. (1951)

Tauber and Wu: Phys. Rev. 93; 295. (1954)

Thieberger and Talmi: Phys. Rev. 102; 923. (1956)

Yamomoto and Steigert: Phys. Rev. 121; 600. (1961)

Index to results illustrated graphically

ACKNOWLEDGEMENTS

I wish to record my thanks to the Department of Scientific and Industrial Research for a three year studentship, without which this work could not have been done.

I have also to acknowledge the assistance provided by the Mathematics Department of Southampton University, and especially by the Sub-department of Computation.

It is a pleasure to express my gratitude to Dr. J.P.Elliott , who has always proved a source of inspiration and with whom I have had many hours of valuable discussion.

Lastly I should like to thank Miss Angela Francillon for her painstaking help in the typing of this thesis. I must, however, retain the responsibility for any errors myself!