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Stability of Nonlinear 2D Systems Described by the Continuous-Time Roesser Model

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Abstract—This paper considers systems with two-dimensional dynamics (2D systems) described by the continuous-time nonlinear state-space Roesser model. The sufficient conditions of exponential stability in terms of vector Lyapunov functions are established. These conditions are then applied to analysis of the absolute stability of a certain class of systems comprising a linear continuous-time plant in the form of the Roesser model with a nonlinear characteristic in the feedback loop, which satisfies quadratic constraints. The absolute stability conditions are reduced to computable expressions in the form of linear matrix inequalities. The obtained results are extended to the class of continuous-time systems governed by the Roesser model with Markovian switching. The problems of absolute stability and stabilization via state- and output-feedback are solved for linear systems of the above class. The solution procedures for these problems are in the form of algorithms based on linear matrix inequalities.

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1. INTRODUCTION

Multidimensional (nD) models characterize systems with dynamics that evolves in $n > 1$ independent directions. As possible examples, multidimensional image processing or data transmission in complicated electrical circuits can be mentioned. This paper studies the case of 2D systems. Exploration of 2D systems is motivated by their wide usage in different fields, in the first place, iterative learning control. Here the first process corresponds to dynamics at a current learning iteration, whereas the second process concerns the dynamics of learning process between successive iterations. For each iteration, the model of dynamics can be continuous or discrete, and learning process proper is described by a discrete model. It is not possible to construct a comprehensive systems theory for nD systems as a simple extension of known results for common (1D) systems; investigators have to develop special methods and approaches [1].

In the case of two dimensional systems with discrete dynamics, the models considered include the Roesser model [2], the Fornasini–Marchesini model [3] and the repetitive process model [1]. The Roesser model originates from image processing problems; here researchers separate out the dynamics of two components of the state vector (called the horizontal and vertical components). The Fornasini–Marchesini model (a doubly indexed dynamical system in the initial terminology of [3]) deals with a single state vector. A repetitive process differs from the Roesser model in the finite duration of its components. The models of repetitive processes are applied to iterative learning control problems, e.g., the paper [4] performed the corresponding experimental studies of 2D systems.

A considerable range of publications on control in 2D systems, including the case of uncertain parameter systems, scrutinized discrete-time linear systems. For instance, the paper [5] explored the robust stability of Fornasini–Marchesini systems through eigenvalue sensitivity analysis. The authors of [6] proposed the frequency approach and the Lyapunov function-based approach to analyze robust stability. In [7, 8] robust stability was studied using linear matrix inequalities (LMIs). The paper [9] solved the robust output-feedback stabilization problem for continuous-time linear Roesser systems.

Recent years demonstrated the appearance of research works focused on nonlinear 2D systems. For instance, the stability of nonlinear Fornasini–Marchesini systems was analyzed in [10]. The publications [11, 12] were dedicated to different types of stability in nonlinear discrete-time Roesser systems. This paper treats 2D systems described by the continuous-time nonlinear state-space Roesser model. Such models naturally arise in nuclear magnetic resonance spectroscopy problems [13], but still have rare occurrence in control problems. The sufficient conditions of exponential stability in terms of vector Lyapunov functions are derived. These conditions are employed to study a certain class of systems comprising a linear plant described by the continuous-time Roesser model with nonlinear feedback loop. By assumption, the existing nonlinearities in the feedback loop meet quadratic constraints. Common systems with such nonlinearities were almost completely examined within the framework of absolute stability theory [14] using the Popov criterion and the Kalman–Yakubovich–Popov Lemma. The present paper extends methods of absolute stability theory to the above class of 2D systems with efficient application of linear matrix inequalities.

Furthermore, the obtained results are generalized to the class of continuous-time nonlinear 2D systems governed by the Roesser model with possible failures. We investigate the absolute stability problem of continuous-time Roesser systems with feedback nonlinearities and possible failures modeled by switching in a finite-state Markov chain. In theory of 1D systems, such models are called Markovian switching systems or random structure systems [15–17]. The results of control theory development for such systems (stability, optimal and robust control) can be found, e.g., in [18–21]. The papers [22, 23] extended some results of 1D Markovian switching systems to the two-dimensional setting, namely, state-feedback stabilization and H_∞-control of discrete-time 2D systems with Markovian switching, described by the discrete-time Roesser model. And finally, linear-quadratic parametrization of stabilizing controls in discrete-time repetitive processes with Markovian switching was performed in [24].

2. STABILITY OF DETERMINISTIC CONTINUOUS-TIME ROESSER SYSTEMS

Consider a dynamic system described by the nonlinear state-space Roesser model

$$\begin{aligned} \frac{\partial}{\partial t_1} h(t_1,t_2) &= f_1(h(t_1,t_2), v(t_1,t_2), t_1, t_2), \\ \frac{\partial}{\partial t_2} v(t_1,t_2) &= f_2(h(t_1,t_2), v(t_1,t_2), t_1, t_2), \end{aligned} \tag{2.1}$$

where $h \in \mathbb{R}^{n_h}$ and $v \in \mathbb{R}^{n_v}$ denote the horizontal and vertical components of state vector, f_1 and f_2 are nonlinear functions such that $f_1(0,0,t_1,t_2) = 0$, $f_2(0,0,t_1,t_2) = 0$ and, under the initial conditions stated below, there exists a unique solution of the system (2.1), which enjoys continuity in t_1, t_2 and boundedness for any bounded $t = t_1 + t_2$. In the case of systems governed by ordinary differential equations, these properties are guaranteed by the well-known Lipschitz condition and linear growth condition.

The boundary conditions have the form $v(t_1, 0) = \hat{v}(t_1)$ for any $t_1 \geqslant 0$ and $h(0, t_2) = \hat{h}(t_2)$ for any $t_2 \geqslant 0$. In the sequel, we adopt two classes of the functions $\hat{v}(t_1)$ and $\hat{h}(t_2)$. The first class unites all functions possessing bounded norms on bounded intervals and vanish beyond them:

$$|\hat{h}(t)| \leqslant M_1, \text{ if } 0 \leqslant t \leqslant T_1; \qquad \hat{h}(t) = 0, \text{ if } t > T_1, \tag{2.2}$$
$$|\hat{v}(t)| \leqslant M_2, \text{ if } 0 \leqslant t \leqslant T_2; \qquad \hat{v}(t) = 0, \text{ if } t > T_2. \tag{2.3}$$

The second class consists of all functions whose norms are bounded above by decreasing exponential functions:

$$|\hat{h}(t)| \leqslant \kappa_1 \exp(-\varepsilon_1 t), \quad |\hat{v}(t)| \leqslant \kappa_2 \exp(-\varepsilon_2 t). \tag{2.4}$$

Here $\kappa_1,\ \varepsilon_1,\ \kappa_2,\ \varepsilon_2$ mean positive constants.

Definition 1. A system described by (2.1) with the boundary conditions (2.2), (2.3) or (2.4) is said to be exponentially stable if the inequality

$$|h(\tau, t-\tau)| + |v(\tau, t-\tau)| \leqslant \beta \exp(-\alpha t) \tag{2.5}$$

holds for $0 \leqslant \tau \leqslant t,\ \alpha > 0,\ \beta > 0$.

According to this definition, along the line $t_1 + t_2 = t$ the norms of the horizontal and vertical components $h(t_1,t_2),\ v(t_1,t_2)$ converge to the equilibrium state $h \equiv 0,\ v \equiv 0$ as $t \to \infty$ not slower than the exponential function with the rate $-\alpha$.

Introduce the vector function

$$V(h,v) = \begin{bmatrix} V_1(h) \\ V_2(v) \end{bmatrix}, \tag{2.6}$$

where $h \in \mathbb{R}^{n_h}$, $v \in \mathbb{R}^{n_v}$, $V_1(0) = 0$, $V_2(0) = 0$, $V_1(h) > 0$, $h \neq 0$, $V_2(v) > 0$, and $v \neq 0$. For this function, define the divergence operator along the trajectories of the system:

$$\mathrm{div} V(h(t_1,t_2), v(t_1,t_2)) = \frac{\partial V_1(h(t_1,t_2))}{\partial t_1} + \frac{\partial V_2(v(t_1,t_2))}{\partial t_2}. \tag{2.7}$$

Actually, the following statement is close to N.N. Krasovskii's results on the design of Lyapunov functions which satisfy special inequalities inherent to quadratic forms [25]. A similar assertion was established in [24] for discrete-time repetitive processes, in [11, 12] for Roesser systems and in [10] for Fornasini–Marchesini systems. In contrast to the 1D case, exponential stability of 2D systems under arbitrary boundary conditions has not yet been established.

Theorem 1. *Consider the system* (2.1) *with the boundary conditions* (2.2), (2.3) *or* (2.4). *Suppose that there exist positive constants* $c_1,\ c_2,\ c_3$ *such that the function* (2.6) *and its divergence* (2.7) *along the trajectories of the system meet the inequalities*

$$c_1|h(t_1,t_2)|^2 \leqslant V_1(h(t_1,t_2)) \leqslant c_2|h(t_1,t_2)|^2, \tag{2.8}$$
$$c_1|v(t_1,t_2)|^2 \leqslant V_2(v(t_1,t_2)) \leqslant c_2|v(t_1,t_2)|^2, \tag{2.9}$$
$$\mathrm{div} V(h(t_1,t_2), v(t_1,t_2)) \leqslant -c_3(|h(t_1,t_2)|^2 + |v(t_1,t_2)|^2). \tag{2.10}$$

Then the system (2.1) *is exponentially stable.*

The proof is given in the Appendix.

3. ABSOLUTE STABILITY OF CONTINUOUS-TIME ROESSER SYSTEMS

Consider a dynamic system described by the continuous-time Roesser model with nonlinear feedback

$$\begin{aligned}\begin{bmatrix}\dfrac{\partial}{\partial t_1}h(t_1,t_2)\\ \dfrac{\partial}{\partial t_2}v(t_1,t_2)\end{bmatrix} &= A\begin{bmatrix}h(t_1,t_2)\\ v(t_1,t_2)\end{bmatrix} + Bu(t_1,t_2),\\ z(t_1,t_2) &= C\begin{bmatrix}h(t_1,t_2)\\ v(t_1,t_2)\end{bmatrix}.\end{aligned}\qquad(3.1)$$

According to the dimensions of the horizontal and vertical variables, the matrices A, B and C have the block structure

$$A = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix},\quad B = \begin{bmatrix}B_1\\ B_2\end{bmatrix},\quad C = [C_1\ C_2],$$

the input variable $u(t_1,t_2)$ takes the form

$$u(t_1,t_2) = \varphi(z(t_1,t_2)),\quad \varphi(0) = 0,\qquad(3.2)$$

and the function $\varphi(z)$ meets the quadratic constraints

$$z^TQz + 2z^TS\varphi(z) + \varphi^T(z)R\varphi(z) \geqslant 0,\quad z\in\mathbb{R}^{n_z},\qquad(3.3)$$

where $Q = Q^T$, $R = R^T$ and S designate matrices of appropriate dimensions. Inequality (3.3) is a standard constraint in absolute stability theory [14]. The following definition is proposed based on this fact.

Definition 2. The system (3.1) is termed absolutely stable in the class of nonlinearities (3.2) if this system enjoys exponential stability for any nonlinear functions $\varphi(z)$ satisfying inequality (3.3).

The problem is formulated as follows: find the absolute stability conditions of the system (3.1) in the class of nonlinearities (3.2) and develop algorithms for efficient numerical verification.

Choose the components of a vector Lyapunov function as the quadratic forms

$$\begin{gathered}V_1(h(t_1,t_2)) = h^T(t_1,t_2)P_1h(t_1,t_2),\quad V_2(v(t_1,t_2)) = v^T(t_1,t_2)P_2v(t_1,t_2),\\ P_1 > 0,\quad P_2 > 0.\end{gathered}\qquad(3.4)$$

To ensure the absolute stability of the system (3.1) with the control law (3.2), the function (2.6) and its divergence must agree with the conditions of Theorem 1 for all $\varphi(z)$ meeting (3.3). Then application of the S-procedure [14, 26] shows it is necessary that

$$\begin{gathered}\mathrm{div}V(h(t_1,t_2),v(t_1,t_2)) + z^T(t_1,t_2)Qz(t_1,t_2)\\ + 2z^T(t_1,t_2)S\varphi(z(t_1,t_2)) + \varphi(z^T(t_1,t_2))R\varphi(z(t_1,t_2))\\ \leqslant -\varepsilon(|h(t_1,t_2)|^2 + |v(t_1,t_2)|^2).\end{gathered}\qquad(3.5)$$

Denote $x(t_1,t_2) = \begin{bmatrix}h(t_1,t_2)\\ v(t_1,t_2)\end{bmatrix}$. Next, evaluate the divergence along trajectories of the system (2.1), extract the perfect square of the sum of the variables x and φ in (3.5) and take advantage of Theorem 1. These operations establish the following theorem.

Theorem 2. *The system* (3.1) *with the control law* (3.2) *meeting* (3.3) *is absolutely stable in the class of nonlinearities* (3.2) *if the following LMIs are feasible in* $P = P_1 \oplus P_2$:

$$\begin{bmatrix} A^T P + PA + C^T QC + \varepsilon I & PB + SC \\ B^T P + C^T S^T & R \end{bmatrix} \leqslant 0, \qquad P = P_1 \oplus P_2 > 0. \tag{3.6}$$

These results can be generalized to the case of uncertain parameter plants described by the affine model. In this case, $A = A(\delta(t_1, t_2))$, $B = B(\delta(t_1, t_2))$, where

$$A(\delta) = A + \sum_{i=1}^{N} \delta_i A_i, \quad B(\delta) = B + \sum_{i=1}^{N} \delta_i B_i, \tag{3.7}$$

$\delta = [\delta_1 \ldots \delta_N]^T$ makes the uncertain parameter vector whose components $\delta_i(t_1, t_2)$ represent bounded functions (below and above):

$$\underline{\delta}_i \leqslant \delta_i \leqslant \overline{\delta}_i. \tag{3.8}$$

Denote by $\boldsymbol{\Delta}$ the set of uncertain parameters; the finite set of its vertices takes the form

$$\boldsymbol{\Delta}_v = \left\{ \delta = \begin{bmatrix} \delta_1 & \ldots & \delta_N \end{bmatrix} : \ \delta_i \in \{\underline{\delta}_i, \overline{\delta}_i\} \right\}. \tag{3.9}$$

Hence, by analogy, if for any $\delta \in \boldsymbol{\Delta}$ the linear matrix inequalities

$$\begin{bmatrix} A(\delta)^T P + PA(\delta) + C^T QC + \varepsilon I & PB(\delta) + SC \\ B(\delta)^T P + C^T S^T & R \end{bmatrix} \leqslant 0, \qquad P = P_1 \oplus P_2 > 0 \tag{3.10}$$

hold true, then the uncertain parameter system (3.1), (3.7) is absolutely stable. Recall the affine character of the above uncertainties. Inequalities (3.10) take place for all $\delta \in \boldsymbol{\Delta}$ iff they are valid merely on the finite set $\boldsymbol{\Delta}_v$, i.e., for $\delta \in \boldsymbol{\Delta}_v$. Therefore, the following theorem is proved.

Theorem 3. *The system* (3.1) *with the control law* (3.2) *satisfying* (3.3) *and uncertain parameters described by the affine model* (3.7) *appears absolutely stable in the class of nonlinearities* (3.2) *if the system of LMIs* (3.10), *where* $\delta \in \boldsymbol{\Delta}_v$, *is feasible in* $P = P_1 \oplus P_2$.

4. STABILITY OF CONTINUOUS-TIME ROESSER SYSTEMS WITH MARKOVIAN SWITCHING

Consider the Roesser system with possible failures:

$$\begin{aligned} \frac{\partial}{\partial t_1} h(t_1, t_2) &= g_1(h(t_1, t_2), v(t_1, t_2), r(t_1, t_2)), \\ \frac{\partial}{\partial t_2} v(t_1, t_2) &= g_2(h(t_1, t_2), v(t_1, t_2), r(t_1, t_2)), \end{aligned} \tag{4.1}$$

where g_1 and g_2 mean nonlinear functions, the boundary conditions $v(t_1, 0) = \hat{v}(t_1)$ and $h(0, t_2) = \hat{h}(t_2)$ represent deterministic functions satisfying the conditions (2.2), (2.3) or (2.4), $r(t_1, t_2)$ $(t_1, t_2 \geqslant 0)$ is a Markov process with the discrete set of states $\mathbb{N} = \{1, \ldots, \nu\}$ and transition rates defined by the expressions

$$\mathrm{Prob}(r(t_1+\tau, t_2) = j \mid r(t_1, t_2) = i) = \begin{cases} \pi_{ij}\tau + o(\tau), & \text{if } j \neq i \\ 1 + \pi_{ii}\tau + o(\tau), & \text{if } j = i, \end{cases} \qquad i, j = 1, \ldots, \nu,$$

$$\mathrm{Prob}(r(t_1, t_2+\tau) = j \mid r(t_1, t_2) = i) = \begin{cases} \omega_{ij}\tau + o(\tau), & \text{if } j \neq i \\ 1 + \omega_{ii}\tau + o(\tau), & \text{if } j = i, \end{cases} \qquad i, j = 1, \ldots, \nu,$$

$$r(\tau_1, 0) = r_1, \quad r(0, \tau_2) = r_2,$$

where $\pi_{ij} > 0$, $\omega_{ij} > 0$ $(i \neq j)$, $\pi_{ii} = -\sum_{i \neq j}^{\nu} \pi_{ij}$, $\omega_{ii} = -\sum_{i \neq j}^{\nu} \omega_{ij}$.

By assumption, for any $r \in \mathbb{N}$: $g_1(0,0,r) = 0$, $g_2(0,0,r) = 0$; moreover, there exists a unique solution of the system (4.1), whose trajectories are almost surely continuous in t_1 and t_2, and $\mathrm{E}[|h(t_1,t_2)|^2 + |v(t_1,t_2)|^2] < \infty$ for any bounded $t = t_1 + t_2$. Such properties of 1D systems with Markovian switching result from the Lipschitz condition and the linear growth condition [17].

Definition 3. The system (2.1) is termed to be mean-square exponentially stable if, under the boundary conditions (2.2), (2.3) or (2.4), we have

$$\mathrm{E}\left[|h(\tau, t-\tau)|^2 + |v(\tau, t-\tau)|^2\right] \leqslant \beta \exp(-\alpha t), \tag{4.2}$$

where E stands for expectation operator, $\alpha > 0$ and $\beta > 0$.

Consider the vector function

$$V(h,v,r) = \begin{bmatrix} V_1(h,r) \\ V_2(v,r) \end{bmatrix}, \tag{4.3}$$

where $h \in \mathbb{R}^{n_h}$, $v \in \mathbb{R}^{n_v}$, $r \in \mathbb{N}$ $V_1(0,r) = 0$, $V_2(0,r) = 0$, $V_1(h,r) > 0$, $h \neq 0$, $V_2(v,r) > 0$, $v \neq 0$, and $r \in \mathbb{N}$. Define some operators as the stochastic analogs of appropriate partial derivatives:

$$\begin{aligned}
\mathcal{D}^{(1)}_{h,v,r} V_1(h(t_1,t_2), r(t_1,t_2)) &= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathrm{E}[V_1(h(t_1+\Delta t, t_2), r(t_1+\Delta t, t_2)) \\
&\quad - V_1(h(t_1,t_2), r(t_1,t_2)) \mid h(t_1,t_2) = h, v(t_1,t_2) = v, r(t_1,t_2) = r], \\
\mathcal{D}^{(2)}_{h,v,r} V_2(v(t_1,t_2), r(t_1,t_2)) &= \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathrm{E}[V_2(v(t_1, t_2+\Delta t), r(t_1, t_2+\Delta t)) \\
&\quad - V_2(v(t_1,t_2), r(t_1,t_2)) \mid h(t_1,t_2) = h, v(t_1,t_2) = v, r(t_1,t_2) = r].
\end{aligned}$$

Introduce the stochastic analog of the divergence operator of the vector function (4.3) along trajectories of the system:

$$\begin{aligned}
&\mathcal{D}_{h,v,r} V(h(t_1,t_2), v(t_1,t_2), r(t_1,t_2)) \\
&= \mathcal{D}^{(1)}_{h,v,r} V_1(h(t_1,t_2), r(t_1,t_2)) + \mathcal{D}^{(2)}_{h,v,r} V_2(v(t_1,t_2), r(t_1,t_2)).
\end{aligned}$$

The following statement is the case.

Theorem 4. *Consider the system* (4.1) *with the boundary conditions* $\hat{v}(t_1)$ *and* $\hat{h}(t_2)$ *satisfying* (2.2), (2.3) *or* (2.4). *Suppose that there exist positive constants* c_1, c_2, c_3 *such that the function* V *and its operator* $\mathcal{D}$ *along trajectories of the system* (4.1) *meet the inequalities*

$$c_1 |h(t_1,t_2)|^2 \leqslant V_1(h(t_1,t_2), r(t_1,t_2)) \leqslant c_2 |h(t_1,t_2)|^2, \tag{4.4}$$

$$c_1 |v(t_1,t_2)|^2 \leqslant V_2(v(t_1,t_2), r(t_1,t_2)) \leqslant c_2 |v(t_1,t_2)|^2, \tag{4.5}$$

$$\mathcal{D}_{h,v,r} V(h(t_1,t_2), v(t_1,t_2), r(t_1,t_2)) \leqslant -c_3(|h|^2 + |v|^2). \tag{4.6}$$

Then the system (2.1) *is mean-square exponentially stable.*

The proof is given in the Appendix.

5. ABSOLUTE STABILITY OF CONTINUOUS-TIME ROESSER SYSTEMS WITH MARKOVIAN SWITCHING

Consider a dynamic system described by the linear Roesser model with possible failures and nonlinear feedback

$$\begin{aligned}\frac{\partial h(t_1,t_2)}{\partial t_1} &= A_{11}(r(t_1,t_2))h(t_1,t_2) + A_{12}(r(t_1,t_2))v(t_1,t_2) + B_1(r(t_1,t_2))u(t_1,t_2),\\ \frac{\partial v(t_1,t_2)}{\partial t_2} &= A_{21}(r(t_1,t_2))h(t_1,t_2) + A_{22}(r(t_1,t_2))v(t_1,t_2) + B_2(r(t_1,t_2))u(t_1,t_2),\\ z(t_1,t_2) &= C(r(t_1,t_2))\begin{bmatrix} h(t_1,t_2)\\ v(t_1,t_2)\end{bmatrix}.\end{aligned} \tag{5.1}$$

Here the input variable $u(t_1,t_2)$ has the form

$$u(t_1,t_2) = \varphi(z(t_1,t_2), r(t_1,t_2)), \quad \varphi(0,r) = 0, \tag{5.2}$$

and the function $\varphi(z,r)$ obeys the quadratic constraints

$$\begin{gathered} z^T Q(r) z + 2z^T S(r)\varphi(z,r) + \varphi^T(z,r)R(r)\varphi(z,r) \geqslant 0,\\ z\in\mathbb{R}^{n_z}, \quad \text{if} \quad r(t_1,t_2) = r,\end{gathered} \tag{5.3}$$

$Q(r) = Q^T(r),\ R(r) = R^T(r)$ and $S(r)$ specify matrices of appropriate dimensions.

Definition 4. The system (5.1) is said to be mean-square absolutely stable in the class of nonlinearities (5.2) if this system enjoys mean-square exponential stability for any nonlinear functions $\varphi(z)$ meeting inequality (5.3).

Similarly to the deterministic case, formulate the following problem: find the mean-square absolute stability conditions of the system (5.1) in the class of nonlinearities (5.2) and develop algorithms for efficient numerical verification. Choose the components of the vector Lyapunov function (4.3) as the quadratic forms

$$\begin{gathered} V_1(h,r)) = h^T P_1(r) h, \quad V_2(v,r) = v^T P_2(r) v,\\ P_1(r) > 0, \quad P_2(r) > 0, \quad r\in\mathbb{N}.\end{gathered} \tag{5.4}$$

To guarantee the absolute stability of the system (5.1) with the control law (5.2), the function (4.3) and its operator $\mathcal{D}$ have to meet the requirements of Theorem 4 for all $\varphi(z)$ satisfying (5.3). Again, application of the S-procedure [14, 26] shows it is necessary that

$$\begin{gathered} \mathcal{D}_{h,v,r}V(h(t_1,t_2), v(t_1,t_2), r(t_1,t_2)) + z^T Q(r) z + 2z^T S(r) + \varphi(z,r)\\ + \varphi^T(z,r)R(r)\varphi(z,r) \leqslant -\varepsilon(|h|^2 + |v|^2),\\ x\in\mathbb{R}^{n_x},\ r\in\mathbb{N}.\end{gathered} \tag{5.5}$$

Denote $\bar P(r) = \operatorname{diag}\,[\sum_{l=1}^{\nu} P_1(l)\pi_{rl} \sum_{l=1}^{\nu} P_2(l)\omega_{rl}]$, $\bar Q(r) = C^T(r)Q(r)C(r) + \varepsilon I$. Evaluate the operator $\mathcal{D}$ along trajectories of the system (5.1) and use Theorem 4 to get an important result.

Theorem 5. *The system* (5.1) *with the control law* (5.2) *meeting* (5.3) *is absolutely stable in the class of nonlinearities* (5.2) *if the LMIs*

$$\begin{bmatrix} A^T(r)P(r) + P(r)A(r) + \bar P(r) + \bar Q(r) & P(r)B(r) + S(r)C(r)\\ B^T(r)P(r) + C^T(r)S(r) & R(r)\end{bmatrix},$$

$$P(r) = P_1(r)\oplus P_2(r) > 0, \quad r\in\mathbb{N}$$

appear feasible in $P(r) = P_1(r)\oplus P_2(r)$.

6. STABILIZATION OF CONTINUOUS-TIME LINEAR ROESSER SYSTEMS WITH MARKOVIAN SWITCHING

6.1. State-Feedback Stabilization

In the sequel, for the sake of compact notation indexes will be adopted, i.e., $A(r), B(r)$ will be replaced by A_r, B_r, etc. Suppose that the control law in (5.1) represents the linear state-feedback

$$u(t_1, t_2) = -K_r x(t_1, t_2), \quad \text{if} \quad r(t_1, t_2) = r. \tag{6.1}$$

According to Theorem 4, for the mean-square exponential stability of the system (5.1), (6.1) it suffices that there exists a matrix

$$P_r = P_{1r} \oplus P_{2r}$$

satisfying the inequalities

$$\begin{gathered} (A_r - B_r K_r)^T P_r + P_r(A_r - B_r K_r) + \bar{P}_r + Q_r < 0, \\ P_r = P_{1r} \oplus P_{2r} > 0. \end{gathered} \tag{6.2}$$

Here $\bar{P}_r = \sum_{l=1}^{\nu} P_{1l}\pi_{rl} \oplus \sum_{l=1}^{\nu} P_{2l}\omega_{rl}, Q_r$ is a nonnegative definite symmetrical matrix, $r \in \mathbb{N}$. The Schur complement theorem [26] states that these inequalities appear feasible in a stabilizing pair (P_r, K_r) iff the LMIs

$$\begin{gathered} \begin{bmatrix} M_{11r} & M_{12r} \\ M_{12r}^T & M_{22r} \end{bmatrix} < 0, \\ X_r = [X_{1r} \oplus X_{2r}] > 0, \quad r \in \mathbb{N}, \end{gathered} \tag{6.3}$$

are feasible in X_r, Y_r, where

$$\begin{gathered} M_{11r} = (A_r X_r - B_r Y_r) + (A_r X_r - B_r Y_r)^T + [X_{1r}\pi_{rr} \oplus X_{2r}\omega_{rr}], \\ M_{22r} = -X_1 \oplus \ldots \oplus -X_{r-1} \oplus -I \oplus -X_{r+1} \oplus \ldots \oplus -X_\nu, \\ M_{12r} = \left[X_{11}\pi_{r1}^{\frac{1}{2}} \oplus X_{21}\omega_{r1}^{\frac{1}{2}}\right] \ldots \\ \left[X_{1r-1}\pi_{rr-1}^{\frac{1}{2}} \oplus X_{2r-1}\omega_{rr-1}^{\frac{1}{2}}\right] X_r Q_r^{\frac{1}{2}} \left[X_{1r+1}\pi_{rr+1}^{\frac{1}{2}} \oplus X_{2r+1}\omega_{rr+1}^{\frac{1}{2}}\right] \ldots \\ \left[X_{1\nu}\pi_{r\nu}^{\frac{1}{2}} \oplus X_{2\nu}\omega_{r\nu}^{\frac{1}{2}}\right]. \end{gathered}$$

The gain matrix of the stabilizing control law has the form $K_r = Y_r X_r^{-1}$. The matrix Q_r makes the analog of a weight matrix in the linear quadratic regulator (LQR) problem. By varying this matrix, one can affect the character of closed-loop processes.

6.2. Output-Feedback Stabilization

Consider the case when control represents the linear output-feedback law

$$u(t_1, t_2) = -F(r)z(t_1, t_2), \quad \text{if} \quad r(t_1, t_2) = r. \tag{6.4}$$

According to Theorem 4, the control law (6.4) guarantees the mean-square exponential stability of the system (5.1) if the LMIs

$$[A(r) - B(r)F(r)C(r)]^{\mathrm{T}} P(r) + P(r)[A(r) - B(r)F(r)C(r)] + \bar{P}(r) < 0, \quad r \in \mathbb{N} \tag{6.5}$$

are feasible in the matrices $F(r)$ and $P(r) = P_1(r) \oplus P_2(r) > 0$, $r \in \mathbb{N}$.

The following theorem provides the parametric description of the stabilizing gain matrices $F(r)$ which satisfy (6.5).

Theorem 6. *A matrix* $F(r)$ *meeting* (6.5) *exists iff there are matrices* $Q(r) = Q(r)^{\mathrm{T}} > 0$, $R(r) = R(r)^{\mathrm{T}} > 0$ *and* $L(r)$, *where* $r \in \mathbb{N}$, *such that*

$$F(r)C(r) = R(r)^{-1}\left[B(r)^{\mathrm{T}}P(r) + L(r)\right], \quad r \in \mathbb{N}.$$

Here $P(r) = P_1(r) \oplus P_2(r)$ *makes a solution for the system of inequalities*

$$A(r)^{\mathrm{T}}P(r) + P(r)A(r) - P(r)B(r)R(r)^{-1}B(r)^{\mathrm{T}}P(r) + \bar{P}(r)$$
$$+ Q(r) + L(r)^{T}R(r)^{-1}L(r) \leqslant 0, \quad r \in \mathbb{N},$$

and $\bar{P}(r) = \sum_{j=1}^{\nu} P_1(j)\pi_{rj} \oplus \sum_{j=1}^{\nu} P_2(j)\omega_{rj}$, $r \in \mathbb{N}$.

The proof of Theorem 6 is similar to the one for discrete-time repetitive processes, see [24]. Evaluation of the gain matrices $F(r)$ bases on a general result from [27]. According to this result, the solution of the optimization problem

$$\operatorname{tr}\sum_{r=1}^{N} W(r) \to \max \tag{6.6}$$

subject to the constraints

$$\begin{bmatrix} A(r)^{\mathrm{T}}W(r) + W(r)A(r) + Q(r) + \bar{W}_r & W(r)B(r) \\ B(r)^{\mathrm{T}}W & R(r) \end{bmatrix} \geqslant 0,$$

$$W(r) = W(r)^{\mathrm{T}} > 0, \quad \bar{W}(r) = \sum_{j=1}^{\nu} W(j)\pi_{rj}, \quad r \in \mathbb{N},$$

does coincide with the positive-definite solution $W(r) = W(r)^{\mathrm{T}}$ to the system of Riccati equations

$$A(r)^{\mathrm{T}}W(r) + W(r)A(r) - W(r)B(r)R(r)^{-1}B(r)^{\mathrm{T}}W(r) + \bar{W}(r) + Q(r) = 0, \quad r \in \mathbb{N}.$$

The same line of reasoning as in [24] brings to the following algorithm for computation of the gain matrices.

Algorithm 1.

1. Choose matrices $Q(r)$ and $R(r)$, $r \in \mathbb{N}$ using the principles of LQR theory [20, 21].
2. Solve the optimization problem

$$\operatorname{tr}\sum_{r=1}^{\nu} P(r) \to \max$$

subject to the constraints in the form of linear matrix equations and inequalities:

$$\left[B(r)^{\mathrm{T}}P(r) + L(r)\right]\left[I - C(r)^{+}C(r)\right] = 0,$$

$$\begin{bmatrix} A(r)^{\mathrm{T}}P(r) + P(r)A(r) + (1+\mu(r))Q(r) + \bar{P} & P(r)B(r) \\ B(r)^{\mathrm{T}}P(r) & R(r) \end{bmatrix} \geqslant 0,$$

$$\begin{bmatrix} \mu(r)Q(r) & L(r)^{\mathrm{T}} \\ L(r) & R(r) \end{bmatrix} > 0,$$

$$\bar{P}(r) = \sum_{j=1}^{\nu} P_1(j)\pi_{rj} \oplus \sum_{j=1}^{\nu} P_2(j)\omega_{rj}, \quad r \in \mathbb{N},$$

where the superscript "+" indicates the Moore–Penrose pseudoinverse.

3. If the optimization problem at Step 2 is feasible, find the stabilizing gain matrix by

$$F(r) = R(r)^{-1}\left[B(r)^{\mathrm{T}}P(r) + L(r)\right]C(r)^{+}, \quad r \in \mathbb{N}. \tag{6.7}$$

4. If the system of LMIs

$$[A(r) - B(r)F(r)C(r)]^{\mathrm{T}}S(r) + S(r)[A(r) - B(r)F(r)C(r)] + \bar{S}(r) < 0, \quad r \in \mathbb{N},$$

where $S(r) = S_{1r} \oplus S_{2r}$ and $\bar{S}(r) = \sum_{j=1}^{\nu} S_1(j)\pi_{rj} \oplus \sum_{j=1}^{\nu} S_2(j)\omega_{rj}$, is feasible in $S(r) = S(r)^{\mathrm{T}} > 0$, then $F(r)$ appears the stabilizing gain matrix.

7. CONCLUSION AND FUTURE RESEARCH

In the context of analysis, the complexity of systems studied in this paper (as any other 2D systems) concerns, first of all, the following. Their equations admit no explicit solution in derivatives (or first differences) of all state variables; thus, it is impossible to find the total increment of Lyapunov functions along trajectories of such systems. At the same time, the obtained results have an interesting interpretation which testifies to the medium level of their conservatism. Really, the condition $\operatorname{div} V(h(t_1, t_2), v(t_1, t_2)) < 0$ ensuing from Theorem 1 implies that the vector field $V(h(t_1, t_2), v(t_1, t_2))$ has a sink in each point on the plane (t_1, t_2) and decreasing flow along speed direction. This interpretation corresponds to standard conceptions of vector field theory and makes a compelling argument for developing the method of vector Lyapunov functions [28] for the above class of systems. Such argument gets substantiated by the recent paper [29]; within the framework of the behavioral approach, it was demonstrated that the property of asymptotic stability (in the sense of possible definitions) for linear difference 2D systems appears equivalent to the existence of a vector Lyapunov function with negative divergence. Meanwhile, this function represents a difficult-to-construct bilinear form. A modification of the inverse theorem for discrete-time nonlinear Roesser systems can be found in [12]. Note that divergent methods were intensively investigated for nonlinear 1D systems, see the monograph [30]. Unfortunately, those ideas gained no further development. Some interesting results in this field were derived independently in [31]; the author proposed a novel approach to asymptotic analysis of nonlinear dynamic systems.

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APPENDIX

Proof of Theorem 1. Consider the line $t_1 + t_2 = t$ on the plane $(t_1,\, t_2)$. Having in mind (2.8) and (2.9), integrate inequality (2.10) along this line to get

$$\begin{aligned} &\int\limits_{t_1+t_2=t} \left[\frac{\partial V_1(h(t_1,t_2))}{\partial t_1} + \frac{\partial V_2(v(t_1,t_2))}{\partial t_2}\right] ds \\ &\leqslant -\lambda \int\limits_{t_1+t_2=t} [V_1(h(t_1,t_2)) + V_2(v(t_1,t_2))] ds \\ &= -\lambda \left[\int\limits_{t_1+t_2=t} V_1(h(t_1,t_2)) ds + \int\limits_{t_1+t_2=t} V_2(v(t_1,t_2)) ds\right], \end{aligned} \tag{A.1}$$

where $\lambda = c_3/c_2$. The first integral in the right-hand side of (A.1) can be rewritten as

$$\int\limits_{t_1+t_2=t} V_1(h(t_1,t_2))ds = \int\limits_0^t V_1(h(t-\tau,\tau))\sqrt{2}d\tau.$$

By virtue of the Leibniz formula,

$$\frac{d}{dt}\left[\int\limits_{t_1+t_2=t} V_1(h(t_1,t_2))ds\right] = \sqrt{2}\left[\int\limits_0^t \frac{\partial}{\partial t}[V_1(h(t-\tau,\tau))]d\tau + V_1(h(0,t))\right].$$

Similarly,

$$\frac{d}{dt}\left[\int\limits_{t_1+t_2=t} V_2(v(t_1,t_2))ds\right] = \sqrt{2}\left[\int\limits_0^t \frac{\partial}{\partial t}[V_2(v(\tau,t-\tau))]d\tau + V_2(v(t,0))\right].$$

Therefore,

$$\frac{d}{dt}\left[\int\limits_{t_1+t_2=t} [V_1(h(t_1,t_2)) + V_2(v(t_1,t_2))]ds\right] \tag{A.2}$$

$$= \int\limits_{t_1+t_2=t} \left[\frac{\partial V_1(h(t_1,t_2))}{\partial t_1} + \frac{\partial V_2(v(t_1,t_2))}{\partial t_2}\right] ds + \sqrt{2}[V_1(h(0,t)) + V_2(v(t,0))].$$

Denote $W(t) = \int_{t_1+t_2=t}[V_1(h(t_1,t_2)) + V_2(v(t_1,t_2))]ds$. Due to (A.1), we have

$$\frac{d}{dt}W(t) + \lambda W(t) - \sqrt{2}[V_1(h(0,t)) + V_2(v(t,0))] \leqslant 0. \tag{A.3}$$

Solve the differential inequality (A.3) to obtain

$$W(t) \leqslant W(0)\exp(-\lambda t) + \int\limits_0^t \exp(-\lambda(t-\tau))\sqrt{2}[V_1(h(0,t)) + V_2(v(t,0))]d\tau$$

$$= \exp(-\lambda t)\sqrt{2}\int\limits_0^t \exp(\lambda\tau)[V_1(h(0,\tau)) + V_2(v(\tau,0))]d\tau.$$

Hence, it follows that

$$\int\limits_0^t \exp(\lambda t)[V_1(h(\tau,t-\tau)) + V_2(v(\tau,t-\tau))]d\tau \tag{A.4}$$

$$\leqslant \int\limits_0^t \exp(\lambda\tau)[V_1(h(0,\tau)) + V_2(v(\tau,0))]d\tau.$$

If the boundary conditions meet (2.2), (2.3), then (A.4) leads to

$$\lim_{t\to\infty}\int\limits_0^t \exp(\lambda t)[V_1(h(\tau,t-\tau)) + V_2(v(\tau,t-\tau))]d\tau$$

$$\leqslant \int\limits_0^T \exp(\lambda\tau)[V_1(h(0,\tau)) + V_2(v(\tau,0))]d\tau < \infty,$$

where $T = \max\{T_1, T_2\}$. The last inequality and formulas (2.8), (2.9) bring to validity of (2.5). In the case of the boundary conditions satisfying (2.4), it appears from (A.4) that

$$\int_0^t \exp(\delta t)[V_1(h(\tau, t-\tau)) + V_2(v(\tau, t-\tau))]d\tau \leqslant \frac{\kappa}{|\lambda - \gamma|}|\exp(-|(\lambda-\gamma)t|) - 1|,$$

where $\kappa = \max\{c_2\kappa_1^2, c_2\kappa_2^2\}$, $\gamma = \min\{2\varepsilon_1, 2\varepsilon_2\}$ and $\delta = \min\{\gamma, \lambda\}$. And so,

$$\lim_{t\to\infty}\int_0^t \exp(\delta t)[V_1(h(\tau, t-\tau)) + V_2(v(\tau, t-\tau))]d\tau \leqslant \frac{\kappa}{|\lambda-\gamma|}| < \infty.$$

Again, by taking into account (2.8) and (2.9), we arrive at (2.5). This concludes the proof.

Proof of Theorem 4. Using the definitions of the operators $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(2)}$, write down the following expressions as the analogs of the classical Leibniz formula:

$$\frac{d}{dt}\int_0^t \mathrm{E}[V_1(h(t-\tau, \tau), r(t-\tau, \tau))]d\tau \tag{A.5}$$
$$= \int_0^t \mathrm{E}\left[\mathcal{D}^{(1)}_{h,v,r}V_1(h(t-\tau,\tau), r(t-\tau,\tau))\right]d\tau + V_1(h(0,t), r_2),$$

$$\frac{d}{dt}\int_0^t \mathrm{E}[V_2(v(\tau, t-\tau), r(\tau, t-\tau))]d\tau \tag{A.6}$$
$$= \int_0^t \mathrm{E}\left[\mathcal{D}^{(2)}_{h,v,r}V_2(v(\tau,t-\tau), r(\tau,t-\tau))\right]d\tau + V_2(v(t,0), r_1).$$

In combination with (4.4) and (4.5), formula (4.6) implies that

$$\begin{aligned}&\mathrm{E}\left[\mathcal{D}^{(1)}_{h,v,r}V_1(h(t_1,t_2), r(t_1,t_2)) + \mathcal{D}^{(2)}_{h,v,r}V_2(v(t_1,t_2), r(t_1,t_2))\right]\\ &\quad\leqslant -\lambda[V_1(h(t_1,t_2), r(t_1,t_2)) + V_2(v(t_1,t_2), r(t_1,t_2))].\end{aligned} \tag{A.7}$$

Next, integrate inequality (A.7) along the line $t_1 + t_2 = t$ and take into consideration that (A.5), (A.6). Similarly to the proof of Theorem 1, we obtain

$$\begin{aligned}&\frac{d}{dt}\int_{t_1+t_2=t} \mathrm{E}\left[V_1(h(t_1,t_2), r(t_1,t_2)) + V_2(v(t_1,t_2), r(t_1,t_2))\right]ds\\ &\qquad - \sqrt{2}[V_1(h(0,t), r_2) + V_2(v(t,0), r_1)]\\ &\leqslant -\lambda\int_{t_1+t_2=t} \mathrm{E}[V_1(h(t_1,t_2), r(t_1,t_2)) + V_2(v(t_1,t_2), r(t_1,t_2))]ds.\end{aligned} \tag{A.8}$$

Designate $\hat{W}(t) = \int_{t_1+t_2=t}\mathrm{E}[V_1(h(t_1,t_2), r(t_1,t_2)) + V_2(v(t_1,t_2), r(t_1,t_2))]ds$. According to (A.8),

$$\frac{d}{dt}\hat{W}(t) + \lambda\hat{W}(t) - \sqrt{2}[V_1(h(0,t), r_2) + V_2(v(t,0), r_1)] \leqslant 0. \tag{A.9}$$

The differential inequality (A.9) is analogous to (A.3). Hence, further reasoning repeats the concluding part of proof in Theorem 1. And we naturally demonstrate validity of (4.2). This completes of the proof.

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