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Fine grained traffic state estimation and visualization

Simon Box MEng PhD, University of Southampton

Xiaoyu Chen PhD, University of Southampton

Simon Blainey BA MSc PhD FRGS MCIHT, University of Southampton

Stuart Munro, Siemens PLC

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Abstract

Tools for visualizing the current traffic state are used by local authorities for strategic monitoring of the traffic network and by everyday users for planning their journey. Popular visualizations include those provided by Google maps and by Inrix. Both employ a “traffic light” colour coding system where roads on a map are coloured green if traffic is flowing normally and red or black if there is congestion.

New sensor technology, especially from wireless sources (smartphones, 802.11p, Bluetooth), is enabling traffic state estimation with higher fidelity allowing the state on the road to be resolved down to the lane level.

A case study is reported in which a traffic micro-simulation test bed is used to generate high fidelity state estimates. An interactive visualization of the fine grained traffic state is presented. The visualization is demonstrated using Google Earth and affords the user a detailed 3D view of the traffic state down to the lane level. Traffic states are also updated and streamed into the visualization in real time at 0.1 Hz so that traffic dynamics can be observed.

1 Introduction

Visualizations of the state of traffic are used by many people and for many purposes. These range from everyday users of the traffic system, who are planning their journey [Inrix, 2013]; to traffic controllers, who monitor the network from control rooms and make critical control decisions [Seymour et al., 2007]; to transport researchers, who use visualization to understand the physics behind dynamic traffic phenomena [Ward and Wilson, 2011].

1.1 Motivation

The design of a useful traffic state visualization depends on information. That is, the information that is available and the information that the user needs. Recent trends in traffic data collection are leading to a significant increase in the amount of information that is available.

In recent history data on the traffic state has been gathered from sensors located in the road infrastructure. These include inductive loop sensors [Sreedevi, 2005], which are buried in the road surface and count the vehicles driving over them; Automatic Number Plate Recognition cameras [Qadri and Asif, 2009] that can detect and re-identify passing vehicles; and microwave detectors [Wood et al., 2006] that detect the presence of waiting vehicles and pedestrians at stop lines and crossings.

Now this set of traditional sensors is being augmented by the addition of new “wireless” data sources. The IEEE 802.11p standard for Wireless Access in Vehicular Environments (WAVE) [Jiang and Delgrossi, 2008] is enabling in-vehicle devices to have a network connection with other devices in vehicles and infrastructure. Along with smart-phones and new blue-tooth detectors [Lees-Miller et al., 2013], this is leading to an ambient cloud of data sources within the transport system that can provide rich information for traffic state estimation.

The problem of fusing data from all the available sources to produce a single coherent estimate of the traffic state has been the subject of much recent research (discussed in Section 2.1) and it is clear that the trend towards greater availability of wireless data will enable the estimation of traffic states with higher fidelity and frequency and finer granularity.

This motivates a visualization tool that is capable of displaying a fine grained estimate of the traffic state. Such a tool may not be suitable for the everyday user planning a trip, because it could lead to “information-

overload”. However it could afford professional traffic controllers a more detailed and up to date image of the traffic state leading to improved decision making. It could also be of value to transportation researchers seeking to understand the detailed dynamics of traffic systems.

1.2 Concept

This paper presents a concept for visualizing a fine grained estimation of traffic state. It describes the development of prototype software to produce the visualization and shows the results of a demonstration of the visualization using a simulation test bed. This sub-section gives an overview of the concept, the detailed development is described in Section 3.

Many approaches for traffic modelling and state estimation use a cell transmission [Daganzo, 1994] framework. Here the road network is divided into small sections (cells) and the traffic state is defined by certain metrics attached to these cells (Figure 4), examples of metrics are number of vehicles in cell i (N_i) and average speed of vehicles in cell i (\bar{V}_i).

The visualization concept is to display the cells on a three-dimensional map of the road network and to give the cells a dimension in height (altitude), which is proportional to one of the metrics defining the traffic state. The resulting effect looking like the static screenshots in Figures 5, 6, 7 and 8.

It should be possible to switch between metrics and also possible to use abstract metrics, which are functions of the basic metrics estimated in the state, for example “heaviness of traffic” or “deviation from normal conditions”. The resulting visualization should have a fast refresh rate to allow the user to perceive the dynamics of the state and be pan-able and zoom-able allowing the user to view the scene from different angles and different distances.

1.3 Contribution

The principal contributions of this paper are:

- A new concept for visualizing the traffic state with fine granularity.
- A case study of the implementation of this concept into visualization software.

- The results of a demonstration of the concept indicating its output in two traffic scenarios generated in a microsimulation model.

2 Background

2.1 Traffic state estimation

Traffic state estimation is the task of estimating *metrics* that describe the performance of the traffic network (e.g. speed, flow, density) and how these metrics vary spatially across the network and also temporally. Metrics are generally estimated using a combination of live and historical data from sensors within the traffic network system.

Traffic state estimation is central to many aspects of traffic monitoring and control. In some of these applications, for example signalized junction control, the estimated state is used as an input to an automated control system [Aboudolas et al., 2010, Box and Waterson, 2012a,b]. However in other applications such as strategic control and analysis, humans need to interpret the traffic state directly so tools for visualizing the traffic state are critical in these tasks.

A large amount of work has been done on traffic state estimation with many techniques being proposed. Examples and summaries can be found in [Wang et al., 2008, Tampere and Immers, 2007, Hegyi et al., 2006].

The state estimation method employed in the work described in this paper is reported in Box et al. [2012, 2013]. The approach employs an Extended Kalman Filter to combine sensor data with predictions from a cell transmission model. The framework for incorporating sensor data allows a wide range of sources, including static detector data (e.g. loops, bluetooth) and *probe* data (e.g. GPS, WiFi, cellphone). The method has been previously demonstrated on urban environment models [Box et al., 2012, 2013].

2.2 GIS-based visualization

As illustrated in Figure 1, visualization in Geographical Information Systems (GIS) can take place at three different layers. From the bottom up, each layer introduces richer visual features and user interactions while increasing requirements on interoperability. GIS-based visualization is capable of presenting a rich-featured and interactive environment, based on accu-

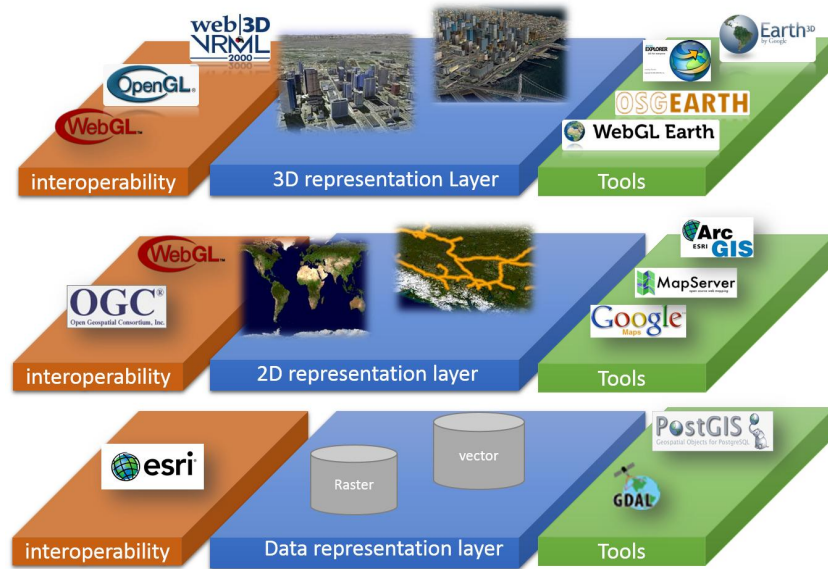


Figure 1: Architectural view of GIS-based visualization techniques and interoperability.

rate real-world geographically-referenced information, which can be shared online to allow end users to interactively engage with phenomena such as environmental hazards and traffic conditions. In recent years web-based map applications such as Google Maps and Bing Maps have enabled users to create custom overlays for a range of geographic data using internal or open source APIs. We characterize such GIS-specific and Web-based visualization solutions which generate 2-D maps to end users as the 2-D representation layer techniques.

GIS can go beyond ‘traditional’ cartographic representation by leveraging state-of-the-art 3D computer graphics and animations to visualize dynamic phenomena. The most popular example, Google Earth, has been applied to research topics ranging from weather [Smith and Lakshmanan, 2006] to public health surveillance [Chang et al., 2009] [Boulos, 2005]. Commercial GIS have also been used to create 3D visualisations of traffic, such as Wang’s [Wang, 2005] use of ArcGIS to drape colour-coded road links on a 3D terrain surface, and Cheng et al’s [Cheng et al., 2010] generation of “wall” maps of traffic delay using the same software. However, the use of such GIS can restrict sharing of data and visualisations, and in order to avoid lock-in to a

specific geo-visualization tool, open-source tools have been developed based on standard computer graphics techniques. For example, OSG Earth is an open-source software library based on OpenGL and Open Scene Graph and enables run-time terrain model generation, while WebGL Earth is another open-source software package based on HTML5 and WebGL enabling on-line visualization of maps and satellite imagery on top of a virtual terrain. Tominski et al. [2012] used OpenStreetMap data to create 3D stacked trajectory visualizations of micro-scale traffic data. These 3D geo-visualization techniques can be combined with 3D animation to allow analysis of dynamic phenomena [DiBiase et al., 1992].

Until recently the real time visualization of transport systems and their operation has been constrained by limits on both computing power and the availability of suitable data. While near-real time information on some transport operations has been available for several decades (for example, the TOPS train location system developed by the Southern Pacific Railroad in the 1960s and later adopted by British Rail) [Simmons and Biddge, 1997], this information was not usually made freely available, and visualization seldom developed beyond the textual. However, a growing quantity of both real time and historic traffic information is now publicly available in many countries as part of ‘open data’ initiatives and this, together with the increasing ubiquity and functionality of GIS, has driven innovation in visualization techniques. Websites provided by both transport operators and third parties now display public transportation data on interactive animated maps showing, for example, the current location of virtually all international flights via www.flightradar24.com and trains on the Belgian rail network via www.railtime.be. Techniques have also been developed for providing animated mapping of road traffic, such as the ArcGIS-based visualization of freeway traffic in San Diego developed by the San Diego Association of Governments [Chung, 2009] based on the PeMS system [Choe et al., 2002], and the online traffic mapping services provided by (for example) Google Maps and INRIX. Similarly, a range of GIS-like visualisation functionality has been incorporated in specialist traffic macro- and microsimulation software. For example, the TransModeler package provides options ranging from the display of quasi-3D individual vehicles at the micro scale through to traffic density maps and heat diagrams at the macro scale [Balakrishna et al., 2011]. Similar functionality is included in a number of other packages, such as Aimsun [Aimsun, 2013], Cube [Cube, 2013], Emme [Emme, 2013], OmniTRANS [OmniTrans, 2013], Paramics [citepparamics13], and PTV Vissim

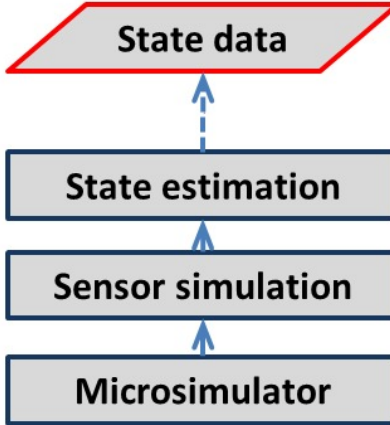


Figure 2: Simulation test bed flowchart.

[VisSim, 2013], meaning that the use of these visualisation techniques, along with others such as the display of traffic as flow bars of varying sizes, has become relatively widespread. This goes beyond the display of traffic flows, with for example Hilton et al. [Hilton et al., 2011] using heat maps to visualise traffic accidents. As the available data becomes increasingly fine-grained and multi-sourced, development of such visualizations seems likely to accelerate further.

3 Development

3.1 Simulation Test Bed

The need for fine grained traffic state visualization is motivated by the availability of high fidelity data on the traffic state (Section 1.1). However this is an emerging trend and these data are not currently readily available. Therefore the preferred method for developing the visualization concept is to *simulate* real time traffic data with high fidelity.

The core of the simulation test bed is S-Paramics “microsimulation” software. This models the accelerations on individual vehicles driving through a traffic network. S-Paramics has been widely used and validated in local authority projects in the UK e.g. Howard [2009], Alexander [2011].

Built around the S-Paramics simulator is a sensor simulation layer which

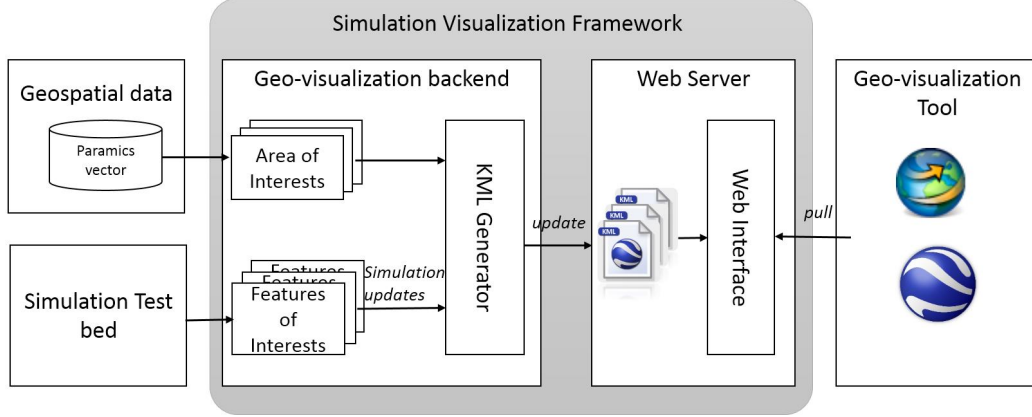


Figure 3: The Design of Simulation Visualization Framework.

can model the output of a large number of sensors including inductive loops, bluetooth, GPS enabled WiFi, GPS enabled smart-phones etc. The Sensor simulation techniques are described in Waterson and Box [2012].

The simulated sensor data are passed to the state estimation layer, which employs an Extended Kalman Filter and Cell transmission model as described in Box et al. [2012, 2013]. The output of the state estimation layer is the estimated metrics attached to each cell in the network, updated at a rate of 0.1 Hz (in simulation time). The flowchart structure for the test bed is shown in Figure 2. The simulation testbed is integrated into the visualization framework (Section 3.2) such that it could be replaced in future by a module which is processing real world data.

3.2 Visualization framework

Figure 3 shows the design of the simulation visualization framework, which presents a 3-D visualization (section 2.2) of the simulated traffic state in real time.

The framework employs two main data sources: the geospatial data comes directly from the S-Paramics Microsimulator and the simulated traffic states come from the simulation test bed (Section 3.1). The geospatial data are processed in order to generate traffic network cells (Figure 4). Notice that the granularity of the cells in this figure is very fine. In general this is the starting point and if lower granularity is required, this is achieved by agglomerating these small cells into larger ones.

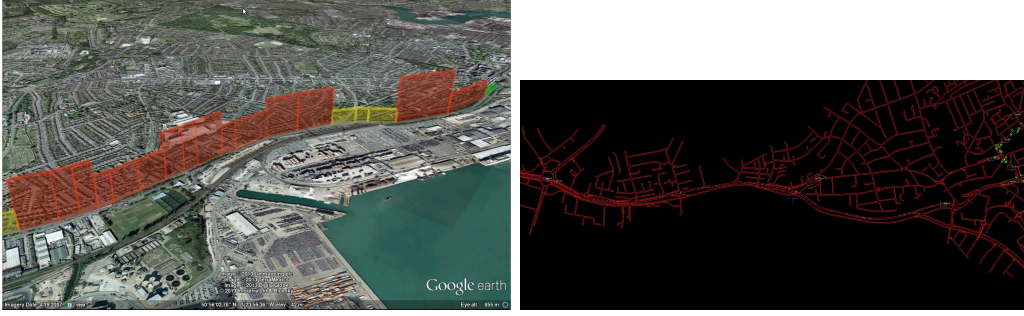


Figure 5: The arterial route scenario, showing on the left a screenshot of the visualization and on the right the network modelled is the simulation test-bed. In this visualization cells are coloured green if they contain $N_i \leq 15$ vehicles, yellow for $15 < N_i \leq 30$ vehicles and red for $30 < N_i$ vehicles.

pair of closely spaced signalized intersections where the A33 joins the A335. The modelled area is shown in Figure 7.

The arterial scenario is designed to show the capabilities of the developed visualization tool working at a granularity of data similar to that which is available in existing visualization tools such as Google traffic or INRIX.

The busy intersection scenario is designed to show the enhanced capabilities of the developed visualization tool to see what is happening “close up” in individual lanes at a complex system of intersections.

As discussed in Section 1.2 and 2.2 the height of the bars in the visualization can be configured to display any metric in the state estimation. As the demonstration of efficacy is not dependent on the metric used all visualizations presented use the same metric for simplicity: the number of vehicles in the cell N_i .

4.1 Arterial route scenario

In the Arterial route scenario a flow of 5500 vehicles/hour is modelled on the A33 into Southampton. This flow rate was calibrated against data supplied by the local authority, specifically data recorded from automatic number plate recognition cameras and inductive loop sensors during the weekday morning rush hour (07:00-09:30) between 09/05/2012 and 19/09/2012. The simulation begins with no vehicles on the road and as the simulation progresses the number of vehicles driving into the simulation can be seen in the

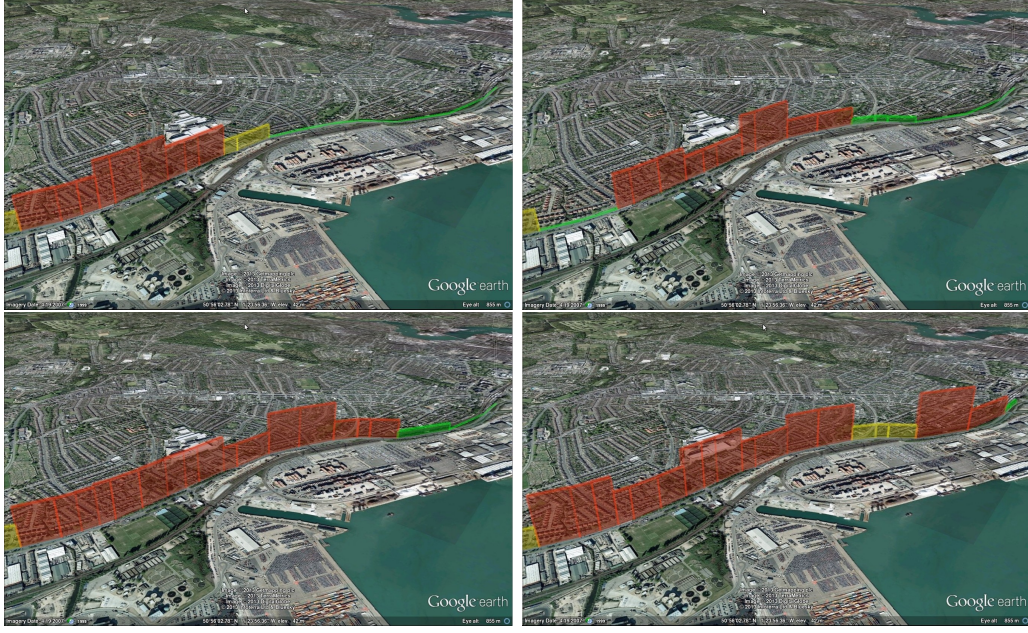


Figure 6: Four progressive screenshots of the arterial route scenario taken at 70, 89, 110 and 130 seconds into the simulation. In this visualization cells are coloured green if they contain $N_i \leq 15$ vehicles, yellow for $15 < N_i \leq 30$ vehicles and red for $30 < N_i$ vehicles.

visualization.

Figure 6 shows four snapshots of the visualization taken at 70, 89, 110 and 130 seconds into the simulation respectively.

The granularity of the visualization in Figure 6 is similar to that of other traffic visualizations. However the use of the third spatial dimension to indicate the number of vehicles in each cell gives a finer level of detail for comparison between cells than a graded colour map in two dimensions.

In the last image in Figure 6 some emergence of traffic dynamics can be observed. The heights of the cells indicate that traffic is not uniformly distributed along the road but “bunched” with some cells highly populated and some cells relatively empty.

The resolution of this information suggest that this visualization could be useful to researchers studying the effects of dynamic instability phenomena on busy arterial routes, for example the “phantom jam” [Wilson, 2008].

4.2 Busy intersection scenario

In the Busy intersection scenario a average flow of 3400 vehicles/hour passes through each of the two junctions. This flow rate was calibrated against data for this area in the same local authority dataset described in the arterial route scenario above. The size of the cells in this scenario are smaller than in the arterial route scenario, with some cells covering a single lane of multi lane road.

Figure 7 shows the visualization and the corresponding screenshot from the microsimulation (showing vehicle locations) at times 1060, 1090 and 1100 seconds into the simulation respectively.

What can be seen is that in the first pair of images at time 1060s is that there are a large number of vehicles queuing to turn right at the left hand junction. These show up in the visualization as a large signal in the cell over that lane.

In the second pair of images at time 1090s the right turning vehicles have been given priority by the traffic lights and the queue has started to discharge, correspondingly the signal in this cell is reduced.

In the last pair of images at time 1100s the queue has almost completely discharged and the signal is low. However the signal has now grown on one of the other arms to the junction, which has been held at a red light in the mean time and a queue has built up.

Figure 7 demonstrates how the fine granularity of detail in this visualization allows the user to resolve detailed individual junction movements and how the distribution of queues varies when the lights change.

Another aspect of the visualization that is difficult to appreciate on paper is the ability of the user to change their viewing angle on the scene and to zoom in and out. This is very helpful in interpreting what is happening at all the lanes around the junctions. Figure 8 shows two alternative views of the scene.

5 Conclusions and Future Work

The trend in increasing fidelity, frequency and granularity of traffic state estimation is driven by new technology, both hardware (smart-phones, 802.11p) and software (filters, models).

The availability of new information motivates visualization tools to help

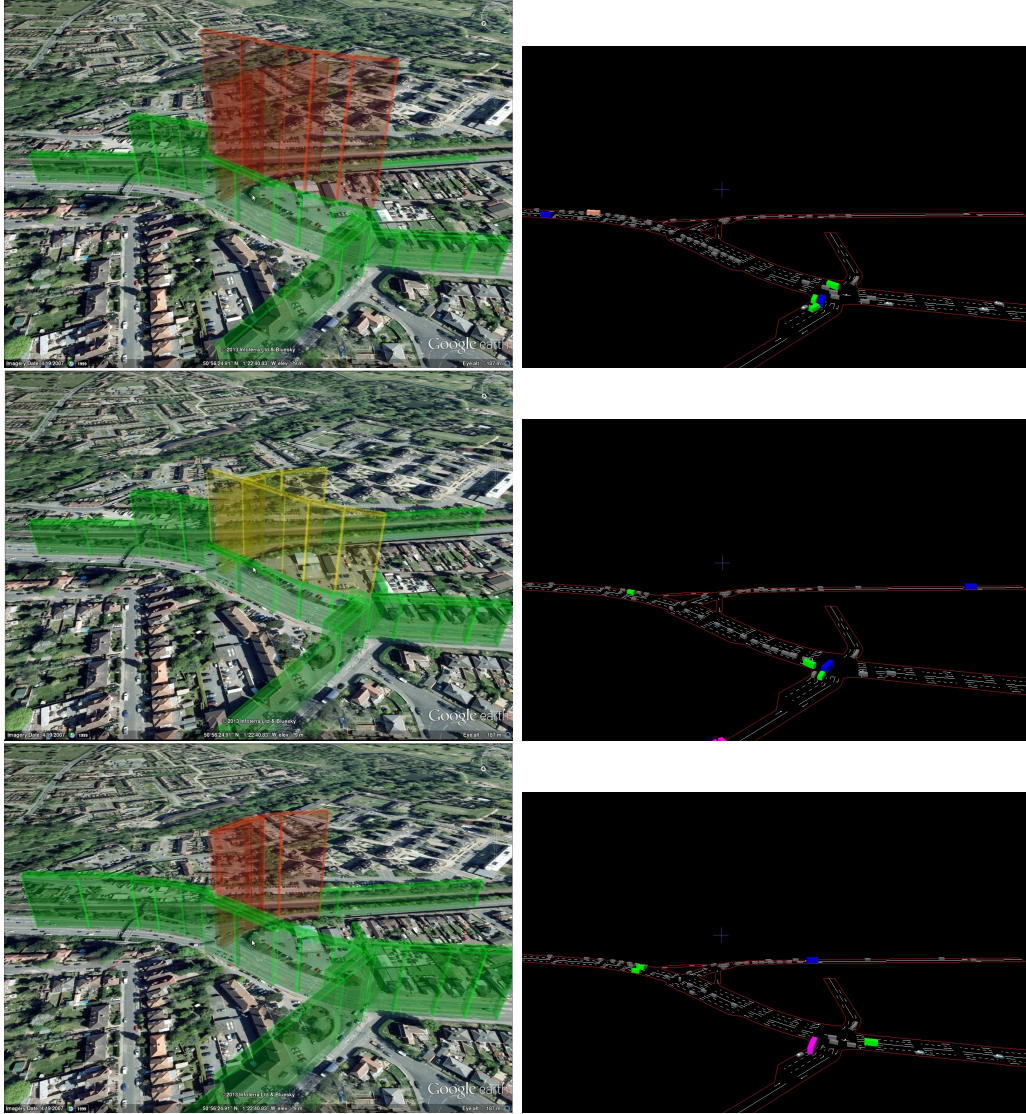


Figure 7: Three progressive screenshots of the busy intersection scenario (on the left) with the corresponding images from the simulation testbed showing actual vehicle locations (on the right). These were taken at 1060, 1090 and 1100 seconds into the simulation respectively. In this visualization cells are coloured green if they contain $N_i \leq 5$ vehicles, yellow for $5 < N_i \leq 10$ vehicles and red for $10 < N_i$ vehicles.

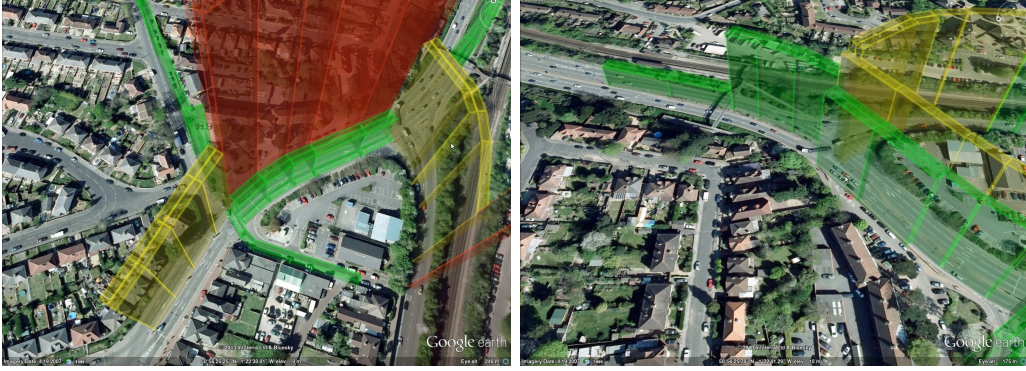


Figure 8: Alternative viewpoints on the busy intersection scenario. In this visualization cells are coloured green if they contain $N_i \leq 5$ vehicles, yellow for $5 < N_i \leq 10$ vehicles and red for $10 < N_i$ vehicles.

humans process and understand this information. This work has presented one such concept for visualizing fine granularity traffic states.

The case study presented here demonstrated the feasibility of a fine grained traffic state estimation based on a concept that builds on existing GIS and computer graphics technologies to produce an interactive, dynamic, three dimensional visualization of the traffic state.

The concept has been demonstrated using a simulation test bed that can produce high fidelity traffic state estimations. This showed that the developed system could visualize fine scale behaviours such as individual lane movements at signalized intersections and dynamic instability phenomena on arterial routes. The ability to visualize these fine scale phenomena may be of value to both practitioners, for strategic control and monitoring and to research investigating dynamic traffic phenomena.

Future work to develop this concept will include experimenting with different metrics in the state estimation to see which are most useful and in which scenarios; experimenting with different colour-maps to represent different aspects of the visualization; and experimenting with textured or patterns on the blocks to represent different information.

Standard Google Earth satellite imagery is used as the surface map over which the blocks are displayed in figures 5, 6, 7 and 8. This can look a little cluttered. Future work will look at alternative surface maps with a cleaner appearance.

Finally the stability and usability of this approach still needs to be tested

on simulations of large networks however it is expected that traditional 2-D visualizations will be preferable at the large scale and that this type of visualization is most suited to in-depth focus on a small area. Dynamically switching between these states would also be something for future work.

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