A holistic approach for the design and assessment of railway tracks

D. Kostovasilis[1], E. Ntotsios[1], M.F.M. Hussein[1], D.J. Thompson[1], G. Squicciarini[1]
[1]Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ,
email: d.kostovasilis@soton.ac.uk, e.ntotsios@soton.ac.uk, m.hussein@soton.ac.uk, djt@isvr.soton.ac.uk,
g.squicciarini@soton.ac.uk

ABSTRACT: In spite of the global financial crisis, considerable investments are being made in railway infrastructure in the UK and many countries around the world. Improvements in the quality and capacity of current services and the development of new railway infrastructure are needed to meet the increasing demand for transferring more people and goods in a more sustainable way. In particular, the performance of the track system is crucial to the successful and cost-effective operation of the railway. This has motivated much scientific research with the aim of better understanding the performance of the railway system, including both existing railway tracks and improved tracks for the future. Much current research on railway track focuses on individual aspects of the design and performance, e.g. track settlement, rail fatigue, ballast degradation, ride quality, maintenance, and noise and vibration. However to achieve substantial advances in railway track design, it is important to consider all these aspects in an integrated way. Changes that can benefit one aspect should not be allowed to have a negative impact on others. To facilitate this, a single tool should be developed or the computational tools that consider individual aspects of the design need to be integrated. The resulting tool can therefore be used to assess the behaviour of railway tracks in a holistic manner. A preliminary version of such a holistic tool is presented here. In this version, fast running models and empirical relationships are put together in order to calculate the performance of a railway track with regard to ride quality, ground-borne noise and vibration and rolling noise. Results for practical case studies are presented and discussed. The paper also highlights the limitations of the preliminary version and the future plans to achieve a reliable and comprehensive tool.

KEY WORDS: holistic approach; multi-criteria; ride quality; ground-borne noise and vibration; rolling noise; frequency domain.

1 INTRODUCTION

In a railway system, vehicles carrying passengers or goods are supported and guided by the track through the wheel/rail interface. Due to the weight of vehicles, high static forces are applied to the railway track structure over a small contact area. Moreover, imperfections on the running surface of the wheel (irregularities, wheel flats etc.) and the rail (joints, head wear, cracks etc.), along with the existence of non-homogeneities (i.e. track stiffness variations) and other factors, give rise to high dynamic loading.

All of the above, as well as issues associated with railway structures, bring the necessity to understand how the different components interact and affect the track structure. For example, reducing the stiffness of the rail pads may result in reduced ground vibration levels but could also increase rolling noise. If this behaviour is properly understood, then different countermeasures can be applied to mitigate for the issues arising and recommendations for future design procedures can be made. In order to understand the effect of the individual components of a railway track in a holistic way, a set of indicators quantifying the overall behaviour of the system needs to be identified. The implementation of these indicators in a single tool, as proposed here, enables the assessment of the impact of the variation in properties of the individual parts of the system, holistically.

Extensive literature exists with regard to investigating individual aspects of railway track design and performance but there is a lack of a more integrated approach as proposed here. In 2000, Zhang et al [1] presented an integrated track degradation tool for the prediction of track behaviour and performance (from a planning point of view) based on rail wear, sleeper, ballast and sub-grade degradation, as well as the interaction between those components. Research on the optimisation of railway track design based on a range of parameters was considered by the European project, EUROBALT, with the main focus being optimising track and vehicle parameters to give improved track geometry behaviour and to minimise maintenance actions [2]. Markine et al. [3] also consider a multi-criteria optimisation, in this case of embedded rail structure slab-track systems. Their investigation was based on the influence of the track design with varying train speeds considering the cost efficiency of the design, minimum noise emission and minimum deterioration at the wheel/rail interface. In a study conducted by Suarez [4] a sensitivity analysis was conducted on the elastic properties of rail vehicle suspensions with regard to their influence on running safety, ride quality and track fatigue, but without taking into account the influence of varying track parameters.

The above studies are examples of work considering multi-criteria optimisation of track or vehicle design, most commonly based on a single parameter evaluation. Nonetheless it is important when trying to achieve optimal track design to consider the effect of all the parameters of the track on all indicators used to evaluate its performance. Such indicators include but are not limited to, track settlement, rail

fatigue, ballast degradation, ride quality, maintenance costs, and noise and vibration.

In this work, a preliminary model is presented for the assessment of ride quality, ground-borne noise and vibration and rolling noise emission from ballasted railway tracks. The track design parameters considered are rail pad stiffness, ballast stiffness and train speed. The results presented are based on a generic inter-city vehicle and a typical UK railway track. In the following sections, the preliminary model will be firstly introduced, describing in brief the indicators considered and the means by which they have been calculated. Then the parameters for the cases considered will be presented and the numerical results will be discussed. Finally, the potential of such a tool, its current limitations and plans for future work are discussed.

2 PRELIMINARY MODEL

A preliminary tool has been developed to show the influence of various track properties on ride quality, ground-borne noise and vibration and rolling noise. The parameters include the railpad stiffness, ballast stiffness and train velocity. This tool utilises previously developed tools and mathematical models as well as empirical relationships. In the following sections a general overview of the process is given along with a brief description of the individual aspects of the holistic tool.

2.1 General overview

For the assessment of ride quality and ground-borne noise and vibration, a frequency domain model has been developed to describe the dynamic behaviour of the railway vehicle, the track and the ground. Figure 1 depicts the railway vehicle and track-form considered. For the vehicle, a 10 degree of freedom rigid-body vehicle model is considered accounting for displacement and rotation of the car-body and bogies, as well as displacements of the wheels. The track form used for this study is a ballasted railway track design. The track is modelled as a continuously supported beam on a two-layer support accounting for rail pads, sleepers and ballast. The track is then further supported on an elastic half-space through a contact strip representing the breadth of the track superstructure. The model used for the track-ground system follows the modelling approach reported by Sheng et al. [5].

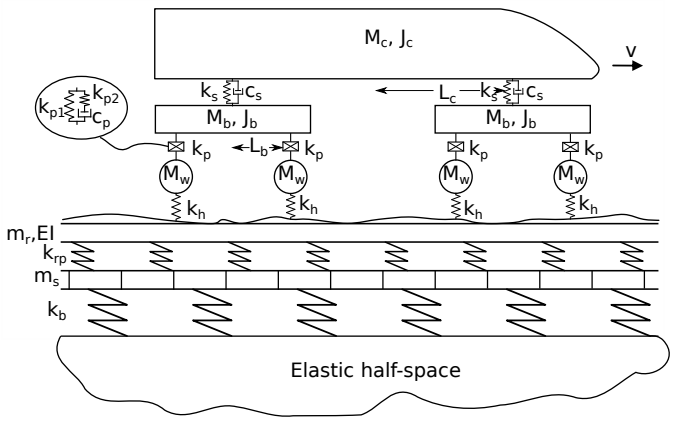

Figure 1. 10-dof vehicle and track layout.

The excitation input results from the vehicle running over irregularities on the wheel-rail surface represented as a stationary random process and described as a Power Spectral Density (PSD) input. The theory of random vibration is utilised in order to obtain the responses of the vehicle and ground. The parameters for the analysis used, further described in Tables 1-3, are taken so as to represent a generic inter-city train running on a typical UK railway track. In the current approach, the vehicle is assumed to be stationary and the irregularities to move with the equivalent vehicle speed in the opposite direction (moving irregularity model). This follows the modelling approach reported by Forrest and Hunt [6] in modelling vibration from underground railways. The model is intended to cover the frequency range up to 250 Hz.

For the assessment of rolling noise, a higher frequency range is required so a different model is used, based on the TWINS software [7] which is further discussed in Section 2.5.

2.2 Excitation mechanism

The excitation of the system originates from irregularities on the wheel-rail contact surfaces, described by their Power Spectral Density. When the vehicle runs over irregularities with a certain wavelength λ at a speed v, the wheels and rails are forced to move vertically relative to each other at the frequency $f = v/\lambda$. Here, two combined idealised spectra are used. The first is the ORE B176 high spectrum described in [8] for the vertical profile of the rails. Due to its limitation in describing smaller wavelengths (associated with higher frequency excitation), it has been combined with the TSI limit spectrum for rail roughness [9], which has been converted from a one-third octave spectrum to a PSD for the current purpose. The two spectra are combined by extrapolating the two spectra into the wavelength range where they are not defined (λ < 2.5m for ORE and λ > 0.25m for TSI) until they coincide. The resulting combined spectrum is shown in Figure 2.

2.3 Ride quality

In order to assess the effect of the dynamic response of the rail vehicle on the passengers, the methodology described in ISO 2631 [10,11] will be used.

Due to the fact that human response to motion varies at different frequencies, an appropriate weighting function needs to be applied. ISO 2631-1:1997 [10] gives a frequency weighting functions for the vibration of standing and seated persons in all three principal axes. The weighing function W_k is used, which is intended for the assessment of standing people. The frequency range of interest for the assessment of ride comfort is 0.5-89 Hz.

For the assessment of ride quality, ISO 2631-1 [10] requires the evaluation of the weighted root-mean-square (r.m.s.) acceleration at the vehicle-human interface, in this case taken to be the floor. Thus, after obtaining the acceleration of the required point in the vehicle for the ith octave band ($a_{i,rms}$), and applying the weighing function for each octave band ($W_{k,i}$), the weighted r.m.s. response is evaluated. The total vibration value is then calculated as:

$$a_W = \sqrt{\sum_i \left(W_{k,i} a_{i,rms}\right)^2} \qquad (1)$$

Approximate limits are given in [10] for the assessment of the undesirable effects with regard to the weighted acceleration.

2.4 *Ground-borne noise and vibration*

Ground-borne noise and vibration are assessed for a notional building located at some distance from the track. Since the vehicle is modelled as stationary (moving irregularity model), the response is calculated at a set of points located at the desired distance from the track and an average taken over the length of the train. It has been shown in [12] that the models based on a moving train produce results in good agreement with those of the moving irregularity model, and thus is sufficient to be used in this application.

Once the forces applied to the ground due to the train-track-ground interaction are calculated, the vibration acceleration of the ground surface at the position of the building is calculated initially, assuming no interaction with the building. This is achieved by using Green's functions for an elastic half-space. Once the acceleration of the ground at the free surface is computed through the frequency spectrum, an empirical procedure is used for evaluating the expected vibration level in a building, according to Nelson [13].

In brief, the process for determining the vibration transmission accounts for coupling losses due to the foundation, amplification of vibration due to floor slabs and the attenuation expected due to vibration transmission from floor to floor. In Nelson [13], graphs are presented for a range of probable values and building types based on measurements conducted by various researchers.

In order to determine the sound pressure level (L_p, dB re 2×10^{-5} Pa) generated by the vibrating floor in a room, the Kurzweil formula as described by Thompson [14] is used, which reads:

$$L_p = L_v - 27dB \tag{2}$$

where L_v is the vibration velocity level in dB re 10^{-9} m/s. Once the sound pressure level is obtained at each band, the overall A-weighted level is determined.

In the next section, the parameters considered for the model will be given along with the results for the criteria specified.

2.5 *Rolling noise*

The module accounting for the rolling noise is based on the TWINS software, described by Thompson et al. [7]. Components of noise radiated by the wheel, rail and sleepers are taken into account. The vehicle is represented only by its wheels, with the sprung mass considered to have negligible effect. In the track model the ground stiffness are considered to have negligible effect on the wheel/track radiated noise. It is noted here that, for the rolling noise calculations, the irregularity input spectrum used accounts for both the rail irregularities (using the TSI limit spectrum) and the wheel roughness (using a typical spectrum for disc-braked wheels).

As the wheel modes are important at high frequencies, these are calculated using a finite element model of a typical inter-city wheel and input as a list of modal parameters. The wheel/rail interaction includes coupling in the lateral as well as the vertical direction. The rolling noise is assessed in terms of the Sound Pressure Level (SPL) at 7.5 m from the track. The effect of a partially reflecting ground surface is included in the model. The model operates with a fine frequency resolution but the results are converted to one-third octave bands for presentation and an overall A-weighted level is determined.

3 RESULTS AND DISCUSSION

In Figure 2, the frequency spectrum of the irregularity is shown for the speed cases considered here, namely 100 km/h (nominal), 50 km/h and 200 km/h.

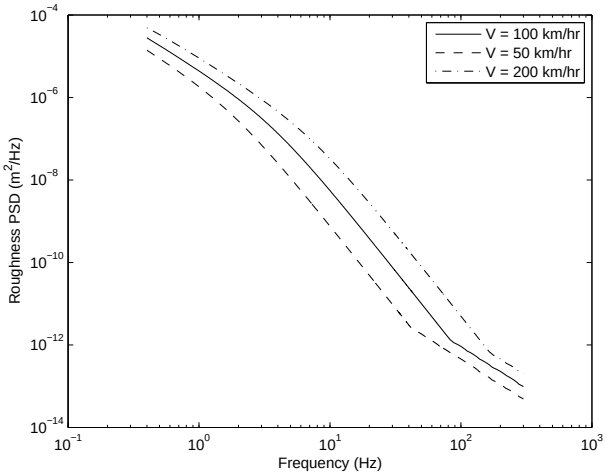

Figure 2. Combined ORE (H) and TSI roughness spectra.

The properties used in the analysis for the vehicle, track and ground are listed through Tables 1-3. In Table 2, the track properties correspond to two rails.

Table 1. Parameters of a generic inter-city train.

Parameter	Value
Body mass, M_c	21,400 kg
Body pitch inertia, J_c	8.3×10^5 kgm^2
Bogie sprung mass, M_b	2707 kg
Bogie pitch inertia, J_b	1.97×10^3 kgm^2
Secondary stiffness, k_s	0.81×10^6 N/m
Secondary damping, c_s	7.4×10^4 Ns/m
Primary stiffness, k_{p1}	0.359×10^6 N/m
Primary damping, c_{p1}	8.4×10^3 Ns/m
Pr. damper stiffness, k_{ps1}	14×10^6 N/m
Half bogie centre length, L_c	8 m
Half bogie wheelbase, L_b	1.3 m
Wheelset mass, M_w	1375 kg
End-of-bogies spacing, L_e	5 m
Number of cars	4
Hertzian contact stiffness, k_h	1.2×10^3 MN/m

In order to investigate the effect of the track stiffness and the train velocity, a nominal value for each of the three parameters (pad stiffness, ballast stiffness and train velocity) has been chosen. These values correspond to the intermediate stiffness for the pads and ballast, and a train velocity of 100 km/h. Based on these parameters, the effect of varying the pad stiffness, the ballast stiffness and velocity has been considered with regard to the ride quality, ground-borne noise and vibration and rolling noise.

Table 2. Track properties.

Parameter	Value
Rail bending stiffness	12.8 MN/m^2
Rail mass	120 kg/m
Rail loss factor	0.02
Rail-pad stiffness	77/369/1080 MN/m^2
Rail-pad loss factor	0.15
Sleeper type	Concrete monobloc
Sleeper mass	462 (370)
Sleeper spacing	0.65
Ballast stiffness	333/1000/3000 MN/m^2
Ballast loss factor	0.1
Ground contact width	2.7 m

Table 3. Ground properties.

Parameter	Value
Density	1800 kg/m^3
P-wave velocity	240 m/s
S-wave velocity	120 m/s
Soil loss factor	0.1

3.1 Track mobility, vehicle mobility and force spectra

In Figure 3 the track mobility and phase is shown.

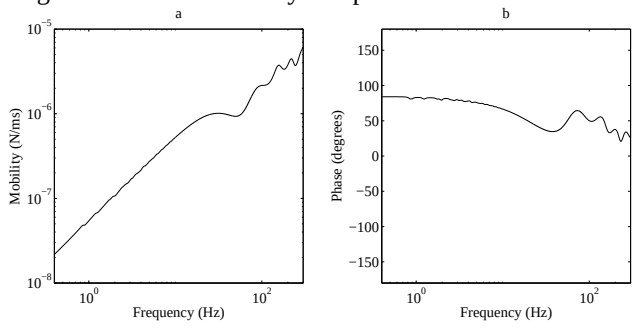

Figure 3. Rail mobility (a) and phase (b) for track without the presence of the ground.

Equivalently, Figure 5 shows the mobilities of the wheels and car-body. The resonances due to the suspension for the parameters considered can be identified at a region below 5 Hz.

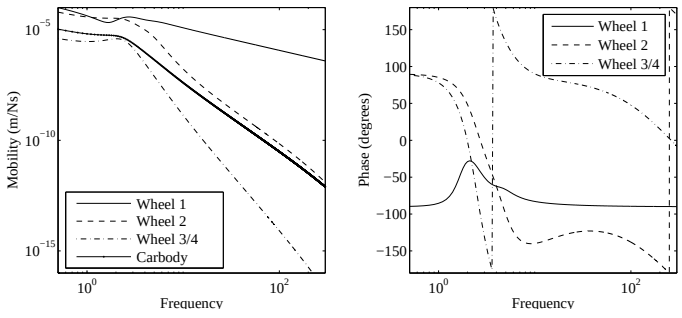

Figure 4. Wheel and car-body mobilities (a) and phase (b).

The force spectra at the wheel/rail interface due to a unit input roughness are shown in Figure 5. In this figure, a clear peak is identified at about 70 Hz which corresponds to the wheel/track resonance. This can also be identified by looking at the mobility of the rail (Figure 3) on top of that of the wheel (Figure 4, solid line), where the two match at approximately 75 Hz.

The frequency at which the wheel/track resonance occurs will change with a change in the track stiffness. For a lower overall stiffness, it will shift to the left (45 Hz for softer pads and 60 Hz for softer ballast) while for a higher overall stiffness it will shift to the right (80 Hz for stiffer pads and 75 Hz for stiffer ballast).

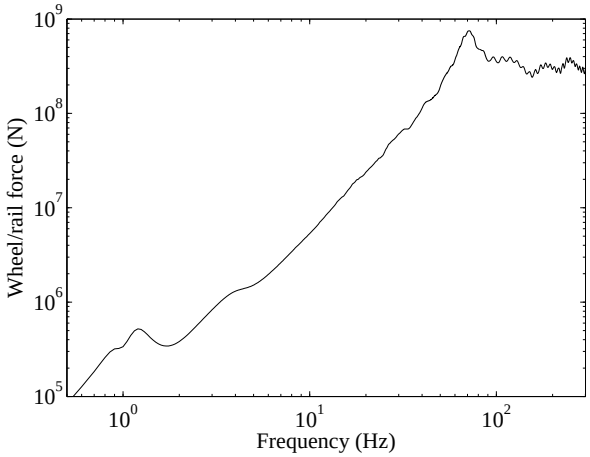

Figure 5. Force spectra at wheel-rail interface

3.2 Ride quality

Figure 6 presents the acceleration spectrum (in one-third octave bands) of the vehicle car-body directly above the leading bogie. In Figure 6a, for the nominal case, there is a peak around 1.25 Hz which can also be identified from the force spectra in Figure 5. Beyond that frequency, fluctuations occur due to the effect of the wheelbase distance, where the wavelength of the irregularities is such that the two bogies are either in phase or out of phase. Considering a single car, these frequencies are at $f_{in}=v/(nL_{bc})$ for a peak (in-phase) and $f_{out}=2v/((2n+1)L_{bc})$ for a trough (out-of-phase).

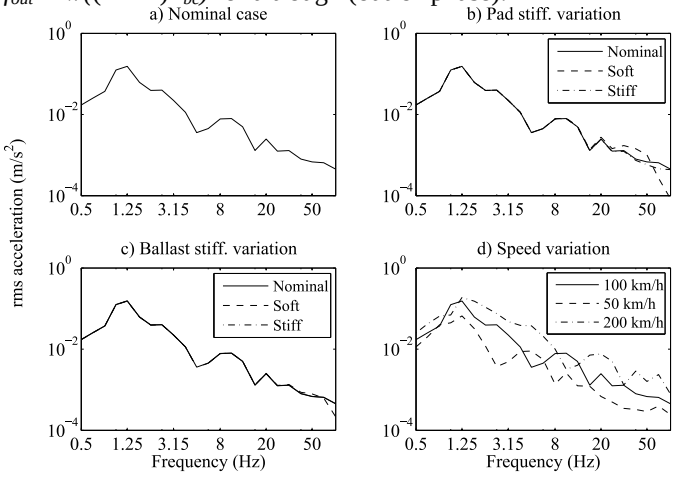

Figure 6. Vehicle vertical acceleration at top of leading bogie.

The effect of decreasing and increasing the track stiffness can be seen in Figure 6b,c where pad and ballast stiffness are modified. In general, the track properties are not expected to affect the ride quality significantly, especially for frequencies below the wheel/track resonance. When the stiffness of the track is decreased (softer pads or reduced ballast stiffness) the wheel/track resonance is lowered and a slight increase in

amplitude is observed. For the current parameters, the pad stiffness has a greater influence than the ballast stiffness. The exclusion of the ground model would have almost no influence on the vehicle car-body for frequencies below the wheel-track resonance, as the track mobility is much smaller than the vehicle mobility in this frequency range.

With regard to the effect of velocity on ride quality, the expected outcome is that ride discomfort increases with increase in velocity. When the speed is increased, the system experiences a larger dynamic excitation at the wheel/rail interface, although the spectra of the force can also change due to the dynamics of the system. An overall increase in vibration is observed in Figure 6d but one can also notice a shift in the frequencies at which the peaks occur. This phenomenon is due to the fact that the frequencies at which the bogies are in and out of phase, discussed previously, depend on the speed and the wavelength. So, for example, for a given wavelength at which all wheels are in phase, a doubling in the speed would lead to a doubling in the corresponding frequency.

The weighted total vibration received for the nominal case is approximately 0.11 m/s^2. The effect of varying the track properties on the total vibration is negligible. Chainging the speed has a noticeable effect, giving a weighted acceleration of 0.05 m/s^2 for 50 km/hr and 0.16 m/s^2 for 200 km/hr. These levels appear to be very small (1/3 of the limit for comfortable ride [10]) which can be attributed to the attenuation afforded by the considered vehicle suspension parameters.

3.3 Ground-borne noise and vibration

For the ground-borne noise and vibration, a single family residence is selected, located at 20 m away from the track. Calculations are performed for the vibration at the first floor level. Figure 7 shows the relative vibration levels for the above specified case.

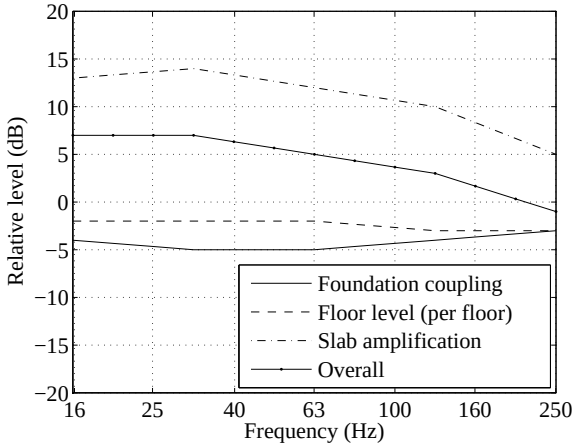

Figure 7. Relative vibration level between ground and receiver (single family residence based on [13]).

Due to the fact that empirical relationships have been used to convert the free field vibration to ground-borne noise and vibration, the conclusions drawn for the two cases are quite similar. The differences between the results for noise and vibration are a) vibration velocity is used for ground-borne noise whereas vibration acceleration is used for ground-borne vibration and b) no weighting has been applied to ground-borne vibration results (W_k weighting could be applied), whereas the A-weighting curve is applied for noise calculations. If one was to plot the insertion loss for the varying cases based on the nominal values, identical results would be found for ground-borne noise and vibration.

Figures 8 and 9 show the vibration and ground-borne noise at the first floor level inside the building. The dominant frequency for the nominal case in both ground-borne noise and vibration is identified at the 63 Hz band, which corresponds to the wheel/track resonance.

Decreasing the track stiffness, results in a shift of the wheel-track resonance to a lower frequency as described before. This can be seen in Figures 8b,c and 9b,c where the response increases at low frequencies for the lower stiffness tracks, followed by a more rapid decay at higher frequencies.

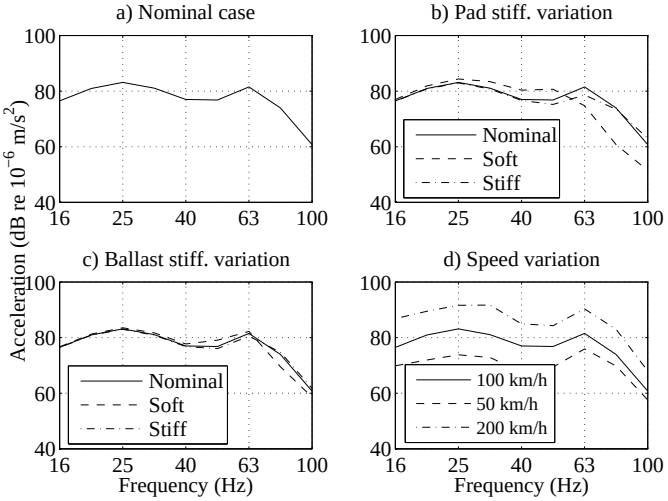

Figure 8. Predictions of ground-borne vibration inside notional building.

Finally the effect of increasing speed in Figures 8d and 9d is to amplify the overall level of ground-borne noise or vibration experienced. It is noticed that the speed has a greater effect at the lower end of the frequency range presented (below 100 Hz) than at higher frequencies. The speed of the train gives the largest variation for the parameters considered.

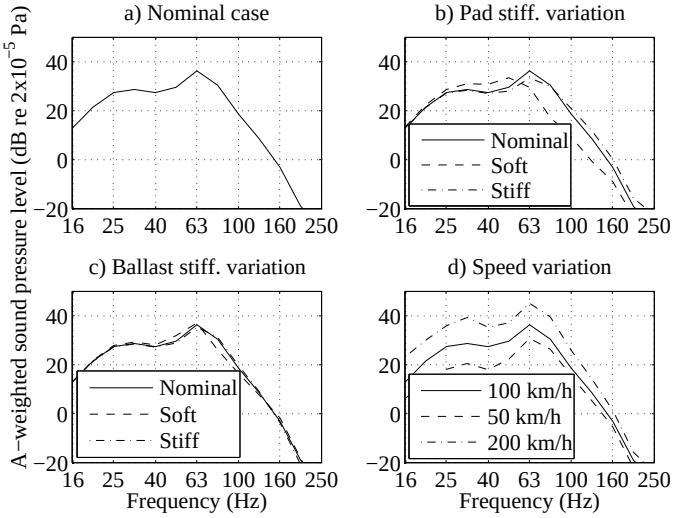

Figure 9. Predictions of ground-borne noise inside notional building.

3.4 Rolling noise

The results obtained for rolling noise are plotted in Figure 10 as total noise level arising from the wheel, rail and sleeper radiation. The sound pressure level is presented at a distance of 7.5 m from the track centreline and a height of 1.2 m above the top of rail. It is noted here, that the parameters from Table 2 for the rolling noise (especially ballast stiffness and damping) have been adjusted to allow for the ground.

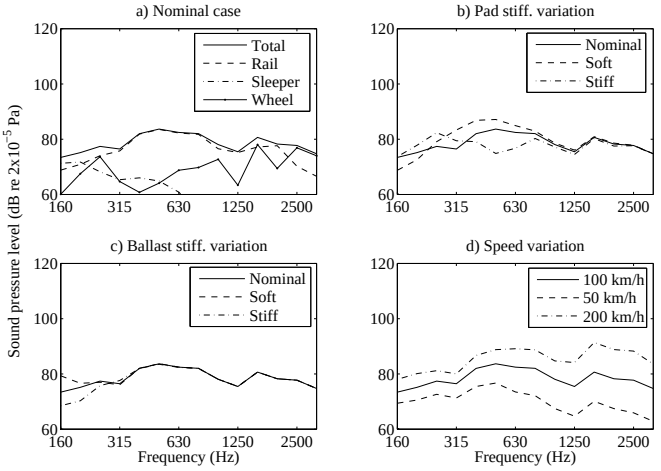

Figure 10. Predicted rolling noise at 7.5 m from track in one-third octave bands.

In general, the sleeper radiation is higher than that of the rail at frequencies up to a few hundred Hertz [14]. In the mid-frequency range (500 Hz – 2 kHz) the rail dominates the radiated noise while at higher frequencies the wheel radiation is most important. When the softer rail pads are used then the rail response increases significantly in the mid-frequency range. When stiffer pads are used, the rail response drops in the low and mid-frequency range, while the sleeper response increases significantly at low frequencies. Table 4 gives the overall A-weighted sound pressure levels for the different configurations.

Table 4. A-weighted sound pressure level for radiated noise.

Case	Level (dBA re $2x10^{-5}$ Pa)
Nominal	89.5
Soft pad	91.2
Stiff pad	88
Soft ballast	89.6
Stiff ballast	89.5
Lower speed	80.2
Higher speed	98

Changing the ballast stiffness mainly affects the results at low frequencies. A reduction in ballast stiffness gives an increase in the sleeper response. As can be seen from Table 4, the effect of ballast stiffness variation is negligible in the overall sound pressure level.

An increase in speed leads to an increase in noise levels at all frequencies. The effect is greatest at higher frequencies (>630 Hz).

Comparing the effect of the track stiffness on rolling noise and ground-borne vibration, it can be seen from Figure 8 and Figure 10 that a softer rail pad will result in a reduction of ground-borne vibration, but an increase in rolling noise. A reduction in ballast stiffness also decreases ground-borne vibration, but the effect on rolling noise is negligible. On the other hand, the effect of velocity is similar for all predictions, showing an increase in undesirable effects with an increase in velocity.

4 CONCLUSION

In this paper, an approach towards a holistic railway track design and assessment has been presented. Using a coupled vehicle/track/ground model developed in the frequency domain and by combining different tools and empirical equations a series of results were presented.

The outputs considered in the preliminary model are ride quality, ground-borne noise and vibration and rolling noise. The impact of changing the rail pad stiffness, ballast stiffness and train velocity on the above criteria was presented. Based on the indicators and parameters analysed, the effect of train velocity has the higher influence on the overall results, followed by the railpad stiffness. The ballast stiffness has much less impact on the outputs specified. The effect of decreasing the track stiffness is to increase the rolling noise whilst reducing the ground-borne noise and vibration.

5 FUTURE WORK

In the current preliminary version of the holistic tool, the ride quality, ground-borne noise and vibration and rolling noise have been considered. In order to give a broader indication of the influence of track parameters, other criteria need to be included such as, for example, rail fatigue, track stresses, settlement etc.

In addition, results were only presented here for one specific type of track (ballasted track). Other track designs, such as slab-track, booted-sleepers, ballast mats etc. should be considered. Other excitation mechanisms may need to be considered in some other cases.

A more detailed ground and building model can also be used in order to improve the predictions of the proposed tool and form a basis for a reliable tool to be used in designing railway tracks.

Although the current version has the above limitations, the potential of such a holistic approach could prove influential on how railway track design and assessment takes place at the moment. For example, it was shown how reducing the pad stiffness to reduce ground-borne vibration can result in an increase in rolling noise. Once developed and validated, this tool can be used for both designing new tracks and assessing existent railway lines. Also, the investigation of mitigation measures would be possible by directly seeing the overall effect of one measure to the whole system.

One negative aspect of the proposed model is that in order to include all aspects as discussed, a computationally demanding tool would have to be developed which would not be practical for repeating computations. Two opportunities based on this approach are to: a) develop a more simplified version of the tool which will be better used in terms of comparative design and b) develop an index based design methodology according to the level of influence of the various parameters to the criteria specified. Then these can be easily used for the preliminary design stage and once the

specifications for the desired track design have been identified, the full model can be used for accurate predictions and detailed suggestions on improving the track performance.

ACKNOWLEDGMENTS

This project has been sponsored by EPSRC and Network Rail. Their support is greatly acknowledged.

REFERENCES

[1] Y.J. Zhang, M.H. Murray and L. Ferreira, Modelling rail track performance: an integrated approach. *Transport Journal*, 141 (4), 187–194, 2000.

[2] EUROBALT: EUropean Research for an Optimised BALlasted Track - Project final report, 2000.

[3] V. Markine, A.P. De Man and C. Esveld, A procedure for design and optimization of a railway track structure. *In Proc. Interactive Conference: Cost Effectiveness and Safety Aspects of Railway Track (UIC)*. Paris, France, 1998.

[4] B. Suarez, J.M. Mera, M.L. Martinez and J.A. Chover, Assessment of the influence of the elastic properties of rail vehicle suspensions on safety, ride quality and track fatigue. *Vehicle System Dynamics*, 51 (2), 280–300, 2013.

[5] X. Sheng, C.J.C. Jones and D.J. Thompson, A theoretical model for ground vibration from trains generated by vertical track irregularities, *Journal of Sound and Vibration*, 272(3-5), 937-965, 2004.

[6] J.A. Forrest and H.E.M. Hunt, Ground vibration generated by trains in underground tunnels, *Journal of Sound and Vibration*, 294, 706–736, 2006.

[7] Thompson, D.J., Hemsworth, B., Vincent, N. Experimental validation of the TWINS prediction programme for rolling noise, Part 1: Description of the model and method, *Journal of Sound and Vibration*, 193, 123-35 1996.

[8] ORE B176, Bogies with steered or steering wheelsets, RP1, Utrecht, Netherlands, 1989.

[9] Technical Specifications for Interoperability, Commission decision of 21 February 2008 concerning a technical specification for interoperability relating to the 'rolling stock' sub-system of the trans-European high-speed rail system (2008/232/CE), 2008

[10] International Standards Organisation, BS ISO 2631 Mechanical vibration and shock, Evaluation of human exposure to whole-body vibration, Part 1 General Requirements, 1997.

[11] International Standards Organisation, BS ISO 2631 Mechanical vibration and shock, Evaluation of human exposure to whole-body vibration, Part 4 Guidelines for the evaluation of the effects of vibration and rotational motion on passenger and crew comfort in fixed-guideway transport systems, 2010.

[12] E. Ntotsios, M.F.M. Hussein and D.J. Thompson, A comparison between two approaches for calculating power spectral densities of ground-borne vibration from railway trains, *Proceedings of the 9th International Conference on Structural Dynamics (EURODYN20014)*, 30th June – 2nd July, Porto Portugal, 2014.

[13] P.M. Nelson (editor), *Transportation Noise Reference Book*, London: Butterworth & Co, 1987.

[14] D.J. Thompson, *Railway noise and vibration: mechanisms, modelling and means of control*, Oxford, UK: Elsevier Science, 2009.