A New Modelling Method for Machine Foundation Coupling System and Its Application to the Control of Power Flow

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A NEW MODELING METHOD FOR MACHINE - FOUNDATION COUPLING SYSTEM AND ITS APPLICATION TO THE CONTROL OF POWER FLOW①

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Abstract A new concept, namely, the equivalent mobility matrix of coupling subsystem is proposed, and the corresponding three-subsystem coupling progressive approach is explored. With the new efficient approach presented, the complexity in dealing with a more complicated dynamic coupling system is greatly reduced. The new modeling method is then combined with the theory of power flow to investigate the dynamics of the overall non-rigid isolation system from the viewpoint of energy. The interaction between the resilient machine of its main modes and the resonant behavior of the flexible foundation on power flow transmission is studied. Taking a machine tool mounted on a multi-story working plant as an example, the dynamic characteristics of the machine-foundation coupling system are analyzed, and their effects on power flow transmission are revealed under various service frequency bands. Some advisable control strategies and the design principle for machinery mounted on flexible structure are proposed.

Key words: Modeling method Power flow Resilient machine Flexible foundation Subsystem Vibration control

0 INTRODUCTION

Vibration transmission from a machine source to flexible supporting structures and the associated sound radiation from the vibrating structures have received great attention in recent years. J. C. Snowdon systematically studied the transmissibility of entirely symmetric isolation systems of rigid machines on non-rigid foundations. E. I. Rivin, and D. B. Debra suggested modeling the elasticity of floor with equivalent mass-spring system while machine mounted on the floor was rigid mass[1,2]. The transmission of vibratory power flow from a vibrating rigid body into a thin supported panel through two mounts has been investigated by Jie Pan et al[3]. In these studies, the machine has conventionally been considered rigid.

However, it is known that machinery and equipment are often built-up assemblies of many components and the contributions to vibration power from the resonant modes of a resilient machine have to be taken into account, especially in the audio frequency range. Since the literature in this area is rather sparse, a definite need exists to reveal the vibration transmission mechanism for including interaction between a resilient machine and a flexible supporting foundation.

In a previous paper, an optimum control model was developed to reduce vibratory power flow

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from an externally excited rigid machine through multiple isolators to a flexible structure[4]. Inthis paper, vibration transmission mechanism of power flow is investigated for a general case-an elastic machine asymmetrically mounted through multiple isolators on a non-rigid foundation. The dynamic characteristics of the flexible machine-foundation system and their effects on the power flow transmission spectra are analyzed. Taking the machine tool as an example, the dynamic characteristics of the coupled system of machine mounted on different type of flexible foundations are investigated under various service frequency bands. Those analyses can explain the reason that the conventional method of the vibration isolation is not functional well in dealing with flexible isolation systems. The advisable strategies to control the power flowing in the coupling system are then proposed so as to improve the effectiveness of isolation at wider frequency band.

1 DESCRIPTION OF THE COUPLING SYSTEM

In engineering practice, a machine is a complicated elastic system. For simplification and comparison, the machine is considered as a lumped element with two degrees of freedom, and the foundation is simulated by two parallel end-clamped beams.

A general model of asymmetric flexible isolation system with N mountings on flexible foundation is shown in Fig. 1, where m_1, m_2 are the effective masses of the machine, and K_1, c_1 are the effective stiffness and damping coefficient of the machine, respectively. The performance of each mount is described by complex stiffness K^*. The internal damping of the foundation is considered to be of the solid type represented by equation $E^* = E(1 + j\eta)$, where η is the damping factor. This model represents the parallel type isolation design in engineering.

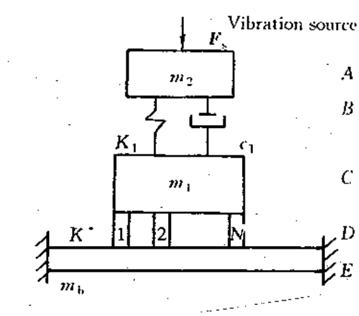


Fig.1 Resilient machine-foundation flexible coupling system

2 THREE-SUBSTRUCTURE COUPLING PROGRESSIVE METHOD

2.1 New modeling method and substructure analysis

A new modeling method, namely, three-substructure coupling progressive approach is developed in this section. The compound system with five substructures shown in Fig. 1 and Fig. 2a can be considered as two equivalent coupling subsystems with three substructures each as shown in Fig. 2b, where C' stands for a new subsystem synthesized by three substructures C, D, and E. The excitation forces of vibration source can be expressed as $\boldsymbol{F}_s$, and the responses of the machine as $\boldsymbol{v}_s$. The mobility matrix of the foundation is still denoted as $\boldsymbol{E}$, and $\boldsymbol{F}_f$, $\boldsymbol{v}_f$ is respectively the transmitted force vector and the corresponding velocity response vector of the foundation at the mounting points.

Based on the substructure mobility synthesis method presented in the previous paper[4], the corresponding dynamic transfer matrixes for subsystem C', which is synthesized by substructures C, D and E, can be derived first as

$$\boldsymbol{F}_f = [\boldsymbol{G}_{11} + \boldsymbol{G}_{12} \cdot \boldsymbol{E}]^{-1}\boldsymbol{F}_C \tag{1}$$

$$\boldsymbol{v}_f = \boldsymbol{E}\boldsymbol{F}_f = [\boldsymbol{e}_{in}]_{N\times N}\boldsymbol{F}_f \tag{2}$$

$$\boldsymbol{v}_C = [\boldsymbol{G}_{21} + \boldsymbol{G}_{22}\boldsymbol{E}]\boldsymbol{F}_f \tag{3}$$

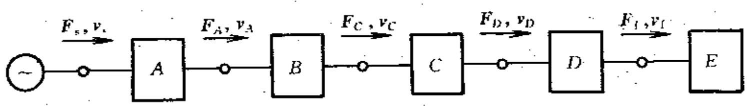
where $e_{in} = \dfrac{j\omega}{m_b}\sum_{k=1}^{\infty}\dfrac{(\beta^*)^4}{(\beta^*)^4 - 1}\cdot\dfrac{\varphi_k(h_i)\varphi_k(h_n)}{\omega_k^2(1+j\eta)} \qquad i, n = 1, 2, \cdots, N \qquad (4)$

$(\beta^*)^4 = (1 + j\eta)(\omega_k/\omega)^2$

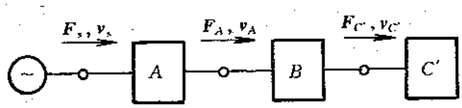
φ_k——Normal mode of flexible foundation

m_b, η——Mass and the damping factor of the beam-like foundation

$$\boldsymbol{G}_{ij} = \sum_{k=1}^{2}\boldsymbol{C}_{ik}\boldsymbol{D}_{kj} \qquad i, j = 1, 2 \tag{5}$$



(a) Connection of five substructures



(b) Equivalent dynamic coupling subsystem

Fig. 2 Schematics of the progressive modeling method

The dynamic transfer matrix for substructure C is also denoted as $\boldsymbol{C}$, and is derived as

$$\begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{M}_{12}^{-1}\boldsymbol{M}_{11} & -\boldsymbol{M}_{12}^{-1} \\ \boldsymbol{M}_{21} - \boldsymbol{M}_{22}\boldsymbol{M}_{12}^{-1}\boldsymbol{M}_{11} & -\boldsymbol{M}_{22}\boldsymbol{M}_{12}^{-1} \end{bmatrix} \tag{6}$$

$$\boldsymbol{M}_{11} = \frac{1}{j\omega m_1}\boldsymbol{I}_a \cdot \boldsymbol{I}_a^{\mathrm{T}} + \frac{1}{j\omega J_1}\boldsymbol{a} \cdot \boldsymbol{a}^{\mathrm{T}} \quad \boldsymbol{M}_{12} = -\frac{1}{j\omega m_1}\boldsymbol{I}_a \quad \boldsymbol{M}_{21} = -\frac{1}{j\omega m_1}\boldsymbol{I}_a^{\mathrm{T}} \quad \boldsymbol{M}_{22} = \frac{1}{j\omega m_1}$$

where $\boldsymbol{I}_a = [1, 1, \cdots, 1]_{1\times N}^{\mathrm{T}}$

$\boldsymbol{a} = [a_1, a_2, \cdots, a_N]_{1\times N}^{\mathrm{T}}$

a_i——Coordinate of the ith isolator with respect to the gravity of the machine

For substructure D consisted of N rubber-metal isolators the transfer matrix is given by

$$\begin{bmatrix} \boldsymbol{D}_{11} & \boldsymbol{D}_{12} \\ \boldsymbol{D}_{21} & \boldsymbol{D}_{22} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{N\times N} & \boldsymbol{O}_{N\times N} \\ (j\omega/K^*)\boldsymbol{I}_{N\times N} & \boldsymbol{I}_{N\times N} \end{bmatrix} \tag{7}$$

From Eqs. (1) and (3) the interfacial velocity $\boldsymbol{v}_C$ and the force $\boldsymbol{F}_C$ can be related to

$$\boldsymbol{v}_C = [\boldsymbol{G}_{21} + \boldsymbol{G}_{22}\boldsymbol{E}][\boldsymbol{G}_{11} + \boldsymbol{G}_{12}\boldsymbol{E}]^{-1}\boldsymbol{F}_C \tag{8}$$

Defining
$$\boldsymbol{M}_C = [\boldsymbol{G}_{21} + \boldsymbol{G}_{22}\boldsymbol{E}][\boldsymbol{G}_{11} + \boldsymbol{G}_{12}\boldsymbol{E}]^{-1} \tag{9}$$

where $\boldsymbol{M}_C$ is the so-called equivallent mobility matrix for the coupled three subsystems. then Eq. (8) can be written as

$$\boldsymbol{v}_C = \boldsymbol{M}_C \cdot \boldsymbol{F}_C \tag{10}$$

With the new concept, Eq. (10) holds the same form as Eq. (2). The system in Fig. 2a now can be considered as a new equivalent three substructures coupled by A, B and C'. With the approach presented above, the dynamic transfer equations for this three coupling substructures can be derived in a similar way as

$$\boldsymbol{F}_C = [\boldsymbol{G}_{11}' + \boldsymbol{G}_{12}' \cdot \boldsymbol{M}_C]^{-1}\boldsymbol{F}_s \tag{11}$$

$$\boldsymbol{v}_s = [\boldsymbol{G}_{21}' + \boldsymbol{G}_{22}' \cdot \boldsymbol{M}_C]\boldsymbol{F}_C \tag{12}$$

where
$$G'_{ij} = \sum_{k=1}^{2} A_{ik}B_{kj} \quad i,j = 1,2$$
$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & j\omega m_2 \\ 0 & 1 \end{bmatrix} \tag{13}$$
$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega/K_1^* & 1 \end{bmatrix} \tag{14}$$
$$K_1^* = K_1(1 + j\omega c_1/K_1) \tag{15}$$

2.2 Generalized dynamic transfer equations

The final governing equations for the transmitted forces acting on the foundation at each mounting point to the known excitation force F_s can be derived in a progressive way as follows
$$F_f = [G_{11} + G_{12} \cdot E]^{-1}[G'_{11} + G'_{12} \cdot M_C]^{-1}F_s \tag{16}$$
$$v_f = \mathbf{E} \cdot [\mathbf{G}_{11} + \mathbf{G}_{12} \cdot \mathbf{E}]^{-1}[\mathbf{G}'_{11} + \mathbf{G}'_{12} \cdot M_C]^{-1}F_s \tag{17}$$
$$v_s = [G'_{21} + G'_{22}M_C][G'_{11} + G'_{12} \cdot M_C]^{-1}F_s \tag{18}$$

Obviously, the complexity in dealing with a more complicated dynamic coupling system is greatly reduced. For periodic structures, where most of the sub-elements are identical, this method becomes even more effective.

3 POWER FLOW TRANSMITTED TO THE FOUNDATION

The transmitted power, superscript tr, is important measures in isolation design and can be calculated by using
$$P_{tr} = \sum_{i=1}^{N} \frac{1}{2}\mathrm{Re}\{F_{fi}^* \cdot v_{fi}\} = \frac{1}{2}\mathrm{Re}\{(F_f^*)^T \cdot E \cdot F_f\} \tag{19}$$
Rewrite Eq. (16) into
$$F_f = T_f \cdot F_s \tag{20}$$
then
$$P_{tr} = 0.5|F_s|^2\mathrm{Re}\{(T_f^*)^T \cdot E \cdot T_f\} \tag{21}$$

Power flow results are also presented in terms of power flow transmission spectrum, and are normally indicated by Q_{tr}. The transmitted power flow spectra in this paper is given by
$$Q_{tr}(\omega) = 0.5\mathrm{Re}\{(T_f^*)^T \cdot E \cdot T_f\} \tag{22}$$

It can be seen from the equations that power flow in flexible machine-foundation system is dependent on the dynamic characteristics of the machine, the mobility of the foundation, and the complicated coupling relations between machine and foundation. In order to reveal the effect that the interaction between the machine of its main modes and the resonant characteristics of the flexible foundation upon power flow transmission, the spectra $Q_{tr}(\omega)$ are calculated under various service conditions, and the results are plotted against frequency f(Hz) in the following sections.

4 NUMERICAL EXAMPLE AND ANALYSIS

To understand better the power flow transmission mechanism of the overall non-rigid machine-foundation coupling system, a numerical example of a machine-tool with two degree of freedom mounted on beam-like foundation, shown in Fig.3, is analysis. Taking the machine-tool in Ref. [5] as an example, the following parameters have been set below so as to have some similarity with real machine tools

$$m_1 = 2\,000 \text{ kg}, \ K_1 = 7.2 \text{ MN/m}, \ c_1 = 1.2\times10^4 \text{ N·s/m}$$

$$m_2 = 100\ \text{kg},\ K_2 = 9.0\ \text{MN/m},\ c_1 = 600\ \text{N}\cdot\text{s/m}$$

The foundation is built with different spans L, width b, height h, and materials of steel or concrete. Power flow transmission spectra are calculated in the frequency range from 2 Hz to 630 Hz, and results are shown in Fig. 4 ~ Fig. 7 decibel scale (dB re: 10^{-6}). Particularly examined and revealed are the dynamic coupling characteristics of the compound flexible system as well as the effects of the resonant behavior of the machine on the power flow transmission.

4.1 Power flow influenced by the dynamics of the non-rigid machine

It can be found from Fig. 4, Fig. 5 that the power transmitted into the foundation is seriously influenced by the effective mass m_1 of the machine, and more power will transmit to the foundation with the decrease of the modal mass of the machine. In the low frequency range, the power flow is mainly influenced by the lower mode of the machine, and the influence of the effective mass of the machine m_2 on the power flow spectra can be ignored. Both theoretical analysis and computer simulation show that a decrease in the modal mass of lower order results in the increase of the power flow (Fig. 5). Therefore, it is proposed that to increase the modal mass m_1 through structural dynamic modification will effectively control the power flow transmission.

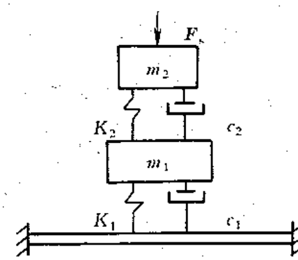


Fig. 3 Machine-tool with two degree-of-freedom on beam-like foundation

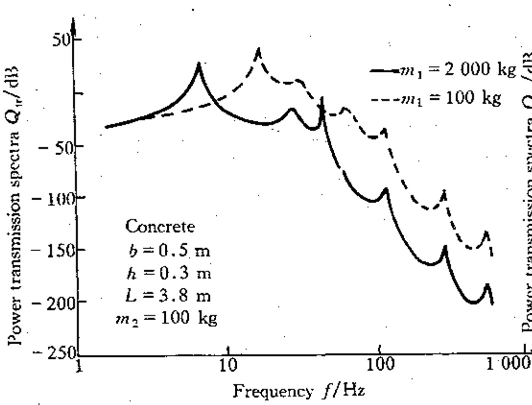


Fig. 4 Power flow influenced by m_1

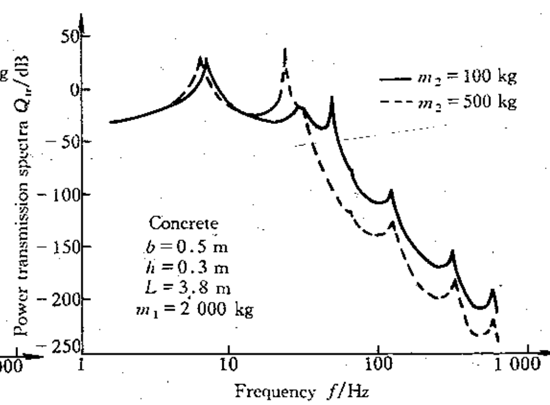


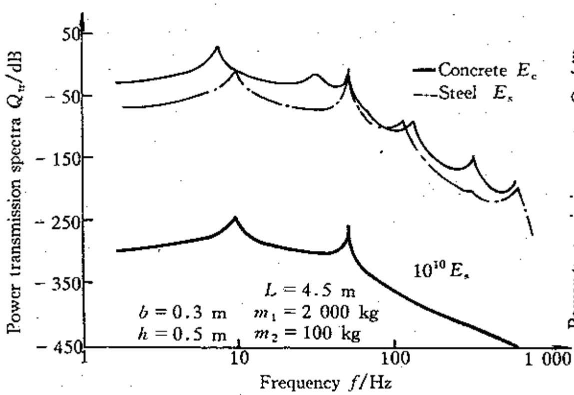
Fig. 5 Power flow influenced by m_2

4.2 Power flow influenced by the stiffness of foundation

In the medium to higher frequency range, the power flowing into the foundation is strongly influenced by the flexibility of the foundation. Fig. 6 shows power flow transmission spectra when machine tool is mounted on supporting foundations of different materials. An increase in the stiffness of the foundation results in the reduction in the power flow at full frequency band (Fig. 6), while an increase of the foundation mass is not so effective(Fig. 7).

4.3 Machine-foundation interaction

The resonant mode of lower order of the machine is significantly influenced by the dynamics of the foundation. With the decrease of the foundation stiffness, the resonant frequency of the coupling system is shifted toward the lower frequency band. For example, when machine is mounted on rigid base structrue, the resonant frequency of the classical isolation system is 10 Hz, however, when mounted on a steel beam with both end clamped, the fundamental frequency of the flexible isolation system is 9 Hz, while coupled with a concrete beam, the frequency is reduced to 7 Hz. This is because that the beam serves as a flexible end condition for the machine bed, therefore, the dynamic interaction between the machine and the foundation should be considered in the isolation design. In other words, the conventional isolation design based on the classical theory is not valid for machinery to be mounted on flexible structures, such as floors, ship decks, train chassis, etc.



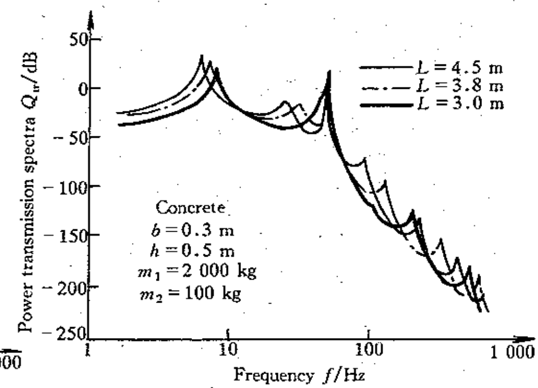


Fig.6 Effect of the foundation stiffness

Fig.7 Effect of the foundation span

Furthermore, theoretical analysis and computer simulation show that the fundamental frequency of the flexible isolation system can be approximately calculated by $\omega_{n1} \approx \sqrt{K_e/m}$, where $m = m_1 + m_2$, $K_e = K_1 K_b/(K_1 + K_b)$, ($K_b$——Effective stiffness of the clamped beam when simplified to a system with one-degree-of-freedom).

The second pronounced peak at frequency of 50 Hz in the power flow spectra is associated with the resonant mode of higher order of the machine. It means that the power is mostly transmitted into the foundation when the subsystem-machine tool is in resonance. Therefore, it is important from the vibration control point of view that to avoid resonant vibration of the flexible machine better isolation will be achieved.

The resonant peaks of power flow at higher frequencies shown in Fig.6 and Fig.7 are caused by the resonance of the flexible foundation. When the service frequency of the machine is in resonant with the beam modes, more vibration energy of the machine will be transmitted to the receiving structure. However, in the case of rigid receiver, the resonant peaks disappear (Fig.6), and the power transmission curve declines rapidly. Again, it is revealed that the influence of foundation on power transmission becomes less important when it is constructed as rigid as possible. It is suggested that the machine be mounted on the locally stiffening area such as main beam of the supporting structure, or near the pillar of the floor of a multistory workshop.

5 CONCLUSIONS

(1) A new concept, i.e. the equivalent mobility matrix is proposed for the first time, and the corresponding three-substructure coupling progressive approach is explored. With the new modeling approach presented, the complexity in dealing with a more complicated coupling system is greatly reduced.

(2) The power flowing into the foundation is seriously influenced by the flexibility of both the machine and its foundation in the medium to higher frequency range. In higher frequency range, the influence of the machimes' modes of higher order is more significant than those of the lower order.

(3) A decrease in the modal mass of lower order of the machine results in the increase of the power flow. Therefore, it is proposed that to increase the modal mass through structural modification will effectively control the power flow transmitted to the foundation.

(4) To effectively control the power transmission, the isolation design principle and the control strategies are proposed as follows: Avoiding the resonant vibration of machine-foundation coupling system, preventing the resonant modes of the subsystems, increasing the stiffness of the supporting foundation, or enhancing indirectly the stiffness ratio of the foundation to machine, optimizing the dynamic parameters of isolators as well as their placement on the flexible foundation. In these ways, the power flow transmission in the service frequency band interested can be significantly reduced.

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