WCCI 2014 Presentation

On-Line Gaussian Mixture Density Estimator for Adaptive Minimum Bit-Error-Rate Beamforming Receivers

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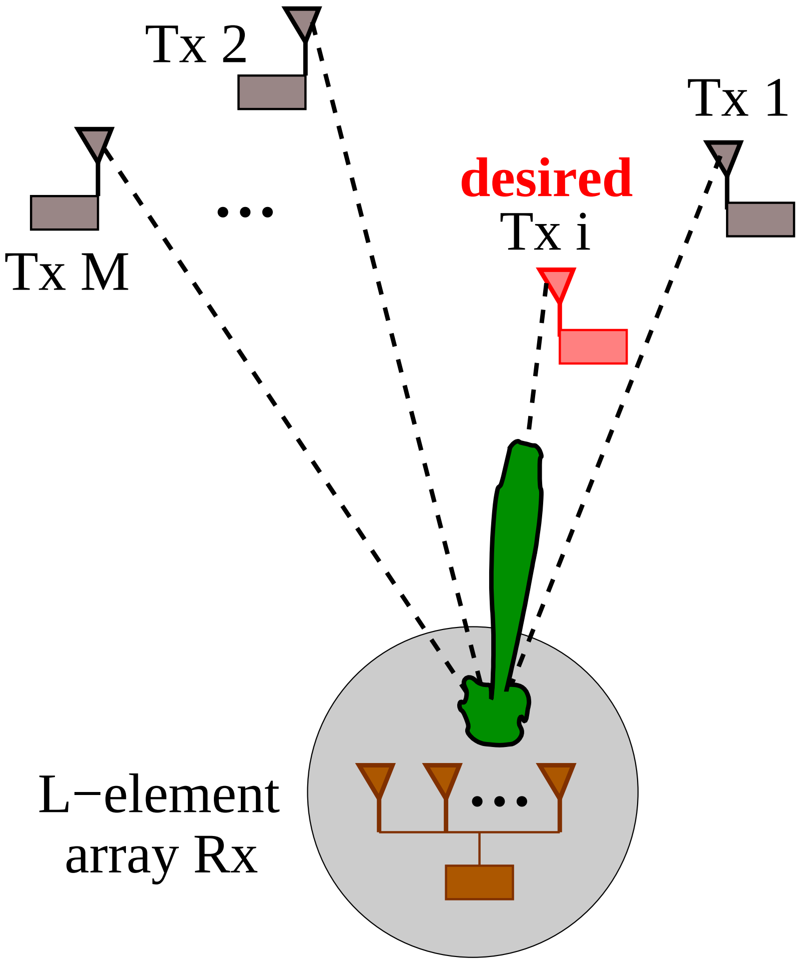
Outline

- Receiver beamforming for **space division multiple access** enabled multiuser communication systems

- Existing state-of-the-art **minimum bit error rate** beamforming with on-line least bit error rate algorithm

- **On-line Gaussian mixture** density estimator for adaptive minimum bit error rate beamforming

Motivations



- **Space division multiple access**: receiver equipped with L-element antenna array to support M single-antenna transmitters
  - classical view: maximise response at desired user **direction** and place nulls at interferers' directions, **must** $L \geq M$
- Standard beamforming is **minimum mean square error** (MMSE)
  - Least mean square algorithm
- State-of-the-art **minimum bit error rate** (MBER), **can be** $L < M$
  - Least bit error rate algorithm

System Model

- M single-transmit-antenna users transmit on same carrier, receiver is equipped with L-element **antenna array**, channels are non-dispersive
- Received signal vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^T$ is

$$\mathbf{x}(k) = \mathbf{P}\,\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

- $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_L(k)]^T$ is noise vector, and **system matrix**

$$\mathbf{P} = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_M\mathbf{s}_M] = [\mathbf{p}_1\ \mathbf{p}_2 \cdots \mathbf{p}_M]$$

- $\mathbf{s}_i$ is **steering vector** of source i, A_i is i-th non-dispersive channel tap, $\mathbf{p}_i$ is ith column of channel matrix $\mathbf{P}$
- User i is **desired** user, and transmitted symbol vector $\mathbf{b}(k) = [b_1(k)\ b_2(k) \cdots b_M(k)]^T$ with QPSK symbol set

$$b_m(k) \in \{b^{[1]} = +1+j,\ b^{[2]} = -1+j,\ b^{[3]} = -1-j,\ b^{[4]} = +1-j\}, 1 \leq m \leq M$$

Beamforming Receiver

- **Beamformer** output with weight vector $\mathbf{w} = [w_1 \ w_2 \cdots w_L]^{\mathrm{T}}$ for user i

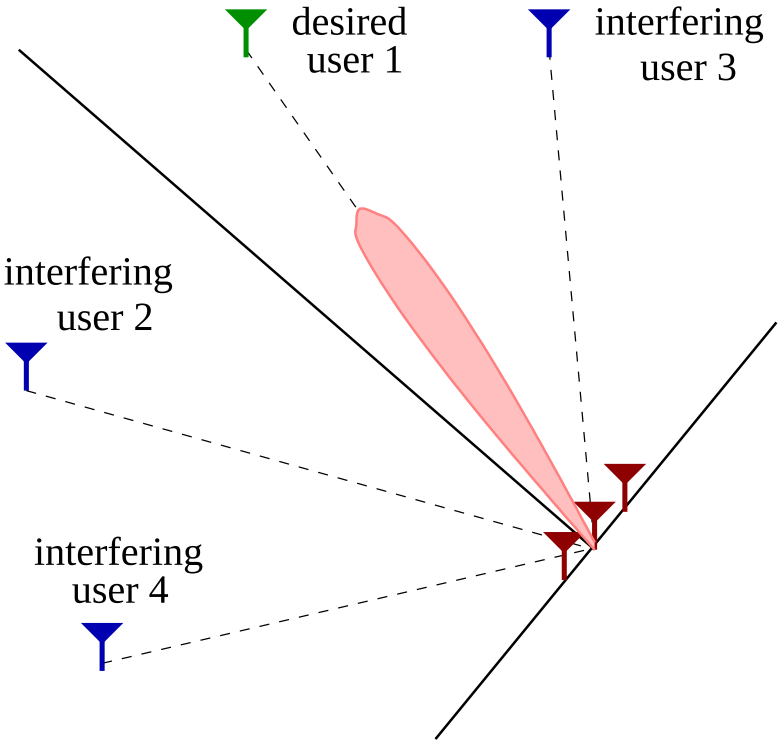
$$y(k) = \mathbf{w}^{\mathrm{H}} \mathbf{x}(k)$$

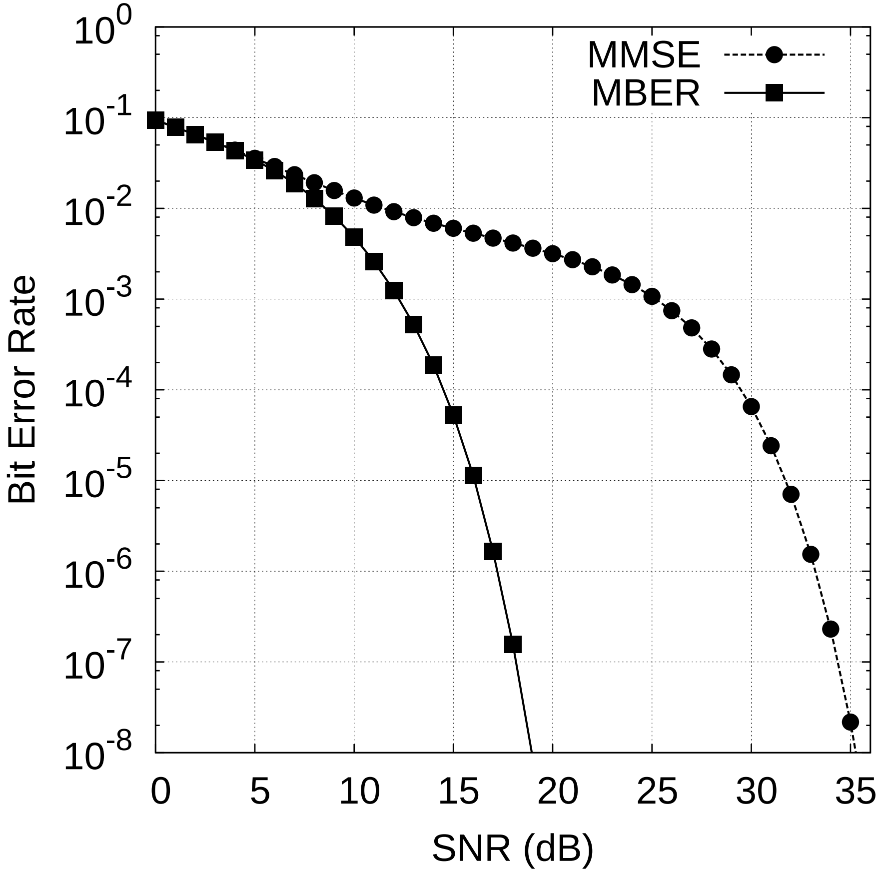
  - Choose appropriate $\mathbf{w} \Rightarrow y(k)$ is a sufficient statistic for estimating $b_i(k)$, i.e. error probability of estimate $\widehat{b}_i(k)$ based on $y(k)$ is small

- **Minimum mean square error**: minimise mean square error $E\left\{\left|\widehat{b}_i(k) - b_i(k)\right|^2\right\} \Rightarrow$ on-line least mean square algorithm
  - Use single sample to form 'instantaneous' MSE, and stochastic gradient descent minimisation of instantaneous MSE leads to LMS

- **Minimum bit error rate**: minimise error probability of $\widehat{b}_i(k) \Rightarrow$ on-line least bit error rate algorithm
  - Use single-sample Gaussian to form 'instantaneous' PDF, and stochastic gradient descent to 'minimise' single-sample based error rate

MMSE versus MBER





- **Three-element** antenna array bermforming receiver for **four-user** system
  - $\text{SIR}_2 = \text{SIR}_3 = 0\,\text{dB}$ and $\text{SIR}_4 = -6\,\text{dB}$: desired user 1 and interferers 2 and 3 have equal power, but interferer 4 has 6 dB more power

Adaptive Minimum Bit Error Rate

- Due to **symmetric** distribution, signal can be shifted to 1st quadrant

$$y_s(k) = y(k) + a\mathbf{w}^{\mathrm{H}}\mathbf{p}_i = \mathbf{w}^{\mathrm{H}}\big(\mathbf{x}(k) + a\mathbf{p}_i\big)$$

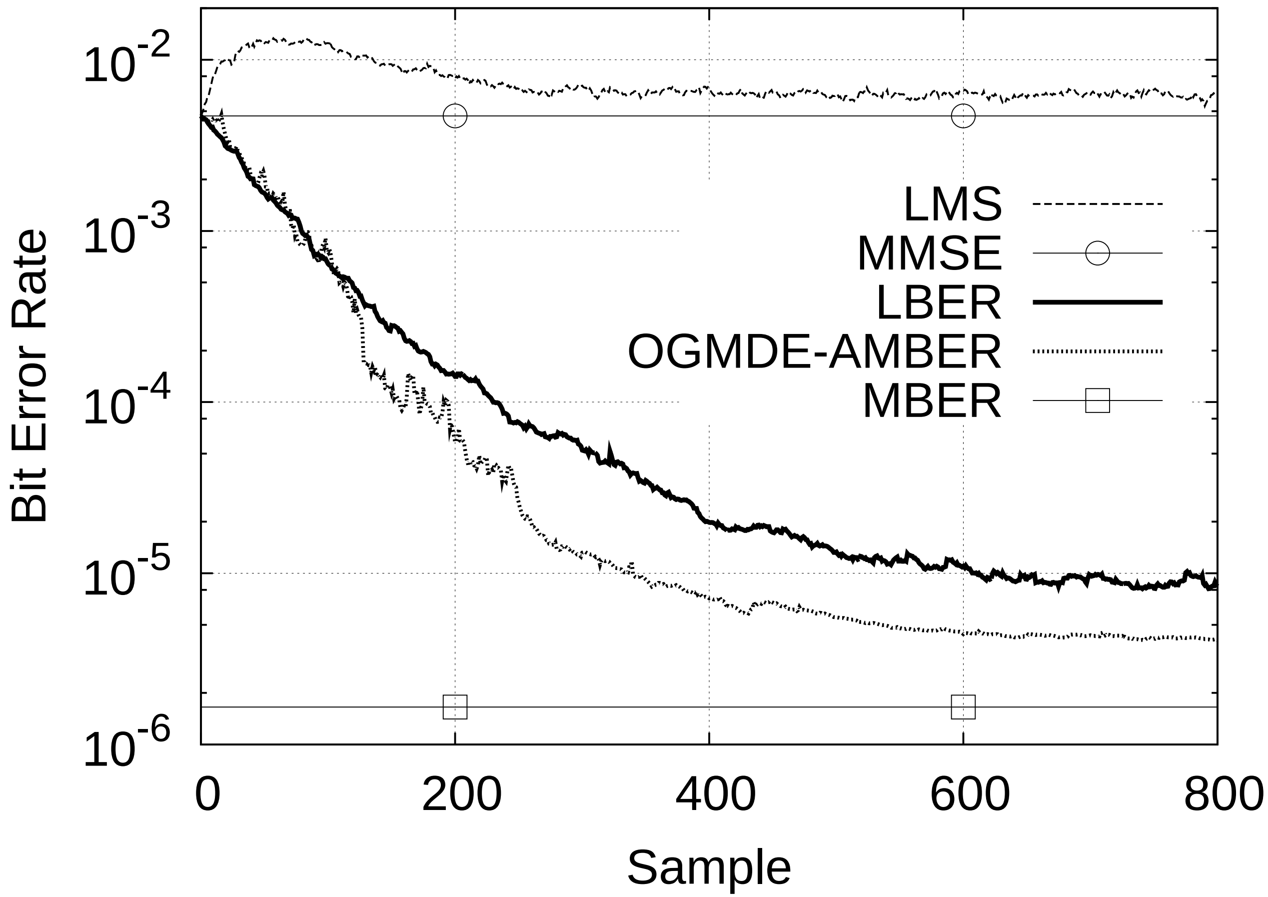
  with $a = \big(1 - \mathrm{sgn}\big(b_{i_R}(k)\big)\big) + \big(1 - \mathrm{sgn}\big(b_{i_I}(k)\big)\big)\mathrm{j}$

- Probability density function of $y_s(k)$ is a large **unknown Gaussian mixture** on signal space, which depends on weight vector $\mathbf{w}$
- **Bit error rate** of beamformer with $\mathbf{w}$, $P_E(\mathbf{w})$, is a sum of error Q-functions $\Rightarrow$ minimising $P_E(\mathbf{w})$ leads to MBER solution
- If **off-line**, block of training data can be used to estimate this unknown PDF, leading to estimate of BER $\widehat{P}_E(\mathbf{w})$ $\Rightarrow$ approximate MBER
- **On-line** requires adaptation sample by sample, and LBER algorithm
  - is on-line single Gaussian density estimator based adaptive MBER $\Rightarrow$ stochastic **single sample** can be seriously influenced by noise

OGMDE-AMBER

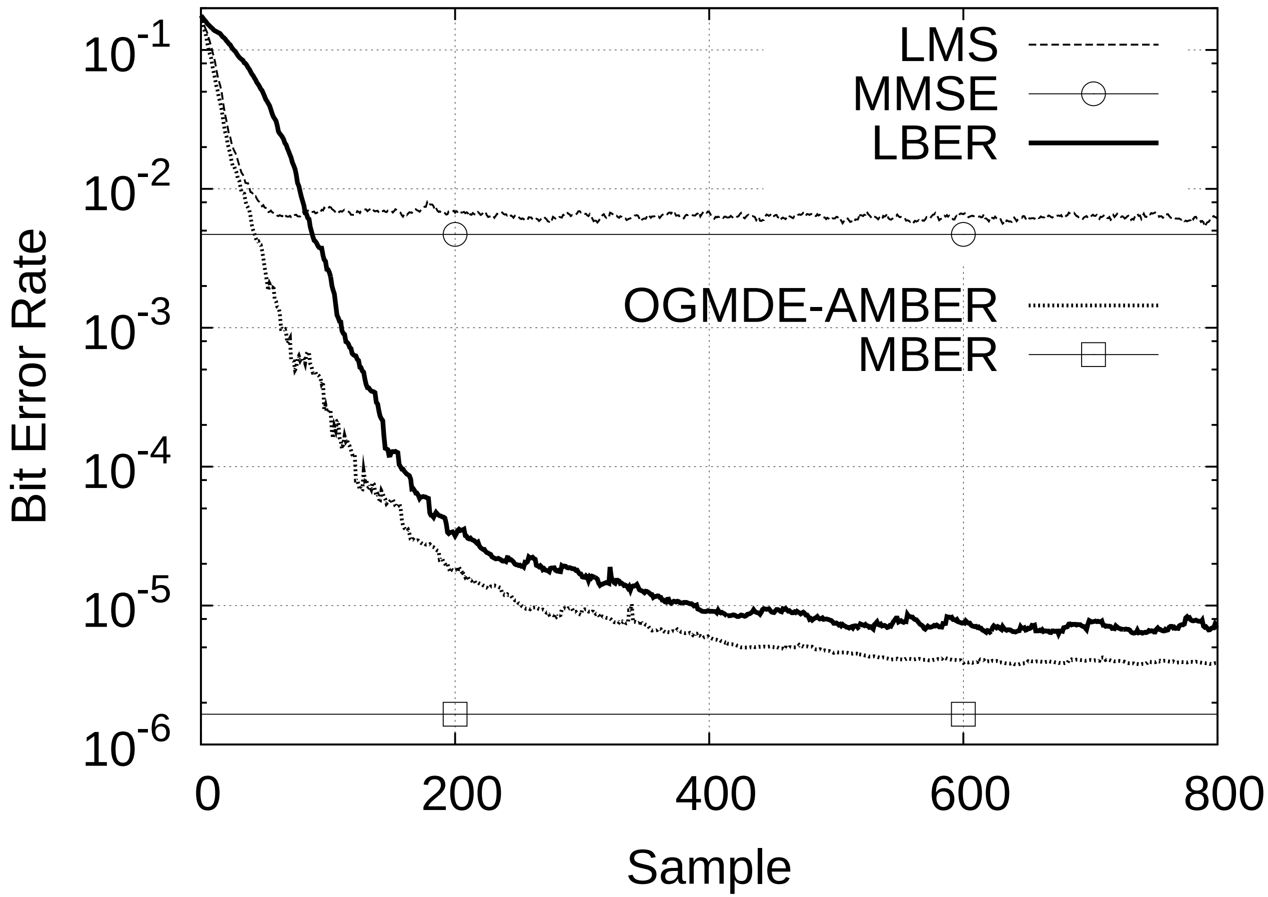
- To reduce noise influence while keeping sample-by-sample adaptation capability with low complexity, we propose OGMDE-AMBER
- On-line Gaussian mixture density estimator consists of **small** number of N Gaussians with means λ_i, kernel widths ρ_i and mixing weights η_i
  - Place a Gaussian kernel on **new sample** $y_s(k)$, and merge it with **nearest** existing mixture component
  - Update mean, kernel width and mixing weight of this **newly merged** mixture component
  - Update means, kernel widths and mixing weights of **rest** mixture components
- Only the error Q-function associated with **newly merged** mixture component contains **new** information $y_s(k)$
  - Adaptive MBER then has similar **sample-by-sample** adaptation

Learning Curves



- Learning curves of **LMS**, **LBER** and **OGMDE-AMBER** ($N = 4$), averaged over 100 runs for 4-user 3-element antenna array system, where SNR $= 17\,$dB, SIR$_2$ = SIR$_3$ $= 0\,$dB and SIR$_4$ $= -6\,$dB, while initial weight vector was set to MMSE solution

Learning Curves



- Learning curves of **LMS**, **LBER** and **OGMDE-AMBER** ($N = 4$), averaged over 100 runs for 4-user 3-element antenna array system, where SNR = 17 dB, $\mathrm{SIR}_2 = \mathrm{SIR}_3 = 0\,\mathrm{dB}$ and $\mathrm{SIR}_4 = -6\,\mathrm{dB}$, while initial weight vector was set to $[0.0 + 0.1\mathrm{j}\ 0.1 + 0.0\mathrm{j}\ 0.1 + 0.0\mathrm{j}]^{\mathrm{T}}$

Conclusions

- ❑ Many applications require to adapt underlying process's probability density function sample-by-sample, with low complexity
- ❑ Adaptive minimum bit error rate linear beamforming receiver for supporting space division multiple access is an example
- ❑ We have proposed a novel on-line Gaussian mixture density estimator aided adaptive MBER beamformer
- ❑ Future work will extend this OGMDE-AMBER to nonlinear beamforming receiver assisted SDMA systems
- ❑ The proposed on-line Gaussian mixture density estimator can readily be applied to other applications