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A METHOD OF ADAPTATION BETWEEN STEEPEST-DESCENT AND NEWTON'S ALGORITHM FOR MULTI-CHANNEL ACTIVE CONTROL OF TONAL NOISE AND VIBRATION

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Active control methods have been applied to a number of practical problems in which it is necessary to control a tonal disturbance. For example, the control of engine noise and vibration in vehicles, propeller noise in aircraft and the vibration produced by reciprocating machinery in many industrial systems. In such applications the steepest-descent algorithm has been widely employed, in part due to its robustness to variations in the plant response. This robustness, however, comes at the expense of a potentially slow convergence speed and this may limit the performance in applications where the disturbance is non-stationary. To improve the speed of convergence, an iterative least-squares algorithm can be employed, such as Newton's algorithm. The convergence of these algorithms is less dependent on the potentially large eigenvalue spread of a multichannel plant matrix and, therefore, can theoretically achieve more rapid convergence. However, these algorithms are significantly less robust to plant response variations and, therefore, their practical performance can be somewhat limited. Generalised algorithms have been presented which combine steepest-descent and Newton's method in order to provide a fixed compromise between convergence and robustness. This paper presents a method of adaptively combining steepest-descent and Newton's method in order to achieve both rapid convergence and robustness to plant response variations. The two algorithms are combined into a single update equation in which a single mixing parameter facilitates a trade-off between the two algorithms. A method of adapting this parameter to minimise the cost function is presented and the performance of the proposed algorithm is assessed through a series of simulations. The proposed combination algorithm is shown to improve the control performance in the presence of plant response variations compared to both the steepest-descent and Newton's algorithms.

1. Introduction

The attenuation of noise and vibration is of significant importance in a wide variety of applications due to factors such as environmental impact, health and safety requirements and the commercial success of noise producing products. In most applications, traditional passive control methods are employed to attenuate noise and vibration; however, at low frequencies the performance of passive control methods are limited by practical constraints such as their size and weight, for example. Under these conditions active control methods become more suitable and consequently there has been a

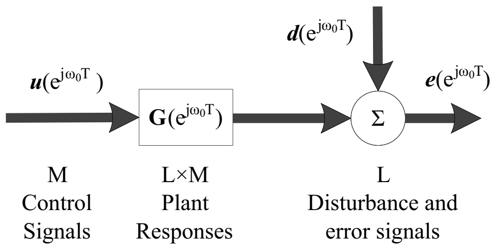


Figure 1: Block diagram of a multichannel tonal control system operating at a frequency of ω_0.

variety of methods developed to control both noise [1] and vibration [2].

In many applications it is of interest to control specific tonal disturbances produced, for example, by internal combustion engines, aircraft propellers, fans, and reciprocating machines. To achieve such control when a number of acoustic or structural modes are excited it is necessary to employ multiple error sensors and control sources. Multichannel control of tonal disturbances has been successfully achieved using both feedforward [3] and feedback [4] control approaches and the relationship between these two approaches has been highlighted by a number of authors [3, 5, 6, 7]. These comparisons aside, there is a general trade-off between the speed of convergence and the robustness of the controller to variations in the system response and this becomes particularly sensitive in the case of a multichannel control system. As a result of this trade-off, a number of different control algorithms have been proposed, which range from the steepest-descent algorithm [8], which is relatively robust to variations in the plant response, to Newton's algorithm, which theoretically provides fast convergence [9]. For example, Cabell and Fuller [10] propose an algorithm based on transformed signals called the principle component least-mean-square (PC-LMS) algorithm which allows the convergence of the individual principal components to be controlled and can thus be tuned to avoid some of the issues of the steepest-descent and Newton's methods.

This paper presents the development of a novel algorithm for the active control of a tonal disturbance in a multichannel control system, as shown in Fig. 1, which adapts between steepest-descent and Newton's algorithms in order to achieve both rapid convergence and robust performance. In section 2 the multichannel tonal control problem is outlined and the limitations of the conventional steepest-descent and Newton's algorithms with respect to convergence speed and robustness are reviewed. In section 3 a method of adaptively combining the steepest-descent and Newton's algorithms is presented and in section 4 the performance of this proposed algorithm is compared to the steepest-descent and Newton's algorithms through a series of simulations. Finally, conclusions are drawn in Section 5.

2. Multichannel Tonal Control Problem

The multichannel tonal control problem is represented by the block diagram shown in Fig. 1. The control system consists of L error sensors, which may be microphones in a noise control problem or accelerometers in a vibration control problem, and M control actuators, which may be loudspeakers in a noise control problem or shakers in a vibration control problem. The aim of the control system is to adjust the amplitude and phase of the vector of control signals, $\mathbf{u}(e^{j\omega_0 T_s})$, at the control frequency ω_0 and sampled at a period of T_s seconds, in order to suppress the L disturbance signals, $\mathbf{d}(e^{j\omega_0 T_s})$. The response between the control actuators and the error sensors is denoted by the $L \times M$ matrix of complex plant responses, $\mathbf{G}(e^{j\omega_0 T_s})$ and, therefore, in the steady state the complex error signals can be expressed as

$$\mathbf{e}(e^{j\omega_0 T_s}) = \mathbf{d}(e^{j\omega_0 T_s}) + \mathbf{G}(e^{j\omega_0 T_s})\mathbf{u}(e^{j\omega_0 T_s}). \tag{1}$$

For the remainder of this paper the dependence on $\omega_0 T_s$ will be dropped for conciseness. The optimal vector of control signals for this multichannel problem is dependent on the cost function to be minimised. In many applications we aim to minimise the cost function defined as the sum of the squared error signals, which is given by

$$J = \mathbf{e}^H \mathbf{e}. \tag{2}$$

If there are more error sensors than control actuators ($L > M$) then the vector of optimal control signals can be calculated as

$$\mathbf{u}_{opt} = -\left[\mathbf{G}^H \mathbf{G}\right]^{-1} \mathbf{G}^H \mathbf{d}. \tag{3}$$

From this equation it is clear that to directly calculate the optimal control signals requires advanced knowledge of the vector of disturbance signals and, in general, this is not practically possible. Therefore, in practice, the control signals are iteratively adjusted in order to minimise the quadratic cost function given by eq. 2. Using a gradient descent approach the control signal vector can be iteratively updated as

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha \mathbf{C} \mathbf{e}(n), \tag{4}$$

where n is the iteration index, α is the convergence coefficient and $\mathbf{C}$ is a complex matrix. The update algorithm given by eq. 4 is derived by assuming that the error signals have reached their steady state values before the next iteration is applied. In practice this means that the algorithm will be very slow to converge and, therefore, it is common practice to compute the next iteration prior to the steady state condition [4]. The algorithms presented and proposed below are thus updated at the sampling frequency, $1/T_s$ Hz, and may therefore be referred to as Instantaneous Harmonic Control (IHC) algorithms.

There are a number of different adaptation algorithms using the gradient descent approach given by eq. 4, with the distinct difference generally being the definition of the complex matrix $\mathbf{C}$. Using the method of steepest-descent to minimise the quadratic cost function leads to $\mathbf{C} = \mathbf{G}^H$ and the resulting steepest-descent update algorithm is given by

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha \mathbf{G}^H \mathbf{e}(n). \tag{5}$$

Alternatively, the control signals can be updated at each iteration using an estimate of the exact least-squares solution, given by eq. 3, and in this case $\mathbf{C} = \left[\mathbf{G}^H \mathbf{G}\right]^{-1} \mathbf{G}^H$ and the update equation is given by

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha \left[\mathbf{G}^H \mathbf{G}\right]^{-1} \mathbf{G}^H \mathbf{e}(n). \tag{6}$$

This will be referred to as Newton's algorithm in the remainder of this paper.

The definition of the matrix $\mathbf{C}$ in the update algorithm will affect both the stability and convergence properties of the algorithm. For example, the convergence of the steepest-descent algorithm has been shown to converge in a series of modes, whose time constants are dependent on the eigenvalues of the Hessian matrix, which in the case of a perfect plant model is $\mathbf{G}^H \mathbf{G}$ [8]. If the eigenvalue spread of this matrix is large, which it can be in a practical system, then the speed of the slow modes of convergence may limit the practical performance of the controller. In such instances it may be desirable to employ a controller whose convergence is less dependent on this eigenvalue spread and this is achieved by Newton's update algorithm given by eq. 6 [9]. In this case the steepest-descent update is essentially pre-multiplied by the matrix $\left[\mathbf{G}^H \mathbf{G}\right]^{-1}$ which compensates for the eigenvalue spread and means that the time constants of the modes of convergence are all equal [9]. In practice, however, the plant response matrix used in the control signal update equation will not be equal to the physical plant matrix and pre-multiplying by $\left[\mathbf{G}^H \mathbf{G}\right]^{-1}$ in Newton's algorithm may cause significant amplification in the components of the control signals corresponding to the smallest singular values [9]. This means that Newton's algorithm is significantly less robust to perturbations in the plant response than the steepest-descent algorithm.

It is clear that the eigenvalue spread of the Hessian matrix limits the performance of the two common gradient descent algorithms given by eqs. 5 and 6 in different ways. For instance, although the widely employed steepest-descent algorithm is robust to plant response variations, its speed of convergence to the optimal solution is limited by the eigenvalue spread of the $\mathbf{G}^H\mathbf{G}$ matrix. Conversely, although the speed of convergence of Newton's algorithm is not affected by this eigenvalue spread and theoretically converges much more rapidly to the optimal solution, it is particularly sensitive to variations in the plant response and, therefore, is practically difficult to implement. Therefore, in the following section a novel algorithm is proposed which attempts to adaptively combine the properties of these two common algorithms.

3. Combined Steepest-Descent and Newton's Method Algorithm

To overcome the speed limitations of the steepest-descent algorithm and the robustness limitations of Newton's algorithm, we seek an update operator matrix, $\mathbf{C}$, for the general gradient descent update algorithm given in eq. 4, which combines the properties of the two algorithms. If the two algorithms were implemented simultaneously, then their outputs could be combined adaptively using the convex combination method defined by Ferrer *et al* [11] in the context of active noise control and more generally by Arenas-Garcia *et al* [12]. However, this requires a significant increase in computational cost [11] and it is not clear whether combining the steepest-descent and Newton's algorithms is a convex optimisation under all conditions. For example, if the MIMO plant response, $\mathbf{G}$, has a large eigenvalue spread, then it is possible that the two algorithms will converge to different solutions in practice and under this condition it is not currently clear how the convex combination method will behave.

To avoid the additional computational cost and the unknown potential issues of the convex combination method, we define a new time varying update operator which combines the Newton's and steepest-descent update operators as

$$\begin{aligned}\mathbf{C}(n) &= \lambda(n)\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H + (1-\lambda(n))\alpha_{SD}\mathbf{G}^H,\\ &= \left[\lambda(n)\left(\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1} - \alpha_{SD}\mathbf{I}\right) + \alpha_{SD}\mathbf{I}\right]\mathbf{G}^H,\end{aligned} \tag{7}$$

where α_N and α_{SD} are the convergence coefficients for the Newton's and steepest-descent algorithms respectively and $\lambda(n)$ is a time varying mixing parameter which allows the algorithm to operate as either Newton's or steepest-descent, or a combination of the two. Substituting eq. 7 into eq. 4 gives the control signal update equation as

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \left[\lambda(n)\left(\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1} - \alpha_{SD}\mathbf{I}\right) + \alpha_{SD}\mathbf{I}\right]\mathbf{G}^H\mathbf{e}(n). \tag{8}$$

If $\lambda(n) = 1$ this combined update algorithm reduces to Newton's algorithm and if $\lambda(n) = 0$ it reduces to the steepest-descent algorithm.

To implement the proposed combined update algorithm it is necessary devise a method to adapt the mixing parameter to achieve the optimal performance. This may be achieved by defining λ as a sigmoid function given by

$$\lambda(n) = \frac{1}{1+e^{-a(n+1)}}, \tag{9}$$

which is bounded between 0 and 1 for real values of a and adapting a according to a gradient-descent type algorithm to minimise the sum of the squared error signals. If a is adapted to minimise the frequency domain cost function given by eq. 2 then it is a complex value and λ will no longer be bounded between 0 and 1. However, if a is updated in the time domain then it is possible to employ the sigmoid mixing parameter method as previously employed in [11] for the convex combination algorithm.

The cost function given by equation 2 can be written in the time domain as

$$\tilde{J}(n) = \tilde{\mathbf{e}}(n)^T \tilde{\mathbf{e}}(n) \tag{10}$$

where $\tilde{\mathbf{e}}(n)$ is the time domain error signal which can be approximated in the steady state as

$$\tilde{\mathbf{e}}(n) = \tilde{\mathbf{d}}(n) + 2\Re\left\{\mathbf{G}\mathbf{u}(n)\mathrm{e}^{j\omega_0 n T_s}\right\}. \tag{11}$$

Substituting for the frequency domain control signal vector using equation 8 gives the time domain error signal vector as

$$\begin{aligned}\tilde{\mathbf{e}}(n) =& \tilde{\mathbf{d}}(n) + 2\Re\Big\{\mathbf{G}\Big[\mathbf{u}(n-1) - \Big[\lambda(n-1)\Big(\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1} - \alpha_{SD}\mathbf{I}\Big) + \alpha_{SD}\mathbf{I}\Big]\cdots \\ & \mathbf{G}^H\mathbf{e}(n-1)\Big]\mathrm{e}^{j\omega_0 n T_s}\Big\}. \end{aligned} \tag{12}$$

Using equations 10 and 12 it is now possible to derive an update equation for the parameter $a(n)$ using the gradient descent approach. This can be written as

$$a(n+1) = a(n) - \mu_a \frac{\partial \tilde{J}(n)}{\partial a(n)} = a(n) - \mu_a \frac{\partial \tilde{J}(n)}{\partial \lambda(n-1)} \frac{\partial \lambda(n-1)}{\partial a(n)} \tag{13}$$

where μ_a is the convergence gain. The first differential term in equation 13 can be evaluated as

$$\frac{\partial \tilde{J}(n)}{\partial \lambda(n-1)} = 4\Re\Big\{\mathbf{G}\Big(\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1} - \alpha_{SD}\mathbf{I}\Big)\mathbf{G}^H\mathbf{e}(n-1)\mathrm{e}^{j\omega_0 n T_s}\Big\}^T \tilde{\mathbf{e}}(n), \tag{14}$$

and the second differential can be evaluated as

$$\frac{\partial \lambda(n-1)}{\partial a(n)} = (1 - \lambda(n-1))\lambda(n-1). \tag{15}$$

The full update equation is thus given by substituting equations 14 and 15 into equation 13 to give

$$\begin{aligned} a(n+1) =& a(n) - \alpha_a \Re\Big\{\mathbf{G}\Big(\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1} - \alpha_{SD}\mathbf{I}\Big)\mathbf{G}^H\mathbf{e}(n-1)\mathrm{e}^{j\omega_0 n T_s}\Big\}^T \cdots \\ & \tilde{\mathbf{e}}(n)(1 - \lambda(n-1))\lambda(n-1) \end{aligned} \tag{16}$$

where α_a is the convergence coefficient.

Although equation 16 could be implemented directly, the effective input to the adaptive update algorithm, which is given by

$$\boldsymbol{\zeta}(n) = \Re\Big\{\mathbf{G}\Big(\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1} - \alpha_{SD}\mathbf{I}\Big)\mathbf{G}^H\mathbf{e}(n-1)\mathrm{e}^{j\omega_0 n T_s}\Big\}^T, \tag{17}$$

will vary with time as it is dependent on the error signal vector which is being minimised. Therefore, as in the conventional LMS algorithm it is prudent to normalise the update algorithm with respect to the input signal power. For this multichannel case the power of the input signals can be estimated as

$$\mathbf{p}(n) = \gamma\mathbf{p}(n-1) + (1-\gamma)\mathrm{Diag}\Big(\boldsymbol{\zeta}(\boldsymbol{n})^T\boldsymbol{\zeta}(\boldsymbol{n})\Big) \tag{18}$$

where Diag$(.)$ denotes the diagonal elements of the matrix. The multichannel normalised update algorithm can then be expressed as in [11] as

$$\begin{aligned} a(n+1) =& a(n) - \alpha_a \Re\Big\{\mathbf{G}\Big(\alpha_N\left[\mathbf{G}^H\mathbf{G}\right]^{-1} - \alpha_{SD}\mathbf{I}\Big)\mathbf{G}^H\mathbf{e}(n-1)\mathrm{e}^{j\omega_0 n T_s}\Big\}^T \cdots \\ & \mathbf{P}(n)\tilde{\mathbf{e}}(n)(1 - \lambda(n-1))\lambda(n-1), \end{aligned} \tag{19}$$

where $\mathbf{P}(n)$ is the power normalisation matrix given by

$$\mathbf{P}(n) = \begin{bmatrix} \frac{1}{p_1(n)} & 0 & \cdots & 0 \\ 0 & \frac{1}{p_2(n)} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & & \frac{1}{p_L(n)} \end{bmatrix}, \tag{20}$$

where $p_l(n)$ is the l-th element of the estimated power input vector given by equation 18.

4. Simulations

In order to compare the performance of the combined algorithm proposed in the previous section to the performance of the conventional steepest-descent and Newton's algorithms, a series of time-domain simulations have been conducted. The simulated control system consists of nine error sensors and five control actuators. The nominal plant response, $\mathbf{G}_0$, has been calculated from the responses measured between five inertial actuators and 9 accelerometers mounted on an arbitrary steel structure. The physical plant responses have been modelled using a bank of finite impulse response filters and the nominal plant response model has been calculated at the control frequency of 103 Hz. The vector of disturbance signals at the control frequency has also been calculated based on a model of the response between an additional actuator mounted to the structure and the 9 accelerometers.

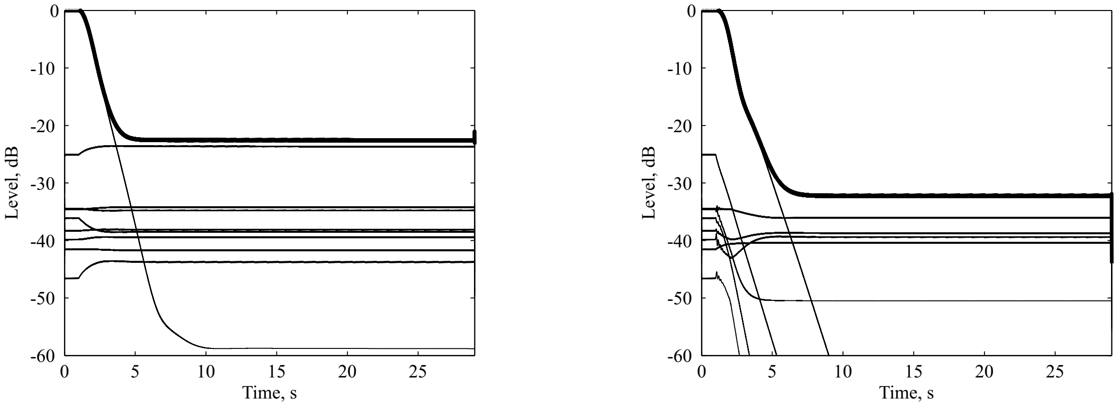
Initially, the performance of the steepest-descent and Newton's algorithms has been simulated for the case when the plant model is equal to the nominal plant response and the convergence coefficients, α_{SD} and α_N, have been defined to give the maximum convergence speed in both cases. The thick lines in the two plots in Figure 2 show the convergence of the sum of the squared error signals for the two algorithms and it is clear that, as expected, Newton's algorithm converges to a better solution more quickly and achieves 10 dB more attenuation. The difference in the performance of the two algorithms, as discussed in Section 2, is due to the eigenvalue spread of the matrix, $\mathbf{G}^H\mathbf{G}$. For the considered plant response the eigenvalue spread is very large, at 5.3×10^6, and is characterised by a single dominant eigenvalue. Based on the discussion presented in Section 2, it is expected that the performance of the steepest-descent algorithm will be severely limited by the slow modes of convergence. To demonstrate this, Figure 2 shows the convergence of the individual 'modes' of convergence for the steepest-descent and Newton's algorithms. From Figure 2a, which shows the results for the steepest-descent algorithm, it can be seen that only the dominant mode is significantly attenuated, whilst the second largest mode, which is enhanced by 1 dB, essentially limits the level of overall control. Conversely, from the results presented in Figure 2b for Newton's algorithm, it can be seen that the five convergent modes are all reduced with an approximately equal time constant and this allows Newton's algorithm to achieve a 10 dB improvement in the overall level of attenuation compared to the steepest-descent algorithm.

From the results presented above it is clear that for the case when the plant model is equal to the physical plant, Newton's algorithm outperforms the steepest-descent algorithm. However, it is also important to consider the effect of a perturbation in the plant response and, therefore, the behaviour of the two algorithms has been simulated for the case when an error has been introduced into the plant model such that the model is given by

$$\mathbf{G} = \mathbf{G}_0 + \mathbf{\Delta}, \tag{21}$$

where $\mathbf{G}_0$ is the nominal plant response and $\mathbf{\Delta}$ is a complex matrix of normally distributed random numbers. This perturbation matrix has been defined as

$$\mathbf{\Delta} = \epsilon(\mathbf{\Delta}_R + j\mathbf{\Delta}_I), \tag{22}$$



(a) Steepest descent algorithm. (b) Newton's algorithm.

Figure 2: The sum of the squared error signals, normalised by the sum of the squared disturbances (thick line), and the individual 'modes' of convergence for the two conventional algorithms with a perfect plant model (thin lines).

where $\mathbf{\Delta}_R$ and $\mathbf{\Delta}_I$ have been defined as matrices of normally distributed random numbers with zero mean and a variance approximately equal to the real and imaginary parts of the elements of the nominal plant matrix $\mathbf{G}_0$ respectively, and ϵ is a scalar gain used to define the level of the perturbation. To characterise the effects of random errors in the plant model, the average performance of the steepest-descent and Newton's algorithms has been calculated for 100 different random perturbation matrices with $\epsilon = 0.1$. The resulting performance of the steepest-descent and Newton's algorithms with and without errors in the plant model is shown in Figure 3.

From the black and red lines in this plot it can be seen that the introduced plant model error does not affect the performance of the steepest-descent algorithm, however, it is clear from comparing the blue and green lines that the plant model error causes Newton's algorithm to diverge. This is consistent with the discussion presented in Section 2, but it is also clear that despite this divergence, Newton's algorithm with plant modelling error initially converges to a better solution than the steepest-descent algorithm. The purple line in Figure 3 shows the performance of the combination algorithm defined by eq 8. The convergence coefficients α_{SD} and α_N are set to the same values as when the two conventional algorithms are implemented independently. From the results presented in Figure 3 it can be seen that the proposed combination algorithm converges to a steady state solution which provides 5 dB more attenuation than the steepest-descent algorithm and does not diverge like Newton's algorithm.

5. Conclusions

Multichannel active control of tonal noise and vibration has been widely implemented in practice and the steepest-descent algorithm has proven to be a practically robust method of adapting the control signals. However, the convergence of the steepest-descent algorithm is limited by the slow modes of convergence which occur when the eigenvalue spread of the $\mathbf{G}^H\mathbf{G}$ matrix is large. This dependence on the eigenvalue spread is avoided in Newton's algorithm, however, this algorithm is significantly more sensitive to errors in the plant response model.

To overcome the limitations of the two update algorithms and combine their positive attributes, a new update algorithm is proposed with a time varying gradient-descent operator. This operator combines the steepest-descent and Newton's update operators via a mixing parameter, λ, such that for $\lambda = 1$ the update algorithm is equal to the Newton's algorithm and for $\lambda = 0$ it is equal to the steepest-descent algorithm. A method of adapting this mixing parameter according to a gradient-descent algorithm to minimise the sum of the squared error signals is proposed.

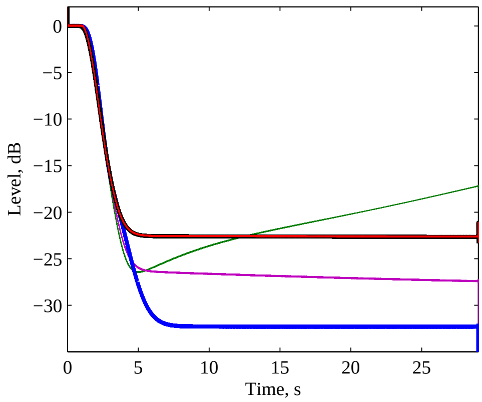


Figure 3: The convergence of the sum of the squared error signals for the steepest-descent algorithm with $\mathbf{G} = \mathbf{G}_0$ (—) and $\mathbf{G} = \mathbf{G}_0 + \boldsymbol{\Delta}$ averaged over 100 random plant error perturbations (—), Newton's algorithms with $\mathbf{G} = \mathbf{G}_0$ (—) and $\mathbf{G} = \mathbf{G}_0 + \boldsymbol{\Delta}$ averaged over 100 random plant error perturbations (—), and the proposed combination algorithm with $\mathbf{G} = \mathbf{G}_0 + \boldsymbol{\Delta}$ averaged over 100 random plant error perturbations (—).

The performance of the proposed algorithm has been evaluated through a series of simulations in which a random plant modelling error has been introduced. Under these conditions the Newton's update algorithm leads to a divergence of the cost function, whereas the proposed algorithm is able to increase the level of control compared to the steepest-descent algorithm whilst remaining stable.

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