Cooperative Electromagnetic Interactions and Linwidth Narrowing in Discrete Metamaterial Systems

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Multiple scattering of the electromagnetic (EM) field from an ensemble of resonators generates interactions which can lead to a cooperative response. In atomic gases, the cooperative response is often washed out due to fluctuations in atomic positions. When these fluctuations are restricted, as in an optical lattice in a Mott-insulator state with precisely one atom per lattice site, a cooperative response can be observed [1]. The ability to fabricate metamaterials whose constituent circuit elements, meta-atoms at fixed positions, interact in prescribed ways with EM fields also permits the construction of systems in which cooperative phenomena emerge.

In this work, we show that the cooperative response of an ensemble of discrete resonators can result in a transmission resonance whose quality factor increases with the size of the system. This behaviour arises from the formation of a collective eigenmode of excitation that possesses a suppressed, or subradiant, emission rate [2]. Our results are in excellent agreement with the experimental observations of Ref. [3]. To describe the metamaterial, we developed a model [4] in which one dynamic variable represents a meta-atom interacting with the EM fields. Collective modes emerge from interactions between meta-atoms mediated by the EM field, each mode with its own resonance frequency and decay rate.

In particular, we consider a 2D array of ASRs arranged in a square lattice. Each ASR consists of two concentric arcs. If currents oscillate in phase, an ASR possesses an electric dipole, while currents moving out of phase produce magnetic dipoles. An asymmetry in arc lengths yields a difference in single meta-atom resonance frequencies $\delta\omega$ which causes a coupling between electric and magnetic dipole oscillations [2]. The metamaterial is illuminated by an incident plane wave propagating normal to the array with electric field $\mathbf{E}_{\text{in}}$ oriented along the ASR electric dipole.

The incident field addresses collective modes which are phase matched, with all ASRs oscillating in sync. One, the collective electric mode, scatters the field resulting in reflection. On the other hand, the magnetic mode in which all magnetic dipoles oscillate in phase (see Fig. 1a) has a subradiant emission rate. Excitation of the magnetic mode at the expense of the electric mode is responsible for the transmission resonance observed in Ref. [3]. Figure 1(b) and (c) shows the numerically calculated properties of the phase matched magnetic mode. The linewidth of the magnetic mode decreases with the size of the system. Figure 1d compares the collective linewidth lifetime with the quality factor of the resonance observed in Ref. [3]. When ohmic losses are accounted for, our theoretical model provides a remarkable agreement with experimental results.

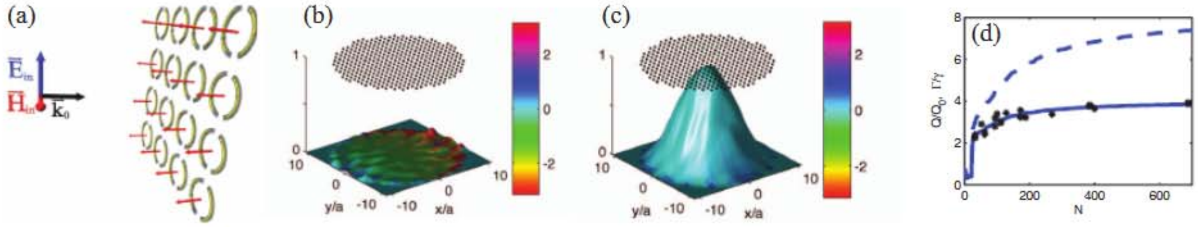

Fig. 1 Excitation of the collective magnetic mode: (a) An illustration of an incident EM field driving the phase-coherent collective magnetic eigenmode. (b) and (c): The numerically calculated uniform magnetic mode for an ensemble of 335 ASRs in which all magnetic dipoles oscillate in phase with minimal contribution from ASR electric dipole excitations. (b) The electric dipole excitation and (c) the magnetic dipole excitations of the ASRs in the uniform magnetic mode. The phase of the excitations are indicated by the colour of the surfaces. The black dots indicate the positions of the ASRs in the array. This mode was calculated for a lattice spacing of $a \simeq 0.28\lambda$. (d) Comparison between experimentally measured transmission resonance quality factors Q/Q_0 (stars) from Ref. [3], where $Q_0 \simeq 4.5$ denotes the single meta-atom quality factor, and numerically calculated resonance linewidth γ of the collective magnetic mode with ohmic loss rate $\Gamma_O \simeq 0.14\Gamma$ (solid line) and $\Gamma_O = 0$ (dashed line), and asymmetry in meta-atom resonance frequencies $\delta\omega = 0.3\Gamma$ where Γ is the single meta-atom emission rate.

References

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