Hybrid contour method/eigenstrain model to predict residual stress in glass

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Abstract

Using finite element (FE) models the residual stress present in a glass sample was constructed using the knowledge of surface deformation resulted in from the stress relaxation along a newly cut plane. The residual stress profile, validated with a scatter light polariscope, was then used to accurately establish the misfit strain (i.e. eigenstrain) of the original glass specimen. The paper shows that once the underlying eigenstrain distribution has been determined, the complete residual stress distribution can simply be determined by incorporating the eigenstrain profile as a misfit strain in an appropriate FE model. The results show that stress depth profile generated in float glass is parabolic. It is also shown that the hybrid model enables modelling residual stress in new geometries (e.g. stress concentrations around the hole) and/or during subsequent loading application, by simply using the knowledge of eigenstrain depth profile.

Keywords: Contour Method, Eigenstrain, Finite Element, Glass, Residual Stress

Introduction

Over the last decades, owing to its unique properties, designers began to use glass as a load bearing structural material in buildings [IStructE (1999)]. One of the greatest difficulties that inhibits accurate prediction of the structural response of commercial glass is the lack of comprehensive analytical/numerical tools to predict residual stress distributions generated due to manufacturing cooling process. The current design guidelines in the UK are lacking in complete design methodology, most of them being based on rules of thumb [IStructE (1999)]. *The Institution of Structural Engineers* design guidelines [IStructE (1999)] provide design permissible tensile stress values (e.g. 28 N/mm^2 for float glass with a thickness up to 6 mm) but recommend that: “In the absence of code-based allowable tensile stresses it is left to the judgment of the engineer what figures to adopt”. This often results in over designed conservative structures.

The misfit strains developed during the cooling of glass in the manufacturing process generate residual stresses. As it is known the effect of residual stresses can be significant in the strength prediction of materials [Withers (2007)]; thus negligence of this can lead to premature failure of the structures. Glass, in particular, is susceptible to brittle fracture failures due to the presence of inevitable surface defects.

The non-crystalline microstructure of the glass makes it impossible to determine the residual stress using conventional experimental methods such as X-ray or neutron diffraction. A scattered light method [Aben et. al. (2008)], which uses the magnitude of birefringence in glass, may be used to determine residual stresses. However, the method is limited by not being able to accurately determine the through thickness stress profiles at certain depths, especially in annealed glass. The current paper presents a validated hybrid contour /eigenstrain method to characterise the full field residual stresses in commercially available float and tempered glass.

Methodology

The contour method, originally developed by Prime (2001) to model residual stresses in metallic components, was used in the present study to determine the residual stress in glass. Although the contour method provides information about the residual stresses, it is not able to advance any details about the cause of the residual stresses (i.e. eigenstrain). Using the knowledge of the residual stress distribution constructed from the contour method, the eigenstrain distribution in the sample was computed from an inverse eigenstrain analysis developed by Korsunsky (2009). The knowledge of the eigenstrain profile was then used to compute the stress concentrations around a hole in plate with new geometry, and/or under applied loads. Fig. 1 presents the step-by-step procedure of the current method.

Original glass sample

↓ Cut sample in half and measure surface displacements

Displacement contour of the cut plane

↓ Finite element analysis

Residual stress

↓ Inverse eigenstrain analysis

Eigenstrain distribution

↓ Finite element model

Residual stress in new geometries/during applied loadings

Figure 1. Step-by-step procedure of contour/eigenstrain hybrid method

Construction of residual stress using contour method

In the contour method, the residual stress distribution is determined by incorporating the surface contour perpendicular to the cut plane as a displacement boundary condition in a static finite element (FE) model representing an initially stress-free half sample of the original specimen. Therefore, the determination of the displacement contour of the cut plane is a prerequisite in the present analysis. The contour method principle illustrated in Fig. 2 shows that after cutting the sample in half (Fig. 2b), the surface deformation (average of surface deformation of both sides is used to account for possible shear effects) is used to construct the residual stress developed in one half of the specimen as a response of forcing back the deformed surface to the initially planar state (Fig. 2c). The contour method proved its success in modelling residual stress in steel and metal alloys in different applications e.g. bent elements [Pagliaro (2008)], welds [Hosseinzadeh (2011)]. A comprehensive discussion of theory of the contour method can be found elsewhere [Pagliaro (2008); Johnson (2008)].

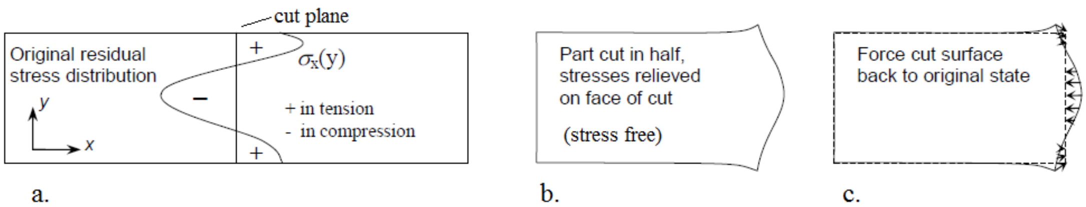

Figure 2. Contour method principle [Prime (2001)]

The analysis of a comercially available float glass sample (Pilkington Glass) of 150 x 100 x 10 mm is discussed below to demonstrate the application of the method to construct residual stress field. Making a cut which is free from defects is not trivial, in particular due to the brittle nature of glass. The most suitable cutting technique used for metal specimens is the wire electric discharge machine (EDM) [Prime (2001)], where the material removal is done by spark erosion. However the method cannot be used to cut glass and different alternatives such as diamond disk cutting and water-jet cutting were investigated. The results revealed that water-jet cutting (done by a comerical contractor) with a jet diameter of ~1 mm and a 80 mesh garnet grade provides a good "cut plane". It should be appreciated that in the experiments sacrificial glass pieces, as shown in Fig. 3a, were used on either side of the main specimen to eliminate the "edge effects".

After the sample was cut into two halves, the displacement contour of both cut surfaces was measured using a 3-D micro-coordinate system which offers great accuracy being able to achieve a vertical resolution of up to 10 nm [Alicona, 2.1.5]. It was observed that the GFM G4 10x objective offers the optimum magnification for determining the displacement contour profile of the sample. The 3-D contour presented in Fig. 3b was done with a sampling distance of 1.75 µm. The measurement was made along the thickness of the sample (10 mm) and accros a width of 1 mm, chosen to eliminate the effects due to potential local defects (e.g. micro-cips).

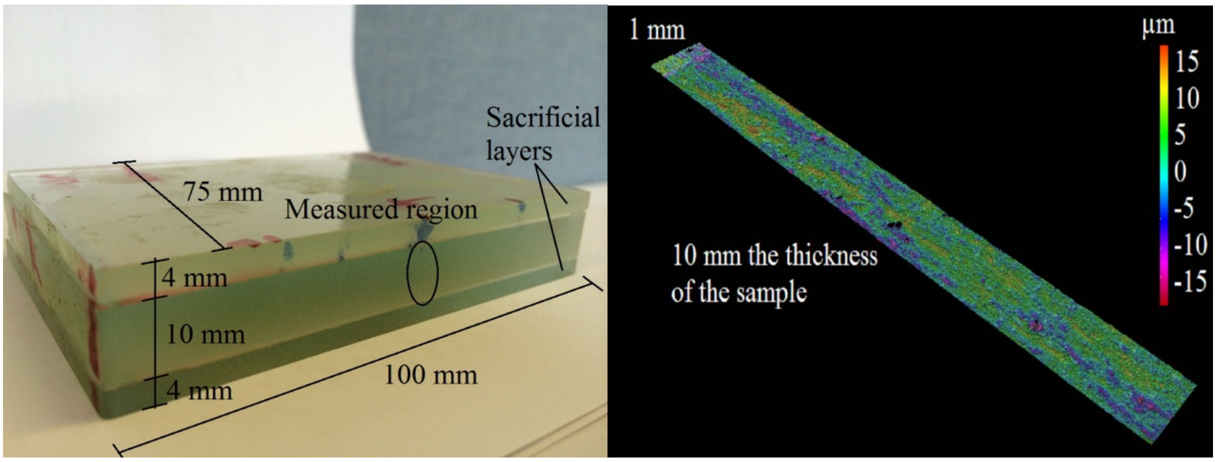

Figure 3. a. Half of the sample after cutting (left), b. 3-D contour of the cut-plane (right)

It should be appreciated that the deformations due to stress relaxation in this specimen are ~1-2 µm. However, the current work is a feasibility study to demonstrate the application of the present hybrid model to predict stresses in tempered glass, which is widely used in commercial applications. Because of the significantly high stresses in tempered glass it is expected that displacement along a cut plane in tempered glass will be more significant than that in an annealed glass specimen.

The surface deformations of each side of the cut shown in Fig. 4a are very similar. Separate measurements across the length of the sample were taken and it was concluded that the surface deformation is mostly uniform in the lateral direction and varies only along the thickness. For instance, Fig.4b shows the measured contour depth profile at three different locations within the left cut plane; all three profiles are very similar. Therefore, it is appropriate to incorporate the surface contour of the cut-plane into a FE model as a polynomial curve that varies in thickness. The average depth profile presented in Fig. 4a was represented by a best fit 2nd order polynomial with coefficients: -0.045, 0.419, -0.616.

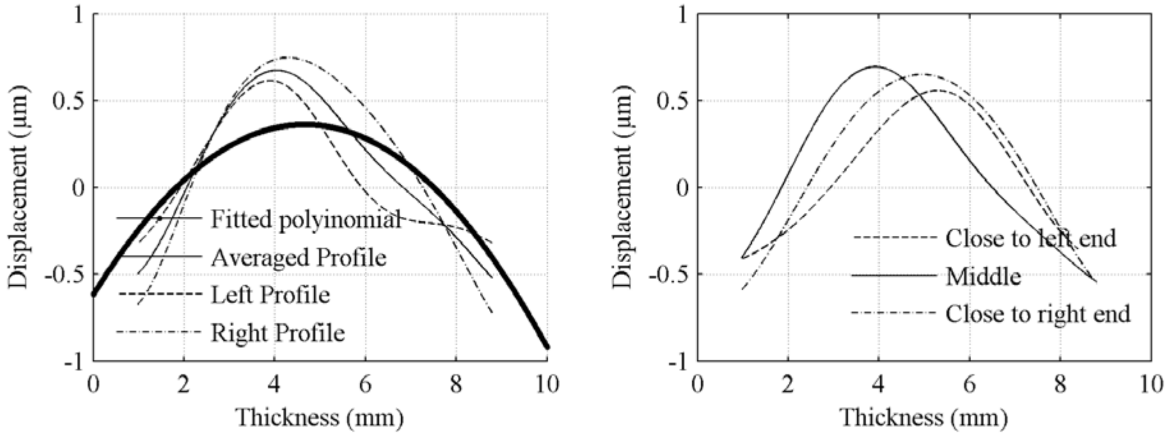

Figure 4. a. Averaged middle profile and fitted polynomial (right), b. Displacement profiles at various locations (left)

ABAQUS/Standard [Abaqus, 6.9-3] was used in the present study to model the residual stress distribution. A 3-D model with 8-noded, linear brick stress elements (3D8R) was used in the simulation. With a linear-elastic behaviour, it was appropriate to assume a material model with Young modulus =70 GPa and a Poisson's ratio =0.23 to characterise material properties of glass. The residual stress distribution was conveniently determined by incorporating the approximate polynomial curve of the displacement contour (Fig. 4a) in a FE model representing the initially stress-free half sample of the original glass specimen.

The residual stress distribution computed using the above FE simulation is presented in Fig. 5a (only the stress component normal to the cut surface is shown here since it is the most relevant stress distribution). The results show a parabolic stress distribution, with tension at the outer surface (~8 MPa) and compression (~7 MPa) at the mid-thickness. The depth of the tension zones on each side is 2 mm (~20% of the specimen thickness) and is balanced by a middle compression zone of 6 mm thick (~60% of the overall thickness). It is worth noting that the "edge effects" due to FE simulation could not be avoided, thus the surface stress predicted from the present FE model may be slightly overestimated. Thus, the above quoted surface tension value is actually the value slightly below/above (~1 mm) the actual surface. Fig. 5b shows that the cumulative force along the depth is zero when integrating the stresses along the depth profile.

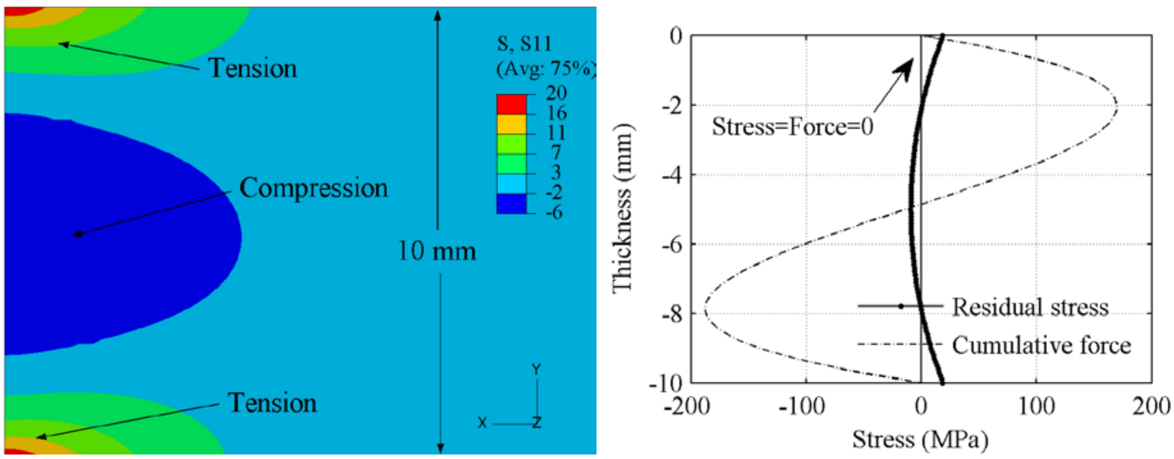

Figure 5. a. Residual stress (left), b. Cumulative force distribution (right)

The unique characteristic of the current method is that the full stress field can be predicted fulfilling the overall equilibrium, compatibility and boundary conditions. It should be noted that the magnitudes of the predicted residual stresses agree with the expected stresses in practice [Geandier et. al (2003)]. Experiments using a scatter light polariscope are currently being undertaken to validate the present results.

Eigenstrain profile estimation using the constructed residual stress distribution

Although, the contour method can be used to model the residual stress in a given glass specimen, a separate experiment programme is required to predict the stress distribution in a new specimen. The residual stress is a response to the eigenstrain developed in the specimen during glass manufacturing process. Thus, as shown hereafter, once the knowledge of the eigenstrain distribution is available the stress state in real-life practical structural elements can be determined in a computationally efficient manner.

The eigenstrain method [Korsunsky (2009)], which is used here to determine the eigenstrain profile, uses a least square approach to determine the unknown eigenstrain distribution based on the residual elastic stresses measured at a finite number of locations. The technique was successfully used [Achintha and Nowell (2011)] to reconstruct the full residual stress field in alloy materials due to laser shock peening.

Assume eigenstrain distribution
(A Chebyshev series)

↓ FE model for each polynomial of the series

Resultant residual stress distribution

↓ Minimisation of the mismatch between the measured and calculated stress

Accurate eigenstrain profile of the sample

↓ Finite element model

Residual stress in new geometries/during applied loadings

Figure 6. Step-by-step procedure of an inverse eiegnstrain analysis of the hybrid method

For the glass sample discussed previously, the eigenstrain distribution was considered to be uniform in the lateral direction, varying only with thickness. Initially an eigenstrain profile represented as a series of Chebyshev polynomials [Mason and Handscomb (2003)] was assumed (although alternative polynomial choices are possible). The number of polynomials in the series is to some extent arbitrary, but it should be large enough to capture the exact form of the eigenstrain distribution accurately. In the present study the analysis was done for a different number of polynomials to ensure that the result is independent of the value chosen. On separate FE models each polynomial of the assumed Chebyshev series was implemented individually and the respective residual stress in the specimen was determined. The response of the specimen to the applied eigenstrain is elastic, thus the resultant residual stress distribution caused by the original assumed eigenstrain distribution is the sum of each individual residual stress. Using a least-square analysis between the predicted stress and the corresponding measured data, the accurate estimate of the actual eigenstrain distribution was established. It should be appreciated that the stress values determined previously using the contour method were used as experimental data in this analysis. Once the best estimate of the eigenstrain distribution has been established, the residual stress distribution can be determined in the usual way by incorporating this eigenstrain distribution in a FE model. The step-by-step procedure of the analysis technique is presented in Fig. 6.

Fig. 7a shows the “best estimate” of the eigenstrain depth profile and Fig. 7b shows the comparison between the residual stress depth profile determined from the earlier contour analysis and that from the eigenstrain analysis. From Fig. 7b it is evident that, as expected, the predictions from the eigenstrain method agree with that constructed from the contour method. The small mismatch between stress profiles is related to the procedure in which the residual stresses were determined. The contour method uses boundary displacement to obtain residual stress and thus the residual stress distribution is correctly predicted only at that edge. In the case of eigenstrain method the full residual stress was determined as an overall response of the model to the eigenstrain profile.

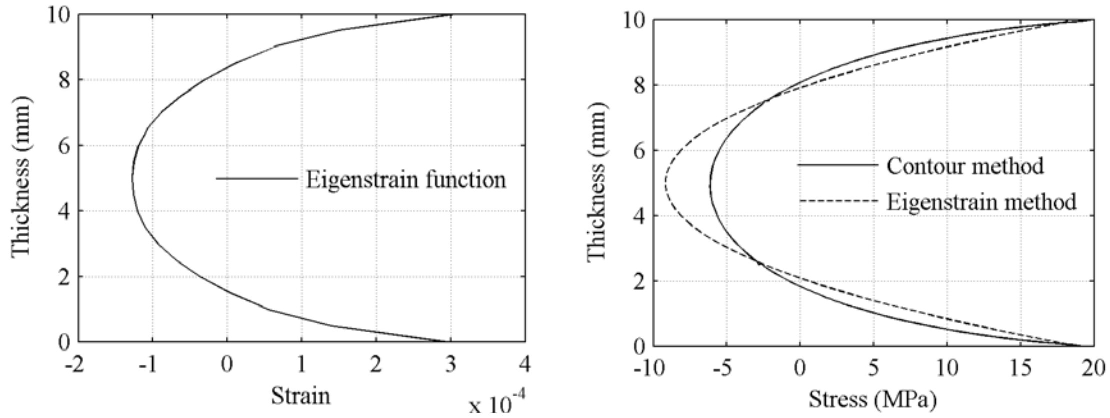

Figure 7. a. Computed eigenstrain distribution (left) b. Comparison of stress profile (right)

Prediction of stress distribution in different structural elements

The knowledge of the eigenstrain distribution allows determining the structural response of real-life structural glass elements of practical geometries and/or under applied loading.

For instance, the knowledge of the eigenstrain profile was used to study the effects due to geometry in a practical glass element. The results of a glass plate (150 x 100 x 10 mm) with a 20 mm diameter central hole, under uni-axial tensile loads of 10 and 20 MPa (X direction) are discussed. Symmetry conditions are used to model only a quarter of the specimen. As it can be seen in Fig. 8a, after the eigenstrain distribution was implemented to the FE model, a full residual stress field (only the middle principal stress is shown in Fig. 8a) was achieved in the sample (e.g. B) fulfilling the equilibrium conditions at the boundaries (e.g. A, C, D).

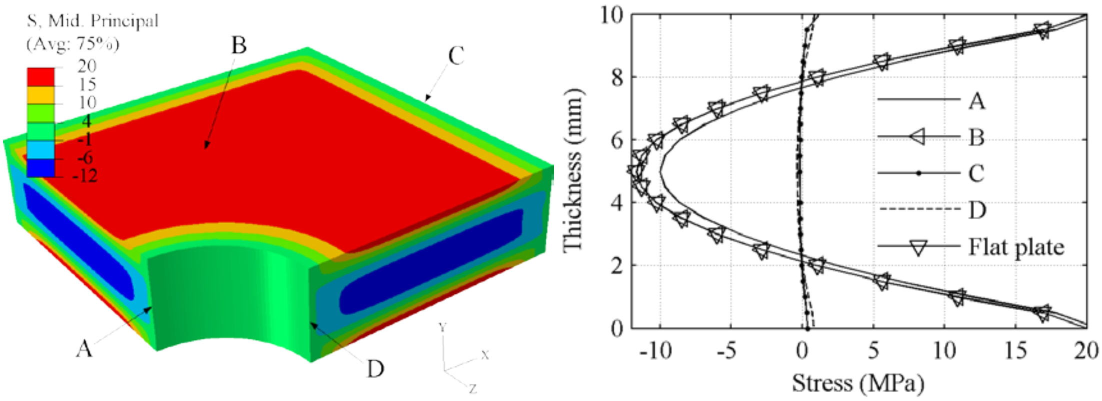

Figure 8. a. Residual stress field (Mid. Principal) b. Stress profile in X direction (S11) at different locations in the sample

Initially the sample has no applied load and the presence of the hole does not influence the residual stress distribution. The results presented in Fig. 8b show that the residual stress in X direction (i.e. along the direction of the applied uni-axial tension) across path A of the hole, and across path B (through thickness) in the plate matches the one of the flat sample without the hole.

Under tensile load, the stress distribution in the sample is no longer uniform; stress concentrations around the hole have developed as can be seen in Fig. 9a. Fig. 9b presents the same sample as before without any initial residual stress. As expected, the magnitude of the stresses in the sample if there was no initial residual stress is lower than that in the sample incorporating residual stresses. From both models it is evident that path A represents a locus where stress concentrations arise, but only the model incorporating initial residual stress is able to provide a comprehensive analysis of stress distribution and evolution in the sample during loading.

It is expected that residual stresses distribution affects the failure of a structural element. If, for example, the exampled considered here was to have an ultimate tensile load of 40 MPa, the FE glass model not incorporating the initial residual stress distribution would result in a satisfying structural design. Whereas, as shown in Fig. 10a, in the case of the FE glass model, in which the residual stress distribution was considered, it is clear that the ultimate limit was reached and the structural element might unexpectedly fail.

Considering the stress distribution along the path A (Fig. 9) was a favourable case, because both models were presenting stress concentration there. If one is to consider a random path away from the edges (path B) then accounting for stress distribution in the analysis makes a significant difference. As can be seen in Fig. 10a the results show that the residual stress (RS) completely changes the stress profile and the magnitude of the surface tension is more than double the value that was computed using a FE model without any initial residual stress. This proves that by incorporating residual stresses into analysis facilitates the modelling of the full stress field generated during subsequent loading. The results presented in Fig. 10b show that even though the step size of the load was constant (10 MPa) the magnitude by which the residual stress distribution (across path A) for each step increased was not constant.

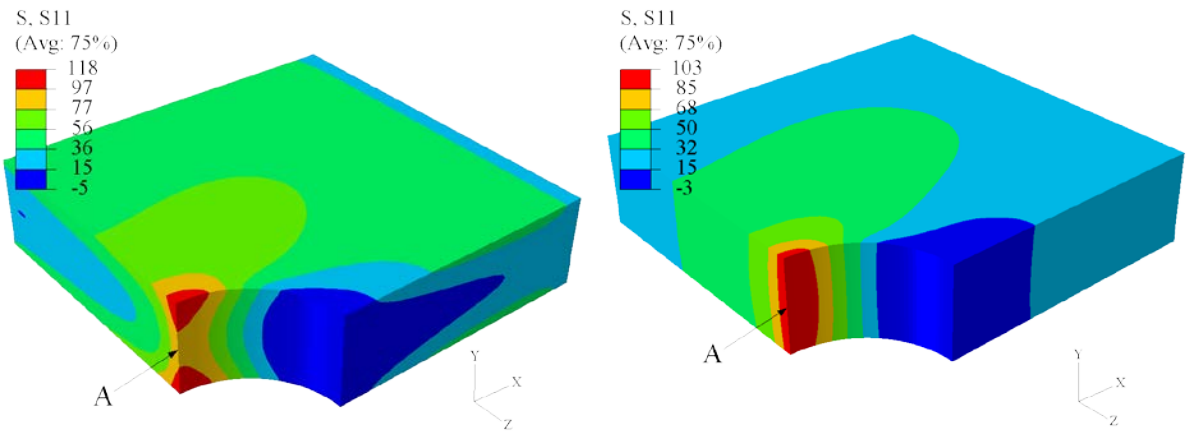

Figure 9. a. Glass sample with residual stress under 20 MPa (X direction) tensile load (left), b. Glass sample without residual stress under 20 MPa (X direction) tensile load (right)

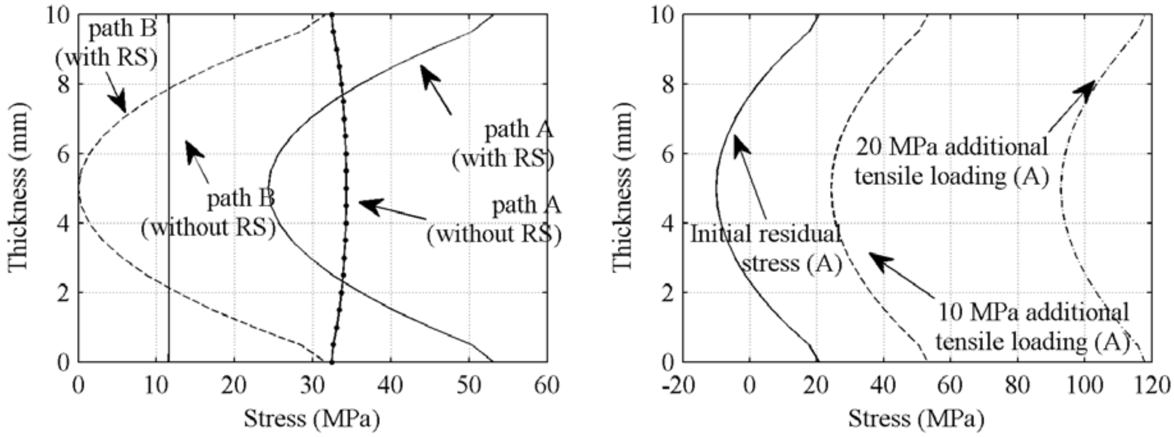

Figure 10. a. Stress profile at 10 MPa tensile loading (left) b. Stress profile along path A for different load steps (right)

Conclusions

This paper presents a hybrid validated experimental/numerical-modelling tool to characterise the residual stress present in commercially available glass. The study shows that the hybrid modelling approach works well to model the residual stresses. It has been shown that, by applying contour-method based finite element models, the full residual stress distribution that satisfies overall equilibrium, compatibility and boundary conditions can be accurately determined. An inverse eigenstrain analysis has been developed to estimate the eigenstrain distribution of glass. The eigenstrain distribution depends only on the glass manufacturing process, therefore, once the knowledge of the eigenstrain depth profile in a given glass type is available, structural response of practical glass elements under service loads can be determined in a computationally efficient manner.

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