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Energy harvesting from a rotational transducer under random excitation

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This paper evaluates the performance of a proposed device for harvesting energy from the vertical motion of boats and yachts under broadband and band-limited random vibrations. The device comprises a sprung mass coupled to an electrical generator through a ball screw. The mathematical equations describing the dynamics of the system are derived. Then by utilizing the theory of random vibration, the frequency response function of the system is obtained. This is used to derive an expression for the mean power produced by the harvester when it is subjected to broadband and band-limited stationary Gaussian white noise. The power expressions are derived in dimensional form to provide an insightful understanding of the effect of the physical parameters of the system on output power. An expression for the optimum load resistance to harvest maximum power under random excitation is also derived and validated by conducting Monte-Carlo simulation. The discussion presented in the paper provides guidelines for designers to maximize the expected harvested power from a system under broadband and band-limited random excitations. Also, based on the method developed in this paper, the output power of a rotational harvester subjected to the vertical excitation of a sailing boat is obtained. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4892452]

I. INTRODUCTION

Harvesting energy from ambient vibrations has been the subject of significant research in recent years, resulting in numerous publications, including a number of review articles and books.[1–5]

The marine environment is known as a significant source of energy. Many vibration energy harvesting systems introduced for this environment rely on a proof mass coupled to an electric generator whose relative movement is, directly or indirectly, caused by the waves.[6,7] However, due to the low frequency of wave oscillations, the relative speed of the proof mass is low and, hence, a direct drive generator can be quite large and expensive relative to the amount of power it produces. Increasing the power density of the generator can be accomplished by exploiting a mechanism to increase the speed to the generator.[8] For example, a screw is an intermediate high efficiency mechanism which can be utilized to convert a low frequency linear movement to a high frequency rotational motion.[9–11] However, reviewing the studies on designing a ball screw based electromagnetic energy harvester for extracting energy in marine environment indicates that they are mostly designed, characterized, and optimized by assuming that the device is under harmonic excitation. On the other hand, in real operating conditions, the device may be subjected to random vibrations which are distributed over a broadband of frequencies.[12]

The theory of random vibration applied to mechanical systems has been studied by a number of researchers.[13–15] Halvorsen[16] used linear random vibration theory to obtain closed-form expression of output power for a general energy harvester model. Adhikari *et al.*[17] used the same approach to derive an expression for the mean normalized harvested power of a piezoelectric based energy harvester. Renaud *et al.*[18] derived closed-form formula describing the power and efficiency of a piezoelectric energy harvester subjected to sinusoidal and random vibrations. It is shown that under random excitation, the optimum generated power is directly proportional to the efficiency of the harvester.

Tang and Zuo[19] studied the performance of single-mass and dual-mass electromagnetic energy harvesters under random force, displacement, velocity, and acceleration. However, in their discussions, no distinction is made between the internal resistance and the load resistance of a generator. Therefore, the derived power formula is the sum of the useful electrical energy and the electrical power loss. In other words, the paper focuses on the power flow from the mechanical environment into the electrical domain rather than the delivery of useful power to an electrical load which is more appropriate. In addition, none of the above mentioned research works investigates the performance of an electromagnetic energy harvesting system under band-limited excitation.

This paper studies the performance of a proposed ball screw based energy harvesting device, shown in Fig. 1, subjected to broadband and band-limited white Gaussian noise. The proposed harvester is designed to be used on a boat or a buoy; in this paper, the analytical solutions within the theory of random vibration are extended to find the average value of the output power and derive an expression for the optimum load resistance to harvest maximum power under broadband white excitation. Also, the expression for output power under band-limited excitation is derived and design criteria to maximize the output power of an energy harvester under random excitation are discussed.

II. SYSTEM MODELING

Fig. 1 shows a drawing of the proposed device. It comprises a sprung mass coupled to an electrical generator via a ball screw. In this base excited mass spring system, the relative

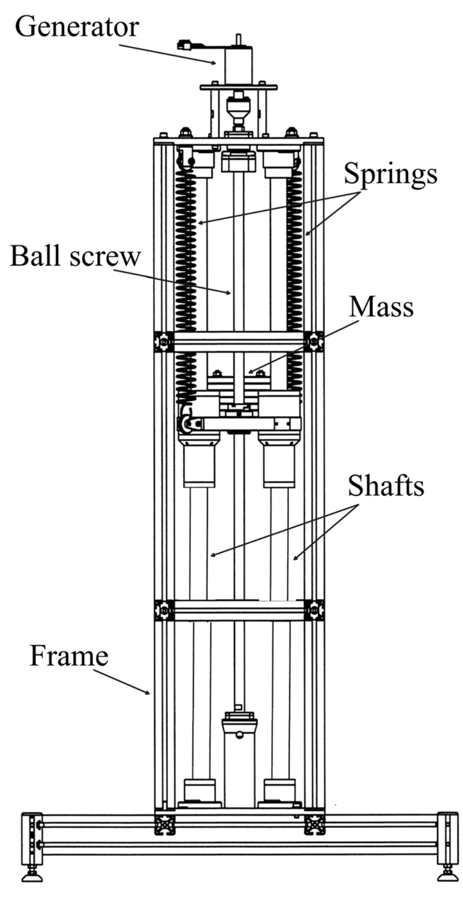

FIG. 1. Drawing of the energy harvesting device.

motion of the mass is produced by the vertical oscillation of a boat or buoy. The boat's vertical motion causes the mass to oscillate relative to the boat which in turn drives a generator through the ball screw coupling. In the analysis of an energy harvesting device, such as the one shown in Fig. 1, it is common to study the non-dimensional model of the system.[20] However, as the goal of this paper is to study the optimal selection of the physical parameters of the system, the dimensional model of the system dynamics are retained.

Fig. 2 shows a free body diagram of the proposed device, where l, m, c_{bg}, k, and F_{EM} are, respectively, the lead size of the ball screw, mass, mechanical damping, overall spring stiffness, and the reflected force due to the generator electromagnetic coupling.

Considering the relative displacement of the mass, the governing differential equation of motion for the system shown in Fig. 2, with respect to the relative displacement of seismic mass $z(t) = x(t) - y(t)$, is

$$M\frac{d^2z(t)}{dt^2} + c\frac{dz(t)}{dt} + kz(t) = -m\frac{d^2y(t)}{dt^2}, \tag{1}$$

where M is the sum of the seismic mass and reflected moment of inertia of rotational components given by

$$M = m + J\left(\frac{2\pi}{l}\right)^2, \tag{2}$$

and c is the reflective damping of the system defined as

$$c = \left(c_{bg} + \frac{K_{tr}^2}{R_t}\right)\left(\frac{2\pi}{l}\right)^2, \tag{3}$$

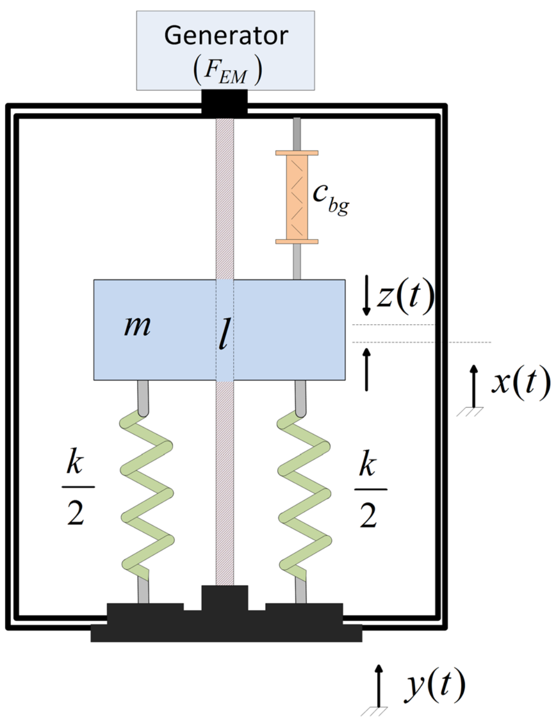

FIG. 2. A free body diagram of an energy harvesting system consisting of a sprung mass coupled to a generator through a ball screw.

where K_{tr} is the generator torque constant and R_t represents the sum of the coil resistance, R_i, and the load resistance, R_l. Ignoring the coil inductance, the current flowing through the coil of the generator is obtained from

$$i(t) = \frac{K_{tr}\,\upsilon(t)}{R_t}, \tag{4}$$

where $\upsilon(t)$ is the rotational speed of the ball screw coupled to the generator and is given by

$$\upsilon(t) = \frac{2\pi}{l}\left(\frac{d(x(t)-y(t))}{dt}\right). \tag{5}$$

III. OUTPUT POWER OF HARVESTER UNDER BROADBAND RANDOM EXCITATION

To evaluate the performance of the energy harvesting device under random vibration, the frequency response function of the device should be derived. In frequency domain, Eq. (1) can be written as

$$Z(\omega)((k - M\omega^2) + jc\omega) = m\omega^2 Y(\omega). \tag{6}$$

Also, from (4) and (5) the relative displacement of seismic mass can be written as

$$Z(\omega) = I(\omega)\frac{R_t}{j\omega\left(\frac{2\pi}{l}\right)K_{tr}}. \tag{7}$$

Substituting (6) into (7) and rearranging it results in

$$R_t\left(k - M\omega^2 + jc\omega\right)I(\omega) = j\left(\frac{2\pi}{l}\right)K_{tr}m\omega^3 Y(\omega). \tag{8}$$

Now, if we assume $Y_A(\omega) = -\omega^2 Y(\omega)$ as the Fourier transform of the base acceleration signal, the transfer function between the load current and the base acceleration signal can be shown to be given by

$$H_{IY_A}(\omega) = \frac{I(\omega)}{Y_A(\omega)} = \frac{-jm\left(\frac{2\pi}{l}\right)K_{tr}\omega}{(k - M\omega^2 + jc\omega)R_t}. \tag{9}$$

The spectral density of the load current is obtained form

$$S_I(\omega) = |H_{IY_A}(\omega)|^2 S_{Y_A}(\omega), \tag{10}$$

where $S_{Y_A}(\omega)$ is the spectral density of the base acceleration signal and is assumed to be constant with respect to frequency, i.e., $S_{Y_A}(\omega) = S_0$.

A. Expected output power

The mean value of the load power is obtained from

$$\begin{aligned} E[P(t)] = E\left[R_l|I|^2\right] &= R_l\int_{-\infty}^{+\infty} S_I(\omega)d\omega = R_l \times \int_{-\infty}^{+\infty} |H_{IY_A}(\omega)|^2 S_{Y_A}(\omega)d\omega \\ &= \frac{m^2 R_l K_{tr}^2 S_0}{(R_i + R_l)^2}\left(\frac{2\pi}{l}\right)^2 \times \int_{-\infty}^{+\infty}\frac{\omega^2}{(k - M\omega^2)^2 + (c\omega)^2}d\omega. \end{aligned} \tag{11}$$

Comparing the integral presented on the right-hand side of (11) with the general form of integral presented in the Appendix, (11) can be written as

$$E[P(t)] = \frac{m^2 R_l K_{tr}^2 S_0}{(R_i + R_l)^2}\left(\frac{2\pi}{l}\right)^2 \times \int_{-\infty}^{+\infty} \frac{\omega^2}{\left[k + c(j\omega) + M(j\omega)^2\right]\left[k + c(-j\omega) + M(-j\omega)^2\right]} d\omega. \quad (12)$$

Now the mean output power can be calculated by utilizing the method presented in the Appendix, which results in the following expression for the output power:

$$E[P] = \frac{S_0 m^2 R_l K_{tr}^2}{M(R_i + R_l)^2}\left(\frac{2\pi}{l}\right)^2 \frac{\pi}{c}. \quad (13)$$

By substituting (2) and (3) into (13), the mean value of output power based on the physical parameters of the energy harvester becomes

$$E[P] = \frac{\pi S_0 m^2 R_l K_{tr}^2}{\left(c_{bg}(R_l + R_i)^2 + K_{tr}^2 (R_l + R_i)\right)\left(m + J\left(\frac{2\pi}{l}\right)^2\right)}. \quad (14)$$

Equation (14) indicates that the expected load power under random excitation is proportional to the square of seismic mass and inversely proportional to the sum of the seismic mass and the reflected moment of inertia of the system's rotating components. This implies that to harvest maximum power from a base excited rotational harvester under random excitation, the moment of inertia of the system should be as small as possible. The optimum value of the load resistance to maximize the output power can be obtained by solving $\frac{\partial}{\partial R_l} E[P] = 0$, which results in

$$R_l = \sqrt{R_i^2 + \frac{K_{tr}^2}{c_{bg}} R_i}. \quad (15)$$

The load resistance shown in (15) is different from the optimum load resistance of an unconstrained electromagnetic energy harvester; hence it is subjected to a single frequency excitation obtained in Ref. 21, i.e., $R_l = R_i + K_{tr}^2/c_{bg}$. However, interestingly, the optimum load resistance shown in (15) is the same as the optimum load resistance when designing a rotational energy harvesting system for a single frequency of harmonically, single frequency, excited with constrained oscillating mass displacement.[22]

B. Simulation

To validate the analytical expression obtained for the optimum load resistance of the rotational electromagnetic energy harvesting system, a Monte-Carlo simulation is conducted. The Monte-Carlo simulation technique is a method that uses a random number sequence to evaluate the characteristics of the system based on a stochastic process.[23] Here, the expected output power is obtained for different values of the load resistance for a system whose parameters are presented in Table I.

In order to simulate the input acceleration, 2000 wide band pseudo-random signals are generated, as follows:

$$\sum_{i=1}^{N} \ddot{y}_i(i.\Delta t) = \sum_{i=1}^{N} \Upsilon_i \sin(\varpi_i(i.\Delta t) + \varphi_i), \quad (16)$$

where variables ϖ_i, Υ_i, and φ_i are independent and normally distributed, respectively, in [0, $\varpi_{\max}$], [0, $\Upsilon_{\max}$], and [0, $\varphi_{\max}$]. In addition, the maximum value of i is defined as N that depends on the duration of the simulation τ and the time step Δt as

TABLE I. Parameters of the energy harvester for Monte-Carlo test.

Parameters	Value
Mass (m)	8 kg
Generator resistance (R_i)	1.01 Ω
Mechanical damping (c_m)	5.36E-5 N s m
Spring stiffness (k)	261 N m^{-1}
Coupling coefficient (K_{tr})	7.39E-3 V s m^{-1}
Ball screw lead (l)	0.016 m
Ball screw moment of inertia (J)	12.0E-5 kg m^2

$$N = \frac{\tau}{\Delta t}. \tag{17}$$

It is known that white noise has a constant spectral value over the whole frequency range. However, in practice for simulation purposes, generating such a signal is not feasible. Here, we assume that $\varpi_{\max} = 400\,\pi$ rad/s, which is much larger than the natural frequency of the simulated system ($\omega_n = \pi$ rad/s), and is thus a reasonable approximation. The simulation is conducted for a period of 20 s, a sampling time of $\Delta t = 0.001$ s, and $\Upsilon_{\max} = 10$ m s^{-2}. The histograms shown in Fig. 3 illustrates the harvested power for different values of load resistance obtained from 2000 sets of random accelerations applied to the system for 20 s. As it is seen, the histogram of the average amount of harvested power for the produced base acceleration varies with the load resistance values. Statistical results of the output power obtained from Monte-Carlo simulation are shown in Table II.

Fig. 4 compares the statistical output of the Monte-Carlo simulation due to different load resistance values with the analytical expected power for each load resistance. As it can be seen, maximum power is transferred to the load resistance for the case where $R_l = 10.2\,\Omega$ which is equal to the optimum load resistance calculated from (15) for the harvester parameters presented in Table I. The Monte-Carlo simulation confirms the calculated value for the optimum load resistance in order to harvest the maximum amount of power when the device is excited by a broadband random acceleration.

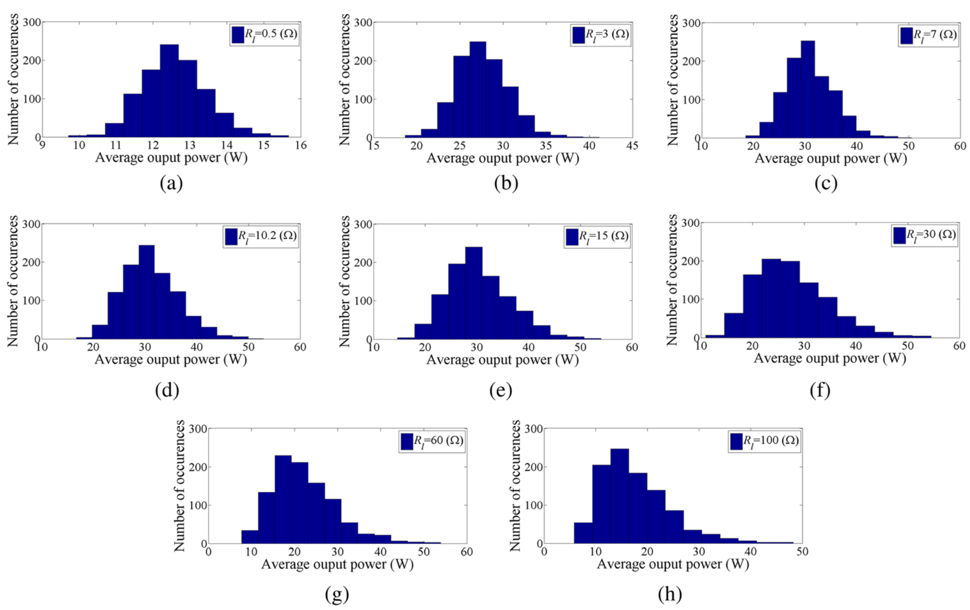

FIG. 3. Histograms of harvested power for 2000 generated random acceleration.

TABLE II. Statistical results of Monte-Carlo simulation.

Load resistance $R_l(\Omega)$	Expected power $\mu = E[P]$ (W)	Standard deviation σ (W)
0.5	12.67	0.86
3.0	27.60	2.96
7.0	30.63	4.53
10.2	31.03	5.28
15.0	30.22	6.04
30.0	27.50	7.13
60.0	22.38	7.32
100.0	17.78	6.66

IV. OUTPUT POWER OF HARVESTER UNDER BAND-LIMITED RANDOM EXCITATION

The system is subjected to a band-limited white noise if $S_{Y_A}(\omega) = S_0$, $\omega_1 \le |\omega| \le \omega_2$. For a band-limited excitation, the power spectral density (PSD) of the load current is

$$S_I(\omega) = \left\{ \begin{array}{cc} \dfrac{m^2 \left(\dfrac{2\pi}{l}\right)^2 K_{tr}^2 \omega^2 S_0}{R_t^2 \left((k - M\omega^2)^2 + c^2\omega^2 \right)} & \omega_1 \le |\omega| \le \omega_2 \\ 0 & \text{elsewhere} \end{array} \right\}. \tag{18}$$

For this condition, the mean value of the output power is obtained from

$$E[P(t)]_{\omega_1 \le |\omega| \le \omega_2} = \int_{-\omega_2}^{-\omega_1} + \int_{\omega_1}^{\omega_2} R_l S_I(\omega) d\omega = \int_{-\omega_2}^{-\omega_1} \dots + \int_{\omega_2}^{\omega_1} \frac{m^2 \left(\dfrac{2\pi}{l}\right)^2 K_{tr}^2 \omega^2 R_l S_0}{(k - M\omega^2)^2 (R_l + R_i)^2 + c^2\omega^2 (R_l + R_i)^2} d\omega. \tag{19}$$

The method presented in the Appendix does not apply to the incomplete integrals shown in (19). However, these integrals can be solved by using a partial fraction expansion method that is presented in Ref. 24. In this method, if $\Phi(\omega)$ is the spectral density of the response function of a stationary random process such that

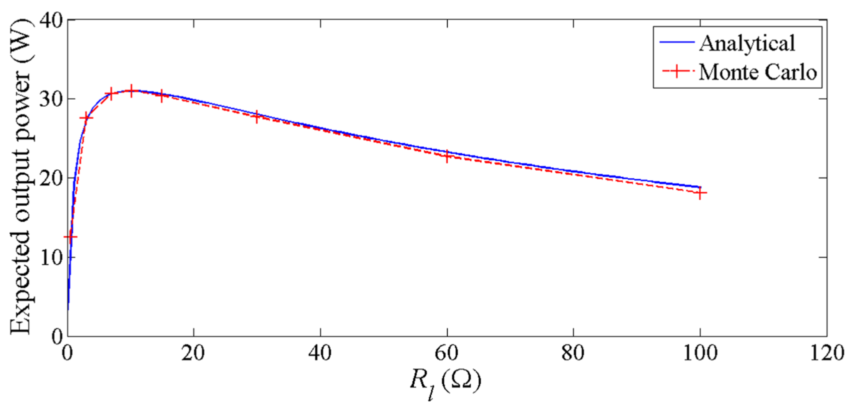

FIG. 4. Comparison of the analytical expected power with the average harvested power in Monte-Carlo simulation for different loads.

$$\Phi(\omega) = \left\{ \begin{array}{ll} \dfrac{\Phi_0}{\left(\omega_n^2 - \omega^2\right)^2 + 4\xi^2\omega^2\omega_n^2} & \omega_1 \le |\omega| \le \omega_2 \\ 0 & \text{elsewhere} \end{array} \right\}, \tag{20}$$

then the result of the second spectral moment is

$$m_2 = \int_{-\infty}^{+\infty} \omega^2 \Phi(\omega) d\omega = \frac{\pi\Phi_0}{2\xi\omega_n}\left[\Delta\left(\frac{\omega_2}{\omega_n}, \xi\right) - \Delta\left(\frac{\omega_1}{\omega_n}, \xi\right)\right], \tag{21}$$

where

$$\Delta\left(\frac{\omega}{\omega_n}, \xi\right) = \frac{1}{\pi}\arctan\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2} - \frac{\xi}{2\pi\sqrt{1-\xi^2}} \times \ln\frac{1 + \left(\frac{\omega}{\omega_n}\right)^2 + 2\left(\frac{\omega}{\omega_n}\right)\sqrt{1-\xi^2}}{1 + \left(\frac{\omega}{\omega_n}\right)^2 - 2\left(\frac{\omega}{\omega_n}\right)\sqrt{1-\xi^2}}. \tag{22}$$

Considering $\omega_n = \sqrt{k/M}$, to calculate the definite integral of (19), it can be written as

$$E[P(t)]_{\omega_1 \le |\omega| \le \omega_2} = \frac{m^2\left(\frac{2\pi}{l}\right)^2 K_{tr}^2 R_l S_0}{M^2 (R_l + R_i)^2} \times \left[\int_{-\omega_2}^{-\omega_1} + \int_{\omega_1}^{\omega_2} \frac{\omega^2}{\left(\omega_n^2 - \omega^2\right)^2 + 4\xi\omega^2\omega_n^2} d\omega\right], \tag{23}$$

where ξ is the damping ratio defined as

$$\xi = \frac{c}{2M\omega_n}. \tag{24}$$

Comparing (23) to (21) shows that the mean value of the output power when the harvester is subjected to a band-limited stationary white noise is given by

$$E[P(t)]_{\omega_1 \le |\omega| \le \omega_2} = \frac{m^2\left(\frac{2\pi}{l}\right)^2 K_{tr}^2 R_l \pi S_0}{M (R_l + R_i)^2 c} \times \left[\Delta\left(\frac{\omega_2}{\omega_n}, \xi\right) - \Delta\left(\frac{\omega_1}{\omega_n}, \xi\right)\right]. \tag{25}$$

The first term in (25) is the mean value of the output power when the system is subjected to a broadband white noise. However, the term presented in the square bracket is the correction

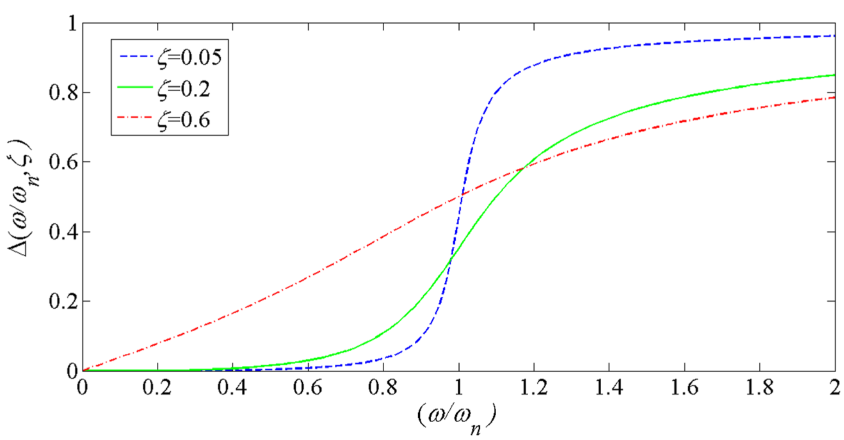

FIG. 5. Variation of $\Delta(\omega/\omega_n, \xi)$ as a function of ω/ω_n for three different values of ξ.

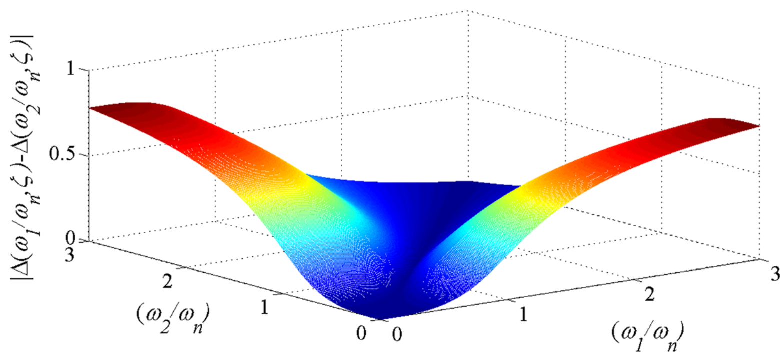

FIG. 6. Correction factor for calculation of the expected output power of energy harvester under band-limited excitation for a device with $\xi = 0.50$.

factor for a band-limited vibration. In other words, for broadband white noise the term $[\Delta(\infty, \xi) - \Delta(0, \xi)]$ tends to unity. However, this term is less than unity when the system is subjected to a band-limited vibration. Fig. 5 shows the behaviour of $\Delta(\omega/\omega_n, \xi)$ for different values of ξ. It is seen that $\Delta(\omega/\omega_n, \xi)$ is a monotonically increasing function of ω/ω_n with values between 0 and 1. This figure shows that for lightly damped systems most variations occur near its natural frequency.

Increasing the damping ratio of the system widens its bandwidth (defined as $2\omega_n\xi$), and reduces the sharpness of $\Delta(\omega/\omega_n, \xi)$ around its natural frequency. Fig. 6 shows the values of the correction factor in (25), for the mean output power of the system under band-limited random excitation. The correction value is presented for the case of $\xi = 0.50$. This is the corresponding damping ratio of the system presented in Table I, for its optimum load resistance of $R_l = 10.2\,\Omega$. This graph illustrates that when the natural frequency of the system is within the band-limited excitation range, i.e., between ω_1 and ω_2, the correction factor is slightly less than unity. However, when the natural frequency of system is outside the excitation band, i.e., when both (ω_1/ω_n) and (ω_2/ω_n) are either less than or greater than unity, the correction factor is very small which drastically reduces the mean value of the expected power. Therefore, from the design point of view, an obvious conclusion is made that in order to harvest maximum output power from band-limited random excitation, the natural of the system should fall in the band-limited sides. However, its optimum value is not the center of the band-limited range.

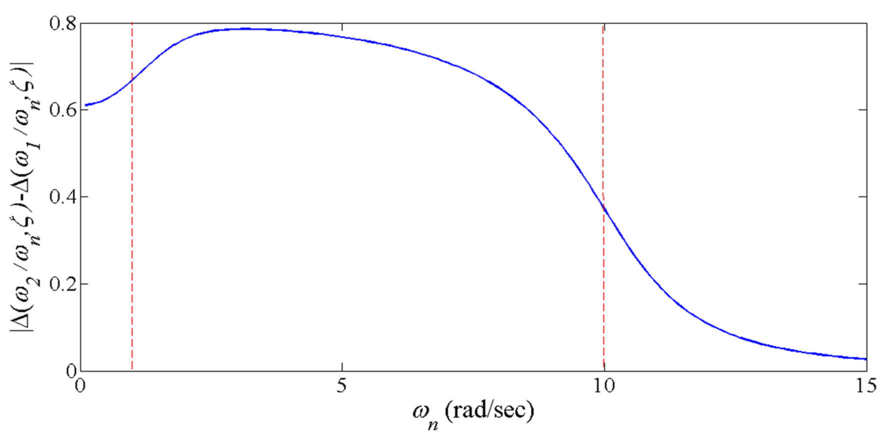

FIG. 7. Correction factor for calculation of the expected output power of energy harvester presented in Table I under band-limited excitation ($\omega_1 = 1$ rad/s and $\omega_2 = 10$ rad/s) versus ω_n.

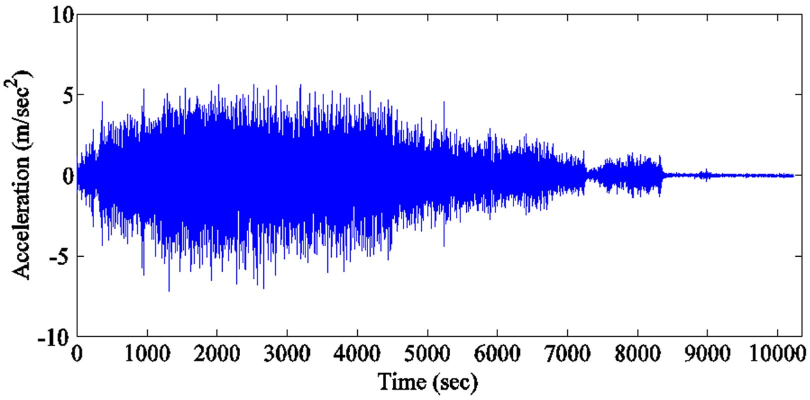

FIG. 8. Typical boat bow vertical acceleration measured while sailing in the English Channel.

The optimum value of ω_n can be obtained from numerical maximization of $[\Delta(\omega_2/\omega_n, \xi) - \Delta(\omega_1/\omega_n, \xi)]$ for a given ω_2 and ω_1. Replacing ξ from (24) in (22) gives the correction factor as a function of ω_n. Considering the physical parameters of the harvester from Table I, and assuming a variable spring stiffness, the correction factor is a function of stiffness through ω_n. Fig. 7 shows the variation of the correction factor for the case when $\omega_1 = 1\,\text{rad/s}$ and $\omega_2 = 10\,\text{rad/s}$. It is seen that for this system the maximum value of the correction factor is 0.79, which is obtained when the natural frequency of the system is $\omega_n = 3.2\,\text{rad/s}$. From (24), the corresponding value of the optimum damping ratio for the system is $\xi = 0.49$. Hence, to harvest the maximum power under band-limited excitation conditions, the designer should ensure that the parameters of the system match its optimally obtained natural frequency. For instance, considering the physical parameters of the above simulated system from Table I and the obtained optimum value of $\omega_n = 3.2\,\text{rad/s}$, the optimum spring stiffness that would maximize the power harvested from band-limited excitation is $k = 271\,\text{N/m}$ôôô

V. HARVESTED POWER IN REAL ENVIRONMENT

This section is dedicated to the estimation of the output power from the recorded random excitation applied to a boat in a real environment. A review of different studies has shown that the vertical movement of typical sailing boats is inherently a narrow band random excitation. This was confirmed by the authors' own boat acceleration measurement obtained while sailing in the English Channel, as shown in Fig. 8. The boat was a double hull catamaran, 34 feet long, 14 feet wide with a total weight of approximately 3.5 tones. To measure the acceleration of the vertical movement of the boat, a micro-machined silicon static accelerometer was positioned about 1 m from the bow. The power spectral density of the recorded acceleration is

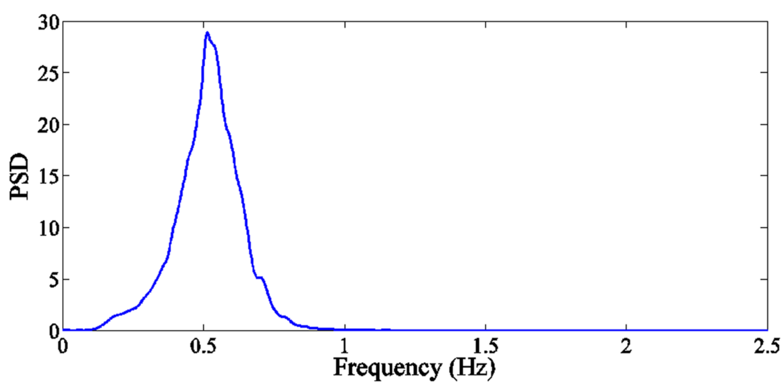

FIG. 9. Power spectral density of the measured acceleration.

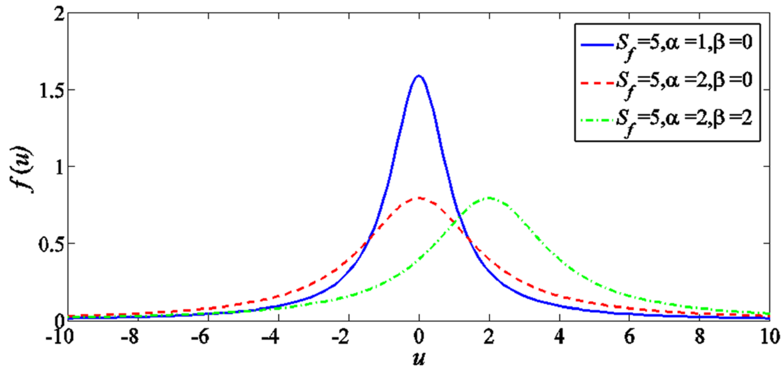

FIG. 10. Cauchy distribution for different values of α, S_f , and β.

shown in Fig. 9 and it is seen that the vertical excitation of boat is not a White noise. The derived expressions for the expected power in Secs. II–IV are helpful to quantify the harvested power under broadband and band-limited white noise excitations. However, the approach described in this paper so far, can be extended to find the mean value of the expected power when the power spectral density of the random excitation is not necessarily constant. For this purpose, the mathematical function corresponding to the power spectral density distribution of the random excitation should be investigated and then the expected output power can be estimated using (11). Note that, here $S_{Y_A}(\omega)$ is not constant and cannot be taken out of the integral term.

Comparing the recorded random excitation shown in Fig. 9 with various distributions[26] indicates that the presented spectral density is very close to a Cauchy distribution with the general form of[25,27,28]

$$f(u;\alpha,\beta) = \frac{\alpha S_f}{\pi\left[\alpha^2 + (u-\beta)^2\right]}, \tag{26}$$

where α is the scale parameter, S_f is the height factor, and β is the location parameter. Fig. 10 shows Cauchy distribution for different values of α, S_f , and β.

Fig. 11 shows the PSD of the recorded acceleration from the boat motion and the mathematical estimation of the PSD plotted based on the Cauchy distribution when the parameters of distribution are $\alpha = 0.52$, $S_f = 48$, and $\beta = 3.2$. It is seen that there is good agreement between the spectral density of the recorded acceleration signal and the mathematical estimated distribution. Therefore, the spectral density of the recorded acceleration can be written as

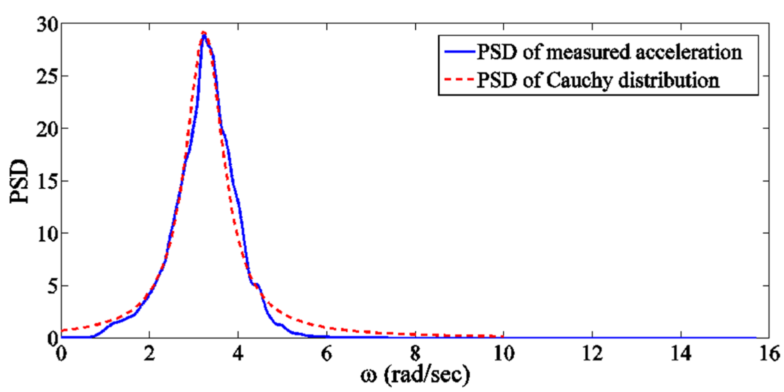

FIG. 11. Fitting the Cauchy distribution on the measured vertical excitation of the boat.

$$S_{Y_A} = \frac{24.96}{\pi\left[0.27 + (u - 3.2)^2\right]}. \tag{27}$$

Replacing (27) in (11), the mean value of the harvested power is obtained from

$$E[P(t)] = \frac{m^2 R_l K_{tr}^2}{(R_i + R_l)^2}\left(\frac{2\pi}{l}\right)^2 \int_{-\infty}^{+\infty} \frac{\omega^2}{(k - M\omega^2)^2 + (c\omega)^2} \cdot \frac{24.96}{\pi\left[0.27 + (\omega - 3.2)^2\right]} d\omega, \tag{28}$$

and the expected output power can be obtained by numerical integration of (28), which can be shown to be 20.45 W for the system with parameters shown in Table I.

VI. CONCLUSION

This paper investigates an energy harvesting device, comprising a sprung mass coupled to an electric generator using a ball screw, when operating under random excitation. Specifically, analytical expressions for the dimensional mean harvested power due to stationary broadband and band-limited white noise excitations are derived. In the case of harvesting energy from a broadband random source, it is shown that the output power is proportional to the square of the weight of the actual mass used in the device. However, the output power is inversely proportional to the moment of inertia of the system's rotating components. Therefore, a system with the lowest moment of inertia would be better when the harvester is subjected to a random excitation. In addition, it is shown that the output power expression is independent of the spring's stiffness.

The optimum load resistance to harvest the maximum power from broadband white noise acceleration is obtained and validated by conducting Monte-Carlo simulation. Furthermore, the closed-form expression of the output power from band-limited random excitation shows that the output power is a function of the physical parameters of the system including the spring stiffness. Therefore, from the derived power expression, the optimum natural frequency of the energy harvester that falls within the excitation band is obtained. Based on this optimum natural frequency, the optimum spring stiffness of the energy harvester can be then obtained.

Also in this paper the profile of the spectral density of the measured acceleration signal of a typical boat is approximated by a Cauchy distribution. The distribution parameters of the spectral density of the acceleration signal are then estimated and subsequently used to calculate the expected power of the proposed energy harvester under real conditions.

APPENDIX: INTEGRAL CALCULATION

Calculation of the integrals in the form of[25]

$$H_n = \int_{\omega_2}^{\omega_1} \frac{\Theta_n(\omega)}{\Psi_n(\omega)\,\Psi_n^*(\omega)} d\omega \tag{A1}$$

where

$$\Theta_n(\omega) = a_{n-1}\omega^{2n-2} + a_{n-2}\omega^{2n-4} + \cdots + a_0, \tag{A2}$$

and

$$\Psi_n(i\omega) = b_n(i\omega)^n + b_{n-1}(i\omega)^{n-1} + \cdots + b_0 \tag{A3}$$

is obtained from

$$H_n = \frac{\pi}{b_n} \frac{\det \begin{bmatrix} a_{n-1} & a_{n-2} & & \dots & & & a_0 \\ -b_n & b_{n-2} & -b_{n-4} & b_{n-6} & \dots & 0 & \dots \\ 0 & -b_{n-1} & b_{n-3} & -b_{n-5} & \dots & 0 & \dots \\ 0 & b_n & -b_{n-2} & -b_{n-4} & \dots & 0 & \dots \\ 0 & \dots & & & \dots & 0 & \dots \\ 0 & 0 & & & \dots & -b_2 & b_0 \end{bmatrix}}{\det \begin{bmatrix} b_{n-1} & -b_{n-3} & \beta_{n-5} & -b_{n-7} & & & \\ -b_n & b_{n-2} & -b_{n-4} & b_{n-6} & \dots & 0 & \dots \\ 0 & -b_{n-1} & b_{n-3} & -b_{n-5} & \dots & 0 & \dots \\ 0 & b_n & -b_{n-2} & b_{n-4} & \dots & 0 & \dots \\ 0 & \dots & & & \dots & 0 & \dots \\ 0 & 0 & & & \dots & -b_2 & b_0 \end{bmatrix}}. \tag{A4}$$

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