Indexed versus nominal government debt under inflation and price-level targeting

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Abstract

This paper presents a DSGE model in which long run inflation risk matters for social welfare. Optimal indexation of long-term government debt is studied under two monetary policy regimes: inflation targeting (IT) and price-level targeting (PT). Under IT, full indexation is optimal because long run inflation risk is substantial due to base-level drift, making indexed bonds a better store of value than nominal bonds. Under PT, where long run inflation risk is largely eliminated, optimal indexation is substantially lower because nominal bonds become a relatively better store of value. These results are robust to the PT target horizon, imperfect credibility of PT and model calibration, but the assumption that indexation is lagged is crucial. A key finding from a policy perspective is that indexation has implications for welfare comparisons of IT and PT.

Keywords: government debt; inflation risk; inflation targeting; price-level targeting.
JEL classification: E52, E63

1. Introduction

Long-term government debt plays an important role in many developed economies. Since contracts of this kind are denominated in nominal terms or imperfectly indexed, unanticipated changes in inflation which are not reversed will lead to fluctuations in real wealth. These fluctuations are important for old generations because they rely on long-term contracts to fund their consumption in retirement. The magnitude of revaluations in long-term contracts due to unanticipated inflation depends crucially upon the amount of long run inflation risk in the economy. This observation motivates a comparison of the costs and benefits of inflation targeting (IT) and price-level targeting (PT) regimes. Under IT, unanticipated shocks to the price level are not reversed by policy, so there is base-level drift. As a result, inflation risk rises with the forecast horizon.[1] By contrast, PT offsets unanticipated shocks to inflation in

[1] That is, the price level follows a random walk. Inflation risk increases with the forecast horizon in this case because inflation between period t and $t+k$ depends on the ratio of the price level in period $t+k$ to that in period t.

order to return the price level to a target path which is known ex ante. Hence, long run inflation risk is largely eliminated under a PT regime.

In this paper, optimal indexation of long-term government debt is studied under IT and PT. Given that government debt accounts for a non-trivial fraction of net nominal wealth in developed economies (Doepke and Schneider, 2006; Meh and Terajima, 2011), this analysis may be important for comparing these two monetary policy regimes. In recent years, both policymakers and academics have become interested in this comparison. Several papers have shown that PT offers short-term stabilisation benefits over IT when agents are forward-looking. Vestin (2006), for example, shows that in the standard New Keynesian model, PT reduces inflation variability for a given level of output gap variability if policy is discretionary. In the same model, optimal commitment implies a stationary price level (Clarida et al., 1999).[2] In light of these results, the Bank of Canada recently conducted a detailed review of the costs and benefits of PT (see Bank of Canada, 2011).[3] However, to the author's knowledge, no paper has studied optimal indexation of long-term debt contracts under IT and PT in a DSGE model where long run inflation risk matters for social welfare. The main contribution of this paper is to provide an assessment of this kind.

An overlapping generations (OG) model in the spirit of Diamond (1965) is calibrated to roughly match the UK economy. The model has two features that make it useful for investigating optimal indexation under long run inflation risk. First, long run inflation risk matters for social welfare because revaluations in the return on government debt due to unanticipated inflation have real implications for consumption. In the model, long run inflation risk affects social welfare by two distinct channels: (i) variations in the real return on government debt lead to variations in retirement consumption, and (ii) inflation risk affects the risk premium the government pays on debt, and therefore the level of taxes. By contrast, only short-term inflation risk matters for social welfare in the standard New Keynesian model (Woodford, 2003). Second, each period in the model lasts 20 years. As a result, inflation risk and equilibrium asset prices can be modelled over a long horizon without introducing a large number of state variables. This feature of the model is important because a second-order perturbation method is used to capture the implications of inflation risk for social welfare, making a numerical solution computationally-intensive.[4]

The main finding of the paper is that full indexation of government debt is optimal under IT, whereas optimal indexation is substantially lower under PT. Intuitively, despite the fact that the payoff on indexed bonds is subject to inflation risk due to a one-year indexation lag, return risk on nominal bonds is much higher since IT implies that cumulative inflation risk over a 20-year horizon is approximately 20 times that at a yearly horizon (due to base-level drift). Under a PT regime, by contrast, long run inflation risk does not increase with the forecast horizon and is therefore kept at annual magnitudes. As a result, nominal bonds become a much better store of value relative to indexed bonds and optimal indexation is substantially lower. The indexation lag is crucial for explaining the lower level of optimal

[2] The issue of whether optimal policy in the New Keynesian model implies price stationarity is controversial. Negative results include Steinsson (2003), Levin *et al.* (2010) and Amano et al. (2012).

[3] This work is surveyed in Ambler (2009), Crawford et al. (2009) and Bank of Canada (2011).

[4] In linear or log-linearized models there is 'certainty equivalence' – i.e. the coefficients of policy functions do not depend on risk (shock volatility). As pointed out by Kim and Kim (2003), failure to account for the effects of risk can lead to spurious welfare reversals.

indexation under PT since substantially reducing long run inflation risk means that even a one-year indexation lag is sufficient to make return risk on indexed and nominal bonds comparable. It is important to note, however, that full indexation is optimal under both IT and PT if the assumption that indexation is lagged is dropped.[5]

The analysis begins with a simplified version of the model in which full indexation is optimal under IT and zero indexation is optimal under PT. Later sections then extend the analysis to more realistic settings. Three main extensions are considered. First, the baseline case assumes that there is target horizon of one year under PT – i.e. policy aims to return the price level to its target path after one year. If this assumption is relaxed so that the price level is returned to target gradually over several years, optimal indexation remains somewhat lower under PT but rises to 44% with a 2-year target horizon, and 76% with a 4-year target horizon. Second, optimal indexation remains lower than 100% under a PT regime with imperfect credibility, but the differential is narrowed somewhat. In particular, optimal indexation rises to 66% under a PT regime with high credibility and 87% under a low credibility regime. Third, introducing moderate correlation of money supply shocks raises optimal indexation under PT to more than 40%, while full indexation remains optimal under IT.

An important finding from a policy perspective is that the outcomes of welfare comparisons of IT and PT regimes can be sensitive to whether indexation is optimised or not. For instance, holding the share of indexed government debt fixed across regimes at the current UK level implies a welfare gain from PT of 0.15% of aggregate consumption, but social welfare is slightly higher under IT when indexation is optimised under both regimes. Moreover, although social welfare and real variables are somewhat less sensitive to the share of indexed government debt under a PT regime, the results suggest that accounting for optimal indexation would lead to more precise estimates of the aggregate and welfare effects of PT. More generally, the results of this study suggest that analyses of IT and PT may produce misleading conclusions if they assume that nominal contracting is fixed across regimes.

The paper is related to two other strands of literature. The first is on the implications of unanticipated inflation. In a seminal paper, Doepke and Schneider (2006) document postwar nominal portfolios in the US and show that unanticipated inflation has substantial redistributional effects through revaluations of nominal assets and liabilities. Meh and Terajima (2011) later examined nominal portfolios in Canada. Building on these two papers, Meh et al. (2010) simulated aggregate and welfare effects from *one-off* episodes of unanticipated inflation in Canada under IT and PT in a quantitative OG model. They find that there are larger redistributional effects under IT because long-term nominal contracts undergo substantial revaluations due to base-level drift. Consequently, induced welfare effects are somewhat larger under IT. However, nominal portfolios are assumed to remain fixed across monetary regimes. As Meh et al. acknowledge, taking into account how nominal portfolios change under PT may be important to reach more precise estimates of its aggregate and welfare effects. Since the current paper allows nominal portfolios to vary, it should provide additional insight into these effects. It also provides a methodology for computing optimal indexation that could be extended to more realistic settings like quantitative OG models.

[5] In practice, the share of indexed government bonds is low in most developed economies. Campbell et al. (2009) report that, in 2008, indexed government bonds were 10% of marketable government debt in the US and around 30% in the UK. In Canada in 2008, indexed bonds were around 6% of marketable government debt and 10% of the outstanding stock of bonds (Department of Finance Canada, 2008).

The second strand of literature to which the paper is related is on optimal indexation of wage contracts. In a seminal paper, Gray (1976) showed that optimal wage indexation increases with the nominal-to-real volatility ratio. More recently, Minford et al. (2003) build a model in which households have an incentive to insure against real wage fluctuations but cannot access financial markets. They find that optimal indexation is lower under a regime that aims at price rather than inflation stability, because nominal wage contracts become relatively better real wage stabilisers. Subsequently, Amano et al. (2007) showed that the same conclusion holds in a model with staggered cohorts of labour-differentiated wage-setters who have unrestricted access to financial markets. An important difference in this paper is that indexation of government debt has important implications for government finances – and hence taxes – through the inflation risk premium.[6] In short, since the government must satisfy its budget constraint, inflation risk has knock-on effects on households and social welfare not present under wage indexation. This problem speaks to the need for a general equilibrium analysis that takes into account the main effects of inflation risk, including those for the government finances. This paper provides such an analysis.

The remainder of the paper proceeds as follows. Sections 2 presents the model and Section 3 describes the IT and PT monetary policy regimes. Section 4 discusses the optimal indexation problem and its numerical solution. In Section 5 the model is calibrated. Section 6 reports the optimal indexation results from the baseline model. Section 7 considers extensions of the baseline model and sensitivity tests. Finally, Section 8 concludes.

2. Model

The model is a version of Diamond's (1965) model where the young save for old age using capital and government bonds. It contains three sectors: a household sector, a government sector, and a sector devoted to production of a single output good. Each sector is described below. This section also discusses equilibrium conditions and social welfare.

2.1 Consumers

An overlapping generations (OG) model with generations that live for two periods is considered. Each generation is modelled as a consumer who inelastically supplies a unit of labour when young and retires when old, leaving no bequests. Let subscripts $\{Y, O\}$ denote, respectively, the young and the old. Each period lasts 20 years. The number of generations born per period is constant and normalized to 1. The real wage income of the young is taxed by the government at a constant rate τ. After-tax wage income is allocated to four assets: indexed government bonds, b^i; nominal government bonds, b^n; capital, k; and money, m.

The young consume and choose an optimal portfolio of assets which pays off in old age. Capital earns a real return r^k, which is taxed by the government at a constant rate τ^k. Indexed bonds pay a risky real return r^i as a result of a one-year indexation lag, and nominal bonds pay a risky real return r^n. Nominal bonds are riskless but for unanticipated inflation over the holding horizon from youth to old age (i.e. 20 years). Consequently, the real return on nominal bonds is $r^n = R/(1+\pi)$, where π is inflation between youth and old age and R is the gross nominal interest rate. Indexed bonds are subject to a one-year indexation lag, so they pay a gross real return $r^i = r\,(1+ \pi^{ind})/(1+\pi)$, where π^{ind} is the inflation rate to which indexed

[6] For a recent survey of the inflation risk premium, see Bekaert and Wang (2010). In the model that follows, indexed debt is risky and the inflation risk premium is defined as the difference between the expected real return on nominal bonds and the expected real return on indexed bonds. Appendix A provides a formal definition.

bonds are linked and r is the ex-ante real interest rate. The interest rates R and r are endogenously determined and ensure that, for each bond, demand is equated to supply.

Money pays a real return $r^m = 1/(1+\pi)$. Positive money demand arises from the legal requirement that young agents hold real money balances of at least $\delta > 0$, so that $m_t \geq \delta$ as in Champ and Freeman (1990). The main advantage of this constraint is that it provides a role for money without requiring that it offer explicit transactions services, so that any differential in optimal indexation under IT and PT can be attributed to the implications of these regimes for long run inflation risk and not the impact of policy on transactions costs or ease of exchange. The constraint binds with equality if $R_t > 1$ for all t, which is assumed to hold.[7]

Hence we have that

$$m_t = \delta, \quad \forall t \tag{1}$$

The budget constraints faced by the generation born in period t are

$$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b^i_{t+1} - b^n_{t+1} - m_t \tag{2}$$

$$\begin{aligned} c_{t+1,O} &= (1-\tau^k)r^k_{t+1}k_{t+1} + r^i_{t+1}b^i_{t+1} + r^n_{t+1}b^n_{t+1} + r^m_{t+1}m_t \\ &= (1-\tau^k)r^k_{t+1}k_{t+1} + [vr^i_{t+1} + (1-v)r^n_{t+1}]b_{t+1} + r^m_{t+1}m_t \end{aligned} \tag{3}$$

where $0 \leq v \leq 1$ is the share of indexed bonds in total bond portfolio, b_{t+1}.[8]

Given the focus in this paper, it is important to use preferences that can potentially match household attitudes to risk. As is well known, standard CRRA preferences cannot match the risk-free rate and risk-premia since they imply that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. Consequently, Epstein and Zin (1989) and Weil (1989) preferences are used here. With these preferences, the elasticity of intertemporal substitution and the coefficient of relative risk aversion can be calibrated separately. In a recent paper, Rudebusch and Swanson (2012) show that this feature enables an otherwise standard New Keynesian model to match the 10-year term premium on nominal bonds without compromising its ability to fit key macro variables.

Consumers solve a maximization problem of the form

$$\max_{\{c_{t,Y},\, z_t,\, c_{t+1,O}\}} U_t = \frac{1}{1-\gamma}\left[c^{\varepsilon}_{t,Y} + \beta\left[E_t c^{1-\gamma}_{t+1,O}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \qquad \text{s.t. (1), (2), (3)} \tag{4}$$

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[7] This condition is easy to prove (see Supplementary Appendix) and was comfortably satisfied in numerical simulations. Results for an economy without money are discussed in Section 7.4.1.

[8] Since bonds have a fixed maturity of one period (i.e. 20 years), the model cannot be used to assess the optimal mix of short-term versus long-term government debt. This is a potentially important channel by which a change in regime from IT to PT could affect social welfare. The model also abstracts from the fact that bonds can be traded prior to maturity. This would enable consumers to avoid some of the inflation risk that a fixed 20-year holding period exposes them to, but a formal analysis of this issue is beyond the scope of this paper.

where $z \equiv (k, b^i, b^n, m)$ is the vector of assets chosen by households, $0 < \beta < 1$ is the private discount factor, γ is the coefficient of relative risk aversion, and $1/(1-\varepsilon)$ is the elasticity of intertemporal substitution.

The first-order conditions are as follows:[9]

$$1 = E_t[sdf_{t+1}(1-\tau^k)r^k_{t+1}] \qquad \text{for capital, } k \tag{5}$$

$$1 = E_t[sdf_{t+1} r^i_{t+1}] \qquad \text{for indexed bonds, } b^i \tag{6}$$

$$1 = E_t[sdf_{t+1} r^n_{t+1}] \qquad \text{for nominal bonds, } b^n \tag{7}$$

$$1 = E_t[sdf_{t+1} r^m_{t+1}] + \tilde{\mu}_t \qquad \text{for money, } m \tag{8}$$

Here $\tilde{\mu}_t$ is the ratio of the Lagrange multiplier on the cash constraint to that on the budget constraint of the young, and

$$sdf_{t+1} \equiv \beta \left(\frac{c_{t,Y}}{c_{t+1,O}} \right)^{1-\varepsilon} \left(\frac{c_{t+1,O}}{(E_t c^{1-\gamma}_{t+1,O})^{1/(1-\gamma)}} \right)^{1-\gamma-\varepsilon} ; \quad r^n_{t+1} \equiv R_t/(1+\pi_{t+1}); \quad r^i_{t+1} = r_t(1+\pi^{ind}_{t+1})/(1+\pi_{t+1}).$$ [10]

2.2 Firms

The production sector consists of a representative firm which produces output using a Cobb-Douglas production function. The share of capital in output is equal to α and the labour share is $1 - \alpha$. The firm hires capital and labour in competitive markets to maximise current profits. Total factor productivity, A, is stochastic and follows an AR(1) process (in logs).

The real wage and the return on capital are given by

$$w_t = y_t - r^k_t k_t = (1-\alpha) A_t k^\alpha_t \tag{9}$$

$$r^k_t = \alpha y_t / k_t = \alpha A_t k^{\alpha-1}_t \tag{10}$$

2.3 Government

The government performs three functions. First, to meet government spending commitments, it taxes wage income of the young at a constant rate $\tau > 0$ and capital income of the old at a constant rate $\tau^k > 0$. Second, it conducts monetary policy by committing to a money supply rule. Third, it sets the total supply of government bonds and chooses the share of indexed debt to maximise social welfare, subject to the monetary policy regime in place.

The government budget constraint is given by

$$\begin{aligned} g_t &= \tau w_t + \tau^k r^k_t k_t + b^i_{t+1} - r^i_t b^i_t + b^n_{t+1} - r^n_t b^n_t + m_t - r^m_t m_{t-1} \\ &= \tau w_t + \tau^k r^k_t k_t + b_{t+1} - [v r^i_t + (1-v) r^n_t] b_t + m_t - r^m_t m_{t-1} \end{aligned} \tag{11}$$

[9] A full derivation of consumers' first-order conditions is provided in the Supplementary Appendix.

[10] The dating on R and r reflects the fact that these prices must clear the markets for nominal and indexed bonds at the date when bonds are purchased, that is, at the end of period t. Note that the inflation rate to which indexed bonds are linked is not equal to the previous period's inflation rate because the indexation lag is *one year*, whereas each period in the model lasts 20 years. For expressions for π and π^{ind}, see Section 3.

where the total supply of government bonds is $b = b^i + b^n$, and the shares of indexed and nominal bonds in the total bond portfolio are constant and equal to v and $1–v$, respectively.

Since the tax rates on wage income and capital are constant, it follows that $\tau^k = a\tau$ for some constant $a > 0$, so that tax policy can be described by the single tax rate τ. The government sets the total supply of bonds to facilitate consumption smoothing between youth and old age. In particular, the total supply of government debt is set so that

$$E_t\,[sdf_{t+1}] = \beta \tag{12}$$

where E_t is the conditional expectations operator.

The bond supply rule in (12) implies a steady-state real interest rate of $1/\beta$ and hence perfect consumption smoothing in the deterministic steady-state. The government sets the nominal money supply according to an IT or PT rule, as discussed in Section 3. The constant tax rate τ is set to meet a long run target ratio of government spending to output:

$$E[g_t/y_t] = G^* > 0 \tag{13}$$

where E is the unconditional expectations operator.

Although τ is constant over time, it will differ across IT and PT since these regimes affect the expected real returns payable on government debt. It should therefore be understood that the tax rate is regime-specific, though this dependence is suppressed in order to minimise notation. Taking into account the equilibrium conditions and the requirement that τ be set so that $E[g_t/y_t] = G^*$, the objective of the government is to choose the share of indexed debt to maximise social welfare. The optimal indexation problem is discussed in Section 4.

2.4 Market-clearing and equilibrium

Capital depreciates fully within a period, an assumption which is empirically reasonable given that each period lasts 20 years. It follows that investment in period t is given by $i_t = k_{t+1}$.

Definition of equilibrium:[11]

A set of allocations and prices $\left\{c_{t,Y}, c_{t,O}, b_t^{i,d}, b_t^{i,s}, b_t^{n,d}, b_t^{n,s}, k_t, m_t^d, m_t^s, g_t, R_t, r_t, r_t^k, w_t, \tau, \tau^k\right\}$ with the following properties for all t:

(1) Allocations $\left\{c_{t,Y}, c_{t+1,O}, b_{t+1}^{i,d}, b_{t+1}^{n,d}, k_{t+1}, m_t^d\right\}$ solve the maximization problem of the generation born at time t and factors of production are paid their marginal products;

(2) The goods, money and bond markets clear:

$$\begin{aligned}
y_t &= c_{t,Y} + c_{t,O} + g_t + k_{t+1} \\
m_t^d &= m_t^s \\
b_t^{i,d} &= b_t^{i,s} \\
b_t^{n,d} &= b_t^{n,s}
\end{aligned}$$

[11] Note that d and s superscripts are introduced in this section to denote demand and supply values. These superscripts are omitted in other sections of the paper in order to avoid unnecessary notation.

(3) The government budget constraint and long run government spending target are satisfied:

$$g_t = \tau w_t + \tau^k r_t^k k_t + b_{t+1}^{i,s} - r_t^i b_t^{i,s} + b_{t+1}^{n,s} - r_t^n b_t^{n,s} + m_t^s - r_t^m m_{t-1}^s$$
$$E[g_t / y_t] = G^*$$

(4) The legal requirement on money holdings is binding: $m_t^d = \delta$

2.5 Social welfare

Welfare is given by the discounted sum of lifetime utilities across all young generations:[12]

$$SW = (1-\omega)E\left[\sum_{t=0}^{\infty} \omega^t U_t\right] = E[U_t] \tag{14}$$

where $0 < \omega < 1$ is the social discount factor, and E is the unconditional expectations operator.

It is clear from (14) that the social discount factor ω will not affect optimal indexation. Consequently, the social discount factor can be left unspecified.

3. Monetary Policy and inflation

The government conducts monetary policy using yearly money supply rules set with annual inflation in mind.[13] It can commit to these rules but cannot control the money supply perfectly and so has imperfect control over inflation. To obtain money supply rules consistent with the 20-year horizon of the model, the implications of these rules are traced out over a 20-year horizon. This section derives expressions for inflation and indexed inflation (π^{ind}) under IT and PT. In the discussion that follows, M_n is the nominal money stock at the end of year n, and ε_n is an IID-normal innovation in year n with mean zero and standard deviation σ.

3.1 Inflation targeting (IT)

Under IT, the yearly nominal money supply grows at the annual target inflation rate, π^*, plus any deviation due to a yearly money supply innovation ε:

$$M_n = M_{n-1}(1+\pi^*)(1+\varepsilon_n) \tag{15}$$

Substituting repeatedly for the past money supply,

$$M_n = M_{n-20}(1+\pi^*)^{20} \prod_{j=n-19}^{n} (1+\varepsilon_j)\,. \tag{16}$$

[12] This social welfare function ignores the utility of the initial old, but this does not alter the main conclusions. The second equality in (14) will hold as long as U_t is stationary. In this paper, both U_t and the remaining equations of the model are approximated using a second-order perturbation method.

[13] The focus on annual inflation is consistent with policymaking in practice and enables the model to capture base-level drift at a yearly horizon (under IT) and its absence (under PT).

It is clear from this equation that the IT money supply rule aims at a constant inflation target and does not attempt to offset past money supply shocks – i.e. 'bygones are bygones'. Given that each period lasts 20 years and the nominal money supply is the end-of-year stock, the implied money supply rule in any period t is

$$M_t = M_{t-1}(1+\pi^*)^{20}\prod_{j=1}^{20}(1+\varepsilon_{j,t}) \tag{17}$$

where the money innovations are indexed by $j = 1,2,\ldots,20$ and M_t is the nominal money stock at the end of period t.

By money market equilibrium $M_t = P_t\, m_t$, where $m_t = \delta$ by the legal requirement on cash holdings. Hence $M_t / M_{t-1} = P_t / P_{t-1} = 1 + \pi_t$. Inflation is period t is therefore given by

$$1+\pi_t = (1+\pi^*)^{20}\prod_{j=1}^{20}(1+\varepsilon_{j,t}) \tag{18}$$

It is clear from (18) that there is base-level drift. As a result, inflation risk accumulates over a 20-year horizon. Note that this rule would stabilise inflation perfectly at the long-term inflation target $(1+\pi^*)^{20}$ in the absence of money supply innovations, consistent with annual inflation of π^* every year. Finally, notice that inflation expectations are anchored at target under IT: $1+E_{t-1}\pi_t = (1+\pi^*)^{20}$.

Since indexed bonds are subject to a one-year indexation lag, the inflation rate to which indexed bond are linked is given by the one-year lag of (18):

$$1+\pi_t^{ind} = (1+\pi^*)^{20}(1+\varepsilon_{20,t-1})\prod_{j=1}^{19}(1+\varepsilon_{j,t}) \tag{19}$$

Equation (19) shows that indexed inflation will covary strongly with actual inflation: they have 19 of 20 shocks in common, with the difference being accounted for by the one-year indexation lag. Consequently, indexed bonds will be excellent stabilisers of long run purchasing power under IT. This point is important for understanding the results that follow.

3.2 Price-level targeting (PT)

Under PT, policymakers aim to stabilise the price level around a target price path whose slope is consistent with an annual inflation target of π^*. The crucial difference relative to IT is that past deviations from the inflation target are offset. The yearly money supply rule therefore includes a correction for the previous year's money supply innovation:[14]

$$\begin{aligned} M_n &= M_{n-1}(1+\pi^*)\frac{(1+\varepsilon_n)}{(1+\varepsilon_{n-1})} \\ &= M_{n-20}(1+\pi^*)^{20}\frac{(1+\varepsilon_n)}{(1+\varepsilon_{n-20})} \end{aligned} \tag{20}$$

where the second equality follows from repeated substitution for the previous money supply.

[14] The intuition can be seen by taking logs: $M_n \approx M_{n-1} + \pi^* + \varepsilon_n - \varepsilon_{n-1}$. M_n and ε_n are defined as above.

This equation implies a period-t money supply rule of the form:

$$M_t = M_{t-1}(1+\pi^*)^{20} \frac{(1+\varepsilon_{20,t})}{(1+\varepsilon_{20,t-1})} \tag{21}$$

Again, $M_t / M_{t-1} = P_t / P_{t-1} = 1 + \pi_t$, so inflation in period t is given by

$$1+\pi_t = (1+\pi^*)^{20} \frac{(1+\varepsilon_{20,t})}{(1+\varepsilon_{20,t-1})} \tag{22}$$

where $\varepsilon_{20,t}$ is the money supply innovation in year 20 of period t.

Notice that the PT money supply rule precludes base-level drift: money supply innovations have a temporary impact on the price level. As a result, long run inflation risk is somewhat lower than under IT.[15] Intuitively, inflation in period t depends on the money supply innovation in year 20 because policy offsets innovations after one year and so cannot offset the innovation in year 20 (the final year of the current period) until the first year of the next period. Inflation in period t also depends on the money supply innovation in year 20 of period t–1, because this must be offset in year 1 of period t in order to correct the past deviation from the target price path. Since rational agents expect past deviations from the target price path to be offset, inflation expectations vary with the past money supply innovation under PT: $1+E_{t-1}\pi_t = (1+\pi^*)^{20}(1+\varepsilon_{20,t-1})^{-1}$.

Since indexed bonds are subject to a one-year indexation lag, the inflation rate to which indexed bonds are linked is given by

$$1+\pi_t^{ind} = (1+\pi^*)^{20} \frac{(1+\varepsilon_{19,t})}{(1+\varepsilon_{19,t-1})} \tag{23}$$

The inflation rate in (23) will not covary with actual inflation *at all*, since yearly innovations to the money supply are uncorrelated.[16] As a result, indexed bonds will tend to be poor stabilisers of purchasing power. The key point is that cumulative inflation over 20 years depends only on two yearly innovations, as all innovations in intervening years are offset in order to return the price level to its target path. Intuitively, because *all* inflation over period t comes from the innovations that hit the economy at the start and end of each 20 year period, these innovations are missed by indexed bonds due to the one-year indexation lag.

4. Optimal indexation

The government chooses the indexation share that maximises social welfare subject to generational budget constraints; the aggregate resource constraint; first-order conditions of households and firms; fiscal policy (i.e. the total bond supply equation and the long run government spending target); and the monetary policy regime in place. As discussed below, the model is solved using a second-order approximation.

[15] The unconditional inflation variance is approximately 10 times higher under IT, as shown in the Supplementary Appendix to this paper.

[16] This assumption is relaxed in Section 7.3. Allowing for correlation between yearly innovations does not overturn the main results regarding optimal indexation unless innovations are highly positively correlated.

The optimal indexation problem of the government can be stated as follows:

$$\max_{v\in[0,1]} SW = E[U_t]$$

subject to Eqs. (1)-(3), (5)-(13), market-clearing, and

$$1+\pi_t = \begin{cases} (1+\pi^*)^{20}\prod_{j=1}^{20}(1+\varepsilon_{j,t}) & \text{under IT} \\ (1+\pi^*)^{20}\dfrac{(1+\varepsilon_{20,t})}{(1+\varepsilon_{20,t-1})} & \text{under PT} \end{cases}$$

$$1+\pi_t^{ind} = \begin{cases} (1+\pi^*)^{20}(1+\varepsilon_{20,t-1})\prod_{j=1}^{19}(1+\varepsilon_{j,t}) & \text{under IT} \\ (1+\pi^*)^{20}\dfrac{(1+\varepsilon_{19,t})}{(1+\varepsilon_{19,t-1})} & \text{under PT} \end{cases} \tag{24}$$

The optimal indexation share that satisfies (24) is computed numerically. To do so, the model was solved using a second-order perturbation approximation in Dynare++ (Adjemian et al., 2011). In particular, social welfare was computed for a discrete number of indexation shares in the interval [0,1]. This was achieved by looping over the parameter v in small steps in Dynare++, with the aid of an algorithm available on Wouter Den Haan's personal webpage.[17]

The problem in (24) is more complicated than might appear at first sight since the researcher must solve for an indexation share that maximises social welfare, subject to the constraint that (13) holds, which pins down a unique tax rate $\tau^*(v_k)$ for each indexation share. The optimal indexation share was therefore computed by simultaneously looping over v and τ in discrete steps in order to find: (i) the tax rate $\tau^*(v_k)$ such that (13) holds for each discrete value of the indexation share $v_1, v_2,\ldots, v_K$ in the interval [0,1]*;* and (ii) the indexation share $v^*(\tau^*(v^*))$ in the set $\{v_1, v_2,\ldots, v_K\}$ that maximises social welfare.

To increase computational efficiency, the tax rate that satisfies (13) for each indexation share v_k was used as an initial guess for the tax rate that would satisfy it for the next indexation share v_{k+1}. Moreover, for $\tau > \tau^*(v_k)$, (13) is violated on the upside (i.e. $E[g_t/y_t] > G^*$), so the loop over τ was ended as soon as $\tau \le \tau^*(v_k)$ was breached. By setting the tolerance level $E[g_t/y_t] - G^*$ at a small positive number, this algorithm speeds up computation and pinpoints $\tau^*(v_k)$ to a good degree of accuracy by approximating it with the last tax rate in the loop.

To understand the indexation results that follow, it is helpful to consider a second-order Taylor expansion of social welfare around the point $c_{t,Y} = E[c_{t,Y}]$ and $c_{t+1,O} = E[c_{t+1,O}]$: [18]

$$\begin{aligned} SW \approx{}& \frac{1}{1-\gamma}\left[(E[c_{t,Y}])^{\varepsilon} + \beta(E[c_{t,O}])^{\varepsilon}\right]^{\frac{1-\gamma}{\varepsilon}} \\ & -\frac{1}{2}\left|\tilde{U}_{C_YC_Y}\right|\mathrm{var}[c_{t,Y}] - \frac{1}{2}\left|\tilde{U}_{C_OC_O}\right|\mathrm{var}[c_{t,O}] - \left|\tilde{U}_{C_YC_O}\right|\mathrm{cov}[c_{t,Y}, c_{t+1,O}] \end{aligned} \tag{25}$$

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[17] The author would like to thank Wouter Den Haan for making this code publicly available.

[18] This expression makes use of the fact that, under stationarity, $E[c_{t+1,O}] = E[c_{t,O}]$ and $\mathrm{var}[c_{t+1,O}] = \mathrm{var}[c_{t,O}]$. The derivatives have the signs indicated under the calibrations considered in this paper.

where $|x| \equiv abs(x)$; $\tilde{U}_{xx}$ is the second derivative of U with respect to x, evaluated at the point $E[x]$; and $\tilde{U}_{xy}$ is the cross-derivative of U with respect to x and y, evaluated at $E[x]$ and $E[y]$.

This expression shows that social welfare increases with mean consumption levels in youth and old age and falls with the consumption variances (due to risk-aversion). There is additionally a covariance term, which will also be driven by the consumption variances.[19] Consequently, any difference in optimal indexation under IT and PT will be due to their impact on mean consumption levels and the consumption variances. The numerical analysis therefore explains optimal indexation with reference to these unconditional moments.

5. Calibration

The model is calibrated for the UK economy. In particular, the parameters of the model are chosen to roughly match key ratios in the data. Expressions for these ratios at the deterministic steady-state are given in Appendix B. Since these ratios depend upon several different parameters, the calibration uses parameter values which are plausible *and* give good overall performance of the model against target ratios.[20] Free parameters are calibrated to match standard values in the literature. The baseline calibration is listed in Table 1.

Table 1 – Baseline calibrated values

α	Capital share	0.263	G^*	Target govt. spending-GDP ratio	0.11
β	Private discount factor	0.70	π^*	Annual inflation target	0.02
γ	Risk aversion	15	ρ_A	TFP persistence	0.40
ε	EIS = 1/(1–ε)	0.74	σ_e	Std(TFP innov.)	0.0557
δ	Real money holdings	0.015	σ	Std (money innov.)	0.0105
a	Capital to labour tax ratio	2.3	v	Share of indexed government debt	0.25

5.1 Aggregate uncertainty

The model contains two aggregate shocks: a money supply shock and a productivity shock. Calibrating the money supply rule requires a standard deviation for the annual money supply innovation. This standard deviation was set at σ = 0.0105 to match the standard deviation of annual CPI inflation from 1997 to 2011 in data from the Office for National Statistics (ONS). This calibration should give the model a good chance of matching the amount of long run inflation risk that would be observed with typical price level shocks under an IT regime.[21]

The productivity shock is given by

$$\ln A_t = (1-\rho_A)\ln A_{mean} + \rho_A \ln A_{t-1} + e_t \tag{26}$$

where e_t is an IID-normal innovation with mean zero and standard deviation σ_e.

[19] This point follows since cov[x, y] = corr[x, y](var[x])$^{1/2}$(var[y])$^{1/2}$.

[20] In some models, key ratios are pinned down by a single parameter so that calibrated values can be set to match target ratios exactly. The model here does not have this property.

[21] Note that, under IT, annualised inflation, $(1+\pi_t)^{1/20}$ has a standard deviation of approximately σ when $\pi^* \approx 0$.

Since there is no convincing empirical evidence that productivity is highly persistent over generational horizons, ρ_A was set at 0.40, which implies moderate persistence. The innovation standard deviation σ_e was set equal to 0.0557. This relatively high value reflects the fact that e_t is the innovation to productivity at a *generational horizon*. As discussed below, the inclusion of aggregate productivity risk means that the coefficient of relative risk aversion can be calibrated to match the Sharpe ratio on capital. The main conclusions regarding optimal indexation are not sensitive to the inclusion of productivity risk.[22]

5.2 The indexation lag

As discussed above, there is a 1-year indexation lag in the model. In the UK, all index-linked gilts issued up until September 2005 were indexed to the Retail Prices Index (RPI) with a lag of 8 months, but all index-linked gilts issued since this time have a 3-month lag. As a result, both types of gilts are currently in existence. The proportion of 8-month gilts will fall over time as debt issued prior to September 2005 reaches maturity. In fact, recent data suggests that around two-thirds of the market is in index-linked gilts with a 3-month indexation lag, and one-third in gilts with an 8-month indexation lag.[23] Crucially, however, the main findings regarding optimal indexation would not be overturned if the indexation lag were one quarter, provided that inflationary shocks were assumed to hit the economy at a quarterly frequency. The baseline assumption of a one-year lag is made primarily to lower the number of money supply innovations, hence making a second-order approximation computationally feasible.

5.3 Model parameters

The preference parameter ε does not affect the deterministic steady-state of the model, as shown by the equations in Appendix B. It was therefore set at -0.35, which implies an elasticity of intertemporal substitution (EIS) of 0.74. This value is consistent with micro studies that estimate an EIS less than 1. The discount factor β was set at 0.70, implying an annual value of 0.982 and hence an annual real interest rate of 1.8% in the deterministic steady-state. The latter is relatively low because matching a real rate of (say) 3% would give an investment-GDP ratio somewhat lower than in the data (see Table 2).[24]

The tax rate on capital (τ^k) was set at 2.3 times the income tax rate (τ), i.e. a = 2.3. A substantially higher tax rate on capital is consistent with UK data from 1970-2005: Angelopoulos et al. (2012) report an average tax rate on capital of 0.44, compared to an average tax rate on labour of 0.27, implying that capital taxes should be around 1.6 times as high as labour taxes. The higher ratio of 2.3 used here ensures that the model does not substantially overshoot the target ratio of investment to GDP, since an increase in the capital tax lowers the demand for capital. The income tax rate τ is set so that the government spending to GDP target, G^*, is met in the stochastic solution of the model. Since government spending is residually determined, G^* was set at 0.11, a value which enables the model to get close to target ratios for consumption, investment and long-term debt (see Table 2).

The parameter α was set at 0.263, implying a share of capital income in GDP of 26.3%. This value is on the low side of standard calibrations, but it helps the model get close to the target

[22] Section 7.4.2 discusses the case where productivity risk is absent.

[23] See the UK Debt Management Office (DMO) website. The data was accessed on 10 May 2013.

[24] Note that the steady-state nominal interest rate is 3.8% per annum due to the 2% inflation target.

ratio of long-term government debt to GDP of 0.10. The annual inflation target π^* was set at 0.02, consistent with the 2% inflation target for the Consumer Prices Index (CPI). The parameter δ was set at 0.015, so that money balances are 3% of steady-state GDP, consistent with UK data on notes and coins (see ONS, 2011). For the purpose of calibration, the indexation share v was set at 0.25, which is similar to the share of index-linked gilts in the UK (see DMO, 2012). It should be noted, however, that the ratios in Table 2 are not sensitive to the indexation share (to two decimal places).

Finally, as shown in Appendix B, the coefficient of relative risk aversion γ does not affect the deterministic steady-state ratios. It was therefore calibrated to match the Sharpe ratio on capital, $E[r^k - r^i]/\text{std}(r^k - r^i)$, in the stochastic solution of the model.[25] The target was set at 0.43 based on Constantinides et al. (2002), who estimate the Sharpe ratio using 20-year holding period returns on US equities and bonds. Accordingly, γ was set at 15, which implies a Sharpe ratio of 0.40.

5.4 Model solution and key ratios

The results in Table 2 show that the model does fairly well against target ratios. The UK investment-GDP ratio has been close to 15% over the past decade (see ONS, 2012) and over the same period the consumption share was around 65%, implying target ratios of 0.15 and 0.65. The calibrated model gives ratios of 0.14 and 0.75 at the deterministic steady-state. The UK debt-to-GDP ratio has averaged around one-third from 2000 to 2011 (ONS, 2011). Together with a 2005 long-term government debt share of around 30%, this implies a target long-term government debt to GDP ratio of around 0.10.[26] On this score, the model gives a ratio of 0.11. Finally, the ratio of cash to GDP matches the UK ratio of 0.03.

Table 2 – Target versus model ratios

Ratio	Target	Definition	Deterministic	Stochastic	Notes
b/y	0.10	Long-term bonds/GDP	0.11	0.11	Target: UK data
i/y	0.15	Investment/GDP	0.14	0.13	Target: UK data
$(c_Y + c_O)/y$	0.65	Consumption/GDP	0.75	0.76	Target: UK data
m/y	0.03	Notes and Coins/GDP	0.03	0.03	Target: UK data
$\frac{E[r^k - r^i]}{std[r^k - r^i]}$	0.43	Sharpe ratio on capital	0	0.40	Target: CDM (2002)

Note: CDM refers to Constantinides et al. (2002). The value of g/y is given by $1 - i/y - c/y$.

6. Baseline results

The model was solved using a second-order perturbation in Dynare++ (see Adjemian et al., 2011). The optimal indexation share under each regime is computed as described in Section 4. To give the welfare results a meaningful interpretation, they are reported as percentage gains or losses in aggregate consumption relative to the case of zero indexation. Formally, the

[25] Returns are annualised here. The Sharpe ratio was computed using the after-tax return on capital.

[26] See historical data on the DMO website. The DMO classifies gilts as ‘long-term’ if maturity exceeds 15 years.

welfare gain is the fractional increase in aggregate consumption, λ, that equates social welfare under zero indexation with that under indexation share v, i.e. $SW_{(0)}\,(1+\lambda)^{1-\gamma} = SW_{(v)}$. The implications of the results for comparisons of IT and PT are also discussed in this section.

Figure 1 shows the relationship between indexation and social welfare under IT and PT, and Figure 2 traces out the impact of indexation on real variables, including the unconditional means and variances of consumption that matter for social welfare; see Equation (25).

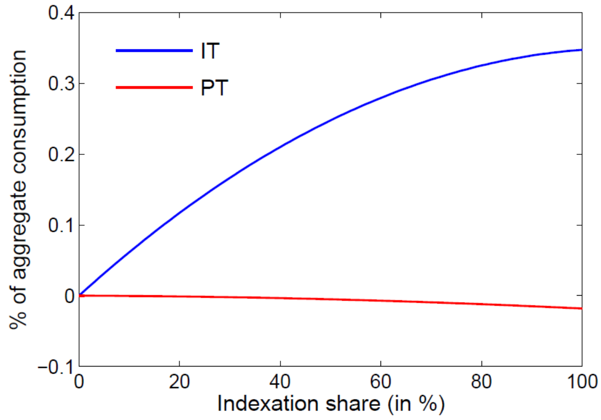

Fig 1 – Social welfare gain relative to zero indexation. Figure reports the welfare gain or loss of indexation at x%, as compared to the case of zero indexation. Units: % of aggregate consumption under each regime.

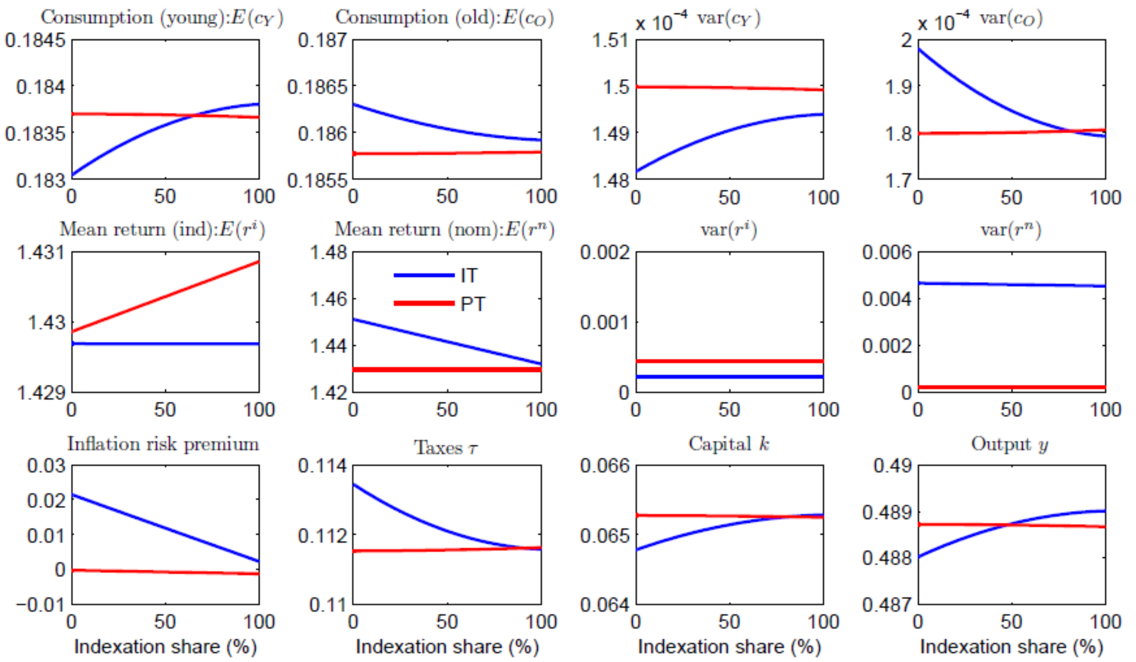

Fig 2 – Indexation and real variables under IT and PT. The inflation risk premium is defined as the difference between the expected real returns on nominal and indexed bonds (i.e. $E(r^n) - E(r^i)$). Returns are non-annualised gross returns and have *not* been converted into percent. All series are unconditional moments.

6.1 Inflation targeting (IT)

We can see from Figure 1 that social welfare increases under IT as the share of indexed debt is increased. As a result, full indexation is optimal. It is also notable that social welfare is sensitive to changes in the indexation share: moving from zero indexation to full indexation raises social welfare by almost 0.35% of aggregate consumption.

Figure 2 helps to shed light on why full indexation is optimal under IT. An important factor is consumption risk in retirement, which is minimised under full indexation and around one-tenth lower than under zero indexation. Old-age consumption risk is important for two reasons. First, as highlighted by Equation (25), it has a negative impact on social welfare due to risk-aversion. Second, lowering old-age consumption risk by increasing indexation brings down the inflation risk premium because it implies a fall in the variance of the stochastic discount factor (*sdf*) and a weaker correlation between the real return on nominal debt and the *sdf*.[27] As a result, lower indexation shares require higher taxes in order to meet the long run government spending target, so that mean consumption by the young and capital accumulation fall. Consequently, mean consumption by the young, capital accumulation and output are maximised at full indexation under IT.

Consumption risk faced by the old is minimised under full indexation of government debt because, as explained in Section 3.1, indexed government debt provides far better insurance against unanticipated inflation than nominal debt under an IT regime. Intuitively, cumulative inflation risk over 20 years is (approximately) 20 times that at a yearly horizon due to base-level drift, so the 1-year indexation lag on indexed debt exposes bondholders to only 1 year of inflation risk, as compared to the full 20 years in the case of nominal debt. To summarize, the young gain from higher mean consumption as indexation is increased, while the old gain from a reduction in consumption risk. And although mean consumption by the old falls as indexation is increased (because the positive inflation risk premium is foregone) and consumption risk for the young rises slightly, the rise in mean consumption by the young and the reduction in consumption risk for the old clearly dominate, so social welfare rises.[28]

6.2 Price-level targeting (PT)

It is clear from Figure 1 that zero indexation of government debt is optimal under PT. Zero indexation is optimal because, as shown by Figure 2, nominal bonds are better stabilisers of purchasing power than indexed bonds – i.e. they have lower return risk. The reasoning for indexed bonds being more risky is as follows. Under a PT regime, the real return on nominal bonds will vary with unanticipated money supply innovations in the final year of each period, as shown by Equation (22). Indexed bonds will also 'miss' such innovations but, due to the indexation lag, they will tend to exacerbate volatility on top of this because their return is linked to the money supply innovation in the penultimate year of each period (see Equation (23)), which is uncorrelated with the innovation in the final year. In other words, the indexation lag becomes an additional source of volatility when money supply innovations are uncorrelated.

[27] Appendix A provides an expression for the inflation risk premium and a brief discussion of the factors that affect its magnitude.

[28] Consumption risk for the young rises as indexation is increased because the demand for indexed bonds is less stable from generation to generation than the demand for nominal bonds. This is because the one-year indexation lag implies variations in the expected real return on indexed bonds, whereas the expected real return on nominal bonds is more stable since expected inflation is constant under IT (see Section 3.1).

Due to the lower level of return risk on nominal bonds under a PT regime, consumption risk for the old is minimised near to zero indexation.[29] The greater riskiness of indexed government debt means that there is a negative inflation risk premium. In other words, the expected real return on indexed debt is higher than the expected real return on nominal debt – in stark contrast to the IT case. As a result, the relationship between taxes and indexation is positive, so that mean consumption by the young is maximised under zero indexation. Because taxes rise as indexation is increased, both capital and output fall as the indexation share rises. In short, the main factors driving optimal indexation work in exactly the opposite direction under PT, because nominal debt is a better store of value than indexed debt.

The one-year indexation lag is crucial in that, in the absence of any lag, full indexation is optimal under PT. The lag is central because cumulative inflation risk over a 20-year horizon is reduced to yearly magnitudes, due to the absence of base-level drift. This makes nominal government bonds rather effective stabilisers of long run purchasing power, while indexed inflation misses the shocks that matter for actual inflation. Lastly, it is important to note that real variables and social welfare are less sensitive to indexation under PT, due to the low level of long run inflation risk. For instance, moving from zero indexation (optimal) to full indexation of 100% (worst outcome) implies a welfare loss of less than 0.02% of aggregate consumption, and consumption risk in retirement rises only modestly.

6.3 Implications of the optimal indexation results

The results above highlight the importance of indexation of government debt for real variables and social welfare. In particular, changes in indexation are associated with substantial changes in real variables and social welfare under IT: moving from zero indexation to full indexation implies a social welfare gain of almost 0.35% of aggregate consumption and a reduction in consumption risk of around one-tenth in retirement.

These figures suggest that moving to full indexation of indexation of government debt would raise social welfare non-trivially in IT economies. This result raises the question of why governments in IT economies issue mainly nominal debt (see Footnote 5). One possible reason to favour nominal debt is that indexed bonds tend to be less liquid due to the high proportion of buy-and-hold investors and a lack of direct hedging instruments; therefore, we would expect indexed bonds to pay a ‘liquidity premium’.[30] In theory, the optimal indexation result could be reversed if the model included a liquidity premium that was sufficiently large. In practice, however, it seems unlikely that illiquidity of indexed bonds alone could account for the low indexation shares we observe, since average estimates of inflation risk premia are substantial and robustly positive in developed economies (Bekaert and Wang, 2010).

Although social welfare and real variables are far less sensitive to indexation under PT, the effects do not appear to be sufficiently small that they could be safely ignored. It is also worth noting that the imperfect credibility results of Section 7.2 suggest that the aggregate and welfare effects of indexation under PT could be somewhat higher than in the baseline analysis. Lastly, it is important to note that indexation has implications for social welfare comparisons of IT and PT. For example, if the indexation share is fixed across regimes at

[29] More specifically, consumption risk in old age is minimised at an indexation share of 3%. Intuitively, holding a small share of indexed bonds diversifies risk because real returns are not strongly positively correlated.

[30] For a discussion of this point in the context of the UK economy, see DMO (2012, p. 30).

25% (which is close to the current UK indexation share) the model implies a PT welfare gain of 0.15% of aggregate consumption, but IT fares slightly better with a welfare gain of almost 0.05% of aggregate consumption when indexation is optimal under both regimes.

7. Extensions

The baseline model assumes that there is a PT target horizon of one year; that the PT regime has perfect credibility; and that money supply shocks are uncorrelated. In this section, these assumptions are relaxed and the implications for optimal indexation are assessed.

7.1 A flexible target horizon under PT

It has been argued that the short-term costs of undoing price level shocks could be reduced by restoring the price level to its target path gradually following deviations (see Gaspar et al., 2007). The analysis in this section therefore assesses the implications for optimal indexation of a PT regime that aims to return the price level to target over several years, rather than after one year as in the baseline model.

For the general case where the price level is returned to target in uniform steps over H years, the implied yearly money supply rule in year n is:

$$M_n = M_{n-1}(1+\pi^*)\frac{(1+\varepsilon_n)}{\prod_{j=1}^{H}(1+\varepsilon_{n-j})^{1/H}} \tag{27}$$

Note that innovations up to H years old enter in the denominator of this rule because each is offset only after H years in total, with a fraction $1/H$ offset each year.

By substitution, and the fact that each period lasts 20 years, Equation (27) implies that

$$M_t = M_{t-1}(1+\pi^*)^{20}\prod_{k=0}^{H-1}\left(\frac{1+\varepsilon_{20-k,t}}{1+\varepsilon_{20-k,t-1}}\right)^{(H-k)/H} \tag{28}$$

Inflation is therefore given by

$$1+\pi_t = (1+\pi^*)^{20}\prod_{k=0}^{H-1}\left(\frac{1+\varepsilon_{20-k,t}}{1+\varepsilon_{20-k,t-1}}\right)^{(H-k)/H} \tag{29}$$

Since the inflation rate to which indexed bonds are linked is the one-year lagged value of actual inflation, indexed inflation for a target horizon of H years is given by

$$1+\pi_t^{ind} = (1+\pi^*)^{20}\prod_{k=0}^{H-1}\left(\frac{1+\varepsilon_{19-k,t}}{1+\varepsilon_{19-k,t-1}}\right)^{(H-k)/H} \tag{30}$$

To see the implications of these expressions, consider setting $H = 2$. In this case, only money supply innovations in years 19 and 20 of period t matter for inflation because innovations in years 1 to 18 will have been offset fully by the end of the period t (i.e. by year 20), given that the target horizon is 2 years. Innovations in years 19 and 20 from the previous period enter in the denominator because they will not have been offset by the end of year 20 of period t–1 and so must be offset at the start of period t to return the price level to its target path.

When H = 2, inflation and indexed inflation are given by

$$1+\pi_t = (1+\pi^*)^{20}\left(\left(\frac{1+\varepsilon_{20,t}}{1+\varepsilon_{20,t-1}}\right)\left(\frac{1+\varepsilon_{19,t}}{1+\varepsilon_{19,t-1}}\right)\right)^{\frac{1}{2}} \tag{31}$$

$$1+\pi_t^{ind} = (1+\pi^*)^{20}\left(\left(\frac{1+\varepsilon_{19,t}}{1+\varepsilon_{19,t-1}}\right)\left(\frac{1+\varepsilon_{18,t}}{1+\varepsilon_{18,t-1}}\right)\right)^{\frac{1}{2}} \tag{32}$$

These equations show that a gradual return of the price level to target has two effects. First, long run inflation risk rises because the price level is allowed to deviate from its target path for longer. Second, actual and indexed inflation are positively correlated, because past deviations from the target price path are 'smoothed' back to target over several years. As a result, indexed bonds become better stabilisers relative to nominal bonds. The results for PT target horizons of 2 and 4 years are reported in Figures 3 and 4.

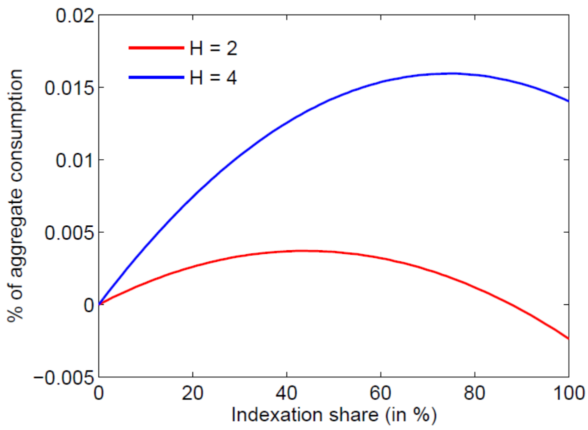

Fig 3 – Social welfare gain relative to zero indexation and as PT target horizon *H* is varied. Figure reports the welfare gain or loss of indexation at x%, as compared to the case of zero indexation. Units: % of aggregate consumption and H is in years. All other parameters have the same values as in the baseline model.

Optimal indexation rises from zero when the PT target horizon is 1 year (the baseline case) to 44% under a target horizon of 2 years and 76% at a 4-year horizon. The reason is that indexed bonds become better stabilisers of purchasing power relative to nominal bonds as the target horizon is increased. This reduction in real return risk is reflected in an inflation risk premium which is initially positive but decreasing, so that substituting towards indexed debt enables taxes to be reduced and mean consumption by the young increased (see Fig 4).

Due to the reduction in return risk on indexed bonds, consumption risk in retirement initially falls as the indexation share is increased, and it is minimised at approximately the optimal indexation shares, rather than at (approximately) zero as in the baseline model.[31] Notably, it is approximately optimal to issue indexed debt up to the point where the inflation risk premium is zero – i.e. where the expected real returns on indexed and nominal debt are equal. Due to the reduction in the riskiness of indexed bonds, the inflation risk premium is initially positive, but it falls as the indexation share is increased because the correlation between the real return on nominal bonds (indexed bonds) and the stochastic discount factor (*sdf*)

[31] Although the return variances on indexed and nominal debt differ slightly, interior solutions minimise consumption risk in old age for diversification reasons – i.e. returns are less than perfectly correlated.

becomes weaker (stronger) as indexation increases, since retirement consumption becomes more dependent on the return on indexed bonds and less so on the return on nominal bonds.[32] When the risk premium is zero, taxes are minimised and mean consumption by the young is maximised.

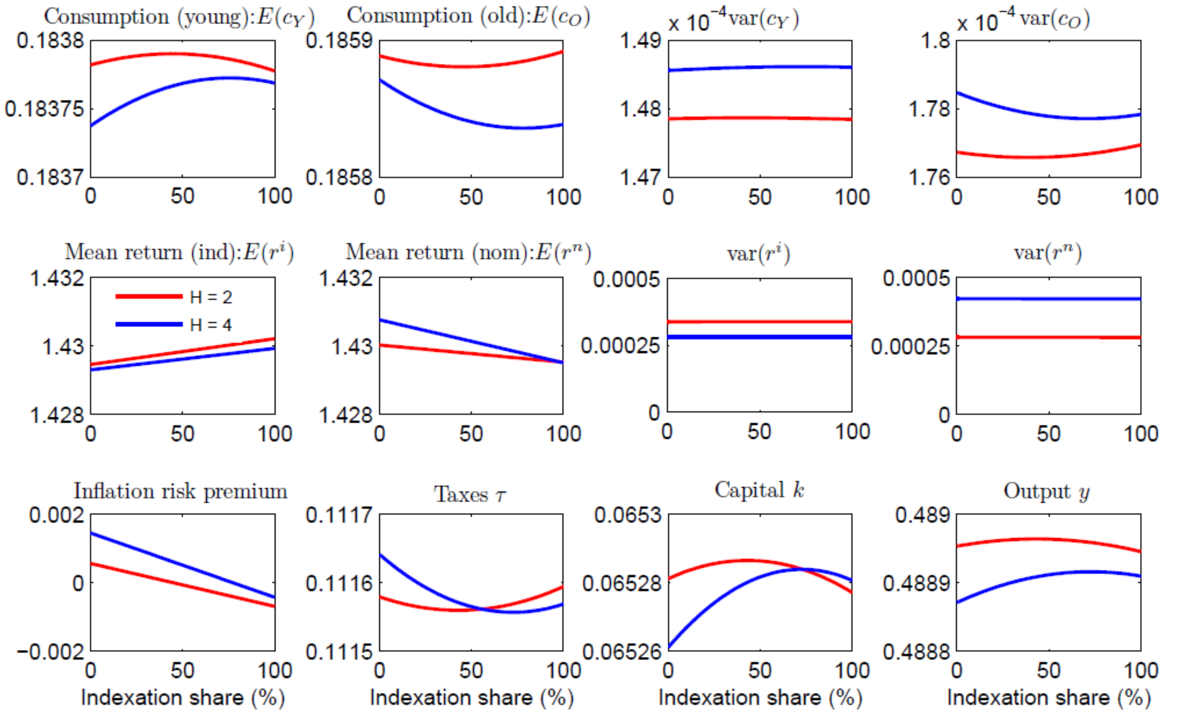

Fig 4 – Indexation and real variables as the PT target horizon *H* is varied. The inflation risk premium is the difference between the expected real returns on nominal and indexed bonds (i.e. $E(r^n) - E(r^i)$). Returns are non-annualised gross returns and have *not* been converted into percent. All series are unconditional moments. All parameters have the same values as in the baseline model. The PT target horizon *H* is in years.

The intuition for optimal indexation being linked to the indexation share at which the inflation risk premium is zero is as follows.[33] On the benefit side, the first-order conditions for bond holdings imply that both bonds offer the same degree of social insurance against inflation risk if and only if the inflation risk premium is zero (see Appendix A). On the cost side, an inflation risk premium of zero implies that the expected cost of issuing additional government debt is the same across bonds, so that the expected real repayment rate on debt, $vE(r^i) + (1-v)E(r^n)$, will be minimised. As a result, optimal indexation implies that both bonds are equally attractive to households and that taxes are minimised.

7.2 Imperfect credibility of PT

The argument that PT would be imperfectly credible is appealing because this regime not been adopted in recent history. As such, imperfect credibility was an important consideration in the Bank of Canada's recent deliberations about the merits of PT (Bank of Canada, 2011).

[32] See Appendix A for an expression for the inflation risk premium in terms of these correlations.

[33] I am grateful to an anonymous referee for suggesting a discussion of this point.

Imperfect credibility of PT has been studied formally by Gaspar et al. (2007) and Masson and Shukayev (2011).[34] Gaspar et al. argue that PT would experience an initial period of imperfect credibility when agents would learn about the new policy regime. They set-up a New Keynesian model with learning and find that an initial period of imperfect credibility is sufficient to turn the net welfare gains from PT negative if agents are slow to learn. Masson and Shukayev build a New Keynesian model where PT operates with an 'escape clause', such that sufficiently large shocks lead to rebasing of the target price path. There are two stable equilibria: one with a low probability of rebasing, and one with a high probability. Consequently, imperfectly-credible PT regimes are long run equilibrium outcomes.

In contrast to these two papers, the model here concentrates on the long run inflation risk channel and is non-linear, so that imperfect credibility will influence aggregate outcomes through the inflation risk premium and not just via inflation expectations. In order to model imperfect credibility, it is assumed that young agents assign a constant probability p_{IT} to the event that that monetary policy will switch back to IT in the next period. Accordingly, agents assign a constant probability $1-p_{IT}$ to the event that the current PT regime will remain in place. The probability p_{IT} can be taken as a measure of credibility of PT, with $p_{IT}=0$ being the perfect credibility case of Section 6.2. It is important to note that although agents assign a positive probability to reversions to IT, no such reversions actually occur in equilibrium. Hence the analysis is one of imperfect credibility and not regime switching.

Given beliefs over regimes $s = \{IT, PT\}$ and period-t information Ω_t, lifetime utility is

$$U_t^{IC} = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\Big[p_{IT}E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT})E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t]\Big]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \tag{33}$$

The first-order conditions are as follows:[35]

$$1 = R_t\,(p_{IT}E[SDF_{t+1(IT)} \mid \Omega_t] + (1-p_{IT})E[SDF_{t+1(PT)} \mid \Omega_t]) \tag{34}$$

$$1 = r_t\,(p_{IT}E[SDF_{t+1(IT)}(1+\pi_{t+1(IT)}^{ind}) \big| \Omega_t] + (1-p_{IT})E[SDF_{t+1(PT)}(1+\pi_{t+1(PT)}^{ind}) \big| \Omega_t]) \tag{35}$$

$$1 = \alpha k_{t+1}^{\alpha-1}([p_{IT}(1-\tau_{IT}^{k})E[sdf_{t+1(IT)}A_{t+1} \mid \Omega_t] + (1-p_{IT})(1-\tau_{PT}^{k})E[sdf_{t+1(PT)}A_{t+1} \mid \Omega_t]) \tag{36}$$

$$1 = (p_{IT}E[SDF_{t+1(IT)} \mid \Omega_t] + (1-p_{IT})E[SDF_{t+1(PT)} \mid \Omega_t]) + \tilde{\mu}_t \tag{37}$$

where $SDF_{t+1(s)} \equiv sdf_{t+1(s)}/(1+\pi_{t+1(s)})$, τ_s^k is the tax rate on capital in regime s, and

$$sdf_{t+1(s)} \equiv \beta\left(\frac{c_{t,Y}}{c_{t+1,O(s)}}\right)^{1-\varepsilon}\left[\frac{c_{t+1,O(s)}}{(p_{IT}E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT})E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t])^{1/(1-\gamma)}}\right]^{1-\gamma-\varepsilon} \tag{38}$$

The model was simulated for two different values of p_{IT} : 0.1 and 0.3. These values represent fixed beliefs that policy will revert to IT next period with probabilities of 10% and 30%. The

[34] See also Kryvtsov et al. (2008).

[35] The government is assumed to set the total bond supply so that the conditionally expected stochastic discount factor across regimes is equal to β (a natural extension of (12)), which implies that $1 = \beta r$ in the deterministic steady-state. The first-order conditions under imperfect credibility are derived in the Supplementary Appendix.

former is interpreted as a situation where PT has high credibility and the latter as a situation of low credibility. The results for these two cases are reported in Figures 5 and 6.

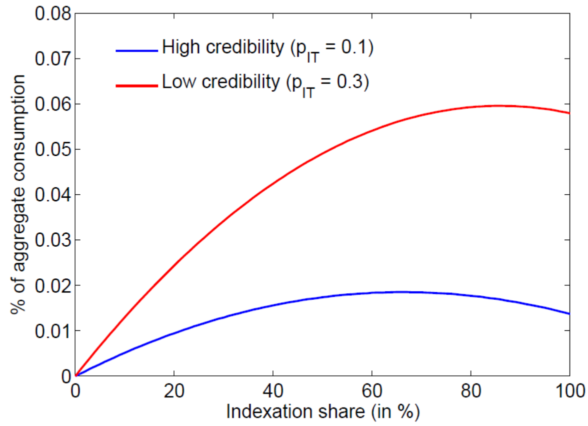

Fig 5 – Social welfare gain from indexation under imperfect credibility of PT. Figure reports the welfare gain or loss of indexation at x%, as compared to the case of zero indexation. Units: % of aggregate consumption under regime. All parameters have the same values as in the baseline model. Social welfare was defined as the unconditional expectation of Equation (33) in both the low and high credibility cases.

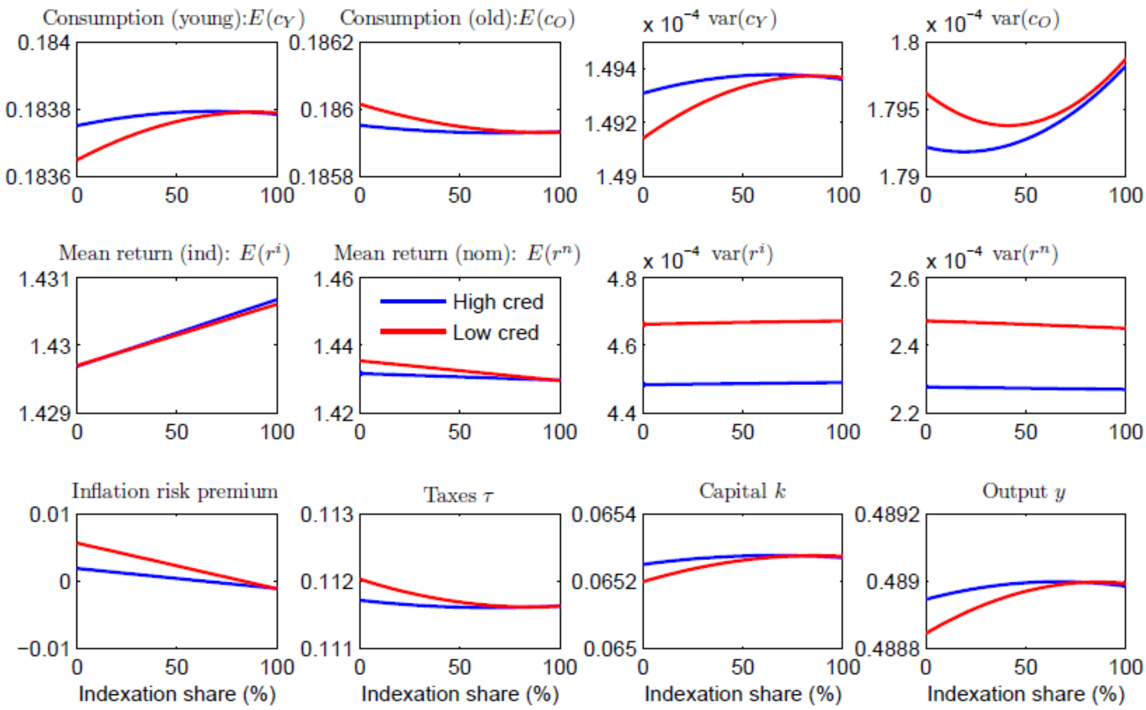

Fig 6 – Indexation and real variables under imperfect credibility of PT. The inflation risk premium is defined as the difference between the expected real returns on nominal and indexed bonds (i.e. $E(r^n) - E(r^i)$). Returns are non-annualised gross returns and have *not* been converted into percent. All series are unconditional moments. 'High cred' refers to the high credibility case: p_{IT} = 0.1. 'Low cred' refers to low credibility: p_{IT} = 0.3.

Introducing imperfect credibility of PT in the baseline model makes it optimal to issue indexed debt. In particular, optimal indexation is 66% under a PT regime with high credibility and 87% under low credibility. Optimal indexation rises because imperfect credibility makes the inflation risk premium positive except at high indexation shares (see Figure 6). The reason is that households attach a positive probability to the event that monetary policy will revert to IT, where long run inflation risk is much higher and nominal bonds are a poor store of value. Accordingly, they require additional risk compensation for holding nominal bonds. In turn, this increase in the inflation risk premium makes it more costly for the government to issue nominal rather than indexed debt, implying higher taxes to meet the long run government spending target and lower mean consumption by the young.

The interior solutions for optimal indexation in Figure 5 balance the welfare loss from this increase in taxes against the welfare gain from the fact that nominal government debt will tend to stabilise old age consumption more effectively than indexed debt given that a PT regime is actually implemented. Intuitively, optimal indexation is higher under a PT regime with low credibility because more weight is attached to the possibility of reversion to IT, so that there is a larger rise in the inflation risk premium. As a result, the increase in taxes necessary to meet the long run government spending target is larger. To avoid this welfare loss, it is optimal to substitute from nominal to indexed debt as PT becomes less credible.

Interestingly, social welfare is somewhat more sensitive to indexation under a PT regime with low credibility. For instance, moving from zero indexation to the optimal indexation share raises social welfare by around 0.06% of aggregate consumption. By comparison, moving from the optimal point of zero indexation to the lowest-welfare equilibrium at full indexation implied a welfare loss of 0.02% of aggregate consumption in the baseline model. Therefore, a high degree of imperfect credibility raises the welfare effects of indexation under a PT regime. The effects of indexation on real variables are also somewhat larger.

7.3 Correlated money supply shocks

The baseline model assumes that yearly money supply innovations are uncorrelated. This assumption is unlikely to be innocuous because it implies that current and past shocks to inflation are uncorrelated, so that indexing with a 1-year lag can potentially be quite costly. In this section, the assumption of uncorrelated innovations is relaxed. In particular, it is assumed that the money supply innovation in any year is positively correlated with the innovation in the previous year. Moderate correlations of 0.5 and 0.7 are considered because empirical evidence suggests that inflation persistence has fallen to moderate levels in the Great Moderation (e.g. Benati, 2008; Minford et al. 2009). Since optimal indexation under IT is unchanged at 100%, this section focuses on the impact of introducing correlated money supply innovations under PT. More specifically, the question is whether correlated innovations substantially close the optimal indexation gap between IT and PT.

The optimal indexation results are reported in Figure 7, whereas Figure 8 shows the relationship between indexation and real variables. Optimal indexation is somewhat higher than in the baseline model at 46% with a correlation of 0.5 and 69% with a correlation of 0.7. Intuitively, correlated money supply innovations should make indexed bonds a better store of value, because they induce a positive correlation between the inflation rate to which bonds are indexed and actual inflation. The results in Figure 8 confirm this intuition: return risk on indexed debt falls as the correlation between money supply innovations is increased, and consumption risk in retirement is minimised at a higher indexation share as a result. Due to this reduction in risk, the inflation risk premium is higher for a correlation of 0.7 than 0.5, so that taxes can be reduced by substituting further towards indexed debt. As a result, both capital and output are higher when money supply innovations are more strongly correlated.

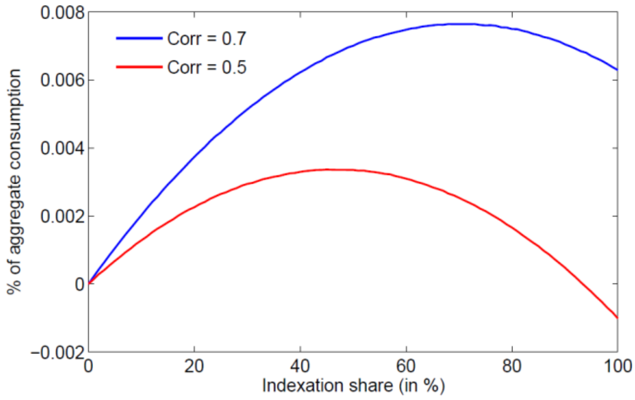

Fig 7 – Social welfare gain from indexation when money supply innovations are correlated (PT). Figure reports the welfare gain or loss of indexation at x%, as compared to the case of zero indexation. Units: % of aggregate consumption. All other parameters have the same values as in the baseline model.

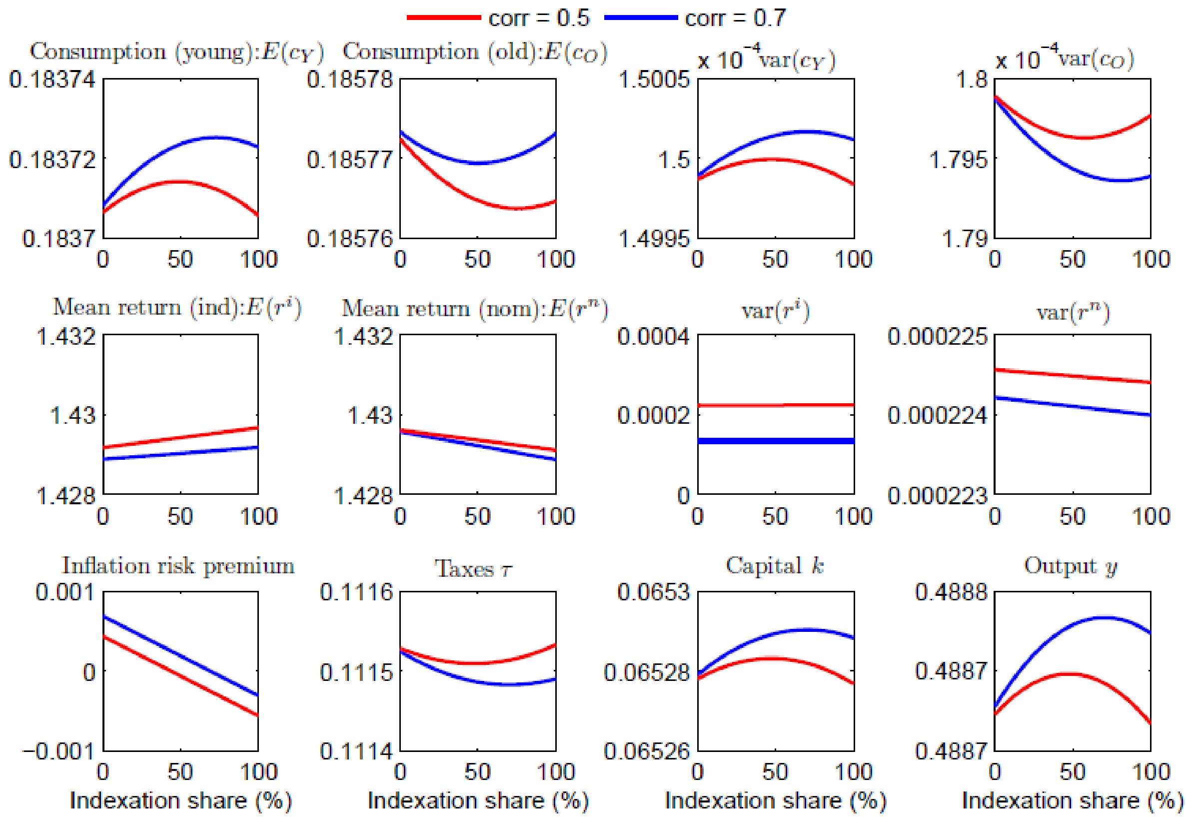

Fig 8 – Indexation and real variables when money supply innovations are positively correlated (PT). The inflation risk premium is defined as the difference between the expected real returns on nominal and indexed bonds (i.e. $E(r^n) - E(r^i)$). Returns are non-annualised gross returns and have *not* been converted into percent. All series are unconditional moments. All parameters have the same values as in the baseline model.

7.4 Sensitivity analysis

This final section is concerned with robustness of results. It considers the cases where money does not enter the model and where productivity risk is absent. Sensitivity to calibrated parameters is also discussed.

7.4.1 Model without money[36]

In the analysis thus far, money has entered the model due to the cash constraint $m_t = \delta$. As a robustness test, the case where $\delta = 0$ was considered, so that money is absent. It is instructive to consider this case as it seems likely that agents would change their money holdings in a regime that affects long run inflation risk. Therefore, the results presented above could potentially be biased. The numerical results for the baseline case are reported in the Supplementary Appendix. Optimal indexation remains at 100% under IT and at 0% under PT, and the quantitative effects on social welfare and real variables are quite similar to those in Figures 1 and 2. The findings of Section 7 are also robust in this direction.

7.4.2 Model without productivity risk

If productivity risk is absent, the real return on capital is known with certainty in the first period of life. As a result, households have access to a risk-free real asset. Does shutting down productivity risk have implications for optimal indexation? In the baseline model, optimal indexation remains at 100% under IT and rises only marginally to 4% under PT (again, the full results are reported in the Supplementary Appendix). Likewise, the optimal indexation results of Section 7 are not sensitive to the inclusion of productivity risk.

7.4.3 Parameter sensitivity analysis

Experimentation with ‘high’ and ‘low’ calibrations of model parameters does not overturn the main conclusions regarding optimal indexation under IT and PT. We can therefore say that the finding that optimal indexation is substantially lower under PT is robust to realistic extensions of the model and alternative parameter calibrations.

8. Conclusion

This paper has investigated optimal indexation of long-term government debt under inflation targeting (IT) and price-level targeting (PT) monetary regimes. These two regimes have very different long run implications. Under IT, inflation risk increases with the forecast horizon, since there is base-level drift. By contrast, a PT regime rules out base-level drift by aiming at a predetermined target price path. As a result, long run inflation risk is largely eliminated under PT, implying that the purchasing power of nominal assets is maintained over long horizons. Optimal indexation was studied in the context of a simple overlapping generations (OG) model that was roughly calibrated to match the UK economy. The model is well-suited for this task because each period lasts 20 years and long run inflation risk matters for social welfare. In order to capture base-level drift under IT – and its absence under PT – the standard OG model was augmented with money supply shocks at a yearly horizon.

The main finding was that if indexed government debt is subject to a one-year indexation lag, then full indexation is optimal under IT, while optimal indexation is substantially lower under PT. To demonstrate this result, the analysis began with a simple version of the model where full indexation is optimal under IT and zero indexation is optimal under PT. Intuitively, real return risk on long-term nominal debt is somewhat higher than on indexed debt under IT,

[36] The author is grateful to an anonymous referee for suggesting this robustness test.

because cumulative inflation risk over a 20-year horizon is approximately 20 times that at a yearly horizon due to base-level drift, while indexed bonds are subject to only a small amount of inflation risk due to the 1-year indexation lag. Under a PT regime, by contrast, cumulative inflation risk is lowered to annual magnitudes, because base-level drift is eliminated. As a result, the amount of risk in the real return on indexed bonds due to the one-year indexation lag can exceed that in the real return on nominal government debt.

The result that optimal indexation is substantially lower under PT holds in more realistic versions of the model where the PT target horizon exceeds one year; PT is imperfectly credible; and where money supply innovations are positively correlated, so that indexing with a one-year lag is less costly than in the baseline model. In each case, moderate levels of indexation become optimal under PT. In the case of a low credibility PT regime there is quite a large impact: optimal indexation rises to 87%, since the belief that policy may revert to IT next period raises the inflation risk premium on government debt substantially. Robustness tests do not overturn these results, but it is important to note that relaxing the assumption that indexation is lagged makes full indexation of government debt optimal under both IT and PT.

An additional finding is that welfare comparisons of IT and PT can be sensitive to whether indexation of government debt is optimised or not. Therefore, changes in bond portfolios may have important welfare implications for comparing IT and PT regimes. Policymakers and researchers should bear this in mind when assessing the case for a change in regime from IT to PT. More generally, the results in this paper suggest that it may be important to account for changes in the structure of nominal portfolios when comparing IT and PT.

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Appendix

Appendix A – The inflation risk premium

This Appendix shows that the difference in expected real returns on bonds can be interpreted as an inflation risk premium, and that this premium depends on consumption risk, return risk and the correlation between real returns and the stochastic discount factor.

The first-order conditions for bond imply that

$$1 = E_t[sdf_{t+1}]E_t[r^i_{t+1}] + \text{cov}_t[sdf_{t+1}, r^i_{t+1}] \tag{A1}$$

$$1 = E_t[sdf_{t+1}]E_t[r^n_{t+1}] + \text{cov}_t[sdf_{t+1}, r^n_{t+1}] \tag{A2}$$

Therefore,

$$\begin{aligned} E_t[r^n_{t+1}] - E_t[r^i_{t+1}] &= -\frac{\text{cov}_t[sdf_{t+1},\ r^n_{t+1} - r^i_{t+1}]}{E_t[sdf_{t+1}]} \\ &= \frac{-\left(\text{cov}_t\left[sdf_{t+1},\ \dfrac{R_t}{1+\pi_{t+1}} \right] - \text{cov}_t\left[sdf_{t+1},\ r_t \dfrac{1+\pi^{ind}_{t+1}}{1+\pi_{t+1}} \right] \right)}{E_t[sdf_{t+1}]} \end{aligned} \tag{A3}$$

where $r^n_{t+1} = R_t/(1+\pi_{t+1})$ and $r^i_{t+1} = r_t(1+\pi^{ind}_{t+1})/(1+\pi_{t+1})$ have been used.

The expression in Equation (A3) can be interpreted as an inflation risk premium because, in general, it is non-zero unless (i) consumers are risk-neutral, or (ii) inflation risk is zero. The numerical analysis in the paper focuses on the unconditional expectation of Equation (A3).

Using the same steps as above, but taking the unconditional expectation of (A1) and (A2) and dropping time subscripts for ease of interpretation, we have:

$$E[r^n] - E[r^i] = -\frac{(\ \text{cov}[sdf, r^n] - \text{cov}[sdf, r^i]\ )}{E[sdf]} \tag{A4}$$

This equation shows that the unconditional inflation risk premium will be zero if and only if bonds offer the same degree of social insurance against inflation risk.

Using the definition of a covariance, the right-hand side of Equation (A4) can be written as:

$$E[r^n] - E[r^i] = -(\ \text{corr}[sdf, r^n]\text{std}[r^n] - \text{corr}[sdf, r^i]\text{std}[r^i]\ )\frac{\text{std}[sdf]}{E[sdf]} \tag{A5}$$

where std[x] denotes the unconditional standard deviation of variable x and corr[x, y] is the correlation between variables x and y.

Equation (A5) shows that the inflation risk premium depends on consumption risk (through the stochastic discount factor standard deviation), real return risk on nominal and indexed bonds, and the correlations between real bond returns and the stochastic discount factor.

Appendix B: Deterministic steady-state and steady-state ratios

Deterministic steady-state

$$c_Y = (1-\tau)(1-\alpha)k^{\alpha} - b - k - m \tag{B1}$$

$$c_O = (1-\tau^k)\alpha k^{\alpha} + rb + \frac{m}{1+\pi} \tag{B2}$$

$$m = \delta \tag{B3}$$

$$\pi = \pi^{ind} = (1+\pi^*)^{20} - 1 \tag{B4}$$

$$r = 1/\beta \tag{B5}$$

$$r^n = r^i = r \tag{B6}$$

$$R = (1+\pi)r \tag{B7}$$

$$A = 1 \tag{B8}$$

$$r^k = \alpha k^{\alpha-1} = r/(1-\tau^k) \tag{B9}$$

$$i = k = (\alpha\beta)^{\frac{1}{1-\alpha}}(1-\tau^k)^{\frac{1}{1-\alpha}} \tag{B10}$$

$$y = k^{\alpha} = (\alpha\beta)^{\frac{\alpha}{1-\alpha}}(1-\tau^k)^{\frac{\alpha}{1-\alpha}} \tag{B11}$$

$$g = [(1-\alpha)\tau + \tau^k\alpha]k^{\alpha} + (1-r)b + \frac{\pi}{1+\pi}m \tag{B12}$$

$$b = \frac{\beta}{1+\beta}[(1-\tau)(1-\alpha) - \alpha(1-\tau^k)]k^{\alpha} - \frac{\beta}{1+\beta}k - \frac{\beta}{1+\beta}\left(\frac{2+\pi}{1+\pi}\right)m \tag{B13}$$

$$b^i = vb = b - b^n \qquad \text{(implying that } b^n = (1-v)b\text{)} \tag{B14}$$

Deterministic steady-state ratios

$$\frac{m}{y} = \frac{\delta}{(\alpha\beta)^{\frac{\alpha}{1-\alpha}}(1-\tau^k)^{\frac{\alpha}{1-\alpha}}} \tag{B15}$$

$$\frac{i}{y} = \frac{k}{y} = \alpha\beta(1-\tau^k) \tag{B16}$$

$$\frac{g}{y} = [(1-\alpha)\tau + \tau^k\alpha] + (1-r)\frac{b}{y} + \frac{\pi}{(1+\pi)}\frac{m}{y} \tag{B17}$$

$$\frac{c}{y} = \frac{c_Y + c_O}{y} = [(1-\tau)(1-\alpha) + \alpha(1-\tau^k)] + (r-1)\frac{b}{y} - \frac{\pi}{(1+\pi)}\frac{m}{y} - \frac{k}{y} \tag{B18}$$

$$\frac{b}{y} = \frac{\beta}{1+\beta}[(1-\tau)(1-\alpha) - \alpha(1-\tau^k)] - \frac{\beta}{1+\beta}\frac{k}{y} - \frac{\beta}{1+\beta}\left(\frac{2+\pi}{1+\pi}\right)\frac{m}{y} \tag{B19}$$

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Supplementary/Online Appendix
(This Appendix is for publication online only)

Section 1 – Derivations and proofs

1.A – Derivation of first-order conditions (baseline model)

Consumers solve a maximisation problem of the form

$$\max_{\{c_{t,Y},\, z_t,\, c_{t+1,O}\}} U_t = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \tag{A1}$$

subject to

$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t$ (Budget constraint of young)

$c_{t+1,O} = (1-\tau^k)r_{t+1}^k k_{t+1} + r_{t+1}^i b_{t+1}^i + r_{t+1}^n b_{t+1}^n + r_{t+1}^m m_t$ (Budget constraint of old)

$m_t = \delta$ (Cash constraint)

where $z_t \equiv (k_{t+1}, b_{t+1}^i, b_{t+1}^n, m_t)$ is the vector of assets chosen by households.

The Lagrangian for this problem is as follows:

$$L_t = E_t\left\{\begin{array}{l} U_t + \lambda_{t,Y}[\,(1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t - c_{t,Y}\,] \\ + \lambda_{t+1,O}[(1-\tau^k)r_{t+1}^k k_{t+1} + r_{t+1}^i b_{t+1}^i + r_{t+1}^n b_{t+1}^n + r_{t+1}^m m_t - c_{t+1,O}\,] + \mu_t(m_t - \delta)\end{array}\right\} \tag{A2}$$

First-order conditions are as follows:

$$c_{t,Y}:\ \frac{\partial U_t}{\partial c_{t,Y}} = \lambda_{t,Y}, \qquad c_{t+1,O}:\ \frac{\partial U_t}{\partial c_{t+1,O}} = \lambda_{t+1,O}, \qquad k_{t+1}:\ \lambda_{t,Y} = E_t[\lambda_{t+1,O}(1-\tau^k)r_{t+1}^k]$$

$$b_{t+1}^i:\ \lambda_{t,Y} = E_t[\lambda_{t+1,O} r_{t+1}^i], \qquad b_{t+1}^n:\ \lambda_{t,Y} = E_t[\lambda_{t+1,O} r_{t+1}^n], \qquad m_t:\ \lambda_{t,Y} = E_t[\lambda_{t+1,O} r_{t+1}^m] + \mu_t$$

By substitution, this system can be reduced to four Euler equations:

$$\frac{\partial U_t}{\partial c_{t,Y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,O}}(1-\tau^k)r_{t+1}^k\right], \qquad \frac{\partial U_t}{\partial c_{t,Y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,O}} r_{t+1}^n\right], \qquad \frac{\partial U_t}{c_{t,Y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,O}} r_{t+1}^i\right],$$

$$\frac{\partial U_t}{\partial c_{t,Y}} = E_t\left[\frac{\partial U_t}{\partial c_{t+1,O}} r_{t+1}^m\right] + \mu_t$$

The partial derivatives of the utility function are as follows:

$$\frac{\partial U_t}{\partial c_{t,Y}} = \left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} c_{t,Y}^{-(1-\varepsilon)} \tag{A3}$$

$$\frac{\partial U_t}{\partial c_{t+1,O}} = \left[c_{t,Y}^{\varepsilon} + \beta \left[E_t c_{t+1,O}^{1-\gamma} \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} \beta \left[E_t (c_{t+1,O}^{1-\gamma}) \right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma} \tag{A4}$$

Dividing (A4) by (A3) gives

$$\frac{\partial U_t / \partial c_{t+1,O}}{\partial U_t / \partial c_{t,Y}} = \frac{\beta \left[E_t (c_{t+1,O}^{1-\gamma}) \right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma}}{c_{t,Y}^{-(1-\varepsilon)}} = \beta \left(\frac{c_{t,Y}}{c_{t+1,O}} \right)^{1-\varepsilon} \left(\frac{c_{t+1,O}}{(E_t c_{t+1,O}^{1-\gamma})^{1/(1-\gamma)}} \right)^{1-\gamma-\varepsilon} \tag{A5}$$

Defining $sdf_{t+1} \equiv \dfrac{\partial U_t / \partial c_{t+1,O}}{\partial U_t / \partial c_{t,Y}}$, the four Euler equations above can be written as in the main text:

$$1 = E_t[sdf_{t+1}(1-\tau^k) r_{t+1}^k] \tag{A6}$$

$$1 = E_t[sdf_{t+1} r_{t+1}^n] \tag{A7}$$

$$1 = E_t[sdf_{t+1} r_{t+1}^i] \tag{A8}$$

$$1 = E_t[sdf_{t+1} r_{t+1}^m] + \tilde{\mu}_t \tag{A9}$$

where $\tilde{\mu}_t \equiv \mu_t / \lambda_{t,Y}$.

1.B – The binding legal constraint on money holdings

Proposition: The constraint binds with strict equality when $R_t > 1$

Proof.

By equations (A7) and (A9), the Lagrange multiplier on the cash constraint is given by

$$\mu_t = \lambda_{t,Y} E_t[sdf_{t+1}(r_{t+1}^n - r_{t+1}^m)] \tag{B1}$$

Since the real return on nominal bonds is $r_{t+1}^n = R_t /(1+\pi_{t+1}) = R_t r_{t+1}^m$, we can say that

$$\mu_t = \lambda_{t,Y} E_t[sdf_{t+1}(R_t - 1) r_{t+1}^m)] = \lambda_{t,Y} (R_t - 1) E_t[sdf_{t+1} r_{t+1}^m] \tag{B2}$$

since R_t is known at the end of period *t*.

The Kuhn-Tucker conditions associated with μ_t are as follows:

$$\{ \mu_t \geq 0 \quad \text{and} \quad \mu_t (m_t - \delta) = 0 \} \tag{B3}$$

The second condition in (B3) is the complementary slackness condition. It implies that the cash constraint will be strictly binding iff $\mu_t > 0$ for all *t*.

Dividing (B2) by $1 = E_t[sdf_{t+1} r_{t+1}^n] = R_t E_t[sdf_{t+1} r_{t+1}^m]$, it follows that $\mu_t = \lambda_{t,Y} (R_t - 1) / R_t$.

Since $\lambda_{t,Y} > 0$ (as the budget constraint of the young will always hold with equality), it follows that $\mu_t > 0$ iff $R_t > 1$ for all *t*. **Q.E.D.**

1.C – Approximate analytical expressions for long run inflation risk under IT and PT

This appendix derives approximate expressions for the inflation variance under IT and PT.

Inflation Targeting (IT)

Under IT, inflation is period t is given by

$$1+\pi_t = (1+\pi^*)^{20}\prod_{j=1}^{20}(1+\varepsilon_{j,t}) \tag{C1}$$

where $\varepsilon_{j,t}$ are IID-normal innovations with mean zero and variance σ^2.

A general non-linear function $g(\varepsilon)$, where ε is a vector of variables, can be approximated by var(g(ε)) ≈ $\sum[g_j'(\mu)]^2\text{var}(\varepsilon_j)$ using the 'Delta method'. Here, μ is the unconditional mean of the vector ε, and g_j' is the first derivative of g(ε) with respect to variable ε_j.

The inflation variance under IT can therefore be approximated as follows:

$$\text{var}(\pi_t) \approx \sum_{j=1}^{20}(1+\pi^*)^{40}\sigma^2 = (1+\pi^*)^{40}20\sigma^2 \tag{C2}$$

Price-level targeting (PT)

Under PT, inflation in period t is given by

$$1+\pi_t = (1+\pi^*)^{20}\frac{(1+\varepsilon_{30,t})}{(1+\varepsilon_{30,t-1})} \tag{C3}$$

where $\varepsilon_{30,t}$ and $\varepsilon_{30,t-1}$ are IID-normal innovations with mean zero and variance σ^2.

Using the same approximation method as above, the inflation variance under PT is given by

$$\text{var}(\pi_t) \approx [(1+\pi^*)^{20}]^2\sigma^2 + [-(1+\pi^*)^{20}]^2\sigma^2 = (1+\pi^*)^{40}2\sigma^2 \tag{C4}$$

Hence the unconditional variance of inflation under IT is (approx.) 10 times that under PT.

1.D – First-order conditions under imperfect credibility

In this case, consumers solve the following problem where $s = \{IT, PT\}$:

$$\max_{\{c_{t,Y},\,z_t,\,c_{t+1,O}\}} U_t^{IC} = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\Big[p_{IT}E[c_{t+1,O(IT)}^{1-\gamma}\,|\,\Omega_t] + (1-p_{IT})E[c_{t+1,O(PT)}^{1-\gamma}\,|\,\Omega_t]\Big]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \tag{D1}$$

subject to

$$c_{t,Y} = (1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t \quad \text{(Budget constraint of young)}$$
$$c_{t+1,O(IT)} = (1-\tau_{IT}^k)r_{t+1}^k k_{t+1} + r_{t+1(IT)}^i b_{t+1}^i + r_{t+1(IT)}^n b_{t+1}^n + r_{t+1(IT)}^m m_t \quad \text{(Budget constraint of old with IT)}$$
$$c_{t+1,O(PT)} = (1-\tau_{PT}^k)r_{t+1}^k k_{t+1} + r_{t+1(PT)}^i b_{t+1}^i + r_{t+1(PT)}^n b_{t+1}^n + r_{t+1(PT)}^m m_t \quad \text{(Budget constraint of old with PT)}$$
$$m_t = \delta \quad \text{(Cash constraint)}$$

where $E[X_{t+1(s)} \mid \Omega_t]$ is the expectation of X_{t+1} under policy *s*, conditional upon period-*t* information, Ω_t.

The Lagrangian for this problem is as follows:

$$L_t = E\left\{\left(\begin{array}{l} U_t^{IC} + \lambda_{t,Y}((1-\tau)w_t - k_{t+1} - b_{t+1}^i - b_{t+1}^n - m_t - c_{t,Y}) + \mu_t(m_t - \delta) \\ + \lambda_{t+1,O(IT)}((1-\tau_{IT}^k)r_{t+1}^k k_{t+1} + r_{t+1(IT)}^i b_{t+1}^i + r_{t+1(IT)}^n b_{t+1}^n + r_{t+1(IT)}^m m_t - c_{t+1,O(IT)}) \\ + \lambda_{t+1,O(PT)}((1-\tau_{PT}^k)r_{t+1}^k k_{t+1} + r_{t+1(PT)}^i b_{t+1}^i + r_{t+1(PT)}^n b_{t+1}^n + r_{t+1(PT)}^m m_t - c_{t+1,O(PT)}) \end{array}\right)\middle|\Omega_t\right\} \quad \text{(D2)}$$

First-order conditions are as follows:

$$c_{t,Y}: \frac{\partial U_t^{IC}}{\partial c_{t,Y}} = \lambda_{t,Y}, \quad c_{t+1,O(IT)}: \frac{\partial U_t^{IC}}{\partial c_{t+1,O(IT)}} = \lambda_{t+1,O(IT)}, \quad c_{t+1,O(PT)}: \frac{\partial U_t^{IC}}{\partial c_{t+1,O(PT)}} = \lambda_{t+1,O(PT)}$$

$$k_{t+1}: \lambda_{t,Y} = E\left([\lambda_{t+1,O(IT)}(1-\tau_{IT}^k)r_{t+1}^k + \lambda_{t+1,O(PT)}(1-\tau_{PT}^k)r_{t+1}^k] \middle| \Omega_t\right)$$

$$b_{t+1}^i: \lambda_{t,Y} = E\left([\lambda_{t+1,O(IT)}r_{t+1(IT)}^i + \lambda_{t+1,O(PT)}r_{t+1(PT)}^i] \middle| \Omega_t\right)$$

$$b_{t+1}^n: \lambda_{t,Y} = E\left([\lambda_{t+1,O(IT)}r_{t+1(IT)}^n + \lambda_{t+1,O(PT)}r_{t+1(PT)}^n] \middle| \Omega_t\right)$$

$$m_t: \lambda_{t,Y} = E\left([\lambda_{t+1,O(IT)}r_{t+1(IT)}^m + \lambda_{t+1,O(PT)}r_{t+1(PT)}^m] \middle| \Omega_t\right) + \mu_t$$

By substitution, this system can be reduced to four Euler equations:

$$\frac{\partial U_t^{IC}}{\partial c_{t,Y}} = E\left[\left(\frac{\partial U_t^{IC}}{\partial c_{t+1,O(IT)}}(1-\tau_{IT}^k)r_{t+1}^k + \frac{\partial U_t^{IC}}{\partial c_{t+1,O(PT)}}(1-\tau_{PT}^k)r_{t+1}^k\right)\middle|\Omega_t\right] \quad \text{(D3)}$$

$$\frac{\partial U_t^{IC}}{\partial c_{t,Y}} = E\left[\left(\frac{\partial U_t^{IC}}{\partial c_{t+1,O(IT)}}r_{t+1(IT)}^n + \frac{\partial U_t^{IC}}{\partial c_{t+1,O(PT)}}r_{t+1(PT)}^n\right)\middle|\Omega_t\right] \quad \text{(D4)}$$

$$\frac{\partial U_t^{IC}}{c_{t,Y}} = E\left[\left(\frac{\partial U_t^{IC}}{\partial c_{t+1,O(IT)}}r_{t+1(IT)}^i + \frac{\partial U_t^{IC}}{\partial c_{t+1,O(PT)}}r_{t+1(PT)}^i\right)\middle|\Omega_t\right] \quad \text{(D5)}$$

$$\frac{\partial U_t^{IC}}{\partial c_{t,Y}} = E\left[\left(\frac{\partial U_t^{IC}}{\partial c_{t+1,O(IT)}}r_{t+1(IT)}^m + \frac{\partial U_t^{IC}}{\partial c_{t+1,O(PT)}}r_{t+1(PT)}^m\right)\middle|\Omega\right] + \mu_t \quad \text{(D6)}$$

The partial derivatives of the utility function are as follows:

$$\frac{\partial U_t^{IC}}{\partial c_{t,Y}} = \Phi c_{t,Y}^{-(1-\varepsilon)} \quad \text{(D7)}$$

$$\frac{\partial U_t^{IC}}{\partial c_{t+1,O(s)}} = \beta \Phi p_s \left(p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t] \right)^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O(s)}^{-\gamma} \tag{D8}$$

where $\Phi \equiv \left[c_{t,Y}^{\varepsilon} + \beta \left[p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t] \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}}$ and $p_{PT} \equiv 1 - p_{IT}$.

Dividing (D8) by (D7) gives

$$\frac{\partial U_t^{IC} \big/ \partial c_{t+1,O(s)}}{\partial U_t^{IC} \big/ \partial c_{t,Y}} = \beta p_s \left(\frac{c_{t,Y}}{c_{t+1,O(s)}} \right)^{1-\varepsilon} \left[\frac{c_{t+1,O(s)}}{\left(p_{IT} E[c_{t+1,O(IT)}^{1-\gamma} \mid \Omega_t] + (1-p_{IT}) E[c_{t+1,O(PT)}^{1-\gamma} \mid \Omega_t] \right)^{1/(1-\gamma)}} \right]^{1-\gamma-\varepsilon} \tag{D9}$$

Defining $sdf_{t+1(s)} \equiv \frac{1}{p_s} \frac{\partial U_t^{IC} \big/ \partial c_{t+1,O(s)}}{\partial U_t^{IC} \big/ \partial c_{t,Y}}$, the four Euler equations above can be written as

$$1 = R_t \left(p_{IT} E[SDF_{t+1(IT)} \mid \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} \mid \Omega_t] \right) \tag{D10}$$

$$1 = r_t \left(p_{IT} E[SDF_{t+1(IT)} (1+\pi_{t+1(IT)}^{ind}) \middle| \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} (1+\pi_{t+1(PT)}^{ind}) \middle| \Omega_t] \right) \tag{D11}$$

$$1 = \alpha\, k_{t+1}^{\alpha-1} \left([\, p_{IT} (1-\tau_{IT}^{k}) E[sdf_{t+1(IT)} A_{t+1} \mid \Omega_t] + (1-p_{IT})(1-\tau_{PT}^{k}) E[sdf_{t+1(PT)} A_{t+1} \mid \Omega_t] \right) \tag{D12}$$

$$1 = \left(p_{IT} E[SDF_{t+1(IT)} \mid \Omega_t] + (1-p_{IT}) E[SDF_{t+1(PT)} \mid \Omega_t] \right) + \tilde{\mu}_t \tag{D13}$$

where $SDF_{t+1(s)} \equiv sdf_{t+1(s)}/(1+\pi_{t+1(s)})$ and $\tilde{\mu}_t \equiv \mu_t \big/ \lambda_{t,Y}$.

End of analytical results; please turn over for the numerical results.

Section 2 – Numerical results

2.A – Results for the baseline model without money

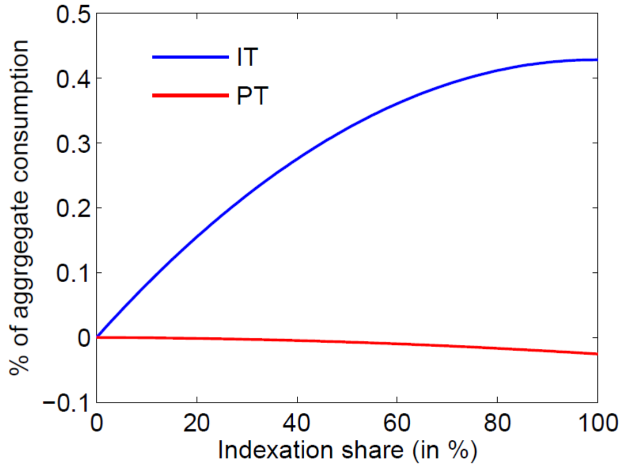

Fig A1 – Social welfare gain from indexation (when money is absent from the model). Figure reports the welfare gain or loss of indexation at x%, as compared to the case of zero indexation. Units: % of aggregate consumption under each regime. All other parameters have the same values as in the baseline model. Optimal indexation shares are equal to 0% under PT and 100% under IT.

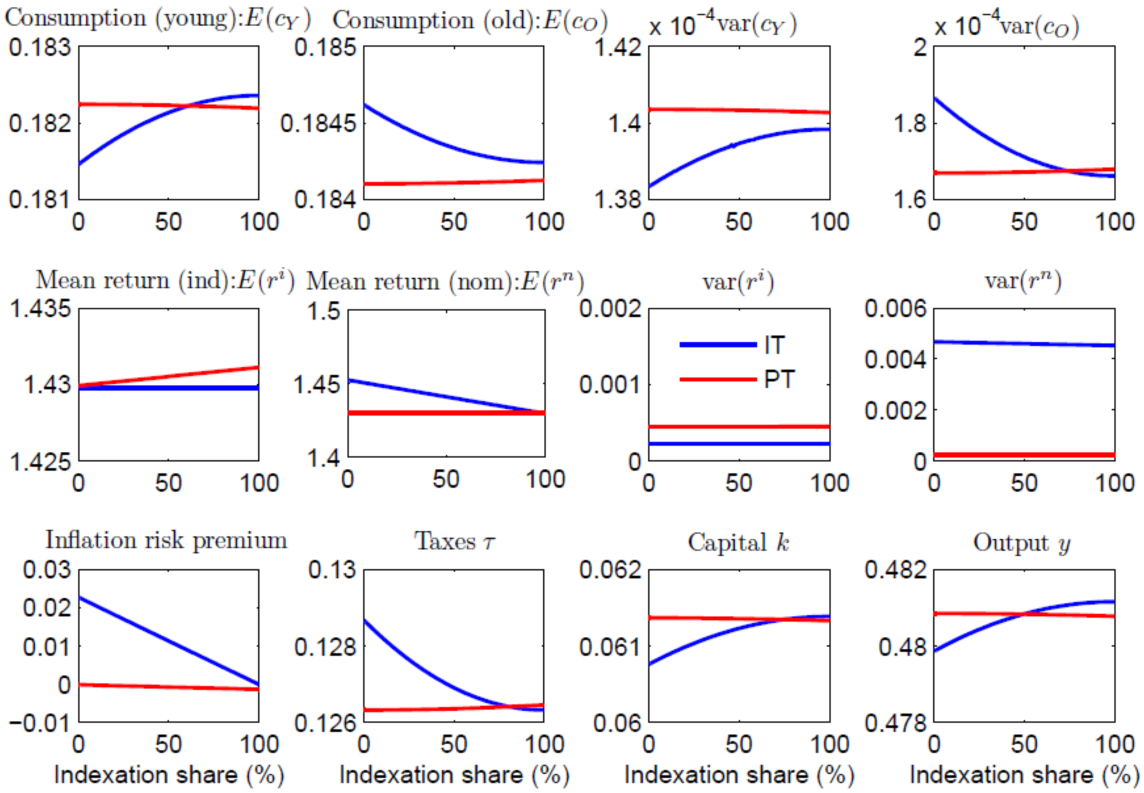

Fig A2 – Indexation and real variables under IT and PT (when money is absent from the model). The inflation risk premium is the difference between the expected real returns on nominal and indexed bonds (i.e. $E(r^n) - E(r^i)$). Returns are non-annualised gross returns and have *not* been converted into percent. All series are unconditional moments. All other parameters have the same values as in the baseline model.

2.B – Results for the baseline model without productivity risk

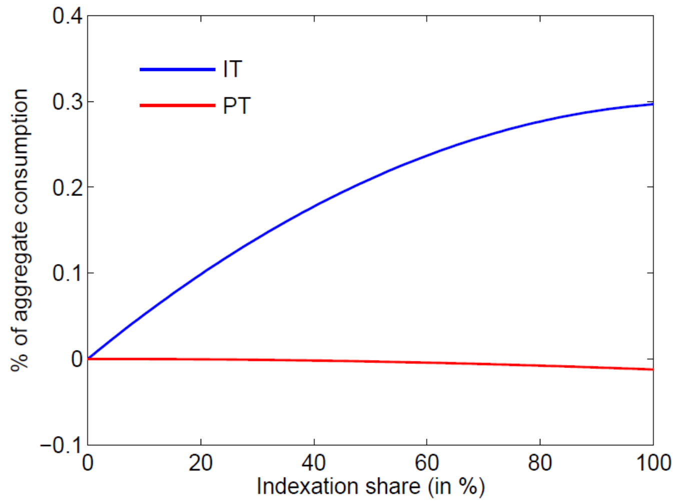

Fig B1 – Social welfare gain from indexation (excluding productivity risk from the model). Figure reports the welfare gain or loss of indexation at x%, as compared to the case of zero indexation. Units: % of aggregate consumption under each regime. All other parameters have the same values as in the baseline model. Optimal indexation shares are equal to 4% under PT and 100% under IT.

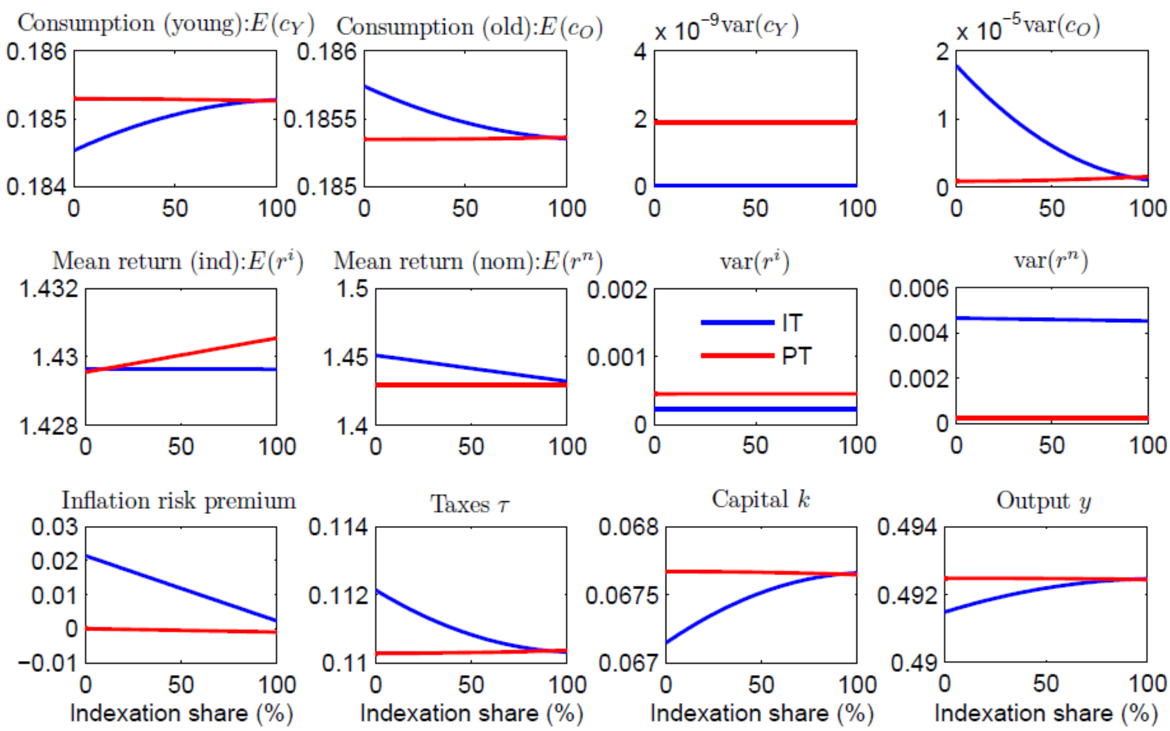

Fig B2 – Indexation and real variables under IT and PT (excluding productivity risk from the model). The inflation risk premium is the difference between the expected real returns on nominal and indexed bonds (i.e. $E(r^n) - E(r^i)$). Returns are non-annualised gross returns and have *not* been converted into percent. All series are unconditional moments. All other parameters have the same values as in the baseline model.