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A FRACTURE-MECHANICS MODEL FOR DEBONDING OF EXTERNAL FIBRE REINFORCED POLYMER PLATES ON REINFORCED CONCRETE BEAMS

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ABSTRACT: A Fracture-mechanics model for debonding of external fibre reinforced polymer plates on reinforced concrete beams is presented. The conventional methods of concrete-FRP interface analysis use finite element models, which require details of unknown and unknowable interface characteristics. The present model assumes flaws in the vicinity of the interface and assesses whether sufficient energy can be released to cause these flaws to propagate. Energy released by an extension of an existing flaw depends on the change of recoverable energy stored in the system. This paper concentrates on the moment-curvature model for a cracked reinforced-concrete beam under a prestress caused by the force in a FRP plate. The use of the proposed model to determine the energy released from the system with the extension of an existing flaw is also presented. The energy required to create the associated new surfaces depends on interface fracture energy which is first reviewed and methods to determine is also discussed.

KEYWORDS: concrete beams, flexural-strengthening, fracture-energy, interface-debonding, moment-curvature, strain energy

1. INTRODUCTION

Reinforced Concrete (RC) beams flexurally strengthened with externally bonded Fibre Reinforced Polymer (FRP) plates often fail by plate debonding, but a rational analytical approach to these failures has yet to be developed. Due to the typical premature and brittle nature of these debonding failures, inadequately designed strengthening applications may become ineffective and reduce the level of safety. Therefore, proper understanding of the concrete-FRP interface debonding is required for safe and reliable application of externally bonded FRP systems for strengthening of RC elements.

Two modes of debonding failures of external FRP plates on RC beams have been observed by the researchers (Figure 1) [1]. The first initiates somewhere in the middle of the beam at an intermediate crack and propagates outwards. Existing models to analyse this mode of debonding are generally based on the results from shear lap tests of FRP plates on small-scale concrete specimens [2], but it is uncertain whether these shear lap tests actually simulate the debonding mechanism of real beams. The other mode of debonding initiates at the plate ends and propagates inwards. Interfacial stress-based models for plate end debonding can be found in the literature but these models are based on linear-elastic theories. Significant cracking is expected at the onset of debonding and hence the applicability of these models is questionable. Finite Element (FE) models have been used for concrete-FRP interface analysis [1] but they require details of interface characteristics unknown and unknowable,

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such as the shape of the adhesive layer, especially at the layer ends; interface texture; shape, size and location of voids in the adhesive layer.

In addition to these strength-based models for debonding of external FRP plates on concrete, some empirical models are also available in the literature. ACI Committee 440 [3] suggests a limit on the FRP plate strain to prevent debonding, based on a database of test results, but the theoretical justification is unclear: these strain limits depend only on the stiffness properties of the FRP plates and do not take account of the concrete-FRP interface properties, width of the FRP plate or the plate curtailment location, all of which should be expected to affect debonding.

Fracture-mechanics models assume that, since flaws are inevitable in the interface, what matters is whether these flaws can propagate. The energy needed to form new surfaces depends on the interface fracture energy and must be compared with the energy released by the system when the flaw extends, which in turn depends on the change of strain energy stored in the system. This paper is primarily concerned with the determination of that strain energy.

It is necessary to determine the strain energy stored in a reinforced concrete (RC) beam, to which is attached a FRP plate that may be debonded over part of its length; that is not a trivial problem. It is well-known that when a RC beam cracks its stiffness does not immediately change to that of a section where the tension concrete can be fully disregarded. Various empirical models, such as Branson's I_{eff} concept [4], have been used to model this behaviour, primarily with a view to being able to predict the deflections of RC beams to check their compliance with code limits. Such models normally work in moment-curvature space (M-κ) rather than stress-strain space (σ-ε) and thus do not attempt to model individual cracks. Two versions are normally presented; one designed to model the overall beam stiffness and the other to determine the local curvature.

There are two complications that have been addressed in the present work. If the FRP plate is bonded to the concrete it behaves as reinforcement, but the plate will not be expected to affect cracking in the same way as internal reinforcement. If unbonded, the FRP plate will act as a prestressing element, inducing both force and moment in the beam (Figure 2). Most existing models do not cope with either of these effects, but if the model is to determine the strain energy in the system it will have to handle both. The model is explained below and its accuracy verified by comparisons with experimental data available in the literature.

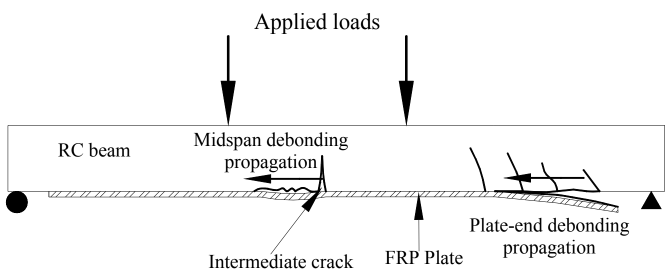

Figure 1. Debonding modes of external FRP plates

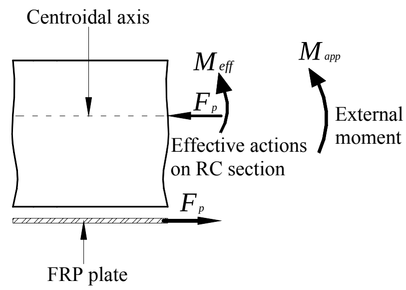

Figure 2. Actions on RC section

2. OVERVIEW OF THE PROPOSED FRACTURE-MECHANICS MODEL

In the proposed model it is assumed that an existing flaw extends by small amount; the energy needed to form the new surfaces (E_F) is compared with the energy released from the system (E_R), which can be determined from energy conservation before and after the flaw extension. The energy needed to form new surfaces can be calculated when the interface fracture energy and the extension of the fracture surface are known. When a beam bends, energy is put into the structure by the loads, some of which is dissipated, whilst some is stored as strain energy, shown schematically in Figure 3. When the flaw extends the energy distribution changes and some of the strain energy can be released. Thus, the elastic strain energy stored in the system before and after the flaw extension is required to determine the corresponding energy released from the system. If $E_R \geq E_F$ the flaw will extend; if not there is insufficient energy for the flaw to propagate.

3. ELASTIC STRAIN ENERGY IN A FRP STRENGTHENED RC BEAM SECTION

The Elastic Strain Energy (ESE) stored in the system before and after the after the flaw extension can be determined if the moment and curvature in the RC beam are known, as well as the strain and force in the FRP plate. However, a rational analytical approach to determine ESE in a FRP plate strengthened RC beam section has yet to be developed. Branson's effective second moment of area (I_{eff}) expression (Equation (1)) indirectly accounts for tension stiffening effects of cracked concrete and successfully predicts deflections and moment-curvature relationships of RC beams [5]. The stiffness is interpolated between the corresponding full- (uncracked I_{un}) and zero- (fully-cracked I_{fc}) tension stiffening states. The interpolation coefficient represents the extent of cracking of the section. M_{cr} and M_{app} in Equation (1) are the moments causing first cracking and the externally applied moment on the section. I_{eff} is taken as I_{un} when $M_{app} \leq M_{cr}$. I_{eff} is the effective second moment of area of the equivalent transformed concrete section of modulus E_c so the effective curvature of the section (κ_{eff}) can be determined from Equation (2).

$$I_{eff} = \left(M_{cr}/M_{app}\right)^4 I_{un} + \left\{1-\left(M_{cr}/M_{app}\right)^4\right\} I_{fc} \quad \Rightarrow if \; M_{app} > M_{cr} \tag{1}$$

$$\kappa_{eff} = M_{app}/\left(E_c \; I_{eff}\right) \tag{2}$$

As described, this model determines the local curvature of the section. A modified form is used when an average I_{eff} of the beam is needed to determine deflection. In this case the exponent in Equation (1) is reduced from 4 to 3. This method has been widely verified for conventional RC beams and will now be extended to deal with the more complex problem of beams with external FRP plates.

4. PROPOSED MOMENT-CURVATURE MODEL FOR FRP STRENGTHENED RC BEAMS

Branson's I_{eff} expression addresses the behaviours of RC beams under nominal service moments where tension steel remains elastic and concrete shows virtually a linear-elastic behaviour in compression. At higher compressive strains concrete is nonlinear. FRP-strengthened RC beams sections are subjected to loads higher than those for which they were originally designed, but it is still possible to define equivalent elastic stiffness (EI_{eq}) of such a section (Equation (3)). κ and M in Equation (3) are the curvature and the effective moment on the RC section about the section's centroidal axis. EI_{eq} represents the equivalent elastic stiffness of the section. The proposed model (Equations (4) and (5)) follows Branson's I_{eff} expression but is based on this EI_{eq} of the sections.

$$EI_{eq} = M/\kappa \tag{3}$$

In normal RC beams the axial force is zero so the centroidal location does not matter, but because of the prestressing effect of the FRP plate the location of the centroid is important. I_{un} and I_{fc} in Equation (1) should be calculated relative to the centroidal axes of the uncracked and fully-cracked sections respectively and I_{eff} is related to the centroidal axis of the imaginary effective section [6]. Sakai and Kakuta [5] presented an empirical expression for the effective centroidal axis depth (α_{eff}) of a partially-cracked RC beam section subjected to an axial force as a transition between uncracked and fully-cracked sections. The applied moments should also be calculated about the relevant centroidal axes to define the corresponding equivalent stiffness from Equation (3).

When the amount of cracking of a RC section increases, the tension-stiffening eventually becomes ineffective. In Branson's original model the stiffness becomes asymptotic to the fully-cracked state. The present model (Equations (4) and (5)) assumes that the beam is fully-cracked at the moment causing first yielding of tension steel and uses a slightly modified form of the interpolation used in Equation (1).

$$EI_{eq-eff} = K\,EI_{eq-un} + (1 - K)\,EI_{eq-fc} \tag{4}$$

$$K = \left(M_{cr-c}/M_{app-c}\right)^m \left[1 - \left\{\left(M_{app-c} - M_{cr-c}\right)/\left(M_{y-c} - M_{cr-c}\right)\right\}^m\right] \Rightarrow \text{ if } M_{cr-c} < M_{app-c} < M_{y-c} \tag{5}$$

The stiffnesses in Equation (4) are calculated at the relevant centroid, but it is found desirable to use a fixed reference axis to calculate *K*, to avoid complications caused by moving the prestressing force. The moments in Equation (5) are calculated relative to the beam's centreline. The exponent m in Equation (5) is taken as 4, but for the equivalent expression used to calculate the movement of the centroid (*A* in Equation (6)) a value of 3.5 is found to give a better fit to the data. When the effective centroidal axis depth (α_{eff}) is known the effective moment on the section can be calculated and then the effective curvature of the section from Equation (3).

$$\alpha_{eff} = A\alpha_{un} + (1 - A)\,\alpha_{fc} \tag{6}$$

The complete derivation of this proposed moment-curvature model and comparison between the model predictions against experimental data available in the literature will be presented elsewhere. From the comparisons shown there (an example is given in Figure 4) it is possible to conclude that the proposed moment-curvature model is accurate and reliable and thus can be used to calculate strain energies in FRP strengthened RC beams.

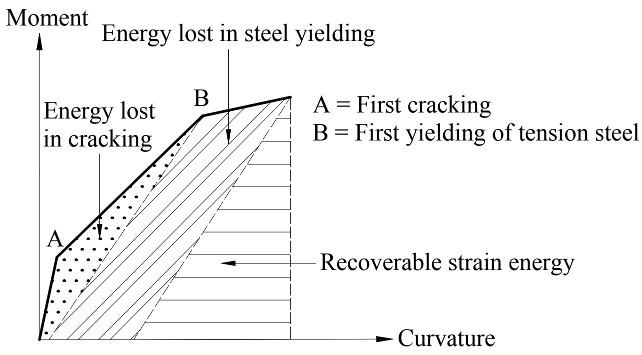

Figure 3. Energy in flexure

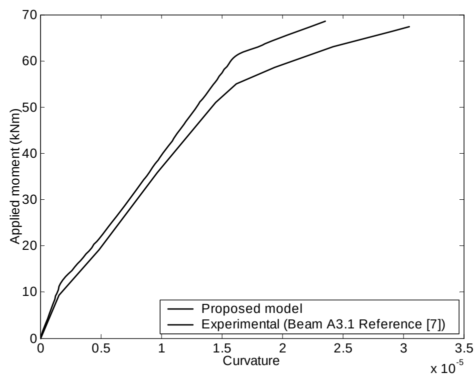

Figure 4. Moment-curvature comparison

5. ENERGY RELEASED FROM THE SYSTEM WITH THE EXTENSION OF FLAW

The Strain Energy (SE) in a FRP strengthened RC beam section consists of three components which are summed in the proposed model. 1) SE in the RC beam due to flexure; 2) SE in the RC beam due to axial force and 3) SE in the FRP plate. Energy released from all of these components with the extension of flaw should be considered; the beam is divided into short segments and the energy releases in each segment are calculated. Since the strain distributions of the segments outside the fractured zone cannot be altered by the extension of the flaw, unless the load changes, it is assumed that only the segments within the fractured zone contribute for the energy released from the system.

It is now possible to consider the energy state of the beam before and after the flaw extension. The elastic strain energy is given by the assumption that the beam would unload elastically from any loaded state, using the effective stiffnesses calculated as in Section 4. There would be some permanent deformation. By accounting for the energy that is dissipated in the concrete, either in flexural-tension cracking, or nonlinear elasticity, and by yielding of the tensions reinforcement, it is possible to determine the elastic strain energy that can be used to extend the fracture. Space does not permit a full description of that process.

Account should also be taken of the strain energy stored in the FRP plate, which can be assumed to remain linearly elastic, and also of the effect of the axial force induced in the RC beam which will have associated strain energy. The full model also accounts for beams which have been unloaded and reloaded.

6. CONCRETE-FRP INTERFACE FRACTURE ENERGY

The energy released by the system when the flaw extends is used to create new surfaces and the propagation path of the existing interface flaw takes the path that requires least energy. Experimental evidence confirms that debonding fractures of external FRP plates on RC beams generally propagate through the concrete just above the interface [8]. However, it is also reported that poor surface preparation prior to the FRP plate bonding or low-strength adhesives can lead failure in the interface or within the adhesive itself [9]. However, with the availability of high-strength adhesives and well-established surface preparation techniques, the proposed model assumes debonding fractures propagate through the concrete. Thus, the interface fracture energy (G_F) should be that of concrete.

The opening mode (Mode I) is the most common fracturing behaviour in concrete. It is reported that "*It would seem impossible to propagate cracks in concrete under pure in-plane shear mode (Mode II) conditions, because of frictional forces. Under mixed mode loading, it appears that concrete fractures locally take place in pure Mode I, although the directions of propagation depend on the mixity conditions*" [10] and in any event overall compatibility conditions should preclude other modes of crack opening. However, with complex stress conditions expected at the interface flaw tips there are consensus among researchers that the fracture of concrete in FRP strengthened RC beams propagates under mixed mode loading (combination of Modes I and II loadings) [8].

Elices et al. [11] have performed extensive FE modelling to predict the fracture paths and load versus crack-mouth-opening-displacement variations in notched concrete specimens under mixed mode loadings. Their numerical simulations were based on Mode I fracture properties of concrete and they reported excellent correlation with the experimental results, which agrees with RILEM Report 5 [12] that says that concrete fractures propagate under pure Mode I conditions even under mixed-mode loading. The present model therefore assumes that debonding fractures of external FRP plates on RC beams propagate as pure Mode I fractures of concrete and uses the Mode I fracture energy (G_{FC}^{I}).

Concrete fracture cannot be explained using Linear Elastic Fracture (LEF) theories due to the existence of a significant nonlinear fracture process zone ahead of a crack tip, which is neglected in the derivation of LEF theories. Therefore, Mode I fracture energy of concrete (G_{FC}^{I}) should be determined using available concrete fracture theories. RILEM Report 5 [12] recommends the use of a two-parameter, effective-crack-size model and Bažant's size effect law to determine G_{FC}^{I} using experimental results from central-notched three-point bending tests. Hillerborg's fictitious crack model can also be used to determine G_{FC}^{I} but this requires a concrete softening curve which is generally obtained by means of stable tensile tests on specimens. These tests are less accurate and less reliable than notch tests because the location of the crack is not known *a priori* and on most occasions multiple cracking occurs due to material heterogeneity [11]. The CEB-FIP Code [13] presents an empirical expression for G_{FC}^{I}, based on available experimental data, as a function of concrete compressive strength and the maximum aggregate size.

If this Mode I fracture energy of concrete (G_{FC}^{I}) is known it can be compared with the energy release rate (G_R) of the system. If $G_R \geq G_{FC}^{I}$ the flaw will extend and failure is to be expected.

7. CONCLUSIONS

The outline of a fracture-mechanics model for debonding of external fibre reinforced polymer plates on reinforced concrete beams has been presented. The paper has shown that a modified version of Branson's effective stiffness model can be used to determine the elastic strain energy stored in the beam. The energy released from the system with the extension of an existing interface-flaw can be calculated and compared with the energy required to create new fracture surfaces. This will decide whether the flaw will propagate. The energy required to create the associated new surfaces depends on the Mode I concrete fracture energy**,** which can be determined from standard concrete fracture tests.

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