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Generalised CP and A_4 family symmetry

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Abstract: We perform a comprehensive study of family symmetry models based on A_4 combined with the generalised CP symmetry $H_{\rm CP}$. We investigate the lepton mixing parameters which can be obtained from the original symmetry $A_4 \rtimes H_{\rm CP}$ breaking to different remnant symmetries in the neutrino and charged lepton sectors. We find that only one case is phenomenologically viable, namely $G^{\nu}_{\rm CP} \cong Z^S_2 \times H^{\nu}_{\rm CP}$ in the neutrino sector and $G^{l}_{\rm CP} \cong Z^T_3 \rtimes H^{l}_{\rm CP}$ in the charged lepton sector, leading to the prediction of no CP violation, namely δ_{CP} and the Majorana phases α_{21} and α_{31} are all equal to either zero or π. We then propose an effective supersymmetric model based on the symmetry $A_4 \rtimes H_{\rm CP}$ in which trimaximal lepton mixing is predicted together with either zero CP violation or $\delta_{CP} \simeq \pm\pi/2$ with non-trivial Majorana phases. An ultraviolet completion of the effective model yields a neutrino mass matrix which depends on only three real parameters. As a result of this, all three CP phases and the absolute neutrino mass scale are determined, the atmospheric mixing angle is maximal, and the Dirac CP can either be preserved with $\delta_{CP} = 0, \pi$ or maximally broken with $\delta_{CP} = \pm\pi/2$ and sharp predictions for the Majorana phases and neutrinoless double beta decay.

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Contents

1 Introduction

After the measurement of the reactor mixing angle θ_{13} by the Daya Bay [1, 2], RENO [3], and Double Chooz [4, 5] reactor neutrino experiments, all three lepton mixing angles θ_{12}, θ_{23}, θ_{13} and both mass-squared differences $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$ have been measured to reasonably good accuracy. Yet within the standard framework of three-neutrino oscillations, the Dirac CP phase and neutrino mass ordering still elude measurement so far. Furthermore, if neutrinos are Majorana particles, there exist two more unknown Majorana CP phases which may play a role in neutrinoless double-beta decay searches. Thus, determining the exact neutrino mass ordering and measuring the Dirac and Majorana CP violating phases are the primary goals of future neutrino oscillation experiments. The CP violation has been firmly established in the quark sector and it is natural to expect that CP violation occurs in the lepton sector as well. It is insightful to note that hints of a nonzero $\delta_{\rm CP}$ have begun to show up in global analysis of neutrino oscillation data [6–8].

What would we learn from the measurements of the lepton CP violating phases? What is the underlying physics? These questions are particularly imperative in view of foreseeable future experimental programs to measure the CP-violation in the neutrino oscillations sector. In the past years, much effort has been devoted to explaining the structure of the lepton mixing angles through the introduction of family symmetries. In this scheme, one generally assumes a non-abelian discrete flavour group which is broken to different subgroups in the neutrino and charged lepton sectors. The mismatch between these two subgroups leads to particular predictions for the lepton mixing angles. For recent reviews, see ref. [9, 10] and ref. [11, 12] for the model building and relevant group theory aspects, respectively. Motivated by this approach one can extend the family symmetry to include a generalised CP symmetry $H_{\rm CP}$ [13–16] which will allow the prediction of both CP phases and mixing angles.

The possibility of combining a family symmetry with a generalised CP symmetry has already been discussed in the literature. For example, the simple $\mu-\tau$ reflection symmetry, which is a combination of the canonical CP transformation and the $\mu-\tau$ exchange symmetry, has been discussed and successfully implemented in a number of models where both atmospheric mixing angle θ_{23} and Dirac CP phase $\delta_{\rm CP}$ were predicted to be maximal [17–23]. Additionally in ref. [24], the phenomenological consequences of imposing both an S_4 flavour symmetry and a generalised CP symmetry have been analysed in a model-independent way. They found that all lepton mixing angles and CP phases depend on one free parameter for the symmetry breaking of $S_4 \rtimes H_{\rm CP}$ to $Z_2 \times {\rm CP}$ in the neutrino sector and to some abelian subgroup of S_4 in the charged lepton sector. Concrete S_4 family models with a generalised CP symmetry have been constructed in refs. [25–27] where the spontaneous breaking of the $S_4 \rtimes H_{\rm CP}$ down to $Z_2 \times {\rm CP}$ in the neutrino sector was implemented. Other models with a family symmetry and a generalised CP symmetry can also be found in refs. [28–30]. In addition, there are other theoretical frameworks comprising both family symmetry and CP violation [31–41].

In this work, we study generalised CP symmetry in the context of the most popular family symmetry A_4[1] (please see ref. [46, 47] for a classification of the A_4 models on the market). The generalised CP transformation compatible with an A_4 family symmetry is clarified, and a model-independent analysis of the lepton mixing matrix is performed by scanning all of the possible remnant subgroups in the neutrino and charged lepton sectors. We construct an effective $A_4 \rtimes H_{\rm CP}$ model, where non-renormalisable operators are involved. The lepton mixing is predicted to be trimaximal pattern in the model, and the Dirac phase is trivial or nearly maximal. Furthermore, this effective model is promoted to a renormalisable one in which the higher order operators are under control.

[1] A_4 models with spontaneous CP violation are proposed in refs. [42–45], where a CP symmetry is assumed to exist at a high energy scale.

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The remainder of this paper is organised as follows. In section 2, we present the general CP transformations consistent with the A_4 family symmetry. In section 3, we perform a thorough scan of leptonic mixing parameters which can be obtained from the remnant symmetries of the underlying combined symmetry group $A_4 \rtimes H_{\rm CP}$. We find that only one case out of all possibilities is phenomenologically viable. This case predicts both Dirac and Majorana phases to be trivial. In section 4 we specify the structure of the model at leading order, and the required vacuum alignment is justified. In subsection 4.3, we analyse the subleading Next-to-Leading-Order (NLO) corrections induced by higher dimensional operators and phenomenological predictions of the model are presented. In section 5, we address the ultraviolet completion of the model which significantly increases the predictability of the theory such that all the mixing angles, CP phases and the absolute neutrino mass scale are fixed. We conclude in section 6. The details of the group theory of A_4 are collected in appendix A and appendices B–D contain the implications of preserving other subgroups of A_4 different than $G_\nu = Z_2$ and $G_l = Z_3$. Finally, appendix E describes the diagonalisation of a general 2×2 symmetric complex matrix.

2 Generalised CP transformations with family symmetry

2.1 General family symmetry group

In general, it is nontrivial to combine the family symmetry G_f and the generalised CP symmetry together because the definition of the generalised CP transformations must be compatible with the family symmetry. Thus, the generalised CP transformations are subject to certain consistency conditions [24, 48, 49]. Namely, for a set of fields φ in a generic irreducible representation $\mathbf{r}$ of G_f, it transforms under the action of G_f as

$$\varphi(x) \xrightarrow{G_f} \rho_{\mathbf{r}}(g)\varphi(x), \qquad g \in G_f\,, \tag{2.1}$$

where $\rho_{\mathbf{r}}(g)$ denotes the representation matrix for the element g in the irreducible representation $\mathbf{r}$, the generalised CP transformation is of the form

$$\varphi(x) \xrightarrow{CP} X_{\mathbf{r}}\,\varphi^*(x')\,, \tag{2.2}$$

where $x' = (t, -\mathbf{x})$ and the obvious action of CP on the spinor indices is omitted for the case of φ being spinor. Here we are considering the "minimal" theory in which the generalised CP transforms the field $\varphi \sim \mathbf{r}$ into its complex conjugate $\varphi^* \sim \mathbf{r}^*$, and the transformation into another field $\varphi'^* \sim \mathbf{r}'^*$ with $\mathbf{r}' \neq \mathbf{r}$ is beyond the present scope since both φ and φ' would be required to be present in pair and correlated with each other in that case. Notice that $X_{\mathbf{r}}$ should be a unitary matrix to keep the kinetic term invariant. Now if we first perform a CP transformation, then apply a family symmetry transformation, and finally an inverse CP transformation is followed, i.e.

$$\varphi(x) \xrightarrow{CP} X_{\mathbf{r}}\,\varphi^*(x') \xrightarrow{G_f} X_{\mathbf{r}}\rho^*_{\mathbf{r}}(g)\varphi^*(x') \xrightarrow{CP^{-1}} X_{\mathbf{r}}\rho^*_{\mathbf{r}}(g)X^{-1}_{\mathbf{r}}\varphi(x)\,, \tag{2.3}$$

the theory should still be invariant since it is invariant under each transformation individually. To make the theory consistent the resulting net transformation should be equivalent

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to a family symmetry transformation $\rho_{\mathbf{r}}(g')$ of some family group element g', i.e.

$$X_{\mathbf{r}}\rho^*_{\mathbf{r}}(g)X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(g'), \qquad g' \in G_f\,, \tag{2.4}$$

where the elements g and g' must be the same for all irreducible representations of G_f. Eq. (2.4) is the important *consistency condition* which has to be fulfilled in order to impose both generalised CP and family symmetry invariance simultaneously. It also implies that the generalised CP transformation $X_{\mathbf{r}}$ maps the group element g into g' and that the family group structure is preserved under this mapping. Therefore eq. (2.4) defines a homomorphism of the family symmetry group G_f. Notice that in the case where $\rho_{\mathbf{r}}$ is a faithful representation, the elements g and g' have the same order, the mapping defined in eq. (2.4) is bijective, and thus the associated CP transformation becomes an automorphism [49]. It is notable that both $e^{i\theta}X_{\mathbf{r}}$ and $\rho_{\mathbf{r}}(h)X_{\mathbf{r}}$ also satisfy the consistency equation of eq. (2.4) for a generalised CP transformation $X_{\mathbf{r}}$, where θ is real and h is any element of G_f. Therefore the possible form of the CP transformation $X_{\mathbf{r}}$ is only determined by the consistency equation up to an overall arbitrary phase and family symmetry transformation $\rho_{\mathbf{r}}(h)$ for a given irreducible representation $\mathbf{r}$. In the following, we investigate the generalised CP transformations consistent with an A_4 family symmetry for different irreducible representations, i.e. $G_f = A_4$.

2.2 A_4 family symmetry

The A_4 group can be generated by two generators S and T, which are of orders two and three, respectively (see appendix A for the details of the group theory of A_4). To include a generalised CP symmetry consistent with an A_4 family symmetry, it is sufficient to only impose the consistency condition in eq. (2.4) on the group generators:

$$X_{\mathbf{r}}\rho^*_{\mathbf{r}}(S)X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(S'), \qquad X_{\mathbf{r}}\rho^*_{\mathbf{r}}(T)X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(T')\,. \tag{2.5}$$

To do this, we start with the faithful triplet representation $\mathbf{3}$. Then the order of S' and T' will be 2 and 3, respectively. Therefore S' and T' can only belong to certain conjugacy classes of A_4. Namely,

$$S' \in 3C_2, \qquad T' \in 4C_3 \cup 4C_3^2 \tag{2.6}$$

It is remarkable that the consistency condition of eq. (2.4) must hold for all representations $\mathbf{r}$ simultaneously. However, because of the models constructed in later sections, we assume that our theory contains only one of the nontrivial singlet irreducible representations (either $\mathbf{1}'$ or $\mathbf{1}''$) in the flavon sector and further restrict ourselves to a minimal case where there exists only one flavon transforming under that nontrivial singlet irreducible representation (in addition to other flavons transforming under the $\mathbf{1}$ and $\mathbf{3}$ representations). However, in these models there does exist a $\mathbf{1}'$ and $\mathbf{1}''$ in the matter sector. Yet, additional symmetry forbids the interchanging of these fields under the generalised CP symmetry. Therefore we have chosen to define a generalised CP symmetry without the interchanging of fields transforming under conjugate representations, e.g. fields transforming under $\mathbf{1}'$ and $\mathbf{1}''$ representations. Then, the element T' can further be constrained by these nontrivial singlet

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representations $\mathbf{1}'$ and $\mathbf{1}''$, where the corresponding generalised CP transformations $X_{\mathbf{1}',\mathbf{1}''}$ are numbers with absolute value equal to 1, and then we have

$$\rho_{\mathbf{1}',\mathbf{1}''}(T') = X_{\mathbf{1}',\mathbf{1}''}\rho^*_{\mathbf{1}',\mathbf{1}''}(T)X^{-1}_{\mathbf{1}',\mathbf{1}''} = \rho^*_{\mathbf{1}',\mathbf{1}''}(T) = \omega^{\mp 2} \tag{2.7}$$

Consequently, the element T' can only be in the conjugacy class $4C_3^2$. In summary, the consistency equation applied to our "minimal" case restricts S' and T' to

$$S' \in 3C_2, \qquad T' \in 4C_3^2. \tag{2.8}$$

For the simple case of $S' = S$ and $T' = T^2$ in the $\mathbf{3}$-dimensional representation, the associated CP transformation satisfying eq. (2.4) can be found straightforwardly:

$$X_{\mathbf{0}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv \mathbb{1}_3\,, \tag{2.9}$$

which is the canonical CP transformation. The remaining eleven possible choices for S' and T' lead to different solutions for $X_{\mathbf{3}}$. These solutions are listed in table 1 and can be neatly summarised in a compact way:

$$X_{\mathbf{3}} = \rho_{\mathbf{3}}(g), \quad g \in A_4\,. \tag{2.10}$$

For the singlet representations $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$, we take

$$X_{\mathbf{1},\mathbf{1}',\mathbf{1}''} = \rho_{\mathbf{1},\mathbf{1}',\mathbf{1}''}(g), \quad g \in A_4\,. \tag{2.11}$$

Therefore the generalised CP transformation consistent with an A_4 family symmetry is of the same form as the family group transformation, i.e.

$$X_{\mathbf{r}} = \rho_{\mathbf{r}}(g), \quad g \in A_4\,. \tag{2.12}$$

Now that we have found all generalised CP transformations consistent with the A_4 family symmetry,[2] we proceed by investigating their implications on lepton masses and mixings.

[2]Had we allowed the flavons to transform under all nontrivial A_4 irreducible representations (call them e.g. $\phi_{\mathbf{1}'}$, $\phi_{\mathbf{1}''}$ and $\phi_{\mathbf{3}}$) then the transformation

$$\phi_{\mathbf{1}'} \to \phi^*_{\mathbf{1}''}, \quad \phi_{\mathbf{1}''} \to \phi^*_{\mathbf{1}'}, \quad \phi_{\mathbf{3}} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \phi^*_{\mathbf{3}} \tag{2.13}$$

could generate an alternate set of 12 other generalised CP transformations. We see that this kind of CP transformation can only be realised if both $\phi_{\mathbf{1}'}$ and $\phi_{\mathbf{1}''}$ are present and are interchanged under the CP transformation.

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$X_{\mathbf{3}}$	$S \to S'$	$T \to T'$	$X_{\mathbf{3}}$	$S \to S'$	$T \to T'$
$X_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		T^2	$\rho_{\mathbf{3}}(T^2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$		T^2
$\rho_{\mathbf{3}}(T^2ST) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & -1 & 2\omega^2 \\ 2\omega^2 & 2\omega & -1 \end{pmatrix}$	S	ST^2	$\rho_{\mathbf{3}}(T^2S) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2\omega & -\omega & 2\omega \\ 2\omega^2 & 2\omega^2 & -\omega^2 \end{pmatrix}$	T^2ST	ST^2
$\rho_{\mathbf{3}}(TST^2) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}$		T^2S	$\rho_{\mathbf{3}}(ST^2S) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega^2 & -\omega & 2 \\ 2\omega & 2 & -\omega^2 \end{pmatrix}$		T^2S
$\rho_{\mathbf{3}}(S) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$		ST^2S	$\rho_{\mathbf{3}}(ST^2) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2 & -\omega & 2\omega^2 \\ 2 & 2\omega & -\omega^2 \end{pmatrix}$		ST^2S
$\rho_{\mathbf{3}}(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$	TST^2	T^2	$\rho_{\mathbf{3}}(TS) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2\omega^2 & -\omega^2 & 2\omega^2 \\ 2\omega & 2\omega & -\omega \end{pmatrix}$	TST^2	T^2S
$\rho_{\mathbf{3}}(STS) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & -\omega^2 & 2 \\ 2\omega^2 & 2 & -\omega \end{pmatrix}$		ST^2	$\rho_{\mathbf{3}}(ST) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2 & -\omega^2 & 2\omega \\ 2 & 2\omega^2 & -\omega \end{pmatrix}$		ST^2S

Table 1. The 12 non-trivial generalised CP transformations consistent with an A_4 family symmetry for the triplet representation $\mathbf{3}$ in the chosen basis determined by the consistency equation $X_{\mathbf{3}}\rho^*_{\mathbf{3}}(g)X_{\mathbf{3}}^{-1} = \rho_{\mathbf{3}}(g')$. These CP transformations realise non-trivial outer automorphisms which *change* the conjugacy class of T from $4C_3$ to $4C_3^2$. Notice that even though they are outer automorphisms they are represented by A_4 group elements, e.g. the mapping $(S,T) \to (S',T') = (S,T^2)$ is acheived via the A_4 identity element X_0 by $X_0\rho^*_{\mathbf{3}}(S)X_0^{-1} = \rho_{\mathbf{3}}(S)$ and $X_0\rho^*_{\mathbf{3}}(T)X_0^{-1} = \rho_{\mathbf{3}}(T^2)$.

3 General analysis of lepton mixing from preserved family and CP symmetries

3.1 General family symmetry

To obtain definite predictions for both the lepton mixing angles and CP violating phases from symmetry, we impose the family symmetry G_f and the generalised CP symmetry $H_{\rm CP}$ simultaneously at high energies. Then the family symmetry is spontaneously broken to the G_ν and G_l subgroups in the neutrino and the charged lepton sector respectively, and the remnant CP symmetries from the breaking of $H_{\rm CP}$ are $H^\nu_{\rm CP}$ and $H^l_{\rm CP}$, respectively. The mismatch between the remnant symmetry groups $G_\nu \rtimes H^\nu_{\rm CP}$ and $G_l \rtimes H^l_{\rm CP}$ gives rise to particular values for both mixing angles and CP phases. As usual, the three generations of the left-handed (LH) lepton doublets are unified into a three-dimensional representation $\rho_{\mathbf{3}}$ of G_f. The invariance under the residual family symmetries G_ν and G_l implies that the

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neutrino mass matrix m_ν and the charged lepton mass matrix m_l satisfy

$$\begin{aligned} \rho_{\mathbf{3}}^T(g_{\nu_i})m_\nu\rho_{\mathbf{3}}(g_{\nu_i}) &= m_\nu, \qquad && g_{\nu_i}\in G_\nu,\\ \rho_{\mathbf{3}}^\dagger(g_{l_i})m_lm_l^\dagger\rho_{\mathbf{3}}(g_{l_i}) &= m_lm_l^\dagger, \qquad && g_{l_i}\in G_l\,. \end{aligned} \tag{3.1}$$

where the charged lepton mass matrix m_l is given in the convention in which the left-handed (right-handed) fields are on the left-hand (right-hand) side of m_l. Moreover, the neutrino and the charged lepton mass matrices are constrained by the residual CP symmetry via

$$\begin{aligned} X_{\mathbf{3}\nu}^Tm_\nu X_{\mathbf{3}\nu} &= m_\nu^*, \qquad && X_{\mathbf{3}\nu}\in H^\nu_{\rm CP},\\ X_{\mathbf{3}l}^\dagger m_lm_l^\dagger X_{\mathbf{3}l} &= (m_lm_l^\dagger)^*, \qquad && X_{\mathbf{3}l}\in H^l_{\rm CP}\,. \end{aligned} \tag{3.2}$$

Since there are both remnant family and CP symmetries, the corresponding consistency equation similar to eq. (2.4) has to be satisfied. Namely, the elements $X_{\mathbf{r}\nu}$ of $H^\nu_{\rm CP}$ and $X_{\mathbf{r}l}$ of $H^l_{\rm CP}$ should satisfy

$$\begin{aligned} X_{\mathbf{r}\nu}\rho^*_{\mathbf{r}}(g_{\nu_i})X^{-1}_{\mathbf{r}\nu} &= \rho_{\mathbf{r}}(g_{\nu_j}), \qquad && g_{\nu_i},g_{\nu_j}\in G_\nu,\\ X_{\mathbf{r}l}\rho^*_{\mathbf{r}}(g_{l_i})X^{-1}_{\mathbf{r}l} &= \rho_{\mathbf{r}}(g_{l_j}), \qquad && g_{l_i},g_{l_j}\in G_l. \end{aligned} \tag{3.3}$$

Given a set of solutions $X_{\mathbf{r}\nu}$ and $X_{\mathbf{r}l}$, we can straightforwardly check that $\rho_{\mathbf{r}}(g_{\nu_i})X_{\mathbf{r}\nu}$ and $\rho_{\mathbf{r}}(g_{l_i})X_{\mathbf{r}l}$ are solutions as well. The invariance conditions of eqs. (3.1)–(3.2) allow us to reconstruct the mass matrices m_ν and $m_lm_l^\dagger$, and eventually determine the lepton mixing matrix U_{PMNS}. Furthermore, if two other residual family symmetries G'_ν and G'_l are conjugate to G_ν and G_l under the element $h\in G_f$, i.e.

$$G'_\nu = hG_\nu h^{-1}, \qquad G'_l = hG_lh^{-1}\,, \tag{3.4}$$

then the associated residual CP symmetries $H^{\nu'}_{\rm CP}$ and $H^{l'}_{\rm CP}$ are related to $H^\nu_{\rm CP}$ and $H^l_{\rm CP}$ as

$$H^{\nu'}_{\rm CP} = \rho_{\mathbf{r}}(h)H^\nu_{\rm CP}\rho^T_{\mathbf{r}}(h), \qquad H^{l'}_{\rm CP} = \rho_{\mathbf{r}}(h)H^l_{\rm CP}\rho^T_{\mathbf{r}}(h)\,, \tag{3.5}$$

and the corresponding neutrino and charged lepton mass matrices are of the form

$$m'_\nu = \rho^*_{\mathbf{3}}(h)m_\nu\rho^\dagger_{\mathbf{3}}(h), \qquad m'_lm'^\dagger_l = \rho_{\mathbf{3}}(h)m_lm_l^\dagger\rho^\dagger_{\mathbf{3}}(h). \tag{3.6}$$

Therefore, the remnant subgroups G'_ν and G'_l lead to the same mixing matrix U_{PMNS} as G_ν and G_l do.

Having completed a general discussion of the implementation of a generalised CP symmetry with a family symmetry, we now concentrate on the case of interest in which the family symmetry $G_f = A_4$ and a generalised CP symmetry $H_{\rm CP}$ consistent with A_4 is imposed. Thus, the theory respects the full symmetry $A_4\rtimes H_{\rm CP}$. In the following, we perform a model independent study of the constraints that these symmetries impose on the neutrino mass matrix, the charged lepton mass matrix and the PMNS matrix by scanning all the possible remnant symmetries $G^\nu_{\rm CP}\cong G_\nu\rtimes H^\nu_{\rm CP}$ and $G^l_{\rm CP}\cong G_l\rtimes H^l_{\rm CP}$. We begin this study with an analysis of the neutrino sector.

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3.2 Neutrino sector from a subgroup of $A_4 \rtimes H_{\rm CP}$

As shown in appendix B, the case $G_\nu = K_4 \cong Z_2 \times Z_2$ is not phenomenologically viable. To resolve this issue, we assume that the underlying symmetry $A_4 \rtimes H_{\rm CP}$ is broken into $G^\nu_{\rm CP} \cong Z_2 \times H^\nu_{\rm CP}$[3] in the neutrino sector [24]. Since the three Z_2 subgroups in eq. (A.6) are related by conjugation as $Z^{(2)} = T^2 Z_2^S (T^2)^{-1}$ and $Z_2^{TST^2} = T Z_2^S T^{-1}$, it is sufficient to only consider $G^\nu_{\rm CP} \cong Z_2^S \times H^\nu_{\rm CP}$, where the element $X_{\mathbf{r}\nu}$ of $H^\nu_{\rm CP}$ should satisfy

$$X_{\mathbf{r}\nu}\rho^*_{\mathbf{r}}(S)X^{-1}_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(S)\,. \tag{3.7}$$

It is found that only 4 of the 12 non-trivial CP transformations are acceptable,[4]

$$H^\nu_{\rm CP} = \left\{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S), \rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)\right\}\,. \tag{3.8}$$

Thus, the neutrino mass matrix is constrained by

$$\rho^T_{\mathbf{3}}(S) m_\nu \rho_{\mathbf{3}}(S) = m_\nu, \tag{3.9}$$

$$X^T_{\mathbf{3}\nu} m_\nu X_{\mathbf{3}\nu} = m^*_\nu\,, \tag{3.10}$$

where eq. (3.9) is the invariance condition under Z_2^S, and it implies that the neutrino mass matrix is of the form

$$m_\nu = \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \tag{3.11}$$

where α, β, γ and ϵ are complex parameters, and they are further constrained by the remnant CP symmetry shown in eq. (3.10). In order to diagonalise the neutrino mass matrix m_ν in eq. (3.11), we first apply the tri-bimaximal transformation U_{TB} to yield

$$m'_\nu = U^T_{TB} m_\nu U_{TB} = \begin{pmatrix} 3\alpha + \beta - \gamma & 0 & -\sqrt{3}\,\epsilon \\ 0 & \beta + 2\gamma & 0 \\ -\sqrt{3}\,\epsilon & 0 & 3\alpha - \beta + \gamma \end{pmatrix}, \tag{3.12}$$

[3] As has been shown in previous work [25], if the remnant family symmetry is $Z_2 = \{1, Z\}$ with $Z^2 = 1$, a consistent CP transformation $X_{\mathbf{r}}$ should satisfy $X_{\mathbf{r}}\rho^*_{\mathbf{r}}(Z)X^{-1}_{\mathbf{r}} = \rho_{\mathbf{r}}(Z')$, $Z' \in Z_2$. For the faithful triplet representation $\mathbf{r} = \mathbf{3}$, Z' will be of the same order as Z. Consequently Z' can only be equal to Z exactly. Thus the consistency equation is uniquely fixed to be $X_{\mathbf{r}}\rho^*_{\mathbf{r}}(Z)X^{-1}_{\mathbf{r}} = \rho_{\mathbf{r}}(Z)$. This means that the generalised CP transformation will commute with Z_2, and the semidirect product will reduce to the direct product.

[4] In ref. [24], the authors chose a different basis and proposed that three cases are admissible for $G_\nu = Z_2^S$ in A_4. Case II of ref. [24] exactly corresponds to $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)\}$ of the present work. However, the CP transformations for their Cases I and III map (S, T) to (S, T) and (S, TS) respectively. They belong to another 12 CP transformations defined in eq. (2.13). Therefore, both $\phi_{\mathbf{1}'}$ and $\phi_{\mathbf{1}''}$ should be present in the Lagrangian to define these CP transformations. Furthermore, the scenario of $X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)$ found in our work was omitted in ref. [24] because the authors required that the CP transformation should be both unitary and symmetric. Although it only needs to be unitary (not necessarily symmetric). However, they claimed that non-symmetric CP transformations consistent with the remnant Z_2 flavour symmetry generally implies a partially degenerate neutrino mass spectrum.

where

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \tag{3.13}$$

Now we return to the investigation of the residual CP symmetry constraint of eq. (3.10). Two distinct phenomenological predictions arise for the different choices of $X_{\mathbf{r}\nu}$:

- $X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)$

  For this case, we see that we can straightforwardly solve eq. (3.10) and find that all four parameters α, β, γ and ϵ are real. Then m'_ν can be further diagonalised by

$$U'^T_\nu m'_\nu U'_\nu = \mathrm{diag}(m_1, m_2, m_3), \qquad U'_\nu = R(\theta) P \, , \tag{3.14}$$

  where P is a unitary diagonal matrix with entries ± 1 or $\pm i$ which renders the light neutrino masses $m_{1,2,3}$ positive, and

$$R(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{3.15}$$

  is a rotation matrix with

$$\tan 2\theta = \frac{\sqrt{3}\,\epsilon}{\beta - \gamma} \, . \tag{3.16}$$

  This diagonalisation reveals that the light neutrino masses $m_{1,2,3}$ are given by

$$\begin{aligned} m_1 &= \left| 3\alpha + \mathrm{sign}\left((\beta - \gamma) \cos 2\theta \right) \sqrt{(\beta-\gamma)^2 + 3\epsilon^2} \right| , \\ m_2 &= |\beta + 2\gamma| \, , \\ m_3 &= \left| 3\alpha - \mathrm{sign}\left((\beta - \gamma) \cos 2\theta \right) \sqrt{(\beta-\gamma)^2 + 3\epsilon^2} \right| . \end{aligned} \tag{3.17}$$

  We conclude that this case is acceptable.

- $X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)$

  In this case, it can be seen that the α of eq. (3.11) is purely imaginary, and the remaining parameters β, γ and ϵ are real. Then the hermitian combination $m'^\dagger_\nu m'_\nu$ turns out to be of the form:

$$m'^\dagger_\nu m'_\nu = \mathrm{diag}\left(-9\alpha^2 + (\beta-\gamma)^2 + 3\epsilon^2, (\beta + 2\gamma)^2, -9\alpha^2 + (\beta-\gamma)^2 + 3\epsilon^2 \right) , \tag{3.18}$$

  which implies $m_1 = m_3$. Clearly, this is not consistent with the experimental observation that the three light neutrinos have different masses. Note that the generalised CP transformations $X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)$ are not symmetric in the chosen basis, and hence we confirm the argument of ref. [24] that non-symmetric CP transformations consistent with the remnant Z_2 family symmetry in the neutrino sector lead to partially degenerate neutrino masses.

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Since the remaining choices $G_\nu = Z_2^{T^2ST}$ or $G_\nu = Z_2^{TST^2}$ are related to the discussed case $G_\nu = Z_2^S$ by conjugation, the corresponding remnant CP symmetry is $\rho_{\mathbf{r}}(T^2)H^\nu_{\rm CP}\rho^T_{\mathbf{r}}(T^2)$ or $\rho_{\mathbf{r}}(T)H^\nu_{\rm CP}\rho^T_{\mathbf{r}}(T)$, respectively, where $H^\nu_{\rm CP}$ is given by eq. (3.8). Then their corresponding neutrino mass matrices are of the form $\rho^*_{\mathbf{3}}(T^2)m_\nu\rho^\dagger_{\mathbf{3}}(T^2)$ or $\rho^*_{\mathbf{3}}(T)m_\nu\rho^\dagger_{\mathbf{3}}(T)$, respectively, with m_ν given in eq. (3.11). Now that we have finished a systematic discussion of the effects of the residual flavour and CP symmetries on the neutrino mass matrix, we turn to analyse their effects on the charged lepton mass matrix.

3.3 Charged lepton sector from a subgroup of $A_4 \rtimes H_{\rm CP}$

In appendices C and D we consider the cases $G_l = Z_2$ and K_4 and show that they are not phenomenologically viable. Here we consider the successful case that G_l is one of the Z_3 subgroups shown in eq. (A.7). Since the four Z_3 subgroups are conjugate to each other, i.e.

$$
\begin{aligned}
&(TST^2)Z_3^T(TST^2)^{-1} = Z_3^{ST}, \quad (T^2ST)Z_3^T(T^2ST)^{-1} = Z_3^{TS}, \quad SZ_3^TS = Z_3^{STS}, \\
&SZ_3^{ST}S = Z_S^{TS}, \quad (T^2ST)Z_3^{ST}(T^2ST)^{-1} = Z_3^{STS}, \quad (TST^2)Z_3^{TS}(TST^2)^{-1} = Z_3^{STS},
\end{aligned}
\tag{3.19}
$$

we choose $G_l = Z_3^T$ for demonstration. Then the combined symmetry group $A_4 \rtimes H_{\rm CP}$ is broken to $G^l_{\rm CP} \cong Z_3^T \rtimes H^l_{\rm CP}$ in the charged lepton sector. The element $X_{\mathbf{r}l}$ of $H^l_{\rm CP}$ should satisfy the consistency equation[5]

$$
X_{\mathbf{r}l}\rho^*_{\mathbf{r}}(T)X^{-1}_{\mathbf{r}l} = \rho_{\mathbf{r}}(T^2)\,. \tag{3.20}
$$

It is found that the remnant CP transformation $H^l_{\rm CP}$ can be

$$
H^l_{\rm CP} = \left\{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(T), \rho_{\mathbf{r}}(T^2)\right\}\,. \tag{3.21}
$$

Similar to the neutrino mass matrix, the charged lepton mass matrix m_l must respect both the residual family symmetry Z_3^T and the generalised CP symmetry $H^l_{\rm CP}$, i.e.

$$
\begin{aligned}
\rho^\dagger_{\mathbf{3}}(T)m_lm_l^\dagger\rho_{\mathbf{3}}(T) &= m_lm_l^\dagger, \\
\rho^\dagger_{\mathbf{3}}(1)m_lm_l^\dagger\rho_{\mathbf{3}}(1) &= (m_lm_l^\dagger)^*,
\end{aligned}
\tag{3.22}
$$

where $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(1)$ from eq. (3.21) has been taken. For the value $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(T)$ or $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(T^2)$, the resulting constraint is equivalent to eq. (3.22). One can easily see that $m_lm_l^\dagger$ is diagonal in this case,

$$
m_lm_l^\dagger = {\rm diag}(m_e^2, m_\mu^2, m_\tau^2)\,, \tag{3.23}
$$

where m_e, m_μ and m_τ are the electron, muon and tau masses, respectively. For the other choices $G_l = Z_3^{ST}, Z_3^{TS}$ and Z_3^{STS}, the corresponding residual CP symmetry and the mass matrix $m_lm_l^\dagger$ follow from the general relations eq. (3.5) and eq. (3.6) immediately with $h = TST^2, T^2ST$ and S, respectively.

[5]The alternative $X_{\mathbf{r}l}\rho^*_{\mathbf{r}}(T)X^{-1}_{\mathbf{r}l} = \rho_{\mathbf{r}}(T)$ is ruled out by the singlet representations $\mathbf{1}'$ and $\mathbf{1}''$ as discussed below eq. (2.7).

3.4 Lepton mixing from $A_4 \rtimes H_{\rm CP}$ broken to $G^{\nu}_{\rm CP} \cong Z^S_2 \times H^{\nu}_{\rm CP}$ and $G^{l}_{\rm CP} \cong Z^T_3 \rtimes H^{l}_{\rm CP}$

In the context of family symmetry and its extension of including generalised CP symmetry, a specific lepton mixing pattern arises from the mismatch between the symmetry breaking in the neutrino and the charged lepton sectors. In this section, we perform a comprehensive analysis of all possible lepton mixing matrices obtainable from the implementation of an A_4 family symmetry and its corresponding generalised CP symmetry by considering all possible residual symmetries $G^{\nu}_{\rm CP}$ and $G^{l}_{\rm CP}$ discussed in previous sections.

Immediately we can disregard the cases predicting partially degenerate lepton masses. Therefore, breaking to the subgroups $G^{\nu}_{\rm CP} \cong K_4 \rtimes H^{\nu}_{\rm CP}$ or $G^{l}_{\rm CP} \cong K_4 \rtimes H^{l}_{\rm CP}$ will be neglected in the following. Furthermore, in order that the elements of G_ν and G_l give rise to the entire family symmetry group A_4, we take G_l to be one of the Z_3 subgroups shown in eq. (A.7). Then, there are $3\times 4 = 12$ combinations for $G_\nu = Z_2$ and $G_l = Z_3$. However, we find that all of these are conjugate to each other.[6] As a result, all possible symmetry breaking chains of this kind lead to the same lepton mixing matrix U_{PMNS}. This important point is further confirmed by straightforward calculations which are lengthy and tedious.

Without loss of generality, it is sufficient to consider the representative values $G_\nu = Z^S_2 = \{1, S\}$ and $G_l = Z^T_3 = \{1, T, T^2\}$, and the original symmetry $A_4 \rtimes H_{\rm CP}$ is broken to $Z^S_2 \times H^{\nu}_{\rm CP}$ in the neutrino sector and $Z^T_3 \rtimes H^{l}_{\rm CP}$ in the charged lepton sector, where $H^{\nu}_{\rm CP} = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)\}$[7] and $H^{l}_{\rm CP} = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(T), \rho_{\mathbf{r}}(T^2)\}$. In this case, $m_l m^{\dagger}_l$ is diagonal as shown in eq. (3.23). Therefore, no rotation of the charged lepton fields is needed to get to the mass eigenstate basis, and the lepton mixing comes completely from the neutrino sector. In the PDG convention [50], the PMNS matrix is cast in the form

$$U_{PMNS} = V\,\mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}), \tag{3.24}$$

with

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}. \tag{3.25}$$

where we use the shorthand notation $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, δ_{CP} is the Dirac CP phase, α_{21} and α_{31} are the Majorana CP phases. Using this PDG convention we find that the resulting PMNS matrix is:

$$U_{PMNS} = U_{TB}R(\theta)P = \begin{pmatrix} \frac{2}{\sqrt{6}}\cos\theta & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}\sin\theta \\ -\frac{1}{\sqrt{6}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta \\ -\frac{1}{\sqrt{6}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta \end{pmatrix} P\,, \tag{3.26}$$

[6] For example, the choice $G'_\nu = Z^{T^2ST}_2, G'_l = Z^{TS}_3$ is conjugate to $G_\nu = Z^S_2, G_l = Z^T_3$ via $G'_\nu = (T^2S)G_\nu(T^2S)^{-1}$ and $G'_l = (T^2S)G_l(T^2S)^{-1}$.

[7] $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)\}$ leads to degenerate light neutrino masses, and it is ignored here.

where as shown previously P is a unitary diagonal matrix with entries ± 1 or $\pm i$ and $R(\theta)$ and U_{TB} are given in eq. (3.13) and eq. (3.15). Hence, the lepton mixing angles and CP phases are

$$\sin\delta_{\rm CP} = \sin\alpha_{21} = \sin\alpha_{31} = 0, \tag{3.27}$$
$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta, \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} = \frac{1}{3\cos^2\theta_{13}}, \quad \sin^2\theta_{23} = \frac{1}{2}\left[1 + \frac{\sqrt{3}\,\sin 2\theta}{2+\cos 2\theta}\right],$$

which implies the three CP phases δ_{CP}, α_{21}, $\alpha_{31} = 0, \pi$, and therefore there is no CP violation in this case. Note that the same results are found in ref. [24].

To summarise the arguments of the preceding section, if one imposes the symmetry $A_4 \rtimes H_{\rm CP}$, which is spontaneously broken to certain residual family and CP symmetries in order to obtain definite predictions for mixing angles and CP phases, then only the symmetry breaking of $A_4 \rtimes H_{\rm CP}$ to $G^{\nu}_{\rm CP} \cong Z_2 \times H^{\nu}_{\rm CP}$ in the neutrino sector and $G^{l}_{\rm CP} \cong Z_3 \rtimes H^{l}_{\rm CP}$ in the charged lepton sector can lead to lepton mixing angles in the experimentally preferred range. However, there is no CP violation in this case. This is consistent with the result found for $S_4 \rtimes H_{\rm CP}$ for the case where $G^{\nu}_{\rm CP} \cong Z^S_2 \times H^{\nu}_{\rm CP}$ with $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)\}$ [25]. For $S_4 \rtimes H_{\rm CP}$ it was possible to achieve maximal CP violation for the case $G^{\nu}_{\rm CP} \cong Z^S_2 \times H^{\nu}_{\rm CP}$ with $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(U), \rho_{\mathbf{r}}(SU)\}$. This case is not directly accessible for $A_4 \rtimes H_{\rm CP}$ since the U generator is absent, although it is accidentally present at LO in the models that we now discuss.

4 Model with A_4 and generalised CP symmetries

Guided by the general analysis of previous sections, we construct an effective model in this section. The predictions of eq. (3.27) are realised if the remnant CP is preserved otherwise the Dirac CP phase is approximately maximal. The model is based on $A_4 \rtimes H_{\rm CP}$, which is supplemented by the extra symmetries $Z_4 \times Z_6 \times {\rm U}(1)_R$. The auxiliary symmetry $Z_4 \times Z_6$ separates the neutrino sector from the charged lepton sector, eliminates unwanted dangerous operators and it is also helpful to produce the mass hierarchy among the charged leptons. As usual both left-handed (LH) lepton doublets l and the right-handed (RH) neutrinos ν^c are embedded into triplet representation $\mathbf{3}$, while the RH charged leptons e^c, μ^c and τ^c transform as the A_4 singlets $\mathbf{1}$, $\mathbf{1}''$ and $\mathbf{1}'$, respectively. All the fields of the model together with their assignments under the symmetry groups are listed in table 2.

It will be seen that in the ensuing model, the $A_4 \rtimes H_{\rm CP}$ symmetry is broken to $Z^S_2 \times H^{\nu}_{\rm CP}$ in the neutrino sector and $Z^T_3 \rtimes H^{l}_{\rm CP}$ in the charged lepton sector at leading order. An accidental Z^U_2 symmetry, which is the $\mu - \tau$ exchange symmetry, arises due to the absence of flavons transforming as $\mathbf{1}'$ or $\mathbf{1}''$. As a result, the leading order (LO) lepton mixing is tri-bimaximal. The next-to-leading order (NLO) corrections will subsequently correct the mixing pattern, bringing it into agreement with experiment. In the following, we begin by analysing vacuum alignment and Yukawa operators of the model at LO, then turn to the NLO analysis.

`Field`	l	ν^c	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	ζ	φ_S	$\xi(\tilde{\xi})$	χ	ρ	φ_T^0	φ_S^0	ξ^0	χ^0	ρ^0
A_4	**3**	**3**	**1**	$\mathbf{1}''$	$\mathbf{1}'$	**1**	**3**	**1**	**3**	**1**	$\mathbf{1}''$	**1**	**3**	**3**	**1**	$\mathbf{1}''$	**1**
Z_4	-1	-1	$-i$	1	i	1	i	i	1	1	1	1	-1	1	1	1	1
Z_6	ω_6^4	ω_6^2	ω_6^2	ω_6^2	ω_6^2	1	1	1	ω_6^2	ω_6^2	ω_6^5	ω_6^3	1	ω_6^2	ω_6^2	ω_6^2	1
$\mathrm{U}(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0	2	2	2	2	2

Table 2. Field content and their transformation rules under the family symmetry $A_4 \times Z_4 \times Z_6$ and $\mathrm{U}(1)_R$, where $\omega_6 = e^{2\pi i/6}$.

4.1 Vacuum alignment

The vacuum alignment problem can be solved by the supersymmetric driving field method introduced in ref. [51]. This approach utilises a global $\mathrm{U}(1)_R$ continuous symmetry which contains the discrete R-parity as a subgroup. The flavon and Higgs fields are uncharged under $\mathrm{U}(1)_R$, the matter fields have R charge equal to $+1$ and the so-called driving fields φ_T^0, φ_S^0, ξ^0, χ^0 and ρ^0 carry two units of R charge. The most general driving superpotential w_d invariant under the family symmetry $A_4 \times Z_4 \times Z_6$ can be written as

$$w_d = w_d^l + w_d^\nu \,, \tag{4.1}$$

where w_d^l is the superpotential for the flavons entering the charged lepton sector at leading order (LO), i.e.

$$w_d^l = f_1 \left(\varphi_T^0 \varphi_T\right) \zeta + f_2 \left(\varphi_T^0 \varphi_T \varphi_T\right) \tag{4.2}$$

and w_d^ν is the superpotential involving the flavon fields of the neutrino sector, i.e.

$$\begin{aligned} w_d^\nu = {} & g_1 \tilde{\xi} \left(\varphi_S^0 \varphi_S\right) + g_2 \left(\varphi_S^0 \varphi_S \varphi_S\right) + g_3 \xi^0 \left(\varphi_S \varphi_S\right) + g_4 \xi^0 \xi^2 + g_5 \xi^0 \xi \tilde{\xi} + g_6 \xi^0 \tilde{\xi}^2 \\ & + g_7 \chi^0 \left(\varphi_S \varphi_S\right)' + g_8 \chi^0 \chi^2 + M_\rho^2 \rho^0 + g_9 \rho^0 \rho^2 \,, \end{aligned} \tag{4.3}$$

where the fields ξ and $\tilde{\xi}$ are defined in such a way that only the latter couples to the combination $\left(\varphi_S^0 \varphi_S\right)$. Notice that $(\ldots)$ indicate a contraction to the singlet $\mathbf{1}$, $(\ldots)'$ a contraction to the singlet $\mathbf{1}'$ and $(\ldots)''$ a contraction to the singlet $\mathbf{1}''$. Moreover, all couplings in w_d are real, since we have imposed the generalised CP H_{CP} as a symmetry of the model. In the SUSY limit, the vacuum alignment is determined by the vanishing of the derivative of the driving superpotential w_d with respect to each component of the driving field, i.e. the $F-$ terms of the driving fields must vanish. Therefore, the vacuum in the charged lepton sector is determined by

$$\begin{aligned} \frac{\partial w_d}{\partial \varphi_{T_1}^0} &= f_1 \varphi_{T_1} \zeta + \frac{2}{3} f_2 \left(\varphi_{T_1}^2 - \varphi_{T_2} \varphi_{T_3}\right) = 0 \,, \\ \frac{\partial w_d}{\partial \varphi_{T_2}^0} &= f_1 \varphi_{T_3} \zeta + \frac{2}{3} f_2 \left(\varphi_{T_2}^2 - \varphi_{T_1} \varphi_{T_3}\right) = 0 \,, \\ \frac{\partial w_d}{\partial \varphi_{T_3}^0} &= f_1 \varphi_{T_2} \zeta + \frac{2}{3} f_2 \left(\varphi_{T_3}^2 - \varphi_{T_1} \varphi_{T_2}\right) = 0 \,. \end{aligned} \tag{4.4}$$

This set of equations admit two inequivalent solutions. The first solution is

$$\langle\zeta\rangle = 0, \qquad \langle\varphi_T\rangle = v_T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \tag{4.5}$$

where v_T is undetermined, and the second solution is

$$\langle\zeta\rangle = v_\zeta, \qquad \langle\varphi_T\rangle = \begin{pmatrix} v_T \\ 0 \\ 0 \end{pmatrix} \quad \text{with} \quad v_T = -\frac{3f_1}{2f_2} v_\zeta\,. \tag{4.6}$$

Note that the phase of v_ζ can be absorbed into the lepton fields. Therefore we can take v_ζ to be real without loss of generality, and then the VEV v_T is real as well. Since the couplings f_1 and f_2 naturally have absolute values of $\mathcal{O}(1)$, the vacuum expectation values (VEVs) v_ζ and v_T are expected to be of the same order of magnitude. In the present work, we choose this solution and shall show that the mass hierarchies among the charged lepton masses can be naturally produced for

$$\frac{v_T}{\Lambda} \sim \frac{v_\zeta}{\Lambda} \sim \mathcal{O}(\lambda^2)\,, \tag{4.7}$$

where λ is of the order of Cabibbo angle $\theta_c \simeq 0.23$. Similarly the $F-$ term conditions for the flavon fields ξ, $\tilde{\xi}$, φ_S and χ are

$$\begin{aligned}
\frac{\partial w_d}{\partial \varphi^0_{S_1}} &= g_1 \tilde{\xi} \varphi_{S_1} + \frac{2}{3} g_2 \left(\varphi^2_{S_1} - \varphi_{S_2}\varphi_{S_3}\right) = 0\,, \\
\frac{\partial w_d}{\partial \varphi^0_{S_2}} &= g_1 \tilde{\xi} \varphi_{S_3} + \frac{2}{3} g_2 \left(\varphi^2_{S_2} - \varphi_{S_1}\varphi_{S_3}\right) = 0\,, \\
\frac{\partial w_d}{\partial \varphi^0_{S_3}} &= g_1 \tilde{\xi} \varphi_{S_2} + \frac{2}{3} g_2 \left(\varphi^2_{S_3} - \varphi_{S_1}\varphi_{S_2}\right) = 0\,, \\
\frac{\partial w_d}{\partial \xi^0} &= g_3 \left(\varphi^2_{S_1} + 2\varphi_{S_2}\varphi_{S_3}\right) + g_4 \xi^2 + g_5 \xi\tilde{\xi} + g_6 \tilde{\xi}^2 = 0\,, \\
\frac{\partial w_d}{\partial \chi^0} &= g_7 \left(\varphi^2_{S_3} + 2\varphi_{S_1}\varphi_{S_2}\right) + g_8 \chi^2 = 0\,.
\end{aligned} \tag{4.8}$$

Disregarding the ambiguity caused by A_4 family symmetry transformations, we find the solution

$$\langle\xi\rangle = v_\xi, \quad \langle\tilde{\xi}\rangle = 0, \quad \langle\varphi_S\rangle = v_S \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle\chi\rangle = v_\chi\,, \tag{4.9}$$

where the VEVs v_ξ, v_S and v_χ are related by

$$v_S^2 = -\frac{g_4}{3g_3} v_\xi^2, \qquad v_\chi^2 = \frac{g_4 g_7}{g_3 g_8} v_\xi^2\,, \tag{4.10}$$

where v_ξ is undetermined and generally complex. Consequently the VEVs v_S and v_χ are complex as well. Since all couplings are real due to the invariance under the generalised CP symmetry $H_{\rm CP}$, the three VEVs v_ξ, v_S and v_χ share the same phase, up to the phase difference 0, π or $\pm\pi/2$ determined by the sign of $g_3 g_4$ and $g_7 g_8$.[8]

Finally, the minimisation equation for the vacuum of ρ is

$$\frac{\partial w_d}{\partial \rho^0} = M_\rho^2 + g_9 \rho^2 = 0\,, \tag{4.11}$$

which leads to

$$\langle \rho \rangle = v_\rho, \quad \text{with} \quad v_\rho^2 = -M_\rho^2/g_9\,. \tag{4.12}$$

Obviously the VEV v_ρ can only be real or purely imaginary depending on the coupling g_9 being negative or positive, respectively. As we shall see, agreement with the experimental data (in particular the measured sizeable θ_{13}) can be achieved if

$$\frac{v_\xi}{\Lambda} \sim \frac{v_S}{\Lambda} \sim \frac{v_\chi}{\Lambda} \sim \frac{v_\rho}{\Lambda} \sim \mathcal{O}(\lambda)\,. \tag{4.13}$$

Thus, there is a moderate hierarchy of order λ between the VEVs of the flavon fields in the neutrino and the charged lepton sectors. This hierarchy can be accommodated since the two sets of VEVs are determined by different minimisation conditions. Now that we have studied the vacuum alignments possible in this model, we proceed by constructing the explicit charged lepton and neutrino mass matrices.

4.2 The model at leading order

From table 2, it is seen that the effective superpotential for the charged lepton masses is given by

$$\begin{aligned} w_l = & \frac{y_\tau}{\Lambda}\left(l\varphi_T\right)'' \tau^c h_d + \frac{y_{\mu_1}}{\Lambda^2}\left(l\varphi_T\varphi_T\right)' \mu^c h_d + \frac{y_{\mu_2}}{\Lambda^2}\left(l\varphi_T\right)' \zeta \mu^c h_d + \frac{y_{e_1}}{\Lambda^3}\left(l\varphi_T\right)\left(\varphi_T\varphi_T\right) e^c h_d \\ & + \frac{y_{e_2}}{\Lambda^3}\left(l\varphi_T\right)'\left(\varphi_T\varphi_T\right)'' e^c h_d + \frac{y_{e_3}}{\Lambda^3}\left(l\varphi_T\right)''\left(\varphi_T\varphi_T\right)' e^c h_d + \frac{y_{e_4}}{\Lambda^3}\left(\left(l\varphi_T\right)_{3_S}\left(\varphi_T\varphi_T\right)_{3_S}\right) e^c h_d \\ & + \frac{y_{e_5}}{\Lambda^3}\left(\left(l\varphi_T\right)_{3_A}\left(\varphi_T\varphi_T\right)_{3_S}\right) e^c h_d + \frac{y_{e_6}}{\Lambda^3}\left(l\varphi_T\varphi_T\right)\zeta e^c h_d + \frac{y_{e_7}}{\Lambda^3}\left(l\varphi_T\right)\zeta^2 e^c h_d + \dots\,, \end{aligned} \tag{4.14}$$

where dots represent the higher dimensional operators which will be discussed later, and all coupling constants are constrained to be real by the generalised CP symmetry. Due to the auxiliary Z_4 symmetry, the relevant electron, muon and tau mass terms involve one flavon, two flavons and three flavons, respectively. Substituting the VEVs of φ_T and ζ in eq. (4.6), a diagonal charged lepton mass matrix is generated with

$$\begin{aligned} m_e &= \left(y_{e_1} + \frac{4}{9}y_{e_4} + \frac{2}{3}y_{e_6}\frac{v_\zeta}{v_T} + y_{e_7}\frac{v_\zeta^2}{v_T^2}\right)\frac{v_T^3}{\Lambda^3}v_d\,, \\ m_\mu &= \left(\frac{2}{3}y_{\mu_1} + y_{\mu_2}\frac{v_\zeta}{v_T}\right)\frac{v_T^2}{\Lambda^2}v_d\,, \qquad m_\tau = y_\tau\frac{v_T}{\Lambda}v_d\,, \end{aligned} \tag{4.15}$$

[8]Note that it is possible to obtain more complicated phase differences by coupling more flavons together in the flavon potential [40, 41]. Consequently the corresponding driving superpotential becomes non-renormalisable. For example, if eq. (4.10) instead appeared schematically as $v_S^3 \sim v_\xi^3$, then phase differences of $\frac{2k\pi}{3}$ and $\frac{(2k-1)\pi}{3}$ with $k = 1, 2, 3$ could be obtained. More generally, if one obtains a relation like $v_S^p \sim v_\xi^p$, then phase differences of $\frac{2k\pi}{p}$ and $\frac{(2k-1)\pi}{p}$ with $k = 1, 2, \dots, p$ could be realised.

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where $v_d = \langle h_d \rangle$. The VEVs of the flavons φ_T and ζ are responsible for the spontaneous breaking of both family symmetry and generalised CP symmetry here. Furthermore, it is obvious that the A_4 family symmetry is broken to the Z_3^T subgroup in the charged lepton sector. As was pointed out in the vacuum alignment of section 4.1, both v_T and v_ζ can be set to be real. Therefore the generalised CP symmetry is broken to $H^l_{\rm CP} = \left\{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(T), \rho_{\mathbf{r}}(T^2)\right\}$ in the charged lepton sector. It is remarkable that the observed charged lepton mass hierarchies are naturally reproduced for $v_T/\Lambda \sim v_\zeta/\Lambda \sim \lambda^2$. In the following, we turn to discuss the neutrino sector. Neutrino masses are generated by the seesaw mechanism [52–58], and the LO superpotential for the neutrino masses, which is invariant under the imposed family symmetry $A_4 \times Z_4 \times Z_6$, is of the form

$$w_\nu = y\,(l\nu^c)\,h_u + y_1\,(\nu^c\nu^c)\,\xi + \tilde{y}_1\,(\nu^c\nu^c)\,\tilde{\xi} + y_3\,(\nu^c\nu^c\varphi_S)\,, \tag{4.16}$$

where all couplings are real because of invariance under the generalised CP transformations defined in section 2. We can straightforwardly read out the Dirac neutrino mass matrix,

$$m_D = y \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u\,, \tag{4.17}$$

where $v_u = \langle h_u \rangle$ is the VEV of the Higgs field h_u. Given the vacuum configuration of eq. (4.9), which breaks the A_4 family symmetry to $G_\nu = Z^S = \{1, S\}$, the Majorana neutrino mass matrix m_M for the heavy RH neutrinos is

$$m_M = \begin{pmatrix} y_1 v_\xi + 2y_3 v_S/3 & -y_3 v_S/3 & -y_3 v_S/3 \\ -y_3 v_S/3 & 2y_3 v_S/3 & y_1 v_\xi - y_3 v_S/3 \\ -y_3 v_S/3 & y_1 v_\xi - y_3 v_S/3 & 2y_3 v_S/3 \end{pmatrix}. \tag{4.18}$$

Notice that this mass matrix also has an accidental Z_2^U symmetry, which is the $\mu - \tau$ exchange symmetry, arising due to the absence of flavons transforming as $\mathbf{1}'$ or $\mathbf{1}''$. It is exactly diagonalised by the tri-bimaximal mixing matrix U_{TB}, i.e.

$$U^T_{TB} m_M U_{TB} = \text{diag}(y_1 v_\xi + y_3 v_S, y_1 v_\xi, -y_1 v_\xi + y_3 v_S)\,. \tag{4.19}$$

Then, the light neutrino mass matrix follows from the seesaw formula

$$m_\nu = -m_D m_M^{-1} m_D^T = U_{TB} \text{diag}(m_1, m_2, m_3) U^T_{TB}\,, \tag{4.20}$$

where

$$m_1 = -\frac{y^2 v_u^2}{y_1 v_\xi + y_3 v_S}, \quad m_2 = -\frac{y^2 v_u^2}{y_1 v_\xi}, \quad m_3 = \frac{y^2 v_u^2}{y_1 v_\xi - y_3 v_S} \tag{4.21}$$

Note that these masses obey the mass sum rule

$$\frac{1}{m_1} - \frac{1}{m_3} = \frac{2}{m_2}. \tag{4.22}$$

However the sum rule will be violated by NLO corrections.

x	α_{21}	α_{31}	$\lvert m_1\rvert$(meV)	$\lvert m_2\rvert$(meV)	$\lvert m_3\rvert$(meV)	$\lvert m_{\beta\beta}\rvert$(meV)	mass order
0.79	0	π	5.83	10.44	50.07	7.36	NO
1.19	0	0	4.43	9.73	49.93	6.20	NO
−2.01	π	0	51.50	52.22	17.33	16.93	IO

Table 3. The LO predictions for the Majorana phases α_{21} and α_{31}, the light neutrino masses $|m_i|(i = 1, 2, 3)$ and the effective mass $|m_{\beta\beta}|$ of the neutrinoless double-beta decay, where $x = y_3v_S/(y_1v_\xi)$. Note that δ_{CP} is undetermined due to vanishing θ_{13} at LO.

Recalling that the charged lepton mass matrix is diagonal, therefore lepton flavour mixing is predicted to be of the tri-bimaximal form at LO. Since the common phase of v_ξ and v_S can always be absorbed by a redefinition of the fields, we can take the product y_1v_ξ to be real without loss of generality. Then y_3v_S will be either real or purely imaginary depending on g_3g_4 being negative or positive, as shown in eq. (4.10). For the case that y_3v_S is imaginary, we can easily check that the remnant CP symmetry in the neutrino sector is $H^{\nu}_{\rm CP} = \{\rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)\}$, and we have $|m_1| = |m_3|$ from eq. (4.21), which implies the light neutrino masses are degenerate. Therefore this case is not phenomenologically viable, and it will be disregarded in the following.

Hence we are left with the case that v_ξ and v_S are of the same phase up to relative sign, and then the generalised CP symmetry is broken to $H^{\nu}_{\rm CP} = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)\}$ at LO. The neutrino mass-squared differences are given by

$$\begin{aligned}
\Delta m^2_{\rm sol} &\equiv |m_2|^2 - |m_1|^2 = \left(\frac{y^2v_u^2}{y_1v_\xi}\right)^2 \frac{x^2+2x}{(1+x)^2}\,, \\
\Delta m^2_{\rm atm} &\equiv |m_3|^2 - |m_1|^2 = \left(\frac{y^2v_u^2}{y_1v_\xi}\right)^2 \frac{4x}{(1-x^2)^2}, \quad \text{for NO}\,, \\
\Delta m^2_{\rm atm} &\equiv |m_2|^2 - |m_3|^2 = \left(\frac{y^2v_u^2}{y_1v_\xi}\right)^2 \frac{x^2-2x}{(1-x)^2}, \quad \text{for IO}\,,
\end{aligned} \tag{4.23}$$

where $x = y_3v_S/(y_1v_\xi)$ is real. Furthermore, the effective mass parameter $|m_{\beta\beta}|$ for the neutrinoless double-beta decay is given by

$$|m_{\beta\beta}| = \left|\frac{y^2v_u^2}{y_1v_\xi}\right| \left|\frac{3+x}{3\,(1+x)}\right|. \tag{4.24}$$

Since the solar neutrino mass squared difference $\Delta m^2_{\rm sol}$ is positive, we need $x > 0$ or $x < -2$. The neutrino spectrum is normal ordering (NO) for $x > 0$ and inverted ordering (IO) for $x < -2$. Imposing the best fit values for the mass splittings $\Delta m^2_{\rm sol} = 7.50 \times 10^{-5}{\rm eV}^2$ and $\Delta m^2_{\rm atm} = 2.473(2.427) \times 10^{-3}{\rm eV}^2$ for NO (IO) spectrum [8], we find three possible values for the ratio x:

$$x \simeq 0.792, 1.195, -2.014\,, \tag{4.25}$$

where the first two correspond to NO, while the last one corresponds to IO spectrum. The corresponding predictions for Majorana phases, the light neutrino masses and $|m_{\beta\beta}|$ are listed in table 3. Note that the Dirac phase can not be fixed uniquely in this case because of the vanishing θ_{13}.

Recall that for $S_4 \rtimes H_{\rm CP}$ it was possible to achieve $\delta_{CP} = \pm\pi/2$ for the case $G^{\nu}_{\rm CP} \cong Z^S_2 \times H^{\nu}_{\rm CP}$ with $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(U), \rho_{\mathbf{r}}(SU)\}$ [25]. Although this case is not directly accessible for $A_4 \rtimes H_{\rm CP}$ since the U generator is absent, we note that at LO the neutrino mass matrix in eq. (4.18) has an accidental $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(U), \rho_{\mathbf{r}}(SU)\}$ CP symmetry. This leads to the same prediction for Majorana phases $\alpha_{21} = 0, \pi$ and $\alpha_{31} = 0, \pi$ as in the $S_4 \rtimes H_{\rm CP}$ model.

4.3 Next-to-Leading-Order corrections

In the following, we study the subleading NLO corrections to the previous superpotentials, which are essential to bring the model into agreement with data. As will be seen, these corrections will produce a non-zero reactor angle θ_{13} whose relative smallness with respect to θ_{12} and θ_{23} is naturally explained by its generation at NLO. The subleading corrections are indicated by higher dimensional operators which are compatible with all symmetries of the model. The NLO contribution to the driving superpotential w^{ν}_d is suppressed by one power of $1/\Lambda$ with respect to the LO terms in eq. (4.3), and it is of the form

$$\begin{aligned}\delta w^{\nu}_d = & \frac{s}{\Lambda}\left(\varphi^0_S\varphi_S\right)'\chi\rho + \frac{r_1}{\Lambda}\rho^0\left(\varphi_S\varphi_S\varphi_S\right) + \frac{r_2}{\Lambda}\rho^0\left(\varphi_S\varphi_S\right)\xi + \frac{r_3}{\Lambda}\rho^0\left(\varphi_S\varphi_S\right)\tilde{\xi} + \frac{r_4}{\Lambda}\rho^0\xi^3 \\ & + \frac{r_5}{\Lambda}\rho^0\xi^2\tilde{\xi} + \frac{r_6}{\Lambda}\rho^0\xi\tilde{\xi}^2 + \frac{r_7}{\Lambda}\rho^0\tilde{\xi}^3\,,\end{aligned} \tag{4.26}$$

where the coupling s and $r_i (i = 1\dots7)$ are real due to the generalised CP symmetry. The LO vacuum configuration is modified to

$$\langle\xi\rangle = v_\xi, \quad \langle\tilde{\xi}\rangle = \delta v_{\tilde{\xi}}, \quad \langle\varphi_S\rangle = \begin{pmatrix} v_S + \delta v_{S_1} \\ v_S + \delta v_{S_2} \\ v_S + \delta v_{S_3}\end{pmatrix}, \quad \langle\chi\rangle = v_\chi + \delta v_\chi, \quad \langle\rho\rangle = v_\rho + \delta v_\rho\,, \tag{4.27}$$

where the VEV of ξ remains undetermined. The new vacuum configuration is determined by the vanishing of the first derivative of $w^{\nu}_d + \delta w^{\nu}_d$ with respect to the driving fields φ^0_S, ξ^0, χ^0 and ρ^0. Keeping only the terms linear in the shift δv and neglecting the term $\delta v/\Lambda$, we find

$$\begin{aligned}\delta v_{S_1} &= \delta v_{S_2} = \delta v_{S_3} = \frac{s g_5}{6 g_1 g_3}\frac{v_\xi}{v_S}\frac{v_\chi v_\rho}{\Lambda} \equiv \delta v_S, \\ \delta v_{\tilde{\xi}} &= -\frac{s}{g_1}\frac{v_\chi v_\rho}{\Lambda}, \quad \delta v_\chi = -\frac{s g_5 g_7}{2 g_1 g_3 g_8}\frac{v_\xi v_\rho}{\Lambda}, \quad \delta v_\rho = \frac{g_4 r_2 - g_3 r_4}{2 g_3 g_9}\frac{v^3_\xi}{\Lambda v_\rho}\,.\end{aligned} \tag{4.28}$$

We see that the three components of φ_S are shifted by the same amount. This implies that the vacuum alignment of φ_S is not changed. The reason for this is that only the neutrino flavon fields φ_S, ξ, $\tilde{\xi}$, χ and ρ instead of φ_T enter into the NLO operators of eq. (4.26). Hence, the remnant family symmetry $Z^S_2 = \{1, S\}$ in the neutrino sector is still preserved. This implies $\langle\varphi_S\rangle \propto (1, 1, 1)$. Furthermore, eq. (4.28) indicates that δv_S, $\delta v_{\tilde{\xi}}$, δv_χ and δv_ρ are of order $\lambda^2\Lambda$, i.e. the shifts of the flavon fields in the neutrino sector are of relative order λ with respect to the LO VEVs. For the driving superpotential w^l_d, the nontrivial subleading operators, whose contributions can not be absorbed via a redefinition of the LO parameters, are of the form:

$$\begin{array}{lll} \left(\varphi_T^0\varphi_T^2\varphi_\nu^3\right)/\Lambda^3, & \left(\varphi_T^0\varphi_T\varphi_\nu^3\right)\zeta/\Lambda^3, & \left(\varphi_T^0\varphi_\nu^3\right)\zeta^2/\Lambda^3, \\ \left(\varphi_T^0\varphi_T^2\varphi_\nu\right)''\chi^2/\Lambda^3, & \left(\varphi_T^0\varphi_T\varphi_\nu\right)''\chi^2\zeta/\Lambda^3, & \left(\varphi_T^0\varphi_\nu\right)''\chi^2\zeta^2/\Lambda^3, \end{array} \tag{4.29}$$

where $\varphi_\nu = \{\varphi_S, \xi, \tilde{\xi}\}$ denotes the flavon involved in the neutrino sector at LO. Therefore subleading contributions to the $F-$terms of the driving field φ_T^0 are suppressed by $\langle\varphi_\nu\rangle^3/\Lambda^3 \sim \lambda^3$ with respect to the LO renormalisable terms in w_d^l. As a result, the vacuum alignment of φ_T acquires corrections of order λ^3:

$$\langle\varphi_T\rangle = v_T \begin{pmatrix} 1+\epsilon_1\lambda^3 \\ \epsilon_2\lambda^3 \\ \epsilon_3\lambda^3 \end{pmatrix}, \tag{4.30}$$

where $\epsilon_i (i = 1, 2, 3)$ are complex numbers with absolute value of $\mathcal{O}(1)$. Inserting this modified vacuum of φ_T into the LO expression of w_l in eq. (4.14), the off-diagonal elements of the charged lepton mass matrix become non-zero and are all suppressed by λ^3 with respect to the diagonal entries. Consequently, the corrected charged lepton mass matrix has the following structure:

$$m_l = \begin{pmatrix} m_e & \lambda^3 m_\mu & \lambda^3 m_\tau \\ \lambda^3 m_e & m_\mu & \lambda^3 m_\tau \\ \lambda^3 m_e & \lambda^3 m_\mu & m_\tau \end{pmatrix}, \tag{4.31}$$

where only the order of magnitude of each non-diagonal entry is reported. Therefore the lepton mixing angles receive corrections of order λ^3 from the charged lepton sector. These can be safely neglected. Another source of correction to the charged lepton mass matrix comes from adding the product φ_ν^3 or $\varphi_\nu\chi^2$ in all possible ways to each term of w_l. However, the introduction of these additional terms changes the charged lepton mass matrix in exactly the same way as the corrections induced by the VEV shifts of φ_T. Therefore, the general structure of m_l shown in eq. (4.31) remains.

Now we turn to study the corrections to the neutrino sector. The higher order corrections to the neutrino Dirac mass are given by[9]

$$\left(l\nu^c\varphi_\nu^3\right)h_u/\Lambda^3 + \left(l\nu^c\varphi_\nu\right)''\chi^2 h_u/\Lambda^3, \tag{4.32}$$

where all possible A_4 contractions should be considered, and we have suppressed all real coupling constants. The resulting contributions are of relative order λ^3 with respect to the LO term $y\,(l\nu^c)\,h_u$ in eq. (4.16) and therefore negligible. The NLO corrections to the RH Majorana neutrino mass are

$$\delta w_\nu = \tilde{y}_1\,(\nu^c\nu^c)\,\delta\tilde{\xi} + y_3\,(\nu^c\nu^c\delta\varphi_S) + y_4\,(\nu^c\nu^c)'\,\chi\rho/\Lambda, \tag{4.33}$$

[9]The operator $(l\nu^c)\,\rho^2 h_u/\Lambda^2$ is omitted here, since its contribution can be absorbed by redefining the LO parameter y of eq. (4.16).

where $\delta\tilde{\xi}$ and $\delta\varphi_S$ indicate the shifted vacua of the flavons $\tilde{\xi}$ and φ_S. They lead to additional contributions to m_M as follows:

$$\delta m_M = \begin{pmatrix} \tilde{y}_1 \delta v_{\tilde{\xi}} + 2y_3 \delta v_S/3 & -y_3\delta v_S/3 + y_4 v_\chi v_\rho/\Lambda & -y_3\delta v_S/3 \\ -y_3\delta v_S/3 + y_4 v_\chi v_\rho/\Lambda & 2y_3\delta v_S/3 & \tilde{y}_1\delta v_{\tilde{\xi}} - y_3\delta v_S/3 \\ -y_3\delta v_S/3 & \tilde{y}_1\delta v_{\tilde{\xi}} - y_3\delta v_S/3 & 2y_3\delta v_S/3 + y_4 v_\chi v_\rho/\Lambda \end{pmatrix}. \quad (4.34)$$

Notice that this mass matrix breaks the accidental Z_2^U symmetry, which is the $\mu - \tau$ exchange symmetry, arising due to the presence of the χ flavon transforming as $\mathbf{1}''$, allowing a non-zero reactor angle. It also breaks the accidental $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(U), \rho_{\mathbf{r}}(SU)\}$ CP symmetry. In fact, since we have fewer parameters in the neutrino mass matrix than in the S_4 case, we cannot preserve an accidental $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(U), \rho_{\mathbf{r}}(SU)\}$ CP symmetry whilst breaking the accidental Z_2^U family symmetry. It will therefore lead to different predictions for Majorana phases $\alpha_{21} \neq 0, \pi$ and $\alpha_{31} \neq 0, \pi$ compared to the $S_4 \rtimes H_{\rm CP}$ model, however with $\theta_{13} \neq 0$ we will allow the possibility that $\delta_{CP} = \pm\pi/2$ which can be understood from the discussion below eq. (4.10).

As shown in eq. (4.12), the VEV v_ρ is real for $g_9 < 0$ and imaginary for $g_9 > 0$. Eq. (4.10) implies that the phase difference between v_χ and v_ξ is 0, π or $\pm\pi/2$ for the product $g_3g_4g_7g_8 > 0$ or $g_3g_4g_7g_8 < 0$, respectively. Hence the combination $v_\chi v_\rho$ is real or purely imaginary once the phase of v_ξ is absorbed by redefining the fields.

First, we consider the case that $v_\chi v_\rho$ is real,[10] i.e. the phase difference between $v_\chi v_\rho$ and v_ξ is 0 or π, and then both $\delta v_{\tilde{\xi}}$ and δv_S will be also real from eq. (4.28). Further recalling that v_ξ and v_S should have a common phase to avoid degenerate light neutrino masses, the NLO contributions carry the same phase (up to relative sign) as the LO contribution from eq. (4.18) in this case. The corrections due to shifted vacuum of $\tilde{\xi}$ and φ_S can be absorbed by a redefinition of the couplings y_1 and y_3 respectively. Thus the RH neutrino mass matrix m_M including NLO contributions can be parametrised as

$$m_M = \begin{pmatrix} \hat{y}_1 v_\xi + 2\hat{y}_3 v_S/3 & -\hat{y}_3 v_S/3 + y_4 v_\chi v_\rho/\Lambda & -\hat{y}_3 v_S/3 \\ -\hat{y}_3 v_S/3 + y_4 v_\chi v_\rho/\Lambda & 2\hat{y}_3 v_S/3 & \hat{y}_1 v_\xi - \hat{y}_3 v_S/3 \\ -\hat{y}_3 v_S/3 & \hat{y}_1 v_\xi - \hat{y}_3 v_S/3 & 2\hat{y}_3 v_S/3 + y_4 v_\chi v_\rho/\Lambda \end{pmatrix}, \quad (4.35)$$

where $\hat{y}_1 = y_1 + \tilde{y}_1 \delta v_{\tilde{\xi}}/v_\xi$ and $\hat{y}_3 = y_3(1 + \delta v_S/v_S)$ are real. The light neutrino mass matrix is given by the seesaw relation

$$\begin{aligned} m_\nu &= -m_D m_M^{-1} m_D^T \\ &= \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}. \end{aligned} \quad (4.36)$$

[10] We could choose $g_9 < 0$ and $g_3g_4g_7g_8 > 0$ such that v_χ and v_ξ have a common phase up to relative sign and v_ρ is real. Consequently the symmetry $A_4 \rtimes H_{\rm CP}$ is broken down to $G^\nu_{\rm CP} = Z_2^S \times H^\nu_{\rm CP}$ in the neutrino sector with $H^\nu_{\rm CP} = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)\}$. On the other hand, this case can also be realised by taking $g_9 > 0$ and $g_3g_4g_7g_8 < 0$ such that v_ρ is imaginary and the phase difference of v_χ and v_ξ is $\pm\pi/2$.

It is the most general neutrino mass matrix consistent with the residual family symmetry $G_\nu = Z_2^S = \{1, S\}$, as is shown in eq. (3.11). The parameters α, β, γ and ϵ can be regarded as real and are given by

$$\begin{aligned}
\alpha &= \frac{\hat{y}_3 v_S}{3\left(\hat{y}_1^2 v_\xi^2 - \hat{y}_3^2 v_S^2 - \hat{y}_1 y_4 v_\xi v_\chi v_\rho/\Lambda + y_4^2 v_\chi^2 v_\rho^2/\Lambda^2\right)}, \\
\beta &= \frac{-3\hat{y}_1^2 v_\xi^2 + \hat{y}_3^2 v_S^2}{3\left(\hat{y}_1^3 v_\xi^3 - \hat{y}_1 \hat{y}_3^2 v_\xi v_S^2 - \hat{y}_3^2 y_4 v_S^2 v_\chi v_\rho/\Lambda + y_4^3 v_\chi^3 v_\rho^3/\Lambda^3\right)}, \\
\gamma &= \frac{2\hat{y}_3^2 v_S^2 + 3\hat{y}_1 y_4 v_\xi v_\chi v_\rho/\Lambda - 3 y_4^2 v_\chi^2 v_\rho^2/\Lambda^2}{6\left(\hat{y}_1^3 v_\xi^3 - \hat{y}_1 \hat{y}_3^2 v_\xi v_S^2 - \hat{y}_3^2 y_4 v_S^2 v_\chi v_\rho/\Lambda + y_4^3 v_\chi^3 v_\rho^3/\Lambda^3\right)}, \\
\epsilon &= \frac{y_4 v_\chi v_\rho/\Lambda}{2\left(\hat{y}_1^2 v_\xi^2 - \hat{y}_3^2 v_S^2 - \hat{y}_1 y_4 v_\xi v_\chi v_\rho/\Lambda + y_4^2 v_\chi^2 v_\rho^2/\Lambda^2\right)},
\end{aligned} \tag{4.37}$$

where the overall factor $y^2 v_u^2$ has been omitted. We note that the ϵ term in eq. (4.36), which is induced by the last term of the NLO corrections in eq. (4.33), is responsible for the non-zero reactor angle θ_{13}. It is suppressed by λ with respect to the tri-bimaximal mixing preserving contributions α, β and γ terms. Neglecting the small contributions from the charged lepton sector, the PMNS matrix is of the form shown in eq. (3.26), and the predictions for lepton mixing angles and CP phases are given in eq. (3.27). Notice that in this case both Dirac and Majorana CP phases are trivial, and there is no CP violation because the neutrino mass matrix is real except for an overall phase.

In this case, the parameters α, β and γ are real, and ϵ is also real instead of imaginary, as would be required in order to have an accidental $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(U), \rho_{\mathbf{r}}(SU)\}$ CP symmetry, therefore it leads to different predictions from the $S_4 \rtimes H_{\rm CP}$ model where $X_{\mathbf{r}\nu} = \{\rho_{\mathbf{r}}(U), \rho_{\mathbf{r}}(SU)\}$ CP symmetry was preserved [25].

The lepton mixing is predicted to be the so-called trimaximal mixing pattern. All the three mixing angles depend on one parameter θ which is of order λ and related to the model parameters via eq. (3.16). Consequently, the reactor angle θ_{13} is of order λ as well in the present model. For the best fit value $\sin^2\theta_{13} = 0.0227$ [8], the rotation angle θ is determined to be $\theta \simeq \pm 0.186$. Consequently we have the solar mixing angle $\sin^2\theta_{12} \simeq 0.341$ and the atmospheric mixing angle $\sin^2\theta_{23} \simeq 0.393$ or $\sin^2\theta_{23} \simeq 0.607$, which are in the experimentally preferred regions.

For the remaining case in which the phase difference between $v_\chi v_\rho$ and v_ξ is $\pm\pi/2$,[11] eq. (4.28) implies that the shifts δv_ξ and δv_S will be imaginary after extracting the overall phase carried by v_ξ. Then, the RH neutrino mass matrix m_M can be parametrised as

$$m_M = y_1 v_\xi \left[(1+ia\lambda)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + (x+ib\lambda)\begin{pmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{pmatrix} + ic\lambda\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right], \tag{4.38}$$

[11]This scenario could be realised by taking $g_9 < 0$, $g_3 g_4 g_7 g_8 < 0$ or $g_9 > 0$, $g_3 g_4 g_7 g_8 > 0$. In this case, the LO residual CP symmetry $H_{\rm CP}^\nu = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)\}$ is broken completely by the VEVs v_χ and v_ρ, although the residual family symmetry $G_\nu = Z_2^S$ is still preserved.

with

$$a = -\frac{i}{\lambda}\frac{\tilde{y}_1 \delta v_{\tilde{\xi}}}{y_1 v_\xi}, \quad x = \frac{y_3 v_S}{y_1 v_\xi}, \quad b = -\frac{i}{\lambda}\frac{y_3 \delta v_S}{y_1 v_\xi}, \quad c = -\frac{i}{\lambda}\frac{y_4 v_\chi}{y_1 v_\xi}\frac{v_\rho}{\Lambda}, \tag{4.39}$$

where x, a, b and c are $\mathcal{O}(1)$ real parameters. To first order in λ, the light neutrino mass matrix followed by a tri-bimaximal transformation is of the form

$$m'_\nu = U^T_{TB} m_\nu U_{TB} = -\frac{y^2 v_u^2}{y_1 v_\xi}\begin{pmatrix} \frac{2+2x-i(2a+2b-c)\lambda}{2(1+x)^2} & 0 & \frac{i\sqrt{3}\,c\lambda}{2(1-x^2)} \\ 0 & 1-i\,(a+c)\,\lambda & 0 \\ \frac{i\sqrt{3}\,c\lambda}{2(1-x^2)} & 0 & \frac{-2+2x+i(2a-2b-c)\lambda}{2(1-x)^2} \end{pmatrix}. \tag{4.40}$$

Following the procedure presented in appendix E, this matrix m'_ν can be diagonalized. After lengthy and tedious calculations, we find that the lepton mixing parameters are modified to

$$\sin\theta_{13} \simeq \left|\frac{c}{2\sqrt{2}\,x}\right|\lambda, \quad \sin^2\theta_{12} = \frac{1}{3} + \mathcal{O}(\lambda^2), \quad \sin^2\theta_{23} = \frac{1}{2} + \mathcal{O}(\lambda^2), \tag{4.41}$$
$$|\sin\delta_{\rm CP}| = 1 + \mathcal{O}(\lambda^2), \quad |\sin\alpha_{21}| \simeq \left|\frac{3c-2b+2x(a+c)}{2(1+x)}\right|\lambda, \quad |\sin\alpha'_{31}| \simeq \left|\frac{x\,(2a-c-2xb)}{1-x^2}\right|\lambda,$$

where $\alpha'_{31} = \alpha_{31} - 2\delta_{\rm CP}$, and the parameter α'_{31} has been redefined to include the Dirac CP phase $\delta_{\rm CP}$. This parametrisation turns out to be very useful and convenient for the analysis of neutrinoless double-beta decay and leptonic CP violation [59]. We note that the higher order contributions to both θ_{12} and θ_{23} are suppressed such that they are rather close to the tri-bimaximal values. The reactor angle θ_{13} is predicted to be of order λ, and thus experimentally preferred value can be achieved. In particular, the Dirac CP violation is approximately maximal with $\delta_{\rm CP} \simeq \pm\frac{\pi}{2}$.

In order to see more clearly the predictions for the lepton mixing parameters, we perform a numerical analysis. The expansion parameter λ is fixed at the indicative value 0.15, and the parameters x, a, b and c are treated as random real numbers of absolute value between 1/2 and 2. The resulting lepton mixing angles and the mass-squared differences $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm atm}$ are required to lie in their 3σ ranges [8]. Correlations among the lepton mixing angles and the CP phases are plotted in figure 1. Obviously we have almost maximal Dirac CP phase δ_{CP}, and the numerical results are consistent with the analytical estimates of eq. (4.41).

5 Ultraviolet completion of the effective model

In the previously discussed effective model, non-renormalisable terms allowed by the symmetries are included in the superpotential w_l of eq. (4.14) and the subleading correction terms. It is generally believed that these effective terms arise from a fundamental renormalisable theory at high energies by integrating out the heavy degree of freedom. In this section, we present a ultraviolet (UV) completion of the effective model, which in general has the advantage of improving the predictability of the effective model. In such UV completed models, the non-renormalisable terms of the previously discussed effective model

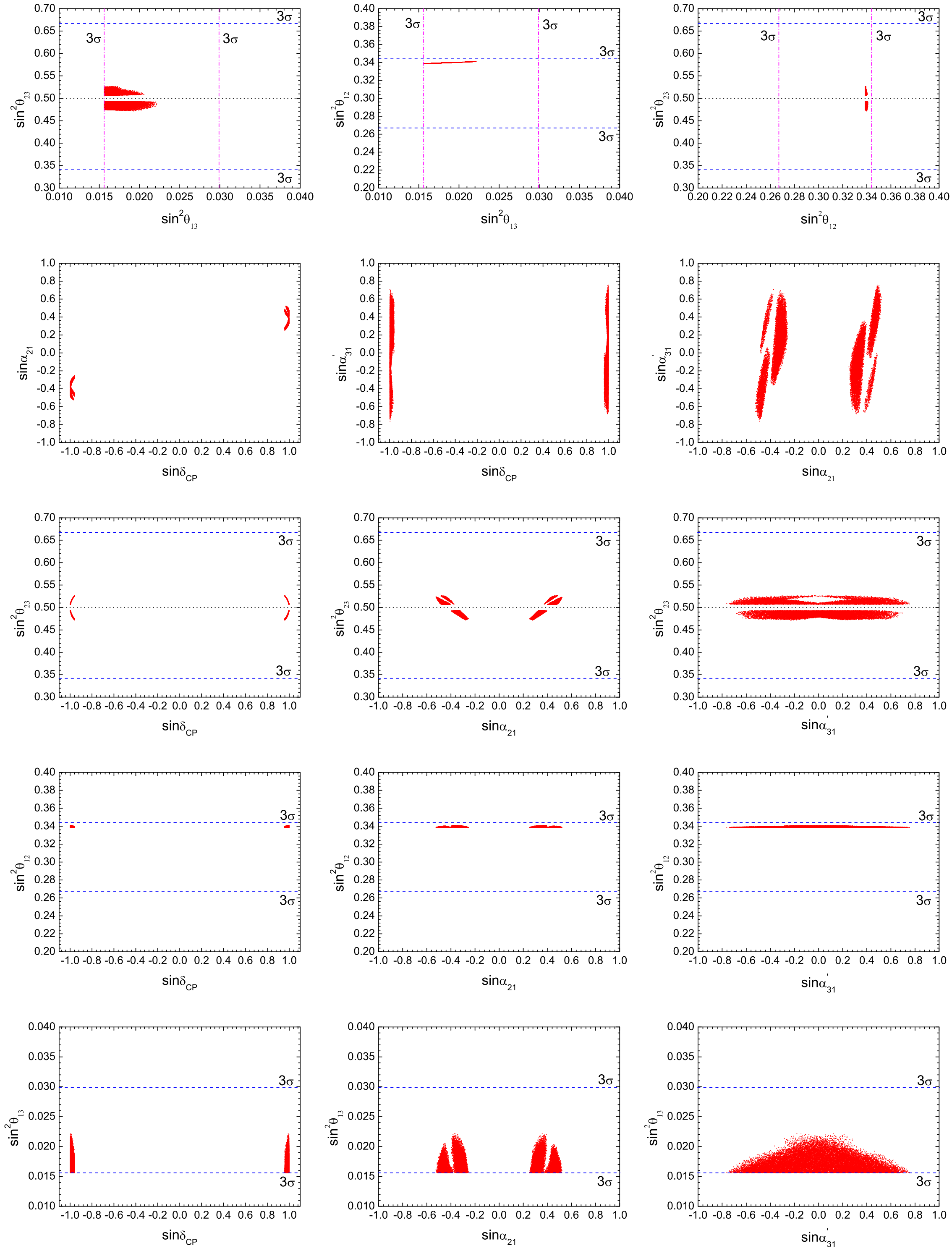

Figure 1. The correlations of different flavour mixing parameters, where the horizontal lines and the vertical ones correspond to the 3σ bound for the mixing angles, which are taken from [8].

Field	Ω_1	Ω_2	Ω_3	Ω_4	Ω_1^c	Ω_2^c	Ω_3^c	Ω_4^c	Σ	Σ^c
A_4	$\mathbf{3}$	$\mathbf{1''}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$
Z_4	-1	i	i	1	-1	$-i$	$-i$	1	-1	-1
Z_6	ω_6^2	ω_6^2	ω_6^2	ω_6^2	ω_6^4	ω_6^4	ω_6^4	ω_6^4	ω_6^5	ω_6
$\mathrm{U}(1)_R$	1	1	1	1	1	1	1	1	1	1

Table 4. The transformation rules of the messenger fields under the family symmetry $A_4 \times Z_4 \times Z_6$ and $\mathrm{U}(1)_R$.

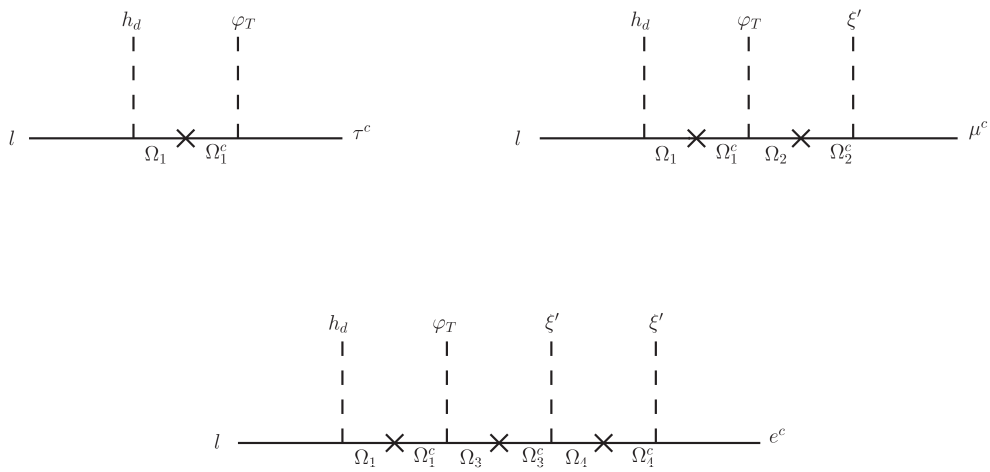

Figure 2. The diagrams which generate the effective operators for the charged lepton masses, where crosses indicate the mass insertions for fermions.

arise from integrating out heavy messenger fields, and some terms included at the effective level will be eliminated if no messenger field exists to mediate them. It is well-known that the UV completion of a low energy effective theory is generally not unique. In this section, we shall present the "minimal" completion of the above effective model in the sense of having the least number of extra messenger fields and the least number of associated (renormalisable) couplings.

To begin, the driving superpotential w_d of eq. (4.1) is already renormalisable, and therefore the vacuum alignment given in eqs. (4.6), (4.9), (4.12) is kept intact. The effective terms for the charged lepton masses in w_l of eq. (4.14) is non-renormalisable. Thus in order to reproduce these terms through the combination of renormalisable terms, we minimally increase the field content to introduce four pairs of messenger fields Ω_i and Ω_i^c $(i = 1, 2, 3, 4)$. The transformation properties of the all the messenger fields under the family symmetry $A_4 \times Z_4 \times Z_6$ are listed in table 4. Notice that these messengers are chiral superfields with non-vanishing hypercharge $+2(-2)$ for Ω_i (Ω_i^c). We can straightforwardly write down the renormalisable charged lepton superpotential

$$\begin{aligned} w_l \;=\; & z_1\,(l\Omega_1)\,h_d + z_2\,(\Omega_1^c\varphi_T)''\,\tau^c + z_3\,(\Omega_1^c\varphi_T)'\,\Omega_2 + z_4\Omega_2^c\zeta\mu^c + z_5\,(\Omega_1^c\varphi_T)\,\Omega_3 + z_6\Omega_3^c\Omega_4\zeta \\ & + z_7\Omega_4^c\zeta e^c + M_{\Omega_1}\,(\Omega_1^c\Omega_1) + M_{\Omega_2}\Omega_2^c\Omega_2 + M_{\Omega_3}\Omega_3^c\Omega_3 + M_{\Omega_4}\Omega_4^c\Omega_4\,, \end{aligned}$$

where all the coupling constants z_i $(i = 1\dots 7)$ and the messenger masses M_{Ω_i} $(i = 1\dots 4)$ are real because of the imposed generalised CP symmetry. Integrating out the heavy messenger fields Ω_i and Ω_i^c, the corresponding Feynman diagrams are shown in figure 2, we obtain the effective superpotential for the charged lepton masses

$$w_l^{\rm eff} = -\frac{z_1 z_2}{M_{\Omega_1}} (l\varphi_T)'' \tau^c h_d + \frac{z_1 z_3 z_4}{M_{\Omega_1} M_{\Omega_2}} (l\varphi_T)' \zeta \mu^c h_d - \frac{z_1 z_5 z_6 z_7}{M_{\Omega_1} M_{\Omega_3} M_{\Omega_4}} (l\varphi_T) \zeta^2 e^c h_d \,. \quad (5.1)$$

Taking into account the vacuum alignments $\langle\varphi_T\rangle = (0, v_T, 0)$ and $\langle\zeta\rangle = v_\zeta$ of eq. (4.6), we obtain a diagonal charged lepton mass matrix with

$$m_e = -z_1 z_5 z_6 z_7 \frac{v_T v_\zeta^2}{M_{\Omega_1} M_{\Omega_3} M_{\Omega_4}} v_d, \quad m_\mu = z_1 z_3 z_4 \frac{v_T v_\zeta}{M_{\Omega_1} M_{\Omega_2}} v_d, \quad m_\tau = -z_1 z_2 \frac{v_T}{M_{\Omega_1}} v_d \,. \quad (5.2)$$

For the neutrino sector, we introduce the messenger fields Σ and Σ^c which are chiral superfields carrying zero hypercharge. The renormalisable superpotential relevant to the neutrino masses reads

$$w_\nu = w_\nu^{LO} + w_\nu^{\Sigma} \,, \quad (5.3)$$

with

$$w_\nu^{LO} = y\,(l\nu^c)\,h_u + y_1\,(\nu^c\nu^c)\,\xi + \tilde{y}_1\,(\nu^c\nu^c)\,\tilde{\xi} + y_3\,(\nu^c\nu^c\varphi_S) \,, \quad (5.4)$$

$$w_\nu^{\Sigma} = x_1\,(\nu^c\Sigma)'\,\chi + x_2\,(\nu^c\Sigma^c)\,\rho + M_\Sigma\,(\Sigma^c\Sigma) \,, \quad (5.5)$$

where all couplings and the mass M_Σ are real due to the generalised CP invariance. The first term of w_ν^{LO} gives rise to the Dirac neutrino mass matrix

$$m_D = y \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u \,. \quad (5.6)$$

The RH neutrino masses receive contributions from both w_ν^{LO} and w_ν^{Σ}, as shown in figure 3. Integrating out the messenger fields Σ and Σ^c leads to the NLO effective operator

$$w_\nu^{NLO} = -\frac{x_1 x_2}{M_\Sigma} (\nu^c\nu^c)'\,\chi\rho \,, \quad (5.7)$$

which corresponds to the last term of the NLO corrections δw_ν in eq. (4.33) with $y_4 = -x_1 x_2 \Lambda / M_\Sigma$. However, the corrections from the shifted vacuum of $\tilde{\xi}$ and φ_S disappear in the present renormalisable model. The reason is that the messenger fields introduced do not affect the driving superpotential, and thus the vacuum alignment is preserved. Combining the contributions from both w_ν^{LO} and w_ν^{NLO}, the RH neutrino mass matrix m_M is given by

$$m_M = \begin{pmatrix} y_1 v_\xi + 2y_3 v_S/3 & -y_3 v_S/3 - x_1 x_2 v_\chi v_\rho / M_\Sigma & -y_3 v_S/3 \\ -y_3 v_S/3 - x_1 x_2 v_\chi v_\rho / M_\Sigma & 2y_3 v_S/3 & y_1 v_\xi - y_3 v_S/3 \\ -y_3 v_S/3 & y_1 v_\xi - y_3 v_S/3 & 2y_3 v_S/3 - x_1 x_2 v_\chi v_\rho / M_\Sigma \end{pmatrix} . \quad (5.8)$$

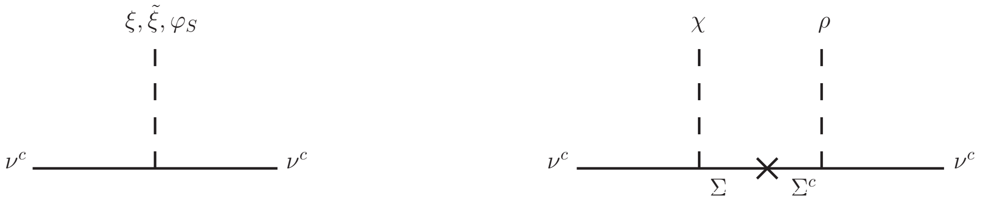

Figure 3. The diagrams for the RH neutrino masses, where crosses indicate the mass insertions for fermions.

Being similar to the effective model, the VEVs v_ξ and v_S should have the same phase up to relative sign otherwise the light neutrino masses will be degenerate at LO. Furthermore, the phase difference between $v_\chi v_\rho$ and v_ξ is 0, π or $\pm\pi/2$, as previously emphasised.

For the former cases, i.e. the phase difference is 0 or π, the light neutrino mass matrix is real once the common phase of v_ξ, v_S and $v_\chi v_\rho$ is absorbed by field redefinition. The resulting PMNS matrix is of the trimaximal form shown in eq. (3.26). Therefore lepton mixing angles compatible with the experimental data can be achieved, and CP is conserved. For the remaining case in which the phase difference of $v_\chi v_\rho$ and v_ξ is $\pm\pi/2$, m_M can be parametrised as in eq. (4.38) with

$$m_M = y_1 v_\xi \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + x \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{pmatrix} + iz \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right], \tag{5.9}$$

where $z = i\frac{x_1 x_2}{y_1}\frac{v_\chi v_\rho}{M_\Sigma v_\xi}$. We can straightforwardly obtain the light neutrino mass matrix from the seesaw formula [52–58] and then apply a tri-bimaximal transformation, i.e.

$$m'_\nu = -U^T_{TB}\left(m_D m_M^{-1} m_D^T\right) U_{TB} = m_0 \begin{pmatrix} \frac{2-2x-iz}{2(1-x^2-z^2-iz)} & 0 & \frac{i\sqrt{3}\,z}{2(1-x^2-z^2-iz)} \\ 0 & \frac{1}{1+iz} & 0 \\ \frac{i\sqrt{3}\,z}{2(1-x^2-z^2-iz)} & 0 & \frac{-2-2x+iz}{2(1-x^2-z^2-iz)} \end{pmatrix}, \tag{5.10}$$

with $m_0 \equiv -y^2 v_u^2/(y_1 v_\xi)$. Notice that the neutrino sector is described by three real parameters m_0, x and z at low energy, and therefore this model is rather predictive. As shown in appendix E, the mass matrix m'_ν can be diagonalised exactly as

$$U'^T_\nu m'_\nu U'_\nu = \text{diag}(m_1, m_2, m_3)\,, \tag{5.11}$$

where the unitary matrix U'_ν is of the form

$$U'_\nu = \begin{pmatrix} e^{i(\pi/4+\phi_1/2)}\cos\theta & 0 & e^{i(\pi/4+\phi_3/2)}\sin\theta \\ 0 & e^{i\phi_2/2} & 0 \\ -e^{-i(\pi/4-\phi_1/2)}\sin\theta & 0 & e^{-i(\pi/4-\phi_3/2)}\cos\theta \end{pmatrix}, \tag{5.12}$$

where the angle θ satisfies

$$\tan 2\theta = \frac{\sqrt{3}z}{2x}\,, \tag{5.13}$$

and the phases $\phi_{1,2,3}$ are given by

$$\begin{aligned}
\phi_1 &= -\arg\left(\frac{z+i(2-2x\cos 2\theta-\sqrt{3}\,z\sin 2\theta)}{1-x^2-z^2-iz}\right)\,,\\
\phi_2 &= \arg\left(1+iz\right)\,,\\
\phi_3 &= -\arg\left(\frac{z+i(2+2x\cos 2\theta+\sqrt{3}\,z\sin 2\theta)}{1-x^2-z^2-iz}\right)\,.
\end{aligned} \tag{5.14}$$

where the overall phase of m_0 has been omitted. Therefore the PMNS matrix is of the form

$$\begin{aligned}
U_{PMNS} =& U_{TB}U'_\nu \qquad (5.15)\\
=& \begin{pmatrix}
\frac{2}{\sqrt{6}}\cos\theta e^{i(\pi/4+\phi_1/2)} & \frac{1}{\sqrt{3}}e^{i\phi_2/2} & \frac{2}{\sqrt{6}}\sin\theta e^{i(\pi/4+\phi_3/2)}\\
\left(-\frac{1}{\sqrt{6}}\cos\theta-\frac{i}{\sqrt{2}}\sin\theta\right)e^{i(\pi/4+\phi_1/2)} & \frac{1}{\sqrt{3}}e^{i\phi_2/2} & \left(-\frac{1}{\sqrt{6}}\sin\theta+\frac{i}{\sqrt{2}}\cos\theta\right)e^{i(\pi/4+\phi_3/2)}\\
\left(-\frac{1}{\sqrt{6}}\cos\theta+\frac{i}{\sqrt{2}}\sin\theta\right)e^{i(\pi/4+\phi_1/2)} & \frac{1}{\sqrt{3}}e^{i\phi_2/2} & \left(-\frac{1}{\sqrt{6}}\sin\theta-\frac{i}{\sqrt{2}}\cos\theta\right)e^{i(\pi/4+\phi_3/2)}
\end{pmatrix}.
\end{aligned}$$

From this, we can immediately extract the lepton mixing angles and CP phases:

$$\begin{aligned}
&\sin^2\theta_{13}=\frac{1}{3}\left(1-\cos 2\theta\right), \quad \sin^2\theta_{12}=\frac{1}{2+\cos 2\theta}=\frac{1}{3\cos^2\theta_{13}}, \quad \sin^2\theta_{23}=\frac{1}{2}\,,\\
&\delta_{\rm CP}=\mathrm{sign}(xz)\frac{\pi}{2}, \qquad \alpha_{21}=\phi_2-\phi_1-\frac{\pi}{2}, \quad \alpha_{31}=\phi_3-\phi_1+\mathrm{sign}(xz)\pi\,.
\end{aligned} \tag{5.16}$$

It is remarkable that this model predicts maximal Dirac CP violation $\delta_{\rm CP}=\pm\frac{\pi}{2}$ and maximal atmospheric neutrino mixing in this case. For the measured values $\sin^2\theta_{13}=0.0227$, the solar mixing angle is predicted to be $\sin^2\theta_{12}\simeq 0.341$ which is compatible with the experimentally allowed regions. Finally, we remark that the light neutrino masses $m_{1,2,3}$ are given by

$$\begin{aligned}
m_1 &= |m_0|\sqrt{\frac{1+x^2+z^2-\mathrm{sign}(x\cos 2\theta)\sqrt{4x^2+3z^2}}{\left(1-x^2-z^2\right)^2+z^2}}\,,\\
m_2 &= \frac{|m_0|}{\sqrt{1+z^2}}\,,\\
m_3 &= |m_0|\sqrt{\frac{1+x^2+z^2+\mathrm{sign}(x\cos 2\theta)\sqrt{4x^2+3z^2}}{\left(1-x^2-z^2\right)^2+z^2}}\,.
\end{aligned} \tag{5.17}$$

As a result, the solar and atmospheric mass-squared splittings are predicted to be

$$\begin{aligned}
\Delta m^2_{\rm sol} &= \frac{\left(x^2-3\right)\left(x^2+z^2\right)+\mathrm{sign}(x\cos 2\theta)(1+z^2)\sqrt{4x^2+3z^2}}{(1+z^2)\left[(1-x^2-z^2)^2+z^2\right]}|m_0|^2\,,\\
\Delta m^2_{\rm atm} &= \frac{2\sqrt{4x^2+3z^2}}{(1-x^2-z^2)^2+z^2}|m_0|^2, \quad \text{for NO}\,,\\
\Delta m^2_{\rm atm} &= \frac{\left(x^2-3\right)\left(x^2+z^2\right)+(1+z^2)\sqrt{4x^2+3z^2}}{(1+z^2)\left[(1-x^2-z^2)^2+z^2\right]}|m_0|^2, \quad \text{for IO}\,.
\end{aligned} \tag{5.18}$$

When we impose the best fit values for the reactor mixing angle $\sin^2\theta_{13}=0.0227$ and the mass-squared differences $\Delta m^2_{\rm sol}=7.50\times10^{-5}\mathrm{eV}^2$ and $\Delta m^2_{\rm atm}=2.473(2.427)\times10^{-3}\mathrm{eV}^2$

(x,z)	$\delta_{\rm CP}$	α_{21}	α_{31}	m_1	m_2	m_3	$\lvert m_{\beta\beta}\rvert$	mass order
$(0.97, 0.44)$	$\pi/2$	0.17π	0.47π	5.43	10.22	50.02	6.37	NO
$(0.97, -0.44)$	$3\pi/2$	1.83π	1.53π					
$(0.81, 0.36)$	$\pi/2$	0.14π	0.73π	5.95	10.51	50.08	7.76	NO
$(0.81, -0.36)$	$3\pi/2$	1.86π	1.27π					
$(-2.17, 0.98)$	$3\pi/2$	1.13π	1.84π	53.46	54.15	22.49	18.90	IO
$(-2.17, -0.98)$	$\pi/2$	0.87π	0.16π					

Table 5. The predictions for the leptonic CP phases, the light neutrino masses $m_i (i = 1, 2, 3)$ and the effective mass $|m_{\beta\beta}|$ of the neutrinoless double-beta decay in the UV completion of the effective model, where the unit of mass is meV.

for normal (inverted) ordering, we find six possible solutions to the parameters x and z:

$$(x, z) \simeq (0.97, \pm 0.44), \quad (0.81, \pm 0.36), \quad (-2.17, \pm 0.98)\,, \tag{5.19}$$

where the first four cases correspond to a normally ordered neutrino mass spectrum, while latter two correspond to inverted ordering. The corresponding predictions for the light neutrino masses and the lepton mixing parameters are presented in table 5.

6 Conclusions

A promising and attractive approach to the well-known family puzzle is to invoke (spontaneously broken) discrete family symmetry to describe the observed patterns. The lepton mixing angles and CP violating phases can be predicted simultaneously from a family symmetry G_f combined with a generalised CP symmetry $H_{\rm CP}$, which is broken to different remnant symmetries in the neutrino and charged lepton sectors. In this work, we have focused on the most popular A_4 family symmetry. For the faithful representation $\mathbf{3}$, we find that the generalised CP symmetry is S_4 which is the automorphism group of A_4. However, only half of these 24 generalised CP transformations are consistent with the nontrivial singlet representations $\mathbf{1}'$ and $\mathbf{1}''$. We performed a comprehensive study of lepton mixing angles and CP phases which can be produced from the original symmetry $A_4 \rtimes H_{\rm CP}$ breaking to different remnant symmetries. Of all the possibilities, we find that only the case with $G^{\nu}_{\rm CP} = Z_2 \times H^{\nu}_{\rm CP}$ and $G^{l}_{\rm CP} = Z_3 \rtimes H^{l}_{\rm CP}$ is phenomenologically viable, in which the second column of the corresponding lepton mixing matrix is proportional to $(1, 1, 1)^T$. Furthermore, there is no CP violation in this case, namely $\delta_{CP} = 0, \pi$, with Majorana phases $\alpha_{21} = 0, \pi$ and $\alpha_{31} = 0, \pi$.

Motivated by this general analysis, we have constructed an effective SUSY model for leptons based on the $A_4 \rtimes H_{\rm CP}$ symmetry with auxiliary $Z_4 \times Z_6$ symmetries. This model reproduces the correct mass hierarchies among the three charged leptons. At LO, the lepton mixing is of the tri-bimaximal form, which is further reduced to trimaximal mixing by the NLO contributions. Consequently the reactor mixing angle arises as a NLO correction, and thus it is of the correct order of magnitude. It is notable that the Dirac phase is

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predicted to be trivial or approximately maximal, namely $\delta_{CP} = 0, \pi$ or $\delta_{CP} = \pm\pi/2$, with Majorana phases α_{21} and α_{31} being more general. For the case $\delta_{CP} = 0, \pi$, the residual symmetry in the neutrino sector is $G^{\nu}_{\rm CP} = Z_2 \times H^{\nu}_{\rm CP}$ with $H^{\nu}_{\rm CP} = \{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)\}$. While for the nearly maximal CP violation case, i.e. $\delta_{\rm CP} \simeq \pm\frac{\pi}{2}$, the generalised CP symmetry is broken completely in the neutrino sector.

Furthermore, we have promoted this effective model to a renormalisable one, where the non-renormalisable terms arise from integrating out heavy messenger fields and some higher dimensional operators included at the effective level are eliminated. As a result, the model becomes rather predictive, and the light neutrino mass matrix depends on only three real parameters which are fixed to reproduce the observed values of $\Delta m^2_{\rm sol}$, $\Delta m^2_{\rm atm}$ and θ_{13}. Then all the other observables including θ_{12}, θ_{23}, Dirac phase $\delta_{\rm CP}$, Majorana phases and the absolute neutrino mass scale are related, leading to the definite predictions shown in table 5. In particular, both the atmospheric mixing angle θ_{23} and Dirac phase $\delta_{\rm CP}$ are maximal.

Acknowledgments

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A Group theory of A_4

A_4 is the even permutation group of four objects. As such, it has 12 elements. Geometrically, it is isomorphic to the symmetry group of a regular tetrahedron. The elements of A_4 can be generated by two generators S and T satisfying the relation:

$$S^2 = T^3 = (ST)^3 = 1\,. \tag{A.1}$$

The 12 elements of A_4 are obtained as 1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2 and T^2ST. Without loss of generality, we can choose

$$S = (14)(23), \qquad T = (123)\,, \tag{A.2}$$

where the cycle (123) represents the permutation $(1,2,3,4) \rightarrow (2,3,1,4)$ and (14)(23) means $(1,2,3,4) \rightarrow (4,3,2,1)$. The A_4 elements belong to 4 conjugacy classes:

$$\begin{aligned}
&1C_1: && 1 \\
&4C_3: && T = (123), && ST = (134), && TS = (142), && STS = (243) \\
&4C_3^2: && T^2 = (132), && ST^2 = (124), && T^2S = (143), && ST^2S = (234) \\
&3C_2: && S = (14)(23), && T^2ST = (12)(34), && TST^2 = (13)(24)\,.
\end{aligned} \tag{A.3}$$

	S	T
1	1	1
1′	1	ω^2
1″	1	ω
3	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$

Table 6. The representation matrices for the A_4 generators S and T in different irreducible representations, where $\omega = e^{2\pi i/3}$ is the cube root of unit.

In the above, we have adopted Schoenflies notation in which mC_n^k denotes a conjugacy class of m elements of rotations by an angle $\frac{2\pi k}{n}$. A_4 has four inequivalent irreducible representations: three singlet representations $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$ and one triplet representation $\mathbf{3}$ which is a faithful representation of A_4. The representation matrices of the generators S and T are listed in table 6. The Kronecker products between various irreducible representations are as follows:

$$\mathbf{1}\otimes R = R, \quad \mathbf{1}'\otimes\mathbf{1}'' = \mathbf{1}, \quad \mathbf{1}'\otimes\mathbf{1}' = \mathbf{1}'', \quad \mathbf{1}''\otimes\mathbf{1}'' = \mathbf{1}',$$
$$\mathbf{3}\otimes\mathbf{1}' = \mathbf{3}, \quad \mathbf{3}\otimes\mathbf{1}'' = \mathbf{3}, \quad \mathbf{3}\otimes\mathbf{3} = \mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{3}_S\oplus\mathbf{3}_A\,, \tag{A.4}$$

where R denotes any A_4 representation, and the subscript S (A) denotes symmetric (anti-symmetric) combinations. For two A_4 triplets $\alpha = (\alpha_1, \alpha_2, \alpha_3) \sim \mathbf{3}$ and $\beta = (\beta_1, \beta_2, \beta_3) \sim \mathbf{3}$, the irreducible representations obtained from their product are:

$$\begin{aligned}
\mathbf{1} &\equiv (\alpha\beta) = \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2\,,\\
\mathbf{1}' &\equiv (\alpha\beta)' = \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1\,,\\
\mathbf{1}'' &\equiv (\alpha\beta)'' = \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1\,,\\
\mathbf{3}_S &\equiv (\alpha\beta)_{\mathbf{3}_S} = \frac{1}{3}\begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix}, \quad \mathbf{3}_A \equiv (\alpha\beta)_{\mathbf{3}_A} = \frac{1}{2}\begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix},
\end{aligned} \tag{A.5}$$

where we have followed the same convention of ref. [51].

Finally A_4 has three Z_2 subgroups, four Z_3 subgroups and one $K_4 \cong Z_2\times Z_2$ subgroup, which can be expressed in terms of the generators S and T as follows:

- Z_2 subgroups

$$Z_2^S = \{1,\ S\}\,, \quad Z_2^{T^2ST} = \{1,\ T^2ST\}\,, \quad Z_2^{TST^2} = \{1,\ TST^2\}\,. \tag{A.6}$$

- Z_3 subgroups

$$\begin{aligned}
Z_3^T &= \{1, T, T^2\}\,, & Z_3^{ST} &= \{1, ST, T^2S\}\,,\\
Z_3^{TS} &= \{1, TS, ST^2\}\,, & Z_3^{STS} &= \{1, STS, ST^2S\}\,.
\end{aligned} \tag{A.7}$$

- K_4 subgroup

$$K_4 = \left\{1, S, T^2ST, TST^2\right\} . \tag{A.8}$$

We note that K_4 is the normal subgroup of A_4, all Z_3 subgroups are conjugate to each other, and all Z_2 groups are conjugate to each other as well.

B Implication of $G_\nu = K_4 \cong Z_2 \times Z_2$

We first show that the remnant subgroup $G_\nu = K_4$ in the neutrino sector can not lead to phenomenologically acceptable lepton mixing angles even if we only impose the A_4 family symmetry. In order to be able to uniquely fix the mixing pattern from the group structure, the residual family symmetry in the charged lepton sector is taken to be Z_3 abelian subgroups. Thus, there are four possible choices for the preserved charged lepton subgroup G_l of A_4 with $G_\nu = K_4$, i.e. $G_l = Z_3^T$, $G_l = Z_3^{ST}$, $G_l = Z_3^{TS}$ or $G_l = Z_3^{STS}$. All four of these combinations lead to the same mixing parameters:

$$\sin^2\theta_{13} = 1/3, \quad \sin^2\theta_{12} = \sin^2\theta_{23} = 1/2, \quad |\sin\delta_{\rm CP}| = 1 . \tag{B.1}$$

The same results have also been found in refs. [60, 61]. Obviously this mixing pattern is not consistent with the present data. This result confirms that it is impossible to generate tri-bimaximal mixing by preserving the complete Klein symmetry group of A_4 in the neutrino sector. In order to produce tri-bimaximal mixing in A_4, one should use flavons transforming as $\mathbf{3}$ not $\mathbf{1}'$ or $\mathbf{1}''$ to break the family symmetry such that only the Z_2^S subgroup together with another accidental Z_2 $\mu-\tau$ symmetry is preserved in the neutrino sector. Moreover, if we choose $G_l = K_4$, the resulting mixing matrix will be the identity matrix up to permutation of rows and columns. This case is clearly not viable.

As an academic exercise to further convince the reader that G_ν can not be K_4 subgroup when considering $G_f = A_4$, it is insightful to investigate the constraints that the residual CP and family symmetries impose on the mass matrices. Considering the K_4 family symmetry first, the following constraints are found on m_ν:

$$\begin{aligned} \rho^T_{\mathbf{3}}(S) m_\nu \rho_{\mathbf{3}}(S) &= m_\nu, \\ \rho^T_{\mathbf{3}}(TST^2) m_\nu \rho_{\mathbf{3}}(TST^2) &= m_\nu\, , \end{aligned} \tag{B.2}$$

because $K_4 = \left\{1, S, TST^2, T^2ST\right\}$ can be generated by S and TST^2. Then, the most general neutrino mass matrix satisfying these equations has the form

$$m_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{13} & m_{11} \\ m_{13} & m_{11} & m_{12} \end{pmatrix} , \tag{B.3}$$

where m_{11}, m_{12} and m_{13} are complex parameters. It can be diagonalised by the unitary transformation

$$U_K = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix} , \tag{B.4}$$

where $\omega = e^{2\pi i/3}$. Thus,

$$U_K^T m_\nu U_K = \text{diag}(m_1, m_2, m_3)\,, \tag{B.5}$$

where

$$m_1 = m_{11}+m_{12}+m_{13}, \quad m_2 = \omega^2 m_{11}+m_{12}+\omega m_{13}, \quad m_3 = \omega m_{11}+m_{12}+\omega^2 m_{13}\,. \tag{B.6}$$

The light neutrino mass matrix m_ν of eq. (B.3) is further constrained by the remnant CP symmetry H^ν_{CP}, as shown in eq. (3.2), and the associated consistency equations are

$$X_{\mathbf{r}\nu}\rho^*_{\mathbf{r}}(S)X^{-1}_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(S'), \quad X_{\mathbf{r}\nu}\rho^*_{\mathbf{r}}(TST^2)X^{-1}_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(g'), \quad S', g' \in K_4\,. \tag{B.7}$$

By considering all possible values for S' and g', we find that all twelve CP transformations of A_4 in eq. (2.12) are acceptable, i.e.

$$X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(g), \quad g \in A_4\,, \tag{B.8}$$

where g is any group element of A_4. We further find that $H^\nu_{\rm CP}$ can be classified into three cases:

- $X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S), \rho_{\mathbf{r}}(TST^2), \rho_{\mathbf{r}}(T^2ST)$

  In this case, m_{11}, m_{12} and m_{13} are constrained to be real, and thus we have the degeneracy $|m_2|^2 = |m_3|^2$. The mass-squared splittings $\Delta m^2_{\rm sol} \equiv |m_2|^2 - |m_1|^2$ and $\Delta m^2_{\rm atm} \equiv \big||m_3|^2 - |m_1|^2(|m_2|^2)\big|$ have been precisely measured to be non-zero,[12] consequently the three light neutrinos should be of different masses. Moreover, partially degenerate light neutrino masses are disfavoured by the recent Planck results [62]. Therefore this case is not viable.

- $X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(T), \rho_{\mathbf{r}}(ST), \rho_{\mathbf{r}}(TS), \rho_{\mathbf{r}}(STS)$

  In this case, the parameters m_{11}, ωm_{12} and $\omega^2 m_{13}$ are required to be real. Therefore, it leads to the degeneracy $|m_1|^2 = |m_3|^2$, which is not compatible with the experimental data.

- $X_{\mathbf{r}\nu} = \rho_{\mathbf{r}}(T^2), \rho_{\mathbf{r}}(ST^2), \rho_{\mathbf{r}}(T^2S), \rho_{\mathbf{r}}(ST^2S)$

  The parameters m_{11}, $\omega^2 m_{12}$ and ωm_{13} have to be real in this case. Therefore, the degeneracy $|m_1|^2 = |m_2|^2$ is produced. This scenario is also not in accordance with three distinct neutrino masses.

As a result, if both the K_4 subgroup and the associated generalised CP symmetry are preserved in the neutrino sector, the neutrino mass matrix is strongly constrained such that the resulting light neutrino mass spectrum is partially degenerate, and the PMNS matrix cannot be determined uniquely. Thus, as determined before from mixing considerations, $G_\nu = K_4$ is not phenomenologically viable.

[12]The atmospheric mass-squared difference $\Delta m^2_{\rm atm} \equiv |m_3|^2 - |m_1|^2$ for the normal ordered neutrino mass spectrum and $\Delta m^2_{\rm atm} \equiv |m_2|^2 - |m_3|^2$ for the inverted ordering.

C Implication of $G_l = Z_2$

In this appendix, we consider the possibility that G_l is a Z_2 subgroup of A_4. It is sufficient to discuss the representative case $G_l = Z_2^S$. As shown in eq. (3.8), the CP symmetry $H^l_{\rm CP}$ consistent with Z_2^S is

$$H^l_{\rm CP} = \left\{\rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S), \rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)\right\} \,. \tag{C.1}$$

The hermitian combination $m_l m_l^\dagger$ is constrained by the remnant symmetry $G^l_{\rm CP} \cong Z_2^S \times H^l_{\rm CP}$ as

$$\begin{aligned}\rho^\dagger_{\mathbf{3}}(S) m_l m_l^\dagger \rho_{\mathbf{3}}(S) &= m_l m_l^\dagger,\\ X^\dagger_{\mathbf{3}l} m_l m_l^\dagger X_{\mathbf{3}l} &= (m_l m_l^\dagger)^* \,,\end{aligned} \tag{C.2}$$

which allows us to straightforwardly reconstruct $m_l m_l^\dagger$. There are two possible scenarios:

- $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S)$

  The mass matrix $m_l m_l^\dagger$ fulfilling eq. (C.2) is of the form

$$m_l m_l^\dagger = \tilde{\alpha}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \tilde{\beta}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \tilde{\gamma}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \tilde{\epsilon}\begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} , \tag{C.3}$$

  where $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\epsilon}$ are real parameters. After performing a tri-bimaximal transformation, we have

$$U^\dagger_{TB} m_l m_l^\dagger U_{TB} = \begin{pmatrix} 3\tilde{\alpha} + \tilde{\beta} - \tilde{\gamma} & 0 & -\sqrt{3}\,\tilde{\epsilon} \\ 0 & \tilde{\beta} + 2\tilde{\gamma} & 0 \\ -\sqrt{3}\,\tilde{\epsilon} & 0 & 3\tilde{\alpha} - \tilde{\beta} + \tilde{\gamma} \end{pmatrix} , \tag{C.4}$$

  which can be further diagonalised by a (1,3) rotation $R(\vartheta)$,

$$R(\vartheta) = \begin{pmatrix} \cos\vartheta & 0 & \sin\vartheta \\ 0 & 1 & 0 \\ -\sin\vartheta & 0 & \cos\vartheta \end{pmatrix} , \tag{C.5}$$

  with $\tan 2\vartheta = \sqrt{3}\,\tilde{\epsilon}/(\tilde{\beta} - \tilde{\gamma})$. The squared charged lepton masses are given by

$$\begin{aligned} m_e^2 &= 3\tilde{\alpha} - \sqrt{(\tilde{\beta} - \tilde{\gamma})^2 + 3\tilde{\epsilon}^2},\\ m_\mu^2 &= \tilde{\beta} + 2\tilde{\gamma},\\ m_\tau^2 &= 3\tilde{\alpha} + \sqrt{(\tilde{\beta} - \tilde{\gamma})^2 + 3\tilde{\epsilon}^2}\,. \end{aligned} \tag{C.6}$$

  In order to account for the observed hierarchies among the charged lepton masses m_e, m_μ and m_τ, a moderate fine-tuning of the parameters $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\epsilon}$ is needed.

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- $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(T^2ST), \rho_{\mathbf{r}}(TST^2)$

  In this case, $m_l m_l^\dagger$ is of the form

$$m_l m_l^\dagger = \begin{pmatrix} R_{11} & R_{12} & R_{12} \\ R_{12} & R_{11} & R_{12} \\ R_{12} & R_{12} & R_{11} \end{pmatrix}, \tag{C.7}$$

  where R_{11} and R_{12} are real. After applying a tri-bimaximal transformation, it becomes

$$U_{TB}^\dagger m_l m_l^\dagger U_{TB} = \mathrm{diag}(R_{11} - R_{12}, R_{11} + 2R_{12}, R_{11} - R_{12}), \tag{C.8}$$

  which implies $m_e^2 = m_\tau^2$. This is obviously not viable.

In the cases of $G_l = Z_2^{T^2ST}$ and $G_l = Z_2^{TST^2}$, we can immediately obtain the corresponding consistent CP transformations and the mass matrix with the aid of the relations in eqs. (3.5), (3.6).

Assuming that $G^\nu_{\mathrm{CP}} \cong Z_2 \times H^\nu_{\mathrm{CP}}$ (the only viable possibility for the neutrino sector) and $G^l_{\mathrm{CP}} \cong Z_2 \times H^l_{\mathrm{CP}}$ (as discussed in this appendix) then the corresponding PMNS matrix is of the form

$$U_{PMNS} = R^\dagger(\vartheta) U_{TB}^\dagger \rho_{\mathbf{3}}^m(T) U_{TB} R(\theta), \quad m = 0, \pm 1\,. \tag{C.9}$$

For $m = 0$, which corresponds to the remnant Z_2 symmetry in G^ν_{CP} and G^l_{CP} being the same, the lepton mixing angles are

$$\sin^2\theta_{13} = \sin^2(\theta - \vartheta), \quad \sin^2\theta_{12} = \sin^2\theta_{23} = 0\,. \tag{C.10}$$

For the case $m = \pm 1$, where the Z_2 factors in G^ν_{CP} and G^l_{CP} are different, the lepton mixing angles are

$$\sin^2\theta_{13} = 1/4, \qquad \sin^2\theta_{12} = \sin^2\theta_{23} = 2/3\,. \tag{C.11}$$

Obviously the predictions in both eq. (C.10) and eq. (C.11) are disfavoured by experimental data. Therefore we exclude the possibility that $G_l = Z_2$.

D Implication of $G_l = K_4$

In this appendix, we discuss the last possibility $G_l = K_4$, which implies that

$$\begin{aligned} \rho_{\mathbf{3}}^\dagger(S) m_l m_l^\dagger \rho_{\mathbf{3}}(S) &= m_l m_l^\dagger, \\ \rho_{\mathbf{3}}^\dagger(TST^2) m_l m_l^\dagger \rho_{\mathbf{3}}(TST^2) &= m_l m_l^\dagger\,. \end{aligned} \tag{D.1}$$

Then, the mass matrix $m_l m_l^\dagger$ is determined to be of the form

$$m_l m_l^\dagger = \begin{pmatrix} \tilde{m}_{11} & \tilde{m}_{12} & \tilde{m}_{12}^* \\ \tilde{m}_{12}^* & \tilde{m}_{11} & \tilde{m}_{12} \\ \tilde{m}_{12} & \tilde{m}_{12}^* & \tilde{m}_{11} \end{pmatrix}, \tag{D.2}$$

where $\tilde{m}_{11}$ is real and $\tilde{m}_{12}$ is complex. It is diagonalised by the unitary transformation U_K of eq. (B.4),

$$U_K^\dagger m_l m_l^\dagger U_K = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2)\,, \tag{D.3}$$

with

$$\begin{aligned} m_e^2 &= \tilde{m}_{11} + \tilde{m}_{12} + \tilde{m}_{12}^*, \\ m_\mu^2 &= \tilde{m}_{11} + \omega\tilde{m}_{12} + \omega^2\tilde{m}_{12}^*, \\ m_\tau^2 &= \tilde{m}_{11} + \omega^2\tilde{m}_{12} + \omega\tilde{m}_{12}^*\,. \end{aligned} \tag{D.4}$$

The hermitian combination $m_l m_l^\dagger$ also respects the CP symmetry H_{CP}^l. As shown in eq. (B.8), all twelve CP transformations are consistent with the K_4 subgroup, i.e.,

$$X_{\mathbf{r}l} = \rho_{\mathbf{r}}(g), \quad g \in A_4\,, \tag{D.5}$$

where $X_{\mathbf{r}l}$ is the element of H_{CP}^l. It is clear that invariance under the action of H_{CP}^l yields

$$X_{\mathbf{3}l}^\dagger m_l m_l^\dagger X_{\mathbf{3}l} = (m_l m_l^\dagger)^*\,, \tag{D.6}$$

which further constrains the parameter m_{12} of eq. (D.2) in various ways for different preserved CP subgroups as follows:

- $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(1), \rho_{\mathbf{r}}(S), \rho_{\mathbf{r}}(TST^2), \rho_{\mathbf{r}}(T^2ST)$

  In this case, the parameter m_{12} is real, which leads to $m_\mu = m_\tau$.

- $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(T), \rho_{\mathbf{r}}(ST), \rho_{\mathbf{r}}(TS), \rho_{\mathbf{r}}(STS)$

  ωm_{12} is constrained to be real, and thus the degeneracy $m_e = m_\tau$ arises.

- $X_{\mathbf{r}l} = \rho_{\mathbf{r}}(T^2), \rho_{\mathbf{r}}(ST^2), \rho_{\mathbf{r}}(T^2S), \rho_{\mathbf{r}}(ST^2S)$

  $\omega^2 m_{12}$ is real in this case, and the relation $m_e = m_\mu$ follows immediately.

Therefore the symmetry breaking $G_{\mathrm{CP}}^l \cong K_4 \rtimes H_{\mathrm{CP}}^l$ leads to partial degeneracy among the charged lepton masses. Hence, it is not viable.

E Diagonalisation of a 2 × 2 symmetric complex matrix

If neutrinos are Majorana particles, their mass matrix is symmetric and generally complex. In the following, we present the result for the diagonalisation of a general 2×2 symmetric complex matrix, which is of the form

$$\mathcal{M} = \begin{pmatrix} a_{11}e^{i\phi_{11}} & a_{12}e^{i\phi_{12}} \\ a_{12}e^{i\phi_{12}} & a_{22}e^{i\phi_{22}} \end{pmatrix}, \tag{E.1}$$

where a_{ij} and ϕ_{ij} $(i,j =, 1, 2)$ are real. It can be diagonalised by a unitary matrix U via

$$U^T\mathcal{M}U = \mathrm{diag}(\lambda_1, \lambda_2)\,, \tag{E.2}$$

where the unitary matrix U can be written as

$$U = \begin{pmatrix} \cos\theta e^{i(\phi+\varrho)/2} & \sin\theta e^{i(\phi+\sigma)/2} \\ -\sin\theta e^{i(-\phi+\varrho)/2} & \cos\theta e^{i(-\phi+\sigma)/2} \end{pmatrix}, \tag{E.3}$$

with the rotation angle θ satisfying

$$\tan 2\theta = \frac{2a_{12}\sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})}}{a_{22}^2 - a_{11}^2}. \tag{E.4}$$

The eigenvalues λ_1 and λ_2 can always set to be positive with

$$\lambda_1^2 = \frac{1}{2}\left\{a_{11}^2+a_{22}^2+2a_{12}^2-\mathcal{S}\sqrt{(a_{22}^2-a_{11}^2)^2+4a_{12}^2\left[a_{11}^2+a_{22}^2+2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})\right]}\right\},$$
$$\lambda_2^2 = \frac{1}{2}\left\{a_{11}^2+a_{22}^2+2a_{12}^2+\mathcal{S}\sqrt{(a_{22}^2-a_{11}^2)^2+4a_{12}^2\left[a_{11}^2+a_{22}^2+2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})\right]}\right\},$$

where $\mathcal{S} = \mathrm{sign}\big(\left(a_{22}^2 - a_{11}^2\right)\cos 2\theta\big)$. Finally the phases ϕ, ϱ and σ are given by

$$\begin{aligned}
\sin\phi &= \frac{-a_{11}\sin(\phi_{11}-\phi_{12}) + a_{22}\sin(\phi_{22}-\phi_{12})}{\sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})}} = \frac{Im\,(\mathcal{M}_{11}^*\mathcal{M}_{12} + \mathcal{M}_{22}\mathcal{M}_{12}^*)}{|\mathcal{M}_{11}^*\mathcal{M}_{12} + \mathcal{M}_{22}\mathcal{M}_{12}^*|},\\
\cos\phi &= \frac{a_{11}\cos(\phi_{11}-\phi_{12}) + a_{22}\cos(\phi_{22}-\phi_{12})}{\sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})}} = \frac{Re\,(\mathcal{M}_{11}^*\mathcal{M}_{12} + \mathcal{M}_{22}\mathcal{M}_{12}^*)}{|\mathcal{M}_{11}^*\mathcal{M}_{12} + \mathcal{M}_{22}\mathcal{M}_{12}^*|},\\
\sin\varrho &= -\frac{\left(\lambda_1^2 - a_{12}^2\right)\sin\phi_{12} + a_{11}a_{22}\sin(\phi_{11}+\phi_{22}-\phi_{12})}{\lambda_1\sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})}},\\
\cos\varrho &= \frac{\left(\lambda_1^2 - a_{12}^2\right)\cos\phi_{12} + a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-\phi_{12})}{\lambda_1\sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})}},\\
\sin\sigma &= -\frac{\left(\lambda_2^2 - a_{12}^2\right)\sin\phi_{12} + a_{11}a_{22}\sin(\phi_{11}+\phi_{22}-\phi_{12})}{\lambda_2\sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})}},\\
\cos\sigma &= \frac{\left(\lambda_2^2 - a_{12}^2\right)\cos\phi_{12} + a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-\phi_{12})}{\lambda_2\sqrt{a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos(\phi_{11}+\phi_{22}-2\phi_{12})}}.
\end{aligned} \tag{E.5}$$

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