An Improved Computational Strategy for Vibration-Proof Structures Equipped with Nano-Enhanced Viscoelastic Devices

E. Ntotsios & A. Palmeri
Loughborough University, United Kingdom

SUMMARY:

Viscoelastic damping devices are effective in mitigating vibrations experienced by Civil Engineering structures subjected to natural actions, such as earthquakes, wind gusts or ocean waves. In this paper, an efficient computational framework for non-classically damped viscoelastic structures is proposed, allowing rheological information on nano-reinforced elastomeric devices to be incorporated in the time-domain dynamic analysis of structures equipped with such components. For thiss purpose, the Generalized Maxwell (GM) model and the Laguerre's polynomial approximation (LPA) can be effectively adopted to represent the relaxation function of the viscoelastic materials, leading to an enlarged state-space model. It is also shown that these models can be used beyond the linear range, provided that the strain-dependent values of their mechanical parameters are identified.

Keywords: Viscoelastic dampers, Modal analysis, Nonlinear dynamics

1. INTRODUCTION

Viscoelastic rubbers have been successfully used in a number of energy dissipation devices and isolation bearings in Civil Engineering structures, such as buildings, bridges or off-shore structures in the attempt of mitigating the dynamic impact of natural actions, including earthquakes, wind gusts or ocean waves (Soong and Dargush, 1997; Zhang and Soong, 1992; Samali and Kwok 1995; Lee, 1997). A concerted effort has been made over the last couple of decades to enhance the performances of natural and synthetic rubbers with a variety of nano-reinforcements. Such viscoelastic materials obey very complicated constitutive laws, and continue to pose questions to researchers in the field. The reinforcing effects of carbon blacks in rubber, for instance, were discovered more than a century ago, and still the dynamic behaviour of structural devices based on this phenomenon (e.g. high-damping rubber bearings) is not completely understood.

It should be also noted that, in spite of a growing body of scientific literature and continuous advances in the field, the vast majority of structural engineers still adopt effective values of stiffness and damping to model the elastomeric components for analysis and design (Lockett, 1972; Jones, 2001; Johnson, 1999; Park, 2001). This approach is certainly straightforward, as the viscoelastic behaviour is represented through an elastic spring in parallel with a viscous dashpot (Kelvin-Voigt model), but it may lead to large inaccuracies, even within the linear range of the devices.

Similarly to the classical eigenvalue analysis for viscously damped structures, it seems therefore desirable to implement a technique able to reduce the size of the matrices for actual buildings provided with viscoelastic devices. Indeed, modal frequencies and modal damping ratios are de facto insufficient to characterize the vibration of such structures in the modal space, but the concept of modal decoupling is still valid. Inaudi and Kelly (1995) tackled this problem in the frequency domain, providing expressions for diagonalisable frequency-dependent stiffness and damping matrices. The same concept of defining non-viscously damped modal oscillators underlies the introduction of the so-

called modal relaxation functions in the time domain. As in Palmeri et al. (2004), for studying the wind-induced vibration of viscoelastically damped buildings, the modal relaxation functions can be viewed as the time-domain counterpart of the frequency dependent stiffness and damping matrices appearing in the frequency domain.

Aim of this paper is to establish an efficient computational framework for non-classically damped viscoelastic structures, allowing accurate rheological information on nano-reinforced elastomeric devices to be incorporated and efficiently used to run non-linear time-domain dynamic analyses of structures equipped with such components. The formulation is developed for both GM (Generalized Maxwell) model and LPA (Laguerre's Polynomial Approximation) technique and requires the strain-dependent values of their mechanical properties. The results of a seismic application to a 3-storey building model equipped with non-linear viscoelastic dampers demonstrate the potential of the proposed numerical scheme to reduce the computational burden associated with the analysis and design of viscoelastically damped structures.

2. LINEAR VISCOELASTIC STRUCTURES

Within the limits of the linear theory, the reaction force $r(t)$ experienced by a viscoelastic device can be expressed in the time domain through a convolution integral (e.g. Lockett, 1972):

$$r(t) = \int_{-\infty}^{+\infty} \varphi(t-s)\dot{u}(s)\,ds \qquad (2.1)$$

where $u(t)$ is the time history of the pertinent deformation of the viscoelastic device and $\varphi(t)$ is its relaxation function, i.e. the time history of the reaction force due to a unit-step deformation applied at the initial time instant $t = 0$. Eqn. (2.1) can be also written as:

$$r(t) = R_0 u(t) + \int_0^t g(t-s)\dot{u}(s)\,ds \qquad (2.2)$$

where R_0 is the equilibrium modulus of the viscoelastic device, representing its purely elastic stiffness, and $g(t)$ the time-varying part the relaxation function, that is:

$$R_0 = \lim \varphi(t) = \varphi(\infty), \quad g(t) = \varphi(t) - \varphi(\infty) \qquad (2.3)$$

2.1. Generalized Maxwell Model

Any LVE (Linear ViscoElastic) system can be approximated through a Generalized Maxwell (GM) model, which consists of an elastic spring, R_0, in parallel with a certain number ℓ of Maxwell's elements, each one given by an elastic spring, R_i, in series with a viscous dashpot $C_i = R_i \tau_i$. The ratio $\tau_i = R_i / C_i$ is the so-called relaxation time of the ith Maxwell's element, which measures the velocity of the unloading process for this rheological unit. It can be shown (e.g. Palmeri et al., 2004), that the time-varying part of the relaxation function in the GM model is given by the superposition of ℓ exponentially decaying functions:

$$g(t) = \sum_{i=1}^{\ell} R_i \exp\left(-\frac{t}{\tau_i}\right) \qquad (2.4)$$

The experimental identification of the ℓ pairs or parameters $\{R_i, \tau_i\}$ of the GM model proves to be quite complicated in real applications. The most popular approach consists in a non-linear regression in the frequency domain, based on the results of small-amplitude vibration tests (Orbey and Dealy, 1991; Syed and Philips, 2000). Unfortunately, this is an ill-posed problem, and the numerical solution is fraught with difficulties.

It has been shown that the reaction force of the GM model can be conveniently expressed in the form (Palmeri et al., 2003):

$$r(t) = R_0 u(t) + \sum_{i=1}^{\ell} R_i \lambda_i(t) \tag{2.5}$$

where the ith additional variable $\lambda_i(t)$, taken as the internal deformation in the ith Maxwell's spring, is ruled by a first-order linear differential equation:

$$\dot{\lambda}_i(t) = \dot{u}(t) - \frac{\lambda_i(t)}{\tau_i} \tag{2.6}$$

2.1. Laguerre's Polynomial Approximation

As an alternative, the relaxation function of any LVE device can be mathematically represented with the help of the ortho-normal properties of the Laguerre's polynomials (Palmeri et al., 2003). By using this approach, termed Laguerre's Polynomial Approximation (LPA), the time-dependent part of the relaxation function is given by a single exponentially decaying function modulated by a polynomial of order ℓ:

$$g(t) = \exp\left(-\frac{t}{\tau_0}\right) \sum_{i=1}^{\ell} R_i L_{i-1}\left(\frac{t}{\tau_0}\right) \tag{2.7}$$

where τ_0 is a characteristic relaxation time of the viscoelastic device; R_i is the ith Laguerre's rigidity; and $L_{i-1}(\cdot)$ stands for Laguerre's polynomial of order $(i-1)$. It can be shown that the parameter τ_0 can be easily estimated from a single relaxation test, while:

$$R_i = \frac{t}{\tau_0} \int_0^{+\infty} g(t) L_{i-1}\left(\frac{t}{\tau_0}\right) dt \tag{2.8}$$

Moreover, the reaction force in the LPA technique can be formally expressed as in Eqn. (2.6), where this time

$$\dot{\lambda}_i(t) = \dot{u}(t) - \frac{1}{\tau_0} \sum_{j=1}^{i} \lambda_j(t) \tag{2.9}$$

2.3. State-Space Equations of Motion for Linear Viscoelastic Systems

The dynamic equilibrium of a linear structure, having n DoF and p added linear viscoelastic dampers, is governed in the time domain by a set of n−coupled integro-differential equations of second order:

$$\mathbf{M} \cdot \ddot{\mathbf{u}}(t) + \mathbf{C} \cdot \dot{\mathbf{u}}(t) + \mathbf{K} \cdot \mathbf{u}(t) + \sum_{j=1}^{p} \mathbf{b}_j \int_0^t \varphi_j(t-s) \mathbf{b}_j^{\mathrm{T}} \cdot \dot{\mathbf{u}}(s) ds = \mathbf{f}(t) \tag{2.10}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are the matrices of mass, viscous damping and elastic stiffness of the structural system without viscoelastic devices, respectively; the jth kernel $\varphi_j(t)$ is the relaxation function of the jth viscoelastic damper; $\mathbf{b}_j$ is its influence vector; and $\mathbf{f}(t)$ is the array of the time-varying external forces acting on the building.

Following the modal analysis proposed by Palmeri et al. (2004), let us consider the transformation of coordinates:

$$\mathbf{u}(t) = \Phi \cdot \mathbf{q}(t) = \sum_{k=1}^{m} \phi_k q_k(t) \tag{2.11}$$

where the rectangular modal matrix $\Phi = [\phi_1 \cdots \phi_m]$, of dimensions $m \times n$, and the m-dimensional array $\mathbf{q}(t) = \{q_1(t) \cdots q_m(t)\}^{\mathrm{T}}$ collect the first $m \leq n$ modal shapes and the corresponding modal coordinates of the structure, respectively (in real applications, $m \ll n$). The kth modal shape, ϕ_k, is evaluated along with the corresponding undamped natural circular frequency, ω_k, as solution of the real-valued eigenproblem:

$$\left[\mathbf{K} + \sum_{j=1}^{r} \mathbf{b}_j \cdot \mathbf{b}_j^{\mathrm{T}} \varphi(\infty) \right] \cdot \phi_k = \omega^2 \mathbf{M} \cdot \phi_k,$$
$$\phi_k^{\mathrm{T}} \cdot \mathbf{M} \cdot \phi_k = \delta_{i,j} \tag{2.12}$$

with $\delta_{i,j}$ being the Kronecker's delta symbol, equal to one when $i = j$, zero otherwise. Interestingly, the summation in the left-hand side of the first of Eqn. (2.12) accounts for the additional stiffness arising in static conditions (i.e. when the circular frequency of vibration ω goes to zero) from the introduction of the viscoelastic dampers.

Pre-multiplying Eqn. (2.10) by Φ^{T}, one obtains the integro-differential equations of motion in the reduced modal space:

$$\ddot{\mathbf{q}}(t) + \Xi \cdot \dot{\mathbf{q}}(t) + \Omega^2 \cdot \mathbf{q}(t) + \int_0^t \mathbf{G}(t-s) \cdot \dot{\mathbf{q}}(s) ds = \Phi^{\mathrm{T}} \cdot \mathbf{f}(t) \tag{2.13}$$

where $\Xi = \Phi^{\mathrm{T}} \cdot \mathbf{C} \cdot \Phi$ is the reduced viscous damping matrix, associated with the inherent dissipation of the building without viscoelastic dampers; $\Omega = \mathrm{diag}\{\omega_1 \cdots \omega_m\}$ is the spectral matrix of the structure, which includes the increase in the stiffness due to the equilibrium modulus of the viscoelastic dampers; and $\mathbf{G}(t)$ is the modal relaxation matrix, taking into account the time-varying part of the viscoelastic kernels, that is:

$$\mathbf{G}(t) = \Phi^{\mathrm{T}} \cdot \left[\sum_{j=1}^{r} \mathbf{b}_j \cdot \mathbf{b}_j^{\mathrm{T}} \left(\varphi_j(t) - \varphi_j(\infty) \right) \right] \cdot \Phi \tag{2.14}$$

For homogeneous structural systems, it is often assumed that the modal shapes are orthogonal with

respect not only to mass and stiffness matrices, $\mathbf{M}$ and $\mathbf{K}$, but also with respect to the viscous damping matrix $\mathbf{C}$. When this condition is met, the structure is said to be classically (or proportionally) damped, and the equations of motion turn out to be decoupled in terms of modal coordinates (Caughey and O'Kelly, 1965). This important result can be extended to the case of buildings in which the viscoelastic dampers are distributed almost homogeneously, e.g. somehow proportionally to the floor masses and/or to the rigidities of the structural frame (Zambrano et al., 1996; Choa et al., 1998; Palmeri and Ricciarelli, 2006; Inaudi and Kelly, 1965). Analytically, this extension implies neglecting the off-diagonal elements of the relaxation matrix $\mathbf{G}(t)$:

$$\mathbf{G}(t) = \text{diag}\left\{g_1(t) \cdots g_m(t)\right\} \tag{2.15}$$

Once the matrices Ξ and $\mathbf{G}(t)$ in Eqn. (2.13) are assumed to be diagonal, the transformation of coordinates of Eqn. (2.11) can be used to decouple the equations of motion in the reduced modal space, so that the kth modal coordinate $q_k(t)$ is ruled by:

$$\ddot{q}_k(t) + 2\zeta_k \omega_k \dot{q}(t) + \omega_k^2 q(t) + \int_0^t g_k(t-s)\dot{q}(s)ds = Q_k(t) \tag{2.16}$$

in which kth modal excitation and kth modal relaxation function are given by:

$$
\begin{aligned}
Q_k(t) &= \phi^{\mathrm{T}} \cdot \mathbf{f}(t), \\
g_k(t) &= \phi_k^{\mathrm{T}} \cdot \left[\sum_{j=1}^r \mathbf{b}_j \cdot \mathbf{b}_j^{\mathrm{T}}\left(\varphi_j(t) - \varphi_j(\infty)\right)\right] \cdot \phi_k
\end{aligned}
\tag{2.17}
$$

Eqn. (2.16) can be viewed as the integro-differential equation governing the motion of the kth modal oscillator of the structure. As the relaxation function $\varphi(t)$ of a real linear viscoelastic device can be represented either through the GM model or the LPA technique, the same can be done for the modal relaxation function $g_k(t)$. This allows a significant reduction in the computational effort, as discussed in the paper by Palmeri and Muscolino (2011).

3. NON-LINEAR VISCOELASTIC STRUCTURES

Introducing now the concept of a non-linear viscoelastic device, let us assume that, due to the presence of nano-reinforcements in the rubber compound, the reaction force $r(t)$ experienced by a viscoelastic device beyond the linear range can be expressed as:

$$r(t) = R_0 u(t) + \left(1 + \alpha u^2(t)\right)\int_0^t g(t-s)\dot{u}(s)ds \tag{3.1}$$

where α is a measure of the non-linearity in the model, i.e. the value $\alpha = 0$ Eqn. (3.1) gives a linear model of Eqn. (2.2), while the higher α, the more non-linear becomes the constitutive law for the viscoelastic device.

For the general case of a building structure, having n DoFs and p added non-linear viscoelastic dampers, the linear and non-linear part of the reaction forces can be listed within the two p-dimensional arrays $\mathbf{r}_0(t)$ and $\mathbf{r}_1(t)$, defined as:

$$\mathbf{r}_0(t) = \mathbf{R}_0 \cdot \mathbf{d}(t) \,, \quad \mathbf{r}_1(t) = \mathbf{R}_1 \cdot \left[\mathbf{I}_r + \mathbf{A} \cdot \mathbf{Q}(\mathbf{d}) \right] \cdot \lambda_1(t) \tag{3.2}$$

where $\mathbf{I}_p$ is the identity matrix of size p; $\mathbf{R}_0 = \mathrm{diag}\{R_{0,1} \ldots R_{0,p}\}$ and $\mathbf{R}_1 = \mathrm{diag}\{R_{1,1} \ldots R_{1,p}\}$ are the diagonal matrices of size p collecting the stiffness coefficients of the linear and non-linear part, respectively, while $\mathbf{A} = \mathrm{diag}\{\alpha_1 \ldots \alpha_p\}$ is the diagonal matrix with the p coefficients α which measure their level of non-linearity; $\mathbf{d}(t) = \mathbf{B}^{\mathrm{T}} \cdot \mathbf{u}(t)$ is the p-dimensional array of the inner deformations associated with the each viscoelastic device; $\mathbf{B} = [\mathbf{b}_1 \cdots \mathbf{b}_p]$ is the $p \times n$ matrix, whose generic column is the influence array of the DoFs on a given internal deformation; and where $\mathbf{Q}(\mathbf{d}) = \mathrm{diag}\{d_1^2 \ldots d_p^2\}$.

In this study, the evolution in time for the internal variables $\lambda_1(t)$ is assumed to be ruled by the following set of linear differential equations:

$$\dot{\lambda}_1(t) = \dot{\mathbf{d}}(t) - \mathbf{D}_1 \cdot \lambda_1(t) \tag{3.3}$$

It is worth noting that, for the sake of simplicity, only one additional internal variable $\lambda_{1,j}(t)$ has been considered for the jth viscoelastic damper. This is also the only case in which GM and LPA coincide. Extension to many additional internal variables, adopting either the GM method or the LPA technique, is straightforward, as it only requires considering more terms in the right-hand side of Eqn. (3.3).

Let us now neglect the contribution of the pure viscous damping, that is it is assumed that $\mathbf{C} = \mathbf{0}$ on the basis that the energy dissipation is mainly provided by the viscoelastic devices. By coupling the equations with the non-linear constitutive laws for the viscoelastic devices, one obtains:

$$\mathbf{M} \cdot \ddot{\mathbf{u}}(t) + \left[\mathbf{K} + \mathbf{B} \cdot \mathbf{K}_0 \cdot \mathbf{B}^{\mathrm{T}} \right] \cdot \mathbf{u}(t) + \mathbf{B} \cdot \mathbf{R}_1 \cdot \left[\mathbf{I}_p + \mathbf{A} \cdot \mathbf{Q}(\mathbf{d}) \right] \cdot \lambda_1(t) = \mathbf{f}(t) \tag{3.4}$$

This equation of motion can be projected onto the modal space by exploiting a transformation of coordinates defined by the same eigenproblem of Eqn. (2.12), which can be written as:

$$\begin{aligned} &\left[\mathbf{K} + \mathbf{B} \cdot \mathbf{K} \cdot \mathbf{B}^{\mathrm{T}} \right] \cdot \phi_k = \omega^2 \mathbf{M} \cdot \phi_k, \\ &\phi_k^{\mathrm{T}} \cdot \mathbf{M} \cdot \phi_k = \delta_{i,j} \end{aligned} \tag{3.5}$$

leading to:

$$\ddot{\mathbf{q}}(t) + \Omega^2 \cdot \mathbf{q}(t) + \Phi^{\mathrm{T}} \cdot \mathbf{B} \cdot \mathbf{R}_1 \cdot \left[\mathbf{I}_p + \mathbf{A} \cdot \mathbf{Q}(\mathbf{d}) \right] \cdot \lambda_1(t) = \Phi^{\mathrm{T}} \cdot \mathbf{f}(t) \tag{3.6}$$

The same modal transformation of coordinates allows us expressing the evolution in time of the internal state variables as:

$$\dot{\lambda}_1(t) = \Phi^{\mathrm{T}} \cdot \mathbf{B} \cdot \dot{\mathbf{q}}(t) - \mathbf{D}_1 \cdot \lambda_1(t) \tag{3.7}$$

Eqns. (3.6) and (3.7) can now posed in the following state-space form:

$$\dot{\mathbf{z}}(t) = \mathbf{F}_0 \cdot \mathbf{z}(t) + \mathbf{F}_1(\mathbf{z},t) \tag{3.8}$$

where $\mathbf{z}(t) = \left\{ \mathbf{q}^\mathrm{T}(t) \ \dot{\mathbf{q}}^\mathrm{T}(t) \ \lambda^\mathrm{T}(t) \right\}$ is the array collecting the state variables of the enlarged problem and

$$\mathbf{F}_0 = \begin{bmatrix} \mathbf{0} & \mathbf{I}_P & \mathbf{0} \\ -\Omega^2 & \mathbf{0} & -\Phi^\mathrm{T} \cdot \mathbf{B} \cdot \mathbf{R}_1 \\ \mathbf{0} & \mathbf{B}^\mathrm{T} \cdot \Phi & -\mathbf{D}_1 \end{bmatrix},$$

$$(3.9)$$

$$\mathbf{F}_1(\mathbf{z},t) = \left\{ \begin{array}{c} \mathbf{0} \\ \Phi^\mathrm{T} \cdot \left\{ \mathbf{f}(t) - \mathbf{B} \cdot \mathbf{R}_1 \cdot \mathbf{A} \cdot \mathbf{Q}(\mathbf{d}) \cdot \lambda_1(t) \right\} \\ \mathbf{0} \end{array} \right\}$$

where $\mathbf{F}_0$ is the matrix of coefficients defining the linear part of the equation of motion, while $\mathbf{F}_1(\mathbf{z},t)$ is the array listing the non-linear terms and the external excitation.

4. NUMERICAL EXAMPLE

To test the potential of the proposed procedure of dynamic analysis to be exploited for the seismic design of structures with non-linear viscoelastic devices, the seismic-induced vibration of the shear-type three-storey frame depicted in Fig. 1 has been studied. The DoFs considered in the analysis are the storey drifts, while the values of three natural modal frequencies are $\omega_1 = 17.2$, $\omega_2 = 48.3$ and $\omega_3 = 69.8$ rad/s, and the inherent viscous damping ratio of the structure is assumed to be zero. Two viscoelastic dampers are located at first and second storey, and connected to the principal moment-resisting frame through a rigid bracing system.
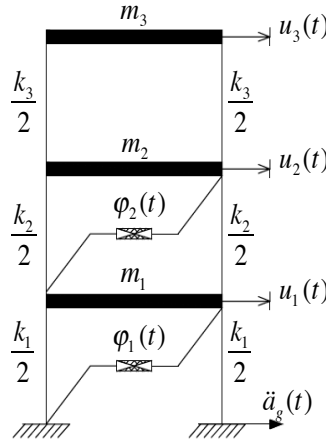

Figure 1. Three-storey frame structure equipped with viscoelastic devices

Two different types of relaxation functions are considered for bottom and first floor, namely types A and B. Type A is the relaxation function represented with the constitutive law of the GM model of a LVE damper described in Section 2, while type B is the relaxation function represented with the non-linear viscoelastic damper presented in Section 3. The relaxation time for both types of relaxation functions was set to be $\tau = 0.01$ s, and the initial value of the relaxation function, described by the coefficients $R_{1,1}$ and $R_{1,2}$ for the first two storeys, is 10 and 5 times larger than the corresponding main stiffness k_1 and k_2 at the same level; moreover, the level of non-linearity coefficient α in Eqn.

(3.1) was set for the device at the first floor equal to half of the coefficient at the ground level, that is $\alpha_2 = \alpha_1/2$.

In a first stage, the ground acceleration has been deterministically modelled as a modulated sinusoidal function:

$$\ddot{a}_g(t) = A_g(t)\sin(\Omega_g(t)t) \tag{4.1}$$

where amplitude and frequency of the input are given by $A_g(t) = (1 + \sin(\pi t/t_f) - t/t_f)^2$ and $\Omega_g(t) = 0.1 + t_f t/t_f$, respectively, $t_f = 5$ s being the final time instant.
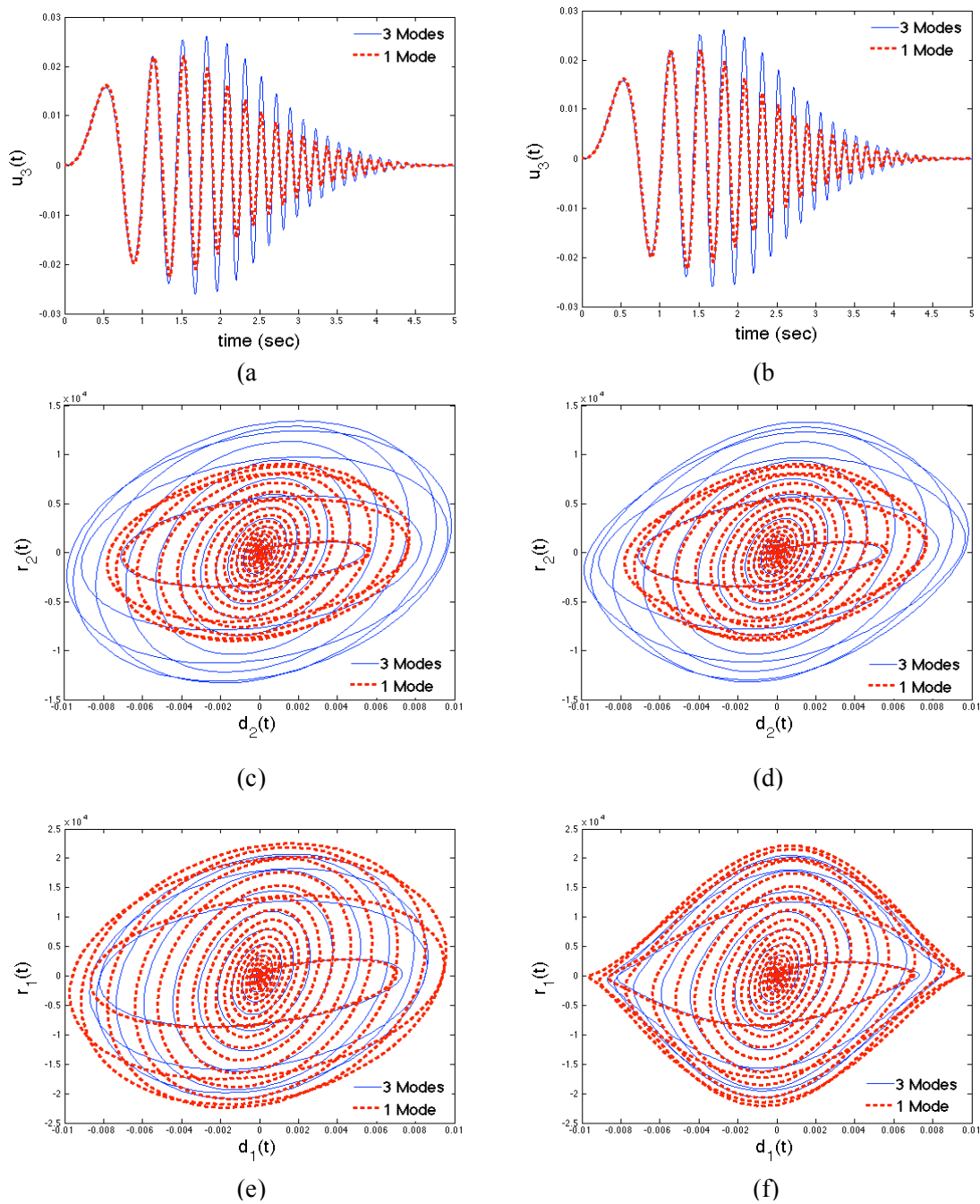

Figure 2. Time histories of displacement response of (a-b) top floor and phase plane response representation of (c-d) middle and (e-f) bottom floor, to sinusoidal input, for (left) linear and (right) non-linear viscoelastic devices

Fig. 2 shows the dynamic response of the objective structure equipped with type-A linear (left) and type-B non-linear (right) viscoelastic devices. More precisely, Figs. 2(a) and 2(b) plot the time histories of the displacement at the top floor of the structure in the two cases, while Figs. 2(c) and 2(d) show the hysteretic loops experienced by the viscoelastic device at the first floor and Figs. 2(e) and 2(f) the corresponding hysteretic loops at the ground floor.

The dynamic response obtained with the full dynamic model, i.e. when all the three modal coordinates were retained (solid thin blue lines), has been compared with the approximate response obtained when a single modal coordinate was used to represent the motion of the structure (dotted thick red lines). The inspection of the six graphs in Fig. 2 clearly reveals that the level of inaccuracy introduced in the analysis while neglecting the higher modes of vibration is substantially the same in the linear and non-linear case. This result seems confirming that projecting the equations of motion onto the modal space is an effective way of reducing the computational effort, not only in the linear range (see Palmeri and Muscolino, 2011), but also in the presence of non-linear viscoelastic devices.
It can be also observed that the shape of the hysteretic loops for the non-linear device in Fig. 2(f) differs significantly from all the other cases, due to the larger value of the non-linearity coefficient α.

In a second stage, the ground acceleration has been stochastically modelled as a stationary Gaussian noise with a broadband spectrum having a cut-off frequency of $\omega = 31.5$ rad/s. The response of the structure to this excitation was calculated while increasing the value of the non-linearity coefficient α_1 for the ground floor.
The time histories of the displacements $u_1(t)$ of the first floor and $u_2(t)$ of the second floor were computed with the proposed numerical scheme, retaining three (solid thin blue lines) and one (dashed thick red lines) modal coordinate, and the corresponding steady-state standard deviations σ_1 and σ_2 were evaluated. Fig. 3 plots the variation of σ_1 and σ_2 with the parameter α_1. Also these numerical results are very promising, since the inaccuracy associated with neglecting the higher modes of vibration seems to be substantially independent of the level of non-linearity introduced in the problem by the viscoelastic devices.
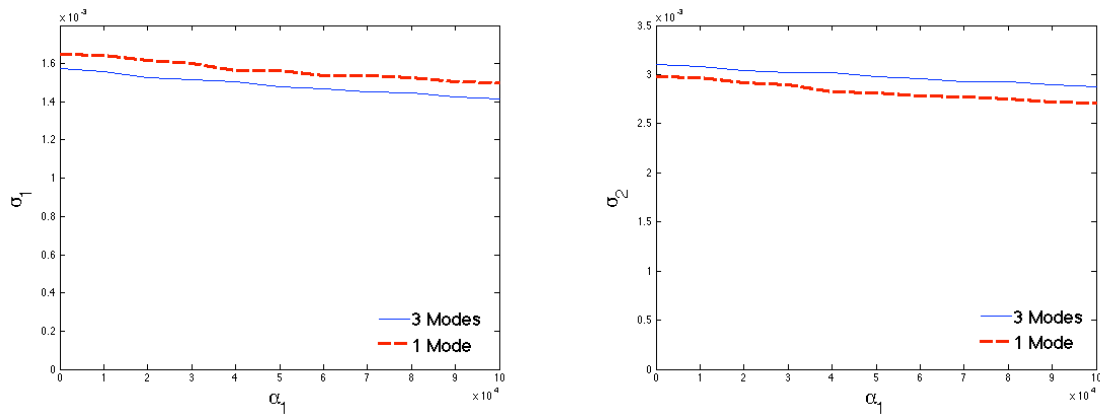

Figure 3. Standard deviation of the response of (a) first and (b) second floor displacement for Gaussian excitation for different levels of non-linearity.

4. CONCLUSIONS

In this paper, an efficient computational framework has been established for the dynamic analysis of non-classically damped structures equipped with linear and non-linear viscoelastic devices. The preliminary results show that the inaccuracy introduced in the numerical solution by reducing the size of the problem in the modal space is substantially independent of the level of non-linearity of the viscoelastic devices. This study should be considered as a first step toward a general strategy to

effectively incorporate accurate rheological information on nano-reinforced elastomeric devices in the non-linear time-domain dynamic analysis of viscoelastically damped structures. Further investigations are currently being developed to validate experimentally the proposed procedure for frames made of composite beams made with different rubber compounds.

REFERENCES

Caughey, T.K. and O'Kelly, M.E.J. (1965). Classical normal modes in damped linear systems. *Journal of Applied Mechanics,Transactions of the ASME* **32**, 583–588

Choa, K.P., Cermaka, J.E., Laib, M.L. and Nielsen, E.J. (1998). Viscoelastic damping for wind-excited motion of a five-story building frame. *Journal of Wind Engineering and Industrial Aerodynamics* **77–78**, 269–281.

Inaudi, J.A. and Kelly, J.M. (1995). Modal equations of linear structures with viscoelastic dampers. *Earthquake Engineering and Structural Dynamics* **24**, 145–151.

Johnson, A.R. (1999). Modeling viscoelastic materials using internal variables. *Shock and Vibration Digest* **31**, 91–100.

Jones, D.I.G. (2001). Handbook of Viscoelastic Vibration Damping. Wiley, Chichester.

Lee, H.H. (1997). Stochastic analysis for offshore structures with added mechanical dampers. *Ocean Engineering* **24**, 817–834.

Lockett, F.J. (1972). Nonlinear Viscoelastic Solids, Academic Press, London, U.K.

Orbey, N. and Dealy J.M. (1991). Determination of the relaxation spectrum from oscillatory shear data. *Journal of Rheology* **35**, 1035–1049.

Palmeri, A., Ricciarelli, F., De Luca, A. and Muscolino, G. (2003). State space formulation for linear viscoelastic dynamic systems with memory. *Journal of Engineering Mechanics* **129**, 715–724.

Palmeri, A., Ricciarelli, F., Muscolino, G. and De Luca, A. (2004). Effects of viscoelastic memory on the buffeting response of tall buildings. *Wind and Structures* **7**, 89–106.

Palmeri, A. and Ricciarelli, F. (2006). Fatigue analyses of buildings with viscoelastic dampers. *Journal of Wind Engineering Industrial Aerodynamics* **94**, 377–395.

Palmeri, A. and Muscolino, G. (2011). A numerical method for the time-domain dynamic analysis of buildings equipped with viscoelastic dampers. *Structural Control and Health Monitoring* **18**, 519-539.

Park, S.W. (2001). Analytical modeling of viscoelastic dampers for structural and vibration control. *International Journal of Solids and Structures* **38**, 8065–8092.

Samali, B. and Kwok, K.C.S. (1995). Use of viscoelastic dampers in reducing wind- and earthquake-induced motion of building structures. *Engineering Structures* **17**, 639–654,

Soong, T.T. and Dargush, G.F. (1997). Passive Energy Dissipation Systems in Structural Engineering. Wiley, Chichester.

Syed, Mustapha SMFD and Phillips, T.N. (2000). Dynamic nonlinear regression method for the determination of the discrete relaxation spectrum. *Journal of Physics D: Applied Physics* **33**, 1219–1229

Zambrano, A., Inaudi, J.A. and Kelly, J.M. (1996). Modal coupling and accuracy of modal strain energy method. *Journal of Engineering Mechanics* **122**, 603–612.

Zhang, R.H. and Soong, T.T. (1992). Seismic design of viscoelastic dampers for structural applications. *Journal of Structural Engineering* **118**,1375–1392.